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Financialization and Endogenous Technological Change: a Post-Kaleckian Perspective

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Abstract

In post-Keynesian literature, Hein (2012a) was the first to incorporate financialization as an influential positive determinant of the rate of technological change. However, financialization is more like a two-edged sword which can affect technological progress negatively as well. We capture both the positive as well as the negative effect of financialization on technological progress which encapsulates the possibility of multiple equilibria. In analyzing the long run of the model we endogenize the financialization parameter as well. We then show how two subsystems (technological progress and financialization dynamics) when interacts with each other, can produce instability and cycles for the whole system. We show that under certain circumstances, higher speed of diffusion of technological innovation, more regulated financial markets, and higher intra-class competition among firms are desirable for stabilizing the economy. Finally, we provide some policy prescriptions for the same.

Keywords: Capital accumulation, Distribution, Financialization, Kaleckian model, Technological progress, Andronov–Hopf bifurcation, Saddle-node bifurcations, Limit cycles

JEL classification: C62, C69, D33, E12, G01, O16, O41

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1 Introduction:

The phenomenon of ‘financialization’ has an important role to play in explaining developments in the world economy (specially for developed countries) over the past four decades. Financial markets and agents play a prominent role in the modern economy. Enormous increases in the size of the financial sector in one hand and deregulation of the sector on the other hand are associated with significantly changed income distribution in this era of financialization. Starting from 15% in 1980s, the financial sector’s share of total profit for US economy tripled in 2007 with a peak of 45% in 2002 (Tomaskovic-Devey and Lin, 2011; Lin and Tomaskovic-Devey, 2013). In case of non-financial sector firms, in the USA, the ratio of financial income to realized profits more than doubled from 15% to 32% with a peak of 42% in 2001(Lin and Tomaskovic-Devey, 2013). While there has been an increase in profit share at the expense of wage share, at the same time the share of rentiers’ income has increased at the cost of the share of the non-financial sectors’ profit within the category of profit itself. In case of wage share, blue-collar labours’ wage share has decreased while that of managerial income has increased dramatically. Both, the share of capital out of national income and the compensation of top corporate executives has increased significantly. Needless to say income inequality has increased tremendously. The shift of power toward rentiers and away from workers (because of the financialization), as Van Arnum and Naples (2013) argue, is one of the primary reason for income inequality.

Financialization has transformed the functioning of the economic system at both the micro and macro levels. For the last four decades for US economy, on the one hand we observe a continuous invention and innovation of new technologies and on the other hand an increasing engagement of non-financial businesses in financial markets. Since the last three decades financial fragility has increased enormously. We have observed the 1992 sterling crisis, the 1994 Mexican peso crisis, the 1997 East Asian financial crisis, the 1999 and 2002 Brazilian financial crisis, the Argentine financial crisis of 2001-02¹ and the latest financial crisis of the USA (2007-08).

The intention of this paper is, first, to focus on how technological progress changes through time, especially in the era of financialization in the context of US economy. Second, to explain how the financialization parameter itself evolves over time. And third, how the interaction between the technological progress and financialization dynamics leads to fragility and instability in the economy. Superiority of our

¹Palley (2013)

analysis over Hein (2012a) lies on the fact that unlike Hein(2012a) (where in the long run the economy always achieve a stable steady-state) by introducing the financialization dynamics and allowing the possibility of nonlinearity of the technological progress dynamics we are able to open up the possibility of long-run instability in our model. While several economists and policymakers have tried to explain the recent financial crisis of the US economy, this paper provides an alternative way of looking into the problem. This paper also seek to explain whether intra-class conflict among firms have any role to play for ensuring stability in the economy, especially when the economy is in a prolonged stagnation.

We focus on the concept of financialization first. After that, we briefly discuss the Keynesian and post-Keynesian literature regarding endogenous technological change. Then we explain the distinctive features and novelty of our analysis compared to the earlier literature. Finally we discuss the outline for the rest of the paper.

‘financialization’ has emerged as a concept like ‘globalization’ for which not only is a unique definition unavailable, but the precise form and usage of it is also unclear. As a result, we find several definitions and various uses of the term. Most acknowledged definition of the term comes from Epstein (2005) to whom “Financialization means the increasing role of financial motives, financial markets, financial actors and financial institutions in the operation of the domestic and international economies”. Orhangazi (2007, 2008) argues for two definition of financialization². At the general level, financialization refers to an increase in the size and significance of financial markets, transactions, and institutions. At a narrower level, however, he uses financialization to designate changes in the relationship between the non-financial corporate sector and financial markets. These latter changes encapsulate, first, an increase in financial investments and hence financial incomes of the NFCs; and second, an increase in financial market pressure on the management of NFCs. This increase in pressure, which is exhibited through governance debates revolving around ‘shareholder value orientation’, results in an increasing transfer of resources from NFCs to financial markets in the

²Among the other proponents of the concept ‘financialization’, for Dumenil and Levy (2004), financialization means the structural change in the post-1980 era characterized by “the growth of financial enterprises, the rising involvement of non-financial enterprises in financial operations, the holding of large portfolios of shares and other securities by households, and so on”. Acknowledging the vagueness of the concept, Stockhammer (2004) narrows down the definition and uses of the concept, particularly in relation to the NFCs. For him, financialization is defined as the engagement of non-financial businesses in financial markets. Krippner (2005) too contends for a relatively narrow definition. As she point out “.....financialization as a pattern of accumulation in which profits accrue primarily through financial channels rather than through trade and commodity production”.

forms of interest payments, dividend payments and stock buybacks. As the intention of this paper is mainly to focus on the long run interaction between the financialization rate and technological progress, to make the model simple and tractable, following Hein (2010, 2012a, 2012b, 2014), in this paper we quite narrowly define the concept of financialization rate as the notion of ‘shareholder value orientation’. Lazonick and O’Sullivan (2000;) extensively discuss the concept of ‘shareholder value’ as a principle of corporate governance in the United States. As they point out, there is a massive “transformation of US corporate strategy from an orientation towards retention of corporate earnings and reinvestment in corporate growth through the 1970s to one of downsizing of corporate labour forces and distribution of corporate earnings to shareholders”³ over the past few decades for satisfying shareholders’ demand for distributed profits and for maintaining high share prices. So, by the notion of ‘shareholder value orientation’ we emphasize on this very change in objective of the managements⁴.

Most of the neo-Keynesian and post-Keynesian literature which treats technological progress as an endogenous phenomenon explain technological progress as positively dependent on the rate of capital accumulation (e.g. Kaldor 1957, 1961, 1966; Rowthorn 1981; Dutt 1990; Taylor 1991; Lavoie 1992 etc). However, a significant amount of post-Keynesian literature considers the technological innovation is being determined by income distribution as well (e.g. Taylor 1991; Cassetti 2003; Naastepad 2006; Dutt 2006, 2013). A basic argument of this literature is that as wage share rises, firms face higher labour costs⁵ and this accelerates the innovations of new labour-saving technologies, so that profit share can be prevented from falling further. According to Dutt (2006, 2013), technological change depends positively on the difference between the growth rate of labour demand and labour supply. A rise in aggregate demand leads to an enhancement of labour employment growth which in turn leads to a faster growth rate of technological (labour-augmenting) change so that the problem of labour shortage is taken care of. This argument is consistent with the impact of distributional variables on technological progress in the sense that a shortage of labour puts an upward pressure on the wage share and this leads to labour-saving changes in technology⁶.

³Lazonick and O’Sullivan (2000; pp. 13)

⁴For more on ‘shareholder value’ see Froud et. al. (2000).

⁵One can argue that as labour costs rise, firms can increase the existing levels of prices. Notwithstanding the fact that it might be possible, as firms face more difficulties in transferring higher costs into prices they feel stronger incentives for adopting labour-saving innovations.

⁶Beyond these two variables (rate of capital accumulation and the wage/profit share), technological progress can be influenced by other phenomena as well. For example in a neo-Schumpeterian post-Keynesian model of growth and distribution, Lima (2000) explores the relationship between market concentration and endogenous technological innovation. Borrowing an idea from Schumpeter (1912, 1942) he argues that higher market power (concentration) by pro-

Using a post-Kaleckian growth model, Hein (2010, 2012a, 2012b, 2014) examines the effect of financialization (through an increase in shareholder power) on the demand regime⁷ and on the productivity regime separately and then on the overall regime of the model. Financialization, which is captured by increasing shareholder power in Hein's model for the analysis of both the demand and the productivity regimes, is considered to be an exogenous variable. When the demand regime is analysed, productivity growth is assumed to be an exogenous variable which is endogenized later for the analysis of the productivity regime. In the analysis of the overall regime, the equilibrium growth rate and the productivity growth both are determined endogenously and finally, the effect of financialization (through a rise in shareholder power) on both the regimes is derived.

Tridico and Pariboni (2018) use an empirical analysis to explain the main causes of labour productivity slowdown in several developed countries. They first explain how financialization⁸ leads to higher income inequality and then considering an extended version of Sylos-Labini's equation⁹ they find the labour productivity growth rate to be positively dependent on the growth rate of GDP and the wage share whereas income inequality and financialization both have a negative impact on it.

The current paper is most closely related to Hein (2012a). Following Bhaduri and Marglin (1990), Hein (2012a) assumes investment decisions to be positively influenced by expected sales and by the profit share as both positively affect the expected profit rate. Distributed profits by reducing the available internal funds and limiting the access to external funds negatively affects investment demand. He

viding more internal financial resources give firms the incentive to spend on innovative activities. On the other hand, high concentration (and hence weak competition) reduces the incentive to innovate as firms with high monopoly power feel less threatened by their rivals. So, as he says, the technological innovation depends non-linearly on market concentration.

Later on, in a post-Keynesian macro-model of accumulation, growth, and distribution Lima (2004) captures the endogeneity of technological innovation. In this literature the rate of labour-saving technological innovation by firms depends non-linearly on the distribution of income. Distribution plays the crucial role as it provides the incentive to innovate and at the same time provides the source (and availability) of funds for innovations. At a low level of wage share, the availability of fund to innovate is high and dominates the incentives to innovate. On the other hand, at a high level of wage share although the incentive for innovation is quite high the availability of funds is low. It is the intermediate level of wage share where the rate of technological innovation is maximum.

⁷In the analysis of demand regime, Hein (2010, 2012a, 2012b, 2014) analyzes the aggregate demand and the rate of capital accumulation where he fixes the labour productivity growth at a constant level. In the analysis of productivity regime he endogenizes the labour productivity growth.

⁸They consider labour flexibility and 'shareholder value orientation' as the main aspects of financialization.

⁹According to Sylos Labini (1999), growth rate of labour productivity depends mainly positively on the growth rate of GNP (Gross National Product), growth rate of wage share, and the growth rate of relative cheapness of labour over capital.

also incorporates technical progress as one of the variables determining the level of investment. In his own language, the explanation is as follows. “Since technical progress is embodied in capital stock, it will stimulate investment. Firms have to invest in new machines and equipment in order to gain from productivity growth that is made available by new technologies” (pp. 482).

An increase in shareholder power, as Hein (2012a) points out, affects the accumulation rate through three channels. First, through the ‘preference channel’¹⁰ which is negative. Second, through the ‘internal finance channel’,¹¹ the overall effect of which is ambiguous. And third, the ‘distribution channel’¹² which also has an ambiguous effect on the capital accumulation. So the overall effect of a rise in shareholder power on the equilibrium accumulation rate is ambiguous. It can be ‘expansive’ i.e. there is a positive impact of a rise in shareholder power on the accumulation rate or it can be ‘contractive’ i.e. an increase in shareholder power will negatively affect the accumulation rate.

For a given capital accumulation rate, a change in shareholder power has a direct positive effect on productivity growth and a negative indirect effect via the profit share. So the overall effect of a rise in shareholder power on productivity growth is ambiguous.

Given the fact that both demand regime and productivity regime are expansive, with a rise in shareholder power, an overall expansive regime can be achieved i.e. capital accumulation and productivity growth both increase in the face of rising shareholder power. Similarly, if both the regimes are contractive, the overall regime will be contractive too.

However, if the demand regime is contractive and the productivity regime is expansive and the contractive effect on the demand regime is relatively weak, we may obtain an overall expansive regime while if the contractive effect on the demand regime is relatively strong then we may obtain an overall contractive regime. If, however, the partial effects on demand regime and productivity regime are neither

¹⁰‘shareholder value orientation’ influences managers’ (here firms’) to shift their preference from retain the profit and reinvest it for enhancing the rate of capital accumulation to downsize the labour forces and distribute the profit to the shareholders. “The preference for growth, and hence the willingness to invest in capital stock, therefore suffers, too” (Hein ; 2012b, pp. 39). This route through which shareholder power works is called the ‘preference channel’.

¹¹Because of ‘shareholder value orientation’, firms are forced to distribute a higher share of profit to the shareholders and hence are left with lower retention ratio. As a result, “internal means of finance for real investment are reduced, and the ability to invest hence suffers” (Hein; 2012b, pp. 39). This route through which shareholder power works is called as ‘internal finance channel’.

¹²The route through which shareholder power influences the distribution of income (between wage share and profit share) can be called as ‘distribution channel’.

too strong nor too weak then an overall intermediate regime is possible i.e. a slow capital accumulation with fast productivity growth may co-exist. Exactly opposite of the above happens if the demand regime is expansive and the productivity regime is contractive.

Hein (2010, 2012a, 2012b, 2014) is the first and to our best knowledge only contribution for the literature who focuses on the impact of financialization on productivity growth (or technological change) from a theoretical perspective. The basic structure of our model is based on Hein (2012a). However, compared to Hein (2010, 2012a, 2012b, 2014) this paper has a few distinct features.

First, in Hein (2012a), technical progress implies an increment in output-labour ratio or labour productivity. In his paper, although the labour productivity is increasing, the wage share is not changing because of that. This is possible only when some implicit assumption is made regarding the fact that wage rate is increasing in accordance with the increment in labour productivity. But for the USA, for last four decades, real wage rate has not grown with the same pace as labour productivity. Thus, it is hard to believe that wage share is unaffected due to a rise in labour productivity. However, our paper is free from this kind of problem.

Second, Hein (2012a) points out that if ‘shareholder value orientation’ goes too far, a potential negative impact of it on labour productivity is possible. Nonetheless, his basic analysis is based on a simple linear positive relationship between ‘shareholder value orientation’ and labour productivity which ensures the unique and stable steady state only. However, in this paper by incorporating both positive and negative effects of financialization on the rate of technological progress, we get a non-linear relationship between those two that allows the existence of multiple equilibria and opens the possibility of instability in the economy. In our analysis of the long run, we provide the rationale for the assumed non-linear relation between degree of financialization and technological progress. This, in our opinion, is more appropriate for explaining developments in the US economy which has become more fragile and unstable for the last several decades.

Third, so far most of the literatures captures financialization as an exogenous parameter and explain its impact on the economy by the change in that very parameter. The novelty of this paper is we are trying to explain how this financialization parameter itself evolves through time (in other words we are trying to endogenize this financialization parameter in the long run). We then show how one stable and one unstable subsystem (represented by technological progress and financialization dynamics) can interact with each other to produce instability and

cycles in the whole system. We show that higher speed of diffusion of technological innovation, more regulated financial markets, and higher intra-class competition among firms are desirable for stabilizing the economy. We discuss some policy prescription for the same as well.

The outline of the rest of the paper is as follows. Section 2 sets up the model, presents a short run analysis including short run comparative statics. Section 3 discusses the long run where we endogenize the financialization parameter and the technological progress. Section 3.1 talks about the possible cases that may arise because of the interaction between financialization and technological progress. This is followed by the section 3.2 where using Andronov-Hopf bifurcation we analyze how the interaction between financialization and technological innovation can produce limit cycles. Section 3.3 discusses the comparative statics. Section 4 offers some concluding remarks.

2 The Model

Our short run analysis is completely based on Hein (2012a). We assume a simple one-sector, closed economy, post-Kaleckian growth model in which the economy consists of workers, rentiers, and firms. There is no government intervention in the economy. For simplicity we assume lack of depreciation of capital and only labour saving and capital-embodied technical progress prevails in the economy. Technical progress thus implies an increment in output-labour ratio or labour productivity ($a = Y/L$)¹³. The rate of capacity utilization (u) is given by the ratio of actual real output to capital stock. As long as the potential output-capital ratio is fixed, the actual output-capital ratio can be used as a proxy for the degree of capacity utilization.

The market is oligopolistic in nature where price (p) is determined by mark-up on prime cost. For simplicity, we assume away the cost of raw materials and overhead cost and consider labour cost as the only cost of production. So price is given by the following equation as

$$p = [1 + m(\Omega)] \frac{wL}{Y}$$

$$\Rightarrow p = [1 + m(\Omega)] \frac{w}{a}; \quad m > 0, \frac{\partial m}{\partial \Omega} \geq 0 \quad (2.1)$$

¹³The capital-labour ratio ($k = K/L$) increases at the same rate as labour productivity, and hence the capital-potential output ratio ($v = K/Y^P$) remains unchanged. Basically we assume a Harrod-neutral technical progress, as in Rowthorn (1981) and Hein (2010, 2012a, 2012b, 2014). In this paper, technological progress and labour productivity growth are used interchangeably.

m denotes the mark-up rate and $a = \frac{Y}{L}$ is labour productivity. Total wage share equals to $\frac{WL}{pY} = \frac{\omega}{a}$, where ω is real wage rate. Ω represents the financialization rate¹⁴. Note that $\Omega \in [0, 1]$.

So, share of profit is $\pi = (1 - \frac{\omega}{a})$. It can be expressed as the ratio of total profit to the nominal level of income as well i.e.

$$\pi = \frac{R}{pY} = \frac{m}{1+m}; \quad \frac{\partial \pi}{\partial \Omega} \geq 0 \quad (2.2)$$

The markup and the profit share both may change with respect to a change in shareholder power vis-à-vis management and labourers¹⁵. A rise in shareholder power (because of mergers, acquisitions and hostile takeovers) can potentially reduce the degree of competition in the goods market and the ‘downsize and redistribute’ strategy and of firms lowers the bargaining power of labour unions in the labour market. Thus an increase in financialization rate (Ω) (which mainly is captured by the idea of a rise in ‘shareholder power’) is associated with an increase in markup in firms’ pricing which is expressed in equation (2.1) and thus it is associated with a rise in the share of profit¹⁶ (equation (2.2)). Rate of profit is expressed as a product of share of profit and the degree of capacity utilization and is expressed in the following equation as

$$r = \frac{R}{K} = \pi u \quad (2.3)$$

A fraction of total profit is retained by the firms (R^F) and the rest is distributed as dividends (paid on equity held by rentiers (R^{Div})) and as interest payment (paid on debt to the rentiers (R^{Int})). Thus total distributed profit (R^R) consists of dividend and the interest payment to the rentiers. This argument is captured by the next equation as

$$R = R^F + R^{Int} + R^{Div} = R^F + R^R \quad (2.4)$$

Dividing both side of the above equation with respect to the nominal value of capital stock we get rate of profit as a summation of firms’ profit rate (r^F) and rentiers’ profit rate (r^R) i.e.

$$r = r^F + r^R \quad (2.5)$$

¹⁴To make the model simple and tractable, following Hein (2010, 2012a, 2012b, 2014), in this paper we quite narrowly define the concept of financialization rate as the notion of ‘shareholder value orientation’.

¹⁵ $\frac{\partial \pi}{\partial \Omega} = \frac{\partial \pi}{\partial m} \frac{\partial m}{\partial \Omega} = \frac{1}{(1+m)^2} \frac{\partial \pi}{\partial \Omega} \geq 0$.

