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Abstract

We experimentally test the qualitatively different equilibrium predictions of two theoretical models of attack and defense of a weakest-link network of targets. In such a network, the attacker's objective is to assault at least one target successfully and the defender's objective is to defend all targets. The models differ in how the conflict at each target is modeled — specifically, the lottery and auction contest success functions (CSFs). Consistent with equilibrium in the auction CSF model, attackers utilize a stochastic "guerrilla-warfare" strategy, which involves attacking at most one target arbitrarily with a random level of force. Inconsistent with equilibrium in the lottery CSF model, attackers use the "guerrilla-warfare" strategy and assault only one target instead of the equilibrium "complete-coverage" strategy that attacks all targets. Consistent with equilibrium in both models, as the attacker's valuation increases, the average resource expenditure, the probability of winning, and the average payoff increase (decrease) for the attacker (defender).

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1. Introduction

In many network applications, such as cyber-security, electrical power grids, or oil pipeline systems, the failure of any individual component in the network may be sufficient to disable the entire network. In the case of a system of dikes on the perimeter of an island, Hirshleifer (1983) coined the term weakest-link to describe this type of intra-network complementarity among components.[1] In addition to networks with physically linked components, political considerations also may create a situation in which physically disjoint components are connected by a form of weakest-link complementarity in preferences. For example, a single terrorist spectacular may allow a terrorist to influence the target audience.[2] This paper experimentally examines two models of attack and defense of a weakest-link network of targets that differ with respect to the choice of contest success function (CSF), i.e., the mapping from the two players' resource allocations to a target into their probabilities of destroying or defending the target, used to model the conflict at each target.

In the attack and defense of a weakest-link network, the nature of the CSF is a key determinant of equilibrium behavior. We focus on two CSFs, lottery and auction, which are two special cases of the general ratio-form contest success function $x_A^r/(x_A^r + x_D^r)$, where x_A and x_D are the attacker's and defender's allocations of force, respectively, and the parameter $r > 0$ is inversely related to the level of noise, or randomness, in the determination of the winner of the conflict (conditional on the players' allocations). In the lottery CSF, $r = 1$, a situation in which the outcome of the conflict at each target is sufficiently noisy. The auction CSF corresponds to the

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[1] Applications of the weakest-link structure include: organizational performance that depends on the weakest-link (Kremer 1993); internet security (Moore et al. 2009); and package auctions in which the objective of some bidders is to obtain all of the goods while for other bidders the objective is to obtain only one good (Milgrom 2007).

[2] As stated in the *Joint House-Senate Intelligence Inquiry into September 11*, 2001 (US Congress 2002), terrorists need to be successful only once in killing Americans and demonstrating the inherent vulnerabilities the victims face.

limiting case where $r = \infty$, a situation in which no noise is present (i.e., the player who allocates the stronger level of force wins).[3]

In Clark and Konrad (2007), the conflict at each target of the weakest-link attacker-defender game features the lottery CSF. Clark and Konrad assume that the exogenous noise generated is independent across targets and demonstrate the existence of a pure-strategy "complete-coverage" equilibrium in which all targets are attacked and defended.[4] In contrast, Kovenock and Roberson (2018) show that, in all equilibria of the game with the auction CSF, the attacker utilizes a stochastic "guerrilla-warfare" mixed strategy, which involves randomly attacking at most one target – where each target is equally likely to be attacked – with a random level of force.[5] Conversely, the defender uses a mixed strategy that stochastically covers all of the targets, allocating a random level of force to each target. That strategy results in a correlation structure of endogenous noise that makes all multiple target attacks payoff dominated by a single target attack.[6] In this paper, we complete the characterization of equilibrium in the lottery CSF version of the game by showing that equilibrium is unique and test the implications of these two models in a laboratory experiment. We employ a two-by-two design that investigates the impact of the CSF (lottery versus auction) and the relative valuation of the attacker's prize (low versus high) on the behavior of attackers and defenders.

[3] However, as noted in the case of a single two-player contest with linear costs − by Baye et al. (1994) and Alcalde and Dahm (2010) − there exist equilibria that are payoff equivalent to the $r = \infty$ case whenever $r > 2$. Ewerhart (2017) has demonstrated that, in fact, in such an environment any Nash equilibrium is payoff and revenue equivalent to the all-pay auction. Thus, the auction CSF case with $\mathrm{r} = \infty$ is a relevant theoretical benchmark for all $r > 2$.
[4] For the attacker, this prediction holds for all parameter configurations. For the defender, this prediction holds if the ratio of the attacker's valuation of success to the defender's valuation of success is below a certain threshold.
[5] See also the related papers Dziubiński and Goyal (2013, 2017).
[6] For almost all configurations of the players' valuations of winning, one of the two players drops out with positive probability by allocating zero resources to each target, with the identity of the dropout determined by a measure of asymmetry in the conflict that takes into account both the ratio of the players' valuations and the number of targets.

The results of our experiment support the theoretical prediction that, under the auction CSF, attackers use a stochastic "guerrilla-warfare" strategy of attacking at most one target, and defenders use a stochastic "complete-coverage" strategy in which all targets are defended. In contrast, under the lottery CSF, instead of the pure-strategy Nash-equilibrium "complete-coverage" strategy, the expenditures of both the attackers and defenders are distributed over the entire strategy space. In fact, under the lottery CSF, attackers utilize a "guerrilla-warfare" strategy of attacking at most one target more than half of the time, instead of using a "complete-coverage" strategy, which is observed less than 25% of the time.

Consistent with predictions, under both CSFs, as the attacker's valuation increases, the attacker's resource expenditure increases and the defender's expenditure declines. As a result, the attacker's probability of winning and the average payoff also increase. However, under both CSFs, both players' average resource expenditures exceed their respective theoretical predictions, as is common in other contest experiments (Dechenaux et al. 2015).

The rest of the paper is organized as follows. In Section 2, we provide a brief review of the multi-battle contest literature. Section 3 presents a theoretical model of the attack and defense game. Section 4 describes the experimental design, procedures and hypotheses. Section 5 reports the results of our experiment and Section 6 concludes.

