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Abstract: The modern New Keynesian literature discusses the stabilizing properties of Taylor-type interest rate rules mainly in the context of complex optimizing models. In this paper we present a simple alternative approach to provide a theoretical rationale for the adoption of the Taylor rule by central banks. We find that the Taylor rule can be derived as the optimal interest rate rule in a classical Barro-Gordon macroeconomic model. The successful practice of central bankers, at the core of the Great Moderation, and currently re-invoked to re-normalize monetary policy after the unprecedented quantitative-easing actions aimed to escape the Great Recession, can perfectly be explained by standard theory, without recourse to more complicated derivations.

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“The Fed decided to hold the interest rate very low during 2003-2005, thereby deviating from the rules-based policy that worked well during the Great Moderation. You do not need policy rules to see the change: With the inflation rate around 2%, the federal funds rate was only 1% in 2003, compared with 5.5% in 1997 when the inflation rate was also about 2%. The results were not good. In my view this policy change brought on a search for yield, excesses in the housing market, and, along with a regulatory process which broke rules for safety and soundness, was a key factor in the financial crisis and the Great Recession”.

John B. Taylor (2015, 1)

1 Introduction

The purpose of this paper is to show a simple and intuitive theoretical foundation for the desirability of interest rate feedback rules in monetary policy design. Many research developments in macroeconomic theory have focused on monetary policy rules. Since the inflationary experience of the 1970s, central banks and academics have tended to seek simple rules that promote transparency and credibility in monetary policy-making. The influential empirical works by Taylor (1993, 1999a) have shown that U.S. monetary policy in the period after 1987—when inflation was successfully stabilized—can be described in terms of an interest feedback rule in the form:

\[
\begin{align*}
  i_t &= r_L + \pi^* + \phi_\pi (\pi_t - \pi^*) + \phi_y (y_t - y_L),
\end{align*}
\]
where $i_t$ denotes the interest rate set by the Federal Reserve—represented by the Federal Funds Rate—$r_L$ is the long-run rate of real interest, $\pi_t - \pi^*$ is the “inflation gap”, i.e., the difference between the actual rate of inflation $\pi_t$ and a target rate $\pi^* > 0$, and $y_t - y_L$ is the output gap, i.e., the log-deviation of “real output” from “potential output”, measured empirically, in Taylor’s original formulation (1993), as a linear trend. The constant $r_L + \pi^*$ is the sum of the average real interest rate and the target rate of inflation. In Taylor (1993), the coefficients reflecting the “strength” of monetary policy, $\phi_\pi$ and $\phi_y$, are set to 1.5 and 0.5, respectively; the target for annual inflation is 2 percent; the average real interest rate is 2 percent. The value of the constant $r_L + \pi^*$, which can be interpreted as the steady-state nominal interest rate if inflation is on target and output at its potential level, is thus 4 percent.

The empirical literature provides formal econometric support for the view that Taylor-type reaction functions effectively mimic the monetary policies followed by several central banks—in particular over the 1980s and the 1990s (see, e.g., Clarida, Galí and Gertler 1998, 2000; Judd and Rudebusch 1998; Gerlach and Schnabel 2000; Orphanides 2003).

Nevertheless, Meltzer (2011) and Taylor (2012) recently point out a remarkable distinction between a “rules-based era”—from 1985 to 2003—and an “ad hoc era”—from 2003 onwards. During the rules-based era—a period featuring the Great Moderation—the Federal Reserve’s monetary policy is well described by the simple Taylor rule (1). By contrast, during the ad hoc era—a period characterized by the occurrence of the Great Recession—the Federal Reserve’s policy displays a significant and protracted deviation from the Taylor
rule. As shown by Poole (2007) and Taylor (2007, 2015), in the period from 2002 to 2006
the Federal Funds Rate stayed 2-3 percentage points below the time path prescribed by
the Taylor rule for any period since 1980s. Ahrend, Cournerde and Price (2008), Ahrend
(2010) and Kahn (2010) reinforce the above evidence by showing the existence of signifi-
cant negative gaps from the Taylor rule over the ad hoc era even for OECD countries as a
group.