¹⁶For a detailed discussion on how financialization affects the markup, share of profits and distributed profits see Hein (2012a) pp. 480.

$$r^R = \frac{R^R}{K}; \quad \frac{\partial r^R}{\partial \Omega} > 0 \quad (2.6)$$

$$r^F = \frac{R^F}{K} \quad (2.7)$$

Following Hein (2012a), we assume that a rise in shareholder power leads to an enhancement in the rentiers' profit rate. As long as there is a given total rate of profit, a given capital-potential output ratio, given income distribution between capital and labour, and a given rate of capacity utilization, a rise in the rentiers' rate of profit leads to a decrease in the firms' profit rate. However, as long as the degree of capacity utilization itself is endogenous, there is a very little scope for the rate of profit to remain constant. Although, in light of a strong contractive macroeconomic effect on the overall profit rate, there is a possibility that a rise in shareholder power can potentially reduce the rentiers' profit rate; for simplicity, in accordance with Hein (2010, 2012a, 2012b, 2014), this possibility is excluded.

We assume workers spend all of their income (which is the wage income only) whereas a fraction (s_r) of rentiers' income is saved. So total savings of the economy consists of saving of the firms (which is essentially the retained profit) and the savings of the rentiers i.e.

$$S = R^F + s_r R^R = R - R^R + s_r R^R = R - (1 - s_r)R^R \quad (2.8)$$

Normalization of the above equation in terms of the existing capital stock yields,

$$\frac{S}{K} = \sigma = \pi u - (1 - s_r)r^R \quad (2.9)$$

Following Hein (2012a) and Bhaduri-Marglin (1990), we assume investment decisions to be positively influenced by expected sales (i.e. by the degree of capacity utilization) and by the profit share as both positively affects the expected profit rate. Distributed profits (i.e. dividends and interest payments to rentiers), by reducing the available internal funds, negatively affects the investment demand while at the same time it imposes restrictions on the access to external funds à la Kalecki (1937). Following Hein (2012a), we assume inventions of new technologies also positively influence the investment demand. This is happening because “firms have to invest in new machines and equipment in order to gain from productivity growth that is made available by new technologies¹⁷”. So, the investment function

¹⁷Hein (2012a).

is

$$\frac{I}{K} = g = \alpha_0 + \alpha_1 u + \alpha_2 \pi - \alpha_3 r^R + \alpha_4 \lambda; \quad \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4 > 0; \quad \frac{\partial \alpha_0}{\partial \Omega} < 0, \quad \frac{\partial r^R}{\partial \Omega} > 0 \quad (2.10)$$

Here λ represents the technological progress or the growth rate of labour productivity. So $\lambda = \frac{\dot{a}}{a} = \hat{a}$.

In accordance with Hein (2012a), we assume increasing shareholder power vis-à-vis management can reduce the available funds for real investment through ‘internal finance channel’ and affects the management’s ‘animal spirit’ through ‘preference channel’ which has been captured by $\frac{\partial r^R}{\partial \Omega} > 0$ and $\frac{\partial \alpha_0}{\partial \Omega} < 0$ respectively.

In the short run equilibrium,

$$\begin{aligned} g &= \sigma \\ \Rightarrow \alpha_0 + \alpha_1 u + \alpha_2 \pi - \alpha_3 r^R + \alpha_4 \lambda &= \pi u - (1 - s_r) r^R \\ \Rightarrow u^* &= \frac{(\alpha_0 + \alpha_2 \pi + \alpha_4 \lambda) + (1 - s_r - \alpha_3) r^R}{(\pi - \alpha_1)} \end{aligned} \quad (2.11)$$

Keynesian stability condition requires responsiveness of investment demand due to one unit change in aggregate demand to be less than that of the savings for the same unit change in aggregate demand, i.e. $\pi > \alpha_1$. Let’s assume the Keynesian stability condition is satisfied. For a meaningful degree of capacity utilization the numerator of the equation (2.11) must be positive i.e. $\alpha_0 + \alpha_2 \pi + \alpha_4 \lambda + (1 - s_r - \alpha_3) r^R > 0$. When $(1 - s_r - \alpha_3) > 0$ the numerator is unambiguously positive. But if $(1 - s_r - \alpha_3) < 0$ then for the numerator to be positive $\alpha_0 + \alpha_2 \pi + \alpha_4 \lambda > |(1 - s_r - \alpha_3) r^R|$ is required. Substituting the short-run equilibrium degree of capacity utilization from (2.11) to (2.10) yields the short run equilibrium growth rate of capital stocks as

$$\begin{aligned} g^* &= \alpha_0 + \alpha_1 u^* + \alpha_2 \pi - \alpha_3 r^R + \alpha_4 \lambda \\ \Rightarrow g^* &= \frac{\pi(\alpha_0 + \alpha_2 \pi + \alpha_4 \lambda) + [\alpha_1(1 - s_r) - \alpha_3 \pi] r^R}{(\pi - \alpha_1)} \end{aligned} \quad (2.12)$$

Lemma 1: $(1 - s_r - \alpha_3) < 0 \rightarrow [\alpha_1(1 - s_r) - \alpha_3 \pi] < 0$

Proof: Suppose $(1 - s_r - \alpha_3) < 0$. $(1 - s_r - \alpha_3) < 0$ and $(\pi - \alpha_1) > 0$ implies $\alpha_1(1 - s_r) < \alpha_1 \alpha_3 < \alpha_3 \pi$ which in turn implies $[\alpha_1(1 - s_r) - \alpha_3 \pi] < 0$. ■

Corollary L1: $[\alpha_1(1 - s_r) - \alpha_3 \pi] > 0 \rightarrow (1 - s_r - \alpha_3) > 0$.

Proof: Suppose $[\alpha_1(1 - s_r) - \alpha_3 \pi] > 0$. $[\alpha_1(1 - s_r) - \alpha_3 \pi] > 0$ and $(\pi - \alpha_1) > 0$ implies $\alpha_1(1 - s_r) > \alpha_3 \pi > \alpha_1 \alpha_3$ which in turn implies $(1 - s_r - \alpha_3) > 0$. ■

Now let's check whether the economy is in a wage-led or profit-led demand regime. Partial differentiation of equation (2.11) w.r.t. π yields

$$\frac{\partial u^*}{\partial \pi} = - \left\{ \frac{(\alpha_0 + \alpha_1 \alpha_2 + \alpha_4 \lambda) + (1 - s_r - \alpha_3) r^R}{(\pi - \alpha_1)^2} \right\} \quad (2.13)$$

Note that if $(1 - s_r - \alpha_3) > 0$, (i.e. when the consumption propensity of the rentiers is greater than the responsiveness of the investment demand due to a unit change in the distributed profit) $\frac{\partial u^*}{\partial \pi}$ is unambiguously negative. However if $(1 - s_r - \alpha_3) < 0$, $\frac{\partial u^*}{\partial \pi} \gtrless 0$ according to whether $|1 - s_r - \alpha_3| \gtrless \frac{\alpha_0 + \alpha_1 \alpha_2 + \alpha_4 \lambda}{r^R}$. There is another way of expressing this. Rearranging the equation (2.11) and differentiating it w.r.t. π we get,

$$\begin{aligned} u^* + (\pi - \alpha_1) \frac{\partial u^*}{\partial \pi} &= \alpha_2 \\ \Rightarrow \frac{\partial u^*}{\partial \pi} &= \frac{\alpha_2 - u^*}{(\pi - \alpha_1)} \end{aligned} \quad (2.14)$$

So $\frac{\partial u^*}{\partial \pi} \gtrless 0$ according to whether $\alpha_2 \gtrless u^*$ i.e. whether the economy is in a wage-led or profit-led demand regime depends on the relative value of equilibrium degree of capacity utilization relative to α_2 .

Differentiating g^* w.r.t. π and rearranging we get,

$$\frac{\partial g^*}{\partial \pi} = \frac{(\alpha_2 \pi - \alpha_1 u^*)}{(\pi - \alpha_1)} \quad (2.15)$$

So $\frac{\partial g^*}{\partial \pi} \gtrless 0$ according to whether $\alpha_2 \pi \gtrless \alpha_1 u^*$.

Proposition 1: A profit-led demand regime implies a profit-led growth regime.

Proof: Suppose the economy is in a profit-led demand regime. So $\alpha_2 > u^*$. $\alpha_2 > u^*$ and $(\pi - \alpha_1) > 0$ together imply $\alpha_2 \pi > \pi u^* > \alpha_1 u^*$ which means the economy is in a profit-led growth regime. ■

Corollary P1: A wage-led growth regime implies a wage-led demand regime.

Proof: Straight forward. ■

Now we discuss about the effect of a rise in growth rate of labour productivity (i.e. an improvement in technological progress) on the aggregate demand and on the equilibrium rate of capital accumulation in the following two propositions.

Proposition 2: *When the economy is in a profit-led demand regime, a rise in labour productivity unambiguously increases the aggregate demand while in the wage-led demand regime, the effect is ambiguous and depends on whether $\alpha_4 \gtrless |(\alpha_2 - u^*) \frac{\partial \pi}{\partial \lambda}|$.*

Proof: Differentiation of the equilibrium degree of capacity utilization w.r.t. λ yields

$$\frac{\partial u^*}{\partial \lambda} = \frac{\alpha_4 + (\alpha_2 - u^*) \frac{\partial \pi}{\partial \lambda}}{(\pi - \alpha_1)}$$

In a profit-led demand regime $(\alpha_2 - u^*) > 0$ and so $\frac{\partial u^*}{\partial \lambda}$ is unambiguously positive. But when the economy is in a wage-led demand regime, the effect of a rise in labour productivity on the aggregate demand is ambiguous and depends on whether $\alpha_4 \gtrless |(\alpha_2 - u^*) \frac{\partial \pi}{\partial \lambda}|$ ■

A unit rise in labour productivity on the one hand increases the investment demand by α_4 amount and on the other hand it enhances the share of profit. When the economy is in a profit-led demand regime, these two channels reinforce each other and as a result, there is an unambiguous positive effect of a rise in labour productivity on the aggregate demand. However, when the economy is in a wage-led demand regime, these two channels work in opposite directions and therefore there is an ambiguous result of a rise in labour productivity on the aggregate demand. If the direct impact of a change in labour productivity on the investment demand is higher than the indirect impact of it through the change in share of profit, labour productivity will have positive effect on the aggregate demand and vice-versa.

Proposition 3: *When the economy is in a profit-led growth regime, a rise in labour productivity unambiguously increases the equilibrium rate of capital accumulation while in the wage-led demand regime, the effect is ambiguous and depends on whether $\alpha_4 \pi \gtrless |(\alpha_2 \pi - \alpha_1 u^*) \frac{\partial \pi}{\partial \lambda}|$.*

Proof: Differentiation of the equilibrium rate of capital accumulation w.r.t. λ yields

$$\frac{\partial g^*}{\partial \lambda} = \frac{\alpha_4 \pi + (\alpha_2 \pi - \alpha_1 u^*) \frac{\partial \pi}{\partial \lambda}}{(\pi - \alpha_1)}$$

In a profit-led demand regime $(\alpha_2 \pi - \alpha_1 u^*) > 0$ and so $\frac{\partial g^*}{\partial \lambda}$ is unambiguously positive. But when the economy is in a wage-led demand regime, the effect of a rise in labour productivity on the aggregate demand is ambiguous and depends on whether $\alpha_4 \pi \gtrless |(\alpha_2 \pi - \alpha_1 u^*) \frac{\partial \pi}{\partial \lambda}|$ ■

The economic intuition behind the result is that a rise in labour productivity has a positive direct impact on the rate of capital accumulation and an indirect effect

through its impact on share of profit. When the economy is in a profit-led growth regime, labour productivity enhances share of profit which in turn enhance the growth rate. As a result, the overall effect of a rise in labour productivity on the equilibrium rate of capital accumulation is positive. However, when the economy is in wage-led growth regime, these two effects (direct and indirect) work in opposite directions and as a consequence we get an ambiguous result. If the direct effect of a change in labour productivity is higher than the indirect effect of it through the change in share of profit, labour productivity will have positive effect on the equilibrium rate of capital accumulation and vice-versa.

Note that our results regarding the effect of a rise in growth rate of labour productivity on the aggregate demand and on the equilibrium rate of capital accumulation is different from Hein (2012a). The effect of a rise in growth rate of labour productivity on the aggregate demand and on the equilibrium rate of capital accumulation are always positive in Hein (2012a).

Now we concentrate on the effect of a rise in financialization rate (or a rise in shareholder power) on the equilibrium degree of capacity utilization and accumulation rate. Rearranging and partially differentiation equation (2.11) w.r.t. Ω we get,

$$\begin{aligned} \frac{\partial \pi}{\partial \Omega} u^* + (\pi - \alpha_1) \frac{\partial u^*}{\partial \Omega} &= \frac{\partial \alpha_0}{\partial \Omega} + \alpha_2 \frac{\partial \pi}{\partial \Omega} + (1 - s_r - \alpha_3) \frac{\partial r^R}{\partial \Omega} \\ \Rightarrow \frac{\partial u^*}{\partial \Omega} &= \frac{\overbrace{\frac{\partial \alpha_0}{\partial \Omega}}^{-} + \overbrace{(\alpha_2 - u^*) \frac{\partial \pi}{\partial \Omega}}^{+/-} + \overbrace{(1 - s_r - \alpha_3) \frac{\partial r^R}{\partial \Omega}}^{+/-}}{(\pi - \alpha_1)} \end{aligned} \quad (2.16)$$

The effect of financialization on the equilibrium degree of capacity utilization via the ‘preference channel’, that has been captured by the expression $\frac{\partial \alpha_0}{\partial \Omega}$, is negative. The impact of financialization via the ‘finance channel’, captured by the third term of the numerator, however, is ambiguous and depends on the rentiers’ propensity to save and on the responsiveness of firms’ investment decision with respect to distributed profits. Higher is the dividend payment, lower be the availability of internal fund for investment. However, higher dividend payment, on the other hand, increases rentiers’ consumption demand that in turn indirectly increases the investment demand. The overall effect of the ‘finance channel’ is hence ambiguous. Finally, the second term, that represents the ‘distribution channel’, is also ambiguous. This is happening due to the fact that any of either the wage-led or the profit-led demand regime can prevail in the economy. If there is a wage-led demand regime in the economy, the ‘distribution channel’ is negative. On the other hand in the economy if there is a profit-led demand regime, the ‘distribution

channel' will be positive.

Proposition 4: $((1 - s_r - \alpha_3) < 0) \wedge (|1 - s_r - \alpha_3| < \frac{\alpha_0 + \alpha_1 \alpha_2 + \alpha_4 \lambda}{r^R}) \longrightarrow \frac{\partial u^*}{\partial \Omega} < 0$

Proof: Suppose $(1 - s_r - \alpha_3) < 0$ and $|1 - s_r - \alpha_3| < \frac{\alpha_0 + \alpha_1 \alpha_2 + \alpha_4 \lambda}{r^R}$. These two together imply $\frac{\partial u^*}{\partial \pi} < 0$ which means (from equation(2.14)) $(\alpha_2 - u^*) < 0$. So, $\frac{\partial \alpha_0}{\partial \Omega} < 0$, $(\alpha_2 - u^*) < 0$, $\frac{\partial \pi}{\partial \Omega} > 0$, $(1 - s_r - \alpha_3) < 0$, $\frac{\partial r^R}{\partial \Omega} > 0$ and $(\pi - \alpha_1) > 0$ together imply $\frac{\partial u^*}{\partial \Omega}$ to be unambiguously negative. ■

From proposition 4 we infer that when $(1 - s_r - \alpha_3) < 0$ and the economy is in the wage-led demand regime, a rise in shareholder power (i.e. financialization) will have a contractionary effect on the aggregate demand (or the equilibrium degree of capacity utilization).

Proposition 5: $((1 - s_r - \alpha_3) > 0) \wedge \left((1 - s_r - \alpha_3) > \frac{-\frac{\partial \alpha_0}{\partial \Omega} - (\alpha_2 - u^*) \frac{\partial \pi}{\partial \Omega}}{\frac{\partial r^R}{\partial \Omega}} \right) \longrightarrow \frac{\partial u^*}{\partial \Omega} > 0$

Proof: Suppose $(1 - s_r - \alpha_3) > 0$. This implies $\frac{\partial u^*}{\partial \pi} < 0$ which means (from equation(2.14)) $(\alpha_2 - u^*) < 0$. Now if $(1 - s_r - \alpha_3) > \frac{-\frac{\partial \alpha_0}{\partial \Omega} - (\alpha_2 - u^*) \frac{\partial \pi}{\partial \Omega}}{\frac{\partial r^R}{\partial \Omega}}$ then equation (2.16) yields $\frac{\partial u^*}{\partial \Omega} > 0$. ■

From proposition 5 it can be inferred that when $(1 - s_r - \alpha_3) > 0$ (which implies the economy is in the wage-led demand regime), a rise in shareholder power will have an expansionary effect on the aggregate demand (or the equilibrium degree of capacity utilization) provided $(1 - s_r - \alpha_3) > \frac{-\frac{\partial \alpha_0}{\partial \Omega} - (\alpha_2 - u^*) \frac{\partial \pi}{\partial \Omega}}{\frac{\partial r^R}{\partial \Omega}}$ holds. That means if the 'finance channel' (which is positive here) is sufficiently large, it can overcompensate the depressing effect of other two channels and hence the impact of a rise in financialization on the aggregate demand will be positive. Although proposition 1, 4 and 5 are not explicitly discussed in Hein (2012a), one can easily derive these results from Hein (2012a).

Now let's focus on the impact of financialization on the equilibrium accumulation rate. Rearranging and partially differentiation equation (2.12) w.r.t. Ω we get,

$$\begin{aligned} g^* \frac{\partial \pi}{\partial \Omega} + (\pi - \alpha_1) \frac{\partial g^*}{\partial \Omega} &= (\alpha_0 + \alpha_2 \pi + \alpha_4 \lambda) \frac{\partial \pi}{\partial \Omega} + \pi \left(\frac{\partial \alpha_0}{\partial \Omega} + \alpha_2 \frac{\partial \pi}{\partial \Omega} \right) \\ &\quad + [\alpha_1(1 - s_r) - \alpha_3 \pi] \frac{\partial r^R}{\partial \Omega} - \alpha_2 r^R \frac{\partial \pi}{\partial \Omega} \\ \Rightarrow \frac{\partial g^*}{\partial \Omega} &= \frac{\overbrace{\pi \frac{\partial \alpha_0}{\partial \Omega}}^{-} + \overbrace{[\alpha_1(1 - s_r) - \alpha_3 \pi] \frac{\partial r^R}{\partial \Omega}}^{+/-} + \overbrace{\left(\frac{\alpha_2 \pi - \alpha_1 u^*}{(\pi - \alpha_1)} \right) \frac{\partial \pi}{\partial \Omega}}^{+/-}}{(\pi - \alpha_1)} \end{aligned}$$

The effect of financialization via the ‘preference channel’, that has been captured by the expression $\pi \frac{\partial \alpha_0}{\partial \Omega}$, is negative. The impact of financialization via the ‘finance channel’, that has been captured by the second term of the numerator, however, is ambiguous and depends on the rentiers’ propensity to save and on the responsiveness of firms’ investment decision with respect to distributed profits as well as to capacity utilization. Higher the dividend payment lower the availability of internal fund for investment. However, higher dividend payment, on the other hand, increases rentiers’ consumption demand that in turn indirectly increases the investment demand. The overall effect of the ‘finance channel’ is hence ambiguous. Finally, the third term, that represents the ‘distribution channel’, is also ambiguous. This ambiguity emerges since any kind of growth regime (wage-led or profit-led) is possible in the economy.

As a final result, whether the impact of financialization on capital accumulation is positive (or ‘expansive’) or negative (i.e. ‘contractive’) depends on the values of different parameters. This argument is encapsulated in the following proposition.

