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# Discriminating Against Captive Customers

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**Abstract**

We analyze a market where some consumers only consider buying from a specific seller while other consumers choose the best deal from several sellers. When sellers are able to discriminate against their captive customers, we show that discrimination harms consumers in aggregate relative to the situation with uniform pricing when sellers are approximately symmetric, while the practice tends to benefit consumers in sufficiently asymmetric markets.

## 1 Introduction

In a market where some customers are "captive" to particular sellers while others can choose freely among alternative offers, is it good or bad for consumers overall if firms can discriminate against their captive customers? Such discrimination is clearly bad for the captives because they are monopolized, but perfect competition then prevails for the custom of non-captives. With uniform pricing, on the other hand, captives get some benefit of competition, but competition is weakened by their presence, making the net effect unclear.

In this paper we show by way of a simple duopoly model that the answer depends on the relative importance of (i) the degree of symmetry between firms and (ii) the ratio of profit to deadweight welfare loss under monopoly. With symmetric firms, discrimination against captive customers harms consumers overall because it does not reduce profits but it widens the variation of profit across consumers. Under the mild regularity condition that consumer surplus is a strictly concave function of profit, this mean-preserving spread of profit is harmful to consumers. It is as though they are risk-averse to profit variation. But if monopoly profit exceeds the associated deadweight loss, the comparison is reversed

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if there is enough asymmetry between firms. That is because uniform pricing, by softening competition, raises profits by enough to make consumers worse off despite their aversion to the greater profit variation that comes with discrimination. The key step in our analysis, following Armstrong and Vickers (2001), is to think of consumer surplus as a function of profit. Familiar concavity arguments then yield welfare results quite directly.

Our model applies to situations where a seller has information about whether or not a prospective customer is able or willing to consider rival sellers for her purchase. For instance, some consumers might use a comparison website to choose between multiple offers while others shop more randomly, and a seller engages in price discrimination if it chooses different prices on the comparison site and when consumers buy from it directly. A chain store may have varying degrees of local competition across its stores, and can choose higher prices in those outlets where consumers are more captive. An insurance seller (say) might initially offer a customer a relatively expensive tariff, especially if she is an existing customer, which is then discounted if the customer says she has found a better deal. An energy firm might offer a range of different tariffs for its (essentially homogeneous) product, where inert customers end up on the most expensive "default" tariff while more active consumers shop around for cheaper (but often short term) offers. Price discrimination in such markets is a live policy issue, as regulators in the energy sector consider whether to force suppliers to put all customers on their cheapest available tariff (or more generally to limit the gap between the cheapest and the default tariffs).

The model we analyse involves a market with homogeneous products where different consumers are able or willing to consider different subsets of firms for their purchase. When firms use uniform pricing, for instance because they have no information about whether customers are captive or not, the equilibrium in a one-shot Bertrand interaction is typically that firms use mixed strategies for their prices and there is price dispersion in the market. (Papers in this tradition include Butters (1977), Varian (1980), and Burdett and Judd (1983).) The paper in this class we follow most closely is Narasimhan (1988), who studies a duopoly model where firms can be asymmetric. The advantage of studying a duopoly market is that it is easily solved, while asymmetric models with more than two firms are currently little understood when firms use uniform prices.

Our paper contributes to the analysis of price discrimination in oligopoly. A feature of some oligopoly models is that, unlike the monopoly case, discrimination reduces equilibrium profit—see, for example, Thisse and Vives (1988) and Corts (1998) for analyses with product differentiation and deterministic prices. The same is true in our main model with asymmetric sellers, but with symmetry equilibrium profits are the same with and without discrimination, which is the key to the mean-preserving spread argument that is central to our analysis. We also provide a modified model where firms see a noisy signal of whether a consumer is captive, where price discrimination instead causes profit and prices to rise. Whereas most of the literature on price discrimination explores the implications of differences of preferences across markets, our baseline model abstracts from this issue to focus on discrimination on the basis of whether or not a consumer is captive. Recent papers that also examine price discrimination not based on consumer preferences include Chen and Riordan (2015) on cost-based differential pricing, and Heidues and Köszegi (2017) on discrimination based on indicators of consumer naivety.