2. Literature review

Most of the existing theoretical work on multi-battle contests features symmetric objectives.[7] However, in applications such as cyber-security and terrorism, objectives are asymmetric with

[7] For a survey, see Kovenock and Roberson (2012). Recent theoretical work on multi-battle/Blotto-type games includes extensions such as: asymmetric players (Roberson 2006; Hart 2008; Weinstein 2012; Dziubiński 2013; Macdonell and Mastronardi 2015), non-constant-sum variations (Szentes and Rosenthal 2003; Kvasov 2007; Hortala-Vallve and Llorente-Saguer 2010, 2012; Roberson and Kvasov 2012; Ewerhart 2018), alternative definitions of

success for the attacker requiring the destruction of at least one target and successful defense requiring the preservation of all targets (Sandler and Enders 2004). Such structural asymmetry is the focus of the two weakest-link attacker-defender games (Clark and Konrad 2007; Kovenock and Roberson 2018) that we test experimentally.

To the best of our knowledge, our study is the first to examine behavior in weakest-link attacker-defender games utilizing both the lottery CSF and the auction CSF. Although most of the existing experimental studies focus on single-battle contests, interest in multi-battle contests is growing.[8] Experimental studies on multi-battle contests have examined how different factors such as budget constraints (Avrahami and Kareev 2009; Arad and Rubinstein 2012), objective functions (Duffy and Matros 2017), information (Horta-Vallve and Llorente-Saguer 2010), contest success functions (Chowdhury et al. 2013), focality (Chowdhury et al. 2016) and asymmetries in resources and battlefields (Arad 2012; Holt et al. 2015; Montero et al. 2016, Duffy and Matros 2017) influence individual behavior in contests. For a recent survey of the experimental literature on contests, see Dechenaux et al. (2015).

Consistent with the previous studies in which allocations are not budget-constrained, we find significant over-expenditure relative to the Nash equilibrium predictions under both the lottery CSF and the auction CSF. However, our most surprising result is that the theoretical prediction that attackers use a "guerrilla-warfare" strategy under the auction CSF also is observed under the lottery CSF. This is surprising because almost all multi-battle contest experiments in the literature find strong qualitative support for the theoretical predictions, even if the precise quantitative

success (Golman and Page 2009; Tang et al. 2010; Rinott et al. 2012), and political economy applications (Laslier 2002; Laslier and Picard 2002; Roberson 2008; Bierbrauer and Boyer 2016; Boyer et al. 2017; Thomas 2018). An extensive theoretical literature also exists on dynamic multi-battle contests. See, for instance, Harris and Vickers (1987), Klumpp and Polborn (2006), Konrad and Kovenock (2009) and Gelder (2014).

[8] The experimental literature on dynamic multi-battle contests also is growing. See, for instance, Deck and Sheremeta (2012, 2015), Mago et al. (2013), Mago and Sheremeta (2017, 2018) and Gelder and Kovenock (2017).

predictions are refuted. A potential explanation as to why attackers use a "guerrilla-warfare" strategy under the lottery CSF is that subjects may find it natural to concentrate resources on just one target since one successful attack is enough to win. Such a heuristic strategy also explains why individual behavior is so close to the theoretical predictions under the auction CSF.

3. The game of attack and defense

The model is formally described as follows. Two risk-neutral players, an attacker A and a defender D, simultaneously allocate resources across n targets. The players' one-dimensional resource expenditures for target i, denoted x_A^i and x_D^i for A and D respectively, are non-negative and are mapped into their respective probabilities of winning target i by the general ratio-form, or Tullock (1980), CSF. Thus, player D wins target i with probability

$$p_D^i\left(x_A^i, x_D^i\right) = \begin{cases} \dfrac{\left(x_D^i\right)^r}{\left(x_A^i\right)^r + \left(x_D^i\right)^r} & \text{if } x_A^i + x_D^i > 0 \\ \dfrac{1}{2} & \text{otherwise} \end{cases}, \tag{1}$$

and player A wins target i with probability $1 - p_D^i\left(x_A^i, x_D^i\right)$. For the lottery CSF, $r = 1$, and for the auction CSF, $r = \infty$.[9]

The attacker and the defender have asymmetric objectives. The defender's objective is to defend all n targets in the network successfully, in which case he receives a "prize" of value v_D. The expected payoff of D conditional on the expenditure $\left(x_A^i, x_D^i\right)$ is:

$$E(\pi_D) = \left(\prod_{i=1}^{n} p_D^i\left(x_A^i, x_D^i\right)\right)v_D - \sum_{i=1}^{n} x_D^i. \tag{2}$$

[9] For the auction CSF game, if the players allocate the same level of the resource to a target, it is assumed that the defender wins the target. However, a range of tie-breaking rules yields similar results. A detailed description of the theoretical model can be found in Clark and Konrad (2007) for the lottery CSF and Kovenock and Roberson (2018) for the auction CSF.

The attacker's objective is to attack at least one target successfully, in which case he receives a prize of value v_A. The corresponding expected payoff of A is:

$$E(\pi_A) = \left(1 - \prod_{i=1}^{n} p_D^i(x_A^i, x_D^i)\right) v_A - \sum_{i=1}^{n} x_A^i. \tag{3}$$

Clark and Konrad (2007) derive a Nash equilibrium for the lottery CSF ($r = 1$). We complete the characterization of equilibrium by showing that equilibrium is unique (see Online Appendix A).

***Proposition* 1:** Under the lottery CSF,

(i) If $v_D \geq (n-1)v_A$, then there exists a unique Nash equilibrium, which is in pure strategies. In equilibrium, player A allocates $x_A^* = \frac{v_A^2 v_D^n}{(v_A+v_D)^{n+1}}$ to every target and player D allocates $x_D^* = \frac{v_A v_D^{n+1}}{(v_A+v_D)^{n+1}}$ to every target.

(ii) If $v_D < (n-1)v_A$, then there exists a unique Nash equilibrium, which is in mixed strategies. In equilibrium, player A allocates $x_A^* = \frac{(n-1)^{n-1}}{n^{n+1}} v_D$ to each target and player D randomizes by allocating $x_D^* = \frac{(n-1)^n}{n^{n+1}} v_D$ to every target with probability $q^* = \frac{v_D}{(n-1)v_A}$ and 0 to every target with the probability $1 - q^*$.