Proposition 6:
$$\left((1 - s_r) > \frac{1}{\alpha_1} \left[\frac{-\pi \frac{\partial \alpha_0}{\partial \Omega} - \left(\frac{\alpha_2 \pi - \alpha_1 u^*}{\pi - \alpha_1} \right) \frac{\partial \pi}{\partial \Omega}}{\frac{\partial r^R}{\partial \Omega}} + \alpha_3 \pi \right] \right) \longrightarrow \frac{\partial g^*}{\partial \Omega} > 0$$

Proof: Suppose $(1 - s_r) > \frac{1}{\alpha_1} \left[\frac{-\pi \frac{\partial \alpha_0}{\partial \Omega} - \left(\frac{\alpha_2 \pi - \alpha_1 u^*}{\pi - \alpha_1} \right) \frac{\partial \pi}{\partial \Omega}}{\frac{\partial r^R}{\partial \Omega}} + \alpha_3 \pi \right]$. This, along with $(\pi - \alpha_1) > 0$, imply $\frac{\partial g^*}{\partial \Omega} > 0$. ■

Following Hein (2012a) we can say that the following conditions together ensure the impact of financialization on capital accumulation to be positive or in other word an ‘expansive’ growth regime will prevail if: (i) a low propensity to save out of rentiers’ income (s_r) (ii) less importance of distributed profits (and hence, internal funds) for firms’ investment decisions i.e. smaller value of α_3 , comparatively lower importance of the ‘preference channel’ for firms’ investment decisions relative to the ‘finance channel’, and (iv) a high responsiveness of investment demand with respect to the profit share. Otherwise the ‘contractive’ demand regime of capital accumulation will be obtained. In the next section, we proceed for the long run dynamics.

3 Long Run:

In this section, we analyse the dynamics of the technological progress and financialization rate. We assume that the short run equilibrium values are always

attained in the long run. The long run equilibrium is defined as where technological progress and financialization rate remain constant over time. Let's first focus on the dynamics of technological progress which can be encapsulated by the following three equations.

$$\dot{\lambda} = \theta[\lambda^d - \lambda]; \quad \theta > 0$$

$$\lambda^d = \xi_0 + \xi_1 g + \xi_2(\Omega - \Omega^2) - \xi_3 \pi; \quad \xi_0, \xi_1, \xi_2, \xi_3 > 0 \quad (3.1)$$

$$\text{So, } \dot{\lambda} = \theta[\xi_0 + \xi_1 g + \xi_2(\Omega - \Omega^2) - \xi_3 \pi - \lambda] \quad (3.2)$$

The rate of technological innovation (or progress) varies according to the difference between the desired rate of technological improvement desired by firms (λ^d), and the actual rate of technological progress, λ . Everything else being unchanged, whenever the desired rate is above the actual rate, the actual rate rises. This kind of specification takes into account the existing lags between the moment when expectations are formed and the moment when they are realized.

The desired rate of technological progress depends positively on the rate of capital accumulation and negatively on the profit share. Beside Hein (2010, 2012a, 2012b, 2014) the first type of explanation can be found in (Kaldor 1957, 1961, 1966; Rowthorn 1981; Dutt 1990; Taylor 1991; Lavoie 1992) and the second type in (Taylor 1991; Cassetti 2003; Lima 2004; Naastepad 2006; Dutt 2006, 2013). Here ξ_1 represents the responsiveness of the desired technological change due to a unit change in the accumulation rate whereas ξ_3 denotes the responsiveness of the same due to a unit change in the share of profit.

Following Hein (2010, 2012a, 2012b, 2014) we assume financialization has an impact on the desired technological progress. He concludes this on the basis of the fact that increasing shareholder power (Jensen/ Meckling 1976), higher demanded dividend payouts by shareholders, weaker ability of firms to obtain new equity finance through stock issues (because if it happens share prices decrease), higher threat of hostile takeovers (Manne 1965), and the financial market-oriented remuneration schemes (Fama 1980) push management to use of the resources more efficiently at their disposal. As Hein (2010, 2012a, 2012b, 2014) argues, this should have positive impacts on labour productivity growth (i.e. on technology) and potential growth of the economy, at least initially. However, according Jensen (2005) and Rappaport(2005), as Hein (2010, 2012a, 2012b, 2014) points out, there may be a negative impact on labour productivity if 'shareholder value orientation' goes too far. In that case, share buybacks and dividend payouts can potentially dominate the productivity-enhancing investment, and management's short-termism under-

mines the efficiency and productivity gains. So the effect of shareholder power on productivity growth may be non-linear. However, in his model, he considers only a directly linear positive partial effect of shareholder power on productivity. Some evidence of negative impact of financialization on technology for the US economy can be found in Lazonick (2014) as well. As he points out that although Exxon Mobil spends about \$21 billion a year on buybacks, spends virtually no money on alternative energy research. In 2013 Intel's expenditures on share repurchasing were almost four times the total 'National Nano-technology Initiative' budget that was launched by the US government in 2001. Same is the story for US pharmaceutical companies. Instead of spending sufficient funds on R&D they are spending more on share buybacks. Novelty of our model is that we consider both the positive and negative impact of financialization on technological progress. At a lower level of financialization, an overall positive impact of financialization on technological progress prevails whereas at the higher level of it the negative effect dominates. This argument is captured by the third term of the right-hand side of the equation (3.1)¹⁸. ξ_2 represents the responsiveness of the desired technological change due to $(\Omega - \Omega^2)$ unit change in the rate of financialization. In other words, $\xi_2(1 - 2\Omega)$ represents the responsiveness of the desired technological change due to an unit change in the financialization rate.

ξ_0 is the autonomous part of desired technological progress which represents all catchall variables other than g , Ω and π . One economic explanation for ξ_0 can be the following. For the sake of simplicity, suppose that there is neither any impact of financialization nor there is distributional effect on the desired rate of technological progress desired by the firms. Under this scenario, when there is a stagnation in the economy, it's the intra-class competition among firms that boosts the desired technological change. When the economy is in the period of stagnation, due to lack of sufficient demand, each firm tries to capture the existing market share by out-competing others¹⁹. For this intra-class competition, they desire higher growth in labour productivity (or higher desired technological progress). This phenomenon is being encapsulated by the parameter ξ_0 .

θ represents the speed of adjustment parameter for the technological change dynamics. Higher is the value of θ , more instantaneous the adjustment of actual technological progress to its desired level be. In line with Bhaduri (2006), we

¹⁸Note that if we assume ξ_0 , ξ_1 and ξ_3 to be zero then the technological progress only depends on the financialization rate. In that case $\lambda \Big|_{\lambda=0} = \xi_2(\Omega - \Omega^2)$ and so first order condition implies $\frac{d\lambda}{d\Omega} = \xi_2(1 - 2\Omega) = 0 \implies \Omega^0 = \frac{1}{2}$. So $\forall \Omega \in (0, \Omega^0)$, $\frac{d\lambda}{d\Omega} > 0$ and $\forall \Omega \in (\Omega^0, 1)$, $\frac{d\lambda}{d\Omega} < 0$. But note that $\forall \Omega \in (0, 1)$, $\lambda > 0$.

¹⁹We borrow this idea from Bhaduri (2006a, 2006b) and Shaikh(1978) whereas they themselves find the idea in Marx

can argue that the speed of adjustment parameter, among many other things, depends on the speed of diffusion of technological innovations which in turn is contingent upon the degree of restrictiveness enforced by patents, copyrights and other intellectual property rights.

In equilibrium $\dot{\lambda} = 0$. That implies

$$\lambda \Big|_{\dot{\lambda}=0} = \xi_0 + \xi_1 g + \xi_2(\Omega - \Omega^2) - \xi_3 \pi \quad (3.3)$$

Putting $\Omega = 0$ in the above equation we get the vertical intercept term as $\lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0)$. $g(0)$ represents the equilibrium value of capital accumulation when there is no financialization at all. Similarly $\pi(0)$ represents the share of profit when the financialization rate is zero. Let's assume $\xi_0 + \xi_1 g(0) - \xi_3 \pi(0) > 0$ i.e. there is a positive vertical intercept for the $\dot{\lambda} = 0$ isocline. When there is no financialization at all, $\xi_0 + \xi_1 g(0) - \xi_3 \pi(0)$ represents the desired technological progress desired by the firms. Alternatively speaking, as long as $\xi_0 + \xi_1 g(0) - \xi_3 \pi(0) > 0$, even if there is lack of financialization, positive technological progress is possible. So the assumption $\lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} > 0$ is quite justified.

Slope of the $\dot{\lambda} = 0$ isocline can be yield by differentiating equation (3.3) w.r.t. Ω as

$$\begin{aligned} \frac{d\lambda}{d\Omega} &= \xi_1 \frac{\partial g}{\partial \lambda} \frac{d\lambda}{d\Omega} + \xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \lambda} \frac{d\lambda}{d\Omega} - \xi_3 \frac{\partial \pi}{\partial \Omega} \\ \Rightarrow \frac{d\lambda}{d\Omega} \Big|_{\dot{\lambda}=0} &= \frac{\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}}{1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}} \end{aligned} \quad (3.4)$$

Differentiating (3.2) partially w.r.t. λ we get,

$$\frac{\partial \dot{\lambda}}{\partial \lambda} = \theta \left[\xi_1 \frac{\partial g}{\partial \lambda} - \xi_3 \frac{\partial \pi}{\partial \lambda} - 1 \right] = \theta P \quad (3.5)$$

Differentiating (3.2) partially w.r.t. Ω we get,

$$\frac{\partial \dot{\lambda}}{\partial \Omega} = \theta \left[\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega} \right] = \theta Q \quad (3.6)$$

Another way of getting the slope of the $\dot{\lambda} = 0$ isocline is

$$\frac{d\lambda}{d\Omega} \Big|_{\dot{\lambda}=0} = -\frac{\theta Q}{\theta P} = -\frac{Q}{P} = \frac{\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}}{1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}} \quad (3.7)$$

Throughout this paper we assume $(1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}) > 0$. The justification of

the assumption is the following. Suppose for the time being that $\frac{\partial \pi}{\partial \lambda} = 0$. Then $(1 - \xi_1 \frac{\partial g}{\partial \lambda}) = \frac{(1 - \xi_1 \alpha_4) \pi - \alpha_1}{(\pi - \alpha_1)}$. Then we can easily justify that $(1 - \xi_1 \frac{\partial g}{\partial \lambda}) > 0$ (i.e. $\{(1 - \xi_1 \alpha_4) \pi - \alpha_1\} > 0$). Treec (2008; pp. 396) mentions that for USA for the period 1982-2004, the value of α_1 is 0.26 and the share of profit (π) for the period 1985-2004 is 30.05% (pp. 375). As stated by Knell (2004), the impact of (investment) demand growth on productivity growth (i.e. ξ_1) is 0.43 while Uni (2007) points out it to be 0.44-0.75. However, Hein and Tarassow (2010) find out it to be 0.11 only. The impact of technological progress on the Investment to capital ratio (α_4) is very small too. So the assumption that $\{(1 - \xi_1 \alpha_4) \pi - \alpha_1\} > 0$ is quite justified. However, if $\frac{\partial \pi}{\partial \lambda} > 0$ then the reasoning for $(1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}) > 0$ is the following.

1. Suppose the economy is in a strong wage-led growth regime. The wage-led growth regime is so strong that here not only $(\alpha_2 \pi - \alpha_1 u^*)$ is negative but also $|(\alpha_2 \pi - \alpha_1 u^*)| \frac{\partial \pi}{\partial \lambda} > \alpha_4 \pi$. So here $\frac{\partial g}{\partial \lambda} < 0$ and hence $(1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda})$ is unambiguously positive.

2. Suppose the economy is in a weak wage-led growth regime so that here $|(\alpha_2 \pi - \alpha_1 u^*)| \frac{\partial \pi}{\partial \lambda} < \alpha_4 \pi$. So here $\frac{\partial g}{\partial \lambda} > 0$. Nonetheless $(1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda})$ is positive here

$$\text{as } \frac{\partial g}{\partial \lambda} = \frac{\overbrace{\alpha_4 \pi}^+ + \overbrace{(\alpha_2 \pi - \alpha_1 u^*)}^- \frac{\partial \pi}{\partial \lambda}}{(\pi - \alpha_1)} < \frac{\alpha_4 \pi}{(\pi - \alpha_1)} \text{ and as a result } (1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}) =$$

$$\frac{\overbrace{\{(1 - \xi_1 \alpha_4) \pi - \alpha_1\}}^+ - \overbrace{\left\{ \xi_1 (\alpha_2 \pi - \alpha_1 u^*) \frac{\partial \pi}{\partial \lambda} \right\}}^-}{(\pi - \alpha_1)} + \xi_3 \frac{\partial \pi}{\partial \lambda} > \frac{\{(1 - \xi_1 \alpha_4) \pi - \alpha_1\}}{(\pi - \alpha_1)} > 0.$$

3. Now suppose the economy is in a profit-led growth regime. Then $\frac{\partial g}{\partial \lambda} =$

$$\frac{\overbrace{\alpha_4 \pi}^+ + \overbrace{(\alpha_2 \pi - \alpha_1 u^*)}^- \frac{\partial \pi}{\partial \lambda}}{(\pi - \alpha_1)} > \frac{\alpha_4 \pi}{(\pi - \alpha_1)} > 0. \text{ So there is a possibility that } (1 - \xi_1 \frac{\partial g}{\partial \lambda} +$$

$$\xi_3 \frac{\partial \pi}{\partial \lambda}) = \frac{\overbrace{\{(1 - \xi_1 \alpha_4) \pi - \alpha_1\}}^+ - \overbrace{\left\{ \xi_1 (\alpha_2 \pi - \alpha_1 u^*) \frac{\partial \pi}{\partial \lambda} \right\}}^-}{(\pi - \alpha_1)} + \xi_3 \frac{\partial \pi}{\partial \lambda} < 0. \text{ If we assume the}$$

effect of a rise in labour productivity on the share of profit (i.e. the distributional effect of a change in labour productivity) is adequately weak, or if we assume ξ_3 is sufficiently large then we may have $(1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}) > 0$. For the sake of simplicity let's assume that in the profit led growth regime, the distributional effect of a change in labour productivity is adequately weak or ξ_3 is sufficiently large so that we get $(1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda})$ to be positive.

As long as $(1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}) > 0$ then $\left. \frac{d\lambda}{d\Omega} \right|_{\lambda=0} \geq 0$ depending on whether $\xi_1 \frac{\partial g}{\partial \lambda} +$

$\xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega} \gtrless 0$. $\left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0} = 0$ if $\Omega = \bar{\Omega} = \frac{\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2 - \xi_3 \frac{\partial \pi}{\partial \Omega}}{2\xi_2} = 0$. So, if $\Omega < \bar{\Omega}$ then $\left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0} > 0$ whereas $\Omega > \bar{\Omega}$ ensures $\left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0}$ to be negative. If $\frac{\partial g}{\partial \Omega} < 0$ then $\bar{\Omega}$ is positive only if $\xi_2 > \xi_2^* = (-\xi_1 \frac{\partial g}{\partial \Omega} + \xi_3 \frac{\partial \pi}{\partial \Omega})$. But if $\frac{\partial g}{\partial \Omega} > 0$ then there is a higher chance of $\bar{\Omega}$ to be positive and hence higher the plausibility of existence of steady state at the downward part of the $\dot{\lambda} = 0$ isocline. So, given the value of $\xi_1, \xi_2, \xi_3, \frac{\partial g}{\partial \Omega}$ and $\frac{\partial \pi}{\partial \Omega}$, it's the degree of financialization that plays a crucial role for determining the slope of the $\dot{\lambda} = 0$ isocline. Higher the financialization rate compared to the critical level of Ω (i.e. $\bar{\Omega}$), the negative the slope of the $\dot{\lambda} = 0$ isocline would be and vice versa.

Now suppose $(1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}) < 0$. Then if $\Omega < \bar{\Omega}$ then $\left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0} < 0$ and if $\Omega > \bar{\Omega}$ then $\left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0} > 0$. Next proposition talks about the possibility of stable equilibrium technological progress provided there is no change in financialization rate at all.

Proposition 7: *For a fixed value of Ω , the steady state technological progress is stable.*

Proof: From equation (3.2) we get,

$$\left. \frac{\partial \dot{\lambda}}{\partial \lambda} \right|_{\Omega \text{ is constant}} = \theta \left[\xi_1 \frac{\partial g}{\partial \lambda} - \xi_3 \frac{\partial \pi}{\partial \lambda} - 1 \right] < 0 \quad (\text{by assumption})$$

So for a fixed values of Ω , the equilibrium technological progress is stable. ■

Let's now focus on the change in the financialization parameter. Financialization parameter changes according to the following set of equations as

$$\dot{\Omega} = \phi[\Omega^d - \Omega]; \quad \phi \in (0, 1), \quad \Omega \in [0, 1], \quad \Omega^d \in [0, 1] \quad (3.8)$$

$$\Omega^d = -\eta_0 + \eta_1 \lambda + \eta_2 g; \quad \eta_0, \eta_1, \eta_2 > 0 \quad (3.9)$$

$$\text{So, } \dot{\Omega} = \phi[-\eta_0 + \eta_1 \lambda + \eta_2 g - \Omega] \quad (3.10)$$

The financialization rate varies according to the difference between the desired rate of financialization (Ω^d) arising from firms, and the actual rate of financialization, Ω . *Ceteris paribus*, whenever the desired rate is above the actual rate, the actual rate rises and vice versa. The beauty with this specification is that it takes care of the existing lags between the moment when expectations are formed and the moment when they are realized.

To encapsulate the idea that for a desired rate of financialization to prevail a

minimum aggregate demand (or a minimum good economic condition) and a minimum technological sophistication are required, a negative constant term ($-\eta_0$) is introduced here.

Technological innovations²⁰ (especially innovations in information and communications technology) has a positive impact on the desired financialization rate. It is taken into account by the second term of the right hand side of the equation (3.9). η_1 represents the responsiveness of the desired financialization rate due to one unit change in technological progress. In the long run a good economic condition, which has been captured here by the rate of capital accumulation, positively influence the financialization rate. During good times firms' and financial institutions' optimism provide the environment of adopting riskier financial innovations and setups which increases the rate of financialization. The justification of this argument can be found in Minsky (1986). Minsky (1986) says "During periods of tranquil expansion, profit-seeking financial institutions invent and reinvent 'new' forms of money, substitutes for money in portfolios, and financing techniques for various types of activity: financial innovation is a characteristic of our economy in good times". Minsky (1986, pp 271) also argues "...during good times, when banks are confronted with a large demand for accommodation by apparently credit worthy clients, the banking system is characterized by innovations that try to circumvent Federal Reserve constraint. That is, bankers aim at having assets and non-equity liabilities grow at least as fast (if not faster) than bank equity, whereas the Federal Reserve tries to have bank liabilities subject to check grow at a slower rate than bank equity." However this characteristic can increase financial instability as is clear by this argument of Minsky (1986, pp 354) "[A]s bankers pursue profits they change the composition of their assets and liabilities; in particular, during good times the interactions between bankers and their borrowing customers increase the weight of assets reflecting speculative and Ponzi finance in the balance sheet of banks."

η_2 represents the responsiveness of the desired financialization rate due to one unit change in the degree of capacity utilization, whereas, ϕ^{21} represents the speed of adjustment of the Ω -dynamics.

²⁰Evidence regarding the impact of technological innovations on financialization can be found in Drummer et. al (2017), Frame and White (2010).

