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Abstract

In basic economic growth models, energy is neglected as a production factor, and output is generated from capital and labor in a single-sector process, with most of growth attributed to an exogenous residual. However, alternative approaches argue for a multi-sector system in which the major contribution to growth comes from increased efficiency in the conversion of energy to more productive forms.

In this work we develop a two-sector model for the economy, featuring an extended energy sector, including all primary-to-final (energy industries) and final-to-useful (end-use devices) exergy conversion processes. Exergy is a thermodynamics concept, accounting for the potential of energy to perform work. Empirical application of the model for a single country (Portugal) requires decomposition and reclassification of national accounts and energy balances, to match empirical data with the model's variables.

Obtained estimates for the price of useful exergy (an intermediate product) facilitate the construction of gross output measures for more accurate growth accounting. Evidence suggests that declining useful exergy prices act as an engine of growth, as previously suggested in the literature. Additionally, useful exergy and capital inputs to non-energy related production act as complements while capital productivity in useful exergy generation declines slightly in the past decade.

Keywords

Two-sector model; extended energy sector; useful exergy; national accounts; energy balances;

Highlights

- A two-sector model, with an innovative extended energy sector, is developed;
- The extended energy sector covers primary-to-final-to-useful exergy conversions;
- National accounts and energy balances are reclassified to match model's variables;
- Declining useful exergy prices support positive feedback cycle as driver of growth;
- Evidence of useful exergy and capital as complementary inputs to non-energy sector;

JEL Codes

Q43; E01; C82; O41; O47; Q41

1. Introduction

Energy is essential to economic activity, and advances in energy consuming technologies, coupled with increasing energy consumption, have characterized industrialization and economic development over the past century. Despite this apparently close relationship, the standard theory of economic growth does not reflect energy availability or prices and fails to satisfactorily explain historical growth and productivity declines following energy crises.

The first and foremost goal of this work is to establish the basis for the development of a macroeconomic growth model that considers the real contribution of energy inputs to production processes and growth. The proposed model is built around several arguments regarding the role of energy in the economy – voiced mostly in the ecological economics literature – as well as recent empirical results supporting those same arguments. With the development of the model, we seek to provide new insights to persistent questions in the field of macroeconomics, such as the explanation for historical economic growth and recent productivity slowdowns verified in industrialized countries, as well as in regard to the relations between energy use, energy efficiency, energy prices, the traditional factors of production (physical capital and human labor), economic output, and growth. Overall, the proposed model aims to provide an improved characterization of economic and growth processes without disregarding an arguably essential input to the economic system: energy.

Section 1.1 begins by exposing the historical economic growth accounting method attached to most basic mainstream growth models (1.1.1) and subsequently highlighting some of the criticism imposed on these models concerning their inability to fully account for growth without an exogenous residual term (total factor productivity, or TFP) and the neglect of energy inputs, as opposed to alternative approaches that posit a central role for energy in economic understanding (1.1.2). Section 1.2 expands on the link between energy use – especially energy efficiency – and economic growth, invoking the thermodynamically defined useful exergy concept as the appropriate measure to account for productive energy uses. This subsection also reviews some of the more recently published evidence for useful exergy consumption as an explanatory factor of economic growth and states the case for the macroeconomic model developed in this work, based on the quantitative and qualitative evidence cited in regard to the relationship between productive energy and growth.

1.1. The main(stream) driver of economic growth: total factor productivity

Basic mainstream economic growth theory generally attributes long-run growth to three major factors: 1) accumulation of capital; 2) increases in labor inputs, i.e. number of workers and/or hours worked; 3) technical change. The contribution from these factors to growth can be measured through growth accounting exercises, breaking down what percentage of growth can be attributed to changes in each of the factors.

1.1.1. Growth accounting and the Solow residual

The Solow-Swan model – developed independently in two pathbreaking papers of the same year (Solow, 1956; Swan, 1956) – has shaped the field of macroeconomics and the way economic growth is approached. At the center of the model is a neoclassical aggregate production function (APF), linking a given level of economic output (Y) with a combination of inputs to production:

$$Y = F(A, K, L) \qquad (1)$$

Physical capital (K) and human labor (L) are generally considered the primary factors of production in modern economies[i]. The term A is a "catch all" factor representing technology, institutional factors, and other forces that account for how productively capital and labor are used. Standard assumptions on the form of the APF include homogeneity of degree one and constant returns to scale, which in turn facilitates the assumption of perfect competition i.e. factors of production are paid their marginal products[ii]. The growth accounting equation is obtained from differentiating the APF in Equation (1) with respect to time, denoting growth rates by $g_i = di/i$:

$$g_Y = (F_A A/Y) \cdot g_A + (F_K K/Y) \cdot g_K + (F_L L/Y) \cdot g_L, \qquad (2)$$

where F_i denotes the partial derivative with respect to factor i. From perfect competition, $F_K = r$ (rents – the marginal product of capital) and $F_L = w$ (wages – the marginal product of labor). Simplifying Equation (2), the share of total income allocated to capital can be denoted by $\alpha = rK/Y$, and the share allocated to labor by $1 - \alpha = wL/Y$.

$$g_Y = (F_A A/Y) \cdot g_A + \alpha \cdot g_K + (1 - \alpha) \cdot g_L \qquad (3)$$

The growth accounting methodology as expressed in Equation (3) – introduced by Robert Solow (Solow, 1957) – decomposes the growth rate of an economy's total output into that which is due to increases in the contributing amount of the factors used, and that which cannot be accounted for by observable changes in factor utilization. In principle, with the exception of the first term, all terms in Equation (3) are directly observable and can be measured using standard national income accounting methods. However, the "technological" term $(F_A A/Y) \cdot g_A$ is not directly observable as it captures improvements in productivity unrelated to changes in the use of factors of production[iii]. This term is commonly referred to in the literature as the "Solow residual", or total factor productivity (TFP).

Empirically, the construction of long time series for labor, capital, and gross domestic product (GDP) data allowed economists to apply the growth accounting technique to virtually every economy in the world (Crafts 2004; van Ark & Crafts 1996; Crafts 2009), and to develop quantitative factor productivity indices (Fabricant, 1954; Abramovitz, 1956; Schmookler, 1962). A common finding from these efforts is that increasing factor inputs – population, savings, and capital accumulation per se – cannot fully account for observed levels of economic growth in most industrialized economies. For the US, using time series data from 1909 through 1949, Solow (1957) estimated that TFP accounted for more than 85% of per-capita growth. The TFP term has also been identified as the major driver of growth within other industrialized countries (Easterly & Levine, 2001).

1.1.2. Limitations of growth accounting and TFP methods

TFP as a "measure of our ignorance"

The detection of TFP indicates that there must be some contribution to growth other than advances in industrializing the economy, i.e. investing in industry towards a more capital-intensive production. While this finding points to innovation rather than capital accumulation as a potential path for growth, TFP should not be solely equated with technical change. As a "measure of our ignorance" (Abramovitz, 1956), TFP encompasses many components, some wanted (technical innovation, institutional and organizational change, shifts in societal attitudes, fluctuations in demand, changes in factor shares), others unwanted (measurement error[iv], omitted variables, aggregation bias, model specification) (Hulten, 2001). Since TFP is measured as a residual, its components are not directly observed, and hence are "lumped together" and cannot be sorted out within the growth accounting framework[v]. The Solow-Swan growth model itself does not seek to explain or derive the empirical residual, rendering it an exogenous black box, outside the model and the influence of economic incentives.

In an attempt to overcome the shortcomings brought by the treatment of innovation (in the form of changes in TFP) as exogenous to the growth model[vi], *endogenous growth theory* – as formulated by Romer (1986) and Lucas (1988) – expands the concept of capital to include knowledge and human capital, thus recognizing that some part of innovation is, in fact, a form of capital accumulation, and therefore endogenous to the model. In these models, growth is mostly due to indefinite investment in human capital which has a spillover effect on the economy and reduces the diminishing returns to capital accumulation (Barro & Sala-i-Martin, 2004). Such models imply that the long-run growth rate of an economy is promoted by policies that increase incentives to innovation.

However, the endogenous approach is not without its critics. The main failings pointed to these theories of growth concern the inability to explain the income divergence between the developing and developed economies (Sachs & Warner, 1997), and the phenomenon of conditional convergence verified in the empirical literature (Parente, 2001). Furthermore, endogenous growth theories also fail to offer a satisfactory explanation for the so-called "productivity conundrum" verified in developed countries. This phenomenon is characterized by a long-term and sustained slowdown of productivity growth, as has been the case for the world's industrialized economies since the Great Recession of late 2000s. This slow rate of growth is even more puzzling because it has not been reversed by the "information revolution" brought by fast advances in digital and other information and communication technologies (ICT)[vii]. Secular stagnation theory attributes this slowdown not to cyclical or short-term depressions, but to a change of fundamental dynamics allowing for the possibility that "*depression may become the normal condition of the economy*" (Harris, 1943). While economists remain puzzled to the causes of this slowdown, possible explanations include Robert Gordon's thesis that the periods of fast economic growth in the last century were fueled by large-scale innovations which are unrepeatable (i.e. the internal combustion engine, electrification, public sanitation) (Gordon, 2017). Other explanations focus on limits to productivity growth imposed by decreased competition, excessive regulation, or a slowing of gains in human capital, or mis-measurements in productivity gains (Baily & Montalbano, 2016). In the endogenous growth view, a productivity slowdown could be due either to a slowdown in the growth rate of generalized capital inputs, or to a decline in the spillover effect. However, neither case is supported by empirical evidence.

Energy inputs and its importance in economic production and growth

The mainstream economic growth models discussed so far focus mainly on what conditions allow for continuing economic growth but are limited in their consideration of energy and its role in economic production and growth. The basic Solow-Swan model does not include energy resources at all[viii], and the endogenous growth literature, with few exceptions, continues to neglect the role of energy as a factor that could constrain or enable economic growth. However, the finiteness of energy resources creates difficulties for the concept of indefinite growth, making it necessary to look more closely at the specific mechanisms that drive long-term growth, especially insofar as they involve energy use and energy efficiency. A suitable theory of economic growth should also be able to explain observations of growth slowdown resulting from energy price spikes[ix] (Hamilton, 2009). The standard growth theory can only justify declines in output as an effect of reduced labor hours – itself a consequence, not a cause, of the declines.

In this sense, Ayres (2001) emphasizes the importance of a positive feedback mechanism as a driver of growth, powered by cheap energy sources. Growth is endogenized with this cycle, which operates as follows: cheaper energy resource inputs, due to discoveries, economies of scale and experience (or learning-by-doing) enable tangible goods and intangible services to be produced and delivered at ever lower cost, i.e. resource (energy) flows are productive; lower costs in competitive markets translate into lower prices for products and services; through price elasticity effects, lower prices encourage demand; since demand for final goods and services corresponds to the sum of factor payments – most of which go back to labor as wages and salaries – it follows that wages of labor tend to increase as output rises; this, in turn, stimulates further substitution of energy resources and mechanical power produced from these resources, for human and animal labor; the continuing substitution drives further increases in scale, experience, learning, and lower energy resource costs still, thus closing the loop. A representation of this positive feedback cycle is given in Figure 1.

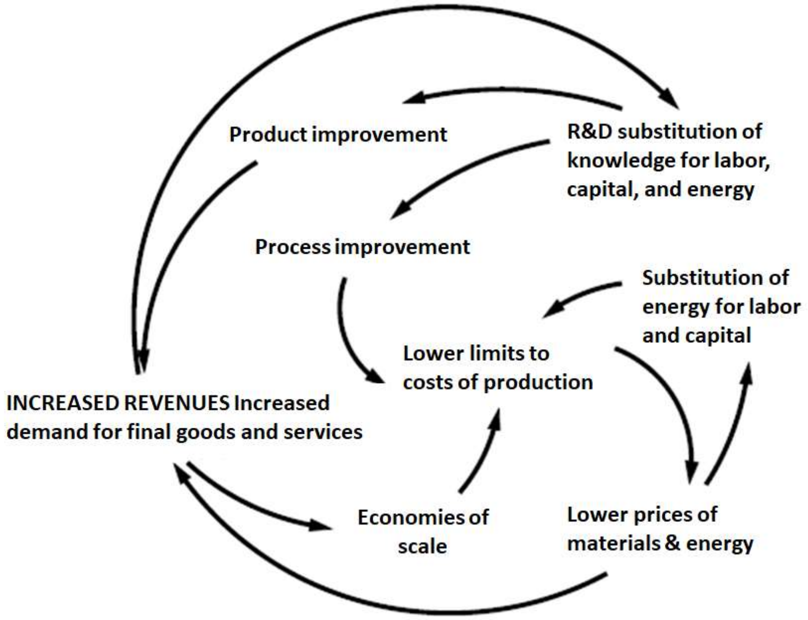


Figure 1 – Generalized positive feedback cycle. [Source: Warr & Ayres, 2006]

Historical evidence for industrialized countries seems to support the argument that economic growth in these countries has been – at least until recently – driven by falling energy prices. The Industrial Revolution of the 18th century was a direct consequence of the development of an energy transformation technology (i.e. the steam engine), and its widespread application to industry and transportation. The effects from this innovation can be understood as a positive feedback loop[x]. The available qualitative and quantitative evidence – according to Ayres & Warr (2005) – supports the argument that physical resource flows (and particularly energy flows) must be a major factor of production. Still, despite its apparent importance to production, energy has been largely ignored by mainstream economic growth theory.

The reasoning behind this neglect of energy inputs in the standard growth theory framework stems from the argument that fuels and materials are not *primary* factors of production (such as capital, labor, and sometimes land), but *intermediate* inputs, which are produced by some combination of capital investment and labor (plus "technology"). This has led to a focus on the primary factors of production, and a much more indirect treatment of the role of energy. The prices paid for all the different inputs are seen as eventually being payments to the owners of the primary inputs for the services provided directly or embodied in intermediate inputs. Payments to capital (rents, interests) and labor (wages, salaries) are adequately represented in standard national accounts, but payments to energy resources are generally underreported in these statistics, and even then, are roughly equated with revenues from the energy industries, corresponding to less than 10% of income (US EIA, 2011; Platchkov & Pollitt, 2011). Denison (1979), among others, relied on this small cost share for energy to argue that energy prices could not have a significant impact on economic output and growth, and hence to justify the omission of (or at most the marginal role attributed to) energy in mainstream growth theory (Aghion & Howitt, 2009; Kümmel & Lindenberger, 2014).

But this conceptualization of growth as essentially independent of energy use is built on a drastic oversimplification of the economic system. For a hypothetical single sector economy consisting of a large number of producers[xi] manufacturing a single all-purpose good using only capital and labor, it is a straightforward exercise to show that the productivity for each factor of production must be proportional to the share of payments received by that factor in national income. This link between factor payments and factor productivities establishes a role for national accounts in production theory. Payments to capital and labor virtually exhaust national income in standard accounts, and the historically stable shares of payments to these two factors (1/3 for capital; 2/3 for labor) constitute a stylized fact of long-term growth, as proposed by Kaldor (1957). Since, as noted, payments to energy – when represented – constitute a considerable smaller share of income, the "cost-share theorem" sketched above necessarily implies that energy's productivity is correspondingly small[xii].

Ayres & Warr (2005) argue that the apparent disagreement between a small observed cost share for energy inputs, and the obvious importance of energy as a factor of production – supported by quantitative evidence (see section 1.2.2 below) can be lifted by establishing a more realistic picture for the economy: as a multi-sector process, producing final output from a chain of intermediates, and not directly from raw materials/energy or, still less, from capital and labor alone. The authors defend that downstream value-added stages of such a multi-sector framework act as productivity multipliers for essential inputs, such as energy, which can then contribute a much larger effective share of the value of aggregate production (i.e. be much more productive), while still receiving a very small share of national income directly. In the real world, a cost-minimizing firm purchases capital, labor, and other intermediate consumables – either embodied in the final product (materials) or used up in the production process (energy) – such that their joint marginal productivity approaches zero. The cost-minimizing strategy for the firm is hence not necessarily related to the marginal productivities of capital and labor per se. The direct cost of capital assets and labor to the firm does not reflect the cost of complementary energy inputs to activate that capital and labor. Hence, even though the production of intermediates, such as processed materials and energy, adds nothing to final output, the idealized microeconomic model of cost-minimizing firms using only capital and labor cannot be easily generalized to the whole economy, as assumed by standard growth theory.

Furthermore, measures of economic output that omit intermediate inputs – namely energy – can significantly impact estimates for TFP in growth accounting exercises (Cobbold, 2003). These measures restrict the role of technological change by assuming that such change affects only the utilization of capital and labor, and improvements in the efficiency of intermediate inputs (energy) cannot be source of TFP growth. On the other hand, an output measure that allows for intermediate inputs as a source of growth – i.e. a gross output-based measure – potentially provides a better indicator of the full extent of disembodied technological change[xiii]. Still, despite its greater intuitive appeal, gross output-based measures impose greater demands on data availability, especially concerning the monetary value attributed to intermediate inputs.

Besides allowing for the role of consumable intermediates (mainly energy) in a multi-sector economic framework, Ayres & Warr (2010) also argue that it is not "raw" energy, as an input, alongside capital and labor, that explains output and drives long-term economic growth. Instead, the authors distinguish between resource energy (primary) and energy actually used productively (useful), and between energy thermodynamically available to perform physical work ("available energy" or "exergy") and energy lost to entropy-generating processes. These concepts are further explored in the next section.