After presenting our general modelling framework in the next section, where we show how price discrimination based on whether a consumer is captive cannot improve industry profit, we specialise the market in section 3 to duopoly. There we show how the impact of price discirmination on consumers depends on the degree of asymmetry between sellers and the degree of "risk aversion" to profit by consumers, where the former makes discrimination more likely to benefit consumers and the latter makes it less likely. Finally, in section 4 we show how the analysis can apply to situations where consumers have different demand curves, and how the results are affected if firms see only a noisy signal of whether a consumer is captive. We show that a noisy signal can convert a symmetric market into a pair of asymmetric markets, and thereby cause profit to rise and consumer surplus to fall.

# 2 A framework

There are $n$ sellers which costlessly supply a homogeneous product. Consumers differ according to which sellers they are able or willing to buy from, and an exogenous fraction consider a given subset $S \subset \{1, ..., n\}$ of sellers. Since consumers who do not consider any sellers play no role in the analysis, suppose all consumers consider at least one seller and normalize the measure of consumers to be 1. A consumer is *captive* to a seller if she considers only that seller. Suppose seller $i = 1, ..., n$ has $\gamma_i$ captive customers, and let $\gamma = \Sigma_{i=1}^{n} \gamma_i$ be the total number of captive customers.

Figure 1 illustrates two patterns of consumer awareness in duopoly, where the left-hand diagram shows a symmetric pattern of consideration sets (where the two sellers have the same number of captive customers), while the right-hand diagram depicts a situation where a smaller seller's potential customers all also consider the larger seller (i.e., the smaller seller has no captive customers). This case of nested reach is relevant when, for instance, the smaller firm is a recent entrant which is considered by only a subset of consumers.

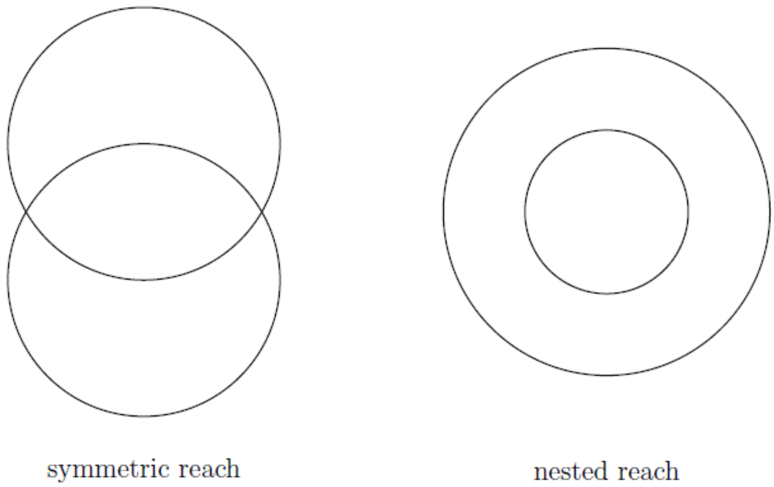


Figure 1: Two industry configurations

Sellers compete in Bertrand manner, and a consumer will buy from the seller she considers with the lowest price. Each consumer demands $q(p)$ units of the product if the price paid is $p$, where $q(\cdot)$ is a smooth and weakly decreasing function when positive. Thus, if a consumer buys from a seller at price $p$ she generates profit $\pi(p) \equiv pq(p)$ for that seller. Denote the profit-maximizing price by $p^*$ and maximum profit by $\pi^* = \pi(p^*)$. A consumer's net surplus if she pays price $p$ is the usual area under the demand curve

$$v(p) = \int_p^\infty q(\tilde{p})d\tilde{p} \ .$$

**Price discrimination**: Suppose all sellers know for sure whether a consumer is captive or not, in which case there is a unique equilibrium and this involves pure strategies. If a consumer is contested, i.e., she considers at least two sellers, then Bertrand competition forces the price to that consumer down to marginal cost, so that $p = \pi = 0$ and the consumer enjoys surplus $v(0)$. When the consumer is captive, her seller will charge the monopoly price $p^*$, so that $\pi = \pi^*$ and the consumer obtains surplus $v(p^*)$. Thus, aggregate consumer surplus in this scenario is $\gamma v(p^*) + (1-\gamma)v(0)$ while aggregate profit is $\gamma\pi^*$.

