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Abstract

Time diversification which is the idea of there being less riskiness over longer investment horizons is examined in this paper. Different from previous studies, this paper contributes to the literature by using the Aumann and Serrano index as a risk measure to examine whether there is any time diversification in stock investment by using the daily return of the S&P 500, the S&P 400, and the NASDAQ with both short and long holding periods and by using the block bootstrapping technique in the simulation. From returns of short (long) holding periods, we conclude that, in general, the riskiness of the shorter (longer) period is statistically greater than that of the longer (shorter) period. Our findings reject the hypothesis of no time diversification effect and reject the geometric Brownie motion process for the returns of different holding periods. The results could be due to short- and medium-term momentum and long-term contrarian. Our findings are useful to academics, investors, and policy makers in their decision making related to time diversification.

Keywords: Time diversification, Economic index of riskiness, Investment horizon

JEL Classification: G11

1. Introduction

Modifying the idea of asset diversification, which diversifies an investment in many risky assets, Thorley (1995) introduced the concept of time diversification on how to spread risky investments over many time periods to reduce risk. If the investment returns are uncorrelated across time periods, it could be argued that the ups and downs of the market tend to even out. Thus, time diversification can create a benefit in risk reduction. Based on this notion, risky assets, such as stocks, would become less risky over longer horizons, and long-term investors could allocate a greater proportion of wealth into stocks. However, time diversification is not as well supported as asset diversification in literature because since Samuelson (1969) first strongly argued against the notion of time diversification, there has been a great deal of debate and research on this issue.

A key element for examining the time diversification effect is the risk measurement. Time diversification could depend on the specific risk measures used.

According to Fabozzi, Focardi, and Kolm (2006), for instance, the normalized risk measure is defined as the return risk divided by the mean return, and the time diversification index (TDI) is defined as the ratio of two normalized risk measures with respect to two different time horizons. If we assume each period return is independently and identically distributed and use variance as a risk measure, then there is no time diversification effect because both mean and variance linearly increase over time. Instead, if we use standard deviation, which increases with the square root of time, then TDI will indicate the existence of time diversification.

Variance, standard deviation, value-at-risk, and several other indicators have been employed for measuring the risk. Although those risk measures are commonly used, Aumann and Serrano (2008) pointed out that they generally do not satisfy the monotonicity with respect to stochastic dominance (SD). The monotonicity is a key property for a good risk measure. If portfolio return A dominates portfolio return B in terms of first-order or second-order SD, and we know that all risk-averse investors prefer A to B, then a risk measure with the monotonicity will indicate that portfolio return A is less risky than portfolio return B. Therefore, a risk measure without the monotonicity can produce riskiness rankings, which contradict economic intuition.

The economic index of riskiness developed by Aumann and Serrano (AS, 2008) is the first risk measure satisfying the monotonicity. Since then, this new risk measure has been applied to various financial issues by replacing traditional risk measures. For instance, Homm and Pigorsch (2012) constructed a new performance measure, which generalizes the Sharpe ratio by replacing standard deviation with the AS index. Furthermore, in studying the hedge with futures, Chen, Ho, and Tzeng (2014) proposed the optimal hedge ratio that minimizes the AS index.

In this paper, we contribute to the literature by using the Aumann and Serrano (AS) index as a risk measure to examine whether there is any time diversification in stock investment and by using the daily return of the S&P 500, the S&P 400, and the NASDAQ with returns of short and long holding periods and by using the block bootstrapping technique in the simulation. Returns of short holding period includes one day, one week, two weeks, and one month while returns of long holding period includes one month, three months, six months, and one year. In addition to the monotonicity, there is another important feature of using the AS index. Under the normality assumption, the AS index equals the variance divided by twice the mean. Thus, if the stock prices are assumed to follow a geometric Brownie motion (GBM) process, there will be no time diversification effect because both the mean return and variance increase with time linearly. However, in stock markets, there is evidence indicating momentum, overreaction, and mean reversion of stock returns, contrary to the GBM. In fact, both Samuelson (1963) and Bodie (1995) argued that time diversification is driven by non-independence or mean reversion. Thus, it is useful to reinvestigate time diversification with the AS index under those phenomena.