Leamer (2007) and Taylor (2010, 2011, 2015) are led to contend that the foregoing
“Global Great Deviation” from rules-based policy making—featuring an excessively accom-
modating stance of monetary policy—“was a key factor in the financial crisis and the Great
Recession” (Taylor 2015, 1)—for it generated boom-bust dynamics in asset prices. The low
interest rates—and the related large amount of liquidity they required to be implemented—
helped determine initially a huge increase in housing demand. Low money market rates,
in particular, stimulated housing finance, featured by variable-rate mortgages. The rise in
housing demand brought about jumps in house price inflation, which, in turn, accelerated
the demand for housing along an upward spiral. With housing prices increasing sharply,
rates on subprime mortgages decreased, thus exacerbating the demand for credit and ex-
cessive risk taking. As the short-term nominal interest rates came back to normal levels,
however, the demand for housing in conjunction with housing price inflation sharply fell,
thereby causing a financial turmoil in the subprime market and the mortgage-backed secur-
rities. The risk in the balance sheets of financial institutions was at the core of the financial
crisis. The monetary policy response to the crisis was characterized by a rapid reduction in
the Federal Funds Rate, excessively sharp compared to the Taylor-rule-framework (Taylor 2009). Now, in the aftermath of unprecedented “quantitative-easing” purely discretionary actions at zero policy rates—expanding reserve balances through large-scale purchases of securities in the attempt to offset the Great Recession—monetary policies of the Taylor-type are strongly re-advocated to “re-normalize monetary policy” (Taylor 2014, 86).

The Taylor rule has historically been derived from empirical observation of the central banks’ successful behavior in triggering and guaranteeing macroeconomic stability, thereby becoming the object of several theoretical investigations. Studies of the relation between Taylor-type rules and aggregate stability typically work from the assumption that the monetary authorities’ decision to follow such rules is exogenous—as suggested by the empirical evidence prevailing over the rules-based era (see, for instance, Christiano and Gust 1999; McCallum 1999; most of the contributions in Taylor 1999b; Carlstrom and Fuerst 2000). The best-known finding from this line of enquiry is that effective economic stabilization and the determinacy of equilibria depend on the so-called “Taylor principle”, according to which the nominal interest rate response to inflation should be more than one-to-one (see Taylor 1999a; Woodford 2001, 2003; Bullard and Mitra 2002; McCallum 2003; Galí 2008).\footnote{For a theoretical treatment of the possible negative consequences of over-aggressive interest rate rules \textit{à la} Taylor, see Benhabib, Schmitt-Grohé and Uribe (2001).} Providing this constraint is satisfied, the Taylor rule is “robust”, in the sense that it performs well with different model specifications (see, e.g., Levin, Wieland and Williams 2003).
The stabilizing properties of Taylor-type rules are often analyzed in the context of dynamic stochastic New Keynesian models, combining optimizing forward-looking behavior, imperfect competition, and nominal rigidities. Svensson (2003) has pointed out, however, that this class of model produces optimal interest rate rules that are “too complex” to be of practical relevance to policy makers. Woodford (2001) has also stressed the fact that, within the class of dynamic stochastic general equilibrium models, “a simple rule [of the form (1)] is unlikely to correspond to fully optimal policy”. Arguably, this may be why the literature assumes the Taylor rule to be exogenously determined.

The issue of theoretical foundations of optimal interest rate rules in the context of standard macroeconomic textbook models, nevertheless, remains largely unexplored. The present paper attempts to offer a simple analytical rationale for the Taylor rule. Specifically, we show that the Taylor rule (1) can be obtained as result of a simple optimization problem within a Barro-Gordon (1983) framework. The historically stabilizing practice of central bankers can thus be explained relying upon a standard and parsimonious analytical path, without recourse to more complicated derivations. We describe the setup and derive the optimality properties of the Taylor rule in Section 2. We discuss further results in Section 3. We provide summary and concluding comments in Section 4.
2 A Simple Foundation of the Taylor Rule

