Optimal Growth Policies in a Two-Sector Model with Financial Market Imperfections

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8 September 2018

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This version: September, 2018

Abstract

This paper studies the pro-growth policies in an endogenous growth model where heterogeneous entrepreneurs face collateral constraints, skilled workers accumulate human capital, and the government intervenes to promote human and physical capital formation. It shows that the model has a balanced-growth path whose rate depends on government policy and financial development level. The theoretical analysis also shows that when the distribution of idiosyncratic productivity is heavy-tailed, the government must subsidize productive entrepreneurs to achieve optimal pro-growth policies.

JEL Classification: E10, E22, E44, H21, O16

Keywords: Heterogeneity; Financial Frictions; Growth Policies

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1 Introduction

This paper addresses the question of how the government should implement its pro-growth policies in an environment where financial markets are imperfect, firm productivity is heterogeneous, and human capital is accumulated by skilled workers. More specifically, the paper investigates the government's pro-growth policies of how to optimally allocate its public spending to promote human capital formation and how to intervene into the capital market to facilitate physical capital accumulation.

When financial markets are imperfect, entrepreneurs even those with high productivity face borrowing collateral constraints, hence, are unable to borrow funds to a desired level to extend production while relatively low productive ones participate in producing goods. Consequently, capital is not efficiently allocated and there are losses in the aggregated productivity. [1] Moreover, because of borrowing collateral constraints, for a given level of financial development the amount of funds that an entrepreneur can borrow depends on his own personal wealth. Consequently, it is intuitive that one of the effective pro-growth policies is to facilitate the personal wealth accumulation of highly productive entrepreneurs so that they can borrow more funds to extend production, hence, improving economic growth. However, sources of economic growth are also driven by another important and significant factor, the improvement in the quality of labor or the human capital formation, in which the government policies also play in an important role via public spending on education, job-training and health programs, etc. As a result, for a given amount of tax revenue, the government faces a tradeoff between those policies that encourage the efficient allocation of funds to highly productive entrepreneurs and those that promote human capital acquisition.

To address the question above, I construct a two-sector endogenous growth model with a continuum of heterogeneous entrepreneurs, a continuum of representative skilled workers, and the government. Entrepreneurs with heterogeneous produc-

[1] In the absence of financial frictions, only entrepreneurs with the highest productivity produce goods, therefore, capital is efficiently allocated to the most productive entrepreneurs and aggregate measured productivity is at its first-best level.

tivity own private firms that produce goods, accumulate personal wealth and face borrowing collateral constraints because of financial frictions. Representative skilled workers accumulate human capital by using their own efforts, the existing human capital stock and public eduction provided by the government. The government also intervenes into the capital input market by subsidizing capital-bill use of actively producing entrepreneurs and collects (capital) income taxes. The constant return to scale in goods production implies that entrepreneurs' profits are a linear function of capital. Therefore, at equilibrium, there exists a threshold of entrepreneurs who are active in producing goods, which depends only on the level of financial deepening. Moreover, the more heterogeneous are entrepreneurs the more distant is the productivity cut-off from the average (mean) productivity level of active entrepreneurs. This in turn implies that capital is less efficiently used because at equilibrium the capital rental rate is equal to marginal product of capital of producers who are at the productivity cut-off. The theoretical analysis shows that when the degree of heterogeneity in entrepreneurs' idiosyncratic productivity is high, the optimal pro-growth policy can only achieved or implemented by appropriately subsidizing active entrepreneurs. It also identifies the conditions under which subsidizing entrepreneurs is required to achieve optimal policy and showing the optimal policy rule between taxing and subsidizing. Intuitively, when the entrepreneurs' productivity distribution is heavy-tailed, hence, under the $i.i.d$ productivity assumption, the share of wealth held by productive entrepreneurs is relatively high. However, because of financial frictions the capital rental rate is relatively low, therefore, subsidizing capital use will lead to a higher rental rate and more efficient aggregated wealth accumulation.

By contrast, when the degree of heterogeneity in entrepreneurs' productivity is not so high so that the marginal producers, namely those who producing at the productivity cut-off, are relatively close to the aggregated average (mean) level of active entrepreneurs, the theoretical analysis shows that subsidizing active entrepreneurs is not necessarily required to achieve the optimal pro-growth policy. In other words, to implement the optimal policy, the government just appropriately utilizes the asset income tax instrument and then spends all their revenues on public education to promote human capital formation.

Related Literature: This paper belongs to the branch of burgeoning literature that studies the interactions between financial frictions, entrepreneurship, resource misallocation, and aggregated productivity (see e.g. Banerjee and Duflo 2005, Buera et al 2011; Buera and Shin 2013; Midrigan and Xu 2014, Moll 2014). [2] It contributes to this branch of literature by incorporating taxation and the human capital accumulation in the spirit of Uzawa-Lucas into an endogenous growth model and shows that government policies and the level of financial development can affect balanced growth path equilibrium. This paper however differs from the traditional Uzawa-Lucas framework by embracing heterogeneous agents and considering the role of public education in human capital formation of skilled workers. [3]

This paper extends from the theoretical framework of Nguyen (2018b) by adding policy instruments to study optimal growth policy. Specifically, Nguyen (2018b) incorporates human capital and public education financed into the theoretical model of Moll (2014) to investigate the effects financial deepening on the accumulation of physical and human capital and working hours in a two-sector endogenous growth model [4]. Nguyen (2018b) analytically shows that in the presence of financial market imperfections, capital taxation exerts U-shaped effects on the balanced growth path rate and that optimal policy rates and balanced-growth path rates are increasing functions of the level of financial deepening. This paper is also close to Otskhoki and Moll (2018) that studies the optimal dynamic Ramsey policies in a standard growth model with financial friction and heterogeneous producers. The differences are that Otskhoki and Moll (2018) does not include human capital and government policy for human capital formation; it just focuses on policies in product and factor markets during the process of transitioning to the steady state equilibrium. This paper addresses optimal policy on the balanced-growth path equilibrium and focuses on the role of human capital formation in the endogenous growth framework. Unlike

[2]See Buera et al. (2015) for a thorough review of this literature.