1.2. (Productive) energy and economic growth: reasoning and evidence

The debate of whether or not energy plays a relevant role in economic production and growth is frequently accompanied by the debate on how to accurately account for and aggregate economically productive energy flows. In section 1.2.1 we review the *useful exergy* metric – based on thermodynamics principles – as a method of aggregating energy flows in the economy. Based on this metric, we briefly review some recent empirical results concerning the relationship between energy use and economic output, and how the adoption of a useful exergy metric to account for productive energy flows can lead to important new insights regarding this relationship.

1.2.1. Productive energy inputs: primary, final, and useful energy and exergy

Traditionally, three levels of energy use in production processes – representing the conversion or energy sources in Nature from less to more usable forms for production (Nakićenović & Gruebler, 1993; Smil, 1994; Summers, 1971) – are differentiated: primary, final, and useful (Nakićenović et al., 1996; Orecchini, 2006; Summers, 1971). Primary energy corresponds to energy carriers as they are recovered and gathered from the natural environment (Haberl, 2001; Nakićenović & Grubler, 1993), e.g. mined coal, collected biomass, crude oil. Final energy corresponds to the flow of energy carriers available for direct use by consumers (industries and households). The final level of energy use mainly accounts for the energy content in output products from the energy sector, e.g. oil derivatives, electricity, biodiesel, or geothermal heat[xiv]. Useful energy corresponds to the last form of energy flows that is directly used to provide energy services (Cullen & Allwood, 2010). This level of energy use is obtained from the conversion of final energy carriers by end-use conversion devices (e.g. motor engines, boilers, ovens, and lamps) (Nakićenović & Grubler, 1993). It corresponds to energy actually used productively in the economy, to provide a final function[xv].

In the first energy transformation stage in the economy – primary-to-final – energy sources are upgraded into more useful forms of energy (Cullen & Allwood, 2010) through conversion processes (e.g. oil refining, coal-fueled electricity generation) and other operation processes characteristic of the energy sector (e.g. extraction, storage, and distribution) (Orecchini &

Naso, 2012; Serrenho, 2013). The second (and last) energy transformation stage – final-to-useful – is carried out in the exact location where energy services are required (in the household or factory) and consists of a large share of one-step conversions (e.g. electricity into motion by an electric motor, or natural gas into steam by a boiler), and of two-step conversions (e.g. gasoline into motion by a car engine into air conditioning low temperature heat). This latter conversion stage is difficult to estimate since every institutional sector, energy carrier, and end-use conversion device must be considered (Nakićenović et al., 1996).

Besides considering useful energy flows opposed to primary of final energy, an adequate description of actually productive energy inputs to the economy must necessarily take into account that energy can be used[xvi] in different forms, e.g. heat, work, light, refrigeration and specific electrical uses. However, these forms are qualitatively different: 1kWh of work can be converted into up to 1 kWh of heat at 30 °C, while the same quantity of heat (1 kWh) can only be converted into at most 0.066 kWh of work[xvii]. Work is in fact the most valuable form of energy transfer, because it is the energy form with the highest efficiency of conversion to any other form of energy. A common and comparable measure for different energy forms – acknowledging differences in both quantity and quality – can be found in the maximum amount of physical work that can be performed by a given amount of energy (in a given form) as the target subsystem approaches thermodynamic equilibrium with a reference state, reversibly. In thermodynamics, this quality-adjusted measure of energy is named exergy, and it is definable not just to more familiar forms of energy but also to all materials.

It is then *useful exergy* – i.e. energy delivered to performs a final function in the economy, measured in exergy terms – that is actually productive, and hence that constitutes the relevant factor of production correlating to economic output and growth. This relationship has been explored in some strands of recent literature, which are briefly reviewed in the next subsection.

1.2.2. Evidence supporting a link between useful exergy and economic growth

Towards a new theory of production and growth

Pioneering the adoption of the useful exergy metric to test energy-related hypothesis for explaining US economic growth since 1900 – and rejecting the neoclassical “cost-share theorem” based on the multi-sector approach illustrated in subsection 1.1.2 – Warr & Ayres (2006) developed a formal model (Resource-Exergy Service, or REXS) to demonstrate that historical US growth can be accurately reproduced, without the need for an exogenous TFP component, when useful exergy is included alongside capital and labor in a Linear Exponential (LINEX) APF[xviii]. On the other hand, when primary energy inputs are considered, a TFP residual component is necessary to fully account for historical US growth. Hence, with several features of their work following (albeit indirectly) from their concept of growth dynamics as a positive feedback cycle (Figure 1), the authors conclude that a significantly better explanation of past economic growth can be obtained by introducing useful exergy as a factor of production, and not primary energy/exergy.

From these results, Warr & Ayres (2006) propose that “technical progress” – as defined by the Solow residual – is almost entirely explained by historical improvements in primary-to-final-to-useful exergy conversion efficiency, which propagates, via cost and price reductions, through the downstream value-added chain. The authors also note a small (but significant) contribution from Information and Communication Technologies (ICT) in recent years, which could, admittedly, also be attributed to qualitative improvements in either capital or labor. The powerful implication drawn from this result is that, if economic growth is to continue without proportional increases in primary energy supply (namely fossil fuels), the exploration of new forms of increasing primary-to-final-to-useful exergy efficiency (i.e. to reduce the primary energy inputs per unit of useful exergy generated) is paramount. In other words, energy (exergy) conservation is likely the main key to long term environmental sustainability.

The novel data on exergy and useful exergy (or work) used by Warr & Ayres (2006) results from a societal level exergy analysis developed by Warr et al. (2010) for Austria, Japan, the UK, and the US, between 1900 and 2000. In this analysis, the authors find that there is a common relationship between useful exergy consumption and economic output (measured as their ratio, or *useful exergy intensity*), characteristic of industrialized economies. While this observation is consistent with the positive feedback cycle proposed by Ayres (2001) (Figure 1), it is also consistent with the more standard view that energy (or exergy) consumption is a consequence, and not a driver, of growth. In order to empirically determine the direction of causality between energy/exergy consumption and output, Warr & Ayres (2010) test for cointegration and Granger-causality in multivariate models including gross domestic product (GDP), capital, labor, and two measures of energy use (primary and useful exergy). Previous empirical research on the energy-growth causality nexus had failed to provide conclusive evidence to unambiguously determine the existence and directionality of causal relationships, and hence of long-term and short-term impacts on energy policy – see the review on this empirical literature by Ozturk (2010), Odhiambo (2010), and Payne (2010). Notwithstanding country-to-country differences, the results for any given country differ with the period studied, the choice of methodology, and the method of aggregation of energy flows. Warr & Ayres (2010) are the first to adopt primary exergy and useful exergy measures to reflect energy quality in addressing the causality issue. They find evidence of unidirectional causality from both exergy measures to economic output, thus concluding that growth is driven by an increased availability of exergy, *and* an increased delivery of useful exergy to the economy. These findings support the Ayres (2001) conception of a positive feedback cycle, and the argument by Ayres & Warr (2010) that conversion efficiency to useful exergy is a driver of growth.

The finding that the marginal productivity of useful exergy is a dominant driver for economic growth does not mean that either capital of labor are unimportant. In fact, the three factors are not likely to be independent, or substitutable, among themselves, as increasing exergy conversion efficiency requires investment in both capital and labor, and in turn capital and labor require useful exergy to operate. Models developed in the field of ecological economics acknowledge thermodynamic considerations which posit minimum energy requirements to a given level of economic production (Stern, 1997), and include realistic constraints on substitution possibilities between energy, capital, and labor (d'Arge & Kogiku, 1973; Gross & Veendorp, 1990; van den Bergh & Nijkamp, 1994; Lindenberger & Kümmel, 2011).

Insights from useful exergy for the Portuguese case

The societal exergy and useful exergy analysis conducted in Warr et al. (2010) has recently been refined and employed to the Portuguese economy by Serrenho et al. (2016)[xix]. As in other such studies, Serrenho et al. (2016) find that the observed increase in useful exergy consumption in the country is due to: 1) increases in primary/final exergy consumed; 2) improvements in aggregate exergy efficiency. The former is a result of a period of fast industrialization, while the latter was mainly a result of the electrification process in the country. Also, despite dramatic changes in the composition of useful exergy consumed throughout the last 150 years in Portugal, useful exergy intensity is remarkably stable for this period, thus signaling an apparent very strong correlation between useful exergy consumption and economic output. This important result provides a powerful motivation to statistically test the relationship between these variables.

This avenue has been explored in Santos et al. (2018), using cointegration analysis to identify statistically significant APFs linking economic output, capital, labor, and both primary energy and useful exergy, and checking for these functions' economic plausibility against selected criteria. Applied to the Portuguese case, the authors are unable to identify statistically significant/economically plausible APFs without useful exergy or including a TFP component. On the other hand, the authors find that only by the inclusion of useful exergy inputs – alongside quality adjusted measures for capital and labor – does an acceptable model emerge from the analysis. This model is composed of two simultaneous cointegrating relationships between the variables: 1) a capital-labor APF consistent with the neoclassical "cost-share theorem"; 2) a constraint linking all factors of production (capital, labor, useful exergy), reflecting the limits to substitution between these factors. Moreover, this model adjusts to historical data without the need for an exogenous TFP component. The work of Santos et al. (2018) provides new insights to the correlation between useful exergy and economic production, without rejecting *a priori* neither the essentiality of (productive) energy to production, nor some of the more standard assumptions of the neoclassical growth models. Furthermore, Granger causality tests also conducted in Santos et al. (2018) support the so-called "growth hypothesis" i.e. that energy (both primary energy and useful exergy) has a unidirectional causal effect on economic output. Taken together, these results are in line with the view that both the quantitative amount of energy consumed, and the efficiency with which energy is converted, transformed, and delivered to end-uses, are important components of economic growth. It also supports the view that most of the contribution from "technical progress" to growth can be attributed to advances in energy/exergy efficiency.

The relation between "technical progress" – as measured by TFP – and final-to-useful aggregate exergy efficiency has been explored in a more direct approach in the context of the MEET2030 project (IST & BCSD, 2018), developed by the Portuguese Business Council for Sustainable Development (BCSD) in partnership with the Instituto Superior Técnico from the University of Lisbon. The goal of this project is to construct energy and economic scenarios for the Portuguese economy, based on the relationship between energy (in fact, useful exergy) and economic growth. By studying the evolution of both final-to-useful exergy efficiency at the aggregate level of the economy, and estimates of TFP for the same period, the project team finds that the two are closely linked, with an approximately constant ratio between the two[xx]. Taking advantage of this rough relationship, the project team is able to construct projections for TFP in the following decades, based on projections for final-to-useful exergy efficiency, these in turn fueled by the participatory contributions from relevant stakeholders of the Portuguese economy. Hence, it is central to this project the view that "technical progress", or TFP, corresponds in fact to progress in the efficiency of useful exergy generation, and that it is increases in this efficiency which will primarily drive future economic growth.

Finally, it should be noted that for Portugal, as for the US (according to Ayres et al., 2003), the rate of exergy efficiency increase has been slowing down since the 1970s. If changes in TFP are indeed attributable to changes in this efficiency – and if TFP is the major driver of economic growth in industrialized economies – then this slowdown in exergy efficiency is likely related to the "productivity conundrum" invoked in section 1.1.2 above. The implication is that permanent productivity slowdown (or secular stagnation) might be avoided if effective measures to increase the rate of exergy efficiency are undertaken.

1.2.3. Overview and aims of the present analysis

Having reviewed several of the major ideas and empirical results tying (actually productive) energy flows to economic production processes and growth, the first and foremost aim of the present work is to form a basis for the development of a macroeconomic growth model that takes into account not only energy flows at different stages of economic processes (namely, primary, final, and useful), but also aggregates these energy flows reflecting qualitative differences among energy inputs, by adopting an appropriate useful exergy metric, based on thermodynamics.

Described in detail in the Section 2 below, the proposed model is based on the arguments by Ayres (2001) and Ayres & Warr (2005) that energy inputs to the economy are productive via a positive feedback mechanism (Figure 1), and that the economy should be characterized as a multi-sector process (in this case, a two-sector process), producing final output not simply from capital, labor, and raw materials/energy, but allowing for intermediate consumables (i.e. useful exergy). The proposed two-sector model also takes into account that the majority of the contribution of so-called "technical progress" (measured as TFP) to historical economic growth can in fact be attributed to both increases in the amount of energy consumed in production, but especially the advances in the conversion efficiency of energy (exergy) into more useful forms. Finally, the proposed two-sector model also acknowledges that an adequate depiction of economic growth should be able to relate slowdowns in productivity with energy use and energy prices (on the other hand, such a model could also empirically validate the mechanism proposed by Ayres 2001, by which a fall of energy prices drives economic growth).

The next section describes the proposed two-sector model in detail and establishes the methodological steps for its application to empirical data (for Portugal).

2. Methodology

In this section we detail the proposed two-sector model approach, and its correspondence with macroeconomic and energy use data, through the decomposition and reclassification of available datasets for consumption and investment expenditure, and energy and useful exergy balances. The framework and methods presented in the following subsections are tested on a single-country case-study (Portugal, 1960-2014), and the obtained results and conclusions are detailed in Section 3.

2.1. Two-sector model for the economy

The two-sector model proposed in our own work – while inspired by the work of Ayres & Warr (2005; 2009) – differs from the latter effort in several aspects. In our description of the economy we look at overall production as a two-stage process, with a separation between energy-related activities, and the remaining economic production activities. Figure 2 illustrates the two-sector model for the economy considered in our analysis.

In our two-sector model, the *Energy Sector* (E-Sector) is responsible for supplying the economic production activities with useful exergy, which can act as both an intermediate to the production of final goods and services or be consumed directly by consumers. The *Non-energy Sector* (NE-Sector) produces all kinds of final investment and non-energy related consumption goods and services in the economy. By isolating all energy conversion and transformation activities in an (abstract) extended energy sector, this model is in line with the Warr & Ayres (2006) argument that a major driver of economic growth is "technical progress", not as an exogenous residual, but rather as concentrated in historical improvements in primary-to-final-to-useful energy/exergy efficiency, which are entirely located in our proposed extended energy sector.

It is assumed, throughout our analysis, that we are dealing with a closed economy, running in continuous time, and populated only by consumers (i.e. households, government, and NPISH[xxi]) and firms. Other components such as imports/exports, taxes and subsidies, as well as capital transfers and net lending/borrowing, are disregarded in our analysis. Markets are assumed to be perfectly competitive, in the sense that economic agents take prices as given.

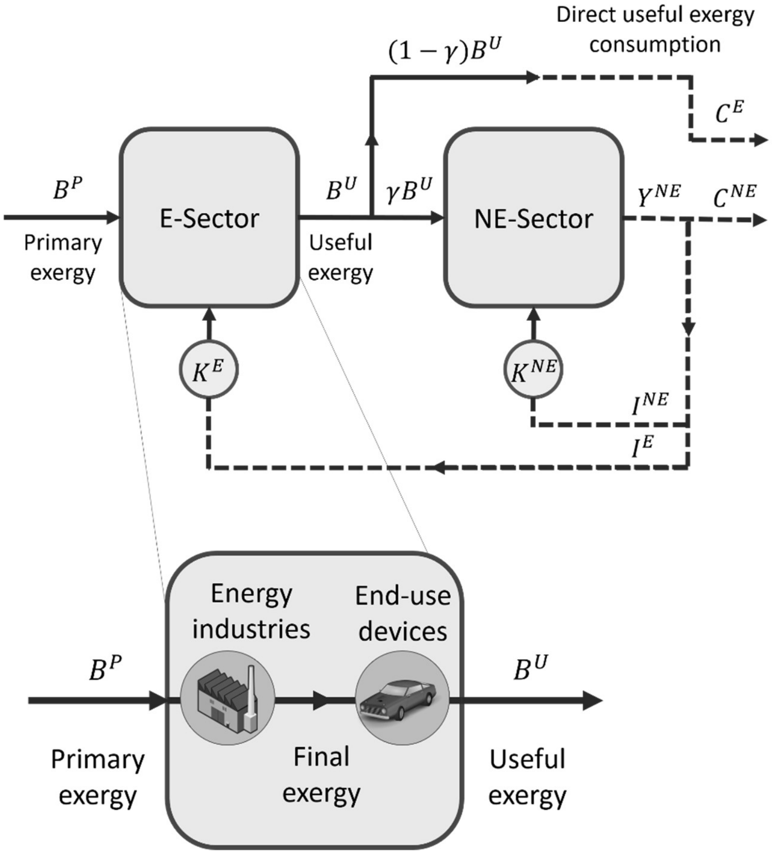


Figure 2 – Two-sector model framework for the economy. Physical flows (e.g. primary/useful exergy) are represented as full lines. Monetary flows (e.g. consumption/investment expenditure) are represented as dashed lines.

Energy enters the economic system through the E-Sector as primary exergy inputs extracted from the environment, $B^P(t)$. In the E-Sector, primary exergy inputs are converted into useful exergy, as a function of the investment in physical capital in this sector, $K^E(t)$. The useful exergy output from the E-Sector, $B^U(t)$, discounts useful exergy consumed within the E-Sector by energy industries own-uses. A constant fraction of the useful exergy output from the E-Sector ($0 < \gamma < 1$) is to be used as a factor of production in the production of goods and services taking place in the NE-Sector, while the remaining useful exergy is to be directly consumed, at a given price, by households, government, and NPISH, $C^E(t)$. The NE-Sector utilizes useful exergy, as well as its own investment in physical capital, $K^{NE}(t)$, to produce final consumption goods and services, $C^{NE}(t)$, and investment in physical capital, $I(t) = I^{E}(t) + I^{NE}(t)$, to supply both economic sectors.