**Uniform pricing**: When sellers either do not know when a consumer is captive, or are not permitted to discriminate against captive customers, a seller must offer the same price to all potential customers. If all consumers are captive ($\gamma = 1$) then all sellers choose the monopoly price, while if no consumer is captive ($\gamma = 0$) all sellers choose the competitive price,and in either of these extremes the outcome is the same with or without price discrimination. When $0 < \gamma < 1$, however, the equilibrium with uniform pricing involves at least some sellers using mixed strategies for their prices. Since aggregate profit is a continuous function of the vector of prices chosen by the sellers, existence of equilibrium is ensured by Dasgupta and Maskin (1986, Theorem 5). Except in symmetric and other special cases—such as the duopoly market studied in section 3—the form of the equilibrium is not known. Moreover, when the equilibrium is known it may not be unique. However, since seller $i$ can always choose the monopoly price and sell at least to its $\gamma_i$ captive customers, in any equilibrium its expected profit must be at least $\gamma_i\pi^*$. Therefore, industry profit profit in any equilibrium with uniform pricing must be at least equal to $\gamma\pi^*$, which was the equilibrium profit with price discrimination.

Stating this result formally:

**Proposition 1** *Industry profit with price discrimination is no higher than industry profit in any equilibrium with uniform pricing.*

Consider the special case of unit demand, i.e., where $q(p) = 1$ if $p \leq 1$ and $q(p) = 0$ for $p > 1$, in which case $p^* = \pi^* = 1$. Then total welfare (profit plus consumer surplus) does not depend on price and is identically equal to 1 regardless of the pricing strategies followed by sellers. Since profit is weakly greater with uniform pricing, we have the following corollary to Proposition 1:

**Corollary 1** *If consumers have unit demand then aggregate consumer surplus with price discrimination is no lower than consumer surplus in any equilibrium with uniform pricing.*

In the next section we put more structure on the model to gain further insight into when price discrimination of this form is harmful or beneficial for consumers and for overall welfare.

# 3 A duopoly market

In broad terms, when sellers engage in price discrimination the result is that the average profit generated from consumers falls but the variability of profit across consumers rises, relative to the regime with uniform pricing. In this section we consider consumer surplus as a function of the profit a consumer generates. In regular cases, this consumer surplus is a concave function of profit, in which case consumers are "risk averse" towards variation in profit, and whether they prefer the regime with price discrimination depends on how much industry profit falls.

In more detail, if $\eta(p)$ denotes elasticity of demand, $\pi'(p)$ has the sign of $1-\eta(p)$, and so $\pi(p)$ is strictly single-peaked in $p$ if

$$\eta(p) \equiv -\frac{pq'(p)}{q(p)} \text{ strictly increases with } p \ , \tag{1}$$

which is assumed henceforth.[1] As before, denote the profit-maximizing price by $p^*$, in which case only prices in the interval $[0, p^*]$ will ever be chosen by sellers. Since profit $\pi(p)$ is strictly increasing in $[0, p^*]$, and since $v(p)$ is strictly decreasing in $p$, we can construct a decreasing function $V(\pi)$ such that if the consumer generates profit $\pi$ she enjoys net surplus $V(\pi)$, so that

$$v(p) \equiv V(\pi(p)) \ . \tag{2}$$

Differentiating (2) shows that $-q(p) = V'(\pi(p))\pi'(p)$, or

$$-V'(\pi(p)) = \frac{1}{1-\eta(p)} \ .$$

In particular, assumption (1) implies $V(\pi)$ is strictly concave on $[0, \pi^*]$. Since profit $\pi(p)$ is strictly increasing over the relevant range $[0, p^*]$, as in Armstrong and Vickers (2001) we can view sellers as choosing the per-consumer profit $\pi$ rather than the price $p$ they ask from their customers, and a consumer buys from the seller with the smallest $\pi$ from the set of sellers she considers.

An important determinant of the impact of price discrimination is whether the deadweight loss associated with monopoly pricing is smaller than monopoly profit. With unit demand there is no deadweight loss from monopoly pricing, while when demand $q(p)$ is

[1] Since unit cost has been normalized to zero, price $p$ is net of cost. With positive cost, our regularity condition (1) is met with constant elasticity of demand.

linear one can check that deadweight loss is precisely half the monopoly profit. More generally, the following result shows that the condition is satisfied provided that demand is not too convex.[2]