Consider a textbook model in which a Phillips curve describes the short-run trade-off between output and inflation, an IS curve relates output to the policy decisions of the central bank—in our case, the fixing of the short-term nominal interest rate—and a loss function describes the central bank’s preferences. If the central bank’s policy instrument is the nominal interest rate, the money supply is determined endogenously via the interest rate target and it is not necessary to specify an LM curve. On the supply-side, (the log of) real output, $y_t$, is given by:

$$y_t = \kappa + \lambda (p_t - w_t) - \varepsilon_t,$$

where $\kappa, \lambda$ are positive parameters, $p_t$ is the (log of) price level, $w_t$ represents (the log of) nominal wages and $\varepsilon_t$ is a white noise random variable that can be interpreted as a cost-push shock. This equation can be interpreted as a specification of the optimal condition for firms. One possible derivation is the following. Assume imperfect competition in the market for goods and a production function which is log-linear with respect to labor input. In this setting, real wages are equal to the marginal product of labor, net of an exogenous gross markup. Equation (2) is a logarithmic version of this relation. At date $t - 1$, workers and firms agree to set nominal wages $w_t$ for period $t$ such that, on average, real wages are maintained at level $\omega$:

$$w_t - E_{t-1} (p_t) = \omega,$$
where $E_{t-1}(\cdot)$ is the conditional expectation based on the information set available at time $t - 1$. This information set does not include the realization of $\varepsilon_t$ at date $t$; wages are not indexed to the value of $\varepsilon_t$. Substituting (3) into (2), we obtain the traditional Phillips curve:

$$y_t = y_L + \lambda [\pi_t - E_{t-1}(\pi_t)] - \varepsilon_t$$

(4)

where $\pi_t = p_t - p_{t-1}$ is the inflation rate and $y_L = \kappa - \lambda \omega$ is the long-run level of output.

On the demand-side, the IS relation is specified as

$$y_t = a - \sigma r_t,$$

(5)

where $a > 0$ represents the exogenous component of aggregate expenditure—which, for simplicity, we treat as a constant—and $\sigma > 0$ measures the semi-elasticity of output with respect to the real interest rate, $r_t$. The link between the real and the nominal rates of interest is provided by the Fisher equation:

$$i_t = r_t + E_t(\pi_{t+1}).$$

(6)

Within the period $t$, wages have no way of adjusting to a supply shock; the monetary authorities, on the other hand, have policy instruments which allow them to respond to the disturbance. In this way, it is possible for monetary policy to produce real effects.

Under a discretionary monetary regime, the goal of stabilization policy is to minimize
the following loss function:

$$L_t = \pi_t^2 + \phi (y_t - y^*)^2,$$  \hspace{1cm} (7)

where $\phi \in [0, \infty)$ is the relative weight, with the weight on inflation normalized to unity, policy-makers attach to stabilizing output—as opposed to inflation stabilization. As in Barro and Gordon (1983), we assume that the monetary authorities target a level of output, $y^*$, strictly larger than $y_L$. Such targeting could reflect, for instance, the presence of imperfect competition leading to equilibrium output below the Pareto-efficient level.

It is worth pointing out that there exists a remarkable difference between the concepts of “optimal” monetary policy in the present macroeconomic setup and “optimal” monetary policy in the New Keynesian Dynamic Stochastic General Equilibrium microfounded optimization framework, whose main advantage is the derived link between private agents’ utility, and hence welfare, and the objective function of the central banker, which acts as social planner. The validity of our analysis, therefore, sticks to the context of the non-microfounded optimization problem under discretion in the classical macroeconomic model of the Barro and Gordon-type. As Walsh (2010) notes, however, one can show that under certain conditions, approximations to the utility of the representative agent in New Keynesian models result in an objective function of the type expressed in (7).

The optimization problem can be divided into two steps: First of all, the central bank chooses $\{y_t, \pi_t\}$ to minimize (7), given (4); in the second step, it uses the IS relation, to determine the optimal interest rate rule, conditional on the values of $y_t$ and $\pi_t$. 
Minimization of (7) leads to the following optimality condition:

$$y_t = y^* - \frac{1}{\lambda_2 \phi} \pi_t.$$  \hfill (8)

Condition (8) states that since an increase in inflation requires a contraction in output, the optimal policy for the central bank is to “lean against the wind”. Substituting the Phillips curve (4) into (8), we obtain the reaction function for the central bank:

$$\pi_t = \frac{\lambda_1 \phi}{1 + \lambda_2 \phi} E_{t-1} (\pi_t) + \frac{\lambda_1 \phi}{1 + \lambda_2 \phi} (y^* - y_L) + \frac{\lambda_1 \phi}{1 + \lambda_2 \phi} \varepsilon_t.$$  \hfill (9)

Applying such a policy involves a systematic attempt by the monetary authorities to produce “surprise” inflation, in order to drive output above its steady state level \(y_L\). The only case in which the inflation rate \(\pi_t = 0\) will be optimal \textit{ex post} is the limiting case of a strict inflation targeting (\(\phi = 0\)).