Another feature of this study is that we focus on the comparison of various short-term investment horizons and their riskiness. Most previous studies on time diversification often used monthly return data to generate holding period returns over one year. However, the investment horizons under investigation in our paper include holding periods of not more than one year. We include one-day, one-week, two-week, one-month, three-month, six-month, and one-year holding period. Daily data is used to generate the different holding period returns. Thus, we use much more historical observations that other studies using monthly data.

From returns of short holding periods, we conclude that, in general, the riskiness of the shorter period is statistically greater than that of a longer period. The findings reject the hypothesis of no time diversification effect and conclude that, in general, the economic index of riskiness for the return in the shorter holding period is greater than that of the longer holding period and the investment of the shorter period is riskier than that of the longer period. On the other hand, from the returns of long holding periods, we reject the hypothesis of no time diversification effect and conclude, in general, the economic index of riskiness for the return in the shorter holding period is greater than that of the longer holding period and the investment of the shorter period is riskier than that of the longer period. Our findings reject the market efficiency hypothesis and reject the geometric Brownie motion process for the returns of different holding periods. Our findings could be due to short- and medium-term momentum and long-term contrarian. Our findings are useful to academics, investors, and policy makers in their decision making related to time diversification.

The remainder of this paper is organized as follows. The AS index and related riskiness indices are introduced in the next section. Then, Section 3 presents data and methodology. Results are discussed in Section 4. The final section is the conclusion.

2. Economic Index of Riskiness

The Aumann and Serrano (AS) index is an economic index of riskiness proposed by Aumann and Serrano (2008) for any risky asset is the reciprocal of the positive risk aversion parameter of an individual with constant absolute risk aversion (CARA), who is indifferent about taking or not taking the risky asset. Under this model setting, the AS index is defined to satisfy the following equation:

$$\mathrm{E}U(W + S_t - S_0) = U(W), \tag{1}$$

where U is an utility function, W is the initial wealth, and S_t is the price of the risky asset at time *t*. Here, the risky asset is assumed to be a stock or a stock portfolio. Assuming no cash dividend, $S_t - S_0$ is the absolute return of holding the stock for the time interval. Aumann and Serrano (2008) constructed the index of riskiness by using an exponential utility function. Thus, the AS index of the risky asset, $\mathrm{AS}(S_t)$ is defined implicitly as follows:

$$\mathrm{E}e^{-(S_t - S_0)/\mathrm{AS}(S_t)} = 1 \ . \tag{2}$$

They proved that the $\mathrm{AS}(S_t)$ is a unique positive number, and any index satisfying the two axioms will be a positive multiple of $\mathrm{AS}(S_t)$ if some of the absolute returns are negative, and the mean of the absolute return is positive. Thus, the AS index does not actually require the CARA utility function. The exponential utility function is merely an easy method to derive the riskiness measurement.

Under the model setup as discussed in the above, the investment risk is an additive risk. However, if the individual places the initial wealth in the risky asset, then the risk becomes a multiplicative risk. For a multiplicative risk, similar to Aumann and Serrano's (2008) approach, Schreiber (2014) defined an economic index of relative riskiness for a risky asset as the reciprocal of the positive risk aversion parameter of an individual with constant relative risk aversion (CRRA) who is indifferent about taking or not taking the risky asset. Under this setup, the index of relative riskiness satisfies the following equation:

$$\mathrm{E}U(W(S_t/S_0)) = U(W). \tag{3}$$

Schreiber (2014) adopted a power utility and derived the index of relative riskiness which, in fact, is equal to the AS index applied to the log return instead of the absolute return. That is, the index of relative riskiness, $\mathrm{RS}(S_t)$ is defined implicitly as follows:

$$\mathrm{E}e^{-(\ln S_t - \ln S_0)/\mathrm{RS}(S_t)} = 1. \tag{4}$$

Under this modeling setting, to measure the relative riskiness of a risky asset, the log return formula should be displayed. The relative riskiness index is used in this paper.