²¹We assume $\phi \in (0, 1)$. Otherwise Ω can exceed its maximum value 1. Let's take an example. Suppose $\Omega^d = \frac{1}{2}$, $\Omega = \frac{1}{4}$ and $\phi = 4$. Then from equation (3.8) we get $\dot{\Omega} = 1$ and hence new financialization rate $\Omega' = \Omega + \dot{\Omega} = \frac{5}{4} > 1$. But this is in contradiction with the assumption that $\Omega \in [0, 1]$.

In equilibrium $\dot{\Omega} = 0$. That implies

$$\lambda \Big|_{\dot{\Omega}=0} = \frac{\eta_0 + \Omega - \eta_2 g}{\eta_1} \quad (3.11)$$

Putting $\Omega = 0$ in the above equation we get the vertical intercept term as $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} \geq 0$. $g(0)$ represents the equilibrium rate of capital accumulation when there is no financialization at all. The horizontal intercept for the $\dot{\Omega} = 0$ isocline is $\Omega \Big|_{\dot{\Omega}=0}^{\lambda=0} = \eta_2 g(\lambda = 0) - \eta_0$. Rearranging the vertical intercept of the $\dot{\Omega} = 0$ isocline we get $[-\eta_0 + \eta_1 \lambda + \eta_2 g(0)]$ which represents the desired financialization rate desired by the firms when there is no financialization at all.

Slope of the $\dot{\Omega} = 0$ isocline can be yield by differentiating and rearranging equation (3.11) w.r.t. Ω as

$$\begin{aligned} \eta_1 \frac{d\lambda}{d\Omega} &= 1 - \eta_2 \frac{\partial g}{\partial \lambda} \frac{d\lambda}{d\Omega} - \eta_2 \frac{\partial g}{\partial \Omega} \\ \Rightarrow \frac{d\lambda}{d\Omega} \Big|_{\dot{\Omega}=0} &= \frac{1 - \eta_2 \frac{\partial g}{\partial \Omega}}{\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}} \end{aligned} \quad (3.12)$$

Differentiating (3.10) w.r.t. λ we get,

$$\frac{\partial \dot{\Omega}}{\partial \lambda} = \phi \left[\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda} \right] = \phi M \quad (3.13)$$

Note that as $\frac{\partial g}{\partial \lambda} \geq 0$. For simplicity let's assume $[\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}]$ is always positive i.e. $M > 0$.

Differentiating (3.10) w.r.t. Ω we get,

$$\frac{\partial \dot{\Omega}}{\partial \Omega} = \phi \left[\eta_2 \frac{\partial g}{\partial \Omega} - 1 \right] = \phi N \quad (3.14)$$

Another way of getting the slope of the $\dot{\Omega} = 0$ isocline is N

$$\frac{d\lambda}{d\Omega} \Big|_{\dot{\Omega}=0} = -\frac{\phi N}{\phi M} = -\frac{N}{M} = \frac{1 - \eta_2 \frac{\partial g}{\partial \Omega}}{\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}} \quad (3.15)$$

Now let's check for a given level of technological progress, how the Ω -dynamics behaves. This analysis is captured by the following proposition.

Proposition 8: *For a fixed value of λ , a contractionary effect of financialization on the rate of capital accumulation implies a stable equilib-*

rium financialization rate. However in case of an expansionary effect of financialization on the rate of capital accumulation, an unstable equilibrium financialization rate is possible.

Proof: From equation (3.10) we get,

$$\left. \frac{\partial \dot{\Omega}}{\partial \Omega} \right|_{\lambda \text{ is constant}} = \phi \left[\eta_2 \frac{\partial g}{\partial \Omega} - 1 \right]$$

If $\frac{\partial g}{\partial \Omega} < 0$ then $\left. \frac{\partial \dot{\Omega}}{\partial \Omega} \right|_{\lambda \text{ is constant}}$ is unambiguously negative and hence for a fixed value of λ , there would be a stable equilibrium level of financialization. However, if $\frac{\partial g}{\partial \Omega} > 0$ then sign of $\left. \frac{\partial \dot{\Omega}}{\partial \Omega} \right|_{\lambda \text{ is constant}}$ would be ambiguous. For $\left| \frac{\partial g}{\partial \Omega} \right| > \frac{1}{\eta_2}$, $\left. \frac{\partial \dot{\Omega}}{\partial \Omega} \right|_{\lambda \text{ is constant}} > 0$ is attained. ■

The diagrams regarding the $\dot{\lambda} = 0$ isocline and the $\dot{\Omega} = 0$ isocline are given in figure 3.1.

In the next subsection we discuss the possible cases that may arise due to the interaction between the financialization and technological progress dynamics.

3.1 Possible Cases:

Because of our assumption, $(1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda})$ is positive which ensures P to be negative.

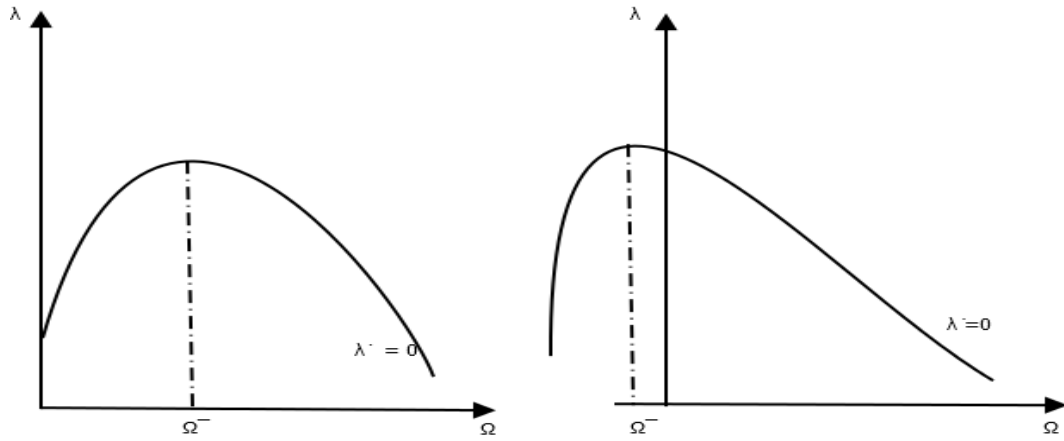
For case 1 we assume $\frac{\partial g^*}{\partial \Omega} < 0$ which in turn implies N to be unambiguously negative.

Case 1.1: Let's assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is negative i.e. $\lambda \left. \frac{\partial \dot{\Omega}}{\partial \Omega} \right|_{\dot{\Omega}=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} < 0$. This is possible when $\eta_2 g(0) > \eta_0$.

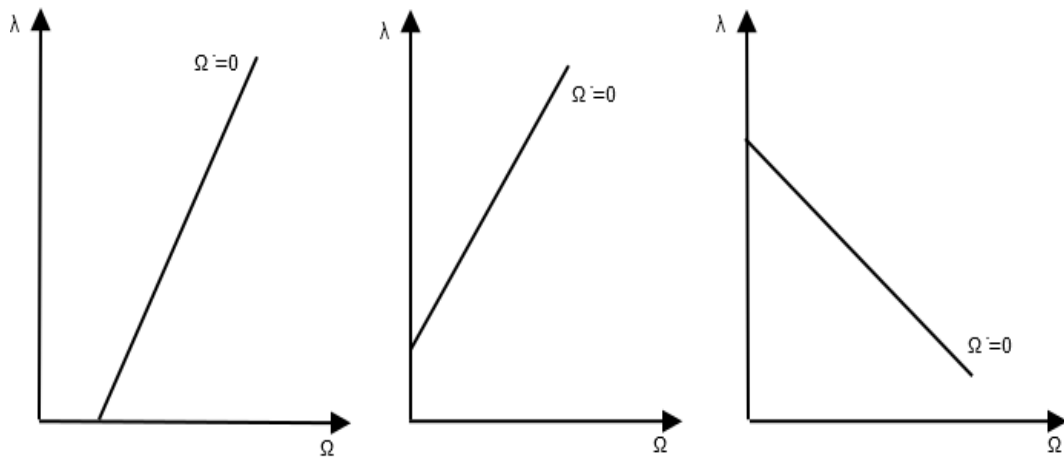
Consider point A: Here $P < 0$ (by assumption), $Q < 0$ (as $\Omega > \bar{\Omega}$), $M > 0$ and $N < 0$ (as $\frac{\partial g^*}{\partial \Omega} < 0$).

At point A, slope of the $\dot{\Omega} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$\begin{aligned} \left. \frac{d\lambda}{d\Omega} \right|_{\dot{\Omega}=0} > 0 > \left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0} \\ \Rightarrow -\frac{\phi N}{\phi M} > -\frac{\theta Q}{\theta P} \end{aligned}$$



(a) Diagram of $\dot{\lambda} = 0$ isocline



(b) Diagram of $\dot{\Omega} = 0$ isocline

Figure 3.1: diagrams of $\dot{\lambda} = 0$ and $\dot{\Omega} = 0$ isoclines

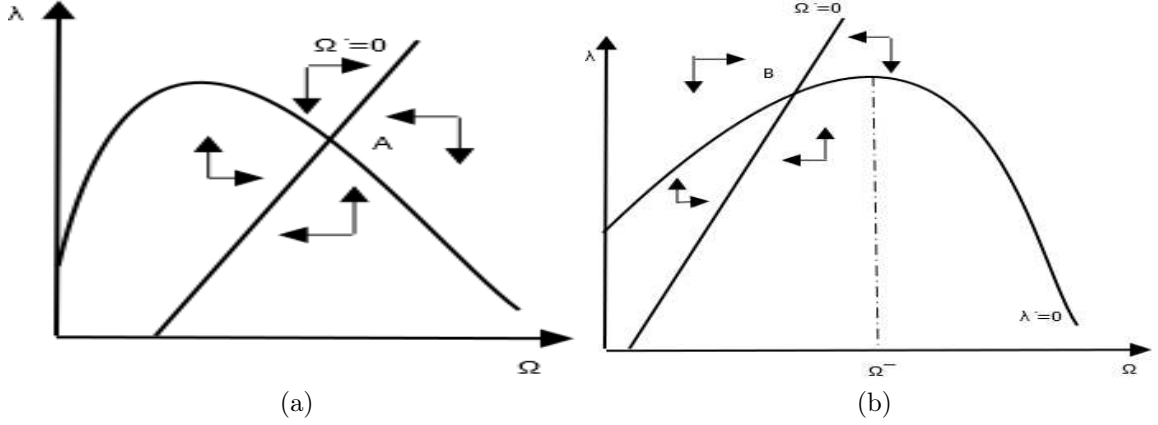


Figure 3.2

$$\Rightarrow \theta\phi(PN - QM) > 0 \quad (\because P < 0 \text{ and } M > 0)$$

So the determinant of the Jacobian matrix $Det(J) = \theta\phi[NP - MQ] > 0$. Trace of the matrix $tr(J) = \theta P + \phi N = [\theta P + \phi Q] < 0$. As a result, point A emerges as a stable steady state. It is represented by figure 3.2(a).

Consider point B: Here $P < 0$ (by assumption), $Q > 0$ (as $\Omega < \bar{\Omega}$), $M > 0$ and $N < 0$ (as $\frac{\partial g^*}{\partial \Omega} < 0$).

At point B, slope of the $\dot{\Omega} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$\begin{aligned} \left. \frac{d\lambda}{d\Omega} \right|_{\dot{\Omega}=0} &> \left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0} > 0 \\ \Rightarrow -\frac{\phi N}{\phi M} &> -\frac{\theta Q}{\theta P} \end{aligned}$$

$$\Rightarrow \theta\phi(PN - QM) > 0 \quad (\because P < 0 \text{ and } M > 0)$$

So the determinant of the Jacobian matrix $Det(J) > 0$. Trace of the matrix $tr(J) = \theta P + \phi N < 0$. Thus point B also is a stable equilibrium which is shown in figure 3.2(b).

Consider point H: Here $P < 0$ (by assumption), $Q = 0$ (as $\Omega = \bar{\Omega}$), $M > 0$ and $N < 0$ (as $\frac{\partial g^*}{\partial \Omega} < 0$).

At point B, slope of the $\dot{\Omega} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$\begin{aligned} \left. \frac{d\lambda}{d\Omega} \right|_{\dot{\Omega}=0} &> \left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0} = 0 \\ \Rightarrow -\frac{\phi N}{\phi M} &> -\frac{\theta Q}{\theta P} \end{aligned}$$

$$\Rightarrow \theta\phi(PN - QM) = \theta\phi PN > 0 \quad (\because P < 0 \text{ and } M > 0)$$

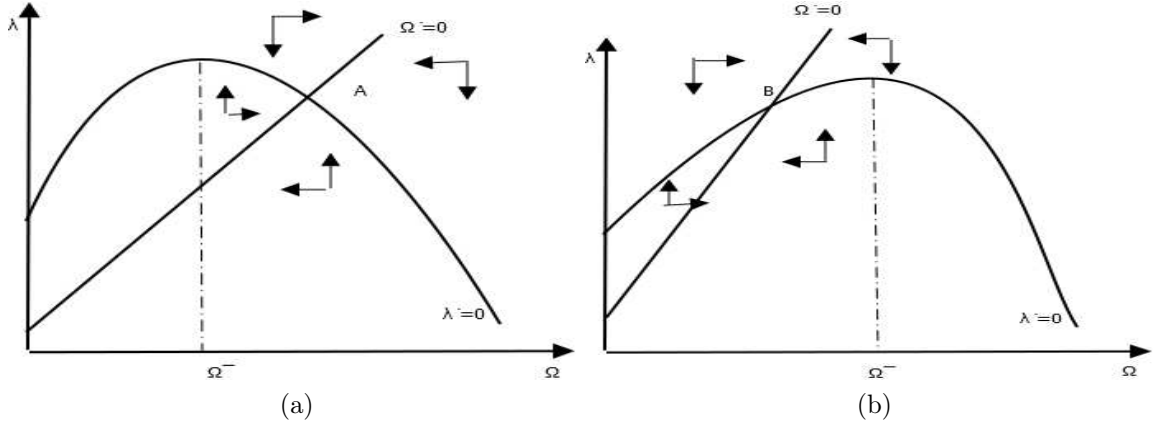


Figure 3.3

So the determinant of the Jacobian matrix $Det(J) > 0$. Trace of the matrix $tr(J) = \theta P + \phi N < 0$. Thus point H also is a stable equilibrium.

From the above three points (A,B & H) we can infer the following proposition

Proposition 9: *In case 1.1, as long as steady state exists, a contractionary effect of financialization on the rate of capital accumulation is sufficient to ensure the stability of the steady state i.e. $(\frac{\partial g^*}{\partial \Omega} < 0) \rightarrow [Det(J) > 0 \wedge tr(J) < 0]$*

Proof: Suppose $\frac{\partial g^*}{\partial \Omega} < 0$. This ensures $N < 0$. For case 1.1 $P < 0$ (by assumption), $Q \gtrless 0$ (as $\Omega \gtrless \bar{\Omega}$) and $M > 0$. For all the above three different types of points, as we see, the determinant of the Jacobian matrix is positive and the trace is negative. As a result, all the above three different types of equilibria are stable steady states. ■

Now focus on the next sub-case.

Case 1.2: Let's assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is positive i.e. $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > 0$. This is possible when $\eta_2 g(0) < \eta$. Let's also assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is less than the vertical intercept of the $\dot{\lambda} = 0$ isocline i.e. $\lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0) > \lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > 0$.

The equilibrium can be either point A, point B or point H. The analysis is same as case 1.1. The diagram for this sub-case is represented by figure 3.3(a) & 3.3(b) respectively.

Next, we discuss the last sub-case.

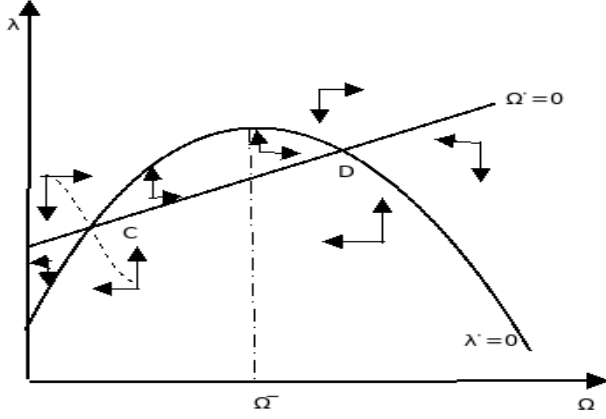


Figure 3.4

Case 1.3: Let's assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is positive and is greater than the vertical intercept of the $\dot{\lambda} = 0$ isocline i.e. $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > \lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0) > 0$.

Consider point C: Here $P < 0$ (by assumption), $Q > 0$ (as $\Omega < \bar{\Omega}$), $M > 0$ and $N < 0$ (as $\frac{\partial g^*}{\partial \Omega} < 0$).

At point A, slope of the $\dot{\Omega} = 0$ isocline is less than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$\begin{aligned} \frac{d\lambda}{d\Omega} \Big|_{\dot{\lambda}=0} &> \frac{d\lambda}{d\Omega} \Big|_{\dot{\Omega}=0} > 0 \\ \Rightarrow -\frac{\theta Q}{\theta P} &> -\frac{\phi N}{\phi M} \\ \Rightarrow \theta \phi (PN - QM) &< 0 \quad (\because P < 0 \text{ and } M > 0) \end{aligned}$$

So the determinant of the Jacobian matrix $Det(J) < 0$ and hence point C is a saddle point.

Consider point D: Here $P < 0$ (by assumption), $Q < 0$ (as $\Omega > \bar{\Omega}$), $M > 0$ and $N < 0$ (as $\frac{\partial g^*}{\partial \Omega} < 0$).

At point A, slope of the $\dot{\Omega} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$\begin{aligned} \frac{d\lambda}{d\Omega} \Big|_{\dot{\Omega}=0} &> 0 > \frac{d\lambda}{d\Omega} \Big|_{\dot{\lambda}=0} \\ \Rightarrow -\frac{\phi N}{\phi M} &> -\frac{\theta Q}{\theta P} \\ \Rightarrow \theta \phi (PN - QM) &> 0 \quad (\because P < 0 \text{ and } M > 0) \end{aligned}$$

So the determinant of the Jacobian matrix $Det(J) > 0$. Trace of the matrix

$tr(J) = \theta P + \phi N < 0$. Thus point D is a stable equilibrium. Diagram for case 1.3 is given in figure 3.4.

For the next case, Case 2, we assume $\frac{\partial g^*}{\partial \Omega} > 0$ but $\frac{\partial g^*}{\partial \Omega} < \frac{1}{\eta_2}$ i.e. N is negative.

Case 2.1: Let's assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is negative i.e. $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} < 0$. This is possible when $\eta_2 g(0) > \eta_0$. Rest of the analysis is same as case 1.1.

Case 2.2: Let's assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is positive i.e. $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > 0$. This is possible when $\eta_2 g(0) < \eta_0$. Let's also assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is less than the vertical intercept of the $\dot{\lambda} = 0$ isocline i.e. $\lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0) > \lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > 0$.

Rest of the analysis is same as case 1.1.

Case 2.3: Let's assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is positive and is greater than the vertical intercept of the $\dot{\lambda} = 0$ isocline i.e. $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > \lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0) > 0$. Rest of the analysis is same as Case 1.3.

From cases 1 & 2 we can infer the following argument. Suppose the vertical intercept of the $\dot{\Omega} = 0$ isocline is less than the vertical intercept of the $\dot{\lambda} = 0$ isocline i.e. $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > \lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0)$. Then the following proposition holds.

