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Piolatto, Amedeo

Toulouse School of Economics - GREMAQ, Università degli Studi di Milano Bicocca

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Publicly provided private goods: education and selective vouchers*

Amedeo Piolatto
Université de Toulouse 1 (GREMAQ and TSE)
and Università di Milano - Bicocca†

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Abstract

The literature on vouchers often concludes that a vouchers-based system cannot be the outcome of a majority vote. This paper shows that, when the value of vouchers and who is entitled to receive them are fixed exogenously, the majority of voters are in favour of selective vouchers. On top of that, as long as the introduction of vouchers does not undermine the existence of the public school system, introducing selective vouchers induces a Pareto improvement. Middle class agents are the only one using vouchers in equilibrium, while the poorest agents in the economy profit from the reduction in public school congestion.

JEL Classification number: H42, I20, I22, I28, I29, D70

Keywords: positive public economics; education; vouchers; voting

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†Visiting scholar at Universitat Autònoma de Barcelona from February to July 2008. E-mail: amedeo.piolatto@univ-tlse1.fr

1 Introduction

Most western countries publicly provide some private goods, such as education. These goods are financed through taxes and offered to all citizens at a lower than competitive price and possibly for free. Households can choose between the public and private supply.

For some of these goods, it is possible to supplement public service consumption with a private one, while for others it is not (either for technical or legal reasons). It is common in the literature to assume that education belongs to this second group of goods. In a first-best world, each consumer chooses his optimal level of quality, which is a priori different for each agent. One drawback of the public school system is that, for equity reasons, all students in a public school receive the same service regardless of their preferences.

The public service is often congested.[1] Offering incentives to students to move to the private sector reduces congestion but also the political support for a high quality public service.[2] It is interesting to investigate whether citizens would agree on the introduction of vouchers.[3] A voting model seems appropriate to forecast how a change in the level of taxation and the use of instruments such as vouchers would be perceived by voters.

The mainstream literature on education generally concludes either that the introduction of vouchers is not welfare improving, or that it cannot be the outcome of a majority vote unless some additional features are introduced

[1] Clearly, public schools are not always congested, but several empirical studies show that this is the case in several countries (e.g. Duncombe, Miner, and Ruggiero (1995), Kokkelenberg, Dillon, and Christy (2008), Ruggiero (1999), Smet (2001) and Wössmann and West (2006)).

[2] If the quality of public school increases, the intensity of support increases too for the people attending it, but here we consider support in terms of number of voters. The people moving to the private sector don't have a direct interest in a high-quality public school.

[3] Vouchers are a sort of cheque exploitable only to purchase private education. They can be "universal" (everybody is entitled to receive them) or "selective" (they are offered only to a subset of the population).

(e.g. peers effects). In my paper I show that this is not the case and that, as long as the value of vouchers is chosen exogenously, their introduction can lead to a Pareto improvement.

A major contribution to the field comes from Epple and Romano (1996a). Most of the successive literature, as well as my paper, can be considered as an extension of their model. Agents vote over the tax to finance public school; consumption of education varies amongst agents in terms of quality rather than quantity. Epple and Romano (1996a) show that to ensure the existence of an equilibrium a single crossing condition has to be imposed and identify two alternative conditions: Slope Rising in Income (SRI) and Slope Decreasing in Income (SDI).[4] Under SRI, at the equilibrium, named "ends against the middle", the richest and poorest households push to reduce the tax, while the medium class does the opposite. Vouchers are not considered. In my paper I only consider the SDI case, for which they conclude that the median voter is always decisive and the poorest in the society form a coalition facing the richest.

In my model, based on Epple and Romano (1996a) and Chen and West (2000), I consider the introduction of selective vouchers as a possible way to reduce congestion (by reducing the price of private education, vouchers might allow some voters to consume it) and to increase the quality level in the public sector.

Chen and West (2000) carry out a positive analysis of the school system, using the same model as Epple and Romano (1996a). Their aim is to compare systems with universal, selective and without vouchers under SDI. The upper threshold to receive selective vouchers is the median income, while in my model it is higher, and the voucher value is equal to the (constant) marginal cost of education, while in my model it is the average its cost. The article concludes that the majority always prefer the no-voucher model to the universal one, while the decisive voter is indifferent between the selective and the no-voucher frameworks and there are no welfare differences. The

[4] For more details on these conditions, see footnote 6 and also page 9.

differences in results with respect to my paper come from the attributes of the selective voucher.

To show that selective vouchers are welfare improving and they are supported by a voters' majority, I exogenously fix their value and the subset of people entitled to receive them. Agents vote over the tax to finance public school. My model is more general then Chen and West (2000): I require neither the cost of public and private education to be the same, nor the marginal cost of a student to be constant. As in Chen and West (2000), I do not consider peer effects,[5] and I concentrate on the SDI condition.[6] Absent vouchers, my model's results are identical with those in Epple and Romano (1996a), which I use as a benchmark. My model shows that in the extreme case, when most of public school students are attracted by vouchers, the public sector collapses and a subset of the population is worse off. Introducing vouchers always induces a Pareto improvement in the more realistic case in which public education continue existing.

Epple and Romano (1998) consider a universal vouchers model with students differing in income and ability. They conclude that vouchers' introduction is supported by a majority of voters and followed by a fall in congestion; all their results rely on the presence of peers effects. Best students attend private schools together with the richest ones. Private schools are attended by either rich or skilled (or both) students. Only a minority of neither-rich-

[5] Introducing peer effects would add a noise to my analysis, since it would be impossible to disentangle the effects of peer effects and of vouchers.

[6] SDI (Slope Decreasing in Income): this monotonicity condition means that the preferred tax by an agent decreases in income. There is no empirical evidence showing that this single crossing property is more realistic than the alternative one (SRI) nor the reverse. Probably the SRI assumption is more reasonable for countries where the life condition of poorest are dramatic and they do not care a lot about education, maybe because they even don't attend school because even very young people have to work. On the opposite, the SDI assumption might be more appropriate for countries where poor people are sufficiently rich to be able to profit of education and consider it as an investment.

I restricted my attention on SDI to keep the model simpler and I let the SRI case for future analysis.

nor-skilful students remain in public schools, where the quality drops along with students' utility. The authors also develop a computational model, calibrated to existing empirical evidence. The market structure and the cost function in this model are very close to mine.

In what follows, the paper is divided into four sections: section 2 describes the basic attributes of the model; section 3 illustrates the voting outcome without vouchers (benchmark case). Section 4 studies the effects of introducing vouchers, while in section 5 I analyse the results of the vote over the tax and show under which conditions selective vouchers induce a Pareto improvement. The last section concludes.