Normality is often a useful case of study. If the log return is normally distributed, then the index of riskiness will be equal to the variance of the return over twice of the mean return. Thus, if the stock prices follow a GBM process with a drift rate μ and a volatility rate σ, then, for a period T, the return will be log normally distributed with mean $\left(\mu - \frac{\sigma^2}{2}\right)T$, variance $\sigma^2 T$, and the riskiness index will be equal to $\sigma^2/\left(\mu - \frac{\sigma^2}{2}\right)$.

Under the GBM model, the riskiness is independent of time, and thus, there is no time diversification effect.

The AS index is further extended by Schnytzer and Westreich (2013) who provide a measure of riskiness for return distributions with either positive or negative expectations. They proposed the index of riskiness $\mathrm{Q} = e^{-\alpha_g}$, where α_g satisfies

$$\mathrm{E}e^{-\alpha_g(\ln S_t - \ln S_0)} = 1. \qquad (5)$$

The index of riskiness satisfies a monotonic increasing function of the AS index when restricted to the return with a positive expectation. The index proposed by Schnytzer and Westreich (2013) is used in this paper to measure risk because there is a possibility of negative mean return in this study.

3. Data and methodology

In section 2, we have discussed that under the GBM model, the riskiness is independent of time, and thus, there is no time diversification effect. In addition, if we assume each period return is independently and identically distributed and use variance as a risk measure, then there is no time diversification effect because both mean and variance linearly increase over time. Instead, if we use standard deviation, which increases with the square root of time, then TDI will indicate the existence of time diversification. Thus, in this paper, we are interested in testing the following hypotheses:

H_0 : there is no time diversification effect in the distributions of returns for different holding periods in terms of the economic index of riskiness, (6)

versus

H_1 : there is time diversification effect in the distributions of returns for different holding periods in terms of the economic index of riskiness. (7)

Some academics and practitioners could be interested in testing the following:

H_{1a} : The economic index of riskiness for the return in the shorter holding period is greater than that of the longer holding period, (8)

and

H_{1b} : The economic index of riskiness for the return in the longer holding period is greater than that of the shorter holding period. (9)

To test whether any of the above hypotheses hold true, we use the daily total returns for indices of S&P 500, S&P 400, and NASDAQ obtained from Datastream in our testing analysis. At the end of July 2016, the available observations are 7455, 5449, and 3352 for S&P 500 (starting on January 4, 1988), S&P 400 (starting on September 12, 1995), and NASDAQ (starting on September 25, 2003), respectively. Daily returns, weekly returns, two-week returns, monthly returns, three-month returns, six-month returns, and one-year returns are calculated by using the formula $\ln(R_{i+t}) - \ln(R_i)$ where R_i is the return at the observed trading day *i* and *t*=1, 5, 10, 21, 63, 126, and 252, respectively.

To generate the distributions of returns with different holding periods, two block bootstrapping methods are used. First, 1000 blocks of each holding period are randomly drawn. Thus, it is a random starting date for each holding period. Second, 1000 dates are randomly picked as the investment starting date i, and the corresponding end date for each holding period is determined thereafter. Thus, based on the returns of the same starting date but different end dates, the returns can be calculated for different holding periods. For both methods, 1000 samples are generated for each holding period return. Later, the two methods are referred to as the same seed and the random seed. The same seed is used to indicate the different holding period returns have the same starting date, while the random seed indicates each starting date is randomly selected.

