Exploring the Determinacy Dynamics in an Open Economy

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Abstract

A crucial theme in macroeconomic dynamics concerns the issue of determinacy, that is, the question of uniqueness or multiplicity of admissible dynamic trajectories. Unlike previous studies which economy addressed this question in a closed, we explore the determinacy dynamics in a small open economy. The structure of the model set forth, is such that it leads to a higher degree characteristic equation which cannot be handled analytically. By using a specific algorithm developed, we solve it and show that a form of a Taylor rule implies, for the parameter space examined, determinate equilibrium dynamics. In line with previous findings on determinacy, the case for a form of flexible Price Level Targeting (PLT), does not only hold in a closed economy but, also, extends with some modification, to a small open economy as well.

Keywords: Taylor Rule, Open Economy Dynamics, Flexible PLT, Determinacy

JEL Classification: C54, E52

1. INTRODUCTION

The advent of the so-called Taylor rules has had a major impact on the positive and normative aspects of monetary policy making (e.g. Koenig et al, 2012). In a seminal paper, Taylor (1993), used a monetary rule to address the question of monetary policy making, in the U.S (1987-1992); the appealing feature of being simple aided the issue of transparency. As an operational and descriptive frame, it became highly popular, sparking off a voluminous literature and a new agenda for further topics of research.
A key normative implication of Taylor’s work, which has led to a distinct strand of literature, is that injudiciously designed rules may cause instability and multiple equilibria (indeterminacy/non-uniqueness). The intuition is simple. It is asserted, in this respect, that in the face, say, of an inflationary aggregate demand rise, the federal funds rate should respond in such a way so that real rates rise unequivocally, in order to stifle the incipient inflationary pressure (commonly known as Taylor’s principle); otherwise the whole process may evolve self-perpetuated, leading to instability.

A number of notable studies, in this strand of literature, have approached the question of (in) determinacy from a variety of perspectives. Specifically, Clarida et al (2000), noted the transition from indeterminacy to determinacy since the early eighties (the Volcker-Greenspan era), owing to the toughening of the anti-inflationary stance. Other major studies, in this line of inquiry are Bullard and Mitra (2002), Kiley (2007) and Coibion and Gorodnichenko (2011). The former derives analytical conditions for determinacy when expectations are viewed as a learning process, while the latter seek to qualify Taylor’s principle regarding determinacy\(^1\), in case where trend inflation grows over time. More recently, Ambler and Lam (2015), in comparing Inflation Targeting (IT) with Price Level Targeting (PLT), conclude for the parameter space examined, indeterminacy does not arise under PLT (contrary to the case of IT). Also, in a broadly similar vein, Dittmar and Gavin (2005), in a RBC model with money, determined that with a zero policy coefficient on inflation, determinacy obtains when the policy coefficient on the price level belongs to the interval \((0, 2)\).

We propose to reconsider the issue, at hand, on two principal counts. First, in the light of earlier findings, instead of inflation targeting, we choose to focus on Price Level Targeting (PLT); several reasons have been adduced in its support together with an overall eclectic

\(^1\): Taylor’s equation was originally specified in real terms; when transformed into nominal, the Taylor principle states that the nominal rate response to inflationary hikes should exceed unity.
assessment (see Cecchetti, 2000; Cote, 2007; Khan, 2009; Ambler, 2010, among others). Further, as Ambler (2010), in particular, has argued, PLT may substitute for commitment if the latter is not possible.

Historically, it has been successfully implemented by the Swedish RiksBank (1931-1937) and, since 2006, has been systematically on the research agenda of the Bank of Canada. Note, moreover, that empirically, Fair (2008), after testing multiple specifications, concludes that the equation for the *price level*, is the preferred specification in a structural price-wage model. Although, PLT has been sparsely analyzed from the determinacy perspective, it has never been done so in an open economy. In view of the particular findings of Dittmar and Gavin (2005) and Ambler and Lam (2015), regarding inflation indeterminacy, we purport to investigate the properties of a Taylor rule, adapted to price level targeting, in a dynamic open economy. The latter integrates the key role of the exchange rate and its various channels of influence, in to the transmission process; in particular, critical is the role of the exchange rate presence in the wage-price sector as well in the demand sector, as my personal research, and the extensive work of Froyen and Guender (2007), Guender (2011) have clearly shown.