2 The model

I consider a model with two normal goods (the numeraire b and educational services X) whose basic setting is as follows:

1. Public and private school are mutually exclusive. Subscript P indicates the public sector and R the private one (e.g. X_P and X_R are respectively the qualities of public and private education).

2. The mass of voters is normalised to one. Each voter has a pupil attending school. Voters' type depends only on income ω (density function $f(\omega)$; support $\omega \in [\omega_{\min}; \omega_{\max}]$). I assume the average income $\overline{\omega} = \int_{\omega_{\min}}^{\omega_{\max}} \omega f(\omega) d\omega$ to be greater than the median one (ω_{med}).[7]

3. Voters' utility function ($U(X, b)$), is separable and strictly concave in both arguments.[8]

[7] Note that, given the normalisation of the population, the average and aggregate income coincide.

[8] This assumption is slightly more restrictive than the one ensuring the single crossing property in Epple and Romano (1996a); the subsequent computations are simplified by this assumption but results and insights are not affected by it.

4. To incorporate congestion[1] in the model, the school cost function is convex in the number of students n, so that $C(X, n) = F + V(n)X$, with $V_n > 0$ and $V_{n,n} > 0$.[9]

5. I assume $V(n_p) = (c_1 n_p + c_2 (n_p)^2)$, where n_p is the number of students attending it; thus the cost function is $C(X_P, n_p) = F + (c_1 n_p + c_2 (n_p)^2)\, X_P$.[10] Without loss of generality I consider that only one public institute is present.[11]

6. In the educational market, the public sector is the dominant firm while the private one is the competitive fringe.

7. Each private-school student can decide the level of educational quality to purchase. The low barriers to entry allow new firms to enter in the market so that the number of students in each school adjusts to always be the efficient one (i.e. for each firm i, $n_i = \arg\min(C(n_i)/n_i)$).

[9] The convexity assumption is controversial if considering education as a pure public good or assuming the cost function to be linear in the number of students. Having an increasing marginal cost is reasonable when we consider the product sold by schools including complementary services (such as school bus, professors office hours, sport facilities...), buildings (close to each other) etc. because the cost of these services might be non-linear (especially in case of capacity constraints). From both Epple and Romano (1998) and Epple, Romano, and Sieg (2006) it seems to be a correct specification. Lazear (2001) offers a survey of the literature on how class' size matters and under which condition it is correlated with education's quality/costs.

[10] This form of cost function, showing decreasing returns to scale, is supported by several empirical studies, e.g. Epple and Romano (1998), Epple, Romano, and Sieg (2006), Kokkelenberg, Dillon, and Christy (2008) and Duncombe, Miner, and Ruggiero (1995).

[11] This is equivalent to assumpe that all public schools have same number of students and quality of service.

This is possible with perfectly mobile students and the same budget for each institute (arbitrage effect), even in presence of idiosyncratic heterogeneity (e.g. different average wealth) and peer effects (whose analysis is beyond the scope of this work).

Passing from the general cost function $\Psi(X_P, \eta) = S + \left(c_1\eta + \psi\eta^2\right) X_P$ (with η the number of students per institute) with k institutes to this one would be simply a matter of renaming some variables.

The quality of one unit of private education X_R is defined in order to normalise the private sector's price (i.e. $q = 1$). [12]

8. Public education is financed via a proportional income tax t paid by all citizens and chosen through majority voting. Without loss of generality, for the sake of simplicity I suppose that the government's budget constraint requires balancing only ordinary (variable) costs and the proportional income tax proceeds.[13]

9. I assume that tax proceeds are firstly used to finance vouchers.[14] The remaining resources are equally shared among public school students (thus, all students attending public school receive the same quality of education X_P).

10. The government proposes a voucher of value v, that a given part of the population is entitled to use to attend a private school. Denoting n_v the number of people using their voucher in equilibrium, the public cost of vouchers is $n_v v$.

By assumption 8, the total public (variable) expenditure for education $(c_1 + c_2 n_p) n_p X_P + n_v v$ must be equal to total public income $t\varpi$. Rearranging the budget constraint, the quality of public schools is indirectly determined by:

[12] By the free entry assumption, q does not depend on the number of students in the private sector. Chen and West (2000) gets the same conclusion through a generic technology to produce education showing decreasing returns to scale. Epple and Romano (1996a) do not need to specify the private sector market structure.

[13] In other words I suppose fixed costs to be covered by *ad hoc* lump sum taxes. This can be explained because fixed costs are infrequent and huge thus they might have to be approved by specific procedures and financed through special public funds. This assumption does not affect results qualitatively.

[14] Fixing a minimal expenditure for public school would imply a higher preferred tax, but it would not qualitatively affect the outcome.

The alternative option (i.e. having total income shared between vouchers and public school) would imply a much less treatable model without adding special insights nor being a more realistic assumption.

$$X_P = \frac{t\overline{\omega} - n_v v}{g n_p} \tag{1}$$

where $g = (c_1 + c_2 n_p)$ is the per-pupil cost of one unit of public education. Clearly, since X_P cannot be negative, we must ensure that $t\overline{\omega} \geq n_v v$.

Households' behaviour can be summarized as follows:

- the problem of an agent choosing private school is[15]

$$\begin{cases} \max\limits_{X_R} U(X_R; b) \\ \quad s.t. \quad b = (1-t)\omega - X_R + v \end{cases}$$

His indirect utility can be written (in its reduced form) as $U^R((1-t)\omega - X_R^* + v)$, where X_R^* is the optimal level of consumption of private education. Clearly, since he does not profit from public education, if he uses vouchers his preferred level of taxation is $t = \frac{n_v v}{\overline{\omega}}$ (the minimum tax to finance them) and otherwise $t = 0$ (see figure 1). His utility is strictly decreasing in the tax.

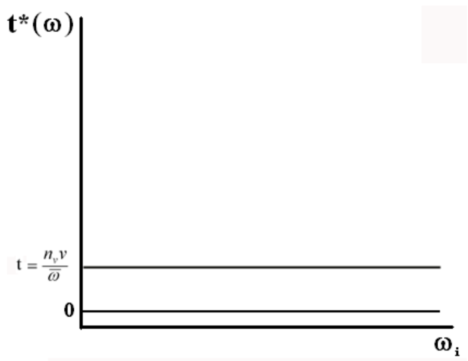

Figure 1: Preferred tax for agents attending private school.

[15]Remember that the price of private education q has been normalised to 1. Moreover, by the definition of voucher, $v \leq X_R$.

