Effect of Aging on Housing Prices: A Perspective from an Overlapping Generation Model

Tianyu Sun and Satish Chand and Keiran Sharpe

UNSW Canberra

10 February 2018

Effect of Aging on Housing Prices: A Perspective from an Overlapping Generation Model

Tianyu Sun[*] , Satish Chand and Keiran Sharpe

ABSTRACT

This paper investigates the effect of aging on housing prices. It provides a theoretical explanation to address the on-going debate about this issue. The analysis demonstrates that aging has divergent effects on housing prices, depending on the net effects of a fall in fertility vis-à-vis a rise in longevity on demand for housing. In addition, the results suggest that aging could cause a turning point in the price dynamics. Before this turning point, aging would boost the prices; however, after this point, the prices are depressed because of aging. Furthermore, inequality of household utility is enlarged during the aging processes.

JEL classification: E21, E31, J11, R21, R31

Keywords: Aging Population; OLG Model; House Prices; Land Prices; Turning Point

[*] Corresponding author.
Email addresses: Tianyu.Sun@student.adfa.edu.au (T. Sun), s.chand@adfa.edu.au (S. Chand), k.sharpe@adfa.edu.au (K. Sharpe).
Postal addresses: School of Business, UNSW Canberra, c/o School of Business, PO Box 7916, Canberra BC 2610, Australia (T. Sun).
School of Business, UNSW Canberra, c/o School of Business, PO Box 7916, Canberra BC 2610, Australia (S. Chand).
School of Business, UNSW Canberra, c/o School of Business, PO Box 7916, Canberra BC 2610, Australia (K. Sharpe).

1. Introduction

Will an aging population depress housing prices? The extant literature provides divergent answers to this question. The first argues that an aging population will have a significant effect on housing prices (Mankiw and Weil 1989, 1991, Bergantino 1998, Takáts 2012), and the second opined that aging would have little or mixed impact (Engelhardt and Poterba 1991, Hendershott 1991, Poterba 2001, Eichholtz and Lindenthal 2014, Hiller and Lerbs 2016). The debate on the impact of aging on housing prices is largely drawn from empirical studies, and remains alive (Poterba 2014). This paper contributes to the extant literature by investigating the effects of aging within the context of an overlapping generations model (OLG) to explain the mixed results from empirical studies estimating the effects of aging on housing prices.

The results show that an aging population can result from a fall in fertility and/or an increase in longevity: housing prices increase only when the net effect of the above is positive. More specifically, a fertility rate lower than the replacement level will depress housing prices while an increase in longevity has the opposite effect. The above explains some of the divergent effects of an aging population on housing prices in different contexts. From this perspective, the seemingly contradictory views from previous empirical research can be reconciled.

Moreover, an increase in longevity with a synchronized fall in fertility can lead to an increase in prices initially with the effect being overturned later, leading to a turning point in price dynamics over time. Specifically, an aging population resulting from an increase in longevity with a decline in fertility can initially boost demand for housing and lead to the increase in prices, but this effect will peter out as the worker population begins to shrink. This finding will cast a new light on the relationship between demography and economic fluctuations.

The results presented above have been obtained using an OLG model that has been drawn from two separate strands in the literature. The first strand is drawn from Iacoviello and Neri (2010), in whose work the housing market is analysed using a Dynamic Stochastic General Equilibrium (DSGE) model. The settings of our model are mainly based on their approach, but are transformed to an OLG model so as to incorporate the age structure of the population. The second strand of our model comes from the research into land prices (Davis and Heathcote 2007, Liu, Wang, and Zha 2013). Their insights are to separate out the effects of land and construction on the price of a house.

In literature, Takáts (2012) has established a small OLG model to investigate the effect of aging on a ‘flat’ asset. By its nature, this flat asset is the same as the land in our model. Comparing with Takáts (2012), our model incorporates the supply side of the housing market, revealing the differences between structure and land. In addition, the price dynamics can be simulated, and the results are explained from the perspective of households' behaviours. This method reveals the existence of a turning point in the price of houses as the population continues to age.

In this way, our paper fits into the emerging literature on housing prices using theoretical models.[1] While this literature varies as to what aspect it focuses on, our work made solid progress in isolating age structure from other factors and examines its effect on housing prices. In addition, our paper also contributes to the literature studying the consequences of aging on assets and savings.[2] The bulk of theoretical work in this area has ignored housing as an asset, a major deficiency given that housing is the largest component of households' wealth. Although the work of Takáts (2012) makes a contribution on this issue, the supply side of the housing market is omitted. The modelling in our paper fills this gap.

We first model the long-term effect of an aging population on housing prices, and then simulate the dynamics of housing prices due to demographic change. Although our work is an exercise that aims to reach general conclusions about the effects of aging, the parameters have to be calibrated to conduct simulations. Here, the parameters are calibrated for China because: (i) housing assets constitute more than 70 percent of Chinese households' wealth (Xie and Jin 2015); and, (ii) China is undergoing rapid aging of its population (Lutz, Sanderson, and Scherbov 2008). Thus, choosing the parameters for China provides us with a typical example without losing the generality of the conclusions.

Our results show that demographic change has a bigger impact on land prices than that for construction, a fact revealed only when the two are looked at separately. This result is consistent with the empirical findings of Davis and Heathcote (2007). Specifically, our simulations show that a decline in the fertility rate of 10 percent from the replacement level for one generation (i.e. 30 years in our model) leads to a fall in the price of construction by 1 percent whereas the price of land falls by 10 percent. In contrast, when longevity increases by 6 percent then the corresponding house and land prices increase by 1 percent and 7 percent, respectively.

We next investigate the effects of a simultaneous change in fertility and longevity on households' utility. Anticipating the results, workers would have greater utility because of

---

[1] See, for example, Iacoviello (2005); Iacoviello and Neri (2010); Liu, Wang, and Zha (2013); Justiniano, Primiceri, and Tambalotti (2015); Ng (2015); Chen and Wen (2017).

[2] See, for example, Ando and Modigliani (1963); Brooks (2000); Brooks (2002); Abel (2001); Abel (2003); Modigliani and Cao (2004); and Curtis, Lugauer, and Mark (2015).

higher house and land ownership per capita; however, the utility of the retirees is worse. Assuming perfect foresight in the model, the consequential housing prices would increase immediately before the demographic changes and decline as the effects of the fall in fertility take over.

The remainder of this paper is organized as follows. Section 2 describes the OLG model, Section 3 presents the assumptions and the calibration of parameters, while Section 4 presents the long-term effects of aging on housing prices. Section 5 investigates household utility and the housing price dynamics. Section 6 concludes.

2. The Model

2.1. Demography

The model's time structure is the same as the classical model of Diamond (1965). The economy consists of two overlapping generations[3], workers and retirees, who are alive in both periods. For each generation, we assume that the households are identical[4].

The population of workers is affected only by the rate of fertility. The fertility rate at time t, denoted by n_t, is given as equation (1), where $N_{t,1}$ and $N_{t-1,1}$ are representing the population of workers in time t and its previous period, respectively. Note that in this stylized model, the fertility rate is equal to the rate of growth of workers.

$$n_t = \frac{N_{t,1}}{N_{t-1,1}} \tag{1}$$

The population of retirees, in contrast, is influenced by both the fertility rate and longevity. Following Blackburn and Cipriani (2002) and Cipriani (2013), let longevity be determined by the share of households that survive until the retirement stage. Specifically, households live their period as workers with certainty, but a fraction will leave the economy at the beginning of their retirement, while the rest will live the remaining period of retirement. This survival function is given as equation (2), where $N_{t-1,1}$ and $N_{t,2}$ represent the number of workers in period $t-1$ and the retirees in period t respectively.

$$\pi_t = \frac{N_{t,2}}{N_{t-1,1}} \tag{2}$$

[3] The model that consists of multiple generations will be left for further study.

[4] Therefore, if we know the population of each generation, then the aggregate and per-capita values can be transformed from one to the each other. For the purposes of illustration, we will present the model mainly using aggregate variables (except the section about households' utility). The equations for individuals are listed in Appendix A.

Based on (1) and (2), the population dynamics of retirees is given as:

$$\frac{N_{t,2}}{N_{t-1,2}} = \frac{n_{t-1}\pi_t}{\pi_{t-1}} \tag{3}$$

2.2. Households

Utility

The households' utility is characterized by (4) as follows:

$$U_t = U\big(U_{t,1}, U_{t+1,2}\big) = U_{t,1} + \pi_{t+1}\, \beta\, E(U_{t+1,2}) \tag{4}$$

Following Diamond (1965), the expected life span utility of a household (U_t) is time separable and is determined by the sum of utility when working, $U_{t,1}$ and that when retired, $U_{t+1,2}$. Two discount factors are 1) the time preference β ($0 < \beta < 1$) and 2) the survival rate π. The form of equation (4) is consistent with the practice that survival rates are involved.[5]

The utility during work ($U_{t,1}$) is given as:

$$U_{t,1} = \ln\big(c_{t,1}\big) + j_h \ln\big(h_{t,1}\big) + j_l \ln(l_{t,1}) - \frac{\tau}{1+\eta}(n_{c,t}^{1+\xi} + n_{h,t}^{1+\xi})^{\frac{1+\eta}{1+\xi}} \tag{5}$$

Following Iacoviello and Neri (2010) and Liu, Wang, and Zha (2013), the terms providing positive utility in (5) are per capita consumption of goods, $c_{t,1}$, structure of houses, $h_{t,1}$, and land, $l_{t,1}$. Here, we argue land is a part of the utility function because it is a necessary component of the housing. For example, the houses that occupy bigger land (villas) will provide extra utility for the residents compared to those with smaller land area (e.g. apartments). The disutility comes from working. Variables $n_{c,t}$ and $n_{h,t}$ represent hours worked in non-housing and housing sectors respectively.

The parameters j_h and j_l represent the preference for house and land respectively, and the parameter τ denotes the dis-preference of labor supply. The parameter $\eta > 0$ ensures that the utility function is concave with respect to labor supply. In addition, the parameter $\xi > 0$ indicates the labor mobility between the two production sectors is imperfect (Horvath 2000, Iacoviello and Neri 2010).

The utility function for the retirees, who do not participate in the labour market, is analogous to (5); that is,

[5] See, for example, Blackburn and Cipriani (2002), Cipriani (2013) and Muto, Oda, and Sudo (2016)

$$U_{t+1,2} = \ln(c_{t+1,2}) + j_h \ln(h_{t+1,2}) + j_l \ln(l_{t+1,2}) \tag{6}$$

where the consumption, house and land per capita of this generation are denoted by $c_{t+1,2}$, $h_{t+1,2}$ and $l_{t+1,2}$ respectively.

Budget Constraint

The budget constraint for each generation is illustrated separately. Although households make decisions based on individual budget constraints, we follow Diamond (1965) to present the aggregate equations here for illustration, and the per capita version of the model is shown in Appendix A.

The aggregate budget constraint for the workers is as follows:

$$C_{t,1} + q_t H_{t,1} + p_{l,t} L_{t,1} = (1 - T)(w_{c,t} N_{c,t} + w_{h,t} N_{h,t} + D_t), \tag{7}$$

where the right side is the aggregate income of the workers and the left side is the expenditures. The total income of workers can be divided into wage, $w_{c,t} N_{c,t} + w_{h,t} N_{h,t}$, and dividend, D_t, income. Workers pay tax of T, and thus the fraction $(1 - T)$ is the share of the total income kept by the workers while the remainder accrues to the retirees. One could rationalize the specification of 'T' in (7) as a pay-as-you-go pension system where the current generation of workers are taxed to fund consumption of the retirees.

Total expenditure is $C_{t,1} + q_t H_{t,1} + p_{l,t} L_{t,1}$. We have excluded saving of households under the assumption that it equals the value of loans to the firms which are owned by the households, and so the returns are in the form of profits.

The aggregate budget constraint for the retirees is as follows:

$$C_{t+1,2} + q_{t+1} H_{t+1,2} + p_{l,t+1} L_{t+1,2} = T(w_{c,t+1} N_{c,t+1} + w_{h,t+1} N_{h,t+1} + D_{t+1}) + q_{t+1}(1 - \delta_h)(H_{t,1} + H_{t,2}) + p_{l,t+1}(L_{t,1} + L_{t,2}) \tag{8}$$

The right part of (8) is the total income of retirees, while the left part is expenditures. The revenues accruing to the retirees have some differences with that of workers because the revenues consist of three parts: 1) the pension payments; 2) the wealth accumulated previously; and 3) inheritances. For the pension system, the retirees receive a share of the wage and profit revenues of all the households as explained above. They also have income from accumulated wealth in the form of houses and lands that they purchased while working.

Moreover, inheritance is an additional component of income of retirees. The inheritance is the wealth that the households didn't consume when they passed away, and is assumed to be transferred to the retirees. The inheritance is in the form of houses (and land) and assumed to be so for three reasons. First, housing is needed by every individual, including the retired. Second, houses cannot be fully consumed by their owners, even at the time of their death. Third, houses are bequeathed to the next generation. For example, during the study of the bequest decision of Australians, Ding (2012) stated that housing assets constitute the bulk of bequests.