[3]See e.g. Barro and Sala-i-Martin, 2014 for the introduction and review of the Uzawa-Lucas model and related literature.

[4]Moll (2014) introduces productivity persistence into the previous work of Angeletos (2007) and Kiyotaki and Moore (2012), and shows that self-financing can undo capital mis-allocation and reduce the long-run steady state TFP losses when the shocks are sufficiently persistent.

Otskhoki and Moll (2018), this paper shows that it is always optimal to impose a positive tax rate on capital income when public education plays a positive role in human capital accumulation. Finally, this paper differs from both Nguyen (2018b) and Otskhoki and Moll (2018) by incorporating endogenous policy instruments that are contingent on entrepreneurs' idiosyncratic productivity and personal wealth, therefore, it is able to address more general policy function forms.

This paper is organized as follows. Section 2 sets up the model economy. Section 3 defines and analyzes the aggregate equilibrium and the balanced-growth path equilibrium. Section 4 then discusses the of government's optimal policy to achieve maximum balanced growth rate. Section 5 concludes.

2 The Model

This is a two-sector growth model with a representative skilled worker and a unit measure of entrepreneurs. Each entrepreneur has a private firm and is indexed by his productivity, z, and personal wealth, a. Heterogeneous firms use effective labor and physical capital to produce homogenous final goods for both consumption and physical capital investment. Human capital is built by skilled worker's efforts, the existing human capital stock and also public eduction provided by the government. The government collects tax to finance its spending.

2.1 Entrepreneurs

All entrepreneurs have the same preferences,

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log c_e(t) dt \tag{2.1}$$

where c_e denotes entrepreneur's consumption.

Each entrepreneur owns a firm. At each time, the firm employs n^d efficiency units of labor at the wage rate, $w(t)$ and rents k^d units of physical capital from a competitive capital market at the rental rate, $r(t)$, to produce homogeneous final goods with the following production technology,

$$y = f(z,k,n) = (zk)^{\alpha} n^{1-\alpha} \tag{2.2}$$

where $\alpha \in (0,1)$ and z denotes idiosyncratic entrepreneurial productivity.

Specifically, we assume each moment entrepreneurs draw z from a Pareto distribution whose distribution function is given by,

$$G(z) = \begin{cases} 1 - \left(\frac{1}{z}\right)^{\varphi} & z \geq 1 \\ 0 & z < 1 \end{cases} \tag{2.3}$$

where $\varphi > 1$ is the shape parameter. Smaller φ corresponds to a heavier tail of the productivity distribution, i.e., a higher fraction of very productive entrepreneurs, therefore, implying a higher degree of productivity heterogeneity among entrepreneurs. We also assume that idiosyncratic productivity z is $i.i.d$ over time as well as across entrepreneurs so that the law of large numbers implies that the population share of type z entrepreneurs is stationary and deterministic.

Each entrepreneur obtains profits from his private firm, which is defined as,

$$\pi(z,k,n) \equiv f(z,k,n) - wn - (1 - \eta_k(z))rk$$

where $\eta_k(z)$ is the capital subsidy rate policy that depends on idiosyncratic productivity of the entrepreneurs.

Each entrepreneur also receives capital returns from his personal assets. Hence, his personal wealth $a(t)$ evolves as follows: [5]

$$\dot{a} = \pi(z,k,n) + (1 - \tau_a(a))ra - c_e \tag{2.4}$$

where $\tau_a(a)$ denotes the tax rate policy on asset income.

The entrepreneur at the same time faces the following borrowing constraint:

$$k \leq \lambda a \tag{2.5}$$

[5] It is straightforward to show that rental capital market setting is equivalent to a setup where entrepreneurs own and accumulate capital and are allowed to trade a risk-free bond. For simplicity, I consider only capital income tax in the main text and the Appendix shows that similar analytical results can be obtained when labor income tax instrument is added.

where $\lambda \geq 1$ denotes the maximum borrowing leverage ratio that reflects the degree of financial deepening. This borrowing constraint states that the maximum amount of capital an individual entrepreneur can borrow is limited by the amount of his personal assets, a, and the efficiency of the financial markets reflected by the maximum borrowing leverage ratio λ. In particular, $\lambda = 1$ expresses financial autarky where entrepreneurs are completely capital self-financed whereas $\lambda = \infty$ denotes perfect financial markets where entrepreneurs can borrow freely.[6]

Each entrepreneur maximizes the profit of his private firm subject to the technology (2.2) and his borrowing constraint (2.5), hence his static optimization problem at each time t can be expressed as follows:

$$\Pi(a, z) = \max_{k,n} \{f(z, k, n) - wn - (1 - \eta_k(z))rk\}$$
$$s.t. \quad k \leq \lambda a$$

The first order condition of this problem with respect to efficiency units of labor requires: $(1-\alpha)(zk)^{\alpha} n^{-\alpha} = w$. Consequently, the implied labor demand becomes, $n^d = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} (zk)$. After substituting the optimal labor demand into the technology equation (2.2) we then obtain the production function that is linear in individual entrepreneurs capital input as: $F(z, k) = \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} (zk)$.