After generating the return distributions, the riskiness index proposed by Schnytzer and Westreich (2013) is employed for measuring the riskiness of the returns for different holding periods. The seven different holding periods are divided into two groups. The first group contains 1, 5, 10, and 21 trading days. The second group includes 21, 63, 126, and 252 trading days. In each group, the riskiness is ranked between returns for any two holding periods. The whole process, including generating return distributions, risk measuring and pair ranking, is repeated 1000 times in our simulation. In addition, the proportion of the riskiness of the shorter period return over that of the longer period return is computed for each pair of returns. Thereafter, hypothesis testing for the null hypothesis of no time diversification is conducted. Under the null hypothesis, the proportion is equal to 0.5, which means the riskiness of the short-term investment is the same as the long-term. The alternative hypothesis is the proportion greater than 0.5, which indicates the long-term investment risk is less than the short-term. This proportion test can be implemented by using the standardized normality z statistic, with critical values of 2.575 and 1.96 for 1% and 5% significant levels, respectively.[1]

4. Empirical Analysis

The basic statistics, including mean, standard deviation, skewness, and kurtosis, of the daily, weekly and monthly returns for S&P 500, S&P 400, and NASDAQ in the same period are exhibited in Tables 1, 2, and 3, respectively. There are two weekly returns, one is based on Monday's total returns and another one is based on Wednesday's total returns of the indices. There are also two monthly returns, one is based on the first date of the month while another one is based on the fifteen day of the month. From the tables, we find that as the time period increases, the mean return and the volatility also increase. The volatilities of S&P 400 and NASDAQ are larger than those of S&P 500 for all different holding periods. The mean return of S&P 400 is generally higher than that of S&P 500, but not higher than that of NASDAQ. Furthermore, we find that the values of the skewness are negative for the returns from different holding periods and from all the indices, implying that the returns from different holding periods and from all the indices are skew to the left. This means that there is a chance investors may lose relative big money than gain big profit if they invest in any of the indices studied in our paper by holding daily weekly, or monthly.

Tables 1-3 also report the values of the AS index for the returns from three different holding periods. According to nature of the riskiness indices, the riskiness of the daily return is the highest, followed by the weekly return, and then the monthly return. This

[1] The z statistic is defined as $(\bar{x} - 0.5)/(\frac{s_x}{\sqrt{n}})$ where $\bar{x}$ is the proportion of the riskiness of the shorter period return over that of the longer period return, $s_x = \frac{n\bar{x}(1-\bar{x})}{n-1}$, and n the sample size 1000. By the large sample theory, The z statistic follows the standard normal distribution.

phenomenon holds true for S&P 500. However, in the cases of S&P 400 and NASDAQ, the riskiness of the shorter period return is still (but not always) higher than those of the returns in longer periods. For example, the Weekly (Monday) returns of both S&P 400 and NASDAQ have the highest riskiness. Also, the riskiness of Monday weekly return is relatively higher than that of Wednesday. This could be due to the weekend effort. Furthermore, for each index, the riskiness of the two weekly returns is not as close as the two monthly returns.

We turn to compare the ranking of the riskiness between indices of any pair of daily, weekly, two-week, and monthly returns and exhibit the results in Table 4.[2] For each pair, we list the proportion of the riskiness of return for the shorter period over that for the longer period in 1000 simulations across the three indices. In the table, we use P(X >Y), where X is shorter than Y and X and Y can be "D", "W", "2W", and "M", representing one day, one week, two weeks, and one month, respectively. The first three rows of Table 4 demonstrate that the proportions of the daily return's riskiness over those for the three longer periods. Using 5% significant level, we confirm that X is significantly greater than Y when P(X >Y) is higher than 0.526. Similarly, using 5% significant level, we confirm that Y is significantly greater than X when P(X >Y) is less than 1 - 0.526 = 0.474. Thus, from Table 4, we conclude that the riskiness of the daily return is statistically greater than those of the weekly, two-week, and monthly returns when we use both same and random seeds. The riskiness of the weekly returns is mostly (except one that is bigger but not significantly bigger) significantly higher than those of the two-week returns.

However, between the weekly and monthly returns, the proportions are all below 0.5, and this indicates the riskiness of the weekly return is not significantly higher than that of the monthly return. Interestingly, we find that nearly all (except two that are smaller but not significantly smaller) are significantly smaller than 0.5, implying that the riskiness of the nearly all monthly returns is significantly higher than those of the weekly returns.