In our model, inheritance comes from two sources: 1) the previous generation of retirees; and 2) the workers who do not survive to retirement. The market values of these two parts are $q_{t+1}(1-\delta_h)H_{t,2}+p_{l,t+1}L_{t,2}$ and $(1-\pi_{t+1})(q_{t+1}(1-\delta_h)H_{t,1}+p_{l,t+1}L_{t,1})$ respectively, where δ_h represents that rate of depreciation of houses. Meanwhile, the market value of house and land purchased during their working period is $\pi_{t+1}(q_{t+1}(1-\delta_h)H_{t,1}+p_{l,t+1}L_{t,1})$. Thus, the total market value of the housing wealth is obtained by adding these parts together and is represented as $q_{t+1}(1-\delta_h)(H_{t,1}+H_{t,2})+p_{l,t+1}(L_{t,1}+L_{t,2})$.

2.3. Firms

The other agent in the economy is the firms. We adapt the model in Liu, Wang, and Zha (2013) where firms are assumed to exist forever, and they have three important functions. The first is that they produce goods for consumption and housing, and this function will be illustrated in the section on technology. Furthermore, the firms invest in and maintain plant and equipment. Finally, firms maximize profits with the proceeds paid to households in the form of dividends.

Profit maximization

Following Liu, Wang, and Zha (2013), firms maximize long-run profits, denoted as follows:

$$Max \sum_t \beta_e^t \ln(D_t) \tag{9}$$

, where D_t represents the total profits of the firms in period t. In addition, the parameter β_e represents the time preference of firms. As is widely accepted in practice, the time preference of firms would be lower than that of the households, i.e. $\beta_e < \beta$, thus they invest (Iacoviello 2005, Liu, Wang, and Zha 2013, Iacoviello 2015).

A major modification from Liu, Wang, and Zha (2013) is that, in our model, the firms are assumed to be owned by all the households as a whole instead of a certain fraction of them[6].

[6] Although firms are owned by households, the aim function of firms shows a lower discount rate. We provide a microeconomic explanation to this setting. In this explanation, the whole economy is viewed as a big firm, and it acts according to our model. However, the whole economy consists of many small firms, and each of them is owned by a panel of households. The time that the panel can own this firm will be no more than the longevity of households (because only living people can be on the panel). If taking the uncertainty into consideration,

This modification will allow us to focus on the households' heterogeneity of age structures without adding the complication of income distribution. Without this modification, as shown in Liu, Wang, and Zha (2013), the entrepreneurs would be introduced as a distinct class from the employees.

Two more assumptions are added; namely: 1) more profit is always better; and 2) stable profit flow is preferred. The first is standard practice in such models, while the second follows from risk-aversion by households who own these firms (Sandmo 1971, Leland 1972, Oh, Rhodes, and Strong 2016). The log-function in (9) satisfies these assumptions.

Technologies

An often cited omission from previous research on the effect of aging population on housing prices is the supply side of the market (Swan 1995). We follow Iacoviello and Neri (2010) and treat the supply of housing as a separable production sector from the non-housing productions.

Non-housing production sector: The non-housing sector uses Cobb-Douglas technology of the form:

$$Y_t = A_{c,t} K_{c,t-1}^{\mu_c} N_{c,t}^{1-\mu_c} \tag{10}$$

The non-housing production is denoted by Y_t, and there are three types of variables involved in the production process: the aggregate capital of non-housing production sector, $K_{c,t-1}$; the labor employed, $N_{c,t}$; and the Total Factor Productivity (TFP) in the non-housing production sector, $A_{c,t}$.

Housing production sector: Similar to that of the non-housing production sector, housing production takes the form:

$$\mathrm{IH}_t = A_{h,t} K_{h,t-1}^{\mu_h} L_{e,t-1}^{\mu_l} N_{h,t}^{1-\mu_h-\mu_l} \tag{11}$$

The houses built by the housing production sector is denoted by IH_t. The construction of houses needs capital $K_{h,t-1}$, land (which is owned by the firms) $L_{e,t-1}$, and labor $N_{h,t}$ while TFP growth is denoted by $A_{h,t}$. Following the work of Iacoviello and Neri (2010), we assume the land is indispensable for the production of housing.

the existent time of a firm (owned by the same panel) will be strictly less than the longevity of households. Thus, the firm owners, although they are households, will be more impatient about the future of firms than their own lives. This explanation is consistent with the reality that the survival rates of firms are less than that of the households. Consequently, the whole economy as a big firm will have a lower discount rate of time than that of the households because it consists of these firms. Regarding the households, we suppose that they have full insurance on their wage and profits, thus their income will only relate with the performance of the whole economy.

Capital Accumulation

We denote the investments in non-housing and housing sectors in period t as $\mathrm{IK}_{c,t}$ and $\mathrm{IK}_{h,t}$ respectively. Following Liu, Wang, and Zha (2013), the capital accumulation processes for the non-housing and housing sectors are assumed to have the following specifications:

$$K_{c,t} = (1 - \delta_{\mathrm{kc}})K_{c,t-1} + \mathrm{IK}_{c,t} - \Phi_{c,t} \tag{12}$$

$$K_{h,t} = (1 - \delta_{\mathrm{kh}})K_{h,t-1} + \mathrm{IK}_{h,t} - \Phi_{h,t} \tag{13}$$

where parameters δ_{kc} and δ_{kh} represent the depreciation rates of capital in the non-housing and housing sectors respectively. In addition, variables $\Phi_{c,t}$ and $\Phi_{h,t}$ denote the adjustment cost during the accumulation[7].

Regarding the adjustment cost in investments, we follow the work of Iacoviello and Neri (2010) and Liu, Wang, and Zha (2013) and assume that they have the following specifications:

$$\Phi_{c,t} = \Phi_c\big(K_{c,t}, K_{c,t-1}\big) = \frac{\phi_{kc}}{2}\left(\frac{K_{c,t}}{K_{c,t-1}} - g_{KC,t}\right)^2 K_{c,t-1} \tag{14}$$

$$\Phi_{h,t} = \Phi_h\big(K_{h,t}, K_{h,t-1}\big) = \frac{\phi_{kh}}{2}\left(\frac{K_{h,t}}{K_{h,t-1}} - g_{KH,t}\right)^2 K_{h,t-1} \tag{15}$$

where ϕ_{kc} and ϕ_{kh} are the parameters that represent the specific frictions in adjusting the capital stocks in non-housing and housing sectors respectively. In addition, variables $g_{KC,t}$ and $g_{KH,t}$ denote the balanced capital growth rates on which the corresponding adjustment cost would be zero.

Budget Constraint

The budget constraint of the firms is as follows:

$$D_t + \frac{K_{c,t}}{A_{k,t}} + K_{h,t} + w_{c,t}N_{c,t} + w_{h,t}N_{h,t} + p_{l,t}L_{e,t} + \frac{\Phi_{c,t}}{A_{k,t}} + \Phi_{h,t} = Y_t + q_t\,\mathrm{IH}_t + \frac{1-\delta_{\mathrm{kc}}}{A_{k,t}}K_{c,t-1} + (1 - \delta_{\mathrm{kh}})K_{h,t-1} + p_{l,t}L_{e,t-1} \tag{16}$$

The right side of the (16) is the resources available to the firms while the left side is the payments for the above. Firms have revenues from: 1) selling non-housing and housing productions; 2) the balance of capital after depreciation, and 3) the market value of land owned

[7] In the long run, the adjustment cost is the accumulation of the adjustment costs in short runs. The adjustment cost of capital includes opportunity costs of underutilized capital, capital obsolete and transition costs among activities (de Córdoba et al. 2006). These costs that happen in the short run will not vanish in the long run. Thus, the adjustment cost in the long run here refers to the sum of them.

by the firm. For the capital in the non-housing sector, as in Iacoviello and Neri (2010), the investment specific technology shock is introduced to the model and denoted by $A_{k,t}$.

The distribution of this wealth can be divided into four parts. Firstly, firms pay profits to households, as denoted by D_t in (16). Secondly, the capital stocks for next period are decided by the firms, which are denoted as $K_{c,t}$ and $K_{h,t}$. Along with this process, the capital adjustment costs are involved ($\Phi_{c,t}$ and $\Phi_{h,t}$). Thirdly, wages are paid to the workers. Lastly, the firms will decide on the amount of land owned in this period.

2.4. Equilibrium

There are four markets in our model, which are 1) the non-housing production market, 2) the housing market, 3) the land market, and 4) the labor market. The markets are perfectly competitive[8]. The equilibrium conditions of the first three markets are:

$$Y_t = C_t + \frac{\mathrm{IK}_{c,t}}{A_{k,t}} + \mathrm{IK}_{h,t} + \Phi_t \tag{17}$$

$$H_t = \mathrm{IH}_t + (1 - \delta_h)H_{t-1} \tag{18}$$

$$L_t = L_{h,t} + L_{e,t} \tag{19}$$

Where $C_t = C_{t,1} + C_{t,2}$, $H_t = H_{t,1} + H_{t,2}$, $L_{h,t} = L_{h,t,1} + L_{h,t,2}$

For the labor market, its equilibrium means the labor supply of households would be equal to the labor demand of firms. This equilibrium amount of working hours has been denoted by $N_{c,t}$ and $N_{h,t}$ in the previous discussion (or per worker, $n_{c,t}$ and $n_{h,t}$).

3. Assumptions and Calibration

3.1. Demographic Assumptions

In this research, we study both the effects of a decline in the fertility rate and an increase in longevity on housing prices. The levels of the decline and increase are arbitrarily chosen; however, the analysis on them will be sufficient to reach the qualitative conclusions relating to our research question.

The timeline chart of the assumed demographic changes is given as follows:

---

[8] The firms have profit because they own capitals.

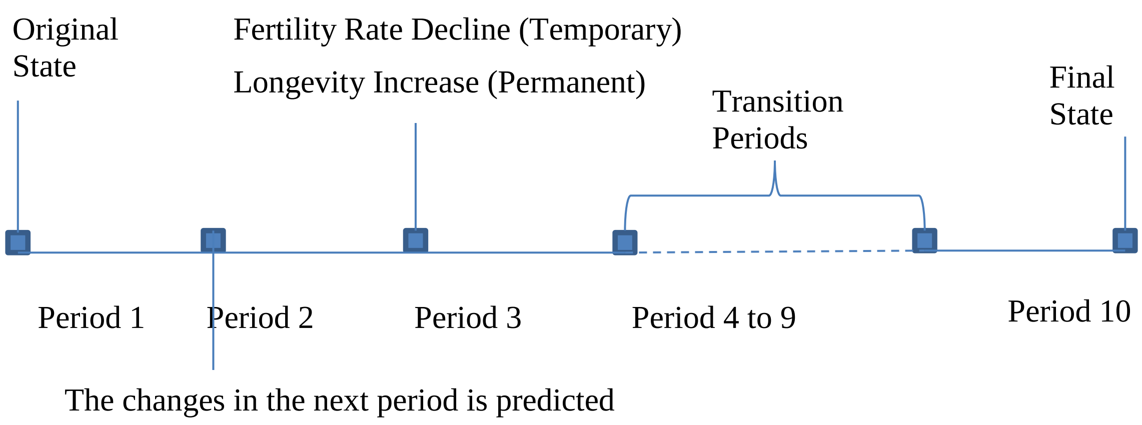

Figure 1. Timeline Chart of the Demographic Changes

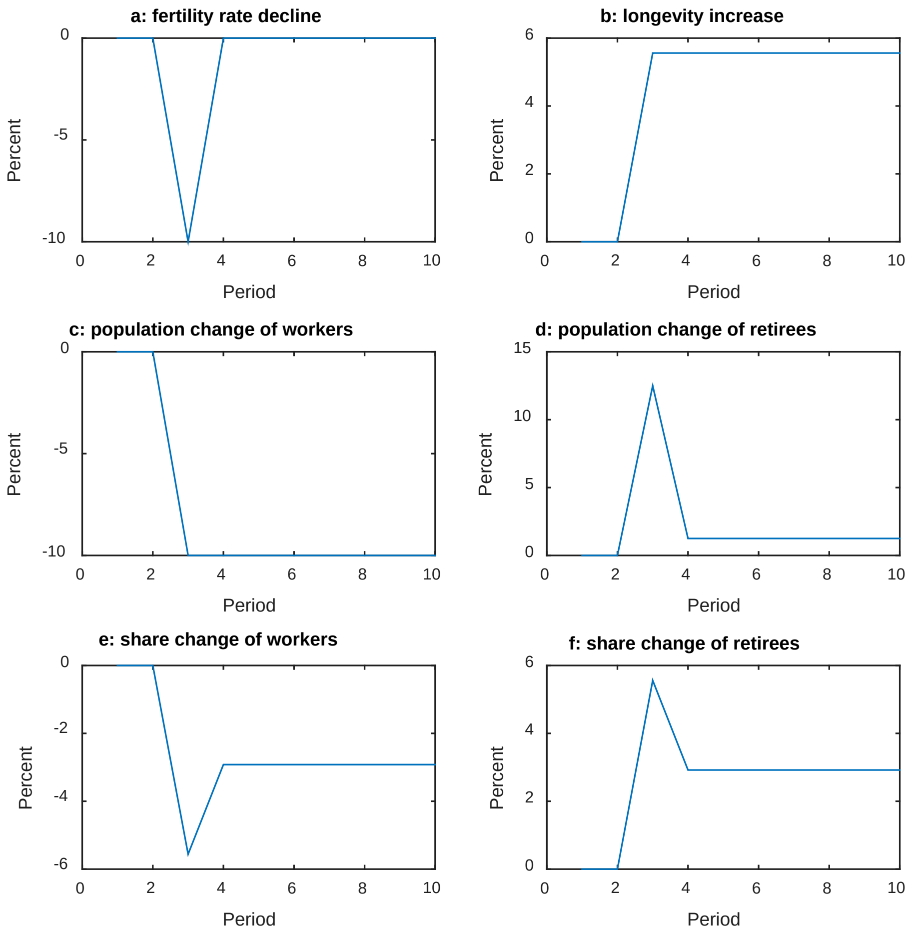

Figure 2. Overall Effect: Demography Changes

Specifically, at the original state, fertility rate (denoted by variable n) is set at the replacement level. The first shock is a fall in the fertility rate of 10 percent in period 3 (Figure 2a), and this fall is foreseen by the agents of the economy beforehand. In period 4, the fertility rate is returned to the replacement level as its original state and kept stable from then on (Figure 2a).

