# Some remarks on a paper of Nils Fröhlich.

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**Abstract**

This note contains some remarks about a generalized theoretical framework for a labour value theory that allows to deduce the different settings considered in [3] for empirical validation from a unified point of view. The formulation strongly suggests that profit rates can be interpreted as labour values for the organisational part of production. Profit rate formation suggests that this part is not measured by a time measure, a suspicion already expressed by Marx in the *Grundrisse* about labour value formation in a situation dominated by applications of the "general intellect".

## 1 Introduction

Fröhlich's paper addresses aspects of theoretical foundation and empirical validation of labour value theories from a macroeconomic point of view. The author elaborates very well the principal logical levels to be distinguished for his goal – the empirical validation of statements about the dynamics of highly aggregated data as, e.g., a common or sectoral profit rate $r$. These levels are

1. explanation of the theoretical framework resp. model to be considered,
2. identification of the kind of statistical data that is suited to adjust the model and draw consequences about the degree of validity of certain hypotheses and
3. evaluation of the practical comparison between predictions of the model and the empirical data.

The second level may be (and is in this case) substituted by a semantically sound mapping of *given* statistical data to the model. This causes an additional problem – this statistical data bound to another model may not give the required selectivity for the model under consideration, and require an estimation about the soundness of the observed differences between model and data in its own way.

Level 1 is a completely theoretical one and addresses clear semantics of notions, logical dependencies within the model – best expressed in adequate mathematical notion, the *lingua franca* of science – and consequences that can be falsified. Fröhlich's main contribution in this part is a detailed discussion and evaluation of existing macroeconomic concepts based on a Leontieff model and picking out those concepts that are suited for further empirical evaluation.

Fröhlich considers different models to be fitted on the same data, the IO tables from the German Federal Bureau of Statistics and German capital stocks. For this purpose he develops a roadmap for level 2 and 3 that can be adjusted also for other theoretical frameworks within a certain range.

These roadmap considerations are – in my view – the main benefit of the paper. They yield two main results. First, they give a practical evaluation of the economic theories under consideration on empirical data and close this way a gap that was claimed to be open for labour value based economic theories for a long time, see, e. g., [6]. Second, they propose a general roadmap for empirical evaluation of similar theories thus setting a certain standard for empirical validation of labour value based theories in general.

The main goal of this note is to propose and justify some modifications of the theoretical framework within level 1 and discuss how to fit them in Fröhlich's roadmap.

## 2 Some general considerations about labour value theory

Fröhlich starts his considerations in [3] "with an economy that is described by a linear, constant-returns-to-scale technology $\{A, l\}$ where $A = (a_{ij})$ is an indecomposable, productive $(n \times n)$-matrix of input coefficients and $l$ is the $(1 \times n)$-vector of direct labour inputs". Later on, $A$ is identified with intersectoral exchange relations as given by the macroeconomic IO tables. On that layer all terms are already counted in monetary units (MU), and it is that perspective, that is addressed, e. g., also in volume 3 of Marx' *Capital*. A constant claim against labour value considerations at this level is that prices and labour values are mixed. This applies also to Fröhlich's formula (1) and the semantics of $\lambda$ since it can only be interpreted in the sense of (2), i. e., as

$$\lambda = l\,(I - A)^{-1} = l + l\,A + l\,A^2 + \ldots,$$

where $l$ counts the labour spent in this "uniform period of production", $l\,A$ counts the labour spent last period, $l\,A^2$ in the prelast etc. This makes only sense for a *static* equilibrium, but Fröhlich's goal is explicitely to study the *dynamics* of profit rates, so his models have to take into account the transfer mechanism of labour spent in different periods (that are related to subsistence badgets sold at the price levels of those periods) to the present etc.

Since Fröhlich starts already at the macroeconomic layer, he does not discuss the level 1 arguments that lead to those aggregated notions. Such arguments are discussed in volume 1 and 2 of Marx' *Capital*, to indicate two more abstraction layers below the layer addressed by Fröhlich. On the first layer notions have to be coined, in particular a clear understanding of the difference between *(exchange) value* and *use value* and their interrelation. On the second level, microeconomic relations (reproduction schemes) for the *production of goods* are considered. Only on the next level interrelations between *production sectors* as the target of macroeconomic investigations come into account.

At those abstraction layers the desired transfer mechanism is elaborated and matrix $A$ comes into account once more, but this time as *matrix of technical coefficients*, i. e., as matrix that describes the *technical* composition of new use values from the use values of the ingredients. The particular formula $x = A\,x + y$ describes, how within a *closed economy* and a given amount of labour $l$ (not yet present here) not only the means of subsistence $y$ can be produced but also the productive context is reproduced.