Proposition 10: *As long as a steady state exists and the intercept of the $\dot{\Omega} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline, a contractionary effect of financialization on the rate of capital accumulation is sufficient to ensure the stability of the steady state. However, if the effect of financialization on the aggregate demand is expansionary but not too strong (i.e. $0 < \frac{\partial g^*}{\partial \Omega} < \frac{1}{\eta_2}$) then also a stable equilibrium can be achieved.*

Proof: See appendix A for the proof. ■

For the next case, case 3, we assume $\frac{\partial g^*}{\partial \Omega} > 0$ and $\frac{\partial g^*}{\partial \Omega} > \frac{1}{\eta_2}$ i.e. N is positive.

Case 3.1: Let's assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is negative i.e. $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} < 0$. This is possible when $\eta_2 g(0) > \eta_0$. No steady state is possible here.

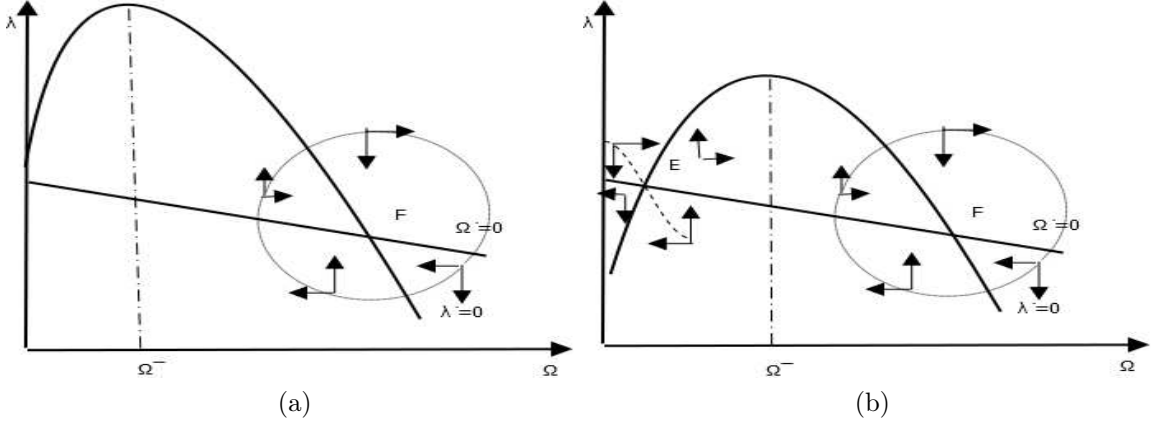


Figure 3.5

Case 3.2: Let's assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is positive i.e. $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > 0$. This is possible when $\eta_2 g(0) < \eta$. Let's also assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is less than the vertical intercept of the $\dot{\lambda} = 0$ isocline i.e. $\lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0) > \lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > 0$.

Consider the only possible steady-state point F: Here $P < 0$ (by assumption), $Q < 0$ (as $\Omega > \bar{\Omega}$), $M > 0$ and $N > 0$ (as $\frac{\partial g^*}{\partial \Omega} > \frac{1}{\eta_2} > 0$).

At point F, slope of the $\dot{\Omega} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$\begin{aligned} 0 > \frac{d\lambda}{d\Omega} \Big|_{\dot{\Omega}=0} > 0 \frac{d\lambda}{d\Omega} \Big|_{\dot{\lambda}=0} \\ \Rightarrow -\frac{\phi N}{\phi M} > -\frac{\theta Q}{\theta P} \\ \Rightarrow \theta \phi (PN - QM) > 0 \quad (\because P < 0 \text{ and } M > 0) \end{aligned}$$

So the determinant of the Jacobian matrix $Det(J) > 0$. Trace of the matrix $tr(J) = \theta P + \phi N \gtrless 0$. Thus point F can be anything: stable or unstable equilibrium. See figure 3.5(a) for the diagrammatic explanation.

Case 3.3: Let's assume that the vertical intercept of the $\dot{\Omega} = 0$ isocline is positive and is greater than the vertical intercept of the $\dot{\lambda} = 0$ isocline i.e. $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > \lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0) > 0$.

Consider point E: Here $P < 0$ (by assumption), $Q > 0$ (as $\Omega < \bar{\Omega}$), $M > 0$ and $N > 0$ (as $\frac{\partial g^*}{\partial \Omega} > \frac{1}{\eta_2} > 0$).

At point E, slope of the $\dot{\Omega} = 0$ isocline is less than the slope of the $\dot{\lambda} = 0$ isocline

i.e.

$$\begin{aligned} \frac{d\lambda}{d\Omega} \Big|_{\dot{\lambda}=0} &> 0 > \frac{d\lambda}{d\Omega} \Big|_{\dot{\Omega}=0} \\ &\Rightarrow -\frac{\theta Q}{\theta P} > -\frac{\phi N}{\phi M} \\ &\Rightarrow \theta\phi(PN - QM) < 0 \quad (\because P < 0 \text{ and } M > 0) \end{aligned}$$

So the determinant of the Jacobian matrix $Det(J) < 0$. Thus point E is a saddle point.

Consider point F: Here the analysis is same as Case 3.2. See figure 3.5(b) for the diagrammatic explanation.

3.2 Andronov-Hopf Bifurcation:

In this sub-section, we discuss the possibilities of emergence of cycle as a solution to the dynamical systems represented by equation (3.2) and (3.10). Consider the steady state F of the case 3.2 and/or 3.3. We get the following proposition.

Proposition 11: *For an appropriate value of the speed of adjustment parameter, θ , the characteristic equation to (3.2) & (3.10) evaluated at the steady state F of the case 3.2 and 3.3 have purely imaginary roots and for the same dynamical system, $\theta = \hat{\theta}$ provides a point of Andronov-Hopf bifurcation²².*

Proof: Provided in appendix B. ■

Note that for $\theta < \hat{\theta}$, the trace become positive and hence we have an unstable equilibrium. However when $\theta > \hat{\theta}$, the equilibrium is stable. When θ falls to $\hat{\theta}$, the system with a stable equilibrium point loses its stability and gives birth to a limit cycle. Similarly if θ rises to $\hat{\theta}$, the system with an unstable steady state produces a limit cycle²³. We already have discussed that the speed of adjustment parameter depends on the speed of diffusion of technological innovations. Bhaduri (2006b) points out that faster diffusion rate of technological innovations is an important parameter for fueling growth whereas at the very same time it has potential to destabilize the steady growth path. In our model, on the contrary, higher speed of diffusion of technological innovation is not necessary for fueling growth (as it cannot stimulate the technological change itself), but is important for stabilizing

²²For further discussion regarding Hopf-bifurcation see Gandolfo (1997), Izhikevich (2007)

²³Note that the limit cycle can arise only when the expansionary effect of financialization on the aggregate demand prevails.

the economy. So government intervention for loosening the degree of restrictiveness enforced various intellectual property rights are desirable for ensuring the stability in the economy.

Proposition 12: *For an appropriate value of the speed of adjustment parameter, ϕ , the characteristic equation to (3.2) & (3.10) evaluated at the steady state F of the case 3.2 and 3.3 have purely imaginary roots and for the same dynamical system, $\phi = \hat{\phi}$ provides a point of Andronov-Hopf bifurcation.*

Proof: Provided in appendix C. ■

Note that for $\phi > \hat{\phi}$, the trace become positive and hence we have an unstable equilibrium. However when $\phi < \hat{\phi}$, the equilibrium is stable. When ϕ rises to $\hat{\phi}$, the system with a stable equilibrium point loses its stability and gives birth to a limit cycle. Similarly if ϕ falls to $\hat{\phi}$, the system with an unstable steady state produces a limit cycle. This speed of adjustment parameter ϕ that is associated with the change in financialization rate, among many other things, depends on governments role on the (de)regulation of the financial markets. Higher the deregulation of the financial sector, higher is the speed of adjustment associated with the change in financialization rate and hence higher is the possibility that the stable system losing its stability produces limit cycle. So government intervention for a more regulated financial market is desirable for ensuring stability of the system (economy).

3.3 Comparative Static:

In this sub-section we investigate how various parameters influence the equilibrium values of technological progress and financialization rate.

3.3.1 Effect of a change in ξ_0 :

Differentiating both side of the equation (3.2) w.r.t. ξ_0 and rearranging we get,

$$P \frac{\partial \lambda}{\partial \xi_0} + Q \frac{\partial \Omega}{\partial \xi_0} = -1 \quad (3.16)$$

Differentiating both side of the equation (3.10) w.r.t. ξ_0 and rearranging we get,

$$M \frac{\partial \lambda}{\partial \xi_0} + N \frac{\partial \Omega}{\partial \xi_0} = 0 \quad (3.17)$$

where $P = (\xi_1 \frac{\partial g}{\partial \lambda} - \xi_3 \frac{\partial \pi}{\partial \lambda} - 1)$; $Q = [\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}]$; $M = [\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}]$; $N = [\eta_2 \frac{\partial g}{\partial \Omega} - 1]$

Rewriting equation (3.16) & (3.17) in matrix form we get,

$$\begin{pmatrix} P & Q \\ M & N \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial \xi_0} \\ \frac{\partial \Omega}{\partial \xi_0} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad (3.18)$$

$$\text{So, } \frac{\partial \lambda}{\partial \xi_0} = \frac{-N}{(NP - MQ)} \quad (3.19)$$

$$\text{and } \frac{\partial \Omega}{\partial \xi_0} = \frac{M}{(NP - MQ)} \quad (3.20)$$

Case 1.1:

Consider point A: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.16) we get $\frac{\partial \lambda}{\partial \xi_0} = \frac{-N}{(NP - MQ)} > 0$ and from equation (3.20) we can say $\frac{\partial \Omega}{\partial \xi_0} = \frac{M}{(NP - MQ)} > 0$. Thus as ξ_0 increases, both the equilibrium value of λ^* and Ω^* upsurge and a fall in ξ_0 has a dwindling effect on both λ^* and Ω^* . When $\frac{\partial g^*}{\partial \lambda} > 0$, impact of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is ambiguous which is shown by the following equation.

$$\frac{dg^*}{d\xi_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \xi_0}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \xi_0}}^+ \right\} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (3.21)$$

However, if there is a strong wage-led growth regime i.e. if $\frac{\partial g^*}{\partial \lambda} < 0$, then the impact of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is unambiguously negative. It is captured by the following equation

$$\frac{dg^*}{d\xi_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^- \overbrace{\frac{\partial \lambda}{\partial \xi_0}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \xi_0}}^+ \right\} < 0 \quad (3.22)$$

The above analysis can be represented through diagram as well. The vertical intercept of the $\dot{\lambda} = 0$ curve is $\lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0)$. So partially differentiating it with respect to ξ_0 yields,

$$\frac{\partial \left(\lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} \right)}{\partial \xi_0} = 1 > 0. \quad (3.23)$$

Slope of the $\dot{\lambda} = 0$ isocline is $\left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0} = \frac{\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2 (1-2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}}{1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}}$ and so the slope is invariant with respect to the change in ξ_0 . Thus when ξ_0 rises, only the vertical intercept increases. So due to a rise in ξ_0 , the $\dot{\lambda} = 0$ isocline shifts to the upward direction. See figure 3.6(a).

Consider point B: Here $P < 0, Q > 0, M > 0, N < 0$. So from equation (3.16) we get $\frac{\partial \lambda}{\partial \xi_0} = \frac{-N}{(NP-MQ)} > 0$ and from equation (3.20) we can say $\frac{\partial \Omega}{\partial \xi_0} = \frac{M}{(NP-MQ)} > 0$. Therefore an expansion in ξ_0 upsurges both the equilibrium value of λ^* and Ω^* whereas a decline in ξ_0 plummets the equilibrium value of λ^* and Ω^* together. See figure 3.6(b) for the diagram. Here also when $\frac{\partial g^*}{\partial \lambda} > 0$ the impact of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is ambiguous (which is shown by equation 3.21) and if there is a strong wage-led growth regime (i.e. if $\frac{\partial g^*}{\partial \lambda} < 0$), then the impact of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is unambiguously negative.

So, from point A and B we can conclude that regardless of the initial value of the financialization rate (i.e. whether $\Omega > \bar{\Omega}$ or $\Omega < \bar{\Omega}$), the effect of a rise in ξ_0 is expansionary for both the equilibrium value of λ^* and Ω^* .

Case 1.3:

Consider point D: Here $P < 0, Q < 0, M > 0, N < 0$. So from equation (3.16) we get $\frac{\partial \lambda}{\partial \xi_0} = \frac{-N}{(NP-MQ)} > 0$ and from equation (3.20) we can say $\frac{\partial \Omega}{\partial \xi_0} = \frac{M}{(NP-MQ)} > 0$. So ξ_0 has a positive impact on both the equilibrium value of λ^* and Ω^* . It is shown in figure 3.6(c). As case 1.1 given that $\frac{\partial g^*}{\partial \lambda} > 0$, here also the impact of a rise in ξ_0 on the equilibrium rate of capital accumulation is ambiguous. However, in a strong wage-led growth regime the impact of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is negative.

Note that if ξ_0 decreases the two equilibria C and D come closer and hence the stability corridor shrinks. Hence the economy is now more fragile and a shock (which could previously be absorbed by the stable equilibrium point) can shift the equilibrium from the stable to the saddle point. For a sufficient fall in ξ_0 , both the equilibria can converge to a saddle point. See figure 3.6(d). So, we can infer that in the period of stagnation, it's the intra-class conflict among firms that plays an important role. It is important not only for the new equilibrium level of λ^* and Ω^* to achieve, but for ensuring the stability as well. In the period of stagnation, a lower intra-class conflict among firms can shrink the stability corridor and for a sufficiently low level of intra-class conflict, ultimately a saddle point can emerge.

Case 2.1:

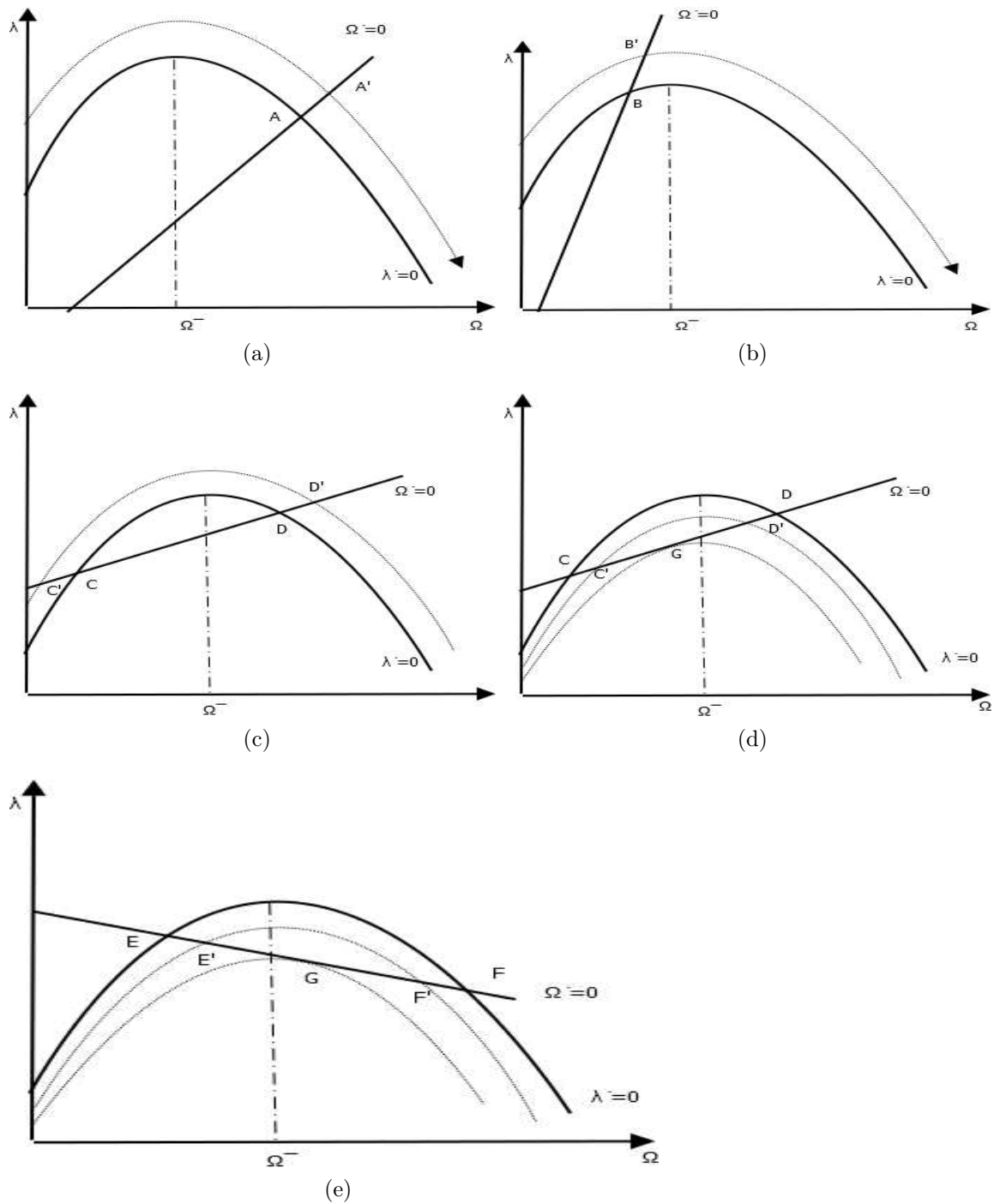


Figure 3.6: Effect of a change in ξ_0

Here $\frac{1}{\eta_2} > \frac{\partial g^*}{\partial \Omega} > 0$. The analysis is same as case 1.1. However, when $\frac{\partial g^*}{\partial \lambda} > 0$, a rise in ξ_0 has a positive effect on the equilibrium rate of capital accumulation which is shown in the following equation.

$$\frac{dg^*}{d\xi_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \xi_0}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \xi_0}}^+ \right\} > 0 \quad (3.24)$$

However, if there is a strong wage-led growth regime then the impact of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is ambiguous. It is captured by the following equation

$$\frac{dg^*}{d\xi_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^- \overbrace{\frac{\partial \lambda}{\partial \xi_0}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \xi_0}}^+ \right\} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (3.25)$$

Case 2.2:

The analysis is same as case 1.1 except the fact that as $\frac{\partial g^*}{d\xi_0} > 0$, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is positive while in a strong wage-led growth regime, the effect of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is ambiguous.

Case 2.3:

The analysis is same as case 1.3 except the fact that as $\frac{\partial \Omega}{d\xi_0} > 0$, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is positive while in a strong wage-led growth regime, the effect of a rise in ξ_0 on the long run equilibrium rate of capital accumulation is ambiguous.

Case 3.2/3.3:

Consider point F: Here $P < 0$, $Q < 0$, $M > 0$, $N > 0$. So from equation (3.16) we get $\frac{\partial \lambda}{\partial \xi_0} = \frac{-N}{(NP-MQ)} < 0$ and from equation (3.20) we can say $\frac{\partial \Omega}{\partial \xi_0} = \frac{M}{(NP-MQ)} > 0$. Thus for an increment in ξ_0 , the equilibrium value of Ω^* rises but λ^* falls and for a decline in ξ_0 , the opposite happens. When $\frac{\partial g^*}{\partial \lambda} > 0$, the impact of a rise in ξ_0 on the equilibrium rate of capital accumulation is, however, ambiguous which is encapsulated by the following equation.

$$\frac{dg^*}{d\xi_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \xi_0}}^- + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \xi_0}}^+ \right\} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (3.26)$$

However, when $\frac{\partial g^*}{\partial \lambda} < 0$, the impact of a rise in ξ_0 on the equilibrium rate of capital accumulation is unambiguously positive.

Note that if ξ_0 decreases, in case 3.3 the two equilibria E and F coming closer shrinks the stability corridor and for an appropriate fall in ξ_0 , both the equilibria unite to a saddle point. See figure 3.6(e).