**Lemma 1** *If $q(p)$ is log-concave then deadweight loss associated with monopoly is smaller than monopoly profit, i.e.,*

$$V(0) - V(\pi^*) - \pi^* < \pi^* \ . \tag{3}$$

**Proof.** Log-concavity implies

$$\log q(p) \leq \log q(p^*) + (p - p^*)\frac{q'(p^*)}{q(p^*)} = \log q(p^*) + \frac{p^* - p}{p^*} \ ,$$

where the equality follows from the first-order condition for $p^*$ to maximize profit. It follows that $q(p) \leq q(p^*)e^{1-p/p^*}$, in which case

$$V(0) - V(\pi^*) - \pi^* = \int_0^{p^*} [q(p) - q(p^*)]dp \leq q(p^*) \int_0^{p^*} [e^{1-p/p^*} - 1]dp = (e-2)\pi^*$$

which is smaller than $\pi^*$. ■

In the remainder of this section we consider a duopoly market, where seller $i = 1, 2$ has $\gamma_i$ captive consumers (and remaining consumers consider both sellers). Thus seller $i$ reaches (i.e., is considered by) $\sigma_i \equiv 1 - \gamma_j$ consumers, and the proportion of seller $i$'s reach which is captive is denoted $\rho_i = \gamma_i/\sigma_i$, or

$$\rho_i = \frac{\gamma_i}{1 - \gamma_j} \ . \tag{4}$$

Throughout the following analysis we label firms so that $\rho_1 \geq \rho_2$ (in which case $\gamma_1 \geq \gamma_2$ and $\sigma_1 \geq \sigma_2$). Suppose that $0 < \rho_1 < 1$, i.e., there are some captive and some contested consumers, in which case the equilibrium with uniform pricing involves mixed strategies, as described in the following standard result:

**Lemma 2** *The unique equilibrium with uniform pricing involves the two sellers choosing profit in the same interval $[\rho_1\pi^*, \pi^*]$, seller 1 has an atom at $\pi = \pi^*$ with probability $(\sigma_1 - \sigma_2)/\sigma_1$ (while seller 2 has no such atom), and seller $i = 1, 2$ obtains profit $\sigma_i\rho_1\pi^*$.*

[2] Note that log-concavity also implies (1). In the proof of this lemma, log-concavity of demand is stronger than required to be sure that deadweight loss with monopoly pricing is smaller than monopoly profit. A weaker, but less familiar, condition which ensures this is that $1/\sqrt{q(p)}$ be convex (or, in the terminology of $\rho$-concavity, $q$ is a $(-1/2)$-concave function).

**Proof.** This result is taken from Narasimhan (1988). For completeness we construct this (unique) equilibrium as follows. Let seller $i$ choose its per-consumer profit $\pi$ according to the CDF $F_i(\pi)$. Then for $i \neq j$ in equilibrium these CDFs need to satisfy

$$\pi \times \sigma_i \left[\rho_i + (1-\rho_i)(1-F_j(\pi))\right] \equiv \sigma_i \rho_1 \pi^*$$

for any $\pi$ in seller $i$'s support. (Here, seller $i$ will always sell to its $\rho_i \sigma_i$ captive customers, and when it chooses profit $\pi$ it will also sell to its $(1-\rho_i)\sigma_i$ contested customers if its rival offers a higher profit, which occurs with probability $1 - F_j(\pi)$.) This defines two functions, $F_1$ and $F_2$, which are increasing on the interval $[\rho_1 \pi^*, \pi^*]$, are both zero at $\pi = \rho_1 \pi^*$, and where $F_2(\pi^*) = 1$ (so seller 2 has no atom at $\pi = \pi^*$) and $1 - F_1(\pi^*) = (\sigma_1 - \sigma_2)/\sigma_1$. Each seller is indifferent over any profit in the interval $[\rho_1 \pi^*, \pi^*]$, and neither seller has an incentive to choose profit outside this interval. ■

We next present our main result, which is that consumers in aggregate prefer uniform pricing if sellers are sufficiently symmetric (as with the left-hand diagram in Figure 1) while they usually prefer price discrimination if sellers are sufficiently asymmetric (as with the right-hand diagram).[3]

**Proposition 2**

*(i) Consumer surplus is higher with uniform pricing than with price discrimination when $\rho_2$ is sufficiently close to $\rho_1$.*

*(ii) If the deadweight loss associated with monopoly is no greater than monopoly profit, then consumer surplus is higher with price discrimination than with uniform pricing when $\rho_2$ is sufficiently close to zero.*

**Proof.** As in section 2, with price discrimination consumer surplus is

$$(1-\gamma)V(0) + \gamma V(\pi^*) \ , \tag{5}$$

while industry profit is $\gamma \pi^*$, where $\gamma = \gamma_1 + \gamma_2$ is the fraction of captive customers in the market. The proof for part (i) finds a lower bound on consumer surplus with uniform pricing and shows when this lower bound is greater than (5), while part (ii) finds an upper