- The utility function of an agent of income ω attending public school is $U(X_p; b)$; replacing b by the after tax income and X_p by equation (1), the indirect utility is:

$$U^P\left(\frac{t\overline{\omega} - n_v v}{g n_p}; (1-t)\omega\right) \tag{2}$$

The preferred level of taxation for this agent is the one that maximises his utility, i.e. $t^*(\omega) = \arg\max_t U^P\left(\frac{t\overline{\omega}-n_v v}{g n_p}; (1-t)\omega\right)$. From the FOC it is possible to indirectly determine how the preferred tax of a voter attending public school changes with his type/income. Since the utility function is assumed to be separable in its two arguments, we have that $\frac{\partial t^*(\omega)}{\partial \omega} > 0$ if and only if $-\omega(1-t)U^P_{22} > U^P_2$ and $\frac{\partial t^*(\omega)}{\partial \omega} < 0$ if and only if $-\omega(1-t)U^P_{22} < U^P_2$. These expressions correspond respectively to the SRI and SDI monotonicity conditions.[16]

Both conditions are widely accepted in the literature; I assume that the SDI assumption (depicted in figure 2) holds. This assumption means that the marginal utility of education is much higher than the one of the numeraire for low levels of consumption, while the opposite is true when an agent is consuming a richer bundle. As a consequence, richer people are less eager to substitute units of the numeraire versus education.

Every agent chooses between public and private school by comparing the two possible levels of utility he can attain. If an agent prefers public school so do all the agents with lower income and if another prefers private school so do all the agents richer than him.[17] As a consequence, the poorest households attend public school while the richer ones prefer private schools.

[16] Assuming one of the two monotonicity assumption is necessary to ensure the existence of an equilibrium. For more details on the SRI and SDI assumptions, see footnote 6 and also Epple and Romano (1996a).

[17] This is true as long as we compare agents all receiving the voucher or if none of them receive it.

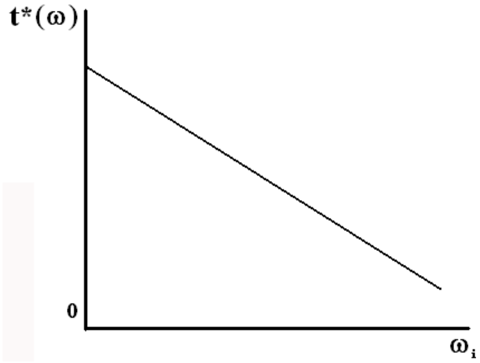

Figure 2: Preferred tax under SDI for public school students (linerar proxy).

It is possible to identify the "indifferent voter(s)" $\widehat{\omega}$, i.e. the voter(s) having the same utility regardless of the type of school attended:

$$U^R\left((1-t)\omega - X_R^* + v\right) = U^P\left(\frac{t\overline{\omega} - n_v v}{g n_p}; (1-t)\omega\right) \tag{3}$$

His identity depends on public school quality and thus on the equilibrium tax t. Denoting by $\underline{\omega}$ the pivotal voter, the equilibrium tax t depends on $\underline{\omega}$, so it is more precise to denote the indifferent voter by $\widehat{\omega}(t(\underline{\omega}))$ or $\widehat{\omega}(\underline{\omega})$ to be more concise.

The two following lemmas allow us to conclude that, once we identify the indifferent voter, all richer agents attend private school and the others the public one.[17]

Lemma 1 *In a given interval $\omega \in [\alpha; \varphi]$ and for $\varphi > \beta > \alpha$, if the agent $\omega = \beta$ prefers the private system so do all those richer than him (i.e. $\omega \in [\beta, \varphi]$).*

Lemma 2 *Similarly to the previous lemma, if $\omega = \beta$ prefers the public system, so do all the poorer agents (i.e. $\omega \in [\alpha, \beta]$).*

The indifferent agent can choose between two bundles: attending public school he can consume more of the numeraire but less education and vice

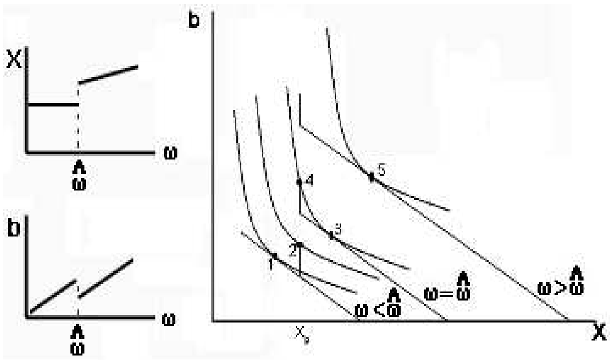

Figure 3: Moving from public to private school: Engel's and indifference curves.

versa (see fig. 3). From the sketch of the indifference curves[18] one can see that for an agent with low income ($\omega < \hat{\omega}$) it is preferable to choose bundle 2 (i.e. attend public school) than choosing the tangency point 1. The indifferent voter $\hat{\omega}$ can choose the tangency point 3 (attending private school) or to consume public school choosing bundle 4. Finally for those agents with sufficiently high income, the tangency point suggests that the best option is to consume bundle 5.

Before considering the solution of the model, I consider the situation when vouchers are not available, which is considered as a benchmark to study the consequences of the introduction of vouchers.

3 The benchmark case (without vouchers)

When vouchers are not available, this model is the same as in Epple and Romano (1996a), except that I don't impose the cost function parameters to

[18] The line represents the budget constraint and the vertical deviation in correspondence to the point X_p is due to the fact that everybody can always attend public school and in that case he can consume all his disposable income to buy the numeraire, thus there is a jump in his consumption.

be the same for the public and private sectors.[19]

The equilibrium results for the no-voucher case are denoted by the superscript nv. Equation (1) becomes $X_p^{nv} = \frac{t\varpi}{g^{nv} n_p^{nv}}$ and (3) is $U^R\left((1-t)\omega - X_R^*\right) = U^P\left(\frac{t\varpi}{g^{nv} n_p^{nv}}; (1-t)\omega\right)$.

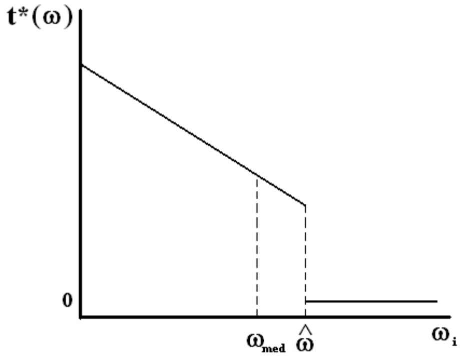

Figure 4: Preferred tax under SDI and no voucher.