Therefore, an entrepreneur's optimization problem becomes:

$$\Pi(a, z) = \max_{k} \left\{ \alpha \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} zk - (1 - \eta_k(z))rk \right\}$$
$$s.t. \quad k \leq \lambda a$$

The optimization conditions of this problem imply that his capital and effective labor demands and profits are linear in personal wealth; and there is a productivity

[6] See Buera and Shin (2013) and Moll (2014) for further discussions of this borrowing constraint and Nguyen (2018a) for the case where the maximum borrowing leverage ratio is endogenous.

cutoff for active entrepreneurs $\underline{z}$ as follows:

$$\Pi(a,z) = \max\{z\pi - (1-\eta_k(z))r, 0\}\,\lambda a$$

$$k^d(a,z) = \lambda a \cdot \mathbb{1}_{\{z \geq \underline{z}\}} \tag{2.6}$$

$$n^d(a,z) = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} z\lambda a \tag{2.7}$$

$$\underline{z}\pi = [1-\eta_k(z=\underline{z})]\,r \tag{2.8}$$

$$\pi \equiv \alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \tag{2.9}$$

At the same time, entrepreneurs also maximize the expected sum of discounted utilities (2.1) subject to the budget constraint (2.4), which can now be rewritten as:

$$\dot{a} = [\lambda \max\{z\pi - (1-\eta_k(z))r, 0\} + (1-\tau_a(a))r]\,a - c_e$$

This in turn implies the optimal consumption rule, $c_e(t) = \rho a(t)$.

$$\dot{a} = s(z)a, \quad where: \tag{2.10}$$

$$s(z) = \lambda \max\{z\pi - (1-\eta_k(z))r, 0\} + (1-\tau_a(a))r - \rho$$

The net amount of tax (asset tax minus capital subsidy) that an entrepreneur with personal wealth $a(t)$ pays to the government is therefore,

$$T_e = \left[\tau_a(a) - \lambda\eta_k(z) \cdot \mathbb{1}_{\{z \geq \underline{z}\}}\right] ra$$

2.2 The Representative Skilled Worker

The representative workers has the following preferences,

$$\int_0^\infty e^{-\rho t} \log c_w(t) dt \tag{2.11}$$

where ρ is the discount rate and c_w is the worker's consumption.

At time t, the worker has one unit of non-leisure time and human capital stock, $h(t)$. He then divides a fraction $u(t)$ of his non-leisure time to supply $u(t)h(t)$ efficiency units of labor at a competitive labor market at a wage $w(t)$. The left fraction of time $1-u(t)$ is used to increase his level of human capital stock via as the following technology: [7]

$$\dot{h} = b(1-u)h^{\phi} g_e^{1-\phi} \tag{2.12}$$

where $\phi \in (0,1)$, b is a parameter that denotes the efficiency of accumulating human capital and g_e is the level of government expenditures on public education and health programs. [8]

As in Moll (2014), we assume that the worker does not have access to financial markets and consume all their labor incomes. Hence, his budget constraint is as follows:[9]

$$c_w(t) = u(t)h(t)w(t) \tag{2.13}$$

The representative worker maximizes the sum of discounted utilities (2.11) subject to the budget constraint (2.13) and human capital accumulation (2.12) while taking the amount of public spending on education $g_e(t)$ as given. After substituting the budget constraint into the instantaneous utility function and denoting the current-value costate variable by $\mu(t)$, the current-value Hamiltonian for the representative worker' optimization problem can be expressed as:

$$H(u,h,\lambda) = \log[wuh] + \mu\left[b(1-u)h^{\phi} g_e^{1-\phi}\right]$$

The F.O.Cs of this optimization state:

$$\frac{1}{u} = \mu b h^{\phi} g_e^{1-\phi}$$
$$\frac{1}{h} + \mu\left(\phi b(1-u)h^{\phi-1} g_e^{1-\phi}\right) = \rho\mu - \dot{\mu}$$
$$\lim_{t\to\infty} e^{-\rho t}\mu(t)h(t) = 0$$

[7] Because ϕ is less than one, (2.12) implies that the production of human capital exhibits diminishing returns to existing level of human capital stock, $h(t)$. When $\phi = 1$ we have the traditional Uzawa-Lucas model setting where the accumulation of human capital depends only on private investment input, $1-u(t)$ and $h(t)$.

[8] Because human capital in this model means the quality of labor, government spending on human capital development can be interpreted generally. It can include not only public education but also spending on public health and other on-the-training programs.

[9] An important implication of this assumption is that since workers do not participate in the capital markets, their behavior is not directly affected by the the process of capital accumulation.

These F.O.Cs together with the evolution equation for h imply that optimal allocating time to work, $u(t)$, will obey the following differential equations:

$$\frac{\dot{u}}{u} - buh^{\phi-1}g_e^{1-\phi} + \rho = 0 \tag{2.14}$$

2.3 The Government

For simplicity, we assume that each time the government have balanced budget. Namely, at each time t, the government collects tax capital (asset) income and spends all on public education and on capital subsidy for active entrepreneurs. The budget constraint of the government then states:

$$\iint \tau_a(a) r a \phi(a,z) dadz = g_e + \iint \left[\lambda \eta_k(z)) \cdot \mathbb{1}_{\{z \geq \underline{z}\}}\right] r a \phi(a,z) dadz \tag{2.15}$$

where $\phi(a,z)$ is the joint distribution of productivity and wealth.