We shall approach the issue at hand, both in analytic and algorithmic terms, but it is through the latter perspective, that more informative light shall be provided. Emerging results and detailed comments thereof, appear in the final section.

The specifics of the skeletal model to be used and the technical issues involved, shall be discussed in what follows.

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2: I. Fisher has been a prominent early proponent.

3: What price level to aim for, is a rather contested issue; there appears some consensus that domestic price level is the right choice, if there exists price stickiness (see for e.g. De Paoli, 2009, among others)
2. THE MODEL

Consider the following dynamic open economy model, suitably adapted to our purposes. We begin with the New Keynesian IS forward-looking equation which has been formulated, in the light of Liu (2006), Gali (2008) and, especially, Guender (2011):

\[
Y_t = -\Phi_1 \left\{ r_t - E_t \left[ \theta P_{t+1} + (1 - \theta)(E_{t+1} + P_{t+1}^f) - \left( \theta P_t + (1 - \theta)(E_t + P_t^f) \right) \right] \right\} + \Phi_2 \left[ E_t + P_t^f - P_t - E_t \left( E_{t+1} + P_{t+1}^f - P_{t+1} \right) \right] + E_t Y_{t+1} + \Phi_3 \left( Y_t^f - E_t Y_{t+1}^f \right) + \nu_t
\]

(2.1)

where: \( CPI = \left( \theta P_t + (1 - \theta)(E_t + P_t^f) \right) \)

\( Y = \) output; \( P = \) Domestic Price Level, \( r = \) nominal interest rate, \( E = \) Domestic currency units per unit of foreign currency, \( P^f = \) Foreign Price Level, \( Y^f = \) Foreign Output; \( \nu_t \sim N(0, \sigma^2) \). Expectations are supposed to be rational, and variables (except for interest rates) are typically in logarithms. The steady state rate of inflation is constant, which in full equilibrium could be taken as zero.

What follows provides a comprehensive model description. Equation (2.1) is the output equation which postulates that output is equal to total consumption, consisting of home goods consumption by domestic households and consumption of home goods relative to foreign ones by foreigners.

As regards the pricing scheme, we follow, unlike earlier studies, the Rotemberg (1982) inter-temporal optimizing framework where firms face intra and inter-temporal quadratic costs of adjustment:

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4: Guender, in particular, spells out with clarity, the micro-founded underpinnings.

5: Inflation targeting countries, set a target ranging between 1.5 and 2.5 percent (Svensson, 1999). Woodford (1999), Cecchetti (2000), discuss when Inflation Targeting amounts to Price Level Path Targeting.
\[
\min(\text{LS}) = E_t \sum_{i=0}^{\infty} \xi_i [\kappa (P_{t+i} - P_{t+i-1})^2 + \mu (P_{t+i} - P_{t+i}^*)^2]
\]
\[
\kappa, \mu > 0 \tag{2.2}
\]

The ensuing conditions (2.3)-(2.3a):

\[
P_t = \alpha_1 P_{t-1} + \alpha_2 P_t^* + \alpha_3 E_t P_{t+1} \tag{2.3}
\]

\[
\alpha_1 + \alpha_2 + \alpha_3 = 1, \alpha_3 = \xi \alpha_1 \text{ where } \xi = \text{discount factor} = 0.99 \tag{2.3a}
\]

with \( \alpha_1 = \kappa / \kappa + \mu + \xi \), \( \alpha_2 = \mu / \kappa + \mu + \xi \), \( \alpha_3 = \xi \kappa / \kappa + \mu + \xi \)

\[
P_t^* = \gamma_1 (Y_t - \bar{Y}) + \gamma_2 (E_t + \sum_{i=1}^{k} Z^f_{it}) \tag{2.4}
\]

obtain after optimizing with respect to \( P_t \) and re-parameterizing, and lay down a scheme comprising: a) a predetermined price variable, b) a forward-looking price term and c) a an ex-post equilibrium” price term, \( P_t^* \), that would have prevailed if there were no price frictions.