The second shock is an increase in longevity, which is presented in Figure 2b. In period 3, the longevity increases and reaches its peak. Thereafter, the longevity is assumed to be constant. Again, the longevity changes are known by the agents of the economy beforehand. The total longevity increase during this transition is 5.56 percent, indicating the corresponding survival rate is raised from 0.8 to 0.9.

By assuming both the changes begin at the same time within the range above, the overall demographic changes are shown in Figure 2. The population of workers has declined (Figure 2c), while the population of retirees has a sharp rise from period 2 to 3, and a fall follows from period 3 to 4 before reaching a stable value in period 4 (Figure 2d). During this process, the proportion of retirees rises from period 2 to 3. After that, it declines and then stays the same (Figure 2f).

3.2. Parameter Calibration

To present the result numerically, the parameters have to be calibrated. In this paper, we calibrate the parameters to fit the case of China. As illustrated above, the case of China will be a typical example when examining the effects of ageing on housing prices.

There are 16 parameters in our model. In Table 1, their calibrated values are listed, as well as the sources of the calibration.

The time preference of households β is calibrated to match China's average real interest rates during 1980–2015[9]. According to World Development Indicator of the World Bank[10], this rate, rounded to two decimal places, is 2 percent, which indicates the annual discount factor of 0.98, and for a period of thirty years, we set $\beta = 0.55$. Besides, following Iacoviello and Neri (2010) and Ng (2015), the annual discount rate of firms is set as 0.958 (=0.98/1.0232), lower than that of the households. In thirty years, the corresponding discount rate of firms β_e is 0.275.

There are three depreciation rates in our model. For the two types of capital in the production sectors, their annual depreciation rates are about 10 percent (Iacoviello and Neri 2010, Ng 2015). In thirty years, this depreciation rate implies that the capitals are fully depreciated. Thus, the corresponding parameters δ_{kc} and δ_{kh} are calibrated as 1. The depreciation rate of houses

---

[9] Follow Blackburn and Cipriani (2002) and Muto, Oda, and Sudo (2016), the survival rates in the utility function do not influence the calibration methods of the parameter of time preference.

[10] World Development Indicator of the World Bank: Real interest rate.
Website: http://data.worldbank.org/indicator/FR.INR.RINR. Date of Access: 21/Apr/2017

is calibrated according to Iacoviello and Neri (2010) as 4 percent annually[11], indicating 70 percent depreciation in 30 years.

The parameters denoting income shares and weight of utility are calibrated according to Ng (2015) and Liu, Wang, and Zha (2013). Their values are listed in Table 1: the capital share in housing sector (μ_h) is set at 0.24[12]. The capital adjustment cost in the non-housing and housing sector is calibrated according to Ng (2015), and the values are reported in Table 1. Among all the parameters, the value of pension share T of China has not been found in similar studies. Here, we use the case of America as an alternative, and the value is calibrated according to the work of Iacoviello and Pavan (2013).

Table 1—Parameter Calibration of the Model

Description	Symbols	Values	Sources
Time preference of households	β	0.55	See text
Time preference of firms	β_e	0.275	See text
Capital share (non-housing sector)	μ_c	0.46	Ng (2015)
Capital share (housing sector)	μ_h	0.24	See text
Land share (housing sector)	μ_l	0.10	Ng (2015)
Adjustment Friction (non-housing sector)	ϕ_{kc}	11	Ng (2015)
Adjustment Friction (housing sector)	ϕ_{kh}	10	Ng (2015)
Capital depreciation (non-housing sector)	δ_{kc}	1	Ng (2015)
Capital depreciation (housing sector)	δ_{kh}	1	Ng (2015)
House depreciation	δ_h	0.7	Iacoviello and Neri (2010)
Weight on housing in utility	j_h	0.12	Ng (2015)
Weight on land in utility	j_l	0.045	Liu, Wang, and Zha (2013)
Dis-preference on Labour supply	τ	1	Ng (2015)
See text	η	0.5	Ng (2015)
See text	ξ	1	Ng (2015)
Pension share	T	0.4	Iacoviello and Pavan (2013)

4. Long-Term Projection

A decline in fertility rate and an increase in longevity are both important underlying forces for aging of the population. We investigate the long-term effects of these two separately at first, and then combine them together to assess the aggregate changes. More concretely, since the

[11] We did not use this parameter value in Ng (2015) because that value indicates the houses will close to be fully depreciated in 30 years, and this is not true in the reality. Thus, we use the parameter value in Iacoviello and Neri (2010) instead.

[12] Comparing with Iacoviello and Neri (2010) and Ng (2015), the intermediate input has been omitted here, and the share of this input is added to the capital.

long run effect could be reflected by trend or steady state changes, we are here looking for the specific values of these changes.

4.1. Fertility Rate Decline

The fertility rate decline will cause trends in the values of variables (see Appendix B). Among them, the trend growth rates of house and land prices have the following closed form solutions (the derivations are provided in Appendix B):

$$g_{q,t} = \frac{1-\mu_h}{1-\mu_c}\gamma_{\mathrm{ac,t}} - \gamma_{\mathrm{ah,t}} + \frac{\mu_c(1-\mu_h)}{1-\mu_c}\gamma_{\mathrm{ak,t}} + \mu_l\left(\gamma_{N,t} - \gamma_{L,t}\right) \tag{20}$$

$$g_{pl,t} = \frac{1}{1-\mu_c}\gamma_{ac,t} + \frac{\mu_c}{1-\mu_c}\gamma_{ak,t} + (\gamma_{N,t} - \gamma_{L,t}) \tag{21}$$

The meanings of the variables are listed in Table 2:

Table 2—Exogenous Variables in Trends

γ_N	Growth rate of worker population
γ_L	Growth rate of residential land area
γ_{ac}	Growth rate of TFP in non-housing sector
γ_{ah}	Growth rate of TFP in housing sector
γ_{ak}	Growth rate of investment specific technology

Specifically, the trend caused by fertility rate changes is reflected by the following equations, which neglect other exogenous variables from equations (20) and (21).

$$g_{q,t} = \mu_l\gamma_{N,t} \tag{22}$$

$$g_{pl,t} = \gamma_{N,t} \tag{23}$$

The only parameter in equation (22) is μ_l, which represents the share of land in house construction. This parameter, as assumed in the previous section, has the range of $0 < \mu_l < 1$. Therefore, the effect of adjusting γ_N would be bigger on the land price than that of the house.

The fertility rate n_t is not shown in these formulas directly; however, the variables γ_N and n_t have the relationship $n_t = \exp(\gamma_{N,t})$. Thus, when $\gamma_{N,t} = 0$, the fertility rate is on replacement level, and worker population stays stable. A negative γ_N implies a decline in fertility rate, and so the worker population contracts. Accordingly, the proportion of retirees would increase commensurately.

In this circumstance, the replacement level of fertility rate is essential for the price trends. If the fertility rate remains higher than the replacement level, then the prices would rise as a result.

In contrast, if the fertility rate stays lower than the replacement level, then the price falls. These effects from a fall in the fertility rate end with a return to its replacement level.

The following result derived from (22) and (23) would sum up the discussion above:

Result 1: *A decline in the fertility rate lowers the house and land prices in the long run. Moreover, when* $0 < \mu_l < 1$*, the effect on the land price would be bigger than that on the house.*

The mechanisms from fertility rate to prices is presented in Figure 3.

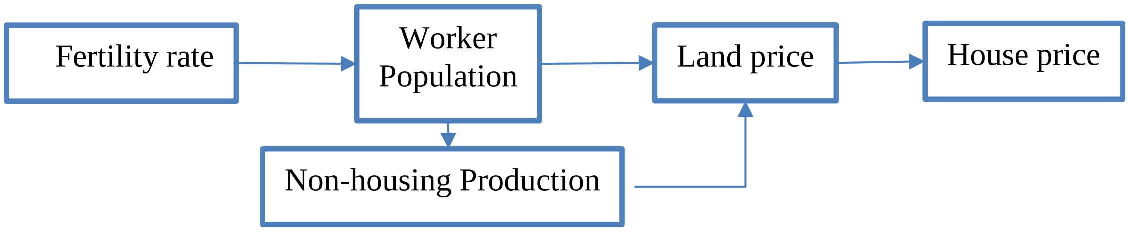

Figure 3. Causal Chain from Fertility Rate to Prices

In the first step, the fertility rate decline would lead to a contraction in the population of workers which then affects land and house prices. We will take the land price as an example to illustrate this effect. First, when worker population declines, the land price would fall because of reduced demand. Second, the decline in the population of workers will lead to a decline in non-housing production. Note that the land price is a relative price and is denoted by the units of non-housing production. Thus, when the output declines, the land price would decline accordingly.

What about the price of houses? First note that house construction is endogenous in our model. Thus, the changes in population and productions may not influence the house price because of the flexibility of supply. However, land is an input in house construction and thus a fall in the price of land will be transmitted to the house price. As shown in (22), the parameter μ_l denotes the share of land, which is also the share of price transmission. Since $0 < \mu_l < 1$, the decline in house price is less than that of land.

Finally, the mechanisms illustrated above would indicate the price trend decline would happen along with the shrinkage of worker population, and the value of the decline can be calculated from equations (22) and (23). Substituting in the parameters from the previous section, the trends of house/land price are shown in Figure 4, where their values decline by 1 percent and 10 percent, respectively.

Figure 4. Price Changes: Long-Term Effect of Fertility Rate Decline

Notes: The corresponding demographic changes are shown in Figure 2a.

4.2. Longevity Increase

Different from the fertility rate, the effect of an increase in longevity has no impact on price trends as shown by equations (20) and (21), thus a change in longevity affects the steady state prices instead.

There are three ways of calculating the effect on steady states: derive the analytical solution, calculate the partial derivative from the above, or use numerical simulations when the above is not practical. The first two have proved difficult thus numerical simulations have been employed. By using parameter values and the changes in longevity, this method permits the calculation of steady values for the endogenous variable.

Here, by using the calibrated parameters and assumptions in the previous section, the numerical solution is presented to show the long-term effect of an increase in longevity. The robustness of the results is tested as explained in Appendix E. The results of the numerical solution are shown in Figure 5. The effects of an increase in the survival rate from 0.8 to 0.9 are calculated and plotted in this figure: it shows a positive elasticity of price to an increase in longevity.

This result indicates that, for plausible calibrations of the structural parameters of the economy, the long-term effects of an increase in longevity on the prices is positive. In addition, as shown in Figure 5, the rise in the price of land is higher than that of the house. Thus, for the given parameters, the long-term effect of an increase in longevity on land price is larger than that on the house.

Figure 5. Price Changes: Long-Term Effects of Longevity Increase

Notes: the survival rate is incremented by 0.1 to derive the corresponding steady state prices for house and land separately.

Result 2: *the long-run effect of an increase in longevity is positive on both house and land prices. Moreover, its effect on the land price is bigger than that on the house price.*

4.3. Overall

We next investigate combined effects of a fall in the fertility rate and an increase in longevity. More concretely, when using the assumed demographic changes in the previous section, their overall effects on the prices are shown in Table 3.

In the long run, the house price rises by 0.07 percent. This rise indicates that, conditional on the values of the assumed parameters, the positive effect on house price from an increase in longevity is greater than the negative effect from the fall in fertility. In contrast, the land price declines by 2.39 percent. Thus, the negative effect of the simulated fall in fertility outweighs the positive effect from the simulated increase in longevity.