Workers and firms understand the strategy in (9), forming their expectations—and setting wages—accordingly. Thus, in a rational-expectations equilibrium:

$$E_{t-1} (\pi_t) = \lambda_1 \phi (y^* - y_L).$$  \hfill (10)

Combining (9) and (10), we can derive equilibrium levels for inflation and output, namely:

$$\pi_t = \lambda_1 \phi (y^* - y_L) + \frac{\lambda_1 \phi}{1 + \lambda_2 \phi} \varepsilon_t.$$  \hfill (11)
\[ yt = y_L - \frac{1}{1 + \lambda^2 \varphi} \varepsilon_t. \]  \hspace{1cm} (12)

Except in the case when \( \varphi = 0 \), this implies a systematic inflation bias, proportional to the deadweight loss of output \( (y^* - y_L) \). A binding commitment to a zero inflation rate would eliminate this bias, but would prevent the policy maker from responding to unpredictable supply shocks. The results (11) and (12) tell us that, in the presence of a cost-push shock, there is a short-run trade-off between inflation and the volatility of output. This we can see by examining the standard deviations for inflation and output, \( \sigma_\pi = \frac{\lambda \varphi}{1 + \lambda^2 \varphi} \sigma_\varepsilon \) and \( \sigma_y = \frac{1}{1 + \lambda^2 \varphi} \sigma_\varepsilon \). The optimal monetary policy reduces the standard deviation of output, as \( \varphi \) increases, at the cost of increased variability in inflation. In the polar cases, for \( \varphi \to 0 \), \( \sigma_\pi \to 0 \) and \( \sigma_y \to \sigma_\varepsilon \); for \( \varphi \to \infty \), \( \sigma_y \to 0 \) and \( \sigma_\pi \to \frac{1}{\lambda} \sigma_\varepsilon \).

In the second step of the maximizing problem, we first substitute the Fisher equation (6) and the optimal condition (8) into (5), yielding

\[ i_t = \mathbb{E}_t (\pi_{t+1}) + \frac{1}{\sigma \lambda \varphi} \pi_t + \frac{1}{\sigma} (a - y^*). \]  \hspace{1cm} (13)

Note that (11) and (12) imply that \( \mathbb{E}_t (\pi_{t+1}) = \pi_t + \lambda \varphi (y_t - y_L) \). Using this fact into (13), one obtains

\[ i_t = \left( 1 + \frac{1}{\lambda \varphi \sigma} \right) \pi_t + \lambda \varphi (y - y_L) + \frac{1}{\sigma} (a - y^*). \]  \hspace{1cm} (14)
The last term in the right-hand side of (14) can be written as

\[
\frac{1}{\sigma} (a - y^*) = \frac{1}{\sigma} (a - y_L) - \frac{1}{\sigma} (y^* - y_L) = r_L - \frac{1}{\lambda \varphi \sigma} \pi^*,
\tag{15}
\]

where \(r_L\) is the long-run real interest rate and \(\pi^* = \lambda \varphi (y^* - y_L)\) is the rate of inflation announced by the central bank. The latter rate can be interpreted as the time-consistent target rate of inflation. Substituting (15) into (14), we obtain the optimal feedback policy for the rate rule:

\[
i_t = r_L + \pi^* + \phi^*_\pi (\pi_t - \pi^*) + \phi^*_y (y_t - y_L),
\tag{16}
\]

where

\[
\phi^*_\pi = 1 + \frac{1}{\lambda \varphi \sigma} > 1,
\]

\[
\phi^*_y = \lambda \varphi > 0.
\]