Production processes within the NE-Sector generate this sector's total output, given by $Y^{NE}(t)$[xxii]. Unlike the model developed by Ayres & Warr (2005; 2009), the assumption that factors of production are used in the same proportions for the energy sector as for the whole economy – is lifted in our proposed two-sector framework.

The total product of the economic system illustrated in Figure 2 then corresponds to the sum of the NE-Sector production output and the monetary value attributed to final directly consumed useful exergy. This total product for the economy corresponds to gross domestic product (GDP). There are two different (standard) approaches to measuring GDP: the *production*, or *value-added* approach; and the *expenditure* approach. The production approach sums the outputs from every class of enterprise and deducts intermediate consumption (the cost of material, supplies, and services used in production) from this value, in order to obtain GDP:

$$Y = Y^{NE} + (1-\gamma)p_{B^U}B^U \qquad (4)$$

The gross output for the whole economic system – as depicted in Figure 2 – is given by $Y^{NE} + p_{B^U}B^U$, in which p_{B^U} corresponds to the price attributed to useful exergy output from the E-Sector. Hence, $\gamma p_{B^U}B^U$ represents the total monetary value associated with intermediate products used up in the production processes within the NE-Sector.

Alternatively, the expenditure approach measures GDP according to the total amount of money spent in the purchase of goods and services. For a closed economy such as our own, this translates as

$$Y = C + I = C^{NE} + C^{E} + I^{NE} + I^{E} \qquad (5)$$

Both alternative approaches to measure GDP should, in principle, yield the same result. When applying the proposed model to empirical data, collected datasets will be decomposed and reclassified according to the variables defined in the expenditure approach, as presented in Equation 5.

The extended energy sector introduced in our work is innovatively defined, when compared to general economic models and accounts. The traditionally defined energy sector is generally associated exclusively with the energy industries, i.e. those involved in the production and sale of energy and energy related products, including fuel extraction, manufacturing, refining, and distribution activities. More specifically, the traditionally defined energy sector includes (Case & Fair, 2007):

- The petroleum industry, including oil companies, petroleum refiners, fuel transport, and end-user sales at gas stations;
- The gas industry, including natural gas extraction and coal gas manufacture, as well as distribution and sales;
- The electrical power industry, including electricity generation and electric power distribution and sales;
- The coal industry;
- The nuclear power industry;

- The renewable energy industry, comprising alternative energy and sustainable energy companies, including those involved in hydroelectric power, wind power, and solar power generation, and the manufacture, distribution, and sale of alternative fuels;
- Traditional energy industry based on the collection and distribution of firewood, the use of which, for cooking and heating, is particularly common in poorer countries.

In the proposed two-sector model presented in our work, the extended energy sector is defined by boundaries that go beyond the traditional energy industries. Namely, this extended energy sector aggregates every single process involved in the conversion of primary energy (or exergy), extracted from the environment, into final energy (exergy) sold to consumers, *and into useful exergy actually used to perform a final function in the economy*. Hence, within the boundaries defined for our extended energy sector, lie not only the primary-to-final energy (exergy) transformation processes that compose the traditional energy industries, but also all final-to-useful exergy transformation processes that occur within any end-use device used in the economy. Concretely, this means that the extended energy sector includes not only machines used in the traditional energy industries (e.g. boilers and turbines used in a coal fired power plant), but also devices used in households and firms (e.g. refrigerators, laptops, automobiles, etc.). In this sense, the defined energy sector (E-Sector) is much broader than the traditional energy sector composed of energy industries only. This extended energy sector encompasses all processes of energy transformation and conversion in the economy, and its output corresponds to useful exergy actually used to perform economic activities and generate economic value.

The efficiency with which useful exergy is generated from primary exergy resources depends on efficiencies which are related with the technological capacity of the extended energy sector. In value terms, output for the extended energy sector (E-Sector) is given by

$$Y^E = p_{B^U} B^U = \tilde{B}^U \qquad (6)$$

A constant positive fraction of the total useful exergy output from the extended energy sector $\gamma \tilde{B}^U$ will be consumed as a factor of production in the APF governing the production of goods and services in the non-energy sector. The remaining fraction $(1-\gamma)\,\tilde{B}^U$ will correspond to useful exergy directly consumed by households, government, and NPISH. In monetary terms, this is written as

$$\begin{cases} Y^E = C^E + \gamma \tilde{B}^U \\ C^E = (1-\gamma)\tilde{B}^U \end{cases} \qquad (7)$$

The novel definition for an extended energy sector within our proposed two-sector framework for the economy implies a redefinition of variables pertaining to consumption and investment expenditure, as well as useful exergy use, by both sectors. In the next sections we propose a simplified first approach to conduct the decomposition and reclassification of expenditure and energy balances datasets according to the two-sector model's variables as defined in our framework.

2.2. Decomposition and reclassification of National Accounts

Macroeconomic national accounts are based on the implementation of complete and consistent accounting techniques for measuring the economic activity of a given territory.

These accounts include detailed underlying measures relying on double-entry bookkeeping. National income and product accounts provide estimates for the value of income and output on an annual basis, including GDP. The expenditure approach focuses on estimating total output through measurement of the amount of money spent by economic agents. For a closed economy disregarding imports and exports:

$$GDP = C + I \qquad (8)$$

In Equation 8, C corresponds to total final consumption expenditure – the sum of private expenditure by households and NPISH, and general government expenditure. The second r.h.s. term I corresponds to gross private domestic investment[xxiii].

In our proposed two-sector model approach, consumption and investment expenditure for the whole economy can be decomposed as consumption and investment expenditure incurred by each of the two sectors. Hence, total consumption expenditure for the whole economy will correspond to the sum of consumption expenditure on goods and services produced in the non-energy sector (C^{NE}), and consumption of useful exergy output from the extended energy sector by households, firms, and NPISH (C^{E}). Analogously, total investment expenditure will correspond to the sum of investment expenditure in physical capital used in NE-Sector production (I^{NE}), and physical capital used in E-Sector generation of useful exergy (I^{E}).

Since the extended energy sector defined in our two-sector framework – as detailed in Section 2.1.1 – encompasses not only primary-to-final energy (exergy) conversion devices found in traditional energy industries, but also final-to-useful exergy conversion devices found in households; and since these latter devices (conventionally accounted as consumer goods) constitute an investment in the physical capital required by the extended energy sector to generate its output (useful exergy) from primary energy resources; it follows that adjustments have to be made when classifying investment expenditure in the extended energy sector.

Concretely, a redefinition of selected (those performing final-to-useful exergy conversion) traditionally-defined consumer goods, produced by the NE-Sector, as investment in physical capital in the extended energy sector is required in order to have a clear-cut separation between the two sectors. Because of this requirement, total consumption and investment expenditure for our proposed two-sector framework will necessarily differ from consumption and investment expenditure as reported in national accounts. Specifically, the redefinition of consumer goods as investment will result in lower values for total consumption expenditure – compared with national accounts – and correspondingly higher values for total investment expenditure. This can be expressed mathematically as

$$\begin{cases} C^{*} = C^{NE} + C^{E} < C \\ I^{*} = I^{NE} + I^{E} > I \\ C^{*} + I^{*} = C + I \end{cases} \qquad (9)$$

Where C and I correspond to total consumption and investment expenditure as defined in national accounts, respectively, and C^{*} and I^{*} corresponds to total consumption and investment expenditure as defined in our two-sector model, respectively. For our proposed two-sector framework, the expenditure approach identity in Equation 8 becomes (without imports and exports)

$$GDP = C^* + I^* + (X - M) \quad (10)$$

The following sections deal – in detail – with the decomposition of national accounts total consumption expenditure C and total investment expenditure I according to final use and asset type, respectively. Each consumption and investment expenditure category is analyzed and allocated to the four disaggregate key variables associated to each sector's consumption and investment expenditure, as defined in our framework (C^{NE}, C^{E}, I^{NE}, I^{E}). This allocation is performed following some simplifying criteria which should not, in principle, affect the general quality of results. A schematic depiction illustrating (through selected examples) the decomposition and reclassification effort detailed in the following sections is presented in Figure 3.

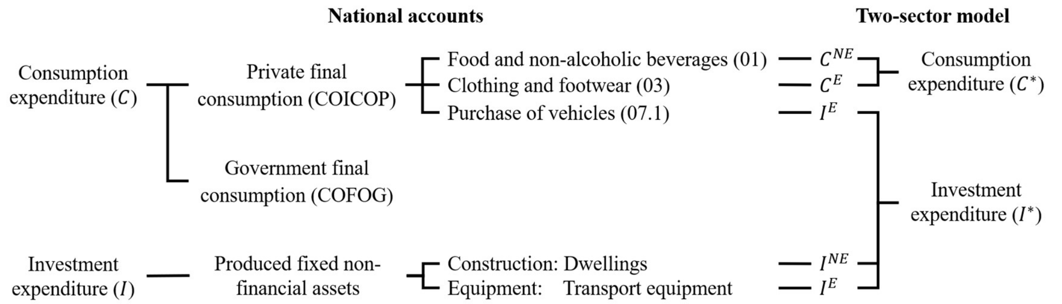


Figure 3 – Diagram representing the decomposition of standard macroeconomic national accounts (left) and reclassification according to the variables defined in the proposed two-sector model with an extended energy sector (right). Consumption expenditure (e.g. private final consumption) is decomposed according to purpose and reclassified as either directly consumed useful exergy (C^E), non-energy related consumption (C^{NE}), or investment in the extended energy sector (I^E). Investment expenditure is decomposed according to type of asset and reclassified as either investment in the non-energy (I^{NE}) or extended energy (I^E) sectors. Total consumption and investment expenditure in the proposed two-sector model then differs from national accounts as in Equation (10).

2.2.1. Total consumption expenditure

Total final consumption expenditure, as defined by macroeconomic national accounts, comprises both the expenditure on goods and services used for the direct satisfaction of individual needs (*individual consumption*), or collective needs of members of the community (*collective consumption*). Total final consumption expenditure therefore comprises;

- Final consumption expenditure by households and NPISH (*private final consumption expenditure*);
- Final consumption expenditure by general government.

The most important component of final consumption expenditure comes from households and includes all durable and non-durable consumer goods. Exception is made of purchases for own-construction or improvements of residential housing, which are instead treated as a component of gross capital formation. Private household consumption is composed of:

- All goods and services purchased by households to satisfy everyday needs;
- Households' consumption of outputs produced by unincorporated enterprises owned by households;
- Imputed rents for services of owner-occupied housing;
- Payments to government units to obtain various kinds of licenses, permits, certificates, passports, etc.;
- Partial payments for products and services provided by the general government;

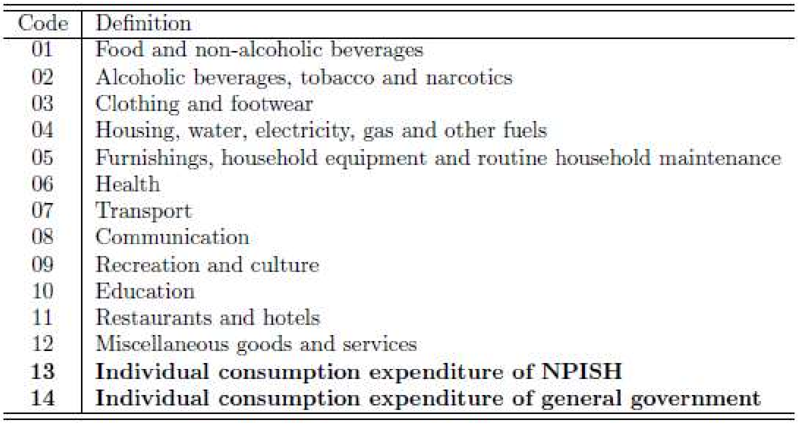
- Explicit and imputed service charges on household uses of financial intermediation services provided by banks, insurance companies, etc.

The classification of individual consumption according to purpose (COICOP) is a reference classification published by the United Nations' Statistics Division. It divides the purpose of individual consumption expenditures incurred by households, NPISH, and general government. The classification units are transactions. Three structure levels are defined in COICOP:

- Structure Level 1: Divisions (two-digit);
- Structure Level 2: Groups (three-digit);
- Structure Level 3: Classes (four-digit).

The first structure level is presented in Table 1.

Table 1 – First COICOP structure level (divisions; two-digit) of private household consumption expenditure (1-12). Divisions 13 and 14 (bolded) represent individual consumption expenditure of non-profit institutions serving households (NPISH) and of general government.



Code	Definition
01	Food and non-alcoholic beverages
02	Alcoholic beverages, tobacco and narcotics
03	Clothing and footwear
04	Housing, water, electricity, gas and other fuels
05	Furnishings, household equipment and routine household maintenance
06	Health
07	Transport
08	Communication
09	Recreation and culture
10	Education
11	Restaurants and hotels
12	Miscellaneous goods and services
13	**Individual consumption expenditure of NPISH**
14	**Individual consumption expenditure of general government**

Divisions 01 through 12 are allocated to consumption expenditure incurred by households. Divisions 13 and 14 are allocated, respectively, to consumption expenditure incurred by NPISH and general government. For the purposes of the analysis conducted in our work, only Divisions 01 through 12 will be accounted as private final consumption expenditure (hence disregarding NPISH consumption expenditure), while general government consumption expenditure will be dealt with further ahead.

Both the INE and EUROSTAT macroeconomic databases provide detailed statistical data on the final consumption expenditure incurred by households disaggregated by COICOP divisions and groups, but not classes. For our analysis, this means that decomposition and aggregation of consumption expenditure datasets according to the variables defined in our proposed two-sector model framework will be conducted at the group level, risking some loss of precision in the obtained results.

Some divisions and/or groups within the COICOP tree can be immediately allocated to a single variable of the proposed two-sector model framework, while other divisions and/or groups include elements which could be allocated to distinct key variables of the two-sector model (in any case no more than two, e.g. the division *05.3 Household appliances* includes refrigerators, space heater, and washing machines – devices that participate in final-to-useful exergy conversion and are hence redefined as investment in the extended energy sector – but also expenditure on the delivery, installation and repair of these devices – which still constitutes consumption expenditure under the two-sector model's definitions). In the latter case, the criteria adopted in our analysis consists of splitting the considered group/division in half and allocating each half with each of the two-sector model's variables which match the group/division's contents.

It is important to note that the redefinition of particular energy-driven consumption goods as investment in the extended energy sector (E-Sector) – the most innovative characteristic of our proposed two-sector model for the economic production – carries some concerns regarding the consistency in accounting consumption and investment expenditure for each sector. The concept of imputed rents applies to any capital good[xxiv] and it must be taken into account when redefining automobiles and appliances, for example, as components of investment expenditure in the extended energy sector productive processes. The full cost of using a fixed asset in production is also measured by the actual or imputed rental on the asset, and not by depreciation alone. Although imputed rents are not explicitly accounted for in our work for each consumer good redefined as investment expenditure in the extended energy sector, it is expected that the effect of these imputed rents is eventually indirectly accounted for when estimating consumption of fixed capital for sector specific investment expenditure in the proposed two-sector model.

Final consumption expenditure incurred by households includes expenditure on services such as electricity, gas, and heat supply. These and other similar expenditures (such as the purchase of fossil fuels) are interpreted in our work as payments for the energy (i.e. exergy) contained within these products, and indirectly, for the useful exergy provided by this exergy content. Hence, the COICOP groups *Electricity, gas and other fuels (04.5)* and *Operation of personal transport equipment (07.2)* (which includes some expenditure on fuels) are allocated to direct useful exergy consumption by households $C^{E(H)}$. The first two COICOP divisions – *Food and non-alcoholic beverages (01)*; *Alcoholic beverages, tobacco and narcotics (02)* – are also straightforwardly allocated to with $C^{E(H)}$, for similar reasoning. These divisions concern payments for food products, which can ultimately be equated with payments for the useful exergy (i.e. muscle work) extracted from the consumption of these food products. It is then assumed that all exergy contained in food products is to be converted to muscle work.

Due to the innovative definition adopted in our work concerning the extended energy sector, all consumption goods participating in the conversion of final energy/exergy to useful exergy are allocated not to consumption expenditure, but to investment expenditure in the extended energy sector (E-Sector) $I^{E(H)}$. In practice, this means that consumption goods such as motorized vehicles and electric appliances will be considered capital investment expenditure in our proposed two-sector model. The groups *Purchase of vehicles (07.1)* and *Telephone and telefax equipment (08.2)* are the only ones allocated to $I^{E(H)}$ in their entirety. Both groups also include components accounting for expenditure in the repair of equipment, but such components are deemed negligible.

Most of the remaining individual COICOP groups and divisions can be allocated in its entirety to consumption expenditure by households on goods and services generated by the non-energy sector (NE-Sector) $C^{NE(H)}$. This is the case for *Education (10)*, *Restaurants and hotels (11)*, and any other group composed uniquely of services – e.g. *Outpatient services (06.2)*. Among COICOP groups containing some elements attributable to one key variable defined in the two-sector model, and other elements attributable to a distinct key variable of the two-sector model, the majority consists of COICOP categories which can be partitioned and allocated to $C^{NE(H)}$ and $I^{E(H)}$. One example is the COICOP group *Audio-visual, photographic and information processing equipment (09.1)*, which includes both physical capital responsible for the conversion of final energy/exergy into useful exergy for other electrical uses (television sets, stereo systems, personal computers, etc.), as well as non-energy related consumption goods (compact disks, lenses, software, etc.). For these cases, the datasets are equally split between the two key variables. A similar methodology will be applied for the decomposition and aggregation of final consumption expenditure of general government categories (COFOG).