Nonetheless, between the two-week and monthly returns, from Table 4 we find that the riskiness of the two-week return is statistically greater than those of the monthly returns in 3 simulations (0.572, 0.550, and 0.532), it is greater but not statistically greater than those of the monthly returns in 2 simulations (0.510 and 0.507) and it is smaller but not statistically smaller than that of the monthly returns in 1 simulation (0.493). Thus, we can conclude that the riskiness of the two-week return is marginally

[2] Readers may refer to Lu, et al. (2018) for Table 4.

statistically greater than those of the monthly returns in general.

The findings displayed in Table 4 basically conclude that in general, the riskiness of the shorter period is statistically greater than that of a longer period, including the daily return is statistically greater than those of the weekly, two-week, and monthly returns and the riskiness of the two-week return is marginally statistically greater than those of the monthly returns in general. The findings reject H_0 in (6) and accept H_1 in (7) and conclude that there is time diversification effect in the distributions of returns for different holding periods in terms of the economic index of riskiness. Actually, the findings accept H_{1a} in (8) and suggest that, in general, the economic index of riskiness for the return in the shorter holding period is greater than that of the longer holding period and the investment of the shorter period is riskier than that of the longer period.

Could this phenomenon hold for even longer periods like one month, three months, six month, and one year? To test whether we can accept t H_{1a} in (8) for a longer holding period, we turn to compare the performance of longer holding periods of one month, three months, six month, and one year displayed in Table 5[3].

Our finding in Table 5 is very interesting. From Table 5, we find that different from the findings displayed in Table 4, most numbers are far below from 0.5, implying that the riskiness of the longer period is strongly greater than that of the shorter period significantly. To be precise, for the S&P 500, all proportions are much less than the critical value, implying that the riskiness of the returns from three months, six month, and one year is statistically greater than that of one month; the riskiness of the returns from both six month and one year is statistically greater than that of three months; and the riskiness of the returns from one year is statistically greater than that of six months for the S&P 500. The findings for the performance of the S&P 500 in the longer period reject H_0 in (6) and accept H_1 in (7) and conclude that there is time diversification effect in the distributions of returns for different holding periods in terms of the economic index of riskiness. However, it rejects H_{1a} in (8) but accept H_{1b} in (9) and conclude that the economic index of riskiness for the return in the longer holding period is significantly greater than that of the shorter holding period and the investment of longer period is riskier than that of the shorter period. Does this conclusion contradicts the conclusion drawn from the findings displayed in Table 4? We note that it is not and we will discuss the issue in the conclusion and inference section.

For another two indices S&P 400 and NASDAQ, except for the pairs between the

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[3] Readers may refer to Lu, et al. (2018) for Table 5.

three-month and the one year, and between the six-month and the one year, we have the same results as the S&P 500. Only the numbers between the three-month and the one year for both S&P 400 and NASDAQ, and the number between the six-month and the one year for the S&P 400 are greater than the critical value 0.526. The rest are all consistent with the findings from the S&P 500. Thus, we can still conclude from Table 5 that, in general, the findings for the performance of the S&P 500, S&P 400 and NASDAQ in the longer period reject H_0 in (6) and accept H_1 in (7) and conclude that there is time diversification effect in the distributions of returns for different holding periods in terms of the economic index of riskiness. However, it rejects H_{1a} in (8) but accept H_{1b} in (9) and conclude that the economic index of riskiness for the return in the longer holding period is significantly greater than that of the shorter holding period and the investment of longer period is riskier than that of the shorter period.

5. Concluding Remarks and Inference

In this paper, the economic index of riskiness is employed to examine whether there is time diversification effect, whether the economic index of riskiness for the return in the shorter holding period is greater than that of the longer holding period, and whether the economic index of riskiness for the return in the longer holding period is greater than that of the shorter holding period. The result could then be used to test market efficiency hypothesis and reject the geometric Brownie motion process for the returns of different holding periods.