[3]This result and the next are stated in terms of the captive-to-reach ratios in (4), rather than in terms of $(\gamma_1, \gamma_2)$ or $(\sigma_1, \sigma_2)$. This is because the market is defined for any $(\rho_1, \rho_2) \in [0,1]^2$, while working with other parameterizations requires extra constraints (such as that $\sigma_1 + \sigma_2 \geq 1$).

bound on consumer surplus with uniform pricing and shows when this upper bound is below (5). In the following analysis we parameterize the market in terms of $(\rho_1, \rho_2)$, in which case the numbers of captive customers and reach can be expressed as

$$\gamma_i = \frac{\rho_i(1-\rho_j)}{1-\rho_1\rho_2} \ ; \ \gamma = 1 - \frac{(1-\rho_1)(1-\rho_2)}{1-\rho_1\rho_2} \ ; \ \sigma_i = \frac{1-\rho_j}{1-\rho_1\rho_2} \ . \tag{6}$$

(i) We show that consumer surplus is higher with uniform pricing than with price discrimination whenever

$$\rho_2 \geq \frac{V(0) - V(\rho_1\pi^*)}{V(0) - V(\pi^*)} \ . \tag{7}$$

Lemma 2 shows that industry profit with uniform pricing is $(\sigma_1+\sigma_2)\rho_1\pi^*$ and the smallest profit offered in equilibrium is $\pi_0 = \rho_1\pi^*$. This industry profit is unchanged if the distribution of profit across consumers is altered so that $\sigma_2$ consumers generate profit $\pi_0$ and the remainder generate profit $\pi^*$. (Formally, $(\sigma_1 + \sigma_2)\rho_1\pi^* = (1-\sigma_2)\pi^* + \sigma_2\pi_0$.) This hypothetical profit distribution is therefore a mean-preserving spread of the true distribution under uniform pricing, in the sense of Rothschild and Stiglitz (1970). Since $V(\cdot)$ is a concave function, aggregate consumer surplus with this hypothetical profit distribution, which is

$$\sigma_2 V(\rho_1\pi^*) + (1-\sigma_2)V(\pi^*) \ ,$$

cannot be greater than the equilibrium consumer surplus with uniform pricing. Since consumer surplus with price discrimination is (5), a sufficient condition for consumers to prefer uniform pricing is

$$\sigma_2 \left[V(\rho_1\pi^*) - V(\pi^*)\right] \geq (1-\gamma)\left[V(0) - V(\pi^*)\right] \tag{8}$$

which from (6) reduces to condition (7). Finally, to check that (7) holds when the two firms are symmetric, observe that when $\rho_2 = \rho_1$ condition (7) requires $V(\rho_1\pi^*) \geq (1-\rho_1)V(0) + \rho_1 V(\pi^*)$, which follows from the concavity of $V(\cdot)$.

(ii) We show that consumer surplus is higher with price discrimination than with uniform pricing whenever

$$\rho_2 \leq 1 - (1+\rho_1)\frac{V(\frac{2\rho_1}{1+\rho_1}\pi^*) - V(\pi^*)}{V(0) - V(\pi^*)} \ . \tag{9}$$

Lemma 2 shows that industry profit with uniform pricing is $\Pi \equiv \rho_1(\sigma_1 + \sigma_2)\pi^*$ and that the larger firm chooses the monopoly profit $\pi^*$ with probability $(\sigma_1 - \sigma_2)/\sigma_1$. Therefore, a consumer will pay $\pi^*$ if she is captive to firm 1 and that firm chooses $\pi^*$, and so the fraction of consumers who pay the monopoly price is $a \equiv \rho_1(\sigma_1 - \sigma_2)$.

Since industry profit consists of the profit from those consumers paying $\pi = \pi^*$ and those paying $\pi < \pi^*$, we have

$$\Pi = a\pi^* + (1-a)\mathbb{E}[\pi \mid \pi < \pi^*]$$

so that

$$\mathbb{E}[\pi \mid \pi < \pi^*] = \frac{\Pi - a\pi^*}{1-a} = \frac{2\rho_1\pi^*}{1+\rho_1} \ ,$$

where the second equality follows after routine manipulation. It follows that expected consumer surplus with uniform pricing satisfies

$$\begin{aligned} \mathbb{E}(V(\pi)) &= aV(\pi^*) + (1-a)\mathbb{E}[V(\pi) \mid \pi < \pi^*] \\ &\leq aV(\pi^*) + (1-a)V\left(\frac{2\rho_1\pi^*}{1+\rho_1}\right) \end{aligned}$$

where the inequality follows from the concavity of $V(\cdot)$ via Jensen's inequality. Therefore, from (5) consumer surplus is higher with price discrimination if

$$(1-\gamma)[V(0) - V(\pi^*)] \geq (1-a)\left[V\left(\frac{2\rho_1\pi^*}{1+\rho_1}\right) - V(\pi^*)\right] \ . \tag{10}$$

Since (6) implies that

$$\frac{1-\gamma}{1-a} = \frac{1-\rho_2}{1+\rho_1} \ ,$$

this inequality can be written as (9), as claimed.