Final consumption expenditure of general government consists of expenditure incurred by government in its production of non-market final goods and services[xxv] and market goods and services provided as social transfers in kind. Included in the final consumption expenditure of general government are:

- Non-market output other than own-account capital formation, which is measured by production costs less incidental sales of governmental output;
- Expenditure on market goods and services that are supplied without transformation and free of charge to households (social transfers in kind).

Analogously to private final consumption expenditure, the United Nations' Statistics Division classifies government expenditure datasets in national accounts according to the purpose for which the funds are used. This is done through the classification of the functions of government (COFOG), which allocates government expenditure for specific uses. It corresponds to the 14th division in the COICOP classification, and exhibits a similar structure level, with transactions as units. The COFOG divisions are presented in Table 2.

Table 2 – First COFOG structure level (divisions; two-digit) of government consumption expenditure.

Code	Definition
01	General public services
02	Defense
03	Public order and safety
04	Economic affairs
05	Environmental protection
06	Housing and community amenities
07	Health
08	Recreation, culture and religion
09	Education
10	Social protection

The first 3 COFOG divisions – *General public services (01)*; *Defense (02)*; *Public order and safety (03)* - plus *Environmental protection (05)*, *Education (09)*, and *Social protection (10)*, can be entirely allocated to the two-sector variable $C^{NE(G)}$, along with the majority of the remaining COFOG groups. Exceptions are the COFOG groups *Transport (04.5)*, *Communication (04.6)*, *Street lighting (06.4)*, and *Medical products, appliances, and equipment (07.1)*, all of which including elements allocated to different key variables of the proposed two-sector model. For these latter COFOG groups, annual values are split in half and allocated equally to the two-sector variables $C^{NE(G)}$ and $I^{E(G)}$. The COFOG group Fuel and energy (04.3) is entirely allocated to direct consumption of useful exergy by the government $C^{E(G)}$. As before, it is assumed that the price paid by the government for fuels can be equated to the price paid for the useful exergy extracted from these fuels.

Total final consumption expenditure in macroeconomic national accounts corresponds to the sum of private final consumption expenditure and general government consumption expenditure. The sum of aggregate two-sector model's variables corresponding to the consumption of non-energy related (NE-Sector produced) goods and services by households and government – $C^{NE(H)}$ and $C^{NE(G)}$ respectively – with the direct consumption of useful exergy by households and government – $C^{E(H)}$ and $C^{E(G)}$, respectively – constitutes total final consumption expenditure under the definitions adopted for the proposed two-sector model framework:

$$\hat{C} = C^{NE} + C^{E} = C^{NE(H)} + C^{NE(G)} + C^{E(H)} + C^{E(G)} \qquad (11)$$

2.2.2. Total investment expenditure

Gross capital formation in macroeconomic national accounts represents investment expenditure in capital assets. Gross capital formation incorporates not only produced capital goods (e.g. machinery, buildings, roads) but also improvements to non-produced assets. Hence, gross capital formation (GCF) constitutes a measure for the additions to the capital stock of buildings, equipment, and inventories – i.e. the addition to the capacity of the economy to produce more goods and income in the future. The components of GCF can be listed as:

- Gross fixed capital formation (GFCF);
- Changes in inventories;
- Acquisition less disposals of valuables.

For the purposes of the analysis presented in our work, only the major components of GCF – i.e. gross fixed capital formation (GFCF) – will be considered for the computation of investment expenditure. This component of GCF incorporates (Schreyer, 2009):

- Acquisition less disposal of new or existing produced assets, such as dwellings, other building structures, machinery and equipment, cultivated assets (e.g. trees and livestock), mineral exploration, computer software, entertainment, literary or artistic originals, and other intangible fixed assets;
- Costs of ownership transfers on non-produced, non-financial assets, such as land and patented assets;
- Major improvements to produced and non-produced, non-financial assets that extend the lives of assets;
- Acquisition that can be in terms of purchase, own-account production, barter, capital transfer in kind, financial leasing, natural growth of cultivated assets and major repairs of produced assets;
- Disposal that can be in terms of sale, barter, capital transfer in kind, or financial lease. Exceptional losses, such as those due to natural disasters, are not recorded as disposal.

Gross fixed capital formation (GFCF) is a component of gross domestic product (GDP) expenditure – as defined by the United Nations' System of National Accounts (Carson & Honsa, 1990), and the International Monetary Fund (IMF) Balance of Payments system – which illustrates how much of new value added in economic production is invested in the productive processes themselves, rather than consumed. It is, like consumption, a flow value[xxvi]. Fixed assets included in statistical measurement range according to their useful purpose in productive processes. Vehicles, for example, constitute fixed assets, but are included in GFCF accounts only if their use in the economy is related to work activities, i.e. within the scope of production. Hence, an automobile purchased for personal use does not constitute GFCF but is rather accounted for as final consumption expenditure of households – Table 4. However, under the innovative definition of variables according to the two-sector modelling framework proposed in our work, automobiles constitute capital assets actively participating in the conversion of final energy/exergy to useful exergy, and therefore constitute investment expenditure in the extended energy sector (E-Sector) of the economy, regardless of being used in economic productive processes or leisure.

Non-produced assets such as land, mineral reserves, and natural resources (water, primary forests, etc.), as well as repair work and purchases of household durable equipment, are excluded from official measures for GFCF. In relevant literature and databases, detailed breakdowns of GFCF are available according to:

- Type of asset (plant, machinery, land improvements, buildings, vehicles, etc.);
- Industry (manufacturing, construction, services, etc.);
- Economic sector (residential versus non-residential, government sector versus private sector, market sector versus non-market sector, etc.).

According to the definitions provided by the European System of Accounts (ESA 95), only produced fixed non-financial assets constitute GFCF – Figure 4.

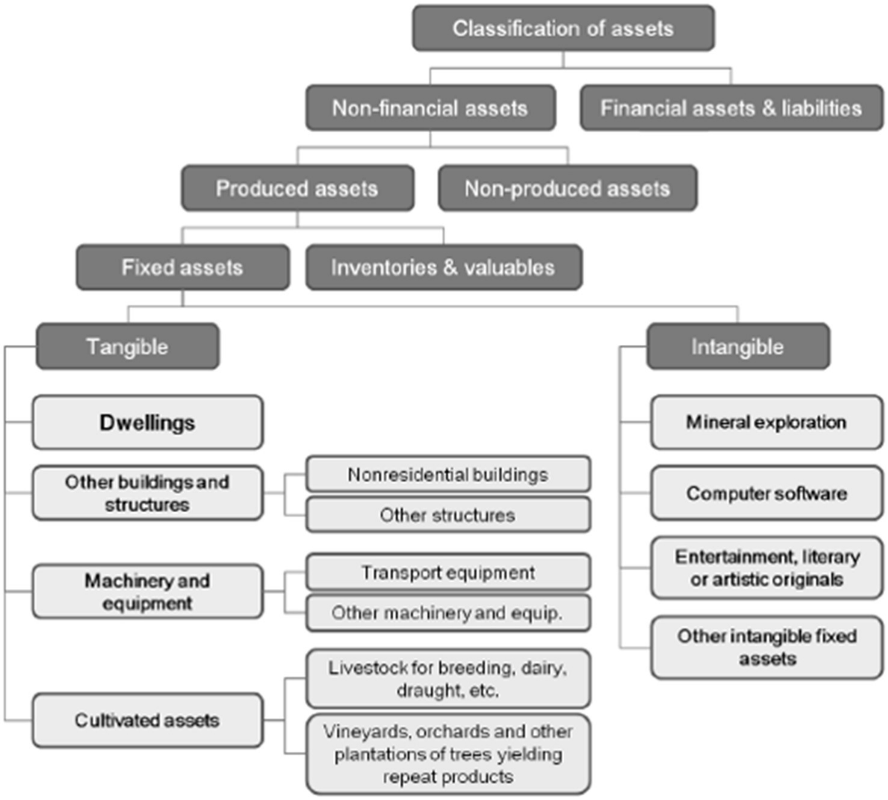


Figure 4 – Classification of fixed assets according to the European System of Accounts (ESA95). Breakdown of non-financial produced fixed assets by type of asset.

For the analysis conducted in our work, GFCF is decomposed in terms of asset type – construction (dwellings, non-residential construction, and civil engineering); and equipment (metal products, machinery and transport equipment) – and aggregated according to the two-sector model's variables concerning investment expenditure in the extended energy sector (E-Sector) $I^{E(C)}$, and in the non-energy sector (NE-Sector) $I^{NE(C)}$. Total investment expenditure in the extended energy sector's production processes I^{E} is equivalent to the sum of consumption expenditure components redefined as investment in this sector – $I^{E(H)}$ and $I^{E(G)}$ – and $I^{E(C)}$.

The GFCF asset categories for construction are entirely allocated to investment expenditure in the non-energy sector production processes $I^{NE(C)}$ – Table 8. The bulk of the dwellings category refers to buildings and structures used for residence. Some types of residences included in this category, such as mobile homes and caravans, could be considered as investment expenditure in the extended energy sector $I^{E(C)}$, due to their participation in the conversion of final energy/exergy to useful exergy. However, these components of the dwellings category in GFCF are considered negligible. The non-residential construction and civil engineering GFCF category concerns warehouse, industrial and commercial buildings, mostly. Other structures such as roads, streets, tunnels, harbors, and airfield runways are also included in this category. For the analysis in our work, the category Other Investment is split in half and each fraction allocated to E-Sector and NE-Sector investment expenditure. Cultivated assets include livestock for breeding, dairy, draught, vineyards, orchards, and other plantations of trees yielding repeat products. Intangible fixed assets consist mainly of mineral exploration, computer software, entertainment and literary/artistic originals. The former constitute investment in the extended energy sector, while the latter constitute investment in the non-energy sector production processes.

Transport equipment is entirely allocated to investment expenditure in the extended energy sector $I^{E(C)}$. This GFCF category includes motor vehicles, motorcycles, railways and tramway locomotives, and aircrafts, all of which participate in the conversion of final energy/exergy to useful exergy – namely in the form of mechanical drive.

The GFCF category for metal products and machinery includes assets that participate in the conversion from final energy/exergy to useful exergy (e.g. office machinery, communication equipment, agricultural machinery, etc.) and also assets which constitute investment expenditure in the NE-Sector. Analogous to the assumptions adopted earlier for COICOP and COFOG categories for consumption expenditure, this asset category is equally split between $I^{NE(C)}$ and $I^{E(C)}$.

The fraction of GFCF allocated to investment expenditure in the NE-Sector constitutes the entirety of investment expenditure in this sector of the proposed two-sector model: $I^{NE} = I^{NE(C)}$. Investment expenditure in the extended energy sector is equal to the sum of the fraction of GFCF allocated to this sector's investment expenditure $I^{E(C)}$, and the consumption expenditure categories redefined as investment expenditure in this sector – $I^{E(H)} + I^{E(G)}$. Total investment expenditure for both sectors of the proposed two-sector model is then

$$\hat{I} = I^{NE} + I^{E} = I^{NE(C)} + I^{E(C)} + I^{E(H)} + I^{E(G)} \qquad (12)$$

2.2.3. Capital stock and depreciation

In order to accurately estimate annual time series for capital stock (K) pertaining to each of the sectors in the proposed two-sector model for the economy – K^{NE} and K^{E} – it is first necessary to take into account the depreciation of capital assets, which requires determining the annual datasets for the consumption of fixed capital (CFC) in each sector – CFC^{NE} and CFC^{E} – as well as the initial values[xxvii] of capital stock for each sector – $K^{NE}(1960)$ and

$K^E(1960)$. The respective annual capital stock datasets for each sector are then computed through a perpetual inventory method[xxviii]:

$$\begin{cases} K^{NE}(t+1) = K^{NE}(t) - CFC^{NE}(t) + I^{NE}(t) \\ K^{E}(t+1) = K^{E}(t) - CFC^{E}(t) + I^{E}(t) \end{cases} \tag{13}$$

Consumption of fixed capital (CFC) is a term used in business and national accounts to measure the amount of fixed capital that is used up, each year, in the process of generating new output. The measurement CFC may also incorporate – beyond actual depreciation charges – expenses incurred in using or installing fixed assets. Through simple association, CFC datasets corresponding to investment expenditure in each of the two sectors considered in our proposed economic model is estimated as:

$$\begin{cases} CFC^{NE} = \frac{I^{NE}}{GFCF} \times CFC \\ CFC^{E} = \frac{I^{E}}{GFCF} \times CFC \end{cases} \tag{14}$$

Initial values for capital stock datasets concerning each of the two sectors – $K^{NE}(1960)$ and $K^E(1960)$ – are determined also by simple association, from data on total capital stock K, $GFCF$, and CFC. Initial values for capital stock in the extended energy sector (E-Sector) and NE-Sector are hence given by:

$$\begin{cases} K^{NE}(1960) = \frac{I^{NE}(1960)}{GFCF(1960)} \times K(1960) \\ K^{E}(1960) = \frac{I^{E}(1960)}{GFCF(1960)} \times K(1960) \end{cases} \tag{15}$$

Annual datasets for capital stock in both sectors of the proposed economic model – K^{NE} and K^E – are computed through the perpetual inventory method presented in Equation 13. Total annual capital datasets (K) are computed as the sum of K^{NE} and K^E annual datasets.

2.3. Decomposition and reclassification of useful exergy balances

Besides the decomposition and reclassification of macroeconomic variables, an analogous decomposition and reclassification for useful exergy used in NE-Sector production, or directly by consumers, is also required for a detailed and accurate depiction of economic production within our two-sector framework. Namely, annual values must be assigned to primary exergy from natural resource inputs to the extended energy sector's productive processes – B^P –, as well as to useful exergy consumed directly by households – γB^U –, and useful exergy used by the productive processes of the non-energy sector – $(1-\gamma)B^U$. The values for these variables can be computed from the analysis of energy balances from mainstream databases, in combination with alternative sources of data, and the application of the methodology developed in Serrenho et al. (2016).

2.3.1. Decomposition of energy consumption

Decomposition of energy consumed by the economy's productive processes begins at country-level energy balances, discriminating between primary energy supply (E^P), gross energy consumption, energy industry own-uses, and final energy consumption. For an accurate and detailed decomposition and reclassification of energy balances according to the two sectors of the proposed economic model – as well as its conversion to useful exergy figures (see Serrenho et al., 2016) – energy input data should be organized by energy carrier (oil and oil products; coal and coal products; natural gas; combustible renewables; electricity and CHP heat; food and feed for humans and working animals; other non-conventional carriers, including for example wind kinetic energy), and by institutional sector (industry; transport; other, including residential; non-energy uses; and energy industries own-uses).

By analyzing a country's energy balances, and their respective decomposition by institutional sector (and further subdivisions), it is possible to obtain estimates for the total final energy (minus energy industries own-uses) that is directly consumed by households and governments – corresponding to the fraction $(1-\gamma)$ – and that which is used in production processes within the NE-sector – corresponding to the fraction γ. The decomposition and aggregation of energy balances under the corresponding two-sector variables is illustrated in Figure 5.

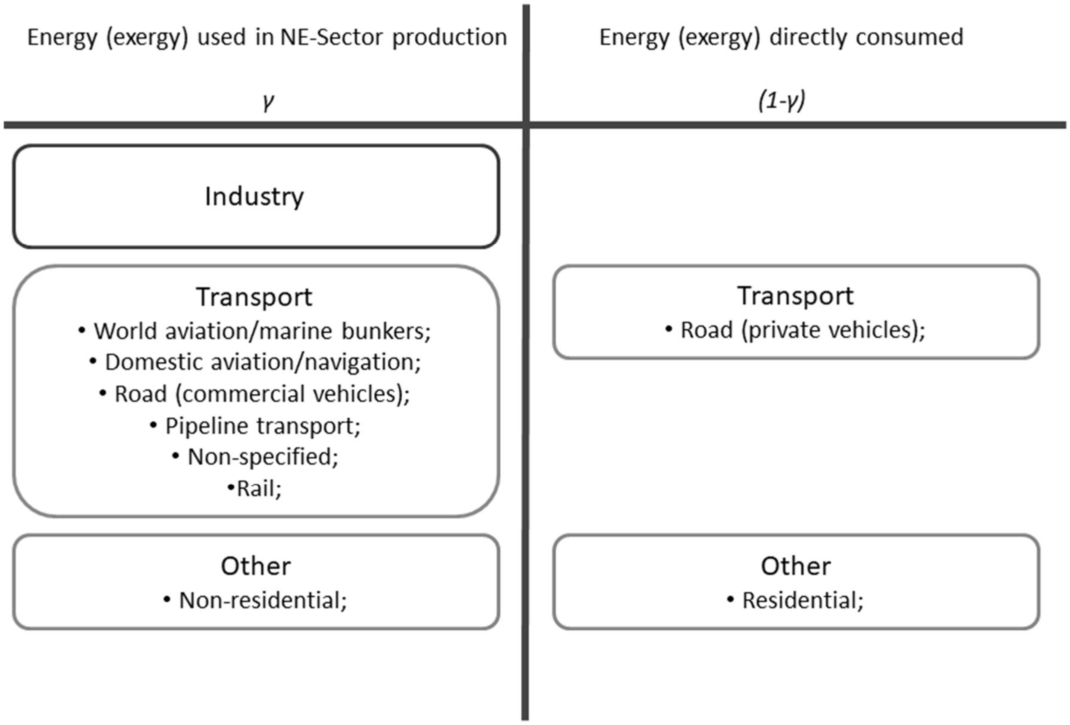


Figure 5 – Disaggregation and reclassification of total final exergy consumption (minus energy industries own-uses) according to direct consumption by households and government, and production processes within the Non-energy Sector.