To test the above hypotheses, we investigate two sets of holding periods. The first set is the holding period no more than one month, including one day, one week, two weeks, and one month. The second set is the holding period no less than one month, including one month, three months, six months, and one year. The return distributions of the different holding periods are generated by block bootstrapping from the daily total return index data of the S&P 500, the S&P 400, and the NASDAQ, and a proportion test is used for testing if the riskiness of longer period's return equals in proportion the riskiness of the shorter period's return.

From the first set of data with holding periods of not more than one month, including one day, one week, two weeks, and one month, we conclude that, in general, the riskiness of the shorter period is statistically greater than that of a longer period, including the daily return is statistically greater than those of the weekly, two-week, and monthly returns and the riskiness of the two-week return is marginally statistically greater than those of the monthly returns in general. The findings reject the hypothesis of no time diversification effect and conclude that there is time diversification effect in the distributions of returns for different holding periods in terms of the economic index of riskiness. The findings suggest that, in general, the economic index of riskiness for the return in the shorter holding period is greater than that of the longer holding period and the investment of the shorter period is riskier than that of the longer period.

From the second set of data with holding periods of not less than one month, including one month, three months, six months, and one year, we conclude that, in general, the riskiness of the shorter period is statistically greater than that of a longer period, including the daily return is statistically greater than those of the weekly, two-week, and monthly returns and the riskiness of the two-week return is marginally statistically greater than those of the monthly returns in general. The findings reject the hypothesis of no time diversification effect and conclude that there is time diversification effect in the distributions of returns for different holding periods in terms of the economic index of riskiness. Our findings and suggest that, in general, the economic index of riskiness for the return in the shorter holding period is greater than that of the longer holding period and the investment of the shorter period is riskier than that of the longer period. The results hold for the S&P 500, the S&P 400, and the NASDAQ.

Both sets of data reject the hypothesis of no time diversification effect, implying that market efficiency hypothesis is rejected and the assumption of geometric Brownie motion process for the returns of different holding periods is rejected. However, the first set of data suggests that, in general, the economic index of riskiness for the return in the shorter holding period is greater than that of the longer holding period while the second set of data make reverse conclusion. Is there any contradiction? We note that there is no contradiction. The first set of data concludes that the economic index of riskiness for the return in the shorter holding period of not more than one month, including one day, one week, two weeks, and one month, this could be because of the short-term momentum effect and short-term underreaction.

On the other hand, the second set of data concludes that the economic index of riskiness for the return in the longer holding period of not less than one month, including one month, three months, six months, and one year, this could be due to the medium-term momentum and long-term contrarian effects and long-term overreaction, see Lam, Liu, and Wong (2010, 2012), Fung, Lam, Siu, and Wong (2011), Fabozzi, Fung, Lam, and Wong (2013), and Guo, McAleer, Wong, and Zhu (2017) and the references therein for more information. Thus, there is no contradiction.

Extension of this paper includes using other risk measures, for example, mean-variance rule (Markowitz, 1952; Wong, 2007; Wong and Ma, 2008; Bai, Liu, and Wong, 2009a,b; Guo and Wong, 2016; Guo, Levy, Lu, and Wong, 2018), Sharpe ratio (Sharpe, 1966; Leung and Wong, 2008), mixed Sharpe ratio (Wong, Wright, Yam, and Yung, 2012), mean-variance ratio (Bai, Hui, Wong, and Zitikis, 2012; Bai, Phoon, Wang, and Wong, 2013), VaR and CVaR (Ma and Wong, 2010), Omega ratio (Guo, Jiang, and Wong, 2017, Guo, Levy, Lu, and Wong, 2018), Kappa ratio (Niu, Wong, and Xu, 2017), Farinelli and Tibiletti ratio (Farinelli and Tibiletti, 2008; Niu, Wong, and Zhu, 2017) to examine the time diversification effect. Academics could also modify the theory developed by Niu, Guo, McAleer, and Wong (2018) to construct the confidence interval for the economic index of riskiness for the returns of different holding periods. One could also include the background risk (Alghalith, Guo, Wong, and Zhu, 2016; Alghalith, Guo, Niu, and Wong, 2017; Guo, Wagener, Wong, and Zhu, 2018) in the study.

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