Proposition 1 can be summarized as follows. If the ratio of the defender's valuation to the attacker's valuation exceeds a threshold, $v_D \geq (n-1)v_A$, then, in equilibrium, the defender uses a pure strategy that defends all targets with the same level of resources, $x_D^* > 0$. However, if $v_D < (n-1)v_A$, then the defender's equilibrium expected payoff is zero and, in equilibrium, the defender engages in the conflict — by allocating $x_D^* > 0$ to each target — with probability $q^* = \frac{v_D}{(n-1)v_A}$. With probability $1 - q^*$, the defender "surrenders" by allocating 0 to all n targets. In contrast, for all parameter configurations the attacker plays a pure strategy. Although the attacker's objective is to destroy at least one target, because of the decreasing returns to expenditure exhibited by the lottery CSF, the equilibrium strategy is to attack all targets with x_A^*.

Kovenock and Roberson (2018) characterize properties of the set of Nash equilibria for the auction CSF ($r = \infty$). They show that all equilibria are in mixed strategies, where a mixed strategy is an n-variate joint-distribution function. That paper completely characterizes the set of equilibrium payoffs and univariate marginal distributions, which are unique for all parameter configurations. These results are summarized as follows:

***Proposition* 2:** Under the auction CSF,

(i) If $v_D \geq nv_A$, then with probability $1 - \frac{nv_A}{v_D}$ player A allocates 0 to every target. With the remaining probability, $\frac{nv_A}{v_D}$, player A randomly attacks a single target with a resource allocation drawn from a uniform distribution over the interval $[0, v_A]$. To each and every target, player D allocates a random level of the resource drawn from a uniform distribution over the interval $[0, v_A]$. The players' sets of equilibrium univariate marginal distribution functions are unique, and for each target j are given by: $F_A^j\left(x_A^j\right) = 1 - \frac{v_A}{v_D} + \frac{x_A^j}{v_D}$ and $F_D^j\left(x_D^j\right) = \frac{x_D^j}{v_A}$, respectively, over the interval $[0, v_A]$.

(ii) If $v_D < nv_A$, player A randomly attacks a single target with a resource allocation drawn from a uniform distribution over the interval $\left[0, \frac{v_D}{n}\right]$. With probability $1 - \frac{v_D}{nv_A}$, player D allocates 0 to every target. With the remaining probability, $\frac{v_D}{nv_A}$, player D allocates to each target a random level of resources drawn from a uniform distribution over the interval $\left[0, \frac{v_D}{n}\right]$. The players' sets of equilibrium univariate marginal distribution functions for every target are unique, and for each target j are given by: $F_A^j(x_A^j) = 1 - \frac{1}{n} + \frac{x_A^j}{v_D}$ and $F_D^j(x_D^j) = 1 - \frac{v_D}{nv_A} + \frac{x_D^j}{v_A}$, respectively, over the interval $\left[0, \frac{v_D}{n}\right]$.

It is important to note that, although this game generates multiple equilibria, there exists a unique set of equilibrium univariate marginal distribution functions. Kovenock and Roberson (2018) also show that the equilibrium joint distribution functions exhibit several distinctive properties. For example, in all equilibria of the auction CSF game, the attacker allocates a strictly positive amount to at most one target while the defender allocates a strictly positive amount to either all targets or to none of them. This particular property provides a striking contrast with equilibrium in the lottery CSF game (see *Proposition 1*) in which the attacker allocates, to every target, a strictly positive amount.

4. Experimental design, procedures and hypotheses

4.1. Experimental design

Table 1 summarizes the experimental design. We employ a two-by-two design, by varying the CSF (*Lottery* versus *Auction*) and the relative valuation of the attacker's prize (*Low* versus *High*). All four treatments involve four targets and two players (attacker and defender). The experimental instructions, shown in Online Appendix B, used a context-neutral language.[10]

In the *Lottery-Low* and *Lottery-High* treatments, the probability that a player wins a given target is equal to the ratio of that player's allocation of resources to the target to the sum of both players' allocations to that target. In all treatments, the defender's valuation of defending all targets is $v_D = 200$ experimental francs. The attacker's valuation of successfully attacking at least one

[10] For example, players were called participants and targets were called boxes.

target is $v_A = 40$ francs in the *Lottery-Low* treatment and $v_A = 80$ francs in the *Lottery-High* treatment.[11] For the parameter configuration in the *Lottery-Low* treatment, Proposition 1 part (i) applies and in the pure-strategy equilibrium the attacker allocates 3.2 tokens to each target and the defender allocates 16.1 tokens to each target. For the parameter configuration in the *Lottery-High* treatment, Proposition 1 part (ii) applies and in equilibrium the attacker allocates 5.3 tokens to each target and the defender allocates 15.8 tokens to every target with probability 0.83 and 0 tokens to every target with probability 0.17.

In the *Auction-Low* and *Auction-High* treatments the winner of each target is determined by the auction CSF, but the remaining features of the model (v_D, v_A, and n) are the same. From Proposition 2 part (i), in any equilibrium of the *Auction-Low* treatment: (a) the attacker utilizes a mixed-strategy that attacks no targets with probability 0.2 and, with probability 0.8, chooses exactly one target to attack at random and stochastically allocates between 0 and 40 tokens to that target, according to a uniform distribution, and (b) the defender randomizes according to a joint distribution function that stochastically allocates between 0 and 40 tokens to each target according to a uniform marginal distribution. In the *Auction-High* treatment, Proposition 2 part (ii) applies. In any equilibrium: (a) the attacker randomly chooses one of the targets to attack and stochastically allocates between 0 and 50 tokens to that target according to a uniform distribution and (b) the defender employs a mixed strategy in which, with probability 0.375 he engages in no defensive efforts and, with probability 0.625, the defender allocates a stochastic number of tokens, uniformly distributed between 0 and 50, to each target.

[11] We chose these parameter valuations to ensure that: (a) the four treatments cover both parts of both propositions, and (b) each subject faced a non-trivial allocation problem in which both the attacker and the defender had a substantial chance of winning some targets.

4.2. Procedures

The experiment was conducted at the Vernon Smith Experimental Economics Laboratory. The computerized experimental sessions were run using z-Tree (Fischbacher 2007). A total of 96 subjects participated in eight sessions, summarized in Table 2. All subjects were Purdue University undergraduate students who participated in only one session of this study. Some students had participated in other economics experiments that were unrelated to this research.

Each experimental session had 12 subjects and proceeded in two parts, corresponding to the lottery and auction treatments.[12] Each subject played for 20 periods in the *Lottery-Low* (*Auction-Low*) treatment and 20 periods in the *Lottery-High* (*Auction-High*) treatment. The sequence was varied so that half of the sessions had the *Lottery-High* (*Auction-High*) treatment first, and half had the *Lottery-Low* (*Auction-Low*) treatment first.