Under the SDI assumption, the median voter is pivotal ($\underline{\omega} = \omega_{med}$); from figure (4) it is intuitive to see why.[20] This means that the voting outcome in the no voucher case is $t^{nv} = t(\omega_{med})$; all and only agents with income lower than the indifferent voter $\widehat{\omega}$ attend public school. The number of households attending public school is $n_p^{nv}(t^{nv}) = \int_{\omega_{\min}}^{\widehat{\omega}} f(\omega)d\omega = F(\widehat{\omega})$.[21]

[19] For more details and proofs of this section results, the reader can see Epple and Romano (1996a) and Glomm and Ravikumar (1998).

[20] The preferences over the tax being weakly decreasing in income, the poorer households would prefer a higher level of taxation than the richer ones. Any proposed $t < t(\omega_{med})$ can be an equilibrium because all households with income $\omega < \omega_{med}$ prefer $t(\omega_{med})$. Similarly, all tax proposals $t > t(\omega_{med})$ are defeated by a majority of voters composed by all agents with income $\omega \geq \omega_{med}$.

[21] Note that agents, in order to forecast the quality of public school, anticipate the value of n_p^{nv} while choosing to attend public or private school and thus to vote for a given level of taxation. It is crucial to ensure that in equilibrium the proportion of voters opting for public services coincides with agents' expectations. Glomm and Ravikumar (1998)'s proposition 2 proves that it always exists a n_p^{nv} solving (3) and being unique. The conditions under which Glomm and Ravikumar (1998) proposition 2 holds are not

The only difference compared to Epple and Romano (1996a) is that public and private school prices (respectively g and q) in my article are not assumed to be the same. All their results concerning the SDI case hold here simply assuming $g = q$. On top of that, even when $g \neq q$, their results are still qualitatively applicable, only the identity of the indifferent voter will differ. In particular, with respect to Epple and Romano (1996a), if $g > q$ the quality of public school is lower and so is $\widehat{\omega}$ (i.e. the indifferent agent between public and private school is poorer) and the opposite is true for $g < p$.

In the next sections, these results are used as a benchmark to grasp the consequences of the introduction of vouchers.

4 Introducing vouchers

The policy maker proposes a voucher of magnitude $v = \frac{t^{nv}\overline{\omega}}{n_p^{nv}}$ to agents with income below $\widehat{\omega}(t^{nv}(\omega_{med}))$ (i.e. those attending public school in the benchmark case) if they attend a private school.[22]

The public budget constraint (1) can be rewritten as

$$X_P = \left(t - \frac{n_v}{n_p^{nv}} t^{nv}\right) \frac{\overline{\omega}}{g n_p} \tag{4}$$

We can expect some agents entitled to receive the voucher to shift to the private sector. Moreover, this implies a reduction in congestion so the quality of public school might increase, attracting some students previously attending a private institute.

Since the price of private education is no longer the same for all agents, it is possible to identify up to two possible indifferent agents: one among voters receiving vouchers and another within the others.

It is preferable to consider separately the two different groups of agents $[\omega_{\min}; \omega_{\max}^v]$ and $[\omega_{\max}^v; \omega_{\max}]$. Lemmas (1) and (2) allow us to construct four

restrictive and apply in all the frameworks I analyse. In particular they need $F(\omega)$ to be a continuous function increasing in ω and $\widehat{\omega}(\omega)$ has to be decreasing in n_p.

[22] The value of the voucher is strictly smaller than the marginal cost of a student at equilibrium in the case without vouchers, i.e. $\frac{t^{nv}\overline{\omega}}{n_p^{nv}} < \frac{(c_1 + 2c_2 n_p^{nv}) t^{nv}\overline{\omega}}{(c_1 + c_2 n_p^{nv}) n_p^{nv}}$.

(possibly empty) subsets:[23] in particular for each of the two previous groups of agents we can have some voters preferring public education and some preferring the privately provided counterpart.

$\widehat{\omega}_L(t) \in [\omega_{\min}; \omega^v_{\max}]$ is the income level for which

$$U^R\left((1-t)\omega - X^*_R + v\right) = U^P\left(\frac{t\varpi - n_v v}{g n_p}; (1-t)\omega\right) \tag{5}$$

while $\widehat{\omega}_R(t) \in [\omega^v_{\max}; \omega_{\max}]$ is such that

$$U^R\left((1-t)\omega - X^*_R\right) = U^P\left(\frac{t\varpi - n_v v}{g n_p}; (1-t)\omega\right) \tag{6}$$

The two previous equations mean that $\widehat{\omega}_L$ and $\widehat{\omega}_R$ are the income levels for which an agent is indifferent between private and public education. Clearly $\widehat{\omega}_L + v \leq \widehat{\omega} \leq \widehat{\omega}_R$.

The two critical levels of income $\widehat{\omega}_L$ and $\widehat{\omega}_R$ can also be seen in Figure 5, which shows qualitatively how utility changes with income for an agent attending private or public school, both without and with vouchers. The quality of public school in the graph is fixed and $X_p > X_p^{nv}$.

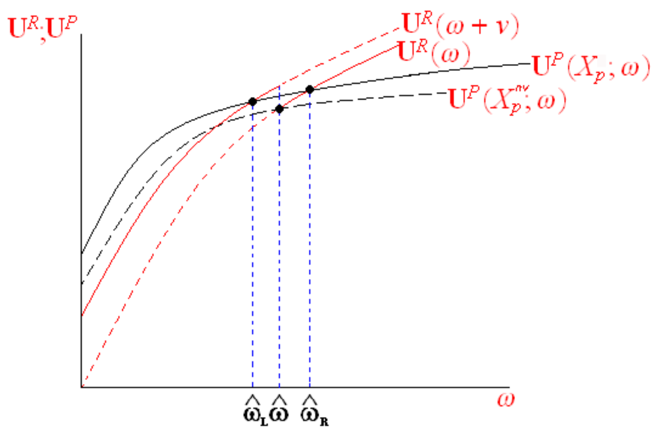

Figure 5: How utility changes with income

[23] Later on I will give the existence conditions for the indifferent agents and thus the four subsets' bounds.

From the graph it is clear that, if the intersection between $U^R(\omega+v)$ and $U^P(X_p;\omega)$ were to the right with respect to the one between $U^R(\omega)$ and $U^P(X_p^{nv};\omega)$ then $\widehat{\omega}_L$ would be greater than $\widehat{\omega}$. Thus, it would not belong to the required interval and all agents in $[\omega_{\min};\omega_{\max}^v]$ would attend public school. Likewise, all agents with income greater than $\widehat{\omega}$ prefer to consume private education when $U^R(\omega)$ and $U^P(X_p;\omega)$ do not cross to the right of $\widehat{\omega}$.