3.3.2 Effect of a change in ξ_1 :

Differentiating both side of the equation (3.2) w.r.t. ξ_1 and rearranging we get,

$$P \frac{\partial \lambda}{\partial \xi_1} + Q \frac{\partial \Omega}{\partial \xi_1} = -g \quad (3.27)$$

Differentiating both side of the equation (3.10) w.r.t. ξ_1 and rearranging we get,

$$M \frac{\partial \lambda}{\partial \xi_1} + N \frac{\partial \Omega}{\partial \xi_1} = 0 \quad (3.28)$$

where $P = (\xi_1 \frac{\partial g}{\partial \lambda} - \xi_3 \frac{\partial \pi}{\partial \lambda} - 1)$; $Q = [\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}]$; $M = [\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}]$; $N = [\eta_2 \frac{\partial g}{\partial \Omega} - 1]$

Rewriting equation (3.27) & (3.28) in matrix form we get,

$$\begin{pmatrix} P & Q \\ M & N \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial \xi_1} \\ \frac{\partial \Omega}{\partial \xi_1} \end{pmatrix} = \begin{pmatrix} -g \\ 0 \end{pmatrix} \quad (3.29)$$

$$\text{So, } \frac{\partial \lambda}{\partial \xi_1} = \frac{-Ng}{(NP - MQ)} \quad (3.30)$$

$$\text{and } \frac{\partial \Omega}{\partial \xi_1} = \frac{Mg}{(NP - MQ)} \quad (3.31)$$

Case 1.1:

Consider point A: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.27) we get $\frac{\partial \lambda}{\partial \xi_1} = \frac{-Ng}{(NP - MQ)} > 0$ and from equation (3.31) we can say $\frac{\partial \Omega}{\partial \xi_1} = \frac{Mg}{(NP - MQ)} > 0$. Subsequently, as ξ_1 improves, both the equilibrium value of λ^* and Ω^* upsurge and when ξ_1 decreases, as a result we have a diminution in the equilibrium value of both λ^* and Ω^* . The impact of a rise in ξ_1 on the equilibrium rate of capital accumulation, provided , is ambiguous which is encapsulated by the following

equation.

$$\frac{dg^*}{d\xi_1} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \xi_1}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \xi_1}}^+ \right\} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (3.32)$$

However, when $\frac{\partial g^*}{\partial \lambda} < 0$, $\frac{dg^*}{d\xi_1} < 0$.

The above analysis can be represented through diagram as well. The vertical intercept of the $\dot{\lambda} = 0$ curve is $\lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0)$. So partially differentiating it with respect to ξ_1 yields,

$$\frac{\partial \left(\lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} \right)}{\partial \xi_1} = g(0) > 0. \quad (3.33)$$

We are not emphasizing on the slope of the $\dot{\lambda} = 0$ isocline as it is not changing the qualitative result significantly. So due to a rise in ξ_1 , the $\dot{\lambda} = 0$ isocline shifts to the upward direction.

Consider point B: Here $P < 0$, $Q > 0$, $M > 0$, $N < 0$. So from equation (3.27) we get $\frac{\partial \lambda}{\partial \xi_1} = \frac{-Ng}{(NP-MQ)} > 0$ and from equation (3.31) we can say $\frac{\partial \Omega}{\partial \xi_1} = \frac{Mg}{(NP-MQ)} > 0$. Thus for an upsurge in ξ_1 , both the equilibrium value of λ^* and Ω^* improve whereas for a fall in ξ_1 , exactly the opposite happens. Here also, when the economy is in profit-led or in a weak wage-led growth regime, the impact of a rise in ξ_1 on the long run equilibrium rate of capital accumulation is ambiguous (which can be shown by the same equation (3.32)). However, in a strong wage-led growth regime, the impact of a rise in ξ_1 on the equilibrium rate of capital accumulation is unambiguously negative.

Case 1.3:

Consider point D: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.27) we get $\frac{\partial \lambda}{\partial \xi_1} = \frac{-Ng}{(NP-MQ)} > 0$ and from equation (3.31) we can say $\frac{\partial \Omega}{\partial \xi_1} = \frac{Mg}{(NP-MQ)} > 0$. Therefore as ξ_1 increases, both the equilibrium value of λ^* and Ω^* rise and when ξ_1 decreases, the equilibrium value of both λ^* and Ω^* fall. It is to be noted that for a reduction in ξ_1 the two equilibria C and D move closer and for an adequate fall in ξ_1 , both the equilibria by converging to each other generate a saddle point. As case 1.1 given that $\frac{\partial g^*}{\partial \lambda} > 0$, here also the impact of a rise in ξ_1 on the equilibrium rate of capital accumulation is ambiguous. However, in a strong wage-led growth regime the impact of a rise in ξ_1 on the long run equilibrium rate of capital accumulation is negative.

Case 2.1:

Here $\frac{1}{\eta_2} > \frac{\partial g^*}{\partial \Omega} > 0$. The analysis is same as case 1.1. However, when $\frac{\partial g^*}{\partial \lambda} > 0$, a rise in ξ_1 has a positive effect on the equilibrium rate of capital accumulation which is shown in the following equation.

$$\frac{dg^*}{d\xi_1} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \xi_1}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \xi_1}}^+ \right\} > 0 \quad (3.34)$$

However, if there is a strong wage-led growth regime then the impact of a rise in ξ_1 on the long run equilibrium rate of capital accumulation is ambiguous. It is captured by the following equation

$$\frac{dg^*}{d\xi_1} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^- \overbrace{\frac{\partial \lambda}{\partial \xi_1}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \xi_1}}^+ \right\} \begin{matrix} \geq \\ < \end{matrix} 0 \quad (3.35)$$

Case 2.2:

The analysis is same as case 1.1 except the fact that as $\frac{\partial g^*}{d\Omega} > 0$, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in ξ_1 on the long run equilibrium rate of capital accumulation is positive while in a strong wage-led growth regime, the effect of a rise in ξ_1 on the long run equilibrium rate of capital accumulation is ambiguous.

Case 2.3:

The analysis is same as case 2.2.

Case 3.2/3.3:

Consider point F: Here $P < 0$, $Q < 0$, $M > 0$, $N > 0$. So from equation (3.27) we get $\frac{\partial \lambda}{\partial \xi_1} = \frac{-Ng}{(NP-MQ)} < 0$ and from equation (3.31) we can say $\frac{\partial \Omega}{\partial \xi_1} = \frac{Mg}{(NP-MQ)} > 0$. Thus as ξ_1 increases, there is an improvement in the equilibrium value of Ω^* while λ^* deteriorates and a reduction in ξ_1 leads, the opposite to happen. When $\frac{\partial g^*}{\partial \lambda} > 0$, the impact of a rise in ξ_1 on the equilibrium rate of capital accumulation is, however, ambiguous which is encapsulated by the following equation.

$$\frac{dg^*}{d\xi_1} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \xi_1}}^- + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \xi_1}}^+ \right\} \begin{matrix} \geq \\ < \end{matrix} 0 \quad (3.36)$$

On the other hand, if $\frac{\partial g^*}{\partial \lambda} < 0$ an increase in ξ_1 unambiguously upsurges the long run equilibrium rate of capital accumulation.

It is worth remembering that a decline in ξ_1 leads the two equilibria E and F to come closer and for a sufficient diminution in ξ_1 , both the equilibria converge to a saddle point.

3.3.3 Effect of a change in ξ_3 :

Differentiating both side of the equation (3.2) w.r.t. ξ_3 and rearranging we get,

$$P \frac{\partial \lambda}{\partial \xi_3} + Q \frac{\partial \Omega}{\partial \xi_3} = \pi \quad (3.37)$$

Differentiating both side of the equation (3.10) w.r.t. ξ_3 and rearranging we get,

$$M \frac{\partial \lambda}{\partial \xi_3} + N \frac{\partial \Omega}{\partial \xi_3} = 0 \quad (3.38)$$

where $P = (\xi_1 \frac{\partial g}{\partial \lambda} - \xi_3 \frac{\partial \pi}{\partial \lambda} - 1)$; $Q = [\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}]$; $M = [\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}]$; $N = [\eta_2 \frac{\partial g}{\partial \Omega} - 1]$

Rewriting equation (3.37) & (3.38) in matrix form we get,

$$\begin{pmatrix} P & Q \\ M & N \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial \xi_3} \\ \frac{\partial \Omega}{\partial \xi_3} \end{pmatrix} = \begin{pmatrix} \pi \\ 0 \end{pmatrix} \quad (3.39)$$

$$\text{So, } \frac{\partial \lambda}{\partial \xi_3} = \frac{\pi N}{(NP - MQ)} \quad (3.40)$$

$$\text{and } \frac{\partial \Omega}{\partial \xi_3} = \frac{-\pi M}{(NP - MQ)} \quad (3.41)$$

Case 1.1:

Consider point A: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.37) we get $\frac{\partial \lambda}{\partial \xi_3} = \frac{\pi N}{(NP - MQ)} < 0$ and from equation (3.41) we can say $\frac{\partial \Omega}{\partial \xi_3} = \frac{-\pi M}{(NP - MQ)} < 0$. Hence as ξ_3 increases, both the equilibria (λ^* and Ω^*) shrink and for a diminution in ξ_3 , the reverse takes place. When the economy is in a profit-led or weak wage-led growth regime, impact of a rise in ξ_3 on the equilibrium rate of capital accumulation is ambiguous (which is encapsulated by the next equation). However, when there is a strong wage-led growth regime in the economy, a positive

effect of a rise in ξ_3 on the long run equilibrium rate of capital accumulation prevails.

$$\frac{dg^*}{d\xi_3} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \xi_3}}^- + \overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \xi_3}}^- \right\} \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \quad (3.42)$$

The above analysis can be represented through diagram as well (See figure 3.7(a)). The vertical intercept of the $\dot{\lambda} = 0$ curve is $\lambda \Big|_{\lambda=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0)$. So partially differentiating it with respect to ξ_3 yields,

$$\frac{\partial \left(\lambda \Big|_{\lambda=0}^{\Omega=0} \right)}{\partial \xi_3} = -\pi(0) < 0. \quad (3.43)$$

Slope of the $\dot{\lambda} = 0$ isocline is $\frac{d\lambda}{d\Omega} \Big|_{\lambda=0} = \frac{\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2 (1-2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}}{1 - \xi_1 \frac{\partial g}{\partial \lambda} + \xi_3 \frac{\partial \pi}{\partial \lambda}}$ and so the slope becomes flatter for all $\Omega < \bar{\Omega}$ and steeper for all $\Omega > \bar{\Omega}$ as there is a rise in ξ_3 . So due to a rise in ξ_3 , the $\dot{\lambda} = 0$ isocline shifts to the downward direction.

Consider point B: Here $P < 0$, $Q > 0$, $M > 0$, $N < 0$. So from equation (3.37) we get $\frac{\partial \lambda}{\partial \xi_3} = \frac{\pi N}{(NP-MQ)} < 0$ and from equation (3.41) we can say $\frac{\partial \Omega}{\partial \xi_3} = \frac{-\pi M}{(NP-MQ)} < 0$. Thus a fall in ξ_3 results in an improvement in both the equilibrium value of λ^* and Ω^* whereas an expansion in ξ_3 brings about the contrary results. See figure 3.7(b) for the diagram. Under the profit-led or a weak wage-led growth regime, the impact of a rise in ξ_3 on the long run equilibrium rate of capital accumulation is ambiguous (which can be shown by the same equation (3.42)). However, under a strong wage-led growth regime a positive effect of a rise in ξ_3 on the long run equilibrium rate of capital accumulation prevails.

Case 1.3:

Consider point D: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.37) we get $\frac{\partial \lambda}{\partial \xi_3} = \frac{\pi N}{(NP-MQ)} < 0$ and from equation (3.41) we can say $\frac{\partial \Omega}{\partial \xi_3} = \frac{-\pi M}{(NP-MQ)} < 0$. As a consequence of an upsurge in ξ_3 , the equilibrium value of λ^* and Ω^* together shrink whereas a decrease in ξ_3 results in the improvement in the equilibrium value of both λ^* and Ω^* . Here also the impact of a rise in ξ_3 on the equilibrium rate of capital accumulation is same as case 1.1.

Note that a rise in ξ_3 causes the two equilibria C and D to come closer and for an appropriate rise in ξ_3 , both the equilibria congregate to a saddle point. See figure 3.7(c) for the diagram.

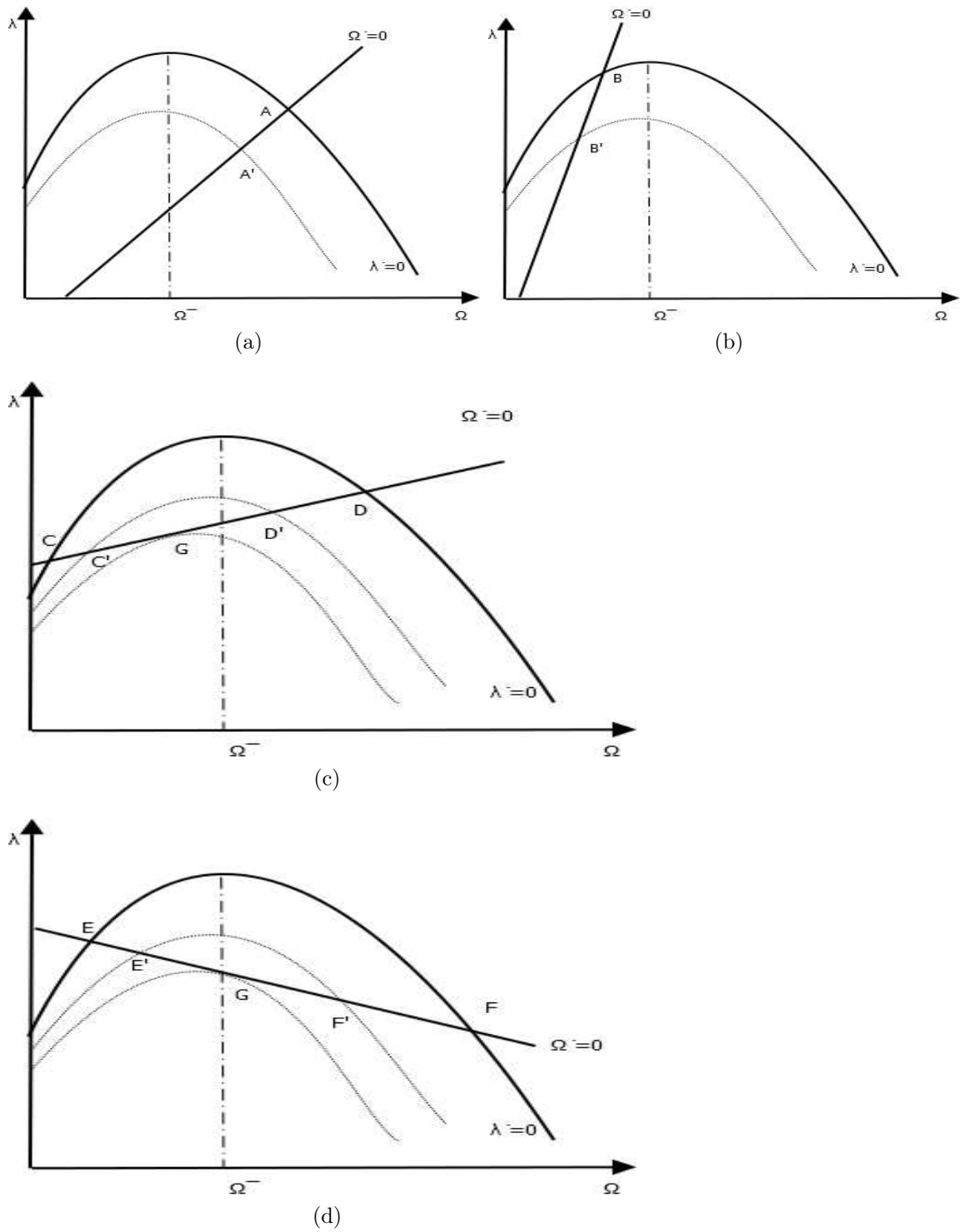


Figure 3.7: Effect of a change in ξ_3

Case 2.1:

Here $\frac{1}{\eta_2} > \frac{\partial g^*}{\partial \Omega} > 0$. The analysis is same as case 1.1. However, a rise in ξ_3 under the profit-led or a weak wage-led growth regime has a negative effect on the equilibrium rate of capital accumulation which is shown in the following equation.

$$\frac{dg^*}{d\xi_3} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \xi_3}}^- + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \xi_3}}^- \right\} < 0 \quad (3.44)$$

When $\frac{\partial g^*}{\partial \lambda} < 0$, a rise in ξ_3 has an ambiguous effect on the equilibrium rate of capital accumulation.

Case 2.2:

The analysis is same as case 1.1 except the fact that as $\frac{\partial g^*}{\partial \Omega} > 0$, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in ξ_3 on the long run equilibrium rate of capital accumulation is negative while in a strong wage-led growth regime, the effect of a rise in ξ_1 on the long run equilibrium rate of capital accumulation is ambiguous.

Case 2.3:

The analysis is same as case 2.2.

Case 3.2/3.3:

Consider point F: Here $P < 0$, $Q < 0$, $M > 0$, $N > 0$. So from equation (3.37) we get $\frac{\partial \lambda}{\partial \xi_3} = \frac{\pi N}{(NP-MQ)} > 0$ and from equation (3.41) we can say $\frac{\partial \Omega}{\partial \xi_3} = \frac{-\pi M}{(NP-MQ)} < 0$. Thus an expansion in ξ_3 has a dwindling effect on Ω^* whereas it upsurges the equilibrium value of λ^* . A deterioration in ξ_3 leads the opposite occurring. The impact of a rise in ξ_3 on the equilibrium rate of capital accumulation under the profit-led or a weak wage-led growth regime is ambiguous which is encapsulated by the following equation

$$\frac{dg^*}{d\xi_3} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \xi_3}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \xi_3}}^- \right\} \begin{matrix} \geq \\ < \end{matrix} 0 \quad (3.45)$$

However, when the economy is in a strong wage-led growth regime, a rise in ξ_3 leads to a deterioration on the long run equilibrium rate of capital accumulation.

Note that if ξ_3 increases the two equilibria E and F come closer and for a sufficient increment in ξ_3 , both the equilibria converge to a saddle point. See figure 3.7(d).