In the first period of each treatment, subjects were randomly and anonymously assigned for the first 10 periods and then changed their assignment for the last 10 periods.[13] Subjects of opposite assignments were re-paired randomly each period to form a new two-player dyad. Each period, each subject allocated a non-negative number of tokens to each of the four targets such that the sum of allocated tokens was weakly less than that subject's valuation. Subjects were informed that all allocated tokens were forfeited. After all subjects made their allocations, the computer displayed the following information: attacker allocation, defender allocation, which targets they won, and individual earnings for the period. In the *Lottery-High* and *Lottery-Low* treatments, the winner was chosen according to the lottery CSF, independently across targets. In the *Auction-High*

[12] Risk aversion preferences also were elicited, along the lines of Holt and Laury (2002). We found no interesting patterns between risk attitudes and behavior in weakest-link contests and omit discussion of this issue.

[13] Role-switching avoids any social preferences, i.e., subjects who were assigned as disadvantaged attackers knew that they would also play the role of the advantaged defenders, and induces better learning, since subjects have an opportunity to learn game strategies in both roles.

and *Auction-Low* treatments, the player who allocated more tokens to a particular target was chosen as the winner of that target.[14]

After completing all 40 decision periods (two treatments), four periods were selected randomly for payment (two periods for each treatment). The sum of the total earnings for those four periods was exchanged at the rate of 26 tokens = $1. Additionally, all players received a participation fee of $20 to cover potential losses. On average, subjects earned $25 each, ranging from $11 to $36, and the payments were in cash. Each experimental session lasted about 80 minutes.

4.3. Hypotheses

Our experiment tests five hypotheses motivated by the theoretical predictions. The first hypothesis addresses the comparative static properties of equilibrium in terms of a change in the attacker's target valuation.[15] The next two describe equilibrium predictions concerning behavior in the *Lottery-Low* and *Lottery-High* treatments. The final two hypotheses describe equilibrium predictions concerning behavior in the *Auction-Low* and *Auction-High* treatments.

Hypothesis 1: Under the lottery and auction CSFs, as the attacker's valuation increases from 40 to 80, the average resource allocation, the probability of winning, and the average payoff increase (decrease) for the attacker (defender).

[14] When both players allocated the same amount to a given target, the computer selected the defender as the winner of that target.
[15] Although the comparative statics results are framed in terms of a change in the attacker's valuation, owing to invariance of preferences with respect to affine transformations of utility, the theoretical benchmark also would apply to a decrease in the unit cost of resource expenditure of the attacker.

Hypothesis 2: In the *Lottery-Low* and *Lottery-High* treatments the attacker uses a "complete-coverage" strategy, which involves allocating a strictly positive and identical level of the resource across all targets.

Hypothesis 3: In the *Lottery-Low* treatment the defender uses a "complete-coverage" strategy. In the *Lottery-High* treatment the defender allocates a strictly positive and identical level of the resource across all targets with positive probability, and a zero level of the resource with the remaining probability.

Hypothesis 4: In the *Auction-Low* and *Auction-High* treatments the attacker uses a stochastic "guerrilla-warfare" strategy, which involves allocating a random level of the resource to at most one target.

Hypothesis 5: In the *Auction-Low* treatment the defender uses a stochastic "complete-coverage" strategy, which involves allocating random positive levels of the resource to all of the targets. In the *Auction-High* treatment the defender follows a stochastic "complete-coverage" strategy with positive probability and also allocates a zero level of the resource to every target with positive probability.

5. Results

5.1. Aggregate behavior

Table 3 summarizes the average allocation of tokens, the probability of winning, and the average payoff of the attacker and the defender in each treatment. Consistent with *Hypothesis 1*, when the attacker's valuation increases from 40 to 80, the average allocation of tokens by the attacker increases from 4.4 to 7.8 under the lottery CSF, and it increases from 4.4 to 7.7 under the auction CSF. The average allocation of tokens by the defender declines from 24.4 to 15.8 under the auction

CSF, but does not decline under the lottery CSF (19.4 versus 19.3). To support those conclusions we estimate panel regressions, reported in the top panel of Table 4, where the dependent variable is the allocation to any given target and the independent variables are a treatment dummy variable (*High*), a period trend (*Period*) and a constant (*Constant*). The model includes a random effects error structure, with the individual subject as the random effect, to account for the multiple allocation decisions made by individual subjects over the course of the experiment. The standard errors are clustered at the session level to account for session effects. The treatment dummy variable is significant in all regressions (p-values < 0.01), except the one where we compare the behavior of the defender in the *Lottery-High* and *Lottery-Low* treatments.

Also, consistent with *Hypothesis 1*, the attacker's probability of winning in the *Lottery-High* treatment (0.68) is higher than his probability of winning in the *Lottery-Low* treatment (0.51), and the probability of winning in *Auction-High* (0.68) is higher than the probability of winning in *Auction-Low* (0.33). The middle panel of Table 4 reports the regression results from a random effects probit model. From that estimation, we see that, for both the auction and lottery CSFs, the attacker's (defender's) probability of winning is higher (lower) in the high attacker valuation treatment (p-values < 0.01).

Finally, consistent with *Hypothesis 1*, from the estimation reported in the bottom panel of Table 4 we see that the defender's (attacker's) payoff in the *Lottery-Low* and *Auction-Low* treatments is higher (lower) than in the *Lottery-High* and *Auction-High* treatments, where the treatment dummy variable is significant in all regressions (p-values < 0.01 for all except the defender in the *Lottery-High* to *Lottery-Low* comparison, which has p-value < 0.05).

Result 1: Consistent with the prediction of *Hypothesis 1*, under the lottery and auction CSF, as the attacker's target valuation increases, the average allocation of tokens, the probability of winning, and the average payoff increase (decrease) for the attacker (defender).

Although the comparative static predictions of the theory are supported by our experiment, significant over-expenditure of resources by both player types is observed in all treatments. In the *Lottery-Low* treatment, the attacker allocates on average 4.4 tokens, instead of the predicted 3.2, and in the *Lottery-High* treatment, the attacker allocates 7.8 tokens, instead of 5.3. The relative magnitude of over-expenditure by the defender is similar: 19.4 tokens instead of 16.1 and 19.3 tokens instead of 13.1. The range of average over-expenditure is 21%-47%. Over-expenditure also is observed in the *Auction-High* and *Auction-Low* treatments; however, the magnitude is around 10%-22%.[16] As a result of significant over-expenditure, in all treatments both player types receive smaller payoffs than predicted (see Table 3).

