



Munich Personal RePEc Archive

# **Endogenous Altruism, Learning by Doing Effect and Impact of Domestic Policies on Child Labour**

Chakraborty, Kamalika and Chakraborty, Bidisha

29 September 2018

Online at <https://mpra.ub.uni-muenchen.de/89229/>  
MPRA Paper No. 89229, posted 01 Oct 2018 17:13 UTC

**Endogenous Altruism, Learning by Doing Effect and Impact of Domestic Policies on  
Child Labour**

**Kamalika Chakraborty**

Jadavpur University, India

**Bidisha Chakraborty**

Jadavpur University, India

**Abstract**

This paper builds an overlapping generations household economy model where child labour is present. We argue that the degree of parental altruism is determined by the level of schooling of the parent. A more educated parent has more willingness to invest in human capital formation of child. These differences in preferences of parent towards offspring's schooling bear significant effects on the long run dynamics of schooling. In this context, we study the efficacy of child labour ban vis-a-vis education subsidy in enhancing schooling and reducing child labour. In an extension of the basic model, we also study the dynamics of schooling in the presence of learning by doing effect in unskilled work.

**Keywords:** child labour, schooling, human capital, child labour ban, education subsidy

**JEL Classification Numbers:** J22, J24, J820, I210

## **1. Introduction**

In recent years, two key papers that emphasize on endogenous parental altruism are Mulligan (1997) and Das (2007). According to Mulligan (1997), parental altruism depends on the time spent with children. The time spent with children is optimally decided by the parents on the basis of utility maximization exercise. Since high wage families have higher opportunity cost of time, high income dynasties will spend less time with children and therefore will be less altruistic. On the contrary, Das (2007) assumes that the degree of parental altruism positively varies with the earning ability of the parent. To capture the positive relationship between the degree of parental altruism and parents' economic status the weight on children's human capital formation in the utility function of the parent is assumed to be an increasing function of the parent's own consumption. A poor parent is likely to attach less weight to children's education than a rich parent. Not only does a poor parent have less ability to invest in children's human capital formation, but also has less willingness. This factor contributes towards perpetuation of lower earning abilities generation after generation.

In the present paper, the degree of parental altruism is endogenously determined too and is assumed to vary with the level of schooling of the parent. Parents derive direct utility from human capital formation of the child. Here the weight assigned to human capital formation of the child is a direct function of the level of parental schooling. As we move from lower educated parents to higher educated parents, perceived utility derived from the schooling of the child increases. This in turn affects the long run dynamics of schooling.

Parental altruism factor plays a major role in ascertaining the child labour status of children in any household. In fact parental decision regarding schooling of the child is one of the key factors affecting human capital formation of child which in turn determines the intergenerational persistence of child labour. In our paper we assume that parents allocate the time of her child between schooling and work. Thus, parental altruism has a direct bearing on child labour.

Along with the endogenous altruism, in an extension of the basic model, we assume learning by doing effect to be present in unskilled work. In the developing countries, apprenticeship is very common among child labour. Often learning by doing occurs through apprenticeship, and in real life, apprenticeship is found mostly in the informal or unskilled sector. Apprenticeship provides vocational education in many fields, e.g. carpentry, farming, masonry, fishing, poultry where knowledge is transmitted through prolonged practice rather than acquiring formal knowledge base. According to World Employment Report 1998-99-“In Kenya, there are more apprentices enrolled in the informal sector than trainees in the formal sector”, while “in Egypt, over 80% of craftsmen in the construction sector acquire their skills through traditional apprenticeship.” According to the report, apprenticeship is common among child labour. According to ILO’s report on Employment Sector (2008), apprenticeship has been providing the traditional solution for developing and financing vocational skills of young people in deprived societies. Estimations suggest that 80% of the skills imparted in the informal economy in West Africa are transferred through apprenticeship. In Benin, in 2005, approximately 2000,000 young apprentices were trained, which represents ten times as many apprentices than students in vocational and technical education. But there are very few child labour papers that capture this learning by doing effect.

In the extended model, we assume experience as child labour pays additional return to unskilled labourers during their adulthood.

A number of rules and conventions have been laid down all over the world to fight child labour. Two most effective policies that may be undertaken to reduce child labour are child labour ban and education subsidy. The pioneer work on child labour by Basu and Van (1998) shows that in case of multiple equilibria in the labour market, a total ban on child labour can take the economy from bad equilibrium to good equilibrium. All working class households will be better off. But if there is only one equilibrium, a total ban may or may not make worker households better off. A partial ban may not always reduce child labour but may reduce only child wage. However utility of the worker household may or may not increase. According to Baland and Robinson (2000) small ban on child labour can be Pareto improving. A ban on child labour reduces the supply of child labour while increasing the supply of adult labour in the future. As a result, current wages of both adults and children are likely to rise and future wages are likely to fall. Thus while children's utility is likely to rise in most cases, parental welfare will increase only when the effect on current wages dominates. The paper by Dessy and Pallage (2001) states that compulsory bans on child labour help sending signals to investors that investment in human capital will be made in the near future and thus skilled labour is likely to be available. Ban or compulsory education will be counterproductive if the cost of investment is very high. Instead a policy that subsidizes technology and imposes compulsory education can help to move the economy from bad equilibrium to a good one. Dinopoulos and Zhao (2007), in their paper on globalisation, show that a ban on child labours benefits adult unskilled workers but hurts adult skilled workers. According to Emerson and Knabb (2006), P.Ranjan (1999, 2001) banning child

labour can reduce dynastic welfare, increase poverty and further accentuate income inequality within society.

This paper builds a theoretical model to examine the relative effectiveness of two types of domestic policies to combat child labour—a child labour ban and an education subsidy. Both domestic as well as international policies may be undertaken to reduce the incidence of child labour. However, in this paper we restrict our analysis only to domestic policies. A number of theoretical papers deal with the effectiveness of domestic policies to reduce child labour. Papers that deal with ban on harmful forms of child labour include Rogers and Swinnerton (2002) and Dessy and Pallage (2005).

There is a small set of literature that deals with the effects of education subsidy on child labour. Emerson and Knabb (2006) show that compulsory education policy may actually reduce welfare. According to Chaudhuri and Mukhopadhyay (2003), a rise in the education subsidy may force the rural workers to migrate to the urban areas with their children. This increases the supply of child labour in the urban sector which further accentuates the problem of urban unemployment of adult labour. Moreover it may raise the level of urban unemployment of adults even when adult labour and child labour are not substitutes to each other. The average income of the urban poor families may also decrease as a consequence. Chaudhuri (2004) states that the effects of increase in education subsidy on child labour depends on relative strength of two effects—namely labour re-allocation effect and the contradictory effect which exerts a downward pressure on the incidence of child labour. Mukherjee and Sinha (2006) and Estevez (2011) argue in favour of education subsidy in improving school attendance. According to Estevez (2011), an education

subsidy will reduce the incidence of child labour, increase the household income and will also indirectly increase the unskilled wage.

Some empirical studies have also been conducted to analyze the impact of domestic policies on child labour<sup>1</sup>. But, none of the papers mentioned so far have theoretically examined the effects of ban and education subsidy on steady state schooling and steady state human capital of child labour. This paper attempts to understand the effects of child labour ban and education subsidy on steady state schooling and steady state human capital of child labour. Moreover this paper studies the relative effectiveness of child labour ban and education subsidy in improving schooling of the child.

The theoretical analysis of this paper helps us to get some interesting results. We find that the relationship between parental schooling and child schooling is monotonically increasing in general but, in the presence of learning by doing effect in unskilled work this relationship is not monotonically increasing. There is no opportunity of full schooling of child of unskilled parent except when unskilled adult wage exceeds expenditure. We also find that banning child labour will increase steady state schooling if the unskilled adult wage exceeds the sum of the schooling cost and subsistence consumption expenditure of the household. If the adult unskilled wage is less than the subsistence consumption expenditure or even if adult unskilled wage exceeds subsistence expenditure but is sufficiently small, the effect of giving education subsidy is higher than child labour ban in enhancing schooling. In the presence of learning by doing effect cycles emerge in the time path of schooling in unskilled sector.

---

<sup>1</sup> Fabre and Pallage (2011), Schultz (2004), Ravallion and Wodon (2000), Krueger and Donohue (2005).