When both thresholds exist, there are four groups of agents, whose preferred choice is represented in Figure 6.

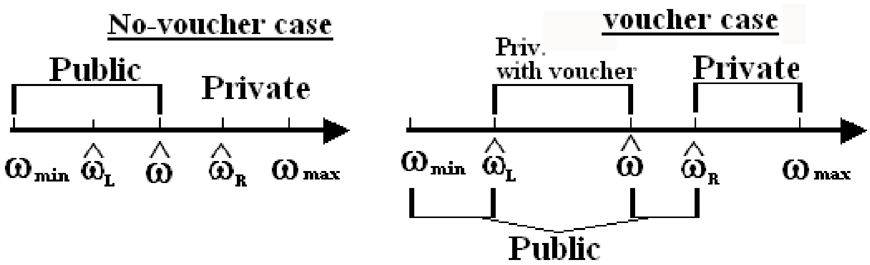

Figure 6: Intervals and choices

Having defined $\widehat{\omega}_L$ and $\widehat{\omega}_R$, it is now possible to precisely define n_v and n_p. The first one is the number of agents using the voucher at equilibrium while the second one is the number of agents attending public school, thus: $n_v=\int_{\widehat{\omega}_L}^{\widehat{\omega}(t^{nv})} f(\omega)d\omega$ and $n_p=\int_{\omega_{\min}}^{\widehat{\omega}_L} f(\omega)d\omega+\int_{\widehat{\omega}(t^{nv})}^{\widehat{\omega}_R} f(\omega)d\omega$.[24]

The following propositions and their corollaries prove that $\widehat{\omega}_L$ and $\widehat{\omega}_R$ exist, that is: $\widehat{\omega}_L\in[\omega_{\min};\omega_{\max}^v]$ and $\widehat{\omega}_R\in[\omega_{\max}^v;\omega_{\max}]$.

Proposition 1 *If, ceteris paribus, the quality of public school increases, the preferred tax for a given level of income falls. Thus the same pivotal voter might choose different tax levels according to the framework.*

Corollary 1 *If $X_p>X_p^{nv}$ then $t(\omega_{med})<t^{nv}(\omega_{med})$ (i.e. if the quality of public school with vouchers is higher than without and the median voter is pivotal in both cases, the tax burden decreases)*

[24]Remind that n_p^{nv} is the number of people attending public school in the no-voucher case and it is defined at page 12.

Proposition 2 *If $\widehat{\omega}_L = \widehat{\omega}$, then $\widehat{\omega}_R = \widehat{\omega}$ and we are back to the case without vouchers. Moreover it cannot be that $\widehat{\omega}_L > \widehat{\omega}$.*

Corollary 2 *A necessary and sufficient condition to have an equilibrium different from the no-voucher case is that $\widehat{\omega}_L < \widehat{\omega}$.*

Corollary 3 *The number of students attending public school when vouchers are available is always weakly smaller than in the case of no-voucher, i.e. $gn_p \leq g^{nv} n_p^{nv}$ with strict inequality as long as $\widehat{\omega}_L < \widehat{\omega}$.*

Proposition 3 *The quality of public school in case of vouchers is larger than without them ($X_P > Xp$) if and only if $\widehat{\omega}_R > \widehat{\omega}$. If $X_p \leq X_p^{nv}$, then $\widehat{\omega}_R = \widehat{\omega}$.*

Corollary 4 *The voting outcome tax can never be higher than the one preferred by the median voter (his preferred tax is the highest that might be supported by at least half of the population). Since $\widehat{\omega}_R > \widehat{\omega}$ only when $X_p > X_p^{nv}$, if we observe $\widehat{\omega}_R > \widehat{\omega}$, the total number of agents attending public school has necessarily to be smaller than in the case without vouchers ($n_p < n_p^{nv}$).*

Proposition 4 *The equilibrium public school quality under vouchers is always greater or equal to the one without vouchers (for a given level of taxation), i.e. $X_p \geq X_p^{nv}$, with strict inequality when $\widehat{\omega}_L < \widehat{\omega}$.*

Proof. See the appendix for the proofs of propositions 1,2, 3 and 4. ■

As a consequence of these propositions, we conclude that either $\widehat{\omega}_L = \widehat{\omega}_R = \omega_{\max}^v$ (vouchers are ineffective), or $\widehat{\omega}_L < \omega_{\max}^v < \widehat{\omega}_R$.

5 The vote over the tax

The tax to finance public school is chosen by households through majority voting. Different scenarios are possible:[25]

[25] Recall that the preferred tax of an agent depends on his choice over public and private education but also on the opportunity to receive a voucher when choosing the private school.

- the preferred tax is decreasing in income for agents attending public school (SDI assumption).
- the preferred tax is $t = 0$ for private school students not using vouchers.
- $t = \frac{n_v v}{\overline{\omega}}$ is the preferred tax of private school students using vouchers; this is exactly the minimum tax to finance the voucher system.[26] With this level of taxation, strictly lower than the one preferred by any public school student, public education disappears.

The voting process outcome depends on the distribution of income and mainly on the relative position of $\widehat{\omega}_L$ with respect to ω_{med}. I analyse separately the cases $\widehat{\omega}_L \geq \omega_{med}$ (subsection (5.1)), when introducing vouchers always induces a Pareto improvement, and $\widehat{\omega}_L < \omega_{med}$ (subsection (5.2)), when we observe a Pareto improvement only as long as the public school system does not collapse, otherwise a minority of the population might be worse off.

5.1 The case of $\widehat{\omega}_L \geq \omega_{med}$

Restricting our attention to the case when $\widehat{\omega}_L \in [\omega_{med}; \widehat{\omega})$,[27] the outcome of the vote is precisely $t = t(\omega_{med})$. In fact, all agents with income $\omega < \omega_{med}$ (by definition half of the population) ask for a tax increase with respect to $t = t(\omega_{med})$, while all agents with income $\omega \in (\omega_{med}; \omega_{\max}]$ are favourable to a decrease in the equilibrium tax.[28] This means that the median voter is pivotal. Figure 7 represents agents' preferred tax in the case of vouchers when $\widehat{\omega}_L \geq \omega_{med}$.

[26] Remind that here vouchers size is fixed, thus voting for a higher tax would be useless for them.

[27] If $\widehat{\omega}_L = \widehat{\omega}$ (that is, given the utility function, vouchers are not sufficiently attractive and in equilibrium any agent use them) we are back to the no-voucher case and the introduction of vouchers is ineffective (see proposition 2).