Significant over-expenditure in our experiment is consistent with previous experimental findings on all-pay auctions and lottery contests (Davis and Reilly 1998; Potters et al. 1998; Gneezy and Smorodinsky 2006; Sheremeta and Zhang 2010; Price and Sheremeta 2011, 2015). Suggested explanations for over-expenditure include bounded rationality (Sheremeta 2011; Chowdhury et al. 2014), utility of winning (Sheremeta 2010; Cason et al. 2012, 2017), other-regarding preferences (Fonseca 2009; Mago et al. 2016), judgmental biases (Shupp et al. 2013), and impulsive behavior (Sheremeta 2016).[17] The same arguments can be made to explain over-expenditure in our experiment.

[16] A standard Wald test, conducted on estimates of panel regression models, rejects the hypothesis that the average expenditures under the lottery CSF are equal to the predicted theoretical values in Table 3 (all p-values < 0.05). Under the auction CSF we can reject the null hypothesis only for the defender (p-value < 0.05).

[17] For a detailed review of possible explanations for the over-expenditure phenomenon see Sheremeta (2013, 2016).

5.2. Behavior of attackers under the lottery CSF

Next, we examine attacker behavior under the lottery CSF, where, in equilibrium, the attacker employs a uniform allocation of tokens across targets. Contrary to *Hypothesis 2*, within each target the allocation of tokens is highly dispersed. Figure 1 displays, by treatment and player type, the empirical univariate marginal cumulative distribution functions of tokens allocated to an arbitrary target. [18] Instead of placing a mass point at 3.2 in the *Lottery-Low* treatment and 5.3 in the *Lottery-High* treatment, the attacker's resources are distributed between 0 and 50.

[Insert Figure 1 here.]

Another inconsistency with *Hypothesis 2* is that, instead of a strictly positive token allocation for each target, the attacker places mass at 0 (see Figure 1). Table 5 shows properties of the strategies used by subjects in the *Lottery-Low* and *Lottery-High* treatments. The attacker frequently uses a "guerrilla-warfare" strategy that assaults at most one target (54% in the *Lottery-Low* treatment and 51% in the *Lottery-High* treatment). A strategy of "complete coverage", allocating a positive amount to all four targets, is used only 24% of time in the *Lottery-Low* treatment and 32% of the time in the *Lottery-High* treatment.[19]

[Insert Table 5 here.]

Result 2: Contrary to the prediction of *Hypothesis 2*, in the *Lottery-Low* and *Lottery-High* treatments, the attacker's allocation of tokens to each target is highly dispersed. Instead of using

[18] We combined the distribution of tokens to each of the four targets into one target, since marginal distributions to each target are identical across targets.

[19] Note that our results are robust to the exclusion of strategies involving drastic over-expenditure. For example, if defining drastic over-expenditure as expenditure greater than twice the expected Nash equilibrium expenditure (which excludes 18% [13%] of attacker strategies in the *Lottery-Low* [*Lottery-High*] treatment), then excluding drastic over expenditure the attacker allocates resources to at most one target 60% of the time in the *Lottery-Low* treatment and 57% of the time in the *Lottery-High* treatment. The attacker allocates a positive amount to all four targets 19% of time in the *Lottery-Low* treatment and 26% in the *Lottery-High* treatment.

the “complete-coverage” strategy, the attacker uses the “guerrilla-warfare” strategy that allocates a positive bid to one or no targets more than 50% of the time.

The fact that the attacker’s resource allocation to each target is very dispersed is consistent with previous experimental studies documenting high variance of individual expenditures in lottery contests (Davis and Reilly 1998; Potters et al. 1998; Chowdhury et al. 2014). Several explanations have been offered for such behavior based on the probabilistic nature of lottery contests and bounded rationality (Chowdhury et al. 2013). Those considerations also could explain the pattern observed in our experiment.

A more novel finding of our study is the use of a “guerrilla-warfare” strategy by the attacker, which is inconsistent with the unique Nash equilibrium in the attack and defense game under the lottery CSF. Also, it appears unlikely that attackers adjust their strategy away from equilibrium because of suboptimal behavior on the part of defenders because, as we discuss later, defenders behave in accordance with the theoretical predictions.

A likely explanation why attackers use a “guerrilla-warfare” strategy is that subjects may find it natural to concentrate resources on the necessary number of targets needed for victory (one in our case). Although such a strategy is not optimal, it is an appealing focal point (Schelling 1960). It has been well documented in the experimental literature that subjects naturally gravitate towards focal points even when doing so is not necessarily in their best interest (Roth 1985; Crawford et al. 2008; Chowdhury et al. 2016). Such a heuristic strategy also can explain why individual behavior is so close to the theoretical predictions under the auction CSF (as we discuss below).

Note that the use of a “guerilla-warfare” strategy appears to come at a cost. If we omit all observed attacker strategies that involve drastic over-expenditure (greater than two times the expected Nash equilibrium level; see footnote 19) and match the remaining observed strategies against a random observed defender strategy from the same treatment, then "guerilla-warfare" strategies appear to perform poorly relative to "complete-coverage" strategies. For the observed attacker strategies that allocate a positive amount to a single target, the payoff is 3.83 (23.39) in the *Lottery-Low* (*Lottery-High*) treatment. For the observed attacker strategies that allocate resources to all targets, this payoff is 6.66 (29.47).[20] In this limited sense, attackers concentrating resources on a single target under the lottery CSF appear to do worse than those spreading resources across all four targets.

5.3. Behavior of defenders under the lottery CSF

Our results concerning the behavior of the defender support *Hypothesis 3*. In particular, theory predicts that in the *Lottery-Low* treatment the defender makes a strictly positive, and uniform, allocation of tokens across targets. Table 5 indicates that, supporting *Hypothesis 3*, the defender allocates tokens to all targets in the *Lottery-Low* treatment 92% of the time. In the *Lottery-High* treatment, theory predicts that the defender covers all of the targets with probability 0.83 and none of the targets with probability 0.17. Consistent with that prediction, the data indicate that the defender covers all of the targets in the *Lottery-High* treatment 84% of the time and none of the targets 12% of the time. [21] However, contrary to *Hypothesis 3*, instead of a uniform allocation across targets, the defender's resources are distributed between 0 and 50 (see Figure 1).