The paper is organized as follows. Section 2 describes the basic model. Section 3 describes the short run equilibrium. Section 4 discusses the long run dynamics. In section 5 we discuss the comparative static results. In section 6 we discuss the policy implications. Section 7 compares the effects of ban and subsidy on child labour. Section 8 discusses the case where learning by doing effect is present in unskilled wage. Concluding remarks are made in section 9.

## **2. The Model**

We consider an economy that consists of identical households in overlapping generations framework<sup>2</sup>. Each household consists of one adult and one child. We consider two parents as one adult and two children as one child. The economy consists of two sectors- a skilled sector and an unskilled sector. In first period agents are children. They may either work in unskilled sector or go to school. In second period, the agent on reaching adulthood may either work in unskilled sector or in the skilled sector. If one individual is employed in skilled sector she gets wage proportional to human capital whereas unskilled sector gives a fixed return. The adult or the parent decides the time allocation of her child between work and schooling. Utility function of the adult depends on family consumption and human capital formation of the child. Parental altruism is endogenously determined. It is an increasing function of the level of parental schooling. More educated parents have more preference towards child's human capital.

Following Glomm (1997), we assume parental choice of human capital investment. The adult decides how much time her child would devote to work in the unskilled sector and how much time for schooling by maximizing utility subject to the budget constraint. Wages earned by the

---

<sup>2</sup>Overlapping generations framework has been adopted by Becker and Tomes (1979), Acemoglu and Pischke (2000), Glomm (1997), Glomm and Ravikumar (1998) and many more.



adult and by the child constitute the total income of the household. If the child joins the skilled sector, on becoming adult, she gets a wage in the skilled sector which is a fixed proportion of the human capital possessed by her  $(\delta h_t)^3$ . In unskilled sector, the adult gets a fixed return 'A'.

A child, by working in the unskilled sector also gets a fixed return which is less than the return obtained by the adults from unskilled sector.

Like Moav (2005), this paper assumes that human capital evolution is independent of physical capital. Human capital accumulation function of a child is assumed to take the following form<sup>4</sup>:

$$h_{t+1} = b s_t \quad (1)$$

where 's<sub>t</sub>' is the time devoted to studies by the child,  $b > 0$  is a positive constant representing education technology.

In case of unskilled parent household income is given by:

$$Y_t = A + A\phi (1 - s_t) , \quad (2)$$

Where  $Y_t$  is total income of the household, A is wage earned by the adult in unskilled sector,  $\phi$  is the fraction of adult wage that a child labour receives. Here  $0 < \phi < 1$  is a positive constant.

The household spends its income on purchasing consumption good and schooling of the child.

So, the budget constraint of the household is given by:

$$A + A\phi (1 - s_t) = p_c c_t + \rho s_t, \quad (3)$$

---

<sup>3</sup>Hare and Ulph (1979) assume that wage rate depends on ability and amount of education received by an individual.

<sup>4</sup>Inclusion of parental human capital in human capital accumulation of child yields nonlinear equations and makes the model very complicated. So, for the sake of simplicity the human capital accumulation of child is assumed to take this form.

where  $p_c$  is the price of the consumption good,  $p_c c_t$  represents the total consumption expenditure and  $p_s s_t$  denotes the expenditure on schooling of the child. When adults work in the skilled sector, household income is given by:

$$Y_t = w_t + A\phi(1-s_t),$$

Where  $w_t$  is the wage earned by the adult in the skilled sector. We assume wage earned in skilled sector ( $w_t$ ) is proportional to the human capital acquired by that individual i.e.  $w_t = \delta h_t$ .

Utility function of an adult of the representative household is defined as follows:

$$U_t = \ln(c_t - \underline{c}) + s_{t-1} \ln(bs_t) \text{ if } c_t \geq \underline{c}$$

$$= -\infty \text{ otherwise} \tag{4}$$

Where  $c_t$  represents consumption,  $\underline{c}$  represents subsistence consumption. The utility function is defined on the range  $c_t \geq \underline{c}$ . Utility depends on consumption of the adult and human capital formation of the child. Higher is the education level of the parent ( $s_{t-1}$ ), more is the importance that she gives to human capital accumulation of the child.

Let us first apply the model in the short run equilibrium context and understand the relationship between parental human capital and schooling of the child.

### 3. Short-run Equilibrium

#### 3.1 Parents working in the unskilled sector

Utility maximization problem of an adult of the representative household working in unskilled sector is to maximize the utility, given by equation (4) subject to the budget constraint given by equation (3) with respect to the decision variables of the household viz,  $c_t$  and  $s_t$

From the first order conditions<sup>5</sup> of the optimization problem, if there exists an interior solution of  $s_t$ , we obtain:

$$s_t = \frac{s_{t-1}[A(1+\varphi)-p_c\underline{c}]}{(A\varphi+\rho)(1+s_{t-1})} \quad (5)$$

$s_t > 0$  if  $A(1+\varphi)-p_c\underline{c} > 0$ .

So  $A(1+\varphi)-p_c\underline{c} > 0$  is a necessary condition for  $s_t$  being positive.

Note that if  $s_{t-1}=0$  then  $s_t=0$  because  $dz/ds_t < 0$ .

$s_t = 1$  if  $s_{t-1} \geq \frac{A\varphi+\rho}{A-\rho-p_c\underline{c}} = s'$ . This implies that lower is the total expenditure of the household ( $\rho+p_c\underline{c}$ ) and higher is the earning of the unskilled adult ( $A$ ), higher is the possibility of full schooling of the child. Therefore, if total earning of the household headed by an unskilled adult exceeds the subsistence expenditure of the household, then the child of that household experiences positive schooling and if total expenditure of the household is low enough and earning of the unskilled adult is sufficiently high, there is higher chance that the child experiences full schooling. Full schooling by a child of an unskilled labourer would be possible to attain only if  $s' < 1$ .

$s' < 1$  implies  $A(1-\varphi) > p_c\underline{c} + 2\rho$

Differentiating  $s_t$  with respect to  $s_{t-1}$  we get

$$\frac{ds_t}{ds_{t-1}} = \frac{A(1+\varphi)-p_c\underline{c}}{(A\varphi+\rho)(1+s_{t-1})^2}$$

Since  $A(1+\varphi)-p_c\underline{c} > 0$  for positive schooling, therefore,  $\frac{ds_t}{ds_{t-1}} > 0$  when  $s_t > 0$ .

---

<sup>5</sup> For detailed derivation please see equations (A.1), (A.2) and (A.3) of Appendix A.

$$\frac{d^2 s_t}{ds_{t-1}^2} = \frac{-2[A(1+\varphi) - p_c \underline{c}]}{(A\varphi + \rho)(1 + s_{t-1})^3} < 0$$

Since more educated parents give more importance to human capital accumulation of the child, it is quite natural that with increase in schooling of the parent, schooling of child will also increase.

### 3.2 Parents working in the skilled sector

When parents work in the skilled sector, the incentive compatibility condition requires that wage earned in skilled sector is higher than the wage earned in unskilled sector. This implies that  $w_{t+1} > A$  which implies that  $\delta(bs_t) > A$  i.e.  $s_t > \frac{A}{\delta b} = \underline{s}$ . This implies that only if  $s_t > \underline{s}$ , then only individuals join the skilled sector.

When adults work in the skilled sector, the budget constraint of the household is given by:

$\delta bs_{t-1} + A\varphi(1 - s_t) = p_c \underline{c} + \rho s_t$  where  $\delta bs_{t-1}$  denotes income of the adult working in the skilled sector.

In this case schooling of the child is given by

$$s_t = \frac{s_{t-1}[\delta bs_{t-1} + \varphi A - p_c \underline{c}]}{(A\varphi + \rho)(1 + s_{t-1})}$$

$s_t > 0$  if  $\delta bs_{t-1} + \varphi A - p_c \underline{c} > 0$  or  $s_{t-1} > \frac{p_c \underline{c} - A\varphi}{\delta b}$ . This implies that higher is the child's earning as fraction of the adult wage ( $A\varphi$ ) and lower is the subsistence expenditure ( $p_c \underline{c}$ ), higher is the possibility of positive schooling of the child.

$$s_t = 1 \text{ if } \delta bs_{t-1}^2 - (p_c \underline{c} + \rho)s_{t-1} - (A\varphi + \rho) \geq 0$$

This implies that there exists a positive value of  $s_{t-1}$  say  $\hat{s}$  for which  $s_t = 1$ .

If  $\underline{s} > \hat{s}$  then all parents who are employed in the skilled sector send their children for full schooling.