3 The Aggregate Equilibrium Dynamics

An equilibrium in this economy is sequences of quantities and factor prices such that (1) the representative worker and each entrepreneur maximize their expected sum of discounted utilities subject to their corresponding budget constraint taking as given equilibrium prices, (2) the government budget constraint (2.15) balances and (3) the factor markets clear at each point in time as follows,

$$\iint k_t^d(a,z)\phi(a,z) dadz = \iint a\phi(a,z) dadz \equiv k(t) \tag{3.16}$$

$$\iint n_t^d(a,z)\phi(a,z) dadz = uh \tag{3.17}$$

Substituting capital demand from (2.6) into (3.16) and recall that z is $i.i.d$ we obtain the following capital market equilibrium equation

$$1 = \lambda \left(1 - G(\underline{z})\right) \tag{3.18}$$

which in turn determines the productivity cut-off $\underline{z} = \lambda^{\frac{1}{\varphi}}$.

The aggregate output denoted by y is then obtained by summing the amounts of homogenous final goods produced by all active entrepreneurs, i.e., entrepreneurs with idiosyncratic productivity higher than the cut-off $\underline{z}$ as:

$$\begin{aligned} y =& \iint f(z,k,l)\phi(a,z)dadz = \iint \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} z\lambda a g(z)\psi(a)dadz \\ =& \frac{\pi}{\alpha}\lambda \int a\psi(a)da \int zg(z)dz = \frac{\pi}{\alpha}\lambda k \int_{\underline{z}}^{\infty} zg(z)dz \\ =& \frac{\pi}{\alpha}\lambda X k, \quad \text{where} \quad X \equiv \int_{\underline{z}}^{\infty} zg(z)dz \end{aligned} \tag{3.19}$$

where $g(z), \psi(a)$ denote the marginal distribution of idiosyncratic productivity and wealth, respectively.

Substituting the optimal labor demand (2.7) into the labor market clearing condition (3.17), we obtain $uh = \iint n_t^d(a,z)\phi(a,z)dadz = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{1-\alpha}} \lambda X k$, which then implies that:

$$\pi = \alpha \left(\lambda X\right)^{\alpha-1} k^{\alpha-1}(uh)^{1-\alpha} \tag{3.20}$$

Plugging this equation back to (3.19) we obtain the aggregate output as follows:

$$y = \left(\lambda X\right)^{\alpha} u^{1-\alpha} k^{[1+\alpha-1]}(h)^{1-\alpha} = Ak^{\alpha}(uh)^{1-\alpha} \tag{3.21}$$

where A is the endogenous measured TFP

$$A(t) \equiv \left(\lambda X\right)^{\alpha} = \mathbb{E}\left[z|z \geq \underline{z}\right]^{\alpha}$$

The wage rate, w, can be obtained by substituting (3.20) into the definition of π (2.9) and is,

$$w = (1-\alpha)Ak^{\alpha}(uh)^{-\alpha} \tag{3.22}$$

The wage rate in (3.22) is equal to the aggregated marginal product of labor implied from the aggregated production (3.21). This is because labor is competitively provided by homogeneous skilled workers. However, in the capital input market, there are distortions because of financial imperfections. In particular, substituting

(3.20) into (2.8) to obtain the capital rental return rate after subsidy, r, as:

$$\begin{aligned} r &= \frac{1}{1-\eta_k(z=\underline{z})}\frac{\underline{z}}{\mathbb{E}\left[z|z\geq\underline{z}\right]}\alpha A k^{\alpha-1}(uh)^{1-\alpha} \\ &= \frac{1}{1-\eta_k(z=\underline{z})}\frac{\underline{z}}{\mathbb{E}\left[z|z\geq\underline{z}\right]}\bar{r} = \frac{1}{1-\eta_k(z=\underline{z})}\frac{\varphi-1}{\varphi}\bar{r} \end{aligned} \tag{3.23}$$

where $\bar{r}$ is the aggregated marginal product of capital implied from implied from the aggregated production (3.21) as,

$$\bar{r} = \alpha A k^{\alpha-1}(uh)^{1-\alpha} \tag{3.24}$$

For the exposition purpose, first consider the case when the government does not intervene into the capital input market, i.e., $\eta_k(z) = 0$, then the capital rental rate is equal to,

$$r_0 = \frac{\underline{z}}{\mathbb{E}\left[z|z\geq\underline{z}\right]}\bar{r} = \frac{\varphi-1}{\varphi}\bar{r} < \bar{r} \tag{3.25}$$

This is a consequence of financial imperfections. Intuitively, because all entrepreneurs including those with high productivity face borrowing constraints, the equilibrium capital rental rate is equal to the marginal product of marginal entrepreneurs (those with productivity cut-off $\underline{z}$). Because in most cases the average productivity of actively producing entrepreneurs, i.e., $\mathbb{E}\left[z|z\geq\underline{z}\right]$, is higher than the productivity cut-off, $r_0 < \bar{r}$ in general.

When we assume that idiosyncratic productivity z follows the Pareto distribution as in (2.3) the rental rate depends on $\frac{\varphi-1}{\varphi}$, where $\varphi > 1$ is the shape parameter and smaller φ corresponds to a heavier tail of the productivity distribution or a higher fraction of very productive entrepreneurs, hence, implying a higher degree of productivity heterogeneity among entrepreneurs. Then it is intuitive that when entrepreneurs are heterogeneous in productivity, i.e. low φ, rental rate is low, which implies that capital is not efficiently used by marginal entrepreneurs.