Finally, we show that the right-hand side of (9) is positive if the deadweight loss from monopoly pricing is less than monopoly profit, i.e., if (3) holds. As

$$\begin{aligned} (1+\rho_1)\left(V(\frac{2\rho_1}{1+\rho_1}\pi^*) - V(\pi^*)\right) &\leq (1+\rho_1)\left(V(0) - V(\pi^*) - \frac{2\rho_1}{1+\rho_1}\pi^*\right) \\ &= V(0) - V(\pi^*) + \rho_1\left[V(0) - V(\pi^*) - 2\pi^*\right] \\ &< V(0) - V(\pi^*) \ , \end{aligned}$$

the right-hand side of (9) is indeed positive. Here, the first inequality follows since total surplus $V(\pi) + \pi$ is maximized at $\pi = 0$, and the second follows from (3). ■

Intuitively, part (i) of this result is true since in near-symmetric markets industry profit is similar when sellers engage in price discrimination and when they cannot. (In either case, industry profit is approximately equal to the number of captive customers times $\pi^*$.) However, the distribution of profit across consumers is riskier with price discrimination—it is either 0 or $\pi^*$—and since consumers are "risk averse" towards variation in profit they are worse off with price discrimination. When sellers are very asymmetric, though, profit is considerably lower with price discrimination. With uniform pricing the seller with many captive customers is unwilling to compete aggressively, and this enables the smaller firm to achieve profit well in excess of its "captive profit" (which is all it can get with price discrimination). Part (ii) of the result describes when this reduction in profit is enough to outweigh the greater variability of profit with price discrimination. Provided that demand is not too convex (e.g., if $q(p)$ is log-concave), then price discrimination benefits consumers with nested reach, when only the larger seller has any captive customers.

In the limit case of unit demand, where $\pi^* = 1$ and $V(\pi) = 1 - \pi$, part (ii) of the result applies in all situations (condition (9) then holds always), as is consistent with Corollary 1. This case corresponds to "risk neutral" preferences over profit, when consumers care only about average profit and not its variation.

Total welfare—industry profit plus consumer surplus—is $V(\pi) + \pi$ which is also a concave function of $\pi$. Therefore, total welfare falls with price discrimination when the two sellers are nearly symmetric, while in asymmetric markets the reduction in average profit caused by discrimination may outweigh the extra riskiness of the distribution of profit. This is formalized in the following result.

**Proposition 3**

*(i) Total welfare is higher with uniform pricing than with price discrimination when $\rho_2$ is close to $\rho_1$.*

*(ii) Total welfare is higher with price discrimination than with uniform pricing when $\rho_1$ is close to 1 and $\rho_2$ is close to zero.*

**Proof.** Using the notation in the proof of Proposition 2, the reduction in industry profit caused by price discrimination is

$$(\sigma_1 + \sigma_2)\rho_1\pi^* - \gamma\pi^* = \sigma_2(\rho_1 - \rho_2)\pi^* = (1 - a)\frac{\rho_1 - \rho_2}{1 + \rho_1}\pi^* \ . \tag{11}$$

(i) We show that total welfare is higher with uniform pricing whenever

$$\rho_2 \geq \frac{V(0) - V(\rho_1\pi^*) - \rho_1\pi^*}{V(0) - V(\pi^*) - \pi^*} \ . \tag{12}$$

Expression (8) shows that the gain in consumer surplus with uniform pricing is at least

$$\sigma_2 \left[V(\rho_1\pi^*) - V(\pi^*)\right] - (1-\gamma)\left[V(0) - V(\pi^*)\right] \ ,$$

and combining this with the change in profit (11) implies that total welfare is higher with uniform pricing if

$$\sigma_2 \left[V(\rho_1\pi^*) - V(\pi^*)\right] - (1-\gamma)\left[V(0) - V(\pi^*)\right] + \sigma_2(\rho_1 - \rho_2)\pi^* \geq 0 \ .$$

After dividing by $\sigma_2$ and noting from (6) that $(1-\gamma)/\sigma_2 = 1 - \rho_2$, shows this is equivalent to (12).