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[20] Attacker strategies involving drastic over-expenditure have smaller expected payoffs than those without and tend to be positively correlated with the number of targets receiving a positive allocation. For example, in the *Lottery-Low* (*Lottery-High*) treatment 49% (72%) of all of the attacker strategies involving drastic over-expenditure are four-target strategies. In looking at the entire sample in the *Lottery-Low* (*Lottery-High*) treatment, the average expected payoff of the observed attacker strategies that allocate resources to only one target is 2.84 (23.11) and the average expected payoff of the observed attacker strategies that allocate resources to all targets is 3.13 (22.83).

[21] If we exclude strategies involving drastic over-expenditure, namely 11% (27%) of the defender strategies in the *Lottery-Low* (*Lottery-High*) treatment, the defender allocates resources to all targets 92% in the *Lottery-Low* treatment and 79% in the *Lottery-High* treatment. In the latter treatment, no targets receive a positive allocation 17% of the time.

Result 3: Consistent with the prediction of *Hypothesis 3*, in the *Lottery-Low* treatment, the defender uses a "complete-coverage" strategy by defending all targets and in the *Lottery-High* treatment the incidence of "complete coverage" is very high, but with the "no coverage" strategy (allocating zero to every target) the next most frequent strategy. Contrary to the prediction, instead of a uniform allocation across targets, allocations are dispersed over the interval [0, 50].

As in the case of attackers, the relatively high dispersion of the defenders' allocations is consistent with previous experimental findings, and could be explained by the probabilistic nature of lottery contests and bounded rationality (Chowdhury et al. 2013).

5.4. Behavior of attackers under the auction CSF

Next, we look at attacker behavior under the auction CSF. Theory predicts that in the *Auction-Low* and *Auction-High* treatments, the attacker employs a stochastic "guerrilla-warfare" strategy, which involves allocating a random amount of the resource to at most one target. Figure 2 displays the empirical univariate marginal cumulative distribution function of the resource allocation to a target and indicates that, in the aggregate, the attacker's behavior is consistent with that prediction. The stochastic "guerrilla-warfare" strategy is characterized by a significant mass point at 0 for the attacker, which is very close to the predicted value (0.75 versus 0.80 in the *Auction-Low* treatment and 0.67 versus 0.75 in the *Auction-High* treatment).[22] Similarly, from Table 6, we see that the attacker allocates tokens to at most one target 89% of the time in the *Auction-Low* treatment and

[22] In calculating the empirical mass points at 0 (Figures 1 and 2), we use an allocation of less than one token as an approximation of 0. That approximation is adopted because the tie-breaking rule favors defenders and therefore may encourage attackers to place a very small allocation on some targets in order to reduce the tie-breaking disadvantage. However, even if we use only 0 allocations to compute mass points at 0, we still get results that are close to the theoretical predictions (in the *Auction-Low* and *Auction-High* treatments, for example, the mass points at 0 for the attackers are 0.6 and 0.5).

81% of the time in the *Auction-High* treatment.[23] These findings provide substantial support for *Hypothesis 4*.

Result 4: Consistent with the prediction of *Hypothesis 4*, in the *Auction-Low* and *Auction-High* treatments, the attacker adopts a stochastic "guerrilla-warfare" strategy, which involves allocating a random amount of the resource to at most one target.

5.5. Behavior of defenders under the auction CSF

We find that defender behavior likewise is consistent with *Hypothesis 5*. In particular, theory predicts that in the *Auction-Low* treatment the defender uses a stochastic "complete-coverage" strategy that allocates a strictly positive amount of resources to each target with probability one. The data indicate that the defender covers all of the targets 87% of the time (see Table 6). Moreover, consistent with the theoretical prediction, in the *Auction-Low* treatment the defender's resources are distributed uniformly between 0 and 40 (see Figure 2). Similarly, in the *Auction-High* treatment defender behavior is consistent with the theoretical prediction that with probability 0.375 the defender engages in no defensive efforts and with probability 0.625 the defender allocates a stochastic number of tokens, uniformly distributed between 0 and 50, to each target. The data indicate that the defender covers all four targets 62% of the time, three targets 2%, two targets 2%, one target 4% and zero targets 30% of the time (see Table 6).[24] Moreover, the defender's allocations are distributed uniformly between 0 and 50 (see Figure 2).

[23] If we exclude strategies involving drastic over-expenditure, which excludes 18% (16%) of attacker strategies in the *Auction-Low* (*Auction-High*) treatment, then the attacker allocates resources to at most one target 87% of the time in the *Auction-Low* treatment and 79% of the time in the *Auction-High* treatment.

[24] The fact that the defender allocates 0 resources to all four targets 30% of the time could, potentially, be explained by subjects changing role assignments after 10 periods and a period 1-10 attacker continuing to behave as an attacker during periods 11-20. However, our results are robust to restricting data to the first 10 periods. Furthermore, excluding strategies involving drastic over-expenditure, which excludes 5% (20%) of defender strategies in the *Auction-Low* (*Auction-High*) treatment, the defender allocates resources to all targets 87% of the time in the *Auction-Low* treatment

Result 5: Consistent with the prediction of *Hypothesis 5*, in the *Auction-Low* treatment, the defender uses a stochastic "complete-coverage" strategy that involves allocating random positive levels of the resource to all of the targets. In the *Auction-High* treatment a high incidence of "complete-coverage" is observed and the "no-coverage" strategy is employed almost a third of the time.

6. Conclusions

This study experimentally investigates behavior in a game of attack and defense of a weakest-link network under two benchmark contest success functions: the auction CSF and the lottery CSF. We find that the auction CSF's theoretical prediction that the attacker uses a stochastic "guerrilla-warfare" strategy is observed under both the auction and lottery CSFs. That is inconsistent with Nash equilibrium behavior under the lottery CSF. However, such behavior is consistent with a simple heuristic strategy of focusing only on the necessary number of targets needed for victory (one in our case). The defender uses a stochastic "complete coverage" strategy under both the auction and lottery CSFs. That finding is consistent with equilibrium behavior under the auction CSF, but the high dispersion of target allocations is inconsistent with equilibrium under the lottery CSF.

A common explanation for the empirical finding that "periods of high terrorism" seem to be relatively infrequent (Enders 2007) is that terrorists face a resource constraint; they therefore cannot attack all of their targets all of the time. Our experiment provides evidence for an alternative explanation. Infrequent "periods of high terrorism" simply may be the result of asymmetric

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and 54% of the time in the *Auction-High* treatment. In the latter treatment, no targets receive a positive allocation 37% of the time.

objectives and strategic interactions between the attackers and defenders within a weakest-link contest environment.