The equation (3.23) implies that by intervening into the capital input market with non-zero capital subsidy rate $\eta_k(z)$ the government can influence the rental rate, therefore affecting the accumulation of wealth and physical capital. In particular, when the subsidy rate for marginal active entrepreneurs is set to $\frac{1}{\varphi}$ then the equilibrium rental rate in the presence of financial imperfections is equal to the the aggregated marginal product of capital implied from implied from the aggregated production $\bar{r}$.

The dynamic equation of the aggregate capital stock is then derived by first aggregating wealth of all entrepreneurs

$$\begin{aligned}
\frac{\dot{k}}{k} &= \frac{1}{k}\iint \dot{a}g(z)\psi(a)dadz \\
&= \frac{1}{k}\iint \left(\lambda \max\left\{z\pi - (1-\eta_k(z))r, 0\right\} + (1-\tau_a(a))r - \rho\right) ag(z)\psi(a)dadz \\
&= \int_0^\infty \left(\lambda \max\left\{z\pi - (1-\eta_k(z))r, 0\right\} + (1-\tau_a(a))r - \rho\right) g(z)dz
\end{aligned}$$

and then dividing entrepreneurs into the inactive group ($z < \underline{z}$) and the active group ($z \geq \underline{z}$), therefore

$$\begin{aligned}
\frac{\dot{k}}{k} &= r - r\int \tau_a(a)\psi(a)da - \rho + \int_{\underline{z}}^\infty \lambda\left\{z\pi - (1-\eta_k(z))r\right\} g(z)dz \\
&= r - r\int \tau_a(a)\psi(a)da - \rho + \pi\lambda\int_{\underline{z}}^\infty zg(z)dz - r - r\lambda\int_{\underline{z}}^\infty \eta_k(z)g(z)dz \\
&= \pi\lambda X - \rho - r\left(\int \tau_a(a)\psi(a)da - \lambda\int_{\underline{z}}^\infty \eta_k(z)g(z)dz\right) \\
&= \alpha\left(\lambda X\right)^{\alpha-1} k^{\alpha-1}(uh)^{1-\alpha}\lambda X - \rho - r\left(\int \tau_a(a)\psi(a)da - \lambda\int_{\underline{z}}^\infty \eta_k(z)g(z)dz\right) \\
&= \alpha A k^{\alpha-1}(uh)^{1-\alpha} - \rho - r\left(\tau - \lambda\eta\right)
\end{aligned}$$

where the third and forth equal signs are implied by the definition of X in (3.21), $\underline{z}$ in (2.8), and π in (3.20) and the effective asset tax rate policy, τ, and effective capital subsidy rate policy,η are defined as follows,

$$\tau \equiv \int \tau_a(a)\psi(a)da \tag{3.26}$$

$$\eta \equiv \int_{\underline{z}}^\infty \eta_k(z)g(z)dz \tag{3.27}$$

The budget constraint of the government becomes

$$
\begin{aligned}
g_e &= \iint \left[\tau_a(a) - \lambda\eta_k(z)\cdot \mathbb{1}_{\{\underline{z}\geq z\}}\right] ra\phi(a,z)dadz \\
&= [(\tau-\lambda\eta)rk] = y\left[\frac{(\tau-\lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\alpha\right]
\end{aligned}
$$

The aggregate equilibrium dynamics of the model economy can be summarized up in the following Lemma.

Lemma 1. *The dynamics of the aggregates and the fraction of non-leisure time assigned to goods production can be expressed as:*

$$
\begin{aligned}
& y = Ak^\alpha(uh)^{1-\alpha} \\
& \dot{k} = \alpha Ak^\alpha(uh)^{1-\alpha} - \left[\rho + ((\tau-\lambda\eta))r\right]k && (3.28) \\
& \dot{h} = b(1-u)h^\phi y^{1-\phi}\left[\frac{(\tau-\lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\alpha\right]^{1-\phi} && (3.29) \\
& \frac{\dot{u}}{u} - buh^{\phi-1}y^{1-\phi}\left[\frac{(\tau-\lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\alpha\right]^{1-\phi} + \rho = 0 && (3.30)
\end{aligned}
$$

where the measured aggregate productivity level are determined by

$$
A = \left(\frac{\varphi}{\varphi-1}\underline{z}\right)^\alpha = \left(\frac{\varphi}{\varphi-1}\right)^\alpha \lambda^{\frac{\alpha}{\varphi}} \qquad (3.31)
$$

The Balanced-Growth Path Equilibrium

Define the physical human capital ratio/intensity $\kappa = \frac{k}{h}$ as the ratio of physical capital stock (per worker) over the average level of human capital and a *balanced-growth path equilibrium* of this economy is established when,

$$
\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\dot{h}}{h} = \gamma, \quad \text{for all t}
$$

where γ denotes the balanced-growth rate.