According to our decomposition and aggregation of energy balances, all final energy (exergy) consumed by the *Industry* sector of the economy will generate useful exergy to be applied in NE-sector production processes. Likewise, the majority of subdivisions in *Transport* and *Other* institutional sectors are also directly allocated to the generation of useful exergy for NE-Sector production processes. Exceptions are the *Transport* subdivision of *Road* – which includes fuels consumed by both private and commercial vehicles – and the *Other* subdivision of *Residential* – which corresponds to the energy consumed in households. Regarding the *Road* subdivision, the distinction between direct consumption $(1-\gamma)$ and use in production γ is made based on the shares of private and commercial vehicles, respectively, registered in a given time period. Regarding the *Residential* subdivision, it is entirely allocated to direct energy consumption $(1-\gamma)$, unlike the remaining subdivisions of the *Other* institutional sector, which are allocated entirely to energy uses in production of goods and services γ.

Non-energy uses are not considered in the empirical analysis undertaken in this work, since the primary goal of these uses is not energetic by definition, but rather material-related. Since they are not involved in the energy-related exploitation of resources[xxix], there is no conversion into useful exergy.

2.3.2. Useful exergy consumption

By adopting the disaggregation and reclassification process described in the previous section, and combining it with the work of Serrenho et al. (2016) on useful exergy accounting, it is possible to allocate the final energy (exergy) supplied by a given energy carrier, to be consumed in a given institutional sector (e.g. the exergy extracted from coal & coal products, and used in the iron & steel industry), with the relevant useful exergy end-uses. Useful exergy end-uses are classified as: heat (high, medium, and low temperature); mechanical drive; light; other electrical uses; and muscle work.

A possible correspondence between economic sectors, energy carriers, and useful exergy end-uses is proposed in Serrenho et al. (2016). In this author's approach, food & feed energy carriers are straightforwardly allocated to muscle work end-uses. Correspondence between other carriers, institutional sectors, and end-uses is carried out by estimating the main useful exergy end-uses in a given institutional sector. For example, exergy extracted from coal & coal products, and consumed by the production processes in the Industry sector are mainly used for high and medium temperature heat generation. The correspondence between energy carriers, institutional sectors and useful exergy end-uses, as proposed by Serrenho et al. (2016) is detailed in Table 3.

Table 3 – Correspondence between final energy carriers, institutional sectors, and useful exergy end-uses. Adapted from Serrenho et al. (2016).

	Industry	Transport	Other	Energy industries own-uses
Coal & coal products	High/medium low heat	Mechanical drive	Low heat	Medium heat
Oil & oil products	High/medium low heat	Mechanical drive	Mechanical drive	Medium heat
Natural gas	High/medium low heat	Mechanical drive	Low heat	High/medium low heat
Combustible renewables	Low heat	Mechanical drive	Low heat	-
Electricity & CHP Heat	Various uses	Mechanical drive	Various uses	Various uses
Food & feed	-	-	Muscle work	-

Given its many applications across institutional sectors (mechanical drive, light, other electrical uses, and heat, across the industry, transport, and other sectors of the economy), the energy carrier related to electricity & CHP heat is the most difficult to accurately allocate relevant end-uses in different institutional sectors.

Given the classification of final energy (exergy) consumption by each of the two sectors in our framework – represented in Figure 5 – and the allocation of corresponding useful exergy by institutional sector, energy carrier, and end-use – summarized in Table 3 – it is possible to construct educated estimates for the fractions of useful exergy directly consumed $(1-\gamma)B^U$, and useful exergy used in NE-Sector production γB^U.

This concludes the simplified disaggregation and reclassification of macroeconomic and energy balances according to two-sector's variables as proposed in our framework for the economy. The next section deals with the empirical application of the methodology discussed in this section to a country-level economy, and the interpretation of obtained outcomes.

2.4. Useful exergy prices

Based on the decomposition and reclassification of national accounts and energy balances described in the sections above, a link between the useful exergy output from the extended energy sector (B^U) and the monetary value attributed to direct consumption of useful exergy generated by that sector (C^E) can be derived, by combining Equations 7-8, as presented in Section 2.1.1, to obtain the following relationship:

$$C^E = (1-\gamma) p_{B^U} B^U, \quad (16)$$

where $(1-\gamma)$ corresponds to the fraction of useful exergy – in physical units – directly consumed by households, firms, and NPISH, while p_{B^U} corresponds to the price attributed to directly consumed unit of useful exergy generated by the extended energy sector. An estimate for this price can then be obtained by rewriting Equation 16 as:

$$p_{B^U} = \frac{C^E}{(1-\gamma)B^U}, \quad (17)$$

By constituting a measure of the monetary value for intermediate inputs in the economy, the estimate for useful exergy prices obtained through Equation 17 can be employed in the computation of gross output-based measures for economic output, possibly providing a solution to the data availability demands imposed by these gross output measures.

3. Results & discussion

The following section presents and discusses the results obtained from the application of the proposed two-sector methodology – detailed in Section 2 – to the Portuguese economy. First, results from the decomposition and reclassification of macroeconomic national accounts are presented, including consumption and investment expenditure, capital stocks, and labor inputs to the economy. Next, results regarding the decomposition and reclassification of energy (and useful exergy) balances are presented and discussed. Finally, results concerning the observed relationships between macroeconomic and energy variables, as defined for the proposed two-sector model of the economy, are presented and interpreted.

3.1. Decomposition and reclassification of National Accounts

For the purposes of this work, the empirical analysis will be restricted to a single country: Portugal. Annual economic data used throughout the empirical analysis – spanning the 54-year period between 1960 and 2014 – is collected from the European Commission's annual macro-economic database (AMECO) and EUROSTAT and compared with additional data collected from the Portuguese national statistics institute (INE), the Bank of Portugal, and the Penn World Tables (Feenstra et al., 2015).

3.1.1. Consumption expenditure

Both the INE and EUROSTAT databases provide statistical data on annual final consumption expenditure by Portuguese households according to COICOP divisions and groups (structure levels 1 and 2, respectively), but not classes (structure level 3). This restricts the aggregation of annual consumption expenditure datasets in our two-sector model to the group level of COICOP, with some loss of detail.

The aggregation of final consumption expenditure from COICOP divisions and groups in terms of variables defined for the two-sector model is conducted as described in Section 2.2.1, and the resulting outcome is presented in Table 4. Here, the percentage of household's final consumption expenditure allocated to C^{NE}, C^{E}, and I^{E} is presented.

Table 4 – Allocation from private household consumption expenditure COICOP divisions and groups, to consumption and investment variables as defined in the two-sector model. Column (1): COICOP divisions (two-digit structure level); Column (2): Selected COICOP groups (three-digit structure level); Column (3) Corresponding two-sector model variables of COICOP structure levels.

Division	Group	Variables
01		$C^{E(H)}$
02		$C^{E(H)}$
03		$C^{NE(H)}$
04	04.1 Actual rentals for housing	$C^{NE(H)}$
	04.2 Imputed rentals for housing	$C^{NE(H)}$
	04.3 Maintenance and repair of the dwelling	$C^{NE(H)}$
	04.4 Water supply and miscellaneous services relating to the dwelling	$C^{NE(H)}$
	04.5 Electricity, gas and other fuels	$C^{E(H)}$
05	05.1 Furniture and furnishings, carpets and other floor coverings	$50-50\%\ C^{NE(H)}\ /\ I^{E(H)}$
	05.2 Household textiles	$C^{NE(H)}$
	05.3 Household appliances	$50-50\%\ C^{NE(H)}\ /\ I^{E(H)}$
	05.4 Glassware, tableware and household utensils	$C^{NE(H)}$
	05.5 Tools and equipment for house and garden	$50-50\%\ C^{NE(H)}\ /\ I^{E(H)}$
	05.6 Goods and services for routine household maintenance	$C^{NE(H)}$
06	06.1 Medical products, appliances and equipment	$50-50\%\ C^{NE(H)}\ /\ I^{E(H)}$
	06.2 Outpatient services	$C^{NE(H)}$
	06.3 Hospital services	$C^{NE(H)}$
07	07.1 Purchase of vehicles	$I^{E(H)}$
	07.2 Operation of personal transport equipment	$C^{E(H)}$
	07.3 Transport services	$C^{NE(H)}$
08	08.1 Postal services	$C^{NE(H)}$
	08.2 Telephone and telefax equipment	$I^{E(H)}$
	08.3 Telephone and telefax services	$C^{NE(H)}$
09	09.1 Audio-visual, photographic and information processing equipment	$50-50\%\ C^{NE(H)}\ /\ I^{E(H)}$
	09.2 Other major durables for recreation and culture	$50-50\%\ C^{NE(H)}\ /\ I^{E(H)}$
	09.3 Other recreational items and equipment, gardens and pets	$C^{NE(H)}$
	09.4 Recreational and cultural services	$C^{NE(H)}$
	09.5 Newspapers, books and stationery	$C^{NE(H)}$
	09.6 Package holidays	$C^{NE(H)}$
10		$C^{NE(H)}$
11		$C^{NE(H)}$
12	12.1 Personal care	$50-50\%\ C^{NE(H)}\ /\ I^{E(H)}$
	12.2 Prostitution	$C^{NE(H)}$
	12.3 Personal effects	$C^{NE(H)}$
	12.4 Social protection	$C^{NE(H)}$
	12.5 Insurance	$C^{NE(H)}$
	12.6 Financial services n.e.c.	$C^{NE(H)}$
	12.7 Other services n.e.c.	$C^{NE(H)}$

Disaggregated annual data on final consumption expenditure by COICOP groups is available in the INE and EUROSTAT databases for the period between 1988 to 2014. Additionally, annual percentages for COICOP divisions and groups in relation to total household consumption expenditure are also provided by these sources. On the other hand, the AMECO database provides statistical data on Portuguese annual private consumption expenditure for a wider period, spanning the decades between 1960 and 2014. However, the AMECO datasets fail to discriminate between COICOP structure levels. Hence, in order to obtain annual datasets for final household consumption expenditure disaggregated by COICOP structure levels, and comprehending the period 1960-2014, we opt to: 1) estimate the percentages in total final consumption expenditure attributed to the two-sector model's variables $C^{NE(H)}$, $C^{E(H)}$, and $I^{E(H)}$, for the period 1988-2014, from INE/EUROSTAT data – Table A1 in the Appendix; 2) extrapolate on the observed trends for these percentages for the period 1960-1987 – Figure 6.

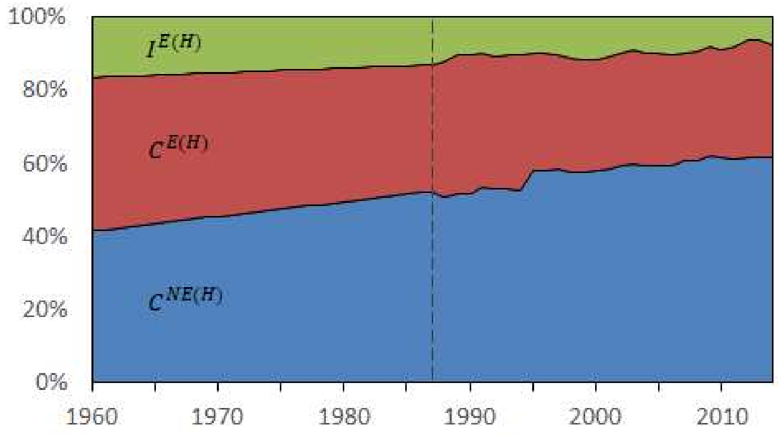


Figure 6 - Percentages of private household consumption expenditure allocated to specific two-sector model consumption and investment variables (1960-2014).

From Figure 6, we observe that the percentage for $C^{NE(H)}$ in total household consumption expenditure exhibits a steady growth from 1988 to 2014. The variation of annual percentages of $C^{NE(H)}$ in household consumption is approximately linear for this period, and hence annual percentages for the period 1960-1987 are extrapolated assuming the same linear trend extending to the past (growing at approximately 0.4% per year). The same process is repeated for the observed percentages for $C^{E(H)}$ in traditional household consumption expenditure[xxx] (shrinking at approximately 0.3% per year). Annual percentages for $I^{E(H)}$ in total final consumption expenditure decrease between 1988 and 2014 and can be determined for the period 1960-1987 from the annual percentages of $C^{NE(H)}$ and $C^{E(H)}$. Combining these percentages of total final consumption expenditure with the annual datasets for private consumption expenditure from the AMECO database, it is possible to obtain a sufficiently accurate dataset for household consumption expenditure under the key variables for the proposed two-sector model.

Analogously to private final consumption expenditure by COICOP structure levels, the available INE/EUROSTAT datasets include government consumption expenditure by COFOG divisions and groups, but not classes. Aggregation of government consumption expenditure under the two-sector model's variables must be undertaken at the group level of COFOG. Table 6 present this aggregation for the Portuguese economy.

The methodology adopted to obtain annual datasets for final government expenditure according to COFOG divisions and groups from the available EUROSTAT, INE, and AMECO databases is analogous to the methodology employed for private consumption expenditure according to COICOP divisions and groups. Percentage annual datasets for every COFOG group – in relation to total government consumption – for the period between 1995 and 2014, are provided in EUROSTAT. In Figure 7, it can be observed that the percentage for $I^{E(G)}$ in government consumption expenditure exhibits a steady (although small) decline from 1995 to 2014. Annual percentages of $I^{E(G)}$ for the period 1960-1994 are extrapolated assuming this same linear trend extending to the past. For the period 1995-2014, the two-sector variable corresponding to government consumption expenditure for the extended energy sector $C^{E(G)}$ exhibits a trendless behavior and its annual values for the remaining years (1960-1994) are assumed to be identical to their respective averages for the 1995-2014 period (a very small percentage – approximately 0.1%). The annual percentages attributed to $C^{NE(G)}$ in total government consumption expenditure are estimated from the annual percentages for the other components. All EUROSTAT percentages and average values are presented in Table A2 in the Appendix.

The decomposition and reclassification of consumption expenditure accounts according to variables for the two-sector model is followed by the computation of total final consumption expenditure for the two-sector framework. This is done according to the formulation presented in Equation 25.

A graphical comparison between total consumption expenditure as presented in macroeconomic national accounts, and total consumption expenditure according to the two-sector model is presented in Figure 8.

The annual variation for total consumption – whether from national accounts or as defined in the two-sector model – is very similar throughout most of the considered period. Both series show the same positive and negative trends for the past decades, including slower growth periods around 1980-85 and 1992-95. Both periods coincide with economic contractions in the country. After 2008, growth in total consumption slows down (and is eventually inverted, in 2010), possibly due to the negative influence in consumption patterns from the economic crisis of that year, as well as the austerity policy measures adopted by the government in its wake.

Figure 8 also presents the percentage of total final consumption – as defined in the two-sector model – allocated to consumption expenditure in the extended energy sector, C^{E}, and the non-energy sector, C^{NE}. Consumption of goods and services produced in the NE-Sector seems to have increased (from 57% in 1960 to 75% of total consumption expenditure by 2014) at the expense of E-Sector consumption (decreasing from 43% in 1960 to 25% of total consumption expenditure by 2014). According to the data, this can be mostly justified by an observable decrease in consumption expenditure from the C^{E} structure levels of *Food and non-alcoholic beverages (01)*, and *Alcoholic beverages, tobacco and narcotics (02)*, accompanied by a combined increase in consumption expenditure from the C^{NE} structure levels of *Imputed rentals for housing (04.2)*, and *Restaurants and hotels (11)*.

Table 6 - Allocation from government consumption expenditure COFOG divisions and groups, to consumption and investment variables as defined in the two-sector model. Column (1): COFOG divisions (two-digit structure level); Column (2): Selected COFOG groups (three-digit structure level); Column (3): Corresponding two-sector model variables of COFOG structure levels.