Table 3—Prices Changes: Long-Term Effect of Aging Population (in percent)

	effect of fertility rate decline	**effect of longevity increase**	**overall effect**
house price	-1.00	1.07	0.07
land price	-10.00	7.61	-2.39

Note: the numbers come from Figure 4 and 5, and the overall effect is the sum of the two. The justification of this method is shown in Appendix D.

These offsetting effects can be deduced from Result 1 and Result 2. Recall from Result 1, the long-term effect of fertility rate decline on prices is negative; however, in Result 2, the long-term effect of longevity increase on prices is positive. Because these two effects move in opposite directions, the net effect depends on which of the above is overwhelming. Furthermore, if we manipulate the extent of fertility rate decline, these opposing effects could be studied more closely for specific parameter values.

As shown in Figure 6, the Cartesian plane consists of the fertility rate decline (y-axis) and longevity increase (x-axis). The line pictured in the plane is a set of points, denoting combinations of fertility rate decline and longevity increase. For house price, on this line, the upward effect from longevity increase equals the downward effect from fertility rate decline. That is, the net effect of an aging population on house prices is zero on this line.

Moreover, the line pictured in Figure 6 is also a dividing line. It divides the quadrant of the plane into two parts. On the right side of the line, the house price will rise in the long run, because of the stronger effect from an increase in longevity. However, on the left side of the line, the house price will decline due to the weight of fertility rate decline.

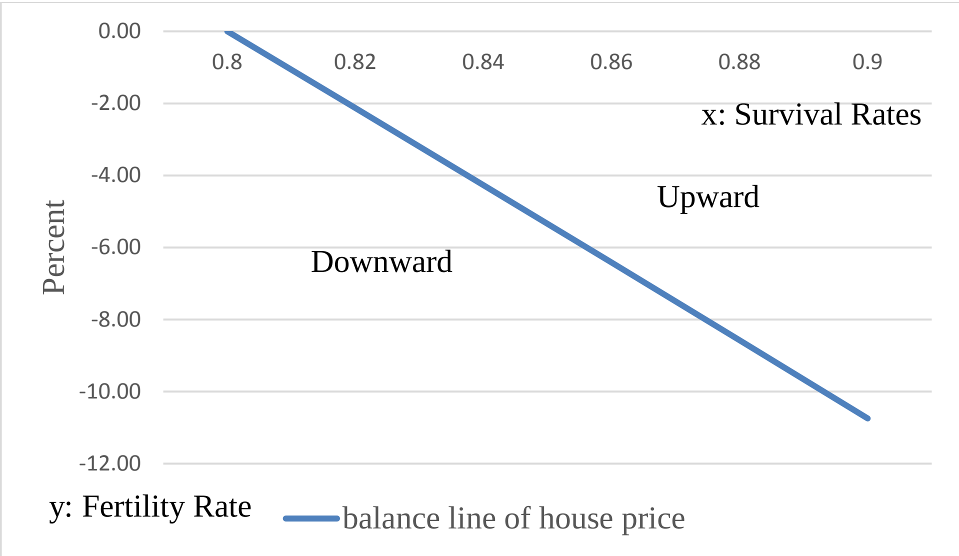

Figure 6. Plane of Aging Population and Balance Line of House Price

Notes: The effect of longevity increase on house price comes from Figure 5, and the corresponding fertility rate decline is calculated according to (22).

For land price, a similar plane is shown in Figure 7, together with a line, on which the long-term effect of aging population is zero. Similar to that of the house price, the quadrant is also divided into two parts by this balance line, and its right / left side would be the upward / downward area for the land price.

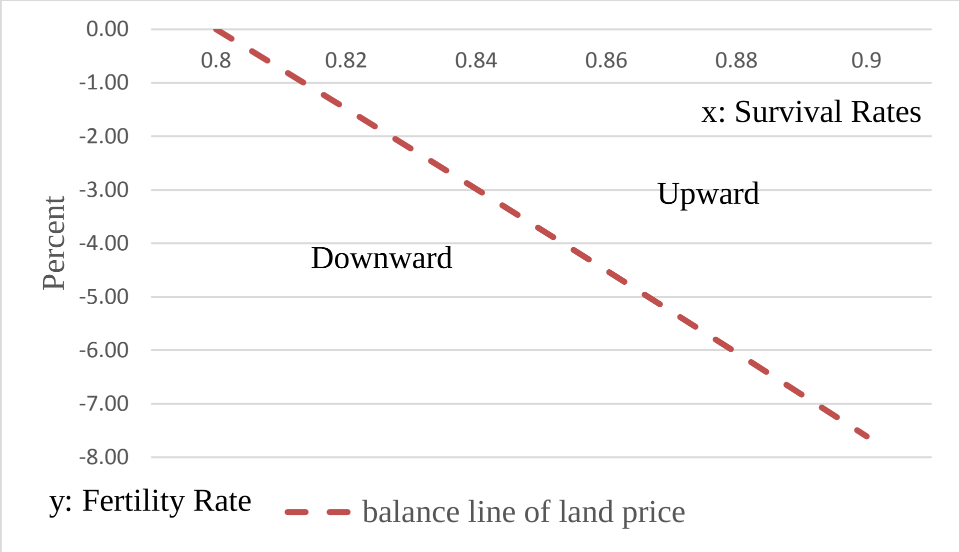

Figure 7. Plane of Aging Population and Balance Line of Land Price

Notes: The effect of longevity increase on land price comes from Figure 5, and the corresponding fertility rate decline is calculated according to (23).

Lastly, notice that the balance lines of house and land prices do not coincide with each other, which is shown in Figure 8. Thus, the zero long-run effect cannot be achieved for both the prices simultaneously. Moreover, the quadrant is divided into three areas by two different lines. Because the areas preserved the properties as in Figure 6 and 7, the right most area is where both land and house prices rise while left-most region is where both prices fall. The middle area lends room for the two prices to diverge: specifically where the price of land drops while that for houses increase. The parameters assumed in the simulations are those for this specific region as shown in Table 3.

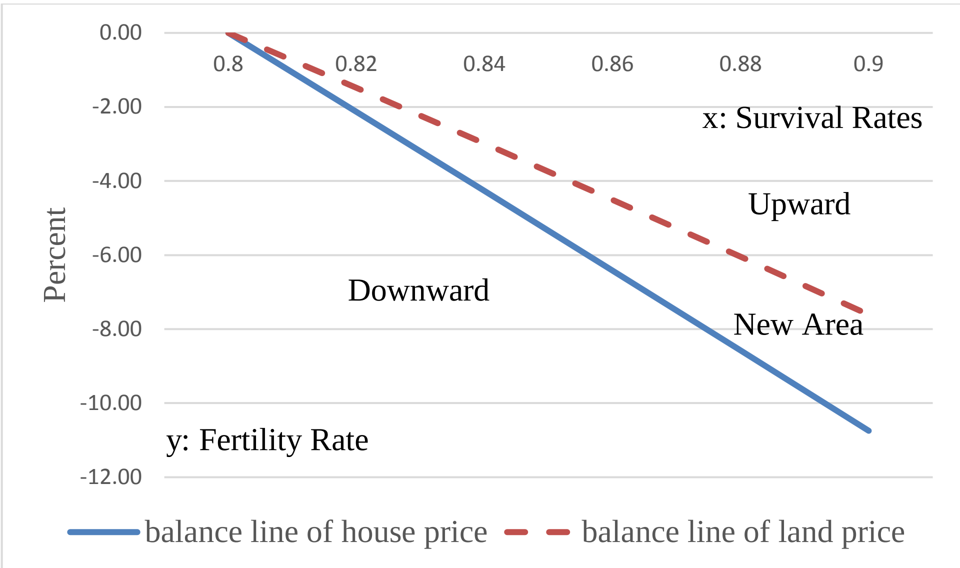

Figure 8. Plane of Aging Population and Balance Line of Land Price

Notes: source from Figure 6 and 7.

To sum up, the discussion above could be highlighted by the following result:

Result 3: *the aging population could have zero effect on either house or land price in the long run.*

5. Simulation

In this section, we will simulate the economic effects of the demographic changes presented in Figure 2. Specifically, the simulations provide the dynamics in terms of household utility and housing prices during the transition periods. From these dynamics, we explain how the demographic changes and economic fluctuations are interconnected.

5.1. Households' Utility

Recall that household utility is determined by the non-housing consumption and house and land owned per capita. If we divide the households into workers and retirees, their per capita utility is shown in Figure 9. The per capita consumption of workers is greater from that of period 1 in periods 2 to 4 (see Figure 9a) when workers own more houses and land both in the long run and short run (see Figure 9c, e). In contrast, the consumption of retirees decreases from period 2 and reaches its lowest point in period 3 (see Figure 9b). Although consumption rises from period 3 to 4, the long run per capita consumption is still below that of period 1 (the original state). Similar situations also happen in both the house and land owned by retirees on a per capita basis (see Figure 9d, f).

The results in Figure 9 show that, relative to period 1 (the original state), the utility of workers is higher; however, the retirees' utility is worse than their original state. We next discuss the retirees' utility since workers' behavior is influenced by the expectations of their retired life.

For retirees, in the long run, the decrease in utility is due to the increase in longevity which in turn raises the population of retirees. Compared with the case of no increase in longevity, the population of retirees rises by 12.5 percent. When the population rises, to keep the same utility level as before, it requires a rise of total income. However, the simulation indicates that in the long-run total income of retirees will rise by less than 1 percent thus per capita income and the utility of retirees fall.

During the transition period, the significant utility loss of retirees is due to the decline in the fertility rate. This decline leads to fewer workers and a concomitant rise in the proportion of retirees. Especially in period 3, the proportion of retirees would reach its peak (see Figure 2f), meaning that the pension income from workers would be shared by more retirees, leading to the most drastic losses in utility. After that, this utility will rise along with a decline in the proportion of retirees (see Figure 2f), and move towards its long run level.

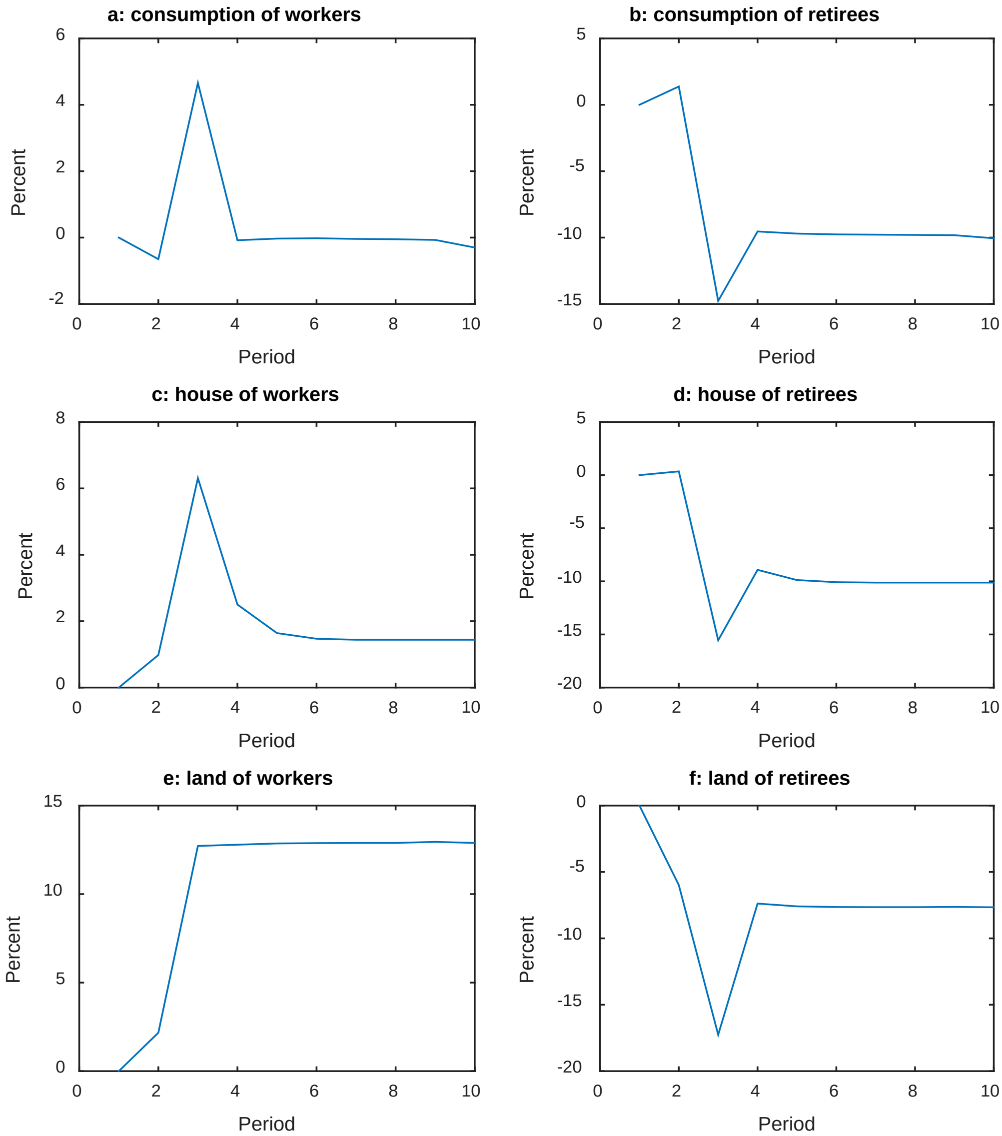

Figure 9. The Utility Dynamics (per capita)

Note: the corresponding demographic changes are presented in Figure 2, both the fertility rate decline and longevity increase happen from period 2 to 3.