This is the target, on that *value* theory has to be developed, even if labour value accounts are not required to distribute the means of subsistence *if the personal needs were known in advance and $y$ is the aggregation per good of those needs.* Unfortunately, this is not the case, and economic theory starts just at this point. Labour value theory assumes at this starting point, that the single source and last resort of explanation of prices is human labour. Humans are working to satisfy their needs, so we have sources (human labour) and sinks (needs). Accounting is done only for work on needs of others, and with the money system clear rules for propagation of acceptance of accounting are established. It are those rules that establish the link between past, present and future in the economic system that was indicated as missing in the above consideration.

So far a very general description of the starting point of labour value theory. The model behind $x = A\,x + y$ (and also Marx' considerations in *Capital*, volume 1 and 2) has a much more restricted target – it models mass production of $n$ different standardized goods using (now $l$ comes into account) standardized labour.

It is one of the main weaknesses of Marx' concept to reduce all complicated labour as multiplied labour to simple labour, since this hides the multipliers from the public. I'll come back to that point later. For the moment, and since standard theory can easily be extended in that direction, we assume that there are also $m$ different standardized kinds of labour. Hence we have to introduce $m$ $(1 \times n)$-vectors $l_1, \ldots, l_m$ that count labour of the different types spent for the production. We collect these row vectors into a $(m \times n)$-matrix $B$ and get $\{A, B\}$ as the *technical* ingredients for the production process under consideration (since $B$ represents the *use value* of the labour to get the production process running).

To get the *use value perspective* complete we have to fix the subsistence badgets for the different kinds of labour, i. e., standardized badgets $b_1, \ldots, b_m$ as $(n \times 1)$-vectors as in [3, formula (6)]. We collect these column vectors into another matrix $C$. Hence $\{A, B, C\}$ are the *technical* ingredients for the corresponding *reproduction* process. These matrices perfectly arrange in a single block matrix

$$U = \begin{pmatrix} A & C \\ B & 0 \end{pmatrix}$$

as explained in [5] in more detail.

## 3 Labour value without capitalists – a restricted framework

For a closed model without capitalists described by $x = A\,x + y$ as discussed in the first part of [3, 2.1] there is another *technical* invariance condition – the net product $y$ should exactly meet the reproduction requirements of the labour $L = B\,x$ spent for the production, i. e.,

$$C \cdot L = C \cdot B \cdot x = y\,. \tag{2}$$

In general we have $C \cdot B \cdot x = y_L$ where $y_L$ is the net product required technically for reproduction of the labour force. Note that such a formula ($b\,l\,x = y$ in Fröhlich's notation) is missing in [3].

For the analogue of [3, formula (3)] we have to introduce the wage rates $w^*$. Since we count

with different standardized kinds of labour $w^*$ becomes a $(1 \times m)$-vector and we get

$$p^* = p^* A + w^* B \tag{3}$$

or in a more compact form

$$\begin{pmatrix} p^* & w^* \end{pmatrix} = \begin{pmatrix} p^* & w^* \end{pmatrix} \cdot \begin{pmatrix} A \\ B \end{pmatrix} . \tag{3a}$$

Multiplying (3) with $x$ from the right and setting $A\,x = x - y$ we get $p^*\,y = w^* B\,x$ and with $y = C\,B\,x$ as just derived $p^*\,C\,B\,x = w^* B\,x$. Since this should hold for all $B\,x$, we get finally

$$p^*\,C = w^* \tag{4}$$

similar to [3, formula (4)]. Note that this formula in Fröhlich's paper should correctly read $w^* = p^*\,b = 1$, but this has no influence on the further argumentation. (4) perfectly completes (3a) to the equation

$$\begin{pmatrix} p^* & w^* \end{pmatrix} = \begin{pmatrix} p^* & w^* \end{pmatrix} \cdot \begin{pmatrix} A & C \\ B & 0 \end{pmatrix} = \begin{pmatrix} p^* & w^* \end{pmatrix} \cdot U\,. \tag{3b}$$

Another word about units: All vectors and matrices are realvalued, i. e., we assume that we fixed (good specific) units $G_i$ to measure. What's about labour units? We see that labour value anyway comes in a natural way as $w^*B$, i. e., as multiplied labour, counted in monetary units MU. The (formal) theoretical arguments remain the same if we differ between labour expense (Arbeitsaufwand), measured in different units $L_i$ for different kinds of labour, and labour value (Arbeitswert, counted in MU).