(ii) We show that total welfare is higher with price discrimination whenever

$$\rho_2 \leq \frac{V(0) - V(\pi^*) - \rho_1\pi^* - (1+\rho_1)\left[V(\frac{2\rho_1\pi^*}{1+\rho_1}) - V(\pi^*)\right]}{V(0) - V(\pi^*) - \pi^*} \ . \tag{13}$$

From (10), the gain in consumer surplus with price discrimination is at least

$$(1-\gamma)[V(0) - V(\pi^*)] - (1-a)\left[V\left(\frac{2\rho_1\pi^*}{1+\rho_1}\right) - V(\pi^*)\right] \ .$$

It follows that total welfare rises with price discrimination if

$$(1-\gamma)[V(0) - V(\pi^*)] - (1-a)\left[V\left(\frac{2\rho_1\pi^*}{1+\rho_1}\right) - V(\pi^*)\right] \geq (1-a)\frac{\rho_1 - \rho_2}{1+\rho_1}\pi^* \ ,$$

which after dividing by $1-a$ becomes the condition

$$\frac{1-\rho_2}{1+\rho_1}[V(0) - V(\pi^*)] - \left[V\left(\frac{2\rho_1\pi^*}{1+\rho_1}\right) - V(\pi^*)\right] \geq \frac{\rho_1 - \rho_2}{1+\rho_1}\pi^*$$

which can be written as (13). When $\rho_1 \approx 1$, the right-hand side of (13) is positive (it is approximately equal to 1). ■

To illustrate Propositions 2 and 3, consider the example with linear demand $q(p) = 2-p$, in which case $p^* = \pi^* = 1$ and $V(\pi) = 1+\sqrt{1-\pi}-\frac{1}{2}\pi$. Figure 2 depicts the impact of price discrimination in terms of $(\rho_1, \rho_2)$, where recall that $\rho_2 \leq \rho_1$. Expression (7) shows that a sufficient condition for uniform pricing to be preferred by consumers overall is that $(\rho_1, \rho_2)$ lies above the upper solid curve, while expression (9) shows that a sufficient condition for price discrimination to be preferred is that $(\rho_1, \rho_2)$ lies below the lower solid curve. Expression (12) shows that total welfare is greater with uniform pricing when $(\rho_1, \rho_2)$ lies above the upper dashed curve, while (13) shows that discrimination raises total welfare if $(\rho_1, \rho_2)$ lies to the right of the lower dashed curve.

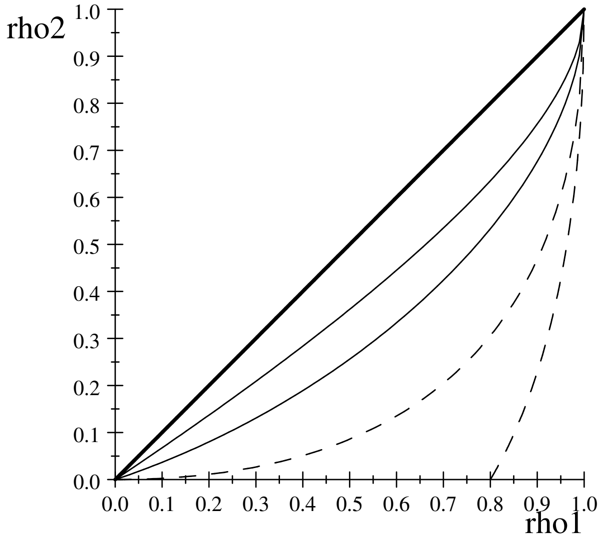


Figure 2: Impact of price discrimination with linear demand

# 4 Extensions

**Heterogeneous demand**: Our model assumed that all consumers had the same demand function, $q(p)$, which is clearly highly restrictive. However, the same analysis applies if consumers have heterogeneous demand functions, provided that their demand was independent of whether or not they are captive. For example, suppose the type-$\theta$ consumer has demand function $q_\theta(p)$, where the distribution for the type parameter $\theta$ is the same regardless of whether the consumer was captive to firm 1, captive to firm 2, or contested. If we write $q(p)$ for the expected (or aggregate) demand function across $\theta$, then provided condition (1) holds for this aggregate demand function, our welfare analysis continues to apply as stated. (Now $\pi(p)$ is expected profit across consumers when a firm chooses price $p$, $v(p)$ is expected consumer surplus with price $p$, and we can still define the function $V(\pi)$ in (2) which relates consumer surplus to profit.)