[28] This is due to the SDI assumption and because part of agents with income $\omega > \omega_{med}$ attends private school.

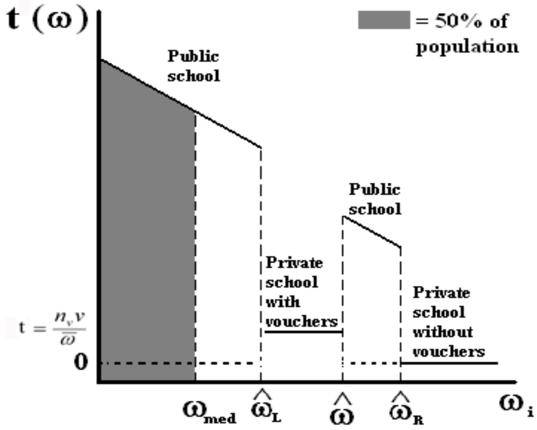

Figure 7: Agents' preferred tax

Note that, even though the median voter is again decisive, by proposition (1) his preferred tax level is lower than in the no-voucher case: $t(\omega_{med}) < t^{nv}(\omega_{med})$. Moreover the public budget constraint is relaxed and the quality of public school necessarily increases.[29] Part of this effect is offset by the arrival of some new students previously attending private school and attracted by the higher public school quality, thus the subset $\omega \in [\widehat{\omega}; \widehat{\omega}_R]$ is non empty. By proposition (3) and its corollary, we know that the number of agents moving from public school is higher than the one of students moving to it and that the final effect is an increase in the quality of the public service (financed through tax proceeds net of vouchers expenditure).

From a welfare standpoint, we observe a Pareto improvement. Intuitively, introducing vouchers the quality of public schools increases making public school students better off. Moreover the tax burden falls, so all citizens are better off. By the Weak Axiom of Revealed Preferences (WARP), all agents changing behaviour while the previous bundle is still affordable must be better off.

[29] Since voucher value is lower than the marginal cost of the most expensive public school students, convincing them to consume private education makes the public budget constraint less binding and increases the quality of the public service.

To be more rigorous, for $\widehat{\omega}_L < \widehat{\omega}$, utility increases for all agents when introducing vouchers:

- $[\omega_{\min}; \widehat{\omega}_L]$: these agents always opt for public school. The quality of public school increases (proposition 4). Since both their disposable income and the public school quality increase, their utility increases as well.

- $[\widehat{\omega}_L; \widehat{\omega}]$: they move from public to private education and use vouchers. If they stuck to public education they would increase their utility (similarly to agents in $[\omega_{\min}; \widehat{\omega}_L]$). If they decide to opt out from public school, by WARP it must be that their utility from attending private school is even higher.

- $(\widehat{\omega}; \widehat{\omega}_R)$: The bundle previously consumed is still affordable. If they modify their choice, by the WARP it means that the new bundle is preferred to the previous one.

- $[\widehat{\omega}_R; \omega_{\max}]$: all the agents in this interval attend private school in both cases. The price they pay to attend private school is the same and the tax decreased. As a consequence, all these households are better off in the voucher case.

To sum up, when the introduction of vouchers is ineffective (i.e. $\widehat{\omega}_L = \widehat{\omega} = \widehat{\omega}_R$) agents are indifferent and when $\widehat{\omega}_L \in [\omega_{med}; \widehat{\omega})$ the selective voucher system strictly Pareto dominates the no-voucher one.

5.2 The case of $\widehat{\omega}_L < \omega_{med}$

When many voters have an income close to $\widehat{\omega}$ and (given the utility function) vouchers are particularly attractive, the introduction of vouchers implies changes in the behaviour of a great number of consumers. All voters with income in the interval $\omega \in [\widehat{\omega}_L; \widehat{\omega}]$ move to the private sector.

If $\widehat{\omega}_L(\omega_{med}) < \omega_{med}$, the poorest part of the population (which is attending public school) is not numerous enough to form a majority. The shift from public to private education implies again (*ceteris paribus*) an increase in the public service quality which attracts a group of voters ($\omega \in [\underline{\omega}; \widehat{\omega}_R]$) who were previously attending a private school. With respect to the previous case (section 5.1) we have an additional element to take into account: if people willing to attend public school are not able to form a coalition of at least half of the population (that is, if $\int_{\omega_{\min}}^{\widehat{\omega}_L} f(\omega)d\omega + \int_{\widehat{\omega}}^{\widehat{\omega}_R} f(\omega)d\omega < 50\%$) public school has not enough support and it collapses. Thus the relative size of each group with respect to the others determines the voting outcome.

Let us define $\underline{\omega} \in (\widehat{\omega}; \widehat{\omega}_R]$ as the income level for which $\int_{\omega_{\min}}^{\widehat{\omega}_L} f(\omega)d\omega + \int_{\widehat{\omega}}^{\underline{\omega}} f(\omega)d\omega = 50\%$[30]. Intuitively the income $\underline{\omega}$ represents the agent whose preferred tax is the "median preferred-tax". This domain of existence ensures that, if it exists then the agent $\underline{\omega}$ attends public school. Agents' preferred tax is summarised in figure 8.

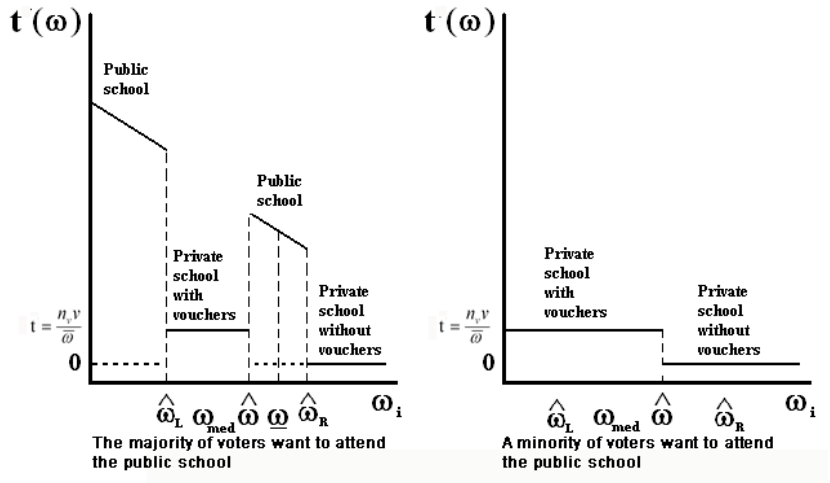

Figure 8: Agents' preferred tax

The existence of $\underline{\omega}$ implies that public school students are forming a coalition able to set the equilibrium tax. The group in favour of no tax is always small enough not to influence the vote outcome alone: when people attending public school are representing more than half of the population ($\underline{\omega}$ exists),

[30]This is equivalent to saying that $\int_{\widehat{\omega}_L}^{\omega_{med}} f(\omega)d\omega = \int_{\widehat{\omega}}^{\underline{\omega}} f(\omega)d\omega$.

they choose the equilibrium tax, otherwise the tax is chosen by the group of agents attending private school and profiting of the voucher (and set to the minimum level to finance vouchers: $t = \frac{n_v v}{\overline{\omega}}$).