Such a slight adjustment of theory has twofold advantage. First, it allows to model closer to reality, since in many industrial places labour is counted not in hours but in piece rates (Stücklohn) and labour value is directly computed by the norm value $n$ in MU per piece. Second, we remind the statement from the *Grundrisse*, where Marx reasons about the consequences of the measure of labour value in a high-tech environment and complains that

> die Schöpfung des wirklichen Reichtums weniger abhängt von der Arbeitszeit und dem Quantum angewandter Arbeit als von der Macht der Agentien, die während der Arbeitszeit in Bewegung gesetzt werden und die selbst wieder [...] in keinem Verhältnis steht zur unmittelbaren Arbeitszeit, die ihre Produktion kostet, sondern vielmehr abhängt vom allgemeinen Stand der Wissenschaft und dem Fortschritt der Technologie. (MEW 42, S. 592)

Hence there is a strong motivation to develop at least the very basics of a labour value framework not bounding labour value to a time measure but remaining open for other kinds of labour value measures. Such a semantic is perfectly covered by the approach developed so far and can be continued much further as explained in [5].

## 4 Labour value with capitalists – a generalized framework

We come to a generalization of [3, formula (6)]. As already explained, a straightforward modification of our explanations so far to a labour value scenario with capitalists has to

modify (2) and substitute $y_L$ at the right hand side. Due to the central role of the *unmodified* formula (2) in the derivation of (4) and (3b) we try another way: Let $B_L$ and $C_L$ denote the matrices $B$ and $C$ as used so far, i. e., $B_L$ is the norm matrix of standardized labour inputs and $C_L$ the matrix of standardized subsistence badgets for the different kinds of labour. Take formally (the semantics will be discussed later) two similar matrices $B_M$ of size $(k \times n)$ and $C_M$ of size $(n \times k)$ and

$$B = \begin{pmatrix} B_L \\ B_M \end{pmatrix}, \quad C = \begin{pmatrix} C_L & C_M \end{pmatrix},$$

such that (2) holds as before. We get immediately by expansion

$$C_L B_L x + C_M B_M x = y$$

and altogether the well known decomposition

$$x = A\,x + C_L B_L x + C_M B_M x = K + L + M$$

of the production output into the replacement $K$ of the goods used in production, the private consumption $L$ and the productive consumption $M$, see, e. g., [2] ($A\,x + C\,x + S\,x$ in Fleissners notion). On the level of labour value (here completely identical with prices) we get

$$p = p\,A + p\,C_L B_L + p\,C_M B_M = p\,A + w_L B_L + w_M B_M$$

with $w_L = p\,C_L$ the price of the (normalized) subsistence goods that should be bought out by the wages $w_L$ per labour kind, and $w_M = p\,C_M$ the price of the (normalized) productive consumption (i. e., investments in the macroeconomic theory) that should be bought out by the profits $w_M$ per "profit kind" (whatever that will be). This is, more or less, an equivalent of [3, formula (17)] and can, together with the settings $w_L$ and $w_M$, compactly be written as

$$\begin{pmatrix} p & w_L & w_M \end{pmatrix} = \begin{pmatrix} p & w_L & w_M \end{pmatrix} \cdot \begin{pmatrix} A & C_L & C_M \\ B_L & 0 & 0 \\ B_M & 0 & 0 \end{pmatrix} = \begin{pmatrix} p & w_L & w_M \end{pmatrix} \cdot U\,. \tag{3c}$$

We end up with a theoretical setting of a mass production where formula (3c) is the central value theoretic link between

- the $n$ standardized kinds of goods to be produced,
- the $m$ standardized kinds of labour required to execute the production and
- the $k$ standardized kinds of "profit making" to organize that production in a sustainable way.

From Leontieff modeling we know that the dual equation should have a use value interpretation, and indeed

$$\begin{pmatrix} x \\ v_L \\ v_M \end{pmatrix} = \begin{pmatrix} A & C_L & C_M \\ B_L & 0 & 0 \\ B_M & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ v_L \\ v_M \end{pmatrix} \tag{5}$$

expands to

$$\begin{aligned} x &= A\,x + C_L v_L + C_M v_M, \\ v_L &= B_L x, \\ v_M &= B_M x, \end{aligned}$$

where $v_L$ is the amount of labour spent measured in labour kind specific non-MU units $L_i$ and $v_M$ is the amount of "profit earned" measured in profit kind specific non-MU units $P_i$ (whatever that means). Both can be mapped via multiplication with $w_L$ resp. $w_M$ to money terms. Whereas $v_L$ has a clear interpretation as gross labour expense (Arbeitsaufwand) the real meaning of $v_M$ remains vague at that point and needs further clarification.