Division	Group	Variables
01		$C^{NE(G)}$
02		$C^{NE(G)}$
03		$C^{NE(G)}$
04	04.1 General economic, commercial and labor affairs	$C^{NE(G)}$
	04.2 Agriculture, forestry, fishing and hunting	$C^{NE(G)}$
	04.3 Fuel and energy	$C^{E(G)}$
	04.4 Mining, manufacturing and construction	$C^{NE(G)}$
	04.5 Transport	$50-50\%\ C^{NE(G)}\ /\ I^{E(G)}$
	04.6 Communication	$50-50\%\ C^{NE(G)}\ /\ I^{E(G)}$
	04.7 Other industries	$C^{NE(G)}$
	04.8 R&D Economic affairs	$C^{NE(G)}$
	04.9 Economic affairs n.e.c.	$C^{NE(G)}$
05		$C^{NE(G)}$
06	06.1 Housing development	$C^{NE(G)}$
	06.2 Community development	$C^{NE(G)}$
	06.3 Water supply	$C^{NE(G)}$
	06.4 Street lighting	$50-50\%\ C^{NE(G)}\ /\ I^{E(G)}$
	06.5 R&D Housing and community amenities	$C^{NE(G)}$
	06.6 Housing and community amenities n.e.c.	$C^{NE(G)}$
07	07.1 Medical products, appliances and equipment	$50-50\%\ C^{NE(G)}\ /\ I^{E(G)}$
	07.2 Outpatient services	$C^{NE(G)}$
	07.3 Hospital services	$C^{NE(G)}$
	07.4 Public health services	$C^{NE(G)}$
	07.5 R&D Health	$C^{NE(G)}$
	07.6 Health n.e.c.	$C^{NE(G)}$
08		$C^{NE(G)}$
09		$C^{NE(G)}$
10		$C^{NE(G)}$

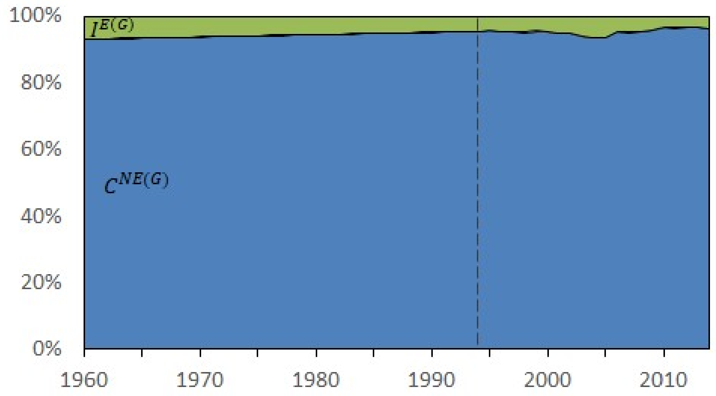


Figure 7 – Percentages of government consumption expenditure allocated to specific two-sector model consumption and investment variables (1960-2014).

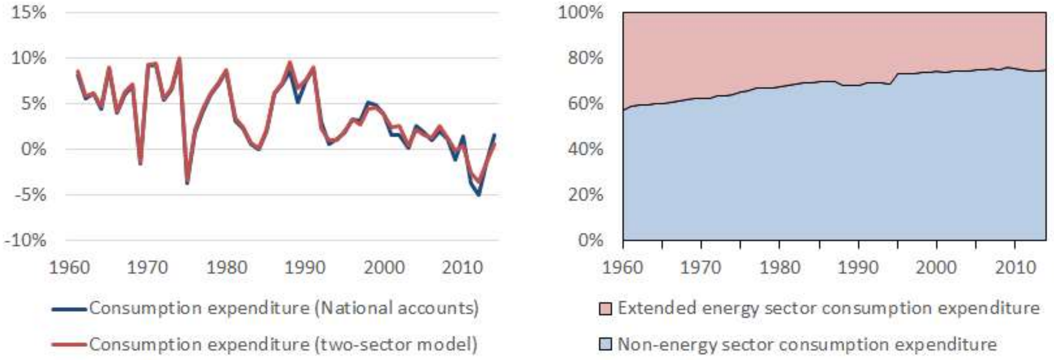


Figure 8 – Decomposition and reclassification of consumption expenditure for the Portuguese economy (1960-2014). Left: annual variation of traditional consumption expenditure (C), and two-sector model consumption expenditure ($\hat{C}$). Right: Shares of E-Sector (C^{E}) and NE-Sector (C^{NE}) consumption expenditure, in relation to total two-sector consumption expenditure ($\hat{C}$).

After 1990, the estimated total consumption expenditure for the two-sector model exhibits a less pronounced variation than the total consumption expenditure in standard national accounts. The difference is due to consumption expenditure items redefined as investment in the extended energy sector, which are absent from the final consumption expenditure series for the two-sector model. Namely, the allocation of the COICOP structure level *Purchase of vehicles* (07.1) entirely to investment expenditure in the extended energy sector, is responsible for this difference.

3.1.2. Investment expenditure

The European Commission's AMECO database provides datasets on country-level GFCF by type of asset – at constant 2010 market prices – between 1960 and 2014, for the Portuguese economy. The types of assets covered by the AMECO database are: construction (dwellings, non-residential construction, and civil engineering); and equipment (metal products, machinery and transport equipment).

According to the ESA95, GFCF incorporates cultivated assets, as well as intangible fixed assets. However, the AMECO database does not provide datasets concerning these types of assets for the Portuguese economy, at constant prices. Both the EUROSTAT and INE databases provide only datasets at current prices, and only for the years between 1995-2014. However, the AMECO database does account for a component of GFCF dubbed *Other Investment*, in which cultivated and intangible assets are likely aggregated together. Future iterations of the work developed here should seek a more detailed accounting of cultivated and intangible assets. Specifically, a detailed accounting of cultivated assets should prove important for earlier decades of Portuguese economic history (1960-1970), a time in which this type of assets have assumed a more relevant role in the economy. The same can be said regarding intangible assets (e.g. software) for later decades. The GFCF decomposition by asset types and aggregation according to two-sector model's investment expenditure variables is represented in Table 8.

Table 8 – Allocation from gross fixed capital formation asset types to investment expenditure variables as defined in the two-sector model. Columns (1) and (2): Categories and subcategories of GFCF by asset type; Column (3): Corresponding two-sector model variables GFCF asset types.

Category	Type of asset	Variable
Construction	Dwellings	$I^{NE(C)}$
	Non-residential construction and civil engineering	$I^{NE(C)}$
Equipment	Transport equipment	$I^{E(C)}$
	Metal products and machinery	50% $I^{NE(C)}$ 50% $I^{E(C)}$
Other Investment		50% $I^{NE(C)}$ 50% $I^{E(C)}$

Analogously to total consumption expenditure, a comparison between annual growth in investment expenditure as defined in national accounts, and investment expenditure as defined in the two-sector model, is presented in Figure 9, along with the percentage of NE-Sector and E-Sector investment expenditure in relation to total investment expenditure as defined in the two-sector model.

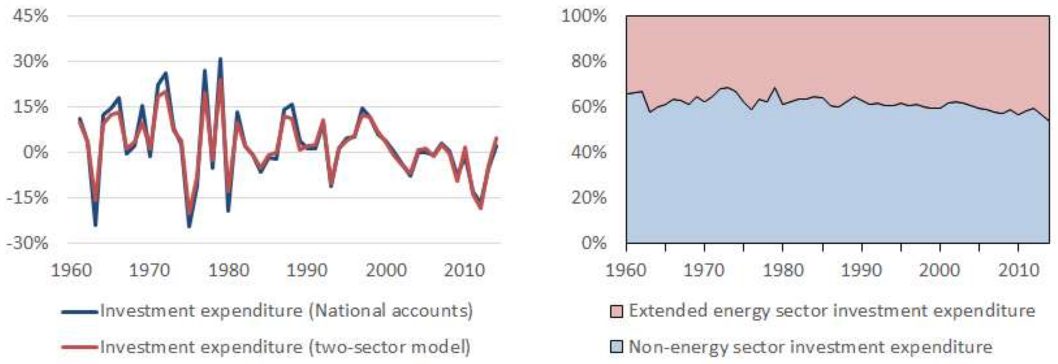


Figure 9 – Decomposition and reclassification of investment expenditure for the Portuguese economy (1960-2014). Left: annual variation of traditional investment expenditure ($GFCF$), and two-sector model investment expenditure ($\hat{I}$). Right: Shares of E-Sector (I^{E}) and NE-Sector (I^{NE}) investment expenditure, in relation to total two-sector investment expenditure ($\hat{I}$).

As described in Section 2.2.2, investment expenditure as defined in the two-sector model incorporates not only traditionally defined investment expenditure, but also selected traditionally defined consumer goods, which participate in energy conversion processes within the extended energy sector. Hence, investment expenditure as defined in the two-sector model will assume higher values than investment expenditure as defined in national accounts, throughout the entire period considered in this analysis. This is due to traditionally defined consumer goods being redefined as investment expenditure in the extended energy sector, I^E.

The graph on the right of Figure 9 shows an increasing share of I^E in total investment expenditure. According to the data, the main contributor to this growth can be found in the *Metal and Machinery* category in GFCF national accounts, which increases significantly throughout this period. Growth in *Medical products, appliances, and equipment (06.1)* – a former consumption expenditure category, now redefined as I^E – also contributes to the increase in investment expenditure in the extended energy sector.

As pointed out by the historical analysis of its economic development (Lains & da Silva, 2005), Portugal has shown a high propensity to invest in the second half of the 20th century. The beginning of the 1960s marked a profound transformation for the Portuguese economy, with higher competition and permeability with respect to external technological innovations. Investment expenditure drops slightly in 1962-63, due to uncertainty generated with the start of the colonial wars. After that, investment expenditure rises, reaching a peak just before 1974, the year of the democratic revolution in the country. The post-revolutionary period (1975-76) is characterized by both political and institutional instability, accounting for a significant drop in investment. The first decade of democracy was accompanied by uncertainty, generated by a slow institutional consolidation, as well as a difficult international conjuncture. A conjectural drop in investment expenditure occurs during the stabilization programme of 1983-85. After joining the European Economic Community (EEC), the Portuguese economy benefited from greater political stability.

3.1.3. Total output for the two-sector model

Consumption and investment expenditure in the two-sector model are combined in order to obtain estimates for aggregate gross domestic product (GDP, minus imports/exports), as well as its composition – throughout the considered period – based on the consumption and investment expenditure variables defined for the two-sector model – C^{NE}, C^E, I^{NE}, and I^E. This is illustrated by Figure 10.

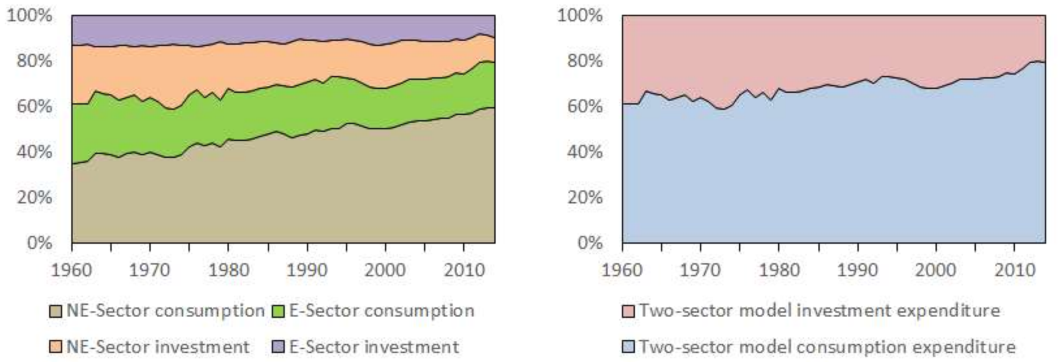


Figure 10 – Two-sector model output for the Portuguese economy (1960-2014). Left: Shares of sector-specific consumption and investment expenditure variables composing the two-sector model's output. Right: Shares of two-sector model's consumption and investment expenditure composing the two-sector model's output.

It can be immediately observed from the graph on the left of Figure 10 that GDP shares regarding expenditure from the extended energy sector (C^E and I^E, in green and purple, respectively) exhibit a less pronounced variation over time than NE-Sector related variables (C^{NE} and I^{NE}, in grey and orange, respectively). The graph shown on the right of Figure 10 suggests a trend towards an increase in total consumption expenditure over the past 50 years for Portugal, at the expense of a complementary decrease in investment expenditure[xxxi]. The graph on the left of Figure 10 illustrates how this increasing trend is mostly due to a rise in consumption expenditure in non-energy sector produced goods and services (C^{NE}, in grey), and a corresponding decrease in investment expenditure in that same sector (I^{NE}, in orange). Meanwhile, the share in GDP of directly consumed useful exergy (C^{E}, in green) remains stable throughout the 50-year period. The share of investment expenditure in the extended energy sector (I^{E}, in purple) decreases slightly in the same period, with a starting value of approximately 13% of total GDP in 1960, and reaching 10% of GDP in the latter years.

In the two-sector model – as shown in Figure 10 (left) – investment expenditure in the non-energy sector is shrinking, while consumption expenditure on goods and services generated by this sector is growing. Based on these results, it would seem that the economy cannot do without expenditure on its exergy inputs.

3.1.4. Capital stocks

Annual capital stock datasets for the two-sector model are built based on Equations 14-16, described in Section 2.2.3. For the Portuguese economy, the AMECO database provides datasets for consumption of fixed capital (CFC), at current prices, between 1960 and 2014. These datasets can be converted into real values – at constant 2010 prices – through an appropriate price deflator[xxxii]. The AMECO database also provides annual datasets on total capital stock, K, from gross fixed capital formation (GFCF) and CFC datasets.

A comparison between capital stock datasets as defined in national accounts, and as defined in the two-sector model, is presented in Figure 11, along with the corresponding shares for capital stock for the two-sector model's non-energy sector and extended energy sector – K^{NE} and K^{E}, respectively.

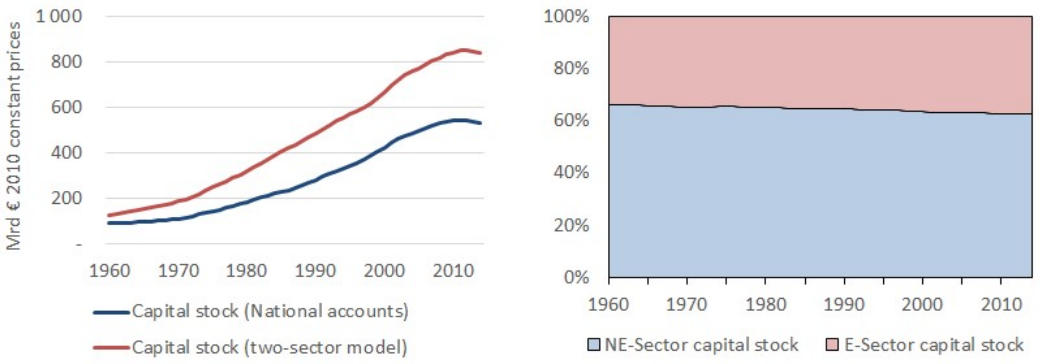


Figure 11 – Reclassification of capital stock used in production, for the Portuguese economy (1960-2014). Left: traditional capital stock estimates (K), and two-sector model capital stock estimates ($\widehat{K}$). Right: Shares of E-Sector (K^{E}) and NE-Sector (K^{NE}) capital stock, in relation to total two-sector capital stock ($\widehat{K}$).

Before interpretation of the observed results in Figure 11, it should be reiterated that both the consumption of fixed capital (CFC) and capital stock annual datasets for the two-sector model are computed, in our work, under rather simplifying assumptions, in order to facilitate the simple analysis intended by the authors. In reality, and since various traditionally consumer goods are redefined as investment expenditure in the two-sector model, a more detailed analysis concerning the decomposition and reclassification of capital stocks and depreciation should be undertaken. The initial values determined for K^{NE} and K^{E}, and used in the perpetual inventory method – Equation 14, Section 2.2.3 – are also the product of simplifying assumptions adopted in our work.

The capital stock series estimated for the two-sector model is expected to assume higher values than the corresponding capital stock series from national accounts, since we are reclassifying several consumption expenditure goods as capital investment in our model. As for the decomposition of capital inputs to each of the two sectors proposed in our model, there seems to be a slight increase in the capital share to the extended energy sector, accompanied by a corresponding decrease in the share of total capital to NE-Sector production. As explored in Section 3.2, this may suggest that capital productivity to the extended energy sector slightly decreases in the considered time period.

3.1.5. Useful exergy

Energy balances for the Portuguese economy are obtained directly from the International Energy Agency's database, for the period between 1960 and 2014. These datasets concern – for all institutional sector – energy extracted from the following energy carriers: coal & coal products, oil & oil products, natural gas, combustible renewables, and electricity & CHP heat. Besides the aforementioned carriers, data on energy extracted from the energy carriers of food for humans, and feed for working animals, are obtained from the work of Henriques (2011). Useful exergy data corresponding to each energy carrier, institutional sector, and end-use is compiled by Serrenho et al. (2016) for the Portuguese economy. One of the most interesting observations made by Serrenho et al. (2016) is that – for Portugal – useful exergy intensity has been remarkably constant in the past 150 years, indicating a close relationship between this energy metric and economic output. Figure 12 shows final and useful exergy intensity for Portugal for the period 1960-2014.

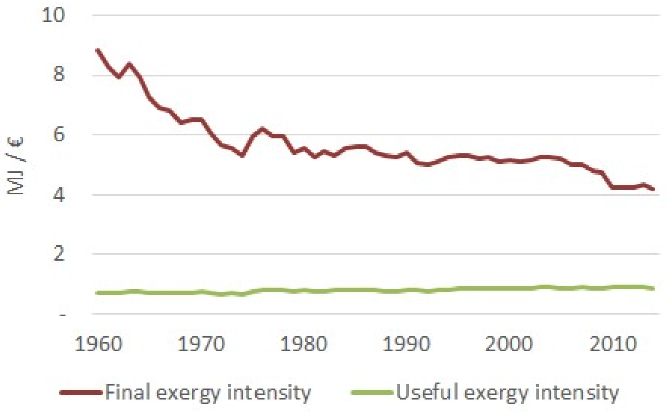


Figure 12 - Final and useful exergy intensities for Portugal (1960-2014). Monetary values in 2010 constant prices.