For the workers, their behavior would be influenced by their expectations about the living standard of their retirement. In the long-run, the higher utility is due to the increased savings of workers. Recall that the utility loss would happen in households' retiree period, the workers with rational expectation would increase their savings to fund their retirement so as to maximize their life-span utility. Specifically, the households, having perfect foresight of their future utility loss, would purchase more house and land when they are workers, and sell them when they retire. This purchase behavior against the future utility loss raises the house and land owned by workers and with it their utility.

During the transition period, the better utility of workers comes from the fertility rate decline. Because of the fertility rate decline, the worker population decreases accordingly. However, the total income of workers will not decrease to the same extent. The key driver here is the wealth stock of the firms, including $K_{c,t-1}$, $K_{h,t-1}$ and $L_{e,t-1}$ (see equation (10) and (11)). When worker population declines, the adjustments of the wealth are not immediate, indicating higher per worker output in goods and houses. According to equation (16), higher per worker output and wealth stock translates into the income of workers through wages and profits. Thus, higher per capita income would accrue to workers, and their utility increases. Along with the adjustment of the wealth stock, workers' utility will tend to converge towards its long run level.

Based on the discussion above, we note that an aging population would increase inequality across generations. The utility of workers increases whereas that of the retirees falls. Especially in the transition periods, this growth in inequality would be significant. In these periods, as has been illustrated above, the significant aging population and the wealth stock adjustment would drive a wedge between the two generations.

The results of this section can be concluded as follows:

Result 4: *the aging population could cause utility inequality across generations. Specifically, the utility of workers would rise while that of retirees would fall.*

5.2. Prices

The price dynamics following the demographic changes presented in Figure 2 is explained next. The house price rises from period 1 to 2, and then declines from period 2 to 4 (see Figure 10). Similarly, the land price also rises from period 1 to 2, and declines from period 2 to 4 (see Figure 11).

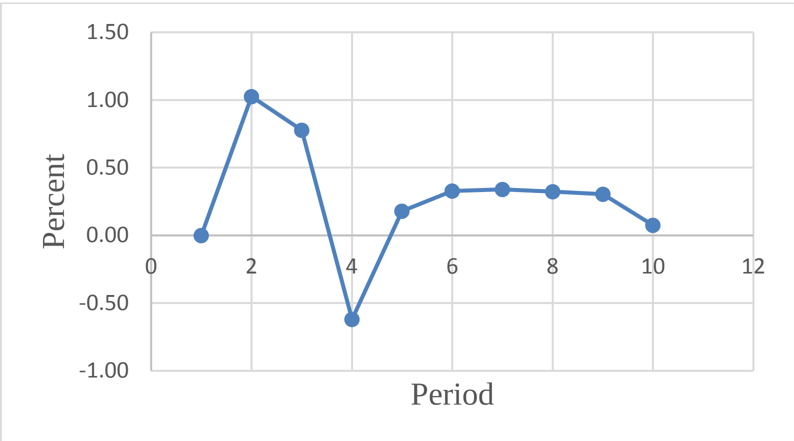

Figure 10. the Overall Dynamics of House Price

Note: the corresponding demographic changes are presented in Figure 2, both the fertility rate decline and longevity increase happen from period 2 to 3. After that, the fertility rate moves back to the replacement level, while the longevity stays stable.

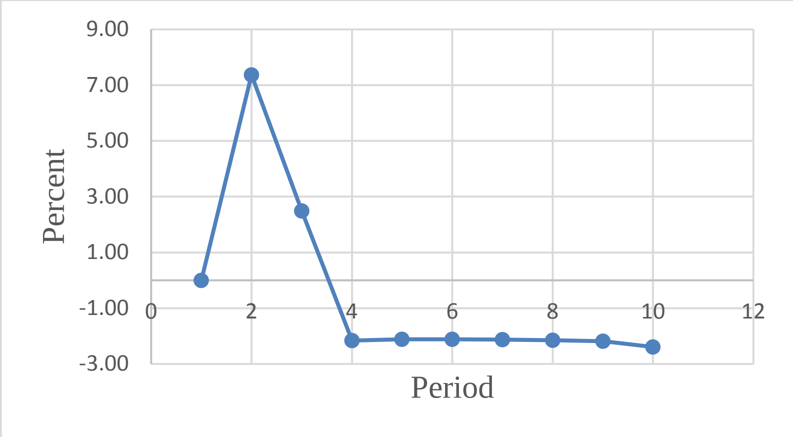

Figure 11. the Overall Dynamics of Land Price

Note: the corresponding demographic changes are presented in Figure 2, both the fertility rate decline and longevity increase happen from period 2 to 3. After that, the fertility rate moves back to the replacement level, while the longevity stays stable.

As shown in Figure 10 and 11, the price dynamics produce a turning point at period 2. Before this point, both prices are rising; however, after this point, both the prices fall. What is the mechanism that connects the demographic change to the price movements?

To explain these dynamics, the Euler Equations of house and land are listed:

$$\tilde{q}_t = \beta E_t(\exp(g_{q,1+t})\,\tilde{q}_{1+t}(1-\delta_h)\,\frac{c_{1,t}}{c_{2,1+t}}) + j_h\,\frac{c_{1,t}}{\tilde{h}_{1,t}} \tag{24}$$

$$\tilde{p}_{l,t} = \beta E_t\left(\exp(g_{\mathrm{pl},1+t})\,\tilde{p}_{l,1+t}\,\frac{c_{1,t}}{c_{2,1+t}}\right) + j_l\,\frac{c_{1,t}}{\tilde{l}_{1,t}} \tag{25}$$

, where the variables with a tilde denote their de-trended values.

According to equations (24) and (25), the price dynamics are explained by the utility changes of households, which are presented in Figure 9. More concretely, we will emphasize that the utility changes are driving prices to rise before the turning point.

As shown in Figure 9b, the per capita consumption of retirees would decline significantly from period 2 to 3. Given the assumption of perfect foresight of households, their behaviors would change accordingly and beforehand. The expected decline in consumption is denoted by $E_t(c_{2,1+t})$. According to (24) and (25), the decline of $E_t(c_{2,1+t})$ will raise the house and land prices ($\tilde{q}_t$ and $\tilde{p}_{l,t}$) beforehand. Therefore, if the expected consumption decline would happen from period 2 to 3, then the price rises would precede period 2. This mechanism explains the price rise before the turning point.

Intuitively, as explained in the household utility section, expectations are the driver of this price rise. When consumption decline is anticipated, the households raise their savings by purchasing more housing and land when they are workers (see Figure 9c, e). It is these purchases which finally lead to the price rise.

After the turning point at period 2, both the prices start decreasing. This decrease is largely driven by the drag from the drop in the fertility rate, which has been explained in the previous section. Here, two issues may be noted.

First, the decline lasts for two periods, from period 2 to 4. This length is different from the fertility rate decline, which only happens from period 2 to 3. Why? The key driver of this phenomenon is the character of stock of wealth, which prevents the price from declining sharply. As discussed previously, the stock of wealth would not adjust immediately against a decline in the population of workers. Therefore, with these wealth stocks, the per capita income of workers in period 3 rises. More house and land would be purchased by using these incomes, and these purchases are supporting the price of these assets. In sum, the stock of wealth is playing role as a buffer for the prices.

Second, during this decline, the house price overshoots before settling to its long run level (see Figure 10, period 3 to 4). Why? The mechanism will be illustrated based on equation (24), in which two reasons are provided. The consumption decline is the main reason for the price decline. As shown in equation (24), the per capita worker consumption $c_{1,t}$ is positively related to the house price. When the consumption declines, the house price would also decline. In our simulation, the consumption decline is shown in Figure 9a from period 3 to 4. Simultaneously, the house price would decline based on equation (24).

The further decline of house price is explained from changes in the house stock. To illustrate, we should notice that the house stock $\tilde{h}_{1,t}$ is not adjusting simultaneously with the consumption (see Figure 9c). Because of the depreciation character of house stock, the adjustment would not be immediate, but take place gradually. Based on (24), a higher $\tilde{h}_{1,t}$ indicates a lower house price. Thus, in period 4, because $\tilde{h}_{1,t}$ is higher than its long run level, the price would have a further fall. This mechanism enriches the discussion about overshooting phenomena that started with Dornbusch (1976).

In sum, connections between demographic changes and price dynamics are presented above, and these connections could be summarized by the following result:

Result 5: *aging population could cause a turning point in housing prices.*

Although this result is based on the specific demographic changes presented in the previous section, it corresponds well with reality and is theoretically grounded as shown in Figure 8– an issue under current investigation.

6. Conclusion

The effect of an aging population on housing prices remains an unresolved issue with mixed empirical findings. This debate began with Mankiw and Weil (1989) and has been ongoing for the past two decades. Here we build an overlapping generations model where aging results from a combination of a decline in fertility and an increase in longevity to simulate effects of the above on housing prices. Our simulations for plausible parameter values give mixed results, depending on which of the above-mentioned overwhelms in terms of the impact of aging on house (and land) prices.

This paper has tackled the impact of aging on housing prices from a theoretical perspective. Our result provides reasons for the mixed results shown through empirical studies. It shows that a decline in the fertility rate depresses housing prices while an increase in longevity does the opposite – the net effect of a simultaneous change in the above two factors depends on which effect is overwhelming. Note that aging is caused by a combination of a fall in fertility with an increase in longevity, but the exact magnitude of the afore-mentioned differs across contexts. More concretely, in the long run, housing prices will decline if the drag of a fall in fertility outweighs the push from an increase in longevity, and vice versa. This result may explain the mixed findings from existing empirical research.

If indeed true, then what does the above imply for the price trends for housing in the future? Our simulations predict a sharp turning point in housing prices in a generation when the upward effect is overwhelmed by the current declines in the rate of fertility. This turning point is revealed by the simulations and explained theoretically. Before this turning point, the prices would rise in anticipation of a longer life span by existing population of workers and thus the need for more wealth to fund retirement. Nevertheless, the decline afterwards is due to the decrease of worker population caused by the lower fertility rate of the current period. Thus, these price declines would continue through the process of aging population caused by the fertility rate decline.

Furthermore, household behavior has also been discussed in this paper, and the analysis showed that the demographic changes would lead to welfare inequality across generations. More concretely, the utility of workers will be higher; however, that of retirees will be lower (see Figure 9). This inequality would be most significant during the transition periods.

In sum, this paper contributes to the literature on the effects of aging – a phenomenon that is spreading across the world – which will affect housing prices but the exact magnitudes will differ by context and change with time. The next challenge, which is part of ongoing research, is to test these predictions using data on aging societies. These findings will deepen our understanding of the relationship between aging and the economy.

Appendix A. per capita Version Model Equations

The per capita version model equations are transformed from the corresponding aggregate equations, and the derivation is based on Eq. (1), (2), (3) in the demography section.