Substituting κ and γ into (3.28), (3.29), (3.30) and note that under a balanced-growth path equilibrium, u and κ stay constant over time, we obtain the following 3 equations for 3 variables, κ, γ and u as,

$$\left[1-\frac{(\tau-\lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\right]\alpha A\left(\frac{\kappa}{u}\right)^{\alpha-1}-\rho=\gamma$$
$$b(1-u)h^{\phi-1}y^{1-\phi}\left[\frac{(\tau-\lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\alpha\right]^{1-\phi}=\gamma$$
$$-buh^{\phi-1}y^{1-\phi}\left[\frac{(\tau-\lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\alpha\right]^{1-\phi}+\rho=0$$

Consequently, the balanced-growth equilibrium fraction of working hour, u and physical-human capital ratio κ are given by:

$$u=\frac{\rho}{\gamma+\rho} \tag{3.32}$$
$$\kappa=\frac{\rho}{\gamma+\rho}\left(\frac{\alpha A}{\gamma+\rho}[1-\theta]\right)^{\frac{1}{1-\alpha}} \tag{3.33}$$

where

$$\theta\equiv\frac{\tau-\lambda\eta}{1-\eta_k(z=\underline{z})}\frac{\varphi-1}{\varphi} \tag{3.34}$$

and the balanced-growth rate γ are determined by the following equation,

$$\begin{aligned}1&=b\frac{1}{\gamma+\rho}\left(\frac{y}{h}\right)^{1-\phi}[\alpha\theta]^{1-\phi}\\&=b\frac{1}{\gamma+\rho}\left[A\left(\frac{\kappa}{u}\right)^{\alpha}u\right]^{1-\phi}[\alpha\theta]^{1-\phi}\\&=b\frac{1}{\gamma+\rho}\left[A\left(\frac{\alpha A}{\gamma+\rho}[1-\theta]\right)^{\frac{\alpha}{1-\alpha}}\frac{\rho}{\gamma+\rho}\right]^{1-\phi}[\alpha\theta]^{1-\phi}\\&=b\frac{1}{\gamma+\rho}\left[\rho\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}\left(\frac{1}{\gamma+\rho}\right)^{\frac{1}{1-\alpha}}\right]^{1-\phi}\left[[1-\theta]^{\frac{\alpha}{1-\alpha}}\alpha\theta\right]^{1-\phi}\end{aligned} \tag{3.35}$$

which is equivalent to the following equations,

$$[\gamma+\rho]^{\frac{1}{1-\alpha}+\frac{1}{1-\phi}}=b^{\frac{1}{1-\phi}}\left[\rho\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}[1-\theta]^{\frac{\alpha}{1-\alpha}}\theta\right] \tag{3.36}$$

Proposition 1. *When the tax and subsidy policies are such that the following condition*

$$[\rho]^{\frac{1}{1-\alpha}+\frac{1}{1-\phi}-1}\leq b^{\frac{1}{1-\phi}}\left[\alpha^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}[1-\theta]^{\frac{\alpha}{1-\alpha}}\theta\right] \tag{3.37}$$

is satisfied then there exists a unique positive balanced-growth path rate equilibrium for this economy.

Intuitively, this Proposition states that for a given tax and subsidy policies τ_a, η_k when financial markets are sufficiently deep, high λ, so that the measured aggregate productivity level A is high and/or the human capital accumulation is sufficiently efficient, high b, so that the condition (3.37) is satisfied then there exists a balanced-growth path rate equilibrium.

Proof. Denote the LHS of (3.36) as $H(\gamma)$, then because $\frac{1}{1-\alpha} + \frac{1}{1-\phi} \geq 2$

$$\frac{dH(\gamma)}{d\gamma} > 0, H(\gamma) \geq H(0) = [\rho]^{\frac{1}{1-\alpha}+\frac{1}{1-\phi}}, \quad \text{for all} \quad \gamma \geq 0$$

Rewrite (3.37) as,

$$[\rho]^{\frac{1}{1-\alpha}+\frac{1}{1-\phi}} \leq b^{\frac{1}{1-\phi}} \left[\rho\alpha^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} [1-\theta]^{\frac{\alpha}{1-\alpha}} \theta\right] \tag{3.38}$$

which equivalently states that the horizontal line representing the RHS of (3.36) interacts the vertical axis at a point above $H(0)$. Consequently, there exists a unique non-negative solution $0 \leq \gamma$ for the equation (3.36).

□

Notice that a higher A leads to a higher RHS of (3.36), hence a higher value for the solution γ of this equation. Consequently, this Proposition also implies that an economy with a higher measured aggregate productivity level thanks to deeper financial markets will have a higher balanced growth path rate and a lower fraction of working hours.

4 Optimal Growth Policies

In the previous section, I assume that tax and subsidy policy rates $\tau_a(a), \eta_k(z)$ are given and identify the condition in which there exists a unique balanced growth path equilibrium. In this section, I discuss the optimal capital taxation policies that maximize the balanced growth path rate. [10] Intuitively, a higher capital subsidy rate and/or a lower asset income tax rate will promote physical capital accumulation but it will reduce government spending on education g_e, hence discouraging human capital acquisition and vice versa. For the exposition purpose, let us denote:

$$\begin{aligned}\Theta(\tau_a, \eta_k) &\equiv [1-\theta]^{\frac{\alpha}{1-\alpha}}\,\theta \\ &= \left[1 - \frac{(\tau-\lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\right]^{\frac{\alpha}{1-\alpha}}\left[\frac{(\tau-\lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\right]\end{aligned} \tag{4.39}$$