All useful exergy obtained from food carriers is allocated to directly consumed useful exergy by households and government $(1-\gamma)B^U$, while useful exergy obtained from animal feed carriers (e.g. agricultural use) is allocated to NE-Sector production processes γB^U. As for other non-conventional energy carriers (e.g. wind and water streams), while relevant data is also available from the work of Henriques (2011), their weight in total final energy consumption is significantly small, and therefore these carriers are disregarded in the present empirical analysis.

Concerning the allocation between energy carriers, institutional sectors and useful exergy end-uses discussed in Section 2.3.2, this is applied to the Portuguese economy, with electricity uses in the Portuguese economy estimated from the evolution of shares of alternative electricity uses reported in the UK and US – see Serrenho et al. (2016).

Figure 13 illustrates the shares of useful exergy (B^U) allocated to direct consumption by households and governments, and NE-Sector production processes, according to the proposed two-sector model for the economy.

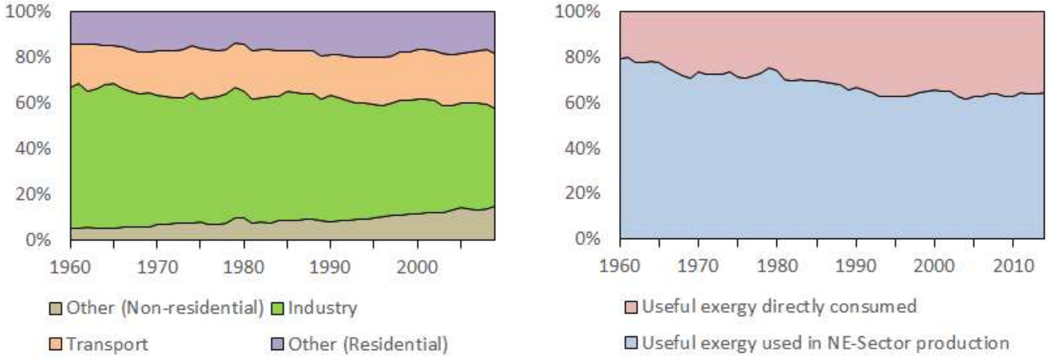


Figure 13 – Decomposition and reclassification of useful exergy balances for the Portuguese economy (1960-2014). Left: useful exergy consumed by institutional sector. Right: shares of useful exergy consumption allocated to NE-Sector production processes (γ) and household and government direct consumption ($1-\gamma$).

The fractions of total useful exergy consumed – discounting energy industries own-uses within the extended energy sector – both directly by households and government ($1-\gamma$), as by production processes within the non-energy sector (γ), appear to be relatively stable throughout the considered time period, for the Portuguese economy. Not surprisingly, the largest share of useful exergy consumed corresponds to useful exergy employed by NE-Sector production – between 62% and 80%. The smallest share, corresponding to useful exergy directly consumed by households and government, oscillates between shares of 20% and 38%. This can be seen on the right graph of Figure 13.

The graph on the left of Figure 13 shows the share of useful exergy consumed by each of the represented institutional sectors of the economy. Initially, the largest share of useful exergy is applied in the Industry sector (approx. 62% by 1960). However, by the end of the period, this percentage drops to approximately 45%, while the shares of useful exergy consumed in the Transport (Other) sector rises from approximately 19% (20%) to 22% (34%) throughout the same period.

3.2. Two-sector model results

This section presents preliminary results obtained by combining observations on the evolution of both macroeconomic and energy use variables for the Portuguese economy – under the proposed decomposition and reclassification of variables in our two-sector framework – in order to obtain new insights to the relationships between these variables.

As described in Section 2.4, an estimate for the price paid for useful exergy directly consumed can be obtained with Equation 16, from the decomposition and reclassification of national accounts and energy balances for a given economy. Based on the results of the decomposition and reclassification methodology for the Portuguese case, it can be inferred that this price paid for directly consumed useful exergy should be decreasing in the long-run.

Dividing both sides of Equation 16 by total output for the two-sector economic model Q, one obtains

$$\frac{C^E}{Q} = (1-\gamma)p_{B^U}\frac{B^U}{Q}, \qquad (18)$$

In Equation 18, the share of total expenditure (identical, in value, to total output Q, and given by Equation 6) devoted to consumption of direct useful exergy generated by the extended energy sector (C^E/Q) is – according to Figure 8 – decreasing throughout the 50-year period studied for the Portuguese economy, with a compound annual growth rate (CAGR) of approximately -0.49%. On the other hand, the fraction of useful exergy – in physical units – generated by the extended energy sector and directly consumed by households, firms, and NPISH $(1-\gamma)$ has – according to Figure 13 increased throughout the same 50-year period, with a CAGR of approximately 1.03%.

Since the last term on the r.h.s. of Equation 18 is approximately constant throughout the period from 1960 to 2014 for the Portuguese economy – as shown in Figure 12 – it follows that in order for Equation 18 to hold for the empirical data observed for the Portuguese economy, the price paid for useful exergy directly consumed p_{B^U} must decrease throughout this period, with a CAGR of approximately -0.38% – Figure 14.

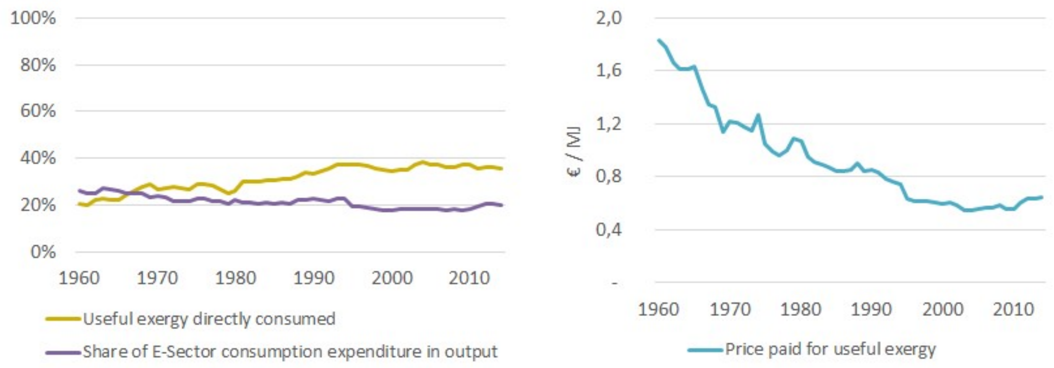


Figure 14 – Left: Share of useful exergy directly consumed by households and government $(1-\gamma)$, in yellow, and share of consumption expenditure associated with the extended energy sector, in total output (C^E/Y), in purple; Right: Estimated price paid for useful exergy directly consumed by households and government (p_{B^U}), in light blue.

In fact, the price paid for useful exergy directly consumed by households, firms, and NPISH is estimated to have fallen from approximately 1.85 € per MJ at the beginning of the period (1960) to approximately 0.65 € per MJ by the end of the period. This fall in the price paid for useful exergy is not uniform throughout the period: most of the decline occurs at the beginning of the 50-year period, between 1960 and 1970; for the next two decades, some peaks occur around 1974-75 and 1978-80, but the overall trend is still decreasing (albeit slowly); there is a significant drop around 1994-96, followed by a period (2000-2010) of stagnation around 0.60 € per MJ; finally, the price paid for useful exergy in the Portuguese economy rises at the very end of the 50-year period, with a CAGR of approximately 3.93% between 2010-14.

Both the 1974 and 1978-80 peaks in the price paid for useful exergy in Portugal can be justified by the oil crisis that occurred in 1973 and 1979 (also called oil "shocks")[xxxiii]. Figure 15 shows the decomposition of useful exergy directly consumed by consumers and used in Ne-Sector production, by type of energy carrier.

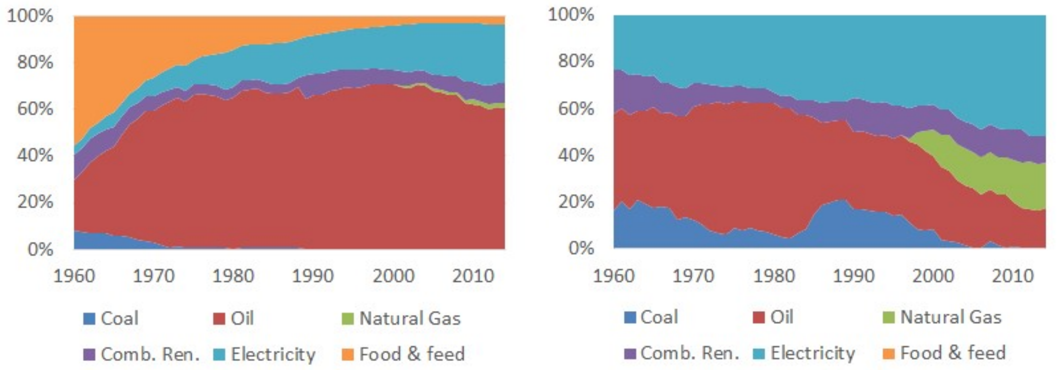


Figure 15 - Shares of useful exergy consumed (in physical units) by type of energy carrier. Left: useful exergy directly consumed by consumers; Right: useful exergy used in NE-Sector production processes.

It can be seen from Figure 15 that the peaks in the price paid for directly consumed useful exergy were likely to be affected by the oil shocks of the 1970s, since useful exergy obtained from oil and oil products constitutes the largest share of directly consumed useful exergy. The second largest energy carrier for directly consumed useful exergy is, at the beginning of the period, food and feed, and, by the end of the period, electricity. As for the decomposition of useful exergy used in NE-Sector production processes by carrier, electricity has a large share (and increasing throughout the period), while oil and oil products become less significant by the end of the period. Coal is also phased out, and natural gas only gains relevance at the end of the period[xxxiv].

The evolution of the estimated price paid for useful exergy in the Portuguese economy is consistent with the argument by Ayres (2001) and Ayres & Warr (2005) that economic growth in industrialized countries has been driven by falling energy prices. Figure 16 compares the evolution of the price paid for useful exergy with the evolution of gross domestic product (GDP) for the Portuguese economy, both in levels and annual growth rates. It is clearly seen that the periods of higher economic growth (e.g. 1960-70) coincide with drops in the price paid for useful exergy. On the other hand, the stagnation in economic growth verified in the last decade for Portugal coincides with a similar stagnation in the price paid for useful exergy. Since 2008, the decline in the GDP has been matched by a rise in the price paid for useful exergy.

As detailed in Section 2.4, the obtained estimates for the price paid for useful exergy (an intermediate input to economic production) can be used to solve data requirement issues in the construction gross output-based measures for economic output, which in turn should lead to improved estimates on TFP and growth.

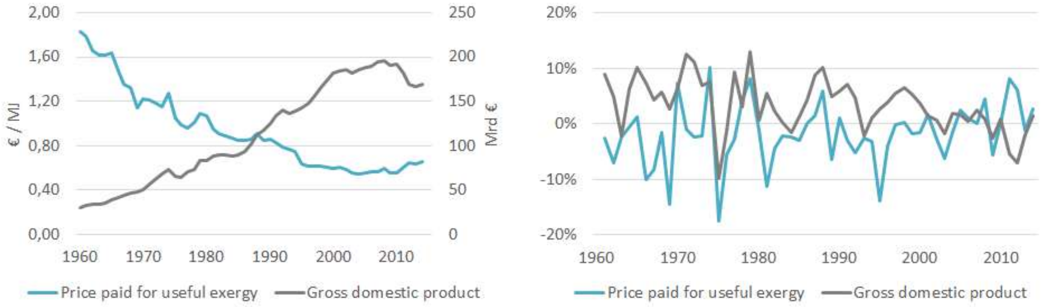


Figure 16 – Left: Comparison of GDP (right axis) and price paid for useful exergy directly consumed (left axis), 1960-2014, in levels. Monetary values at 2010 constant prices. Right: Comparison of GDP and price paid for useful exergy, 1960-2014, in growth rates.

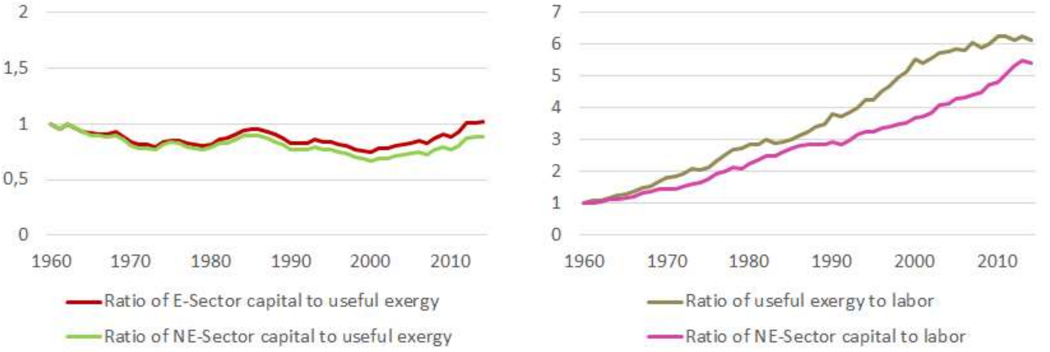


Figure 17 – Relationship between inputs to economic production under the two-sector model. Left: ratio of capital inputs to the extended energy sector (red), and capital inputs to the non-energy sector (green), to total useful exergy inputs; Right: ratios of useful exergy inputs (brown), and non-energy sector capital inputs (pink) to labor inputs. All ratios normalized to the initial year (1960 = 1).

Preliminary observations made with the developed two-sector framework applied to empirical data for the Portuguese economy also concern the ratios of inputs and output to each of the two defined sectors for the economy. Figure 17 (left graph) shows the ratios of capital inputs to the NE-Sector by the useful exergy inputs to that sector (K^{NE}/B^{U}), and capital inputs to the extended energy sector by the useful exergy output from that sector (K^{E}/B^{U}). The apparent stability, throughout the 50-year period, of the K^{NE}/B^{U} ratio suggests that capital and useful exergy inputs to the NE-Sector of the economy might have a complementarity relationship: capital assets require activation by useful exergy in order to be productive. On the other hand, as shown in Figure 16 (right graph), the same relationship does not hold for capital and labor inputs to the NE-Sector (K^{NE}/L), nor for useful exergy and labor inputs to the NE-Sector (B^{U}/L). While the proportion of capital to useful exergy used in the NE-Sector production processes remains fairly constant, the proportions of capital or useful exergy to labor inputs in this sector increases significantly, suggesting the progressive substitution of both capital assets and useful exergy for human labor in production for this sector.

On the other hand, the ratio of capital inputs to the extended energy sector by the useful exergy output from this sector – while also approximately constant throughout the 50-year period – slightly increases, particularly from the 2000s onwards, at a CAGR of approximately 2.23% for the period 2000-2014. The inverse of this ratio is a measure of capital productivity in the extended energy sector, and it decreases at the same approximate CAGR for the same period 2000-2014. This suggests that, over the last two decades of the Portuguese economy, capital invested in the extended energy sector – for the production of useful exergy – has become slightly less productive. One possible justification for this could be tied to the investment in less final-to-useful exergy efficient capital in the extended energy sector. Serrenho et al. (2016), in their useful exergy accounting analysis for the Portuguese economy, indicate a proliferation of individual automobiles in the country after 1980 as a possible explanation for the stagnation of aggregate final-to-useful exergy efficiency (overall efficiency of gasoline powered vehicles is low – around 10%). Since, in our proposed model, purchase of automobiles is accounted as investment in capital for the extended energy sector, the popularization of these vehicles would translate into a decrease in capital productivity for this sector in later decades. Likewise, the proliferation of lower temperature heat uses (with final-to-useful exergy efficiencies ranging from 9-26%) could also justify this decrease in capital productivity for the extended energy sector.

4. Conclusions and future work

Observation and empirical evidence suggest that there is a close relationship linking energy use in productive processes and economic development. However, this relationship is either downplayed or completely disregarded in the most basic theories of economic growth, which at most acknowledge energy consumption as a cause – not a driver – of growth. These models traditionally consider physical capital and human labor as the major primary factors of production, but growth accounting exercises applied to many developed economies reveal that an exogenous residual term – total factor productivity, or TFP – is responsible for the largest share of historical economic growth. This term, not being directly observable (unlike capital and labor inputs), encompasses many components, as well as measurement errors, which cannot be sorted out in the basic theories of economic growth. Nevertheless, it is typically corresponded with “technical change” or “innovation” in the economics literature. A strand of models seeks to endogenize the TFP residual, by treating it as a form of capital accumulation, but these approaches are themselves limited in their inability to justify the slowdown in productivity verified for several developed countries (which some argue to be the new permanent state of these economies, a kind of “secular stagnation”), and in their treatment of energy resources.

By misrepresenting the role of energy in the economy, standard models of economic growth struggle to account for economic crises resulting from energy crises (e.g. the oil shocks of the 1970s) and to acknowledge the constraint on productive activities that the finiteness of energy resources poses. The reasoning for this downplay on the importance of energy in standard models lies in a series of simplifying assumptions that ultimately tie the small share of payments attributed to energy (an intermediate product) in national accounts to its actual productive power, via its output elasticity. One such assumption is that the cost-minimizing behavior of different firms, using only capital and labor, can be generalized to the whole – single-sector – economy. An alternative to this view, providing a more realistic picture of the economy, is to consider production of output as a multi-sector process, from not only capital and labor (nor raw materials and energy), but a chain of intermediates allowing to reconcile a small share for a given factor of production (energy) to contribute a much larger effective share to the value of aggregate production. This is the view argued by, among others, Ayres (2001) and Ayres & Warr (2010), who propose that the declining price of energy has been and still is a driver of growth, through a positive feedback cycle. The authors also argue that the appropriate metric to account for productive energy in the economy is useful exergy, which distinguishes between stages of energy flows (primary-to-final-to-useful) and for energy quality, as energy thermodynamically available to perform physical work (exergy).