Differentiating  $s_t$  with respect to  $s_{t-1}$  we get

$$\frac{ds_t}{ds_{t-1}} = \frac{\delta bs_{t-1}^2 + 2\delta bs_{t-1} + A\varphi - p_c \underline{c}}{(A\varphi + \rho)(1 + s_{t-1})^2}$$

$$\frac{ds_t}{ds_{t-1}} > 0 \text{ if } s_t > 0.$$

$$\frac{d^2 s_t}{ds_{t-1}^2} = \frac{2[\delta b - A\varphi + p_c \underline{c}]}{(A\varphi + \rho)(1 + s_{t-1})^3} > 0.$$

In the next section, we study the long run dynamics of schooling and human capital.

## 4. Long run Dynamics

### 4.1 Dynamics of schooling

Putting  $s_t = s_{t-1} = s_u^*$  in the expression of  $s_t$  in equation (5) we obtain the steady state schooling in the unskilled sector we get

$$s_u^* = \frac{A - p_c \underline{c} - \rho}{A\varphi + \rho}.$$

If  $0 < s' < 1$ , then  $s_u^* > 1$  and if  $0 < s_u^* < 1$ , then  $s' > 1$ .

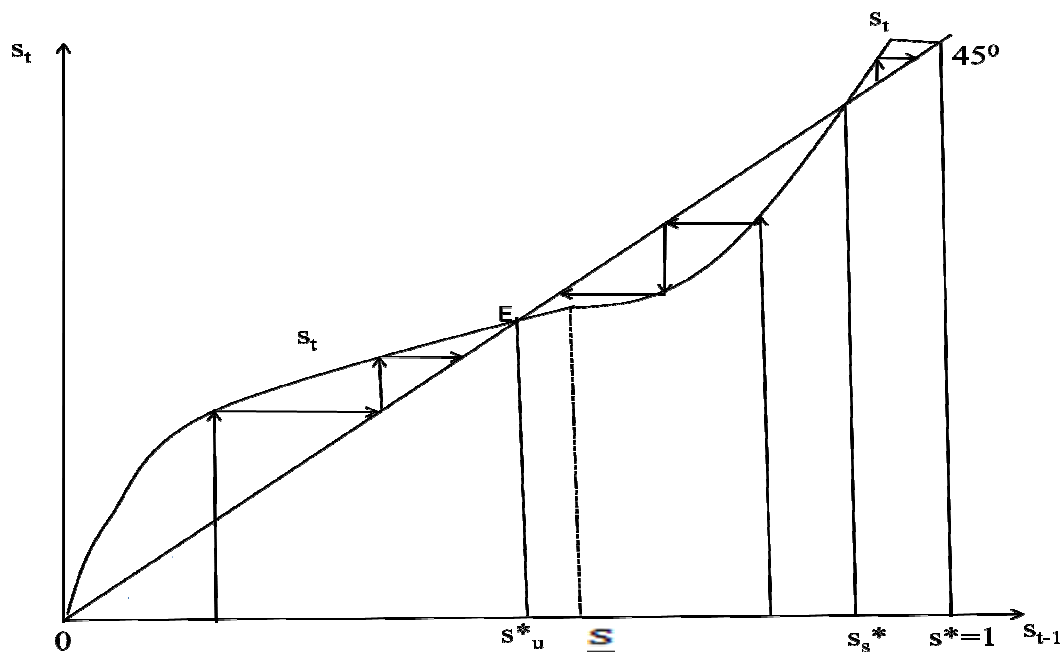
$s' < 1$  implies  $A(1 - \varphi) > p_c \underline{c} + 2\rho$

**Proposition 1:** When unskilled adult wage is low compared to the expenditure, there exists interior equilibrium  $s_u^*$ , full schooling equilibrium for unskilled parent cannot occur; and if unskilled adult wage is sufficiently high, full schooling equilibrium occurs, interior equilibrium does not exist.

Similarly, we obtain the steady state schooling in the skilled sector

$$S_s^* = \frac{p_c c + \rho}{\delta b - A\phi - \rho}$$

We know that, in case of  $\underline{s} < \hat{s}$  skilled parent may send her child for partial schooling. In this case, the dynamics of  $s_t$  for both unskilled and skilled sectors would be similar and that is shown in the following diagram:



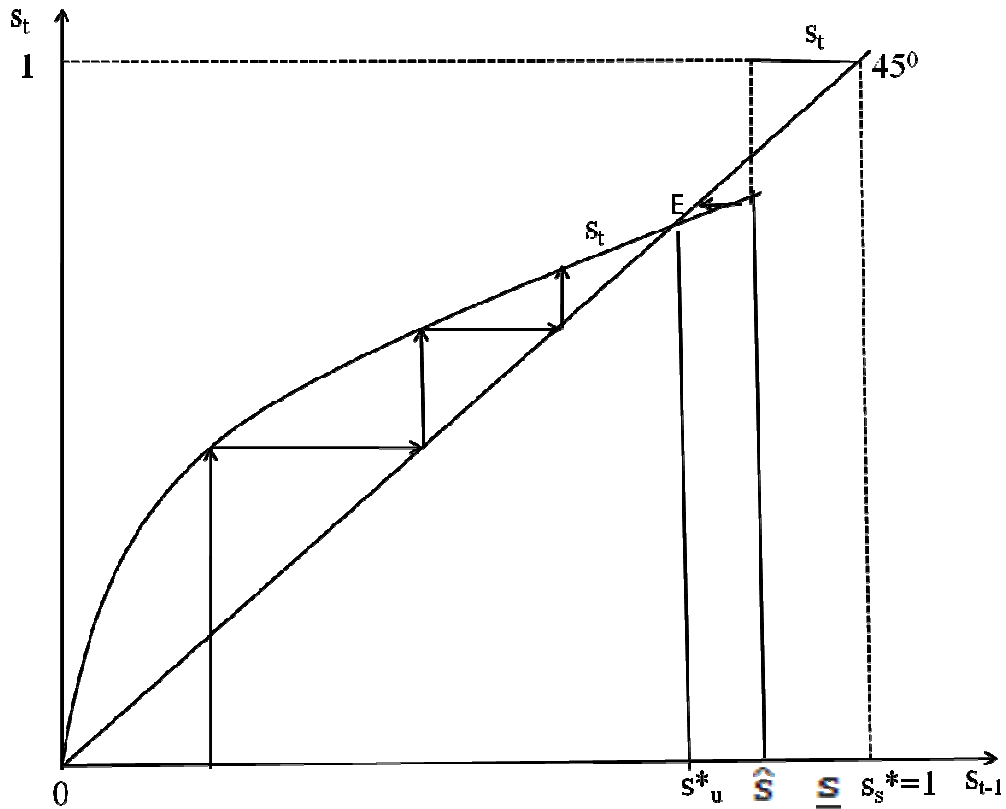
**Figure 1:** Dynamics of schooling for both unskilled sector and skilled sector when skilled parent may send her child for partial schooling

In Figure 1 we demonstrate the dynamics of schooling for both unskilled and skilled sectors when skilled parent may send her child for partial schooling. Time path of schooling is stable and convergent in nature in unskilled sector. The steady state schooling in unskilled sector is denoted by  $s_u^*$ . Unskilled parent always sends her child for partial schooling. There are two equilibria in

the skilled sector out of which one is unstable and the other is stable. The unstable equilibrium schooling in skilled sector is denoted by  $s_s^*$ . Beyond  $s_s^*$ ,  $s_t$  keeps on rising till it reaches full schooling. Below  $s_s^*$ ,  $s_t$  is first falling, then rising and finally converges to steady state schooling in unskilled sector ( $s_u^*$ ). At this equilibrium, skilled parent would send her child for partial schooling. The stable equilibrium is the full schooling equilibrium. Full schooling is denoted by  $s^*=1$ .

**Proposition 2:** When  $\underline{s} < \hat{s}$  and  $s' > 1$ , schooling of the child of an unskilled parent would always converge to stable equilibrium E. If schooling of a skilled parent is below  $s_s^*$ , schooling of her child will converge to steady state schooling of unskilled sector ( $s_u^*$ ). Beyond  $s_s^*$ , schooling of child will keep on increasing till it reaches full schooling.