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Table 1: Experimental parameters and theoretical predictions

Treatment	Player	Value	Average allocation	Expected payoff	Probability of winning
Lottery-Low	*A*	40	3.2	7.8	0.52
	D	200	16.1	32.2	0.48
Lottery-High	*A*	80	5.3	37.8	0.74
	D	200	13.1	0.0	0.26
Auction-Low	*A*	40	4.0	0.0	0.40
	D	200	20.0	40.0	0.60
Auction-High	*A*	80	6.3	30.0	0.69
	D	200	15.6	0.0	0.31

Average allocation in the *Auction-Low* and *Auction-High* treatments are calculated based on equilibrium mixed strategies.

Table 2: Experimental sessions

Session Number	Design	Matching protocol	Participants per session	Periods per treatment
1-2	*Lottery-Low* → *Lottery-High*	Strangers	12	20
3-4	*Lottery-High* → *Lottery-Low*	Strangers	12	20
5-6	*Auction-Low* → *Auction-High*	Strangers	12	20
7-8	*Auction-High* → *Auction-Low*	Strangers	12	20

Table 3: Average allocation, probability of winning and payoff by treatment

Treatment	Player	Value	Average allocation		Probability of winning		Expected payoff	
			Predicted	Actual	Predicted	Actual	Predicted	Actual
Lottery-Low	*Attacker*	40	3.2	4.4 (2.5)	0.52	0.51 (0.50)	7.8	2.7 (18.6)
	Defender	200	16.1	19.4 (10.7)	0.48	0.49 (0.50)	32.2	20.6 (98.4)
Lottery-High	*Attacker*	80	5.3	7.8 (4.3)	0.74	0.68 (0.47)	37.8	23.6 (37.3)
	Defender	200	13.1	19.3 (13.0)	0.26	0.32 (0.47)	0.0	-14.1 (90.1)
Auction-Low	*Attacker*	40	4.0	4.4 (3.5)	0.40	0.33 (0.47)	0.0	-4.5 (16.8)
	Defender	200	20.0	24.4 (12.8)	0.60	0.67 (0.47)	40.0	36.2 (82.1)
Auction-High	*Attacker*	80	6.3	7.7 (4.6)	0.69	0.68 (0.47)	30.0	23.2 (33.5)
	Defender	200	15.6	15.8 (15.2)	0.31	0.32 (0.47)	0.0	1.7 (85.6)

Standard deviation in parentheses.

Table 4: Panel estimation testing *hypothesis 1*

Treatments	*Lottery-Low* and *Lottery-High*		*Auction-Low* and *Auction-High*	
Player	*Attacker*	*Defender*	*Attacker*	*Defender*
Dependent variable	*Average allocation*			
High	3.36***	-0.08	3.26***	-8.57***
[1 if high value]	(1.27)	(1.97)	(0.45)	(3.29)
Period	1.16**	4.86**	1.21**	6.48***
[inverse period trend]	(0.57)	(2.00)	(0.58)	(2.07)
Constant	4.19***	18.55***	4.22***	23.22***
	(0.44)	(1.87)	(0.48)	(1.65)
Dependent variable	*Probability of winning*			
High	0.47***	-0.48***	0.92***	-0.95***
[1 if high value]	(0.13)	(0.13)	(0.11)	(0.11)
Period	-0.20	0.20	-0.30	0.35
[inverse period trend]	(0.22)	(0.23)	(0.27)	(0.28)
Constant	0.06	-0.06	-0.40***	0.40***
	(0.07)	(0.07)	(0.05)	(0.05)
Dependent variable	*Expected payoff*			
High	20.91***	-34.69**	27.70***	-34.48***
[1 if high value]	(2.22)	(17.22)	(2.00)	(10.60)
Period	-12.53***	-4.30	-9.07***	3.44
[inverse period trend]	(3.14)	(13.73)	(2.37)	(10.29)
Constant	4.98***	21.41*	-2.87	35.58***
	(0.84)	(11.49)	(1.85)	(6.45)
Observations	960	960	960	960

* significant at 10%, ** significant at 5%, *** significant at 1%. All models include a random effects error structure, with the individual subject as the random effect, to account for the multiple decisions made by the subject over the course of the experiment. The standard errors are clustered at the session level to account for session effects.

Table 5: Strategies used in the *Lottery-Low* and *Lottery-High* treatments

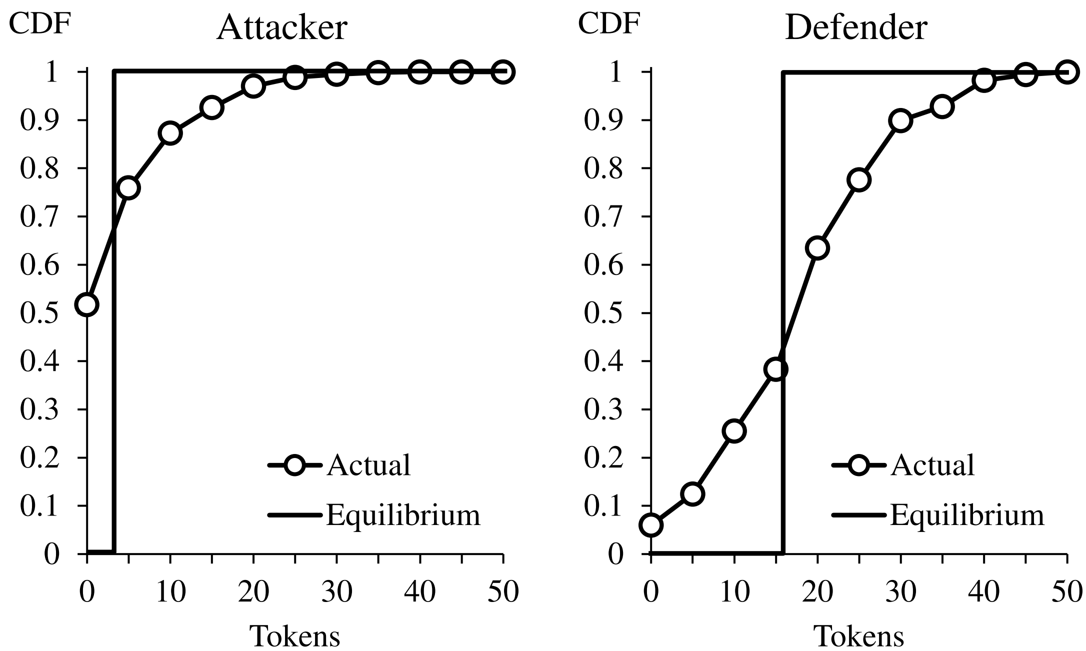
Treatment	Player	Frequency of allocating tokens to				
		0 Targets	1 Target	2 Targets	3 Targets	4 Targets
Lottery-Low	*Attacker*	0.10	0.44	0.14	0.08	0.24
	Defender	0.05	0.01	0.01	0.01	0.92
Lottery-High	*Attacker*	0.05	0.46	0.13	0.04	0.32
	Defender	0.12	0.01	0.01	0.02	0.84