The equilibrium when the majority of voters attend public school If $\underline{\omega} \in [\widehat{\omega}; \widehat{\omega}_R]$, by construction $\underline{\omega}$ is pivotal: all agents with income belonging to the intervals $[\omega_{\min}; \widehat{\omega}_L]$ and $[\widehat{\omega}\,; \underline{\omega}]$ prefer a higher tax than $\underline{\omega}$, and by construction they represent half of the population. Those in $[\widehat{\omega}_L; \widehat{\omega}]$ and $[\underline{\omega}; \widehat{\omega}_R]$ would prefer a lower but positive tax; the remaining $(\omega > \widehat{\omega}_R)$ ask for no tax at all. Given that $\underline{\omega} > \omega_{med}$, by the SDI assumption the equilibrium tax decreases with respect to equilibria in sections (3) or (5.1).

For the existence of this equilibrium, it is necessary for $\widehat{\omega}_R$ to be greater than $\widehat{\omega}$ (otherwise $\underline{\omega} \notin [\widehat{\omega}; \widehat{\omega}_R]$). By proposition (3) we can conclude that quality of public education has necessarily increased and it follows that a strict Pareto improvement occurred.

All agents' disposable income increased $(t(\underline{\omega}) < t^{nv}(\omega_{med}))$, thus agents attending a private school (i.e. $\omega \in [\widehat{\omega}_L; \widehat{\omega}]$ and $\omega > \widehat{\omega}_R$) are necessarily better off than without vouchers. The poorest agents in the population $(\omega < \widehat{\omega}_L)$ pay less taxes and receive a better public service.

People in $\omega \in [\widehat{\omega}; \widehat{\omega}_R]$ could stick to the private market and consume a better bundle with respect to the one consumed without vouchers (since the tax decreased), if they move to the public sector, by WARP we can conclude that they are better off.

All agents being strictly better off, we conclude that in this framework the introduction of vouchers lead to a strict Pareto improvement.

The equilibrium when the majority of voters do not attend public school When agents willing to attend public school are less than 50%, the decisive voter belongs to the group of people attending private school and profiting of the voucher. The minimum tax to finance vouchers for all agents entitled to receive them ($t = \frac{n_p^{nv} v}{\overline{\omega}}$) wins any pairwise comparison. Replacing v by its value we obtain $t = \frac{n_p^{nv} v}{\overline{\omega}} = t^{nv}$. Every former student of the public school is receiving the average social cost of a public student in the no voucher case.

For this solution to be a stable equilibrium at least half of the population has to be better off, otherwise this level tax could not win against the proposal of having no vouchers. All people with income $\omega > \widehat{\omega}$ are indifferent, since the tax does not change with respect to the benchmark.

People with income $\omega \in [\widehat{\omega}_L; \widehat{\omega}]$ are always better off (by WARP).

Concerning people with income $\omega < \widehat{\omega}_L$, they all receive the same voucher to be spent for private education. Three frameworks are possible for them:

1. Private school market price (q) is lower than the average cost of producing public education ($AC(X_P)$) in the no-voucher case. It is socially optimal to dismantle public school and distribute vouchers. Agents are better off: this solution is a Pareto improving equilibrium and public school disappears.

2. $q = AC(X_P)$. They are indifferent (they consume the same amount of both goods). This equilibrium weakly Pareto dominates the no-voucher case and public school disappears.

3. $q > AC(X_P)$. They are strictly worse off (they consume the same quantity of numeraire but receive a worse educational service). Here a minority of the population is worse off ($\omega < \widehat{\omega}_L$), another is better off ($\omega \in [\widehat{\omega}_L; \widehat{\omega}]$) and the remaining ($\omega > \widehat{\omega}$) are indifferent. For this framework to be an equilibrium (i.e. for voters to accept the introduction of vouchers), at least half of voters should agree on vouchers, which means that a substantial part of the richest agents has to form a coalition with the middle class against the lower class.

6 Conclusions

The aim of this work was to investigate the implications of introducing selective vouchers and in particular if this change would be accepted by the majority of voters. The main contributions of this work are to show:

1. that the usual conclusion that the median voter is always decisive under the assumption of SDI is not robust to the introduction of vouchers.

2. that in addition to the known types of coalition ("lower class versus higher class" and "middle class versus the others") we can have a third type of coalition where part of the bottom-middle class joins the coalition of the richest agents to ask for a reduction in taxes while the top-middle class forms a coalition with the poorest voters to increase taxes.

3. that the introduction of vouchers always induces a Pareto improvement unless if, introducing them, the public sector collapses and meanwhile the market price of private education is higher than the average cost of producing public education. In this case the poorest subset of the population would be hurt by the introduction of vouchers.

4. that the introduction of vouchers should always be supported by a large majority (under the hypothesis of rational agents).[31]

5. the middle class is the one who directly profits from vouchers; the poorest class is the one bearing their costs when public school collapses. The richest class always weakly profits from the introduction of vouchers (through taxes reduction).

My model is qualitatively robust to different specifications (such as Constant Return to Scale production functions) as long as we choose the value of the voucher exogenously at a value high enough to attract public students but smaller than their cost to the society.

These results seem to show that the introduction of vouchers should be welcomed by voters; nevertheless in many western countries (especially in

[31] The majority of voters always profits from the introduction of vouchers; in most cases, all agents in the society do.

Europe) vouchers are not very popular. In Switzerland a referendum against vouchers has been voted, in Italy the debate over vouchers has been almost immediately stopped because of the strong hostility showed by many political parties. Which are the reasons for that? Probably a combination of different factors generated this aversion towards vouchers: on the one hand, especially in certain countries, private institutes have religious (and often even political) orientations and vouchers are perceived as a way to subsidise that *credo* or a way to diffuse specific cultures or principles.

Another reason for the failure of vouchers in Europe might be that generally only universal vouchers have been proposed and, as we know from the literature, universal vouchers are more likely to decrease the quality of the public service and reduce redistribution.