When  $\underline{s} > \hat{s}$  the skilled parent always sends her child for full schooling. In this case, there exists unique steady state equilibrium  $s_s^* = 1$ . The dynamics of  $s_t$  for both unskilled and skilled sectors in this case is shown in the following diagram:



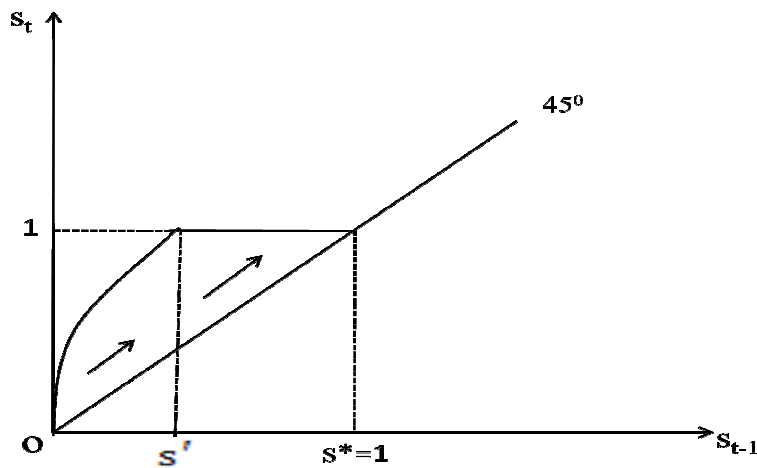
**Figure 2:** Dynamics of schooling for both unskilled sector and skilled sector when skilled parent always sends her child for full schooling

In Figure 2, we demonstrate the dynamics of schooling for both unskilled and skilled sectors when  $\underline{s} > \hat{s}$  and skilled parent always sends her child for full schooling.  $s_t$  is throughout rising with rise in  $s_{t-1}$ . Below  $\hat{s}$ , there is a unique steady state equilibrium schooling ( $s_u^*$ ) in unskilled sector denoted by E which is stable in nature. If  $s_{t-1}$  is below  $s_u^*$  schooling keeps on rising till it reaches  $s_u^*$ . Beyond  $s_u^*$  schooling keeps on falling till it reaches  $s_u^*$ . All parents who are employed in skilled sector send their children for full schooling. Steady state schooling in skilled sector i.e.  $s_s^* = 1$ .



**Proposition 3:** When  $\underline{s} > \hat{s}$ , a skilled parent always sends her child for full schooling. If  $s' > 1$ , the schooling of child of an unskilled parent would converge to the stable equilibrium E.

If  $s' < 1$ , full schooling would be the only equilibrium schooling of a child. This is shown in the following figure:



**Figure 3:** Dynamics of schooling for both unskilled sector and skilled sector when both skilled and unskilled parents send their children for full schooling

## 4.2 Dynamics of human capital

In this section, we discuss dynamics of human capital. Since human capital accumulation function is given by  $h_t = bs_{t-1}$ ,  $s_t = s_{t-1}$  implies that  $h_t = h_{t+1}$  i.e. when schooling is at steady state ( $s_t = s_{t-1} = s^*$ ) human capital will also be in steady state ( $h_t = h_{t+1} = h^*$ ). The time path of human capital will be similar to that of schooling. Here, growth rate of human capital is constant. The comparative static results which hold true for steady state schooling will hold true for steady state human capital as well. Therefore,  $\frac{dh^*}{d\phi}$ ,  $\frac{dh^*}{d\rho}$  and  $\frac{dh^*}{dA}$  will have same signs as  $\frac{ds_u^*}{d\phi}$ ,  $\frac{ds_u^*}{d\rho}$  and  $\frac{ds_u^*}{dA}$ .

This is true for both the case of skilled parent and unskilled parent.

## 5. Comparative static analysis when parents work in the unskilled sector

Note that, in the case where parents work in unskilled sector the followings hold true<sup>6</sup>:

i)  $\frac{ds_u^*}{d\varphi} < 0$  if  $A > \rho + p_c \underline{c}$ . This implies that if the adult unskilled wage exceeds the sum of the schooling cost and subsistence expenditure of the household, steady state schooling of child of unskilled parent increases with fall in child wage.

ii)  $\frac{ds_u^*}{d\rho} < 0$ . This implies that with fall in schooling cost steady state schooling in the unskilled sector increases.

iii)  $\frac{ds_u^*}{dA} > 0$ . This implies that with increase in unskilled adult wage, steady state schooling will increase.

**Proposition 4:** The interior steady state schooling of a child of an unskilled parent increases with increase in unskilled adult wage but decreases with increase in education cost. It increases with fall in child wage only if the adult unskilled wage exceeds the sum of the schooling cost and subsistence expenditure of the household.

## 6. Policy implications

There are many policy options to redress the issue of child labour. In this paper we discuss the two most popular measures world wide – child labour ban and education subsidy. When a child labour ban is perfectly enforced, it forces firms to withdraw children from work. However,

---

<sup>6</sup> For detailed derivation please see equations (A.6), (A.7) and (A.8) of Appendix A.

governments in countries where child labour is present, often do not have enough capacity and resources to perfectly enforce regulations on child employment (Edmonds and Shresthra (2012)). According to a simple model by Basu (2005), when bans are imperfectly enforced, they raise the cost of hiring children, as employers anticipate facing stiff fines or other penalties when caught using child labour. Thus, when imperfectly enforced, bans may simply lower the wages that children are paid. In our paper the effects of imperfectly enforced ban are studied from the comparative static exercise.

From the comparative static exercise in our model we get the result that  $\frac{ds_u^*}{d\varphi} < 0$  if  $A > \rho + p_c \underline{c}$ .

Now, imperfectly enforced ban on child labour implies fall in child wage ( $\varphi$ ). This implies that banning child labour will increase steady state schooling if the unskilled adult wage exceeds the sum of the schooling cost and subsistence consumption expenditure of the household. When unskilled adult wage exceeds the total expenditure of the household, child wage is no longer necessary to meet subsistence requirements of the household. In such a situation, banning child labour has positive impact on steady state schooling. Since child wage is no longer necessary to meet the subsistence requirements of the household, even if child wage falls due to child labour ban, still steady state schooling increases. However, if  $A < \rho + p_c \underline{c}$ , then  $\frac{ds_u^*}{d\varphi} > 0$ . In this case, unskilled adult wage is not enough to cover the total expenditure of the household. In this case, banning child labour will hurt the household since household depends on child wage to cover the subsistence expenditure of the household. So banning child labour, in this case, is not a good proposition.

From comparative static exercise in our model we also get the result that  $\frac{ds_u^*}{d\rho} < 0$ . This implies that fall in schooling cost always leads to rise in steady state schooling. So an education subsidy, which reduces schooling cost, will invariably have a positive impact on steady state schooling.

Next we carry out a comparative study between the effects of ban and subsidy on improving schooling of the child.

### 7. Comparison between the effects of ban and subsidy

Now we compare the effects of ban and education subsidy on steady state schooling in the case where parents work in the unskilled sector.

$$\left| \frac{ds^*}{d\rho} \right| - \left| \frac{ds^*}{d\varphi} \right|$$

$$= \frac{A(\varphi + \rho) + (1-A)(A - p_c \underline{c})}{(A\varphi + \rho)^2}$$

If  $1 < A < p_c \underline{c}$ , then the above expression is positive.

Again if  $p_c \underline{c} < A < 1$  then also the above expression is positive.

This implies that if adult unskilled wage is less than subsistence consumption expenditure or even if adult unskilled wage exceeds subsistence expenditure but is less than one, the effect of giving education subsidy is higher than child labour ban in enhancing schooling.

**Proposition 5:** If adult unskilled wage is less than subsistence consumption expenditure or even if adult unskilled wage exceeds subsistence expenditure but is sufficiently small, the effect of giving education subsidy is higher than child labour ban in enhancing schooling.

## 8. Model with learning by doing effect in unskilled wage

It is well known that workers can improve their productivity by repetition of the same work done. Dessy and Pallage (2005), in their paper on worst forms of child labour, consider the learning by doing effect in the human capital accumulation function. There are many other papers which have emphasized on the learning by doing effect. However, these articles do not deal with the issue of child labour<sup>7</sup>. In the present paper, we consider learning by doing effect in unskilled work. We assume that the unskilled workers earn an additional income as an adult if they worked as child labour at their young age. An individual who works as a child earns an additional income after joining unskilled sector as an adult due to positive learning by doing effect. All other assumptions remain same as previous model.