Complexity is enhanced, as the scheme encompasses, both, backward-looking as well as forward-looking terms. Equation (2.4) is derived from neoclassical first principles and flows from a price-flexible equilibrium frame, where \( P_t^* \) is given in terms of a mark-up augmented marginal cost.

It has been assumed that the nominal wage rate component of the marginal cost, follows a scheme indexed to the CPI, with allowance being made for the outcome to be endogenously affected, by the sluggishness or tightness in the labour market, captured by \( Y_t - \bar{Y} \). The marginal cost is further extended to include the domestic currency price of foreign inputs, which enter a Cobb-Douglas production function:

\[
\bar{Y}_i = \tilde{N} \delta_i \tilde{Z}_i \quad i = 2, 3, \ldots \tag{2.5}
\]

where ( \( \tilde{X} = \text{real variable}, X = \ln \tilde{X} \), \( \bar{Y} = \text{real output}, \tilde{N} = \text{labour}, \tilde{Z}_i = \text{foreign input } i \).

---

\(^6\) The \( \gamma \)'s are composite parameters and the relevant mark-up is assumed to vary inversely with output.
The derived marginal cost equation involves standard calculus, but it is lengthy due to the presence of more than two substitutable factor inputs. Foreign currency prices are denominated in terms of a leading currency.

Equation (2.4) is a contemporaneous linear feedback rule which may be labeled as flexible price targeting. Note, as Taylor did, that both policy reaction coefficients are positive, indicating a policy “against the wind” vis-à-vis the specified policy objectives. Taylor presented empirical estimates in his description of U.S monetary policy making, refraining however, from a normative suggestion regarding the size of $\Phi_p$ relative to $\Phi_Y$, except for the requirement that $\Phi_p > 1$; on the whole, his approach was basically empirical.

We stipulate the policy rule:

$$r_t = \bar{r} + \bar{\pi} + \Phi_p (P_t - \bar{P}) + \Phi_Y (Y_t - \bar{Y})$$

(2.6)

where $\bar{r}$ is the natural rate of interest, and $\bar{\pi}$ is the steady inflation rate which may be constant or zero. Under PLT, however, the latter would be conceptually more appropriate. It would be instructive, at this point, to see the mechanics of PLT and its subtle difference with IT, when steady state inflation is positive (see, also, Cover and Pecorino (2005)).

Let us suppose that:

$$P_t^T = P_{t-1}^T + \bar{\pi}$$

(2.7)

$$\pi_t^T = P_t^T - P_{t-1}^T$$

or

$$\pi_t^T = P_t^T - P_{t-1}^T = P_{t-1}^T - P_{t-1}^A + \bar{\pi}$$

(2.7a)

$$\pi_t^T \equiv \tilde{\pi}_t^T = -(P_{t-1}^A - P_{t-1}^T)$$

(2.7b)

$P_t^T$ = policy target of $P$ at $t$, $P_{t-1}^A$ = actual price level at $t - 1$,

$\tilde{\pi}_t^T$ = target inflation relative to the steady state

This shows that under PLT, there is a linkage between past and present, known as the “memory” property. If $P_{t-1}^T > P_{t-1}^A$, then at time $t$, target inflation is higher than steady state inflation,
international financial linkages are depicted through an uncovered interest parity (UIP) condition, tying exchange rates with interest rates. The exchange rate, in particular, plays a key role, since it exerts a dual transmission effect, affecting (in opposite directions), both, the sector of production (the foreign inputs effect) and the sector of demand (the price competitiveness effect).