Households: For each generation, the budget constraint is shown by variables in lower case denoting per capita amount of that generation.

$$c_{t,1} + q_t h_{t,1} + p_{l,t} l_{t,1} = (1-T)(w_{c,t} n_{c,t} + w_{h,t} n_{h,t} + d_t) \tag{A1}$$

$$c_{1+t,2} + q_{1+t} h_{1+t,2} + p_{l,1+t} l_{1+t,2} = q_{1+t}(1-\delta_h)(\frac{h_{t,1}}{\pi_{1+t}} + \frac{\pi_t h_{t,2}}{\pi_{1+t} n_t}) + T(d_{1+t} + n_{c,1+t} w_{c,1+t} + n_{h,1+t} w_{h,1+t}) \frac{n_{1+t}}{\pi_{1+t}} + p_{l,1+t}(\frac{l_{h,t,1}}{\pi_{1+t}} + \frac{\pi_t l_{t,2}}{\pi_{1+t} n_t}) \tag{A2}$$

Firms: For firms, the variables in lower case represent the amount per worker.

$$y_t = A_{c,t}\,(\frac{k_{c,t-1}}{n_t})^{\mu_c}\, n_{c,t}^{1-\mu_c} \tag{A3}$$

$$\mathrm{ih}_t = A_{h,t}\,(\frac{k_{h,t-1}}{n_t})^{\mu_h}\,(\frac{l_{e,t-1}}{n_t})^{\mu_l}\, n_{h,t}^{1-\mu_h-\mu_l} \tag{A4}$$

$$k_{c,t} = (1-\delta_{\mathrm{kc}})\frac{k_{c,t-1}}{n_t} + \mathrm{ik}_{c,t} - \phi_{c,t} \tag{A5}$$

$$k_{h,t} = (1-\delta_{\mathrm{kh}})\frac{k_{h,t-1}}{n_t} + \mathrm{ik}_{h,t} - \phi_{h,t} \tag{A6}$$

$$\phi_{c,t} = \phi_c(k_{c,t}, k_{c,t-1}) = \frac{\phi_{kc}}{2}(\frac{k_{c,t} n_t}{k_{c,t-1}} - G_{kc,t})^2 \frac{k_{c,t-1}}{n_t} \tag{A7}$$

$$\phi_{h,t} = \phi_h(k_{h,t}, k_{h,t-1}) = \frac{\phi_{kh}}{2}(\frac{k_{h,t} n_t}{k_{h,t-1}} - G_{kh,t})^2 \frac{k_{h,t-1}}{n_t} \tag{A8}$$

$$d_t + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + w_{c,t} n_{c,t} + w_{h,t} n_{h,t} + p_{l,t} l_{e,t} + \frac{\phi_{c,t}}{A_{k,t}} + \phi_{h,t} = y_t + q_t\,\mathrm{ih}_t + \frac{1-\delta_{\mathrm{kc}}}{A_{k,t}}\frac{k_{c,t-1}}{n_t} + (1-\delta_{\mathrm{kh}})\frac{k_{h,t-1}}{n_t} + p_{l,t}\frac{l_{e,t-1}}{n_t} \tag{A9}$$

Equilibrium:

$$y_t = c_t + \frac{\mathrm{ik}_{c,t}}{A_{k,t}} + \mathrm{ik}_{h,t} - \phi_t \tag{A10}$$

$$h_t = \mathrm{ih}_t + (1-\delta_h)\frac{h_{t-1}}{n_t} \tag{A11}$$

$$l_t = l_{h,t} + l_{e,t} \tag{A12}$$

where $c_t = c_{t,1} + \frac{\pi_t}{n_t} c_{t,2}$, $h_t = h_{t,1} + \frac{\pi_t}{n_t} h_{t,2}$, $l_{h,t} = l_{h,t,1} + \frac{\pi_t}{n_t} l_{h,t,2}$.

Appendix B. Trends

The Method of Calculating Trends

When we say a variable has trend, we mean it satisfies the following equation:

$$\tilde{X}_t = \frac{X_t}{G_t} \tag{B1}$$

Where $\tilde{X}_t$ and G_t are denoting the steady state and trend of the variable X_t at period *t*.

In an equation consists of multiple variables, their trends could be related with each other. In this study, these relationships will be shown by using the growth rates of trends. For computational convenience, we define the growth rates of variables by their log-difference, i.e.:

$$g_{x,t} = \ln(X_t) - \ln(X_{t-1}) \tag{B2}$$

And thus

$$\frac{X_t}{X_{t-1}} = \exp(g_{x,t}) \tag{B3}$$

When the growth rate is small, $g_{x,t}$ is approximately equal to $\frac{X_t - X_{t-1}}{X_t}$.

In addition, the calculations of trend relationships are based on the following two assumptions:

1) The de-trended variables (steady states) do not have the same trend.
2) Trends are determined by exogenous variables.

Based on the above assumptions, we discuss the relationship of trends in two kinds of equations: linear and Cobb Douglas form. The linear equation has the form as follows:

$$X_t = aX_{1,t} + bX_{2,t} \tag{B4}$$

The parameters a and b are constant or exogenous variables without trend. Supposing the trend of X_t is G_t, the above equation can be derived as follows:

$$\frac{X_t}{G_t} = a\frac{X_{1,t}}{G_{1,t}}\frac{G_{1,t}}{G_t} + b\frac{X_{2,t}}{G_{2,t}}\frac{G_{2,t}}{G_t}$$

$$\tilde{X}_t = a\tilde{X}_{1,t}\frac{G_{1,t}}{G_t} + b\tilde{X}_{2,t}\frac{G_{2,t}}{G_t}$$

The $G_{1,t}$ and $G_{2,t}$ are trends of $X_{1,t}$ and $X_{2,t}$ respectively. We will prove that $G_{1,t} = G_{2,t} = G_t$. If not, suppose the trend growth rates are positive constants[13], i.e. $G_t = (\exp(g))^t > 0, G_{1,t} = (\exp(g_1))^t > 0, G_{2,t} = (\exp(g_2))^t > 0$, the equation can be rewritten as follows:

$$\tilde{X}_t = a\tilde{X}_{1,t}(\frac{\exp(g_1)}{\exp(g)})^t + b\tilde{X}_{2,t}(\frac{\exp(g_2)}{\exp(g)})^t$$

The inequality of trends indicates that the growth rates are not equal. If $g_i < g, i = 1, 2$, the steady states must have trend included to satisfy the equation and thus violate the assumption one. The situation is the same if $g_i > g, i = 1, 2$.

In case $g_1 > g$ and $g_2 < g$, the absolute value of $X_{1,t}$ will grow faster than that of X_t. To hold the equation, trend in steady states have to be included and thus violate the assumption (1) again. The proof of other situations are similar.

According to the second assumption, the trends are determined by exogenous variables, thus the relationship among the trends can be described by certain equations. In the linear case above, this relationship can be written as $G_{1,t} = G_{2,t} = G_t$. Because this equation holds for every period *t*, their growth rates should equal to each other, i.e.:

$$g_{1,t} = g_{2,t} = g_t \tag{B4'}$$

The Cobb-Douglas form equation that we are focusing is shown as follows:

$$X_t = cX_{1,t}^a X_{2,t-1}^b \tag{B5}$$

Divide both sides of the equation by G_t (the trend of X_t) and we derive the equation as follows:

$$\tilde{X}_t = \frac{X_t}{G_t} = c\frac{X_{1,t}^a X_{2,t-i}^b}{G_t} = c\left(\frac{X_{1,t}}{G_{1,t}}\right)^a \left(\frac{X_{2,t-1}}{G_{2,t}}\right)^b \frac{G_{1,t}^a G_{2,t}^b}{G_t} = c\tilde{X}_{1,t}^a \left(\frac{\tilde{X}_{2,t-1}}{\exp(g_{2,t})}\right)^b \frac{G_{1,t}^a G_{2,t}^b}{G_t}$$

With this derivation, we want to prove $G_t = G_{1,t}^a G_{2,t}^b$. If not, the term $\frac{G_{1,t}^a G_{2,t}^b}{G_t}$ will be more or less than 1. Suppose the growth rates of trends are constants, the value of this term will continue rising or diminishing. To hold the equation, the steady states will have trend involved and thus violate the second assumption.

Taking log-difference operation on both sides of $G_t = G_{1,t}^a G_{2,t}^b$, we have:

$$lnG_t - lnG_{t-1} = a\big(lnG_{1,t} - lnG_{1,t-1}\big) + b(lnG_{2,t} - lnG_{2,t-1})$$

According to Eq. (B2), the above equation can be written as:

$$g_t = a\, g_{1,t} + b\, g_{2,t} \tag{B5'}$$

In case that the equation is:

[13] Because the trends are determined, the growth rates have to be determined too.

$$X_t = X_{1,t} X_{2,t} \tag{B6}$$

The derivation is similar, and we have:

$$g_t = g_{1,t} + g_{2,t} \tag{B6'}$$

The Trends in the Model

The exogenous variables in our model has been listed in Table 2. Among them, we assume the longevity increase have no trend effect on other variables, and its effects are all captured by steady state changes. In addition, we assume that the per worker labour supplies, $n_{c,t}$ and $n_{h,t}$, do not have trends. The trends of other variables are calculated as follows.

The per worker land area is calculated as:

$$l_t = \frac{L_t}{N_{1,t}}$$

Denote the growth rates of worker population and land area by $\gamma_{N,t}$ and $\gamma_{L,t}$ respectively, the per worker land area growth rate can be calculated according to Eq. (B5') as:

$$g_{l,t} = \gamma_{L,t} - \gamma_{N,t} \tag{B7}$$

In case that the land area is fixed, we will assume that $g_{L,t} = 0$ and omit it in the equations. Here, we keep this variable in equations to check its relationships with the trends of other variables.

According to Eq. (A12) and (B4'), the variables $l_{e,t}$ have the same trend growth rate as l_t, i.e. $g_{l,t}$. Based on Eq. (A9) and (B4'), the terms $p_{l,t} l_{e,t}$ and $\frac{k_{c,t}}{A_{k,t}}$ have the same growth rate as the variable y_t, and this relationship can be described by:

$$g_{y,t} = g_{pl,t} + g_{l,t} = g_{kc,t} - \gamma_{ak,t} \tag{B8}$$

We have known $g_{l,t}$ (see Eq. (B7)), if we can describe $g_{y,t}$ using exogenous variables, the trend growth rate of land price $g_{pl,t}$ would be calculated according to the above equation.

Use Eq. (A3) and (B5'), $g_{y,t}$ can be calculated as:

$$g_{y,t} = \gamma_{ac,t} + \mu_c g_{kc,t} \tag{B9}$$

Use Eq. (B8) and (B9), the $g_{y,t}$ can be described as:

$$g_{y,t} = \frac{1}{1-\mu_c} \gamma_{ac,t} + \frac{\mu_c}{1-\mu_c} \gamma_{ak,t} \tag{B10}$$

Thus, according to Eq. (B8), the trend growth rate of land price is:

$$g_{pl,t} = \frac{1}{1-\mu_c} \gamma_{ac,t} + \frac{\mu_c}{1-\mu_c} \gamma_{ak,t} + (\gamma_{N,t} - \gamma_{L,t}) \tag{B11}$$

Next, we will derive the trend growth rate of house price. According to Eq. (A4) and (B5'), the trend growth rate of ih_t is:

$$g_{ih,t} = \gamma_{ah,t} + \mu_h g_{kh,t} + \mu_l g_{l,t} \tag{B12}$$

From Eq. (A9), (B4') and (B6'), we can get:

$$g_{y,t} = g_{kh,t} = g_{ih,t} + g_{q,t} \tag{B13}$$

Based on Eq. (B7), (B10), (B12) and (B13), the trend growth rates of house and house price are:

$$g_{ih,t} = \gamma_{\mathrm{ah,t}} + \frac{\mu_h}{1-\mu_c}\gamma_{\mathrm{ac,t}} + \frac{\mu_c\,\mu_h}{1-\mu_c}\gamma_{\mathrm{ak,t}} - \mu_l(\gamma_{N,t} + \gamma_{L,t}) \tag{B14}$$

$$g_{q,t} = \frac{1-\mu_h}{1-\mu_c}\gamma_{\mathrm{ac,t}} - \gamma_{\mathrm{ah,t}} + \frac{\mu_c(1-\mu_h)}{1-\mu_c}\gamma_{\mathrm{ak,t}} + \mu_l\big(\gamma_{N,t} - \gamma_{L,t}\big) \tag{B15}$$

Assuming that all the exogenous variables are zero except fertility rate, the trend growth rates above can be written as follows:

$$g_{l,t} = -\gamma_{N,t} \tag{B16}$$

$$g_{pl,t} = \gamma_{N,t} \tag{B17}$$

$$g_{ih,t} = -\mu_l\gamma_{N,t} \tag{B18}$$

$$g_{q,t} = \mu_l\gamma_{N,t} \tag{B19}$$

Because $g_{y,t} = 0$ in this circumstance, according to budget constraint equations in Appendix A, the variable such as wages, profits, consumptions and capitals also have no trend. Meanwhile, based on the equations of the Equilibrium in Appendix A, the variables $l_{h,t,1}$, $l_{h,t,2}$ and $l_{h,t}$ have the same trend growth rate as l_t. Similarly, the variables $h_{t,1}$, $h_{t,2}$ and h_t have the same trend growth rate as ih_t.

Appendix C. De-trended Equations and First Order Conditions

Using the equations in Appendix B, the equations in Appendix A can be written in de-trended form. We list these equations as follows, as well as the de-trended first order conditions of households and firms.