(3c) and (5) are very strong hints, that a semantic interpretation (that needs further clarification, see [5]) of profits as "wages" of entrepreneurship (not for the entrepreneur as person but for him as "fungierender Kapitalist" in the sense of Marx) is the correct way to handle the problems with profits in a labour value theory. Note that entrepreneurship is clearly "human work on others needs" (note that in the last resort only real persons have legal capacity), but not directed on the production of goods but of social relations. Such work is claimed to be "unproductive" in classical texts (including Fröhlich's). See [7] for a fundamental criticism of such a position.

For further reference we denote $P = (p\ w_L\ w_M)$ and $X = (x\ v_L\ v_M)^T$, so that (3c) and (5) can shortly be written as

$$P = P\,U \qquad \text{and} \qquad X = U\,X\,. \tag{6.S}$$

These equilibrium conditions can easily be rewritten in dynamic iteration relations

$$P^{(t)} = P^{(t-1)}\,U \qquad \text{and} \qquad X^{(t)} = U\,X^{(t-1)}\,, \tag{6.D}$$

where $P^{(t)}$ and $X^{(t)}$ denote the corresponding vectors after production step $t$ that simultaneously transforms via $U$ the use values from state $X^{(t-1)}$ to $X^{(t)}$ and the exchange values from $P^{(t-1)}$ to $P^{(t)}$, see [2] once more for the corresponding iteration schemes.

## 5 Microeconomical and macroeconomical models

The fundamental relations and equations developed so far describe the economic relations on a very detailed microeconomical level and with the focus on the production of *goods*. Goods and their use value are the *target* of labour value theory, and Marx emphasizes in the Fetisch-Kapitel that commodities only *appear* to have value, but the point is that value is a *social relation between producers* executed on the back of those goods.

On the macroeconomical level the perspective changes and the interrelation of sectoral productions are studied via the exchange relations of (priced) real goods between those sectors. Most of the macroeconomical labour value theories considered by Fröhlich are based on models that arize from direct aggregation of the microeconomical flows of priced goods and claim that these aggregations are ruled by the same fundamental concepts and relations as on the microeconomical level, if the corresponding notions are properly interpreted.

The only different approach considered by Fröhlich is the probabilistic approach [1] of Farjoun and Machover. It considers the macroeconomical aggregation process as a stochastic one on the great number of basic transactions $\tau \in T$ (I use $\tau$ instead of $t$ in Fröhlich's paper not to confuse that with the notion for time or iteration steps) described by the microeconomical rules.

One may ask if there is a deeper link between these theoretical considerations and ways of bookkeeping and accounting in the real world. This is indeed the case and opens the eyes for the embedding target of (Fröhlich's interpretation of) the approach of Farjoun and Machover. Indeed, the relations and estimations given with (6) are not only the basis for academic research but also for bookkeeping and accounting in the real world. Goods are exchanged by producers, and each such transaction is reflected in the bookkeeping and accounting of the producers involved. Moreover, this bookkeeping has to be done by well defined rules, is subject to reporting compulsion and social control by governmental financial authorities and constitutes the basis of the macroeconomical statistical data. It's not the transactions themselves but that bookkeeping, i. e., the local accounting of finished and pending (until the advanced capital has completely revolved) calculations based on the *local* world model of each producer-accounter who stands by her own health and life responsible for her decisions. Even more, *there is no global world model* beyond the abstraction(s) of these communicatively coupled local world models.

All this gives rise to emphasize not only on the transactions $\tau \in T$ as binary relations between producers but also on the producers themselves. This yields a place-transition structure well known under the name of *Petri nets*. Moreover, considering transitions (=transactions) *and places* gives the right framework to address not only flow terms but also stock terms as addressed in [3, 3.1]. Petri net theory supplies a whole bundle of concepts and approaches to deal with flows in such networks. See [5] for the basics of a decentralized approach to a labour value theory based on Petri nets.

The approach of Farjoun and Machover can easily be extended in such a direction. Moreover, we see that there are several degrees of freedom to adapt [3, formula (25)] for different scenarios. I'll come back to this in the next section.