The UIP equation takes either of two equivalent versions:

\[ r_t = r_t^f + E_t^i e_{t+1} - E_t \]  
\[ (2.8a) \]

\[ r_t - E_t \pi_{t+1} = r_t^f - E_t \pi_t^f - E_t \Delta (R_t R_t)_{t+1} \]  
\[ (2.8b) \]

Where \( RER = \) Real Exchange Rate (\( E + P^d - P \)), \( r_t^f \) is the foreign rate of interest, \( \pi_{t+1}^f \) is foreign rate of inflation, \( E \) is the familiar expectations symbol, and \( \Delta \) is the difference operator. For our purposes, it is deemed appropriate to retain the original version.

All foreign variables (except for \( Z_t \)) are assumed to follow a white noise pattern. These assumptions have been made with a view to reduce the dimensions involved, and concentrate on a (second order) three difference equations system, where the forward expectations of variables are given in terms of current and lagged counterparts. We denote with a hat all variables relative to their steady state and make more compact the expectations notation. Substituting equation (2.6) into (2.1) and (2.8a) and, moreover, (2.4) into (2.3), performing all the necessary algebra, and collecting forward expectations on the left-hand side, we obtain our canonical model in matrix notation:
\[
\begin{bmatrix}
\dot{y}_{t+1} \\
\dot{\hat{p}}_{t+1,1} \\
\dot{\hat{e}}_{t+1,1}
\end{bmatrix}
= C_{10} \begin{bmatrix}
\dot{y}_t \\
\dot{\hat{p}}_t \\
\dot{\hat{e}}_t
\end{bmatrix}
+ C_{30} \begin{bmatrix}
\dot{\hat{p}}_{t-1} \\
\dot{\hat{e}}_{t-1}
\end{bmatrix}
\]

Where \( C_{10}, C_{20} \) and \( C_{30} \) are as follows:

\[
C_{10} = \begin{bmatrix}
1 & \Phi_1 \theta + \Phi_2 & \Phi_1 (1 - \theta) - \Phi_2 \\
0 & \alpha_3 & 0 \\
0 & 1 & 1
\end{bmatrix}
\]

\[
C_{20} = \begin{bmatrix}
1 + \Phi_1 \Phi_y & \Phi_1 \Phi_p + \Phi_2 \theta + \Phi_2 & \Phi_1 (1 - \theta) - \Phi_2 \\
-\alpha_2 \gamma_1 & 1 & -\alpha_2 \gamma_2 \\
\Phi_y & \Phi_p & 1
\end{bmatrix}
\]

\[
C_{30} = \begin{bmatrix}
0 & 0 & 0 \\
0 & -\alpha_1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
C_{10}^{-1} = \begin{bmatrix}
1 & \alpha_3 \\
0 & \frac{1}{\alpha_3} & 0 \\
0 & \frac{1}{\alpha_3} & 1
\end{bmatrix}
\]

Pre-multiplying by \( C_{10}^{-1} \), we obtain the following canonical form of the model:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{y}_{t+1} \\
\dot{\hat{p}}_{t+1,1} \\
\dot{\hat{e}}_{t+1,1}
\end{bmatrix}
- C_{10}^{-1} C_{20} \begin{bmatrix}
\dot{y}_t \\
\dot{\hat{p}}_t \\
\dot{\hat{e}}_t
\end{bmatrix}
- C_{10}^{-1} C_{30} \begin{bmatrix}
\dot{\hat{p}}_{t-1} \\
\dot{\hat{e}}_{t-1}
\end{bmatrix}
= 0
\tag{2.10}
\]

Let \( M = \begin{bmatrix}
\hat{y}_t \\
\hat{p}_t \\
\hat{e}_t
\end{bmatrix} \), then the homogenous solution is \( M = \sum_{i=0}^{6} n_i m_i \lambda_i^t \) where \( n_i \) the \( i \) eigen vector and \( \lambda_i \) the \( i \) eigen value which obtains, from a 6th degree polynomial equation having common factor \( \lambda^2 \). Factoring it out, we derive a 4th degree polynomial equation (with a constant term inclusive of \( \Phi_p \Phi_y \)).