Households:

$$\tilde{c}_{t,1} + \tilde{q}_t\tilde{h}_{t,1} + \tilde{p}_{l,t}\tilde{l}_{t,1} = (1-T)(\widetilde{w}_{c,t}\tilde{n}_{c,t} + \widetilde{w}_{h,t}\tilde{n}_{h,t} + \tilde{d}_t) \tag{C1}$$

$$\tilde{c}_{1+t,2} + \tilde{q}_{1+t}\tilde{h}_{1+t,2} + \tilde{p}_{l,1+t}\tilde{l}_{1+t,2} = \tilde{q}_{1+t}(1-\delta_h)\left(\frac{\tilde{h}_{t,1}}{\pi_{1+t}\,exp(g_{h,t+1})} + \frac{\pi_t\tilde{h}_{t,2}}{\pi_{1+t}n_t\,exp(g_{h,t+1})}\right) + T(\tilde{d}_{1+t} + \tilde{n}_{c,1+t}\widetilde{w}_{c,1+t} + \tilde{n}_{h,1+t}\widetilde{w}_{h,1+t})\frac{n_{1+t}}{\pi_{1+t}} + \tilde{p}_{l,1+t}\left(\frac{\tilde{l}_{h,t,1}}{\pi_{1+t}\,exp(g_{l,t+1})} + \frac{\pi_t\tilde{l}_{t,2}}{\pi_{1+t}n_t\,exp(g_{l,t+1})}\right) \tag{C2}$$

First Order Conditions of Households:

The households choose the following variables: $c_{t,1}, h_{t,1}, l_{t,1}, n_{c,t}, n_{h,t}, c_{1+t,2}, h_{1+t,2}, l_{1+t,2}$. The utility maximization of households have the first-order conditions as follows:

$$\frac{\tilde{q}_t}{\tilde{c}_{t,1}} = \frac{j_h}{\tilde{h}_{t,1}} + \frac{\beta(1-\delta_h)\, exp\left(g_{q,t+1}\right)\, \tilde{q}_{1+t}}{\tilde{c}_{1+t,2}} \tag{C3}$$

$$\frac{\tilde{q}_{1+t}}{\tilde{c}_{1+t,2}} = \frac{j_h}{\tilde{h}_{1+t,2}} \tag{C4}$$

$$\frac{\tilde{p}_{l,t}}{\tilde{c}_{t,1}} = \frac{j_l}{\tilde{l}_{h,t,1}} + \frac{\beta\, exp\left(g_{pl,t+1}\right)\, \tilde{p}_{l,1+t}}{\tilde{c}_{1+t,2}} \tag{C5}$$

$$\frac{\tilde{p}_{l,1+t}}{\tilde{c}_{1+t,2}} = \frac{j_l}{\tilde{l}_{h,1+t,2}} \tag{C6}$$

$$\frac{(1-T)\tilde{w}_{c,t}}{\tilde{c}_{t,1}} = \tau \tilde{n}_{c,t}^{\xi} (\tilde{n}_{c,t}^{1+\xi} + \tilde{n}_{h,t}^{1+\xi})^{\frac{\eta-\xi}{1+\xi}} \tag{C8}$$

$$\frac{(1-T)\tilde{w}_{h,t}}{\tilde{c}_{t,1}} = \tau \tilde{n}_{h,t}^{\xi} (\tilde{n}_{c,t}^{1+\xi} + \tilde{n}_{h,t}^{1+\xi})^{\frac{\eta-\xi}{1+\xi}} \tag{C9}$$

Firms:

$$\tilde{y}_t = A_{c,t} \left(\frac{\tilde{k}_{c,t-1}}{n_t}\right)^{\mu_c} \tilde{n}_{c,t}^{1-\mu_c} \tag{C10}$$

$$\widetilde{\text{ıh}}_t = A_{h,t} \left(\frac{\tilde{k}_{h,t-1}}{n_t}\right)^{\mu_h} \left(\frac{\tilde{l}_{e,t-1}}{n_t\, exp(g_{l,t+1})}\right)^{\mu_l} \tilde{n}_{h,t}^{1-\mu_h-\mu_l} \tag{C11}$$

$$\tilde{k}_{c,t} = (1-\delta_{\text{kc}}) \frac{\tilde{k}_{c,t-1}}{n_t} + \widetilde{\text{ık}}_{c,t} \tag{C12}$$

$$\tilde{k}_{h,t} = (1-\delta_{\text{kh}}) \frac{\tilde{k}_{h,t-1}}{n_t} + \widetilde{\text{ık}}_{h,t} \tag{C13}$$

$$\tilde{\phi}_{c,t} = \phi_c\left(\tilde{k}_{c,t}, \tilde{k}_{c,t-1}\right) = \frac{\phi_{kc}}{2} n_t \left(\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}} - 1\right)^2 \tilde{k}_{c,t-1} \tag{C14}$$

$$\tilde{\phi}_{h,t} = \phi_h\left(\tilde{k}_{h,t}, \tilde{k}_{h,t-1}\right) = \frac{\phi_{kh}}{2} n_t \left(\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}} - 1\right)^2 \tilde{k}_{h,t-1} \tag{C15}$$

Here, $g_{KC,t}$ and $g_{KH,t}$ are assumed to equal to the fertility rate n_t.

$$\tilde{d}_t + \tilde{k}_{c,t} + \tilde{k}_{h,t} + \tilde{w}_{c,t}\tilde{n}_{c,t} + \tilde{w}_{h,t}\tilde{n}_{h,t} + \tilde{p}_{l,t}\tilde{l}_{e,t} + \tilde{\phi}_{c,t} + \tilde{\phi}_{h,t} = \tilde{y}_t + \tilde{q}_t\, \widetilde{\text{ıh}}_t + (1-\delta_{\text{kc}}) \frac{\tilde{k}_{c,t-1}}{n_t} + (1-\delta_{\text{kh}}) \frac{\tilde{k}_{h,t-1}}{n_t} + p_{l,t} \frac{\tilde{l}_{e,t-1}}{n_t\, exp(g_{l,t})} \tag{C16}$$

First Order Conditions of Firms:

The firms will decide on the following variables: $k_{c,t}, k_{h,t}, d_t, n_{c,t}, n_{h,t}, l_{e,t}$. The first order condition with respect to $k_{c,t}$ is:

$$\frac{\beta_e\left(\frac{1-\delta_{\mathrm{kc}}}{n_{t+1}}-\tilde{\phi}^{(0,1)}(\tilde{k}_{c,t+1},\tilde{k}_{c,t})+\tilde{r}_{c,t+1}\right)}{\tilde{d}_{t+1}}=\frac{\frac{1}{A_{k,t}}+\tilde{\phi}^{(1,0)}(\tilde{k}_{c,t},\tilde{k}_{c,t-1})}{\tilde{d}_t} \tag{C17}$$

Where:

$$\tilde{r}_{c,t+1}=\tilde{y}^{(1,0)}(\tilde{k}_{c,t},\tilde{n}_{c,t+1})=\mu_c\,\frac{\tilde{y}_{t+1}}{\tilde{k}_{c,t}} \tag{C18}$$

$$\tilde{\phi}^{(0,1)}\left(\tilde{k}_{c,t+1},\tilde{k}_{c,t}\right)=-\frac{\phi_{kc}n_{t+1}}{2}\left(\left(\frac{\tilde{k}_{c,t+1}}{\tilde{k}_{c,t}}\right)^2-1\right) \tag{C19}$$

$$\tilde{\phi}^{(1,0)}(\tilde{k}_{c,t},\tilde{k}_{c,t-1})=\phi_{kc}n_t\left(\frac{\tilde{k}_{c,t}}{\tilde{k}_{c,t-1}}-1\right) \tag{C20}$$

Similarly, the first order condition with respect to $k_{h,t}$ is:

$$\frac{\beta_e\left(\frac{1-\delta_{\mathrm{kh}}}{n_{t+1}}-\tilde{\phi}^{(0,1)}(\tilde{k}_{h,t+1},\tilde{k}_{h,t})+\tilde{r}_{h,t+1}\right)}{\tilde{d}_{t+1}}=\frac{1+\tilde{\phi}^{(1,0)}(\tilde{k}_{h,t},\tilde{k}_{h,t-1})}{\tilde{d}_t} \tag{C21}$$

Where:

$$\tilde{r}_{h,t+1}=q_{1+t}\widetilde{\imath h}^{(1,0,0)}(\tilde{k}_{h,t},\tilde{n}_{h,t+1},\tilde{l}_{e,t})=\mu_h\,\frac{\tilde{q}_{1+t}\widetilde{\imath h}_{t+1}}{\tilde{k}_{h,t}} \tag{C22}$$

$$\phi^{(0,1)}\left(\tilde{k}_{h,t+1},\tilde{k}_{h,t}\right)=-\frac{\phi_{kh}n_{t+1}}{2}\left(\left(\frac{\tilde{k}_{h,t+1}}{\tilde{k}_{h,t}}\right)^2-1\right) \tag{C23}$$

$$\phi^{(1,0)}\left(\tilde{k}_{h,t},\tilde{k}_{h,t-1}\right)=\phi_{kh}n_t\left(\frac{\tilde{k}_{h,t}}{\tilde{k}_{h,t-1}}-1\right) \tag{C24}$$

The first order condition with respect to $\tilde{l}_{e,t}$ is:

$$\frac{\tilde{p}_{l,t}}{\tilde{d}_t}=\frac{\beta_e\,exp\left(g_{pl,t+1}\right)\left(\frac{\tilde{p}_{l,1+t}}{n_{1+t}}+\tilde{r}_{l,t+1}\right)}{\tilde{d}_{1+t}} \tag{C25}$$

Here,

$$\tilde{r}_{l,t+1}=\tilde{q}_{1+t}\widetilde{\mathrm{\imath h}}^{(0,0,1)}\left(\tilde{k}_{h,t},\tilde{n}_{h,1+t},\tilde{l}_{e,t}\right)=\mu_h\,exp\left(g_{pl,t+1}\right)\,\frac{\tilde{q}_{1+t}\widetilde{\imath h}_{t+1}}{\tilde{l}_{e,t}} \tag{C26}$$

The first order conditions with respect to $n_{c,t}$ and $n_{h,t}$ are:

$$\widetilde{w}_{c,t}=(1-\mu_c)\,\frac{\tilde{y}_t}{\tilde{n}_{c,t}} \tag{C27}$$

$$\widetilde{w}_{h,t}=(1-\mu_h-\mu_l)\,\frac{\tilde{q}_t\widetilde{\imath h}_t}{\tilde{n}_{h,t}} \tag{C28}$$

Equilibrium:

$$\tilde{y}_t=\tilde{c}_t+\widetilde{\mathrm{\imath k}}_{c,t}+\widetilde{\mathrm{\imath k}}_{h,t}+\tilde{\phi}_t \tag{C29}$$

$$\tilde{h}_t=\widetilde{\mathrm{\imath h}}_t+(1-\delta_h)\,\frac{\tilde{h}_{t-1}}{n_t\,exp(g_{h,t})} \tag{C30}$$

$$\tilde{l}_t=\tilde{l}_{h,t}+\tilde{l}_{e,t} \tag{C31}$$

where $\tilde{c}_t=\tilde{c}_{t,1}+\frac{\pi_t}{n_t}\tilde{c}_{t,2}$, $\tilde{h}_t=\tilde{h}_{t,1}+\frac{\pi_t}{n_t}\tilde{h}_{t,2}$, $\tilde{l}_{h,t}=\tilde{l}_{h,t,1}+\frac{\pi_t}{n_t}\tilde{l}_{h,t,2}$.

Appendix D. Method in Calculating Dynamics of Variables

When we calculate the dynamics of a variable, we first calculate the growth rates of trends and steady states separately, and then add them together. For example, if we interest in the growth rate of a variable, this growth rate can be represented as follows:

$$g_t = g_{Trend,t} + g_{SS,t} \tag{D1}$$

Where the variable $g_{Trend,t}$ denotes the growth rate of trend at time t, and the growth rate of steady state at time t is denoted by $g_{SS,t}$. The proof of this method is as follows.

Suppose we have a time varying variable X and the trend of this variable is denoted by G, then the steady state $\tilde{X}$ satisfies the equation as follows:

$$\tilde{X}_t = \frac{X_t}{G_t}$$

Suppose the time is discrete, we take log operation on both sides and derive as follows:

$$\ln(\tilde{X}_t) = \ln(X_t) - \ln(G_t)$$

The same equation in the previous period is:

$$\ln(\tilde{X}_{t-1}) = \ln(X_{t-1}) - \ln(G_{t-1})$$

Subtract the above two equations, we have the following equation:

$$\ln(X_t) - \ln(X_{t-1}) = (\ln(G_t) - \ln(G_{t-1})) + \left(\ln(\tilde{X}_t) - \ln(\tilde{X}_{t-1})\right)$$

According to Eq. (B2), the equation above can be rewritten as:

$$g_t = g_{Trend,t} + g_{SS,t}$$

Similarly, if we interest in the changes of a variable comparing with its original state, it can be calculated by the following equation:

$$\ln(X_t) - \ln(X_0) = (\ln(G_t) - \ln(G_0)) + \left(\ln(\tilde{X}_t) - \ln(\tilde{X}_0)\right) \tag{D2}$$

As shown above, the overall changes can be represented by the sum of the changes in trend and steady states.   #

Appendix E. Robust Test

In this section, we will examine the robustness of long-term effect of an increase in longevity on housing prices. Specifically, we change the calibrated parameter values to assess their effects on the numerical outcomes. More concretely, each calibrated parameter incremented by 10 percent and the effects on house (and land) prices assessed. To make the results comparable to the original one, the same demographic change would be applied, i.e. survival rate rise from

0.8 to 0.9. If the above results about longevity increase is robust, then the parameter changes should only make modest changes on the result.