## 6 Fröhlich's models within the generalized framework

Fröhlich's models fit into the generalized framework given with our formula (6) with $m = k = 1$ in the following way:

**1.** The basic labour value accounting in [3, formula (1)] can be obtained with $B_L = l,\ w_L = 1$ and $w_M = 0,\ B_M$ arbitrary:

$$\lambda = \lambda\, A + w_L B_L + w_M B_M = \lambda\, A + l$$

Note in particular, that in the generalized setting such a labour value accounting is a *price* theory deduced from (6) since it counts not the labour (time) really spent but a socially averaged *accounted labour* (Stundenzettel). The difference between Arbeitsaufwand and Arbeitswert becomes much more clear if several kinds of labour are considered and we retain the $w_L$ as coupling factors between the (technically determined) matrix of Arbeitsaufwand $B_L$ and the corresponding Arbeitswerte, even if the latter are measured not in MU but in hours.

On the other hand, as pointed out in [3, 3.1], only *monetary* labour values are empirically available, so that [3, formula (1)] is best generalized to

$$\lambda = \lambda\, A + w_L B_L \tag{1.G}$$

with "labour skill coefficients" $w_L$ and "labour extend matrix" $B_L$. Both are not directly observable but only the aggregate $w_L B_L$ as monetary labour values (per kind of standardized labour). Whereas – within a time measure based labour value theory – $B_L$ has a clear *technical* meaning as standardized time required for the standardized skilled worker to execute a certain standardized task the *technical* meaning of $w_L$ remains vague even if labour values are not considered as monetary labour values. For monetary labour values the character of $w_L$ as *socially negotiated coefficients* is obvious. This shows once more the character of labour value counts as a *price* theory and that any "true price theory" within a labour value theoretical framework (i. e., net prices, production prices, observed prices etc.) is necessarily a secondary price theory to be estimated on the basis of that primary price theory. This is the semantic core of the ratios and probability weight function considered in [3, formula (25)]. As indicated with (1.G) this approach can easily be reformulated within the generalized framework.

**2.** We obtain value prices in [3, formula (9)] with $B_M = B_L = l$ and $w_L = w,\ w_M = e\, w$:

$$p_v = p_v\, A + w_L B_L + w_M B_M = p_v\, A + w\, l + (e\, w)\, l$$

As already mentioned in the first section it is not a good idea to rewrite $p_v = (\cdots)\,(I - A)^{-1}$ since it mixes price conditions at different times and applies only to *static* equilibria. I'll come back to that point below.

**3.** We obtain neoricardian prices of production ([3, formula (14)]) with $B_M = l,\ B_L = p_n\, A$ and $w_L = w,\ w_M = r$:

$$p_n = p_n\, A + w_L B_L + w_M B_M = p_n\, A + w\, l + r\, p_n\, A\,.$$

This formula is a direct interpretation of $r$ as "wage rate" of capitalists' labour based on a labour value measure of the "power of agents" commanded, i. e., of capital circulant (!) $p_n\, A$ only that is bound to the production process.

**4.** [3, Formula (17)] is the very general decomposition

$$p = p\, A + w_L B_L + w_M B_M = p\, A + w\, l + \pi\,. \tag{7}$$

To explore the dynamics of labour value and price system it is a bad idea to vertically integrate costs as in [3, formula (18) ff.] since it adds prices of different accounting periods and hence is only valid if the reproduction scheme is not only technically stable but also the prices within the exchange relations are static. Such a theory is not suited to reason about the dynamics of, e. g., profit rates.

(7) indicates the correct way: Assuming the technical conditions to be constant, i. e., $A$, $B_L$, $B_M$ being time independent, we get $P = P\,U$ and $P = (I - U)^{-1}$, where $U$ is the matrix of technical *and* reproductional conditions at use value level. Since we get the formal inverse

$$(I - U)^{-1} = \begin{pmatrix} Q & Q\,C \\ B\,Q & I + B\,Q\,C \end{pmatrix} \quad \text{for} \quad U = \begin{pmatrix} A & C \\ B & 0 \end{pmatrix} \quad \text{with} \quad Q = (I - (A + C\,B))^{-1}\,,$$

we see that the eigenvalues not of $(I - A)$ but of $(I - (A + C\,B))$ are really driving the (theoretical) business. Note that the matrix product $C\,B$ in this term is "wrongly coupled", since $B\,C$ has a clear semantical meaning within the reproduction process of labour force.

The main problem is to get estimates for the entries of the matrices $B$ and $C$ that are not directly observable (as, by the way, also $A$).

**5.** To generalize [3, formula (37)] and also for the approach of Farjoun and Machover one has to develop a notion of ratio between two different generalized price theories. It is the matter of further work to understand how Fröhlich's interpretation of Farjoun and Machover and the more general criticism on the special way as Farjoun and Machover set up a *Probabilistic Political Economy* fit into such a generalized framework.

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