Because the RHS of (3.36) can be written as $b^{\frac{1}{1-\phi}}\left[\rho\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}\right]\Theta(\tau_a, \eta_k)$, a higher value for $\Theta(\tau_a, \eta_k)$ will lead to a higher balanced growth path rate. Moreover, the partial derivative of $\Theta(\tau_a, \eta_k)$ with respect to θ is given by,

$$\begin{aligned}\frac{\partial}{\partial\theta}\Theta(\tau_a, \eta_k) &= [1-\theta]^{\frac{\alpha}{1-\alpha}-1}\left[\frac{-\alpha}{1-\alpha}\theta + 1 - \theta\right] \\ &= [1-\theta]^{\frac{\alpha}{1-\alpha}-1}\left[1 - \frac{1}{1-\alpha}\theta\right] \\ &= [1-\theta]^{\frac{\alpha}{1-\alpha}-1}\left[1 - \frac{\tau-\lambda\eta}{1-\eta_k(z=\underline{z})}\frac{\varphi-1}{\varphi(1-\alpha)}\right]\end{aligned} \tag{4.40}$$

Consequently, we obtain the following Proposition dictating the optimal policy rule for pro-growth policies in this economy.

Proposition 2. *The optimal policy $\tau_a(a)$ and $\eta_k(z)$ obey the following rule:*

$$\frac{\tau-\lambda\eta}{1-\eta_k(z=\underline{z})} = \frac{\varphi(1-\alpha)}{\varphi-1} \tag{4.41}$$

where $\tau \equiv \int \tau_a(a)\psi(a)da$ and $\eta \equiv \int_{\underline{z}}^{\infty} \eta_k(z)g(z)dz$.

Proof. The proof is straightforward from (4.40). □

Corollary 1. *When $\frac{\varphi(1-\alpha)}{\varphi-1} \leq 1$, the optimal pro-growth policy can be implemented just by utilizing the asset income tax instrument $\tau_a(a)$.*

[10] Because of balanced government budget, the determination of η_k implicitly determines government spending on eduction g_e.

Proof. When $\frac{\varphi(1-\alpha)}{\varphi-1} \leq 1$, by setting $\eta_k(z) = 0, \ \forall z$ then (4.41) implies: $\tau = \frac{\varphi(1-\alpha)}{\varphi-1} \leq 1$. It is then straightforward that there always exists an optimal asset income tax rate, e.g., a flat rate $\bar{\tau_a}$ that satisfies this condition. □

Intuitively, this Corollary states that when the productivity of entrepreneurs is not so heterogeneous, specifically, $\varphi \geq \frac{1}{\alpha}$, or $\frac{\varphi(1-\alpha)}{\varphi-1} \leq 1$ then the optimal pro-growth policy can be implemented by appropriately taxing on asset income and then spending on public education.

Next, we will discuss the case when it is infeasible to implement the optimal pro-growth policy without utilizing the capital subsidy instrument $\eta_k(z)$. The reason is straightforward. Because $\tau \leq 1$ so when $\frac{\varphi(1-\alpha)}{\varphi-1} > 1$ or $\varphi < \frac{1}{\alpha}$, namely the productivity of entrepreneurs is relatively heterogeneous the condition (4.41) can not be satisfied with only one asset income tax instrument $\tau_a(a)$. To focus on how and when the capital subsidy policy should be implemented we assume from here that the effective asset income rate τ is given and is less than one. Rewriting the equation (4.41) as,

$$\left[\frac{\varphi(1-\alpha)}{\varphi-1}\eta_k(z=\underline{z}) - \lambda\int_{\underline{z}}^{\infty} \eta_k(z)g(z)dz\right] = \frac{\varphi(1-\alpha)}{\varphi-1} - \tau \tag{4.42}$$

which then implies the following Proposition.

Proposition 3. *When $\frac{\varphi(1-\alpha)}{\varphi-1} > 1$, for a given tax policy τ, the optimal pro-growth subsidy policy $\eta_k(z)$ is set such that:*

$$\lambda\int_{\underline{z}}^{\infty} \frac{\eta_k(z)}{\eta_k(z=\underline{z})}g(z)dz < \tau \tag{4.43}$$

$$\left[\frac{\varphi(1-\alpha)}{\varphi-1} - \lambda\int_{\underline{z}}^{\infty} \frac{\eta_k(z)}{\eta_k(z=\underline{z})}g(z)dz\right]\eta_k(z=\underline{z}) = \frac{\varphi(1-\alpha)}{\varphi-1} - \tau \tag{4.44}$$

Proof. First, under the condition (4.43) there exists a solution $0 < \eta_k(z = \underline{z}) < 1$ that satisfies the optimal rule (4.44). Second, for given τ, λ and the productivity distribution $G(z)$, we need to show that there exists an optimal subsidy policy $\eta_k(z) < 1$ for all $z \geq \underline{z}$.

For instance, consider the following subsidy policy:

$$\eta_k(z) = \eta_k(z = \underline{z}) \left(\frac{\underline{z}}{z}\right)^{\mu}, \quad \mu > 0 \tag{4.45}$$

then we have $\lambda \int_{\underline{z}}^{\infty} \frac{\eta_k(z)}{\eta_k(z=\underline{z})} g(z)dz = \frac{\varphi}{\varphi+\mu}$. Consequently, (4.43) is satisfied if we set

$$\mu > \varphi \left(\frac{1}{\tau} - 1\right) \tag{4.46}$$

And, the capital subsidy rate for the marginal entrepreneurs at the productivity cut-off is,

$$\eta_k(z = \underline{z}) = \frac{\frac{\varphi(1-\alpha)}{\varphi-1} - \tau}{\frac{\varphi(1-\alpha)}{\varphi-1} - \frac{\varphi}{\varphi+\mu}} \tag{4.47}$$