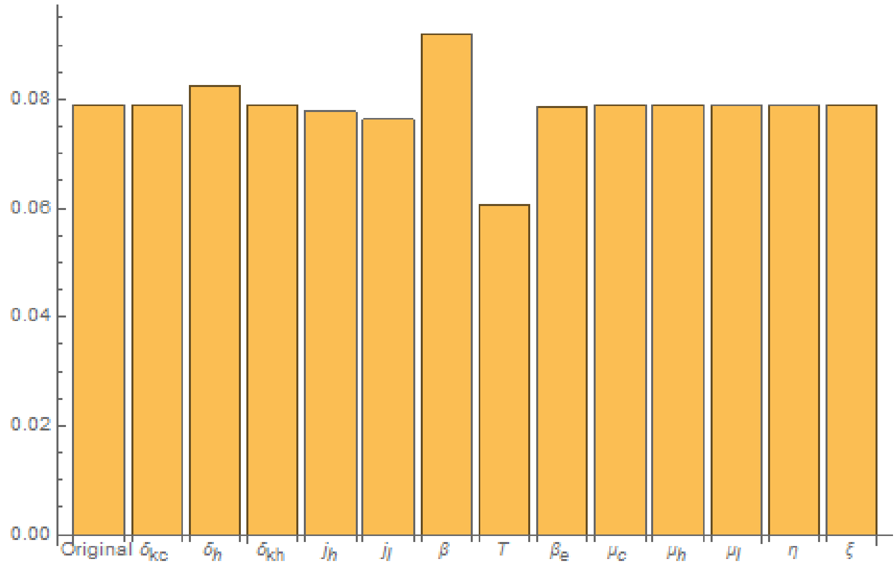

Figure E1. Robust Test: Effect of Longevity Increase on Land Price

Notes: From left to right, the first bar in chart is the land price change when parameters are calibrated to their original values. Then, we change the parameter values one by one. Each parameter is increased by 10 percent comparing with their original values. For example, we could increase the capital depreciation rate of non-housing sector by 10 percent, and the other parameter values are unchanged. In this situation, the effect of longevity increase on land price is shown in the second bar.

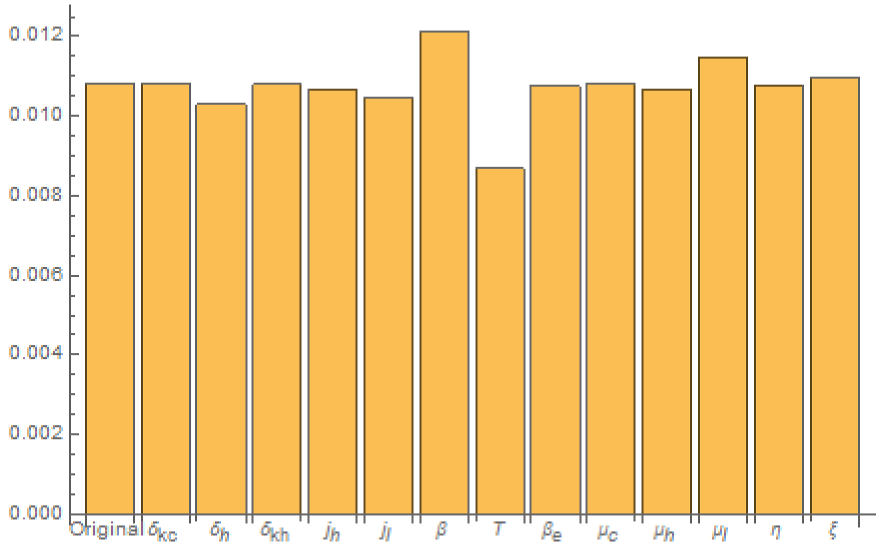

Figure E2. Robust Test: Effect of Longevity Increase on House Price

Notes: From left to right, the first bar in chart is the house price change when parameters are calibrated to their original values. Then, we change the parameter values one by one. Each parameter is increased by 10 percent comparing with their original values. For example, we could increase the capital depreciation rate of non-housing sector by 10 percent, and the other parameter values are unchanged. In this situation, the effect of longevity increase on house price is shown in the second bar.

For the land price, the tests for robustness are shown in Figure E1. Comparing with the original result, which is shown at the left most, the price changes are modest. The biggest difference lay on the time preference β and tax rate T, however, none of their effects is bigger than 2 percent. Thus, the numerical solution of land price changes is considered to be robust.

Similarly, the test for robustness of house price to changes in longevity is shown in Figure E2. Like that of the land price, the longevity increase effect on house price would only be affected modestly by parameter value changes. Therefore, the result on the house price is also robust in this circumstance.

References

Abel, Andrew B. 2001. "Will Bequests Attenuate the Predicted Meltdown in Stock Prices When Baby Boomers Retire?" *Review of Economics and Statistics* 83 (4):589-595. doi: 10.1162/003465301753237678.

Abel, Andrew B. 2003. "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security." *Econometrica* 71 (2):551-578. doi: 10.1111/1468-0262.00417.

Ando, Albert, and Franco Modigliani. 1963. "The "Life Cycle" Hypothesis of Saving: Aggregate Implications and Tests." *The American Economic Review* 53 (1):55-84.

Bergantino, Steven M. 1998. "Life cycle investment behavior, demographics and asset prices." Ph.D., Dept. of Economics, Massachusetts Institute of Technology.

Blackburn, Keith, and Giam Pietro Cipriani. 2002. "A model of longevity, fertility and growth." *Journal of Economic Dynamics and Control* 26 (2):187-204. doi: http://dx.doi.org/10.1016/S0165-1889(00)00022-1.

Brooks, Robin. 2000. WHAT WILL HAPPEN TO FINANCIAL MARKETS WHEN THE BABY BOOMERS RETIRE? : Society for Computational Economics.

Brooks, Robin. 2002. "Asset-market effects of the baby boom and Social-Security reform." *The American Economic Review* 92 (2):402-406.

Chen, Kaiji, and Yi Wen. 2017. "The Great Housing Boom of China." *American Economic Journal: Macroeconomics* 9 (2):73-114. doi: doi: 10.1257/mac.20140234.

Cipriani, Giam Pietro. 2013. "Population aging and PAYG pensions in the OLG model." *Journal of Population Economics* 27 (1):251-256. doi: 10.1007/s00148-013-0465-9.

Curtis, Chadwick C., Steven Lugauer, and Nelson C. Mark. 2015. "Demographic Patterns and Household Saving in China." *American Economic Journal: Macroeconomics* 7 (2):58-94. doi: doi: 10.1257/mac.20130105.

Davis, Morris A., and Jonathan Heathcote. 2007. "The price and quantity of residential land in the United States." *Journal of Monetary Economics* 54 (8):2595-2620. doi: http://dx.doi.org/10.1016/j.jmoneco.2007.06.023.

de Córdoba, Santiago Fernández, Sam Laird, Jean-Christophe Maur, and José María Serena. 2006. "Adjustment Costs and Trade Liberalization." In *Coping with Trade Reforms: A Developing Country Perspective on the WTO Industrial Tariff Negotiations*, edited by Sam Laird and Santiago Fernández de Córdoba, 66-85. London: Palgrave Macmillan UK.

Diamond, Peter A. 1965. "National Debt in a Neoclassical Growth Model." *The American Economic Review* 55 (5):1126-1150. doi: 10.2307/1809231.

Ding, Jie. 2012. "Australian retirees' preferences between consumption, government pension, bequest and housing." PhD Conference in Economics and Business.

Dornbusch, Rudiger. 1976. "Expectations and Exchange Rate Dynamics." *Journal of Political Economy* 84 (6):1161-1176. doi: 10.1086/260506.

Eichholtz, Piet, and Thies Lindenthal. 2014. "Demographics, human capital, and the demand for housing." *Journal of Housing Economics* 26:19-32. doi: http://doi.org/10.1016/j.jhe.2014.06.002.

Engelhardt, Gary V., and James M. Poterba. 1991. "House prices and demographic change." *Regional Science and Urban Economics* 21 (4):539-546. doi: http://dx.doi.org/10.1016/0166-0462(91)90017-H.

Hendershott, Patric H. 1991. "Are real house prices likely to decline by 47 percent?" *Regional Science and Urban Economics* 21 (4):553-563. doi: http://dx.doi.org/10.1016/0166-0462(91)90019-J.

Hiller, Norbert, and Oliver W. Lerbs. 2016. "Aging and urban house prices." *Regional Science and Urban Economics* 60:276-291. doi: http://doi.org/10.1016/j.regsciurbeco.2016.07.010.

Horvath, Michael. 2000. "Sectoral shocks and aggregate fluctuations." *Journal of Monetary Economics* 45 (1):69-106. doi: http://dx.doi.org/10.1016/S0304-3932(99)00044-6.

Iacoviello, Matteo. 2005. "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle." *The American Economic Review* 95 (3):739-764.

Iacoviello, Matteo. 2015. "Financial business cycles." *Review of Economic Dynamics* 18 (1):140-163. doi: http://dx.doi.org/10.1016/j.red.2014.09.003.

Iacoviello, Matteo, and Stefano Neri. 2010. "Housing Market Spillovers: Evidence from an Estimated DSGE Model." *American Economic Journal: Macroeconomics* 2 (2):125-64. doi: doi: 10.1257/mac.2.2.125.

Iacoviello, Matteo, and Marina Pavan. 2013. "Housing and debt over the life cycle and over the business cycle." *Journal of Monetary Economics* 60 (2):221-238. doi: http://dx.doi.org/10.1016/j.jmoneco.2012.10.020.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti. 2015. "Credit Supply and the Housing Boom." *National Bureau of Economic Research Working Paper Series* No. 20874. doi: 10.3386/w20874.

Leland, Hayne E. 1972. "Theory of the Firm Facing Uncertain Demand." *The American Economic Review* 62 (3):278-291.

Liu, Zheng, Pengfei Wang, and Tao Zha. 2013. "Land-Price Dynamics and Macroeconomic Fluctuations." *Econometrica* 81 (3):1147-1184. doi: 10.3982/ECTA8994.

Lutz, Wolfgang, Warren Sanderson, and Sergei Scherbov. 2008. "The coming acceleration of global population ageing." *Nature* 451 (7179):716-719. doi: http://www.nature.com/nature/journal/v451/n7179/suppinfo/nature06516_S1.html.

Mankiw, N. Gregory, and David N. Weil. 1989. "The baby boom, the baby bust, and the housing market." *Regional Science and Urban Economics* 19 (2):235-258. doi: http://dx.doi.org/10.1016/0166-0462(89)90005-7.

Mankiw, N. Gregory, and David N. Weil. 1991. "The baby boom, the baby bust, and the housing market A reply to our critics." *Regional Science and Urban Economics* 21 (4):573-579. doi: http://dx.doi.org/10.1016/0166-0462(91)90021-E.

Modigliani, Franco, and Shi Larry Cao. 2004. "The Chinese Saving Puzzle and the Life-Cycle Hypothesis." *Journal of Economic Literature* 42 (1):145-170.

Muto, Ichiro, Takemasa Oda, and Nao Sudo. 2016. "Macroeconomic Impact of Population Aging in Japan: A Perspective from an Overlapping Generations Model." *IMF Economic Review* 64 (3):408-442. doi: 10.1057/imfer.2016.6.

Ng, Eric C. Y. 2015. "Housing market dynamics in China: Findings from an estimated DSGE model." *Journal of Housing Economics* 29:26-40. doi: http://dx.doi.org/10.1016/j.jhe.2015.05.003.

Oh, Sechan, James Rhodes, and Ray Strong. 2016. "Impact of cost uncertainty on pricing decisions under risk aversion." *European Journal of Operational Research* 253 (1):144-153. doi: http://dx.doi.org/10.1016/j.ejor.2016.02.034.

Poterba, James M. 2014. "Retirement security in an aging population." *The American Economic Review* 104 (5):1-30.

Poterba, James M. 2001. "Demographic Structure and Asset Returns." *Review of Economics and Statistics* 83 (4):565-584. doi: 10.1162/003465301753237650.

Sandmo, Agnar. 1971. "On the Theory of the Competitive Firm Under Price Uncertainty." *The American Economic Review* 61 (1):65-73.

Swan, Craig. 1995. "Demography and the demand for housing A reinterpretation of the Mankiw-Weil demand variable." *Regional Science and Urban Economics* 25 (1):41-58. doi: http://dx.doi.org/10.1016/0166-0462(94)02074-Q.

Takáts, Előd. 2012. "Aging and house prices." *Journal of Housing Economics* 21 (2):131-141. doi: http://doi.org/10.1016/j.jhe.2012.04.001.

Xie, Yu, and Yongai Jin. 2015. "Household Wealth in China." *Chinese Sociological Review* 47 (3):203-229. doi: 10.1080/21620555.2015.1032158.