Let, \( A(\lambda) \) be the 4th degree polynomial:

\[
\lambda^4 + A_3 \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0
\tag{2.11}
\]

LaSalle (1986), provides with clarity the conditions which if and only if they hold, then stability exists. The conditions are:
\[(b)|A_4| < 1 \cdot (c)|A_5 + A_6| < 1 + A_5 + A_6\]  

(2.12)

Where in our case, \(A_{33} = f_3(A, \Psi)\)  
\(A_{23} = f_2(A, B, \Gamma, \Delta, \Psi)\)  
\(A_{13} = f_1(A, B, \Gamma, \Delta, \Psi)\)  
\(A_{03} = f_0(A, \Gamma, \Delta, \Psi)\)  
\(\Psi = (\gamma_1, \gamma_2, \alpha_1, \alpha_2, \alpha_3)\)  

\[A = 1 + \Phi_1 \Phi_Y a_2 \gamma_1 \left[\frac{\Phi_1 \theta + \Phi_2 - (\Phi_1(1-\theta) - \Phi_2)}{a_3}\right] - \Phi_Y (\Phi_1(1-\theta) - \Phi_2)\]
\[B = \Phi_1 \Phi_p + \Phi_1 \theta + \Phi_2 + \frac{[\Phi_1(1-\theta) - \Phi_2] - (\Phi_1 \theta + \Phi_2)}{a_3} - [\Phi_Y (1-\theta) - \Phi_2] \Phi_p\]
\[\Gamma = a_2 \gamma_2 \left[\frac{\Phi_1 \theta + \Phi_2 - (\Phi_1(1-\theta) - \Phi_2)}{a_3}\right]\]
\[\Delta = a_1 \left[\frac{\Phi_1 \theta + \Phi_2 - (\Phi_1(1-\theta) - \Phi_2)}{a_3}\right]\]

By construction two roots are equal to zero. We shall appeal to the Blanchard-Khan (1980) theorem, as the basis of our analysis; in our case, three characteristic roots/eigenvalues must be outside the unit circle, since there exists a triplet of non-predetermined variables. Analytically, the emerging LaSalle conditions are extremely intractable to draw conclusions from. Accordingly, the only route is to approach the problem algorithmically, a theme to which we now turn.
3. RESULTS AND DISCUSSION

We begin by stipulating that $\Phi_P$ and $\Phi_Y$ belong to an interval, broadly in line with topical literature (Dittmar and Gavin, 2005; Ambler and Lam, 2015). To derive the four characteristic roots, satisfying the Blanchard-Khan theorem, we make use the following set of calibration values$^7$:

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\Phi_1$</th>
<th>$\Phi_2$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.40</td>
<td>0.25</td>
<td>0.05</td>
<td>0.1</td>
<td>0.25</td>
<td>13</td>
<td>0.5</td>
</tr>
</tbody>
</table>

$(\Phi_P \in [0, 2]) (\Phi_Y \in [0, 2])$

The proposed algorithm is a device specifically developed to perform a grid search using 0.1 as a step in deriving $\Phi_Y$ and $\Phi_P$. The result is a map of correspondence between policy coefficients and a determinate equilibrium outcome.

Solutions obtain (discretely) across the entire spectrum examined and enlighten the question of determinacy regarding the nature of macroeconomic dynamics in the open economy$^8$. What we observe is that at the lower end of spectrum, $\Phi_P$ is lower than $\Phi_Y$, while at the other (higher) end the spectrum, we obtain the more typical result where $\Phi_P$ is greater than $\Phi_Y$. In general, however, the value of $\Phi_Y$ has been found to be higher than in other topical studies. It stands to reason to surmise, that at the lower end of the spectrum, a higher $\Phi_Y$ makes up for the lower value of $\Phi_P$, so that a sufficient overall response is secured$^9$. Our results, apart from the higher $\Phi_Y$ value, are broadly conformable with those of Ambler and Lam (2015). Algebraically put, we have:

$$\left[0 \leq \Phi_P \leq 1.9\right], \left(0 < \Phi_Y \leq 1.7\right)$$

$^7$: Some values have been adopted from Froyen and Guender (2007, p.268); appropriate references cited, have also provided useful guidance. The value of $\theta$ has been taken from World Bank Statistics.