When energy inputs to the economy are measured as useful exergy, empirical evidence suggests that historical economic growth can be almost completely explained for the US, without the need for an exogenous TFP multiplier. Thus, a better explanation for historical growth trends can be obtained, based on directly measurable variables. Additionally, the adoption of the useful exergy metric removes some of the ambiguity in statistical tests – conducted mostly in the ecological economics literature – assessing the direction of causality between energy use and economic output. The unidirectional causal effect observed from useful exergy consumption to output supports the positive feedback cycle hypothesis by Ayres (2001) and Ayres & Warr (2010). These results also support the argument that "technical progress" – or TFP, the major driver of economic growth – is almost entirely explained by historical improvements in primary-to-final-to-useful exergy conversion efficiency.

Recent results obtained for the Portuguese economy support these findings. Most notably, the societal useful exergy analysis conducted by Serrenho et al. (2016) detects a strong correlation between useful exergy consumption and economic output for the country, despite dramatic changes in both the economy and the composition of energy consumption. Picking up on this proximity, Santos et al. (2018) have explored, econometrically, the relationships between output, capital, labor, and useful exergy, and found that useful exergy can constrain the other factors of production (and growth), without invalidating neoclassical assumptions regarding shares of payments and productive power. The possible correspondence between TFP and useful exergy efficiency has also been investigated and adopted in the construction of economic and energy scenarios for Portugal, under the MEET2030 project.

Based on the above insights, the present work proposes a two-sector modelling framework for the economy, in which all energy/exergy conversion activities are aggregated under an abstract "extended energy sector", which generates useful exergy (an intermediate product) and provides it to consumers and other productive processes. The definition of the extended energy sector is innovative, in the sense that, because it aggregates not only the traditional energy industries (primary-to-final energy conversion) but also end-use devices (final-to-useful) purchased by either households, firms, or institutions, these end-use devices constitute investment in physical capital in this sector. Hence, the extended energy sector boundary is defined not in terms of economic sectors (energy industries, residential, transport, …) but in terms of the stage of energy flows in which energy actually becomes productive to the economy, while still being accounted for in physical units (Joule).

The definition of the proposed model in these terms, when applied to empirical macroeconomic and energy data (in this work for the Portuguese economy), forces the decomposition and reclassification of standard national accounts and energy balances to match the two-sector model's variables. This is a central issue in the present work, which seeks to establish a methodology for this reclassification, to be improved upon and straightforwardly adopted to other economies. In this methodology, standard national accounts on consumption and investment expenditure are first decomposed – the former according to private/government spending and purpose; the latter according to type of physical capital asset – and posteriorly allocated to each of the consumption and investment variables defined in the two-sector model. The main issue in this approach lies in the redefinition of items which are accounted as consumer goods in standard national accounts, but which actively participate in the conversion of final-to-useful exergy in the economy, and hence are reclassified as investment in the extended energy sector (e.g. automobiles, electrical appliances, etc.). This constitutes the most innovative aspect of the present work. Energy/exergy balances are decomposed by energy carrier, institutional sector, and end-use, in order to be allocated to production of non-energy goods and services, or direct consumption.

The methodology for decomposition and reclassification of macroeconomic and energy accounts according to the proposed model's variables – applied to Portuguese data – results in a set of interesting preliminary observations obtained from this first iteration of the model. These empirical observations in turn provide insights to the relationship linking energy use and economic production.

First, by comparing the estimates obtained in the context of the model for the evolution of useful exergy directly consumed by households, government, and NPISH (i.e. not used in production), both in monetary and physical terms, an estimate for the price paid for useful exergy directly consumed in the economy can be estimated. The evolution for this estimated price shows that it has been declining for most of the studied period but stabilizes in the last two decades. When compared with the evolution of economic output (GDP) for the same period, these two series mirror each other, in the sense that significant drops in the price paid for useful exergy correspond to periods of intense economic growth, while stagnation in both series occurs simultaneously. This provides the arguments of Ayres (2001) and Ayres & Warr (2005) – that a declining price for energy (actually useful exergy) is a major driver of economic growth – with important empirical evidence. It is also consistent with the argument that the phenomenon of productivity slowdown in industrialized economies can be traced to the efficiency of energy(exergy) conversion to more productive forms. The obtained estimates for the price paid for useful exergy (an intermediate input) allow to solve data availability constraints imposed on the construction of gross output-based measures of economic output, which in turn allow for the computation of better indicators for "disembodied technological change", i.e. TFP, and growth.

A second preliminary observation made with the proposed model concerns the ratios of capital inputs to each of the two defined sectors in the model, and useful exergy (as an output, for the extended energy sector; as an input, to the non-energy sector). These ratios provide empirical evidence in support of: 1) the possible complementarity relationship between capital and useful exergy used in NE-Sector production processes (i.e. capital needs to be activated by useful exergy in order to be productive[xxxv]); 2) the possible declining capital productivity in the extended energy sector (i.e. more capital is required to generate the same amount of useful exergy, likely due to less efficient end-uses such as mechanical drive for automobiles.

Both preliminary outcomes from this first application of the proposed two-sector model for the Portuguese economy beg further investigation but constitute nonetheless interesting insights to the modeling of the relationships between energy use and economic production. Furthermore, the two-sector model developed in this work is taken as a stepping stone for richer and more complex future models combining energy/exergy flows and economic production, aiming to better understand (and model) the relationships between energy use and efficiency, macroeconomic factors of production, and economic growth. Despite the already insightful preliminary results obtained with the present model, there is much room for improvement. As a final note, we list some paths for the future development of this model, both in terms of its theoretical construct, and in terms of its application to real macroeconomic and energy data.

- The present two-sector model for the economy relies on simplifying assumptions that both facilitate its analysis, but also distance it from economic reality. Namely, as presented in Section 2.1, we consider a closed economy, with no labor inputs. In order to improve the current model, we must also consider imports and exports to each of the two sectors, as well as a division of labor inputs to both sectors. These improvements to the model will require, for the purposes of empirical application, additional disaggregated data on imports/exports, and hours worked;
- The decomposition and reclassification of national accounts to match the model's energy and macroeconomic variables, as presented in this work, also relies on simplifications, motivated by the lack of more detailed data regarding consumption and investment expenditure. Future iterations of the application of this model to empirical data require: 1) consumption expenditure decomposition at the class structure level of COICOP and COFOG; 2) explicitly accounting for imputed rents to any consumption expenditure good redefined as capital investment expenditure; 3) for expenditure and energy use items that should be split between two or more variables of the model, resort to proxies (if additional data is unavailable) to estimate the share of expenditure allocated to each variables (for example, as was done for energy use for road transport), instead of assuming a "rough" 50-50 split; 4) additional detail to estimates for the consumption of fixed capital (i.e. depreciation) of capital inputs to each of the two sectors;
- The methodological approach to decompose and reclassify national accounts and energy balances exposed in this paper should also be expanded to other countries, in order to either confirm the preliminary results obtained for the Portuguese economy or uncover characteristic observations for different countries. This can be done for countries for which useful exergy accounting has been performed (Serrenho et al., 2014), and depending on the level of detail in each countries' macroeconomic national accounts.

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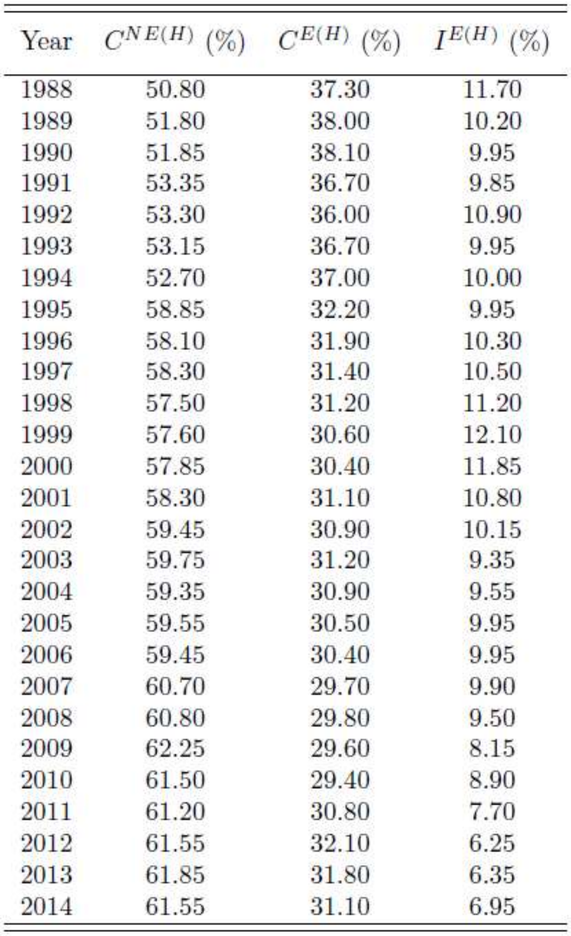
Appendix

Table A1 – Percentages of private household consumption expenditure allocated to specific two-sector model consumption and investment variables (1988-2016).



Year	$C^{NE(H)}$ (%)	$C^{E(H)}$ (%)	$I^{E(H)}$ (%)
1988	50.80	37.30	11.70
1989	51.80	38.00	10.20
1990	51.85	38.10	9.95
1991	53.35	36.70	9.85
1992	53.30	36.00	10.90
1993	53.15	36.70	9.95
1994	52.70	37.00	10.00
1995	58.85	32.20	9.95
1996	58.10	31.90	10.30
1997	58.30	31.40	10.50
1998	57.50	31.20	11.20
1999	57.60	30.60	12.10
2000	57.85	30.40	11.85
2001	58.30	31.10	10.80
2002	59.45	30.90	10.15
2003	59.75	31.20	9.35
2004	59.35	30.90	9.55
2005	59.55	30.50	9.95
2006	59.45	30.40	9.95
2007	60.70	29.70	9.90
2008	60.80	29.80	9.50
2009	62.25	29.60	8.15
2010	61.50	29.40	8.90
2011	61.20	30.80	7.70
2012	61.55	32.10	6.25
2013	61.85	31.80	6.35
2014	61.55	31.10	6.95

Table A2 - Percentages of government consumption expenditure allocated to specific two-sector model consumption and investment variables (1995-2014).

Year	$C^{NE(G)}$ (%)	$C^{E(G)}$ (%)	$I^{E(G)}$ (%)
1995	95.70	0.10	4.20
1996	95.45	0.00	4.55
1997	95.30	0.10	4.60
1998	95.00	0.10	4.90
1999	95.50	0.10	4.40
2000	95.20	0.10	4.70
2001	94.75	0.10	5.15
2002	94.75	0.10	5.15
2003	93.90	0.10	6.00
2004	93.40	0.00	6.60
2005	93.35	0.10	6.55
2006	95.10	0.10	4.80
2007	95.05	0.10	4.85
2008	95.30	0.10	4.60
2009	95.55	0.10	4.35
2010	96.45	0.20	3.35
2011	96.40	0.10	3.50
2012	96.60	0.10	3.30
2013	96.75	0.10	3.15
2014	96.20	0.10	3.70

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[i] Although land and natural resources can also be included.

[ii] Perfect competition also enables prices to be taken as given.

[iii] In developing his model, Solow (1957) was careful to define his technical change term as any shift to the APF, whether from slowdowns, speedups, educational improvements, or other.

[iv] Jorgenson & Griliches (1967) famously advanced the hypothesis that careful measurement of the traditional factors of production – capital and labor – should significantly reduce the TFP residual. The authors then adopt quality-adjusted measures of capital services, and schooling-corrected human labor. While criticized by Denison (1974), the Jorgenson & Griliches (1967) approach contributed to tied data development, growth accounting, and production theory firmly together. Still, the quality adjustment of factors of production is insufficient to "explain away" all of TFP (van Ark, 2014).

[v] Econometrics techniques can be adopted to disaggregate the TFP residual into terms corresponding to technical innovation, efficiency, institutional forces, and others. See, for example, Denny & Fuss (1983).

[vi] Carrying the implication that investment in research and development (R&D) has no systematic predictable effect on economic growth.

[vii] As Solow (1987) put it: "We see the computer age everywhere but in the productivity statistics".

[viii] This model has been subsequently extended with nonrenewable resources, renewable resources, and waste assimilation services. However, these extensions have been applied mostly in the context of environmental sustainability, and not macroeconomics.

[ix] Such as the stagnant economic growth periods resulting from the 1970s oil crisis.

[x] e.g. in coal mining: coal represented a cheap fuel alternative to increasingly scarce charcoal; application of steam engines to coal mining and iron smelting brought down the costs of coal energy, as well as iron and steel; real prices of downstream industrial goods began to fall, along with transportation costs; thus coal itself and all types of manufactured goods became cheaper for all consumers, which resulted in increased intermediate and final demand, and closing the loop.

[xi] Small cost-minimizing "price-taker" firms, operating in a competitive free labor market, with diminishing returns to both labor and capital. Under this approach, what is true for each of these firms must be true for the collectivity of all firms, i.e. the economy as a whole.

[xii] As pointed out by Kümmel et al. (2008), under the assumptions of the cost-share theorem – and assuming that energy inputs receive roughly 5% (<10%) of total income in payments – the energy crisis resulting from the oil shocks of 1973-75, which produced a 5.2% decrease in energy input to the US

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economy, would have only caused a decrease of US output of 0.26%. In reality, the decrease in US output was closer to 1% (four times higher). Hence, the impact from energy crises on economic output is difficult to justify under most neoclassical growth models.

[xiii] Stern & Kander (2012) adopt a gross output-based measure for output in an APF modelling approach to the Swedish economy.

[xiv] It also accounts for energy carriers in secondary form produced by decentralized generation systems, which are not part of the energy sector, e.g. residential solar thermal boilers or stand-alone wind turbines. On the other hand, the direct use of secondary energy carriers by the energy sector is not considered final energy use by the International Energy Agency.

[xv] e.g. the amount of electricity supplied to a lightbulb (final energy) actually converted in the lightbulb to provide light to a room (as opposed to the final energy dissipated as heat by the lightbulb, which does not correspond to the final economic function of the lightbulb).

[xvi] Technically, saying that energy is "used up" is incorrect, since, from the first law of thermodynamics, energy is a conserved quantity, and hence cannot be consumed but only transformed from available to unavailable forms. The correct term for what is meant by energy in most discussions (including the economics literature) is *exergy*, which is defined further ahead in the main text.

[xvii] Assuming that the environment is at 10 °C.

[xviii] The LINEX is an APF in which output depends linearly on energy and exponentially on factor quotients. It was originally derived by Kümmel et al. (1985) by choosing simple mathematical forms for each factor's output elasticities (based on plausible assumptions on asymptotic behavior) and performing partial integrations.

[xix] Among the improvements introduced by Serrenho et al. (2016) to the established useful exergy accounting methodology, the most significant is the disaggregation of final-to-useful efficiencies for different electricity end-uses, instead of considering an aggregate average electricity final-to-useful efficiency (Ayres & Warr, 2005). Also, Serrenho et al. (2016) is one of the only societal exergy analysis to cover the full transition of the economy from rural, through industry, and to services.

[xx] This ratio – taken in logarithms – is of approximately 1.87, implying that a 1% increase in final-to-useful exergy efficiency leads to a 1.87% increase in TFP.

[xxi] Non-profitable institutions serving households.

[xxii] Consumption, investment, physical capital stock, and output are defined in monetary terms (EURO-PTE). Exergy and useful exergy are defined in energy units (joule).

[xxiii] Gross capital formation – GCF.

[xxiv] It is most commonly used in reference to home ownership.

[xxv] Except gross fixed capital formation (GFCF).

[xxvi] Measured over an interval of time, as opposed to a stock value, which is measured at a specific moment in time.

[xxvii] Within the time period under consideration, i.e. 1960-2016.

[xxviii] This method tracks the existing stock of fixed assets through estimation of the amount of fixed assets installed – a result of GFCF undertaken in previous years – that have survived up until the current period.

[xxix] To assess the overall exergy flow in a country, material flow analysis would be needed in order to account for the total chemical exergy of the materials used. For more on this subject see Serrenho et al. (2016), and Ayres & Warr (2009).

[xxx] This is undoubtedly a crude estimation, especially considering the critical behavior of macroeconomic variables before, during, and immediately after the Portuguese Democratic Revolution in 1974.

[xxxi] In the form of GFCF.

[xxxii] The price deflator adopted in our own work was that related to gross fixed capital formation (GFCF) datasets provided in the AMECO database.

[xxxiii] Portugal was one of the countries included in the oil embargo by the members of the Organization of Arab Petroleum Exporting Countries, for their alleged support for Israel in the Yom Kippur War. The embargo was lifted on March 1974. On April 1974, the Democratic Revolution took place in Portugal, a military coup which overthrew the authoritarian regime then in place.

[xxxiv] Natural gas was only introduced in Portugal in 1997.

[xxxv] To quote Steve Keen: "capital without energy is a sculpture".