In this case, the income of the household headed by unskilled parent is given by:

$$Y_t = [A + (1-s_{t-1})\underline{h}] + A\phi (1-s_t)$$

The budget constraint of the household headed by an unskilled adult is now given by:

$$[A + (1-s_{t-1})\underline{h}] + A\phi (1-s_t) = p_c c_t + \rho s_t$$

In the case of an unskilled parent, schooling of child is given by  $s_t = \frac{s_{t-1}[A + (1-s_{t-1})\underline{h} + \phi A - p_c c]}{(A\phi + \rho)(1 + s_{t-1})}$

Differentiating  $s_t$  with respect to  $s_{t-1}$  we get

$$\frac{ds_t}{ds_{t-1}} = \frac{A + (1-s_{t-1})\underline{h} + A\phi - p_c c - \underline{h} s_{t-1}(1 + s_{t-1})}{(A\phi + \rho)(1 + s_{t-1})^2}$$

<sup>7</sup>See e.g. Lucas (1988), Mao (2012), Parente (1994), Hippel and Tyre (1993) etc.

$$\frac{ds_t}{ds_{t-1}} > 0 \text{ if } [-\underline{h}s_{t-1}^2 - 2s_{t-1}\underline{h} + \{A(1+\varphi) + \underline{h} - p_c\underline{c}\}] > 0$$

$$\text{or } s_{t-1} < \pm \sqrt{\frac{A(1+\varphi)}{\underline{h}} + 2 - \frac{p_c\underline{c}}{\underline{h}}} - 1 = M$$

We ignore the negative term since  $s_{t-1}$  cannot be negative.

$$\frac{d^2s_t}{ds_{t-1}^2} = \frac{-2[\underline{h}(1+s_{t-1})^2 + \{A(1+\varphi) + (1-s_{t-1})\underline{h} - p_c\underline{c} - \underline{h}s_{t-1}(1+s_{t-1})\}]}{(A\varphi + \rho)(1+s_{t-1})^3}$$

$$\frac{d^2s_t}{ds_{t-1}^2} < 0 \text{ if } \underline{h}(1+s_{t-1})^2 + A(1+\varphi) + (1-s_{t-1})\underline{h} - p_c\underline{c} - \underline{h}s_{t-1}(1+s_{t-1}) > 0$$

$$\text{or } A(1+\varphi) + 2\underline{h} - p_c\underline{c} > 0$$

$$\frac{d^2s_t}{ds_{t-1}^2} > 0 \text{ if } A(1+\varphi) + 2\underline{h} - p_c\underline{c} < 0$$

We assume  $A(1+\varphi) + 2\underline{h} - p_c\underline{c} > 0$  otherwise  $\sqrt{\frac{A(1+\varphi)}{\underline{h}} + 2 - \frac{p_c\underline{c}}{\underline{h}}}$  becomes an imaginary number.

If  $M \geq 1$ ,  $s_{t-1}$  is always less than  $M$ . This implies that when  $M \geq 1$   $\frac{ds_t}{ds_{t-1}} > 0$  always.

Now  $M \geq 1$  implies  $\frac{A(1+\varphi) - p_c\underline{c}}{\underline{h}} \geq 2$ .

The above condition will hold true if  $A(1+\varphi) - p_c\underline{c} > 0$ ,  $A(1+\varphi)$  is high and  $\underline{h}$  is low.

If  $M < 0$ , then this condition is never satisfied. Equilibrium does not exist.

Therefore if  $M$  is a fraction, then  $\frac{ds_t}{ds_{t-1}} > 0$  till  $M$  is reached and thereafter  $\frac{ds_t}{ds_{t-1}} < 0$ .

Now  $M$  is a fraction when  $0 < M < 1$ . This implies  $-1 < \frac{A(1+\varphi) - p_c c}{h} < 2$ .

If  $A(1 + \varphi) > p_c c$  but  $A(1+\varphi)$  is low and  $h$  is high then this inequality is likely to be satisfied.

$\frac{d^2 s_t}{ds_{t-1}^2} < 0$  throughout.

If total earnings of the household run by unskilled parents exceed subsistence consumption expenditure, but are low and learning by doing effect is high, then below a particular level of parental schooling there is positive relationship between parental schooling and schooling of the child. But beyond that level of parental schooling, schooling of the child decreases with increase in parental level of schooling.

The reason of obtaining such result is when in spite of going to school for a quite long time, parents are still working in unskilled sector, they lack motivation for sending their children to school. Moreover, as unskilled parents went to school themselves, during their adulthood, they are losing a part of the income that they would have earned had they worked as child labour. Below a particular level of parental schooling, parental schooling and child schooling are positively related because it is assumed that more educated parents derive more satisfaction from sending their children to school. But given low levels of earnings of the household and high learning by doing effect in unskilled sector, beyond a particular level of parental schooling there is a negative relationship between parental level of schooling and child schooling. This leads us to the next proposition.

**Proposition 6:** The relationship between child schooling and parental schooling is monotonically increasing in general. However, if learning by doing is present in unskilled sector, though child schooling initially increases with parental schooling but decreases afterwards.

In the case of a skilled parent, the household income and budget constraint of the household remains same as in the case of absence of learning by doing effect.

In the case of a skilled parent, schooling of the child is given by  $s_t = \frac{s_{t-1}[\delta b s_{t-1} + \varphi A - p_c c]}{(A\varphi + \rho)(1 + s_{t-1})}$

Differentiating  $s_t$  with respect to  $s_{t-1}$  we get

$$\frac{ds_t}{ds_{t-1}} = \frac{\delta b s_{t-1}^2 + 2\delta b s_{t-1} + A\varphi - p_c c}{(A\varphi + \rho)(1 + s_{t-1})^2}$$

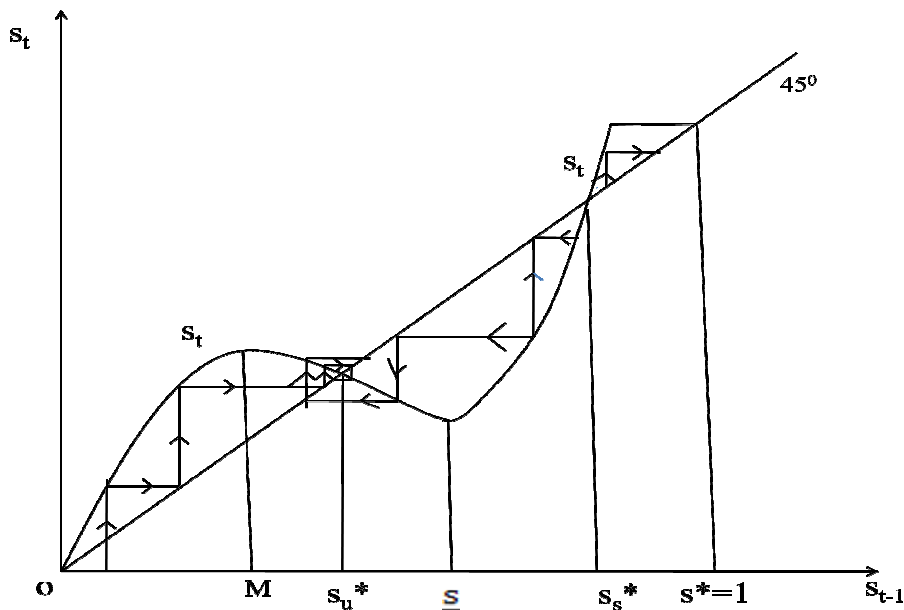
$$\frac{ds_t}{ds_{t-1}} > 0 \text{ if } s_t > 0.$$

$$s_s^* = \frac{p_c c + \rho}{\delta b - A\varphi - \rho}$$

The dynamics of skilled sector remains unchanged as the previous model where learning by doing was not present.

If  $\underline{s} < \hat{s}$  the dynamics of  $s_t$  in both unskilled and skilled sectors in the presence of learning by doing effect in unskilled work are shown in the following diagram:





**Figure 4:** Dynamics of schooling for both skilled and unskilled sectors in the presence of learning by doing effect

In Figure 4, we demonstrate the case of unskilled parent where  $M$  is a fraction. In case of unskilled parent, time path of  $s_t$  is convergent till  $M$  is reached, but once  $M$  is crossed, the time path becomes oscillating but remains convergent. This oscillating nature of the time path of schooling may be explained in the following manner: Suppose in period 1, parental level of schooling is low. However, parent sends her child to school for higher number of hours in expectation that her child may get job in the skilled sector in future. In the next period when this child becomes adult and finds that in spite of high schooling, she is still working in the unskilled sector and thus receiving lower wage as compared to what she could have earned had she worked as a child labour (we assume that learning by doing effect is present in case of unskilled work), she will send her child to school for lesser number of hours. This process is repeated in consecutive periods till schooling in unskilled sector converges to the steady state  $s_u^*$ . Thus in

this model parental preferences regarding schooling of a child and learning by doing effect generate cycles in the time path of schooling in unskilled sector even when there are no external shocks.<sup>8</sup> There is only one steady state level of schooling in the case of unskilled parent represented by  $s_u^*$  in figure 4. This equilibrium is a stable equilibrium.