Hence, the optimal policy takes the form of the Taylor rule (1): The nominal interest rate is a function of both the inflation gap and the output gap, and does not depend on any other variables. Equation (16) is an “implicit” and “direct” instrument rule in Woodford (2003, 544-547)’s sense. It shows that, so long as current inflation is in line with the bank’s long term objective and output does not deviate from its long-term potential level, the optimal nominal rate of interest is equal, according to the Taylor rule, to the long-term level, that is \(r_L + \pi^*\). It should also be clear that the optimal policy requires the application of the “Taylor principle”, \(\phi^*_\pi > 1\). This constraint ensures that whenever policy-makers
observe symptoms of inflationary pressure, they will tighten policy sufficiently to ensure an increase in the real rate of interest. This is clear from equation (8), which we can rewrite in the form:

\[ y_t - y_L = -\frac{1}{\lambda \varphi} (\pi_t - \pi^*) \]  

(17)

In synthesis, when inflation is above target, the optimal policy is to raise real interest rates and thereby to decrease demand.

3 Discussion

The foregoing section delivers the key point of the paper in a direct way. Nevertheless, from a more global policy perspective, it is worth pointing out that the optimal interest rate rule we have obtained in the context of the Barro-Gordon theoretical setup—fully in line with the Taylor-rule-framework—appears to be consistent only with the pre-crisis interest-rate actions undertaken by the Federal Reserve, which are notably based on a “dual” mandate. Alternative central banks’ policy frameworks—such as the “medium-term” strict inflation target, allowing for output and inflation volatilities around the objective in the short run, and characterizing, for example, ECB and Bank of England’s mandates—seem to be not coherent with the model.

Next we show, on the other hand, that it is possible to reconcile the present approach to interest rate policy design with a more general policy system of “constrained discretion”, which permits sufficiently bounded inflation and output fluctuations around the inflation
target. To see this, substitute (12) into (16) to obtain

\[ i_t = r_L + \pi^* + \phi^*_n (\pi_t - \pi^*) - \phi^*_v \varepsilon_t, \]

(18)

where

\[ \phi^*_v = \frac{\phi^*_v}{1 + \lambda^2 \varphi} \]

\[ = \frac{\lambda \varphi}{1 + \lambda^2 \varphi} > 0. \]

It emerges that the interest rate rule expressed by (18) is consistent with an inflation rate targeting policy framework, whereby interest-rate and thus output movements along which inflation returns to the target following the occurrence of a shock are allowed.

4 Summary and Concluding Remarks

Much of the traditional literature on monetary economics focuses on simple reaction rules, tying policy decisions to a small set of economic variables or indicators. One well-known rule, which has attracted growing attention in modern monetary theory and policy, is the Taylor rule, in which the nominal rate of interest fixed by central banks depends on inflation and the output gap. Such a policy rule provides a useful framework for describing and evaluating monetary policies. For example, the so-called “Great Deviation” from the Taylor rule, generated by the Federal Reserve’s accommodating monetary policy over the
period 2002-2006, is argued to be responsible of the subsequent Great Recession, mainly
because it triggered boom-bust patterns in asset prices.

Getting back to the rules-based monetary policy approach of the Taylor-type is cur-
rently often advocated. Monetary theory derives the stabilizing properties of Taylor rules
almost exclusively within complex optimizing models of the New Keynesian-type. In these
frameworks, however, rules as simple as the standard Taylor rule are rarely fully optimal.
In this paper we have shown how a commonly used macroeconomic textbook model appears
to be capable of yielding the Taylor rule as a result of a simple optimal monetary policy
problem. The practice of central bankers—inducing the Great Moderation and recently
re-invoked to re-normalize monetary policy after the unconventional quantitative-easing
programmes—can be well-founded in the context of a stripped-down classical macroeco-
nomic environment, of the kind outlined by Barro and Gordon (1983), without recourse to
more complicated derivations.

The framework of analysis we have employed conveys the main argument of the paper
in a direct and transparent way. It deliberately abstracts from many central features
of modern macroeconomic theories and macroeconomic policies, such as the role of non-
 rational expectations, information and learning, several heterogeneities and imperfections,
the role of housing and banking, institutional arrangements, the interactions with fiscal
policy and, therefore, aspects of coordinated actions with monetary policy under various
mandates. Our setup could be used as a fruitful benchmark for more complex analysis
along these lines.
In conclusion, from our analysis it appears that useful insights on the design of monetary policy rules of practical relevance for policy makers can still be gained working with simple and highly intuitive macroeconomic models.

References