3.3.4 Effect of a change in η_0 :

Differentiating both side of the equation (3.2) w.r.t. η_0 and rearranging we get,

$$P \frac{\partial \lambda}{\partial \eta_0} + Q \frac{\partial \Omega}{\partial \eta_0} = 0 \quad (3.46)$$

Differentiating both side of the equation (3.10) w.r.t. η_0 and rearranging we get,

$$M \frac{\partial \lambda}{\partial \eta_0} + N \frac{\partial \Omega}{\partial \eta_0} = 1 \quad (3.47)$$

where $P = (\xi_1 \frac{\partial g}{\partial \lambda} - \xi_3 \frac{\partial \pi}{\partial \lambda} - 1)$; $Q = [\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}]$; $M = [\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}]$; $N = [\eta_2 \frac{\partial g}{\partial \Omega} - 1]$

Rewriting equation (3.46) & (3.47) in matrix form we get,

$$\begin{pmatrix} P & Q \\ M & N \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial \eta_0} \\ \frac{\partial \Omega}{\partial \eta_0} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (3.48)$$

$$\text{So, } \frac{\partial \lambda}{\partial \eta_0} = \frac{-Q}{(NP - MQ)} \quad (3.49)$$

$$\text{and } \frac{\partial \Omega}{\partial \eta_0} = \frac{P}{(NP - MQ)} \quad (3.50)$$

Case 1.1:

Consider point A: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.46) we get $\frac{\partial \lambda}{\partial \eta_0} = \frac{-Q}{(NP - MQ)} > 0$ and from equation (3.50) we can say $\frac{\partial \Omega}{\partial \eta_0} = \frac{P}{(NP - MQ)} < 0$. Thus as η_0 increases, the equilibrium value of λ^* rises while the equilibrium value of Ω^* falls. For a decrease in η_0 exactly opposite happens. When $\frac{dg^*}{d\lambda} > 0$, the impact of a rise in η_0 on the equilibrium rate of capital accumulation is, however, unambiguously positive which is encapsulated by the following equation. Nonetheless, when $\frac{dg^*}{d\lambda} < 0$, a rise in η_0 has an ambiguous effect on long run g^* .

$$\frac{dg^*}{d\eta_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_0}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \eta_0}}^- \right\} > 0 \quad (3.51)$$

The above analysis can be represented through diagram as well (See figure 3.8(a)). The vertical intercept of the $\dot{\Omega} = 0$ curve is $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1}$. So partially differ-

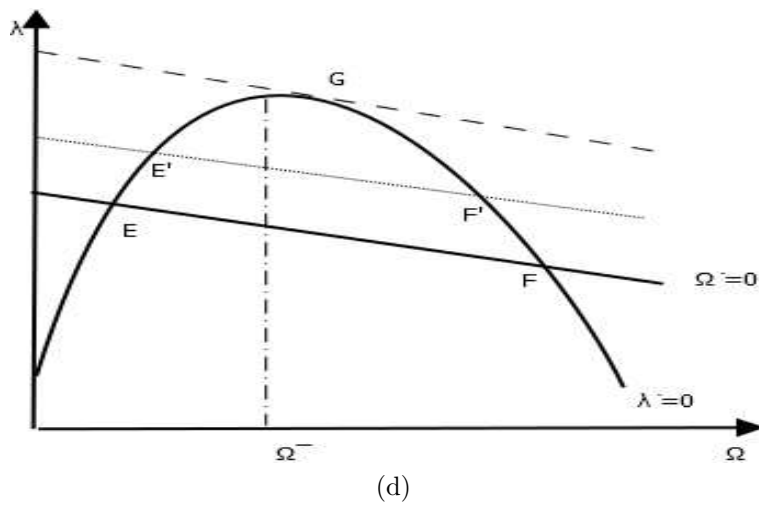
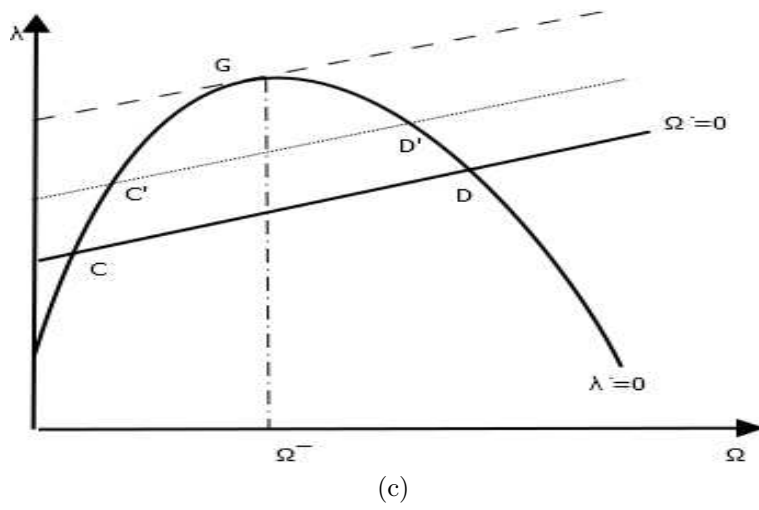
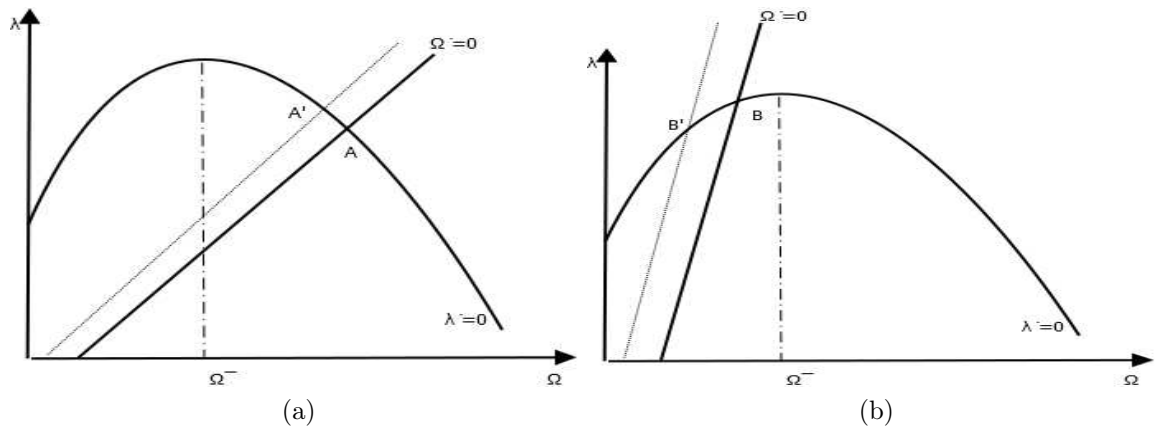


Figure 3.8: Effect of a change in η_0

entiating it with respect to η_0 yields $\frac{\partial \left(\lambda \Big|_{\substack{\Omega=0 \\ \dot{\Omega}=0}} \right)}{\partial \eta_0} = \frac{1}{\eta_1} > 0$. The horizontal intercept for the $\dot{\Omega} = 0$ isocline is $\Omega \Big|_{\substack{\lambda=0 \\ \dot{\Omega}=0}} = \eta_2 g(\lambda = 0) - \eta_0$. So partially differentiating it with respect to η_0 yields $\frac{\partial \left(\Omega \Big|_{\substack{\lambda=0 \\ \dot{\Omega}=0}} \right)}{\partial \eta_0} = -1 < 0$. Slope of the $\dot{\Omega} = 0$ isocline is $\frac{d\lambda}{d\Omega} \Big|_{\dot{\Omega}=0} = \frac{1 - \eta_2 \frac{\partial g}{\partial \Omega}}{\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}}$ and so the slope is invariant with respect to the change in η_0 . Thus when η_0 rises, the vertical intercept increases and the horizontal intercept decreases.

Consider point B: Here $P < 0$, $Q > 0$, $M > 0$, $N < 0$. So from equation (3.46) we get $\frac{\partial \lambda}{\partial \eta_0} = \frac{-Q}{(NP - MQ)} < 0$ and from equation (3.50) we can say $\frac{\partial \Omega}{\partial \eta_0} = \frac{P}{(NP - MQ)} < 0$. Thus as η_0 decreases, both the equilibrium value of λ^* and Ω^* rise and when η_0 increases, the equilibrium value of both λ^* and Ω^* fall. See figure 3.8(b) for the diagram. The impact of a rise in η_0 on the equilibrium rate of capital accumulation under the profit-led or a weak wage-led growth regime is ambiguous which can be represented by the following equation as

$$\frac{dg^*}{d\eta_0} = \left\{ \begin{array}{c} \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_0}}^- + \overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \eta_0}}^- \end{array} \right\} \begin{array}{c} \geq 0 \\ < 0 \end{array} \quad (3.52)$$

Nonetheless, under a strong wage-led growth regime a rise in η_0 has a positive effect on the long run equilibrium rate of capital accumulation.

Case 1.3:

Consider point D: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.46) we get $\frac{\partial \lambda}{\partial \eta_0} = \frac{-Q}{(NP - MQ)} > 0$ and from equation (3.50) we can say $\frac{\partial \Omega}{\partial \eta_0} = \frac{P}{(NP - MQ)} < 0$. Thus as η_0 increases, the equilibrium value of λ^* rises and Ω^* falls and when η_0 decreases, the equilibrium value of both λ^* reduces and Ω^* rises. A rise in η_0 on the equilibrium rate of capital accumulation has same effect as point A of case 1.1.

Note that if η_0 increases the two equilibria C and D come closer and for a sufficient fall in η_0 , both the equilibria converge to a saddle point. See figure 3.8(c).

Case 2.1:

Here $\frac{1}{\eta_2} > \frac{\partial g^*}{\partial \Omega} > 0$. The analysis is same as case 1.1. However, when $\frac{dg^*}{d\lambda} > 0$, a rise in η_0 has an ambiguous effect on the equilibrium rate of capital accumulation

for point A which is shown in by the following equation as

$$\frac{dg^*}{d\eta_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_0}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \eta_0}}^- \right\} \begin{matrix} \geq \\ < \end{matrix} 0 \quad (3.53)$$

However, when $\frac{dg^*}{d\lambda} < 0$, the effect of a rise in η_0 on the long run equilibrium rate of capital accumulation is unambiguously negative.

On the other hand, for point B, when $\frac{dg^*}{d\lambda} > 0$, a rise in η_0 has an unambiguously negative effect on the equilibrium rate of capital accumulation (which is shown in by the following equation) while in a strong wage-led growth regime η_0 has an ambiguous effect on the long run g^* .

$$\frac{dg^*}{d\eta_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_0}}^- + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \eta_0}}^- \right\} < 0 \quad (3.54)$$

Case 2.2:

The analysis is same as case 1.1 except the fact that here for point A, as $\frac{\partial g^*}{d\Omega} > 0$, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in η_0 on the long run equilibrium rate of capital accumulation is ambiguous while in a strong wage-led growth regime, the effect of a rise in η_0 on the long run equilibrium rate of capital accumulation is unambiguously negative.

However, for point B, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in η_0 on the long run equilibrium rate of capital accumulation is unambiguously negative while in a strong wage-led growth regime, the effect of a rise in η_0 on the long run equilibrium rate of capital accumulation is ambiguous.

Case 2.3:

The analysis is same as case 1.3 except the fact that at point D, when $\frac{\partial g^*}{\partial \lambda} > 0$, the impact of a rise in η_0 on the equilibrium rate of capital accumulation is ambiguous, while in the strong wage-led growth regime, the impact of a rise in η_0 on the equilibrium rate of capital accumulation is unambiguously negative.

Case 3.2/3.3:

Consider point F: Here $P < 0$, $Q < 0$, $M > 0$, $N > 0$. So from equation (3.46) we get $\frac{\partial \lambda}{\partial \eta_0} = \frac{-Q}{(NP-MQ)} > 0$ and from equation (3.50) we can say $\frac{\partial \Omega}{\partial \eta_0} =$

$\frac{P}{(NP-MQ)} < 0$. Thus as η_0 increases, the equilibrium value of λ^* rises and Ω^* falls and when η_0 decreases, the equilibrium value of λ^* reduces and Ω^* rises. Here, under the profit-led or a weak wage-led growth regime the impact of a rise in η_0 on the equilibrium rate of capital accumulation is unambiguous and is encapsulated by the following equation as

$$\frac{dg^*}{d\eta_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_0}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \eta_0}}^- \right\} \begin{matrix} \geq 0 \\ < 0 \end{matrix} \quad (3.55)$$

However, under the strong wage-led growth regime there is a negative effect of a rise in η_0 on the long run equilibrium rate of capital accumulation.

Note that if η_0 increases the two equilibria E and F come closer and for a sufficient fall in η_0 , both the equilibria converge to a saddle point. See figure 3.8(d) for the diagram.

3.3.5 Effect of a change in η_1 :

Differentiating both side of the equation (3.2) w.r.t. η_1 and rearranging we get,

$$P \frac{\partial \lambda}{\partial \eta_1} + Q \frac{\partial \Omega}{\partial \eta_1} = 0 \quad (3.56)$$

Differentiating both side of the equation (3.10) w.r.t. η_1 and rearranging we get,

$$M \frac{\partial \lambda}{\partial \eta_1} + N \frac{\partial \Omega}{\partial \eta_1} = -\lambda \quad (3.57)$$

where $P = (\xi_1 \frac{\partial g}{\partial \lambda} - \xi_3 \frac{\partial \pi}{\partial \lambda} - 1)$; $Q = [\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}]$; $M = [\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}]$; $N = [\eta_2 \frac{\partial g}{\partial \Omega} - 1]$

Rewriting equation (3.56) & (3.57) in matrix form we get,

$$\begin{pmatrix} P & Q \\ M & N \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial \eta_1} \\ \frac{\partial \Omega}{\partial \eta_1} \end{pmatrix} = \begin{pmatrix} 0 \\ -\lambda \end{pmatrix} \quad (3.58)$$

$$\text{So, } \frac{\partial \lambda}{\partial \eta_1} = \frac{\lambda Q}{(NP - MQ)} \quad (3.59)$$

$$\text{and } \frac{\partial \Omega}{\partial \eta_1} = \frac{-\lambda P}{(NP - MQ)} \quad (3.60)$$

Case 1.1:

Consider point A: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.56) we get $\frac{\partial \lambda}{\partial \eta_1} = \frac{\lambda Q}{(NP-MQ)} < 0$ and from equation (3.60) we can say $\frac{\partial \Omega}{\partial \eta_1} = \frac{-\lambda P}{(NP-MQ)} > 0$. Thus as η_1 increases, the equilibrium value of λ^* falls and Ω^* rises and when η_1 decreases, the equilibrium value of λ^* increases and Ω^* falls. When $\frac{dg^*}{d\lambda} > 0$, the impact of a rise in η_1 on the equilibrium rate of capital accumulation is, however, unambiguously negative which is encapsulated by equation (3.61). Nonetheless, when $\frac{dg^*}{d\lambda} < 0$, a rise in η_1 has an ambiguous effect on long run g^* .

$$\frac{dg^*}{d\eta_1} = \left\{ \frac{\overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_1}}^-}{\frac{\partial \lambda}{\partial \eta_1}} + \frac{\overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \eta_1}}^+}{\frac{\partial \Omega}{\partial \eta_1}} \right\} < 0 \quad (3.61)$$

The above analysis can be represented through diagram as well (see figure 3.9(a)). The vertical intercept of the $\dot{\Omega} = 0$ curve is $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1}$. So partially differentiating it with respect to η_1 yields,

$$\frac{\partial \left(\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} \right)}{\partial \eta_1} = -\frac{\eta_0 - \eta_2 g(0)}{\eta_1^2} > 0 \quad (\text{as } \eta_0 - \eta_2 g(0) < 0 \text{ here}) \quad (3.62)$$

The horizontal intercept for the $\dot{\Omega} = 0$ isocline is $\Omega \Big|_{\dot{\Omega}=0}^{\lambda=0} = \eta_2 g(\lambda = 0) - \eta_0$. So partially differentiating it with respect to η_1 yields,

$$\frac{\partial \left(\Omega \Big|_{\dot{\Omega}=0}^{\lambda=0} \right)}{\partial \eta_1} = 0 \quad (3.63)$$

Slope of the $\dot{\Omega} = 0$ isocline is $\frac{d\lambda}{d\Omega} \Big|_{\dot{\Omega}=0} = \frac{1 - \eta_2 \frac{\partial g}{\partial \Omega}}{\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}}$ and so the slope decreases with respect to the change in η_1 . Thus when η_1 rises, the vertical intercept increases, slope decreases and at the same time the horizontal intercept remains unchanged. So due to a rise in η_1 , the $\dot{\Omega} = 0$ isocline pivots around the horizontal intercept clockwise.

Consider point B: Here $P < 0$, $Q > 0$, $M > 0$, $N < 0$. So from equation (3.56) we get $\frac{\partial \lambda}{\partial \eta_1} = \frac{\lambda Q}{(NP-MQ)} > 0$ and from equation (3.60) we can say $\frac{\partial \Omega}{\partial \eta_1} = \frac{-\lambda P}{(NP-MQ)} > 0$.

Thus as η_1 increases, both the equilibrium value of λ^* and Ω^* rise and when η_1

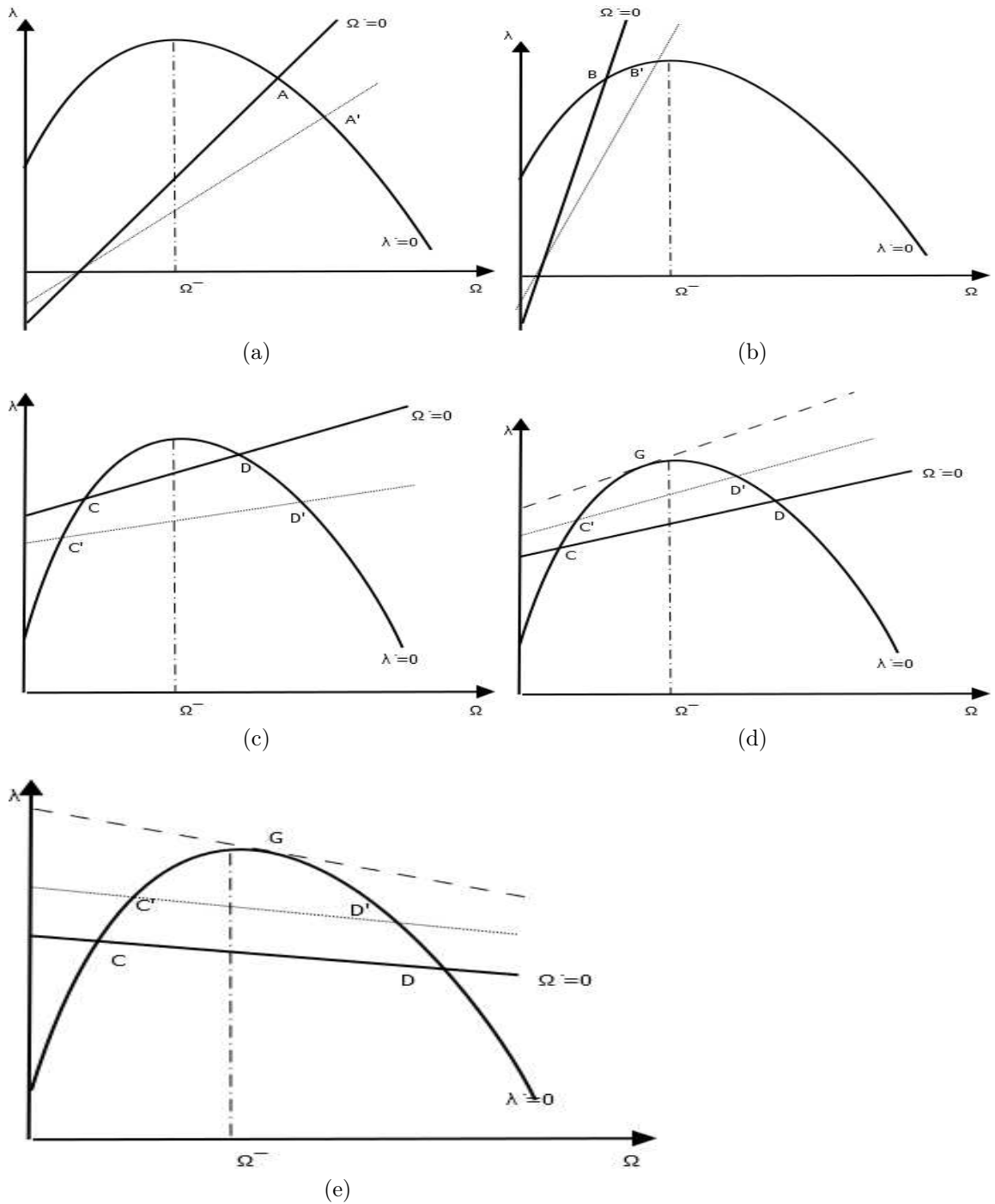


Figure 3.9: Effect of a change in η_1

decreases, the equilibrium value of both λ^* and Ω^* fall. See figure 3.9(b) for the diagram. The impact of a rise in η_1 on the equilibrium rate of capital accumulation under the profit-led or a weak wage-led growth regime is ambiguous which can be represented by the following equation as

$$\frac{dg^*}{d\eta_1} = \left\{ \frac{\overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_1}}^+}{\partial \lambda \partial \eta_1} + \frac{\overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \eta_1}}^+}{\partial \Omega \partial \eta_1} \right\} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (3.64)$$

Nonetheless, under a strong wage-led growth regime a rise in η_1 has negative effect on the long run equilibrium rate of capital accumulation.