The individuals would join skilled labour force if  $w_{t+1} > A + (1 - s_t)h$  i.e.  $s_t > \underline{s} = \frac{A+h}{\delta b+h}$ . When the parental level of schooling lies between  $s_u^*$  and  $\underline{s}$ , schooling of child keeps on falling till it converges to the unskilled level steady state schooling  $s_u^*$  in an oscillating manner.

In skilled sector, schooling of child always increases with increase in schooling of the parent. There are two equilibria in the skilled sector.  $s_s^*$  denotes the level of child schooling corresponding to the unstable equilibrium. Below  $s_s^*$ ,  $s_t$  in skilled sector keeps on falling and eventually converges to steady state equilibrium of unskilled sector in an oscillating manner. Beyond  $s_s^*$ , schooling of the child keeps on increasing and will eventually converge to  $s_t=1$ . Hence, the dynasties having parental skill level between  $\underline{s}$  and  $s_s^*$  may end up in the situation where next generations will be working as unskilled labour. Lower is  $s_s^*$ , lower is the parental level of human capital required to launch the economy on the path of steady growth of schooling. Lower  $s_s^*$  is thus good for the economy. Increase in education cost ( $\rho$ ), child wage ( $A\phi$ ) and subsistence consumption expenditure ( $p_c c$ ) thus leads to higher  $s_s^*$  which is not good for the economy. Increase in responsiveness of wage to human capital ( $\delta$ ) and improvement in education technology (rise in  $b$ ) lead to lower  $s_s^*$  which is good for the economy. The full schooling

---

<sup>8</sup> Zhang (2015), Zhang (2014), Croix and Licandro (1999), Croix (2001) have found oscillations in human capital accumulation but in different context.

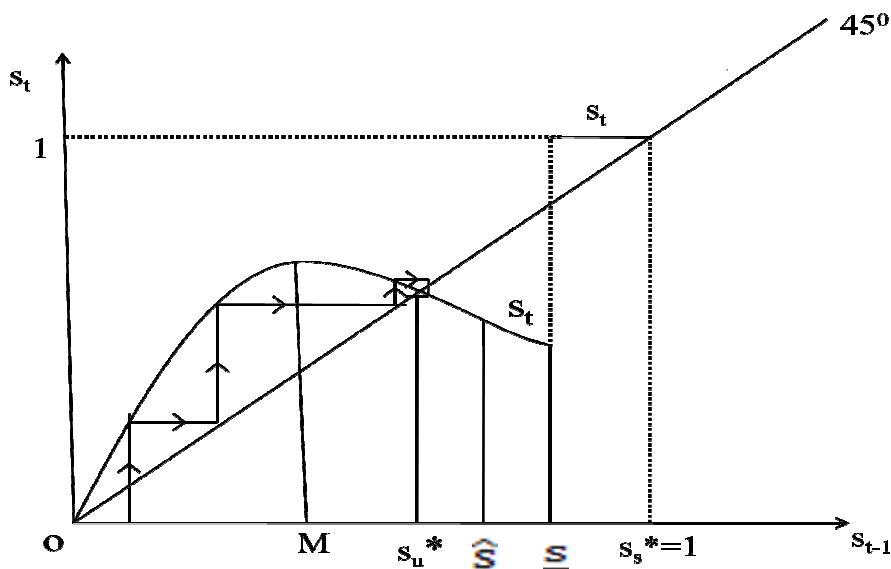
equilibrium is a stable equilibrium and the steady state schooling corresponding to the full schooling equilibrium is denoted by  $s^*=1$ .

**Proposition 6:** In the case of an unskilled parent (parental skill below  $\underline{s}$ ), the time path of schooling of the child is steadily convergent in nature when approached from below steady state level of parental schooling  $s_u^*$  and beyond that  $s_u^*$ , the time path of schooling is convergent but oscillating in nature. In the case of a skilled parent, the dynamics remain similar to previous model.

There exists one positive value of  $s_{t-1}$  say  $\hat{s}$  for which  $s_t = 1$ .

If  $\underline{s} > \hat{s}$  then all parents who are employed in skilled sector send their children for full schooling.

The dynamics of schooling for this case is shown in the following diagram:



**Figure 5:** Dynamics of schooling for skilled sector when  $\underline{s} > \hat{s}$

In Figure 5, till M is reached, time path of  $s_t$  is convergent but once M is crossed, the time path becomes oscillating but convergent. The explanation for oscillatory time path of schooling in

Figure 5 is similar to the explanation given for that in Figure 4. There is unique equilibrium in the unskilled sector. The steady state level of schooling corresponding to this equilibrium is  $s_u^*$ . Parents who work in skilled sector always send their children for full schooling. Steady state schooling in skilled sector is  $s_s^* = 1$ .

The comparative static results and the policy implications remain same in the presence of learning by doing effect as in the case of absence of learning by doing effect.

## 9. Conclusion

In this paper we seek to explain the long run dynamics of schooling and intergenerational persistence of child labour in an overlapping generations household economy model based on limited parental altruism. Here, the level of parental schooling determines her willingness to invest in the human capital formation of the child. A parent who does not undergo schooling herself will never send her child to school. The relationship between parental schooling and child schooling is monotonically increasing in both unskilled and skilled sectors in absence of learning by doing effect in unskilled work. However, in the presence of learning by doing effect in unskilled work this relationship is not monotonically increasing.

As far as the long run dynamics of schooling in the basic model is concerned, when we assume that schooling required to be engaged as skilled worker is less than the critical level of parental schooling beyond which parents send their children for full schooling, i.e.  $\underline{s} < \hat{s}$  and when unskilled adult wage is low compared to the expenditure, i.e.  $s' > 1$ , schooling of a child of an unskilled parent converges to the interior stable equilibrium that implies partial schooling. There

is no opportunity of full schooling of the child of an unskilled parent. For skilled parent, schooling of a child of a skilled parent converges to the interior stable equilibrium of the unskilled sector, if schooling of the skilled parent is below a critical level and if it is beyond the critical level, schooling of the child keeps on increasing till full schooling is reached. The partial schooling equilibrium in the skilled sector is an unstable equilibrium. The full schooling equilibrium is a stable equilibrium. When  $\underline{s} > \hat{s}$  and  $s' > 1$  the skilled parent always sends her child for full schooling and in unskilled sector there is unique stable steady state equilibrium of schooling. However here also there is no opportunity of full schooling of child of an unskilled parent. Only when  $s' < 1$  i.e. unskilled adult wage is high compared to expenditure, full schooling would be the only equilibrium schooling of all children. In the presence of learning by doing effect cycles emerge in the time path of schooling in unskilled sector. However, the dynamics in skilled sector remain same as in the basic model without learning by doing effect.

The comparative static exercise in the basic model yields the result that if the adult unskilled wage is less than the subsistence consumption expenditure or even if adult unskilled wage exceeds subsistence expenditure but is sufficiently small, the effect of giving education subsidy is higher than child labour ban in enhancing schooling. Thus banning child labour may not be the solution for improving schooling outcome in such a situation.

Our paper focuses on the supply side of the labour market while the demand side has been ignored. We have not considered existence of any credit market in our model. Hence, existence of credit market imperfections could not be studied in this context. We consider a closed

economy model. So examining the effects of international policies against child labour is beyond the scope of this paper. All these may be considered for future research.

## **References**

Acemoglu, D. and J.S. Pischke (2000) "Changes in the wage structure, family income, and children's education" *NBER Working Paper Number 7986*.

Apprenticeship in the informal economy in Africa: Workshop report, Geneva, 3-4 May 2007 / International Labour Office, Skills and Employability Department. - Geneva: ILO, 2008.

Baland, J. and J. Robinson (2000) "Is Child Labour Inefficient?" *Journal of Political Economy*, **108**, 663-679.

Basu, K. (2005) "Child labour and the law: Notes on possible pathologies" *Economics Letters*, **87(2)**, 169-174.