**Less precise information:** Our model assumed that sellers possess accurate information about whether a consumer was captive or not—in effect, in which segment on the Venn diagram in Figure 1 a consumer is located—and a natural question is how the results change when sellers have noisier information about a consumer's options. To discuss this, suppose for simplicity that information about a given consumer is public, so that the two sellers have the same information about each consumer. Suppose also that sellers are symmetric, where the total fraction of captives is $\gamma$ (so each seller has $\gamma/2$ captive customers).

At least two kinds of noisy consumer information can be considered. First, sellers might have information about whether a consumer is likely to be captive or not, but not to *which* seller she is captive. Such information preserves symmetry between sellers (so that conditional on sellers seeing a signal the market looks like the left-hand diagram on Figure 1), and equilibrium profit is the same as when sellers use uniform prices. Sellers set high prices when the customer is likely to be captive and low prices when she is likely to be contested, with the result that the distribution of profit across consumers is again a mean-preserving spread relative to the regime with uniform pricing, and consumers in aggregate are harmed by this form of price discrimination. Thus, price discrimination based on information of this form has the same qualitative implications as in our main model.

Alternatively, information might reveal to which seller a consumer is captive (if she is captive), in which case competition for the consumer is tilted in favour of that seller. Because competition is often less intense in asymmetric markets, information of this form may increase profit and raise prices. To illustrate, consider a scenario where all consumers are initially "attached" to one seller or the other (but not both), in equal numbers. A proportion $\gamma$ of a seller's attached customer base is is captive to that seller, while the remainder is footloose and will buy from the rival if its price is lower. (For instance, erstwhile regional energy monopolies with an existing customer base could be permitted to serve each other's markets, or, more generally, sellers have a base of existing customers.) Suppose it is common knowledge to which seller a consumer is attached (but not whether the consumer is captive). By construction, if a consumer is known to be attached to one seller she cannot be captive to the rival, and so the market segment of consumers attached to a given seller looks like the right-hand diagram on Figure 1. The policy issue is whether or not a seller should be permitted to set different prices to its own customer base and to customers attached to the rival.

If price discrimination is not permitted, Lemma 2 shows that each seller chooses profit with the same CDF $F(\pi)$ which satisfies

$$\pi \times [\tfrac{1}{2}\gamma + (1-\gamma)(1-F(\pi))] = \tfrac{1}{2}\gamma\pi^*$$

so that

$$1 - F(\pi) = \frac{\gamma}{2(1-\gamma)}\left(\frac{\pi^*}{\pi} - 1\right) . \tag{14}$$

Suppose next that sellers can set different prices to the two customer bases. If a consumer is attached to seller $i$, Bayes' rule implies that this consumer is captive to seller $i$ with probability $\gamma$ and otherwise she considers both sellers. Lemma 2 implies that the two sellers then choose CDFs $F_i$ and $F_j$ which respectively satisfy

$$\pi \times [\gamma + (1-\gamma)(1-F_j(\pi))] = \gamma\pi^* \ ; \ \pi \times [(1-\gamma)(1-F_i(\pi))] = \gamma(1-\gamma)\pi^*$$

so that

$$1 - F_i(\pi) = \gamma\frac{\pi^*}{\pi} \ ; \ 1 - F_j(\pi) = \frac{\gamma}{1-\gamma}\left(\frac{\pi^*}{\pi} - 1\right) \ . \tag{15}$$

Here, firm $i$ sets higher prices, in the sense of first-order stochastic dominance, than firm $j$ since the consumer might be captive to firm $i$ and cannot be captive to firm $j$. More strikingly, with price discrimination both sellers choose higher prices, in the sense of first-order stochastic dominance, than they do with uniform pricing in (14). Intuitively, seller $i$ raises its price since it has a greater proportion of captives relative to the market with uniform pricing, and this enables the rival too to raise its prices.

Thus, permitting price discrimination of this form induces sellers to raise their prices relative to the regime with uniform pricing. A market that is symmetric under uniform pricing is converted to a mirror pair of asymmetric markets by price discrimination. The result is that equilibrium prices rise, and all consumers are made worse off. This contrasts with our main model (with symmetric sellers), where price discrimination made the distribution of profit riskier, and benefitted the contested consumers, but average profit was unaffected. The example therefore illustrates that, with noisy information about consumer captivity, freedom to engage in price discrimination may affect not only the variability of profits but also the effective degree of market asymmetry and hence the competitive intensity.

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