□

Intuitively, this Proposition states that when the distribution of the idiosyncratic productivity is heavy-tailed (low φ) and capital is relatively less efficiently utilized by marginal producers as compared to the average/mean of all active producers so that the capital rental rate is relatively low then the government should subsidize active entrepreneurs. Alternatively, when the share of wealth held by active entrepreneurs is relatively high then the pro-growth policy is to subsidize these entrepreneurs for more efficient physical capital accumulation. [11]

However, the condition (4.43) also states that it is infeasible to set the same flat subsidy rate $\bar{\eta}k$ for all active entrepreneurs because in this case, (4.43) implies that $1 < \tau$, which is a contradiction. For policy feasibility, the government need either to discriminate them according to their idiosyncratic productivity as in (4.45) or limit the range of active entrepreneurs who can receive capital subsidy as follows:

$$\eta_k(z) = \bar{\eta}_k, \quad \text{for} \quad \underline{z} \leq z \leq \tilde{z} \tag{4.48}$$

where $\bar{\eta}_k$ and $\tilde{z}$ are to be determined.

Then we have $\lambda \int_{\underline{z}}^{\infty} \frac{\eta_k(z)}{\eta_k(z=\underline{z})} g(z)dz = 1 - \frac{\lambda}{\tilde{z}^{\varphi}}$. Hence, (4.43) implies that $\tilde{z}$ must satisfy

$$\tilde{z} < \left(\frac{\lambda}{1-\tau}\right)^{\frac{1}{\varphi}} \tag{4.49}$$

[11] Under the $i.i.d$ assumption for productivity z, the distribution of the share of wealth held by productivity type z is the same with the distribution of productivity z.

5 Concluding remarks

This paper studies the optimal growth policies in an environment where heterogeneous entrepreneurs face borrowing collateral constraints, skilled workers accumulate human capital, and the government provides public education to facilitate human capital formation and intervenes in the capital market to promote economic growth. The theoretical analysis shows that public policy can influence the balanced-growth path equilibrium and the optimal growth policies depend on the degree of productivity heterogeneity. There are several possible extensions from this paper. One is to have more tax instruments and to allow dynamic inter-temporal government budget constraints so that the government can participate directly into the capital market and conduct optimal policies with more freedom. Another is to add more objectives to the optimal policies such as reducing distortions in the capital markets and optimizing entrepreneurs and/or workers' welfare in richer settings.

A Appendix: Adding Labor Income Tax

In this Appendix, I add the labor income tax instrument into the model in the main text and demonstrate that all analytical results are similar. In particular, when the government impose income tax rate, τ_n, the budget constraint of the representative skilled worker becomes,

$$c_w(t) = (1 - \tau_n)u(t)h(t)w(t) \tag{A.50}$$

and the budget constraint of the government then states:

$$\tau_n uhw + \iint \tau_a ra\phi(a,z)dadz = g_e + \iint \left[\lambda\eta_k \cdot \mathbb{1}_{\{\underline{z} \geq z\}}\right] ra\phi(a,z)dadz \tag{A.51}$$

Therefore, the Lemma 1 in the main text is replaced by

Lemma 2. *The dynamics of the aggregates and the fraction of non-leisure time* *assigned to goods production can be expressed as:*

$$y = Ak^{\alpha}(uh)^{1-\alpha}$$

$$\dot{k} = \alpha Ak^{\alpha}(uh)^{1-\alpha} - \left[\rho + ((\tau - \lambda\eta))r\right]k \tag{A.52}$$

$$\dot{h} = b(1-u)h^{\phi}y^{1-\phi}\left[\tau_n(1-\alpha) + \frac{(\tau - \lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\alpha\right]^{1-\phi} \tag{A.53}$$

$$\frac{\dot{u}}{u} - buh^{\phi-1}y^{1-\phi}\left[\tau_n(1-\alpha) + \frac{(\tau - \lambda\eta)}{(1-\eta_k(z=\underline{z}))}\frac{\varphi-1}{\varphi}\alpha\right]^{1-\phi} + \rho = 0 \tag{A.54}$$

where the measured aggregate productivity level are determined by

$$A = \left(\frac{\varphi}{\varphi-1}\underline{z}\right)^{\alpha} = \left(\frac{\varphi}{\varphi-1}\right)^{\alpha}\lambda^{\frac{\alpha}{\varphi}} \tag{A.55}$$

Therefore, the equation that determines the balanced-growth path rate, γ, becomes,

$$[\gamma+\rho]^{\frac{1}{1-\alpha}+\frac{1}{1-\phi}} = b^{\frac{1}{1-\phi}}\left[\rho\alpha^{\frac{1}{1-\alpha}}A^{\frac{1}{1-\alpha}}\left[1-\theta\right]^{\frac{\alpha}{1-\alpha}}\left(\frac{\tau_n(1-\alpha)}{\alpha}+\theta\right)\right] \tag{A.56}$$

where

$$\theta \equiv \frac{\tau-\lambda\eta}{1-\eta_k(z=\underline{z})}\frac{\varphi-1}{\varphi}$$

which then implies that the equilibrium balanced growth path rate depends positively on the following value that is the function of τ_n, τ_a, η_k:

$$\Theta_1(\tau_n,\tau_a,\eta_k) \equiv [1-\theta]^{\frac{\alpha}{1-\alpha}}\left[\frac{\tau_n(1-\alpha)}{\alpha}+\theta\right] \tag{A.57}$$

It is straightforward to obtain similar analytical results with the main text from the above equations.

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