Basu, K. and P. Van (1998) "The Economics of Child Labour" *The American Economic Review*, **88**, 412-27.

Becker, G.S. and N. Tomes (1979) "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility" *Journal of Political Economy*, **87 (6)**.

Chaudhuri, S.(2004) “Incidence of Child Labour, Free Education policy, and Economic Liberalisation in a Developing Economy” *The Pakistan Development Review*, **43(1)**, 1-25.

Chaudhuri, S. And U. Mukhopadhyay (2003) “Free Education Policy and Trade Liberalization: Consequences on Child and Adult Labour Markets in a Small Open Economy” *Journal of Economic Integration*, **18(2)**, 336-359.

Croix, D. (2001) “Growth dynamics and education spending: The role of inherited tastes and abilities” *European Economic Review*, **45**, 1415-1438.

Croix, D. And O. Licandro (1999) “Life expectancy and endogenous growth” *Economic Letters*, **65**, 255-263.

Das, M. (2007) “Persistent inequality: An explanation based on limited parental altruism” *Journal of Development Economics*, **84**, 251-270.

Dessy, S. and S.Pallage (2001) “Child labor and Coordination failures” *Journal of Development Economics*, **65(2)**, 469-476.

Dessy, S. and S.Pallage (2005) “A Theory of the Worst Forms of Child Labour” *The Economic Journal*, **115(500)**, 68-87.

Dinopoulos, E. and L. Zhao (2007) "Child Labor and Globalization" *Journal of Labor Economics*, **25(3)**, 553-579.

Edmonds, E. V. and M. Shrestha (2012) "Impact of minimum age of employment regulation on child labour and schooling" *IZA Journal of Labor Policy*, **1(14)**, 2-28.

Emerson, P.M. and S.D. Knabb (2006) "Opportunity, Inequality and the Intergenerational Transmission of Child Labour" *Economica, New Series*, **73(291)**, 413-434.

Estevez, K. (2011) "Nutritional Efficiency Wages and Child Labour" *Economic Modelling*, **28**, 1793-1801.

Fabre, A. and S. Pallage (2011) "Child Labor, Idiosyncratic Shocks, and Social Policy" *CIRPEE Working Paper Number* **11-15**.

Glomm, G.(1997)"Parental choice of human capital investment"*Journal of Development Economics*, **53**, 99-114.

Glomm G. and B. Ravikumar (1998) "Increasing returns, human capital, and the Kuznets curve" *Journal of Development Economics*, **55**, 353-367.

Hare, P.G. and D.T. Ulph (1979) "On Education and Distribution" *Journal of Political Economy*, **87(5)**.



Hippel, E.V. and M. Tyre (1993) “How “Learning by Doing” is Done: Problem Identification in Novel Process Equipment” SSM WP# BPS 3521-3593.

Krueger, D. and J.T. Donohue (2005) “On the distributional consequences of child labor legislation” *International Economic Review*, **46(3)**.

Lucas, R.E. (1988) “On the Mechanics of Economic Development” *Journal of Monetary Economics*, **22**, 3-42.

Mao, Z.Y. (2012) “Learning-by-Doing and its implications for Economic Growth and International Trade”.

Moav, O. (2005) “Cheap children and the Persistence of Poverty” *Economic Journal Royal, Economic Society*, **115(500)**, 88-110.

Mukherjee, D. and U. Sinha (2006) “Schooling, Job prospect and Child Labour in a Developing Economy” *mimeo: Indian Statistical Institute, Kolkata*.

Mulligan, C. (1997) “Parental Priorities and Economic Inequality” *University of Chicago Press, Chicago*.

Parente, S.L. (1994) “Technology Adoption, Learning-by-Doing and Economic Growth” *Journal of Economic Theory*, **63**, 346-369.

Ranjan, P. (1999) “An economic analysis of child labour” *Economic Letters*, **64**, 99-105.

Ranjan, P. (2001) “Credit constraints and the phenomenon of child labor” *Journal of Development Economics*, **64**, 81-102.

Ravallion, M. and Q.T. Wodon (2000) “Does Child Labour Displace Schooling? Evidence on Behavioural responses to an Enrollment Subsidy” *The Economic Journal*, **110(462)**.

Rogers, C.A. and K.A. Swinnerton (2002) “A Theory of Exploitative Child Labor”

Schultz, T. P. (2004) “School subsidies for the poor: evaluating the Mexican Progresa poverty program” *Journal of Development Economics*, **74**, 199-250.

World Employment Report (1998-99), Training in the informal sector, <https://www.gdrc.org/informal/6-informaltraining.html>.

Zhang, W.B. (2014) “Endogenous population with human and physical capital accumulation” *International Review of Economics*, Springer, **61(3)**, 231-252.

Zhang, W.B. (2015) “Economic Oscillations with Endogenous Population, Human Capital and Wealth”.

## Appendix A: Learning by doing is absent

In the case of unskilled parent the Lagrangian function is

$$Z = \ln(c_t - \underline{c}) + s_{t-1} \ln(bs_t) + \lambda [A + A\varphi(1-s_t) - p_c c_t - \rho s_t] + \theta(c_t - \underline{c})$$

where  $\lambda$  is the Lagrange multiplier. The decision variables of the household are  $c_t$  and  $s_t$ . The first order conditions for maximization of utility are given by:

$$\frac{\delta Z}{\delta c_t} = \frac{1}{c_t - \underline{c}} - \lambda p_c + \theta = 0 \quad (\text{A.1})$$

$$\frac{\delta Z}{\delta s_t} = \frac{s_{t-1}}{s_t} - \lambda(A\varphi + \rho) = 0 \quad (\text{A.2})$$

$$\theta \geq 0, \theta(c_t - \underline{c}) = 0 \quad (\text{A.3})$$

From (A.1) and budget constraint  $A + A\varphi(1-s_t) = p_c c_t + \rho s_t$ , we get

$$\frac{1}{A + A\varphi(1-s_t) - p_c \underline{c} - \rho s_t} = \lambda \quad (\text{A.4})$$

From (A.2) and (A.4) we get,

$$s_t = \frac{s_{t-1} [A(1+\varphi) - p_c \underline{c}]}{(A\varphi + \rho)(1 + s_{t-1})} \quad (\text{A.5})$$

$$\frac{ds_{\underline{u}}^*}{d\varphi} = \frac{-A[A - p_c \underline{c} - \rho]}{(A\varphi + \rho)^2} \quad (\text{A.6})$$

$$\frac{ds_{\underline{u}}^*}{d\rho} = \frac{-[A(1+\varphi) - p_c \underline{c}]}{(A\varphi + \rho)^2} \quad (\text{A.7})$$

$$\frac{ds_{\underline{u}}^*}{dA} = \frac{\rho + p_c \underline{c} \varphi + \rho \varphi}{(A\varphi + \rho)^2} \quad (\text{A.8})$$

In the case of skilled parent the Lagrangian function is  $Z = \ln(c_t - \underline{c}) + s_{t-1} \ln(bs_t) + \lambda [\delta bs_{t-1} + A\varphi(1 - s_t) - p_c c_t - \rho s_t] + \theta(c_t - \underline{c})$

where  $\lambda$  is the Lagrange multiplier. The decision variables of the household are  $c_t$  and  $s_t$ . The first order conditions for maximization of utility are given by:

$$\frac{\delta Z}{\delta c_t} = \frac{1}{c_t - \underline{c}} - \lambda p_c + \theta = 0 \quad (\text{A.9})$$

$$\frac{\delta Z}{\delta s_t} = \frac{s_{t-1}}{s_t} - \lambda(A\varphi + \rho) = 0 \quad (\text{A.10})$$

$$\theta \geq 0, \theta(c_t - \underline{c}) = 0 \quad (\text{A.11})$$

From (A.9) and budget constraint  $\delta bs_{t-1} + A\varphi(1 - s_t) = p_c c_t + \rho s_t$ , we get

$$\frac{1}{\delta bs_{t-1} + A\varphi(1 - s_t) - p_c \underline{c} - \rho s_t} = \lambda \quad (\text{A.12})$$

From (A.10) and (A.12) we get,

$$s_t = \frac{s_{t-1} [\delta bs_{t-1} + \varphi A - p_c \underline{c}]}{(A\varphi + \rho)(1 + s_{t-1})} \quad (\text{A.13})$$

$s_t$  for skilled parent -  $s_{t-1}$  for skilled parent  $> 0$  if  $s_{t-1} > \frac{p_c \underline{c} + \rho}{\delta b - A\varphi - \rho} = s_s^*$ . This implies that once  $s_t$  crosses  $s_s^*$ ,  $s_t$  will be greater than  $s_{t-1}$  i.e.  $s_t$  curve will lie above the  $45^\circ$  line in case of skilled sector.