Table 6: Strategies used in the *Auction-Low* and *Auction-High* treatments

Treatment	Player	Frequency of allocating tokens to				
		0 Targets	1 Target	2 Targets	3 Targets	4 Targets
Auction-Low	*Attacker*	0.28	0.61	0.04	0.01	0.06
	Defender	0.06	0.02	0.02	0.03	0.87
Auction-High	*Attacker*	0.11	0.70	0.05	0.02	0.12
	Defender	0.30	0.04	0.02	0.02	0.62

Figure 1: CDF of tokens in the *Lottery-Low* and *Lottery-High* treatments

The *Lottery-Low* treatment



The *Lottery-High* treatment

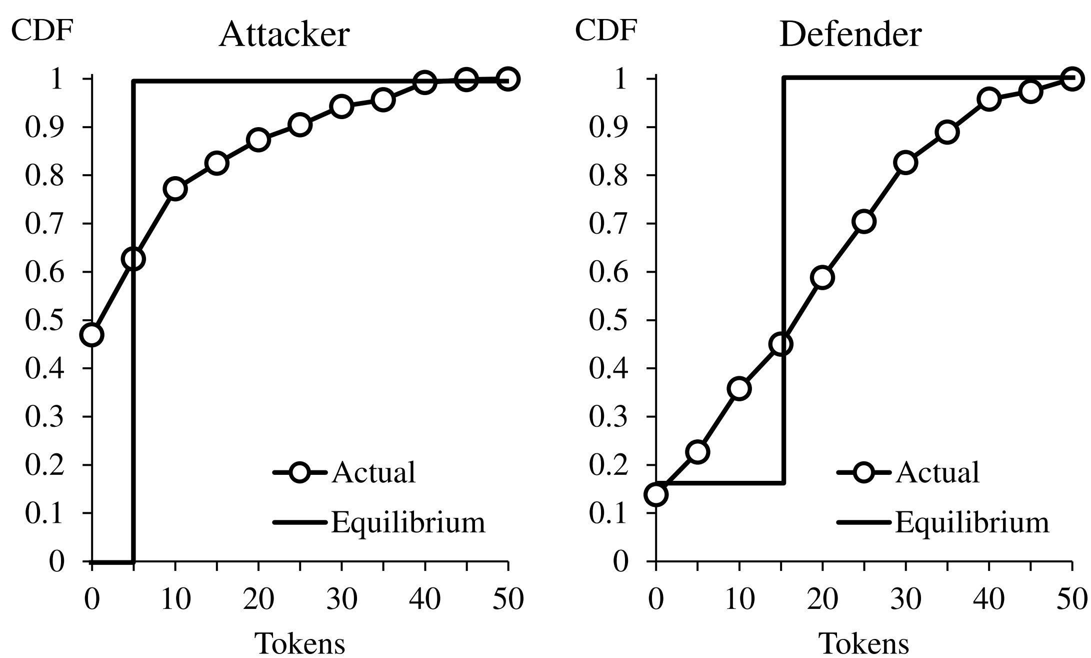
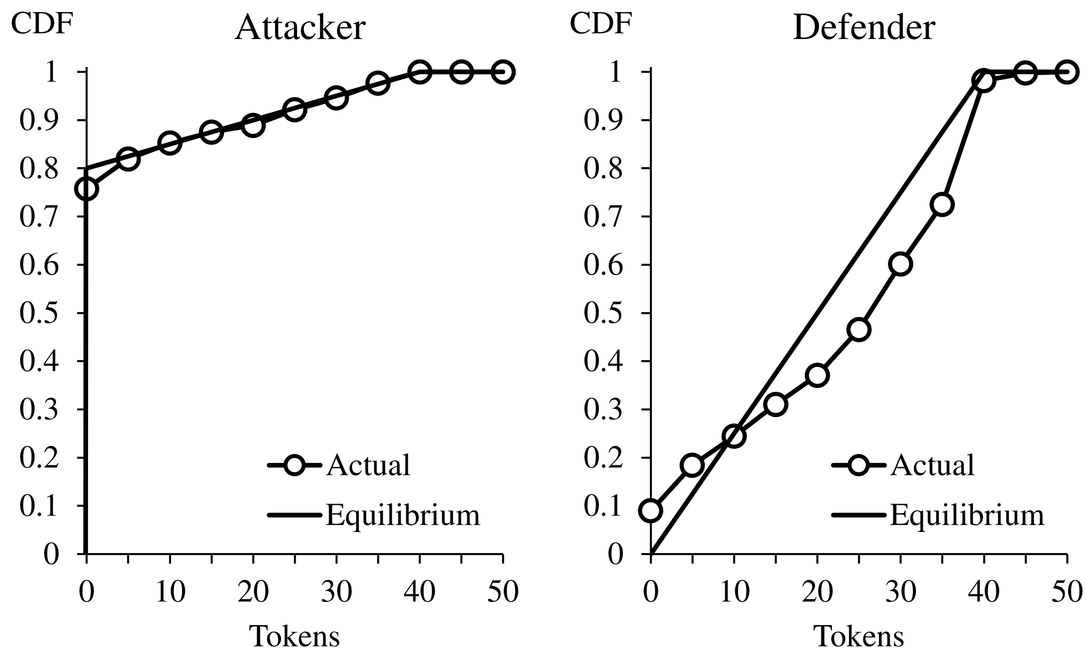


Figure 2: CDF of tokens in the *Auction-Low* and *Auction-High* treatments

The *Auction-Low* treatment



The *Auction-High* treatment