$^8$: On the whole, there obtain 47 solutions regarding the admissible roots. These are discretely different, but only marginally so.

$^9$: The inter-relationship, between $\Phi_P$ and $\Phi_Y$, emerges in analytical terms, in the determinacy condition derived by Bullard and Mitra (2002). It could be argued that our finding is the algorithmic counterpart.
Our perspective is normative in nature and not descriptive. Operationally, however, concerns have been expressed about a stronger response to the output gap, for the reasons outlined in McCallum (2001). According to our findings, the Taylor monetary rule should be normatively construed as a flexible policy framework, where for admissible values of policy coefficients, one may eclectically choose the more appropriate combination, in the face of the economic conjuncture. In fact, this is not incongruous with pragmatic policy experience.

Froyen and Guender (2007) and Guender (2011) have found drastic implications (from a stabilization perspective) for their results in the topics explored, depending on the presence or not, of an exchange rate channel in the pricing scheme\(^{10}\). Motivated by this, we repeat our analysis by setting \(\gamma_2\) equal to zero and examine the implications for the determinacy issue. The deeper essence of the foregoing results does not change much. The whole picture changes only quantitatively, noting that, on average, \(\Phi_Y\), is higher than before.

Widespread openness is what it matters, in sense that the exchange rate permeates, both, demand and supply sectors, in contrast to a fully closed economy. Analytical conditions for determinacy are extremely difficult to explore analytically, which makes compelling the algorithmic approach. By contrast, the closed economy case studied by Bullard and Mitra (2002), had a set-up consisting of (a first order) 2x2 system, in which case, they analytically derived that \(\Phi_P > 1\) (when \(\Phi_Y = 0\)).

Using the determinacy criterion, our results imply that opening up the economy, on all counts, favors the case for a flexible PLT. Further support for this, comes from another source (Batini and Yates, 2003, p.294) whose analysis, is notably, predicated not on the

\(^{10}\) : Guender (2011), for example, showed that when \(\gamma_2 > 0\), monetary policy from a timeless perspective is not equivalent to policy under discretion unlike the case of a closed economy.
determinacy criterion, but on that of the minimum variance of target variables. The fact that their result is derived through the use of a different procedure, adds robustness to our finding. It is found, in this connection, that openness enhances the performance of a rule approaching a flexible PLT relative to other policy targets, *on condition* that, policy makers follow a simple, contemporaneous policy rule, as the one used by Taylor (1993). Yet further support for PLT (relative to IT), comes from another source, (Guender and Oh, Do Yoon, 2006), who find a more favorable trade-off with output under the former regime.

Reverting, after this useful digression, to our own line of inquiry, we notice that only at the higher end of our value spectrum, the precept of the Taylor Principle, stressing a robust nominal rate response, holds mutatis-mutandis, to the case of the price level (in a dynamic open economy). This, in its own right, qualifies earlier findings concerning the closed economy.

In summary, our piece has demonstrated that determinacy is compatible with the case where a Taylor rule (adapted to a flexible form of PLT) in in operation in an open economy. Both goals (price and output stability) are significant; the relation between the policy coefficients depends on the low or high end of their value spectrum. Taylor’s principle, as such, receives only partial support. Further, it is shown that the Taylor framework defines a normative menu of admissible policy choices; the question of which may be selected, depends on the economic conjuncture. The case for a flexible PLT is supported, in a setting where thorough openness, has drastic implications relative to those of a closed economy.
REFERENCES