Now  $s_t$  for skilled parent at  $\underline{s} = s_t$  in case of unskilled parent at  $\underline{s}$ . This implies that the  $s_t$  curve will be continuous at  $s_t = \underline{s}$ .

## Relation between $s_u^*$ , $\underline{s}$ and $s_s^*$

$$s_u^* = \frac{A - p_c \underline{c} - \rho}{A\varphi + \rho}$$

$$\underline{s} = \frac{A}{\delta b}$$

$$s_s^* = \frac{p_c \underline{c} + \rho}{\delta b - A\varphi - \rho}$$

$$s_s^* > \underline{s} \text{ if } \frac{p_c \underline{c} + \rho}{\delta b - A\varphi - \rho} > \frac{A}{\delta b}$$

$$s_u^* < \underline{s} \text{ if } \frac{A(\delta b - A\varphi - \rho) - \delta b(p_c \underline{c} + \rho)}{\delta b(A\varphi + \rho + \underline{h})} < 0$$

$$\text{Now if } \frac{p_c \underline{c} + \rho}{\delta b - A\varphi - \rho} > \frac{A}{\delta b}, \text{ then } s_u^* < \underline{s}$$

$$\text{Therefore if } \frac{p_c \underline{c} + \rho}{\delta b - A\varphi - \rho} > \frac{A}{\delta b} \text{ then } s_s^* > \underline{s} > s_u^*$$

$$\text{In our paper we assume that } \frac{p_c \underline{c} + \rho}{\delta b - A\varphi - \rho} > \frac{A}{\delta b}.$$

$$\text{Therefore } s_s^* > \underline{s} > s_u^*.$$

## Appendix B: Learning by doing is present

In case of unskilled parent the Lagrangian function is

$$Z = \ln(c_t - \underline{c}) + s_{t-1} \ln(bs_t) + \lambda [A + (1-s_{t-1})\underline{h}] + A\varphi(1-s_t) - p_c c_t - \rho s_t + \theta(c_t - \underline{c})$$

where  $\lambda$  is the Lagrange multiplier. The decision variables of the household are  $c_t$  and  $s_t$ . The first order conditions for maximization of utility are given by:

$$\frac{\delta Z}{\delta c_t} = \frac{1}{c_t - \underline{c}} - \lambda p_c + \theta = 0 \quad (\text{B.1})$$

$$\frac{\delta Z}{\delta s_t} = \frac{s_{t-1}}{s_t} - \lambda(A\varphi + \rho) = 0 \quad (\text{B.2})$$

$$\theta \geq 0, \theta(c_t - \underline{c}) = 0 \quad (\text{B.3})$$

From (B.1) and budget constraint  $[A + (1-s_{t-1})\underline{h}] + A\varphi(1-s_t) = p_c c_t + \rho s_t$ , we get

$$\frac{1}{A + (1-s_{t-1})\underline{h} + A\varphi(1-s_t) - p_c c_t - \rho s_t} = \lambda \quad (\text{B.4})$$

From (B.2) and (B.4) we get,

$$s_t = \frac{s_{t-1}[A + (1-s_{t-1})\underline{h} + \varphi A - p_c \underline{c}]}{(A\varphi + \rho)(1+s_{t-1})} \quad (\text{B.5})$$

$$s_u^* = \frac{A + \underline{h} - p_c \underline{c} - \rho}{A\varphi + \rho + \underline{h}}$$

$$\frac{ds_u^*}{d\varphi} = \frac{-A[A + \underline{h} - p_c \underline{c} - \rho]}{(A\varphi + \rho + \underline{h})^2} \quad (\text{B.6})$$

$$\frac{ds_u^*}{d\rho} = \frac{-[A(1+\varphi) + 2\underline{h} - p_c \underline{c}]}{(A\varphi + \rho + \underline{h})^2} \quad (\text{B.7})$$

$$\frac{ds_u^*}{dA} = \frac{\rho + p_c \underline{c} \varphi + \rho \varphi + \underline{h}(1-\varphi)}{(A\varphi + \rho + \underline{h})^2} \quad (\text{B.8})$$

In case of skilled parent the Lagrangian function is  $Z = \ln(c_t - \underline{c}) + s_{t-1} \ln(bs_t) + \lambda [\delta bs_{t-1} + A\varphi(1-s_t) - p_c c_t - \rho s_t] + \theta(c_t - \underline{c})$

where  $\lambda$  is the Lagrange multiplier. The decision variables of the household are  $c_t$  and  $s_t$ . The first order conditions for maximization of utility are given by:

$$\frac{\delta Z}{\delta c_t} = \frac{1}{c_t - \underline{c}} - \lambda p_c + \theta = 0 \quad (\text{B.9})$$

$$\frac{\delta Z}{\delta s_t} = \frac{s_{t-1}}{s_t} - \lambda(A\varphi + \rho) = 0 \quad (\text{B.10})$$

$$\theta \geq 0, \theta(c_t - \underline{c}) = 0 \quad (\text{B.11})$$

From (B.9) and budget constraint  $\delta bs_{t-1} + A\varphi(1-s_t) = p_c c_t + \rho s_t$ , we get

$$\frac{1}{\delta bs_{t-1} + A\varphi(1-s_t) - p_c \underline{c} - \rho s_t} = \lambda \quad (\text{B.12})$$

From (B.10) and (B.12) we get,

$$s_t = \frac{s_{t-1} [\delta bs_{t-1} + \varphi A - p_c \underline{c}]}{(A\varphi + \rho)(1 + s_{t-1})} \quad (\text{B.13})$$

$$s_S^* = \frac{p_c \underline{c} + \rho}{\delta b - A\varphi - \rho}$$

$s_t$  for skilled parent -  $s_{t-1}$  for skilled parent  $> 0$  if  $s_{t-1} > \frac{p_c \underline{c} + \rho}{\delta b - A\varphi - \rho} = s_S^*$ . This implies that once  $s_t$  crosses  $s_S^*$ ,  $s_t$  will be greater than  $s_{t-1}$  i.e.  $s_t$  curve will lie above the 45° line in case of skilled sector.

Now  $s_t$  for skilled parent at  $\underline{s} = s_t$  in case of unskilled parent at  $\underline{s}$ . This implies that the  $s_t$  curve will be continuous at  $s_t = \underline{s}$ .

### Relation between $s_u^*$ , $\underline{s}$ and $s_s^*$

$$s_u^* = \frac{A+h-p_c\underline{s}-\rho}{A\varphi+\rho+h}$$

$$\underline{s} = \frac{A+h}{\delta b+h}$$

$$s_s^* = \frac{p_c\underline{s}+\rho}{\delta b-A\varphi-\rho}$$

$$s_s^* > \underline{s} \text{ if } \frac{p_c\underline{s}+\rho}{\delta b-A\varphi-\rho} > \frac{A+h}{\delta b+h}$$

$$s_u^* < \underline{s} \text{ if } \frac{(A+h)(\delta b-A\varphi-\rho) - (\delta b+h)(p_c\underline{s}+\rho)}{(\delta b+h)(A\varphi+\rho+h)} < 0$$

Now if  $\frac{p_c\underline{s}+\rho}{\delta b-A\varphi-\rho} > \frac{A+h}{\delta b+h}$ , then  $s_u^* < \underline{s}$

Therefore if  $\frac{p_c\underline{s}+\rho}{\delta b-A\varphi-\rho} > \frac{A+h}{\delta b+h}$  then  $s_s^* > \underline{s} > s_u^*$

In skilled sector,  $s_t$  (at  $s_{t-1} = \underline{s}$ ) -  $\underline{s} = \frac{\underline{s}[\underline{s}(\delta b-A\varphi-\rho) - (p_c\underline{s}+\rho)]}{(A\varphi+\rho)(1+\underline{s})}$

Since we assume in our paper  $\frac{p_c\underline{s}+\rho}{\delta b-A\varphi-\rho} > \frac{A+h}{\delta b+h}$ , therefore  $[s_t \text{ in skilled sector (at } s_{t-1} = \underline{s}) - \underline{s}] < 0$ .

This implies that  $s_t$  at  $\underline{s}$  lies below the 45<sup>0</sup> line in skilled sector.