Finally, a more substantial problem concerns the value of the voucher. A voucher of small amount is ineffective and a too large one implies that the public sector is no longer supported by the majority of the population. In my model a benevolent social planner fixes the value of the voucher at a value for which the public budget constraint is relaxed when some students use them. If we let people decide over the value of vouchers, we can expect to have very different results, compared to those of my model, in particular it is possible for the vouchers' value to be higher than the public-school-student's social cost or so small that nobody would be interested in using them.

Appendix

A The effects of a change in the tax

Most variables are affected by a change in the income tax. Intuitively, if the tax rate falls the first impact on the model is that on the one side the public investment in education ($t\overline{\omega}$) falls and, on the other hand, the disposable income ($(1-t)\omega$) of all the agents increases. Both these effects imply that opting for private school becomes more attractive. Concerning the first one, the reasons are obvious, while for the second one they are slightly more subtle: an increase in the disposable income leads to an increase in the consumption of b for everybody, but since the quantity of b consumed by people attending public school is higher, by the concavity of the utility function, the increase in utility for people attending public school is lower than for those preferring private education. Since private school becomes more attractive, a greater number of agents switches from the public to the private system (which means that the income of the two indifferent voters decreases). The number of voters using vouchers increases, tightening even more the public budget constraint. Simultaneously, the number of people attending public school having fallen, the per-capita public expenditure increases (since gn_p drops) making public education more attractive.

To sum up, the impact on the quality of public school from a change in the tax is a priori undetermined. On the one side, a drop in the tax rate implies that the budget available for public school is lower. On the other side this provokes a decrease in the number of people attending public school (both because public school becomes less attractive and because agents' disposable income increases). When $\frac{\partial X_p}{\partial t} \geq 0$, it means that a reduction in the tax rate decreases public expenditure for education and the consequent shrinkage in the number of people attending public school is not enough to offset it (in other words, demand for public school is inelastic), thus the per-capita expenditure will also plunge. The reverse is true for $\frac{\partial X_p}{\partial t} < 0$.

B Proof of proposition (1)

For a given revenue $\widetilde{\omega}$, the preferred tax $t(\widetilde{\omega}) = \arg\max_t U^P(\frac{t\overline{\omega}-n_v v}{gn_p}; (1-t)\widetilde{\omega})$. If, for any reason, the first argument (X_p) increases, its marginal utility of education (U_1^p) decreases. At equilibrium by definition the optimal tax equalises the marginal utility of both arguments ($U_1^p = U_2^p$), which means that the marginal utility of the numeraire falls (thus the numeraire consumption has to increase) and so the tax drops.

C Proof of proposition (2)

If $\widehat{\omega}_L = \widehat{\omega}$, nobody uses the voucher, $n_v = 0$ and $X_p = \frac{\overline{\omega} t^{nv}}{g n_p}$. The number of students attending public school could not be lower than in equilibrium in the no-voucher case, which implies that $g n_p \geq g^{nv} n_p^{nv}$. This make public school (weakly) less attractive than in the no-voucher case, thus all the households with income $\omega > \widehat{\omega}$ (who were already preferring the private system) confirm their choice. If $X_R \succ_\omega X_P$ for all $\omega > \widehat{\omega}$, then $g^{nv} n_p^{nv} = g n_p$ and thus $X_p^v = X_p$ and we are back to the equilibrium case without vouchers.

Finally, it cannot be that $\widehat{\omega}_L > \widehat{\omega}$. This would result in $n_v = 0$ and $g^v n_p^v = g n_p$; this would imply that $X_P^{nv} = X_P$ and so that $\widehat{\omega}_L = \widehat{\omega}$, which is a contradiction. This proves that always $\widehat{\omega}_L \leq \widehat{\omega}$.

D Proof of proposition (3)

$\underline{X_P > X_P^{nv} \Leftarrow \widehat{\omega}_R > \widehat{\omega}}$: if $\widehat{\omega}_R > \widehat{\omega}$, agents in the interval $(\widehat{\omega}; \widehat{\omega}_R)$ are attending public school in presence of vouchers while they were attending private schools before. The introduction of vouchers does not imply changes in the disposable income of agents with income above $\widehat{\omega}$, thus the original consumption bundle remains affordable. By the WARP, if we observe a change in this agents' behaviour, it must be that the new bundle is preferred. Since the numeraire consumption is constant, it must be that the quality of school consumed increased, thus $X_P > X_R > X_P^{nv}$.

$\underline{X_P^{nv} = X_P \Rightarrow \widehat{\omega}_R = \widehat{\omega}}$: when $X_P^{nv} = X_P$, for agents in $(\widehat{\omega}; \omega_{\max}]$ nothing has changed. By simply replacing X_P by X_P^{nv} in (6) we are back to the condition in (3) and thus, by definition, the solution of the problem is $\widehat{\omega}$.

$\underline{X_P > X_P^{nv} \Rightarrow \widehat{\omega}_R > \widehat{\omega}}$: by definition, $\widehat{\omega}_R$ is the level of income for which the left and right hand sides of equation (6) are equal. For $X_P^{nv} = X_P$ then $\widehat{\omega}_R = \widehat{\omega}$. Increasing X_P, public school becomes more attractive (i.e. the right hand side is bigger than the left one). Only an increase in the level of income can re-establish the equality. Such an increase leads to a higher consumption of the numeraire both in case of consumption of public school and of the private one; given the concavity of the utility function, the marginal increase is higher on the left hand side than on the right one, which insures that for a sufficiently large increase in $\widehat{\omega}_R$, the equality holds again.

E Proof of proposition (4)

By proposition (2), $\widehat{\omega}_L$ cannot be greater than $\widehat{\omega}$. Two different scenarios are possible: $\widehat{\omega}_L = \widehat{\omega}$ or $\widehat{\omega}_L < \widehat{\omega}$.

Proof by contradiction. Suppose $\widehat{\omega}_L < \widehat{\omega}$ and $X_P \leq X_P^{nv}$: by proposition (3), $\widehat{\omega}_R = \widehat{\omega}$ and thus a) $n_p = (n_p(t^{nv}) - n_v)$, b) ω_{med} is decisive, c) $t > t^{nv}$ and d) $g < g^{nv}$ (since $\widehat{\omega}_L < \widehat{\omega}$).

Then

$$\left[\frac{t^{nv}}{g^{nv}n_p^{nv}} - \left(\frac{t - \frac{n_v}{n_p^{nv}}t^{nv}}{gn_p}\right)\right]\overline{\omega} > 0$$

a necessary condition for that (since $t > t^{nv}$) is $n_p g + n_v g^{nv} > n_p^{nv} g^{nv}$.For this to be true it must be that $g > g^{nv}$ which is impossible.

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