Case 1.3:

Consider point D: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.56) we get $\frac{\partial \lambda}{\partial \eta_1} = \frac{\lambda Q}{(NP-MQ)} < 0$ and from equation (3.60) we can say $\frac{\partial \Omega}{\partial \eta_1} = \frac{-\lambda P}{(NP-MQ)} > 0$. Thus as η_1 increases, the equilibrium value of λ^* decreases and Ω^* rises (see figure 3.9(c)) and when η_1 decreases, the opposite happens. A rise in η_1 on the equilibrium rate of capital accumulation has same effect as point A of case 1.1.

Note that if η_1 decreases the two equilibria C and D come closer and for a sufficient fall in η_1 , both the equilibria converge to a saddle point. See figure 3.9(d) for the anagrammatic explanation.

Case 2.1:

Here $\frac{1}{\eta_2} > \frac{\partial g^*}{\partial \Omega} > 0$. The analysis is same as case 1.1. However, when $\frac{dg^*}{d\lambda} > 0$, a rise in η_1 has an ambiguous effect on the equilibrium rate of capital accumulation for point A which is shown in by the following equation as

$$\frac{dg^*}{d\eta_1} = \left\{ \frac{\overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_1}}^-}{\partial \lambda \partial \eta_1} + \frac{\overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \eta_1}}^+}{\partial \Omega \partial \eta_1} \right\} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (3.65)$$

However, when $\frac{dg^*}{d\lambda} < 0$, the effect of a rise in η_1 on the long run equilibrium rate of capital accumulation is unambiguously positive.

On the other hand, for point B, when $\frac{dg^*}{d\lambda} > 0$, a rise in η_1 has an unambiguously positive effect on the equilibrium rate of capital accumulation (which is shown in by equation (3.66)) while in a strong wage-led growth regime η_1 has an ambiguous

effect on the long run g^* .

$$\frac{dg^*}{d\eta_1} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_1}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \eta_1}}^+ \right\} > 0 \quad (3.66)$$

Case 2.2:

The analysis is same as case 1.1 except the fact that here for point A, as $\frac{\partial g^*}{\partial \Omega} > 0$, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in η_1 on the long run equilibrium rate of capital accumulation is ambiguous while in a strong wage-led growth regime, the effect of a rise in η_1 on the long run equilibrium rate of capital accumulation is unambiguously positive.

However, for point B, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in η_1 on the long run equilibrium rate of capital accumulation is unambiguously positive while in a strong wage-led growth regime, the effect of a rise in η_1 on the long run equilibrium rate of capital accumulation is ambiguous.

Case 2.3:

The analysis is same as case 1.3 except the fact that at point D, when $\frac{\partial g^*}{\partial \lambda} > 0$, the impact of a rise in η_1 on the equilibrium rate of capital accumulation is ambiguous, while in the strong wage-led growth regime, the impact of a rise in η_1 on the equilibrium rate of capital accumulation is unambiguously positive.

Case 3.2/3.3:

Consider point F: Here $P < 0$, $Q < 0$, $M > 0$, $N > 0$. So from equation (3.56) we get $\frac{\partial \lambda}{\partial \eta_1} = \frac{\lambda Q}{(NP-MQ)} < 0$ and from equation (3.60) we can say $\frac{\partial \Omega}{\partial \eta_1} = \frac{-\lambda P}{(NP-MQ)} > 0$. Thus as η_1 increases, the equilibrium value of λ^* decreases and Ω^* rises and when η_1 decreases, the opposite happens. Here, under the profit-led or a weak wage-led growth regime the impact of a rise in η_1 on the equilibrium rate of capital accumulation is unambiguous and is encapsulated by the following equation as

$$\frac{dg^*}{d\eta_1} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_1}}^- + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \eta_1}}^+ \right\} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (3.67)$$

However, under the strong wage-led growth regime there is a positive effect of a rise in η_1 on the long run equilibrium rate of capital accumulation.

Note that if η_1 decreases the two equilibria E and F come closer and for a sufficient fall in η_1 , both the equilibria converge to a saddle node. See figure 3.9(e).

Proposition 13: For an appropriate value of η_1 , the characteristic equation to (3.2) & (3.10) evaluated at the steady state G of the case 1.3 and/or 3.3 has one zero root and for the same dynamical system, $\eta_1 = \hat{\eta}_1$ provides a point of saddle-node bifurcation.

Proof: Provided in appendix D. ■

This proposition ensures that there is only one path which is stable whereas the rest are unstable. This causes the transition between stability and instability of the equilibrium.

3.3.6 Effect of a change in η_2 :

Differentiating both side of the equation (3.2) w.r.t. η_2 and rearranging we get,

$$P \frac{\partial \lambda}{\partial \eta_2} + Q \frac{\partial \Omega}{\partial \eta_2} = 0 \quad (3.68)$$

Differentiating both side of the equation (3.10) w.r.t. η_2 and rearranging we get,

$$M \frac{\partial \lambda}{\partial \eta_2} + N \frac{\partial \Omega}{\partial \eta_2} = -g \quad (3.69)$$

where $P = (\xi_1 \frac{\partial g}{\partial \lambda} - \xi_3 \frac{\partial \pi}{\partial \lambda} - 1)$; $Q = [\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}]$; $M = [\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}]$; $N = [\eta_2 \frac{\partial g}{\partial \Omega} - 1]$

Rewriting equation (3.68) & (3.69) in matrix form we get,

$$\begin{pmatrix} P & Q \\ M & N \end{pmatrix} \begin{pmatrix} \frac{\partial \lambda}{\partial \eta_2} \\ \frac{\partial \Omega}{\partial \eta_2} \end{pmatrix} = \begin{pmatrix} 0 \\ -g \end{pmatrix} \quad (3.70)$$

$$\text{So, } \frac{\partial \lambda}{\partial \eta_2} = \frac{gQ}{(NP - MQ)} \quad (3.71)$$

$$\text{and } \frac{\partial \Omega}{\partial \eta_2} = \frac{-gP}{(NP - MQ)} \quad (3.72)$$

Case 1.1:

Consider point A: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.68) we get $\frac{\partial \lambda}{\partial \eta_2} = \frac{gQ}{(NP - MQ)} < 0$ and from equation (3.72) we can say $\frac{\partial \Omega}{\partial \eta_2} = \frac{-gP}{(NP - MQ)} > 0$. Thus as η_2 increases, the equilibrium value of λ^* falls and Ω^* rises and when η_2 decreases, the equilibrium value of λ^* increases and Ω^* falls. When

$\frac{dg^*}{d\lambda} > 0$, the impact of a rise in η_1 on the equilibrium rate of capital accumulation is, however, unambiguously negative which is encapsulated by the following equation. Nonetheless, when $\frac{dg^*}{d\lambda} < 0$, a rise in η_0 has an ambiguous effect on long run g^* .

$$\frac{dg^*}{d\eta_2} = \left\{ \frac{\overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_2}}^-}{\frac{\partial \lambda}{\partial \eta_2}} + \frac{\overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \eta_2}}^+}{\frac{\partial \Omega}{\partial \eta_2}} \right\} < 0 \quad (3.73)$$

The above analysis can be represented through diagram as well (see figure 3.10(a)). The vertical intercept of the $\dot{\Omega} = 0$ curve is $\lambda \Big|_{\substack{\Omega=0 \\ \dot{\Omega}=0}} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1}$. So partially differentiating it with respect to η_2 yields $\frac{\partial \left(\lambda \Big|_{\substack{\Omega=0 \\ \dot{\Omega}=0}} \right)}{\partial \eta_2} = -\frac{g(0)}{\eta_1} < 0$. The horizontal intercept for the $\dot{\Omega} = 0$ isocline is $\Omega \Big|_{\substack{\lambda=0 \\ \dot{\Omega}=0}} = \eta_2 g(\lambda = 0) - \eta_0$. So partially differentiating it with respect to η_2 yields $\frac{\partial \left(\Omega \Big|_{\substack{\lambda=0 \\ \dot{\Omega}=0}} \right)}{\partial \eta_2} = g(0)$. Slope of the $\dot{\Omega} = 0$ isocline is $\frac{d\lambda}{d\Omega} \Big|_{\dot{\Omega}=0} = \frac{1 - \eta_2 \frac{\partial g}{\partial \Omega}}{\eta_1 + \eta_2 \frac{\partial g}{\partial \lambda}}$ and so the change in slope is ambiguous with respect to the change in η_2 . Thus, finally we can conclude that when η_2 rises, the $\dot{\Omega} = 0$ isocline shifts toward rightward direction (although the shift may not be parallel).

Consider point B: Here $P < 0$, $Q > 0$, $M > 0$, $N < 0$. So from equation (3.68) we get $\frac{\partial \lambda}{\partial \eta_2} = \frac{gQ}{(NP - MQ)} > 0$ and from equation (3.72) we can say $\frac{\partial \Omega}{\partial \eta_2} = \frac{-gP}{(NP - MQ)} > 0$. Thus as η_2 increases, both the equilibrium value of λ^* and Ω^* rise and when η_2 decreases, the equilibrium value of both λ^* and Ω^* fall. See figure 3.10(b) for the diagram. The impact of a rise in η_2 on the equilibrium rate of capital accumulation under the profit-led or a weak wage-led growth regime is ambiguous which can be represented by the following equation as

$$\frac{dg^*}{d\eta_2} = \left\{ \frac{\overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_2}}^+}{\frac{\partial \lambda}{\partial \eta_2}} + \frac{\overbrace{\frac{\partial g^*}{\partial \Omega}}^- \overbrace{\frac{\partial \Omega}{\partial \eta_2}}^+}{\frac{\partial \Omega}{\partial \eta_2}} \right\} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (3.74)$$

Nonetheless, under a strong wage-led growth regime a rise in η_2 has a negative effect on the long run equilibrium rate of capital accumulation.

Case 1.3:

Consider point D: Here $P < 0$, $Q < 0$, $M > 0$, $N < 0$. So from equation (3.68) we get $\frac{\partial \lambda}{\partial \eta_2} = \frac{uQ}{(NP - MQ)} < 0$ and from equation (3.72) we can say $\frac{\partial \Omega}{\partial \eta_2} = \frac{-uP}{(NP - MQ)} >$

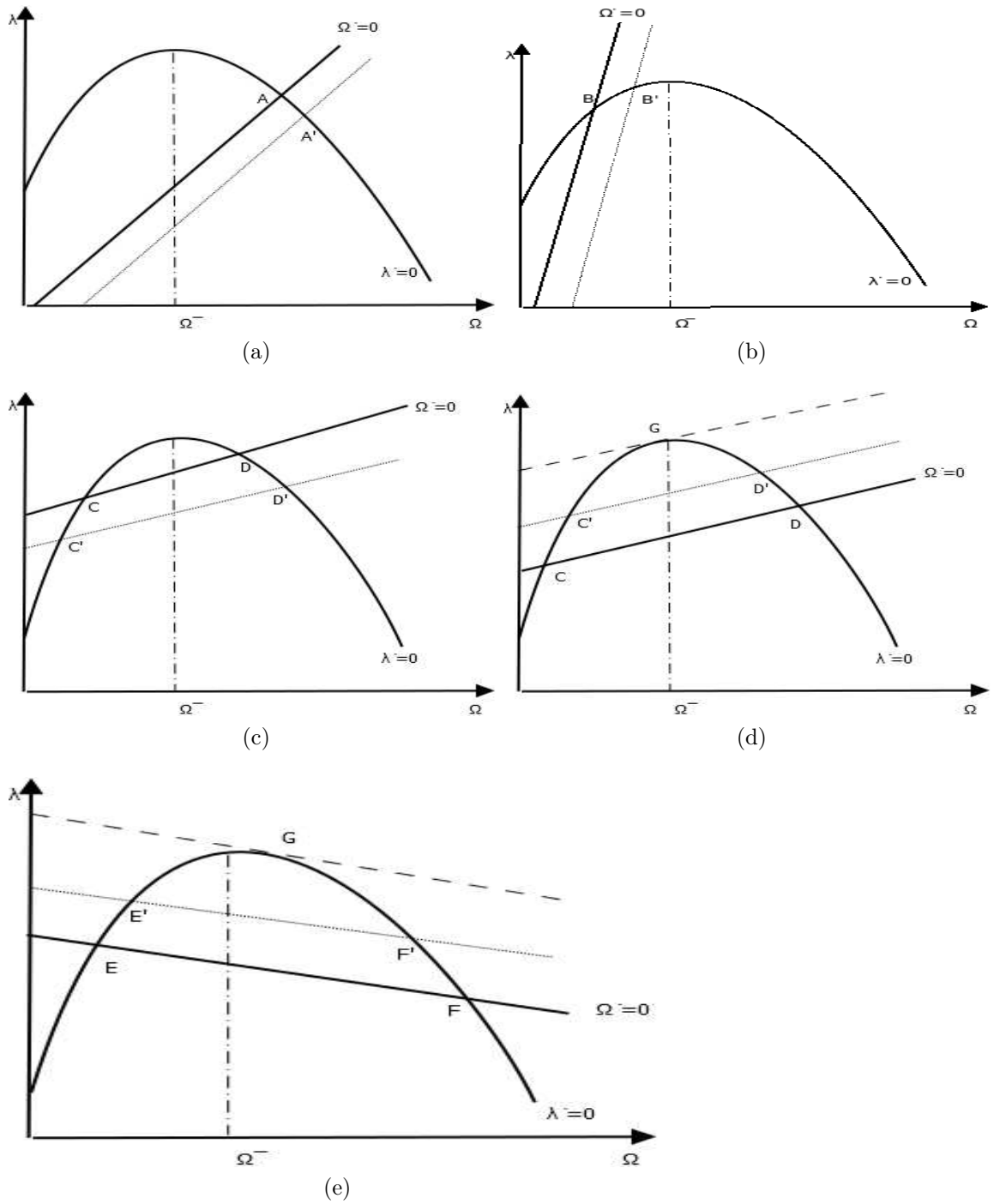


Figure 3.10: Effect of a change in η_2

0. Thus as η_2 increases, the equilibrium value of λ^* falls and Ω^* rises (see figure 3.10(c)) and when η_2 decreases, the equilibrium value of λ^* rises and Ω^* falls. A rise in η_1 on the equilibrium rate of capital accumulation has same effect as point A of case 1.1.

Note that if η_2 decreases the two equilibria C and D come closer and for a sufficient fall in η_2 , both the equilibria converge to a saddle point. See figure 3.10(d).

Case 2.1:

Here $\frac{1}{\eta_2} > \frac{\partial g^*}{\partial \Omega} > 0$. The analysis is same as case 1.1. However, when $\frac{dg^*}{d\lambda} > 0$, a rise in η_2 has an ambiguous effect on the equilibrium rate of capital accumulation for point A which is shown in by the following equation as

$$\frac{dg^*}{d\eta_2} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_2}}^- + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \eta_2}}^+ \right\} \begin{matrix} \geq \\ < \end{matrix} 0 \quad (3.75)$$

However, when $\frac{dg^*}{d\lambda} < 0$, the effect of a rise in η_2 on the long run equilibrium rate of capital accumulation is unambiguously positive.

On the other hand, for point B, when $\frac{dg^*}{d\lambda} > 0$, a rise in η_2 has an unambiguously positive effect on the equilibrium rate of capital accumulation (which is shown in by equation (3.76)) while in a strong wage-led growth regime η_2 has an ambiguous effect on the long run g^* .

$$\frac{dg^*}{d\eta_2} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_2}}^+ + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \eta_2}}^+ \right\} > 0 \quad (3.76)$$

Case 2.2:

The analysis is same as case 1.1 except the fact that here for point A, as $\frac{\partial g^*}{d\Omega} > 0$, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in η_2 on the long run equilibrium rate of capital accumulation is ambiguous while in a strong wage-led growth regime, the effect of a rise in η_2 on the long run equilibrium rate of capital accumulation is unambiguously positive.

However, for point B, when $\frac{\partial g^*}{\partial \lambda} > 0$ impact of a rise in η_2 on the long run equilibrium rate of capital accumulation is unambiguously positive while in a strong wage-led growth regime, the effect of a rise in η_2 on the long run equilibrium rate of capital accumulation is ambiguous.

Case 2.3:

The analysis is same as case 1.3 except the fact that at point D, when $\frac{\partial g^*}{\partial \lambda} > 0$, the impact of a rise in η_2 on the equilibrium rate of capital accumulation is ambiguous, while in the strong wage-led growth regime, the impact of a rise in η_2 on the equilibrium rate of capital accumulation is unambiguously positive.

Case 3.2/3.3:

Consider point F: Here $P < 0$, $Q < 0$, $M > 0$, $N > 0$. So from equation (3.68) we get $\frac{\partial \lambda}{\partial \eta_2} = \frac{gQ}{(NP-MQ)} < 0$ and from equation (3.72) we can say $\frac{\partial \Omega}{\partial \eta_2} = \frac{-gP}{(NP-MQ)} > 0$. Thus as η_2 increases, the equilibrium value of Ω^* rises but λ^* falls and when η_2 decreases, the opposite happens. Here, under the profit-led or a weak wage-led growth regime the impact of a rise in η_2 on the equilibrium rate of capital accumulation is unambiguous and is encapsulated by the following equation as

$$\frac{dg^*}{d\eta_2} = \left\{ \overbrace{\frac{\partial g^*}{\partial \lambda}}^+ \overbrace{\frac{\partial \lambda}{\partial \eta_2}}^- + \overbrace{\frac{\partial g^*}{\partial \Omega}}^+ \overbrace{\frac{\partial \Omega}{\partial \eta_2}}^+ \right\} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (3.77)$$

However, under the strong wage-led growth regime there is a positive effect of a rise in η_2 on the long run equilibrium rate of capital accumulation.

Note that if η_2 decreases the two equilibria E and F come closer and for a sufficient fall in η_2 , both the equilibria converge to a saddle point. See figure 3.10(e).

4 Conclusion:

In this paper we deal with a post-Kaleckian growth model in which in the long run technological progress and the financialization rate evolve endogenously. First, we examine the short-run stability condition and comparative statics. We observe that in the economy, both wage-led and profit-led demand regimes as well as growth regimes are possible. When the consumption propensity of the rentiers is greater than the responsiveness of investment demand to a unit change in distributed profit (which ensures the economy to be in the wage-led demand regime), a higher ‘finance channel’ is sufficient to ensure an expansionary effect of financialization on the aggregate demand provided that it is sufficiently strong compared to the other two channels. Similarly, when rentiers’ consumption propensity is smaller than the responsiveness of the investment demand to a unit change in

distributed profits, financialization has a contractionary effect on the aggregate demand. These results are not explicitly mentioned in Hein (2012a). Nonetheless, one can easily derive these results from Hein as well.

Consistent with Hein (2012a), we find that the following conditions together ensure the impact of financialization on capital accumulation to be expansionary: (i) a low propensity to save out of rentiers' income, (ii) weak effects of distributed profits on firms' investment decisions, (iii) comparatively lower importance of the 'preference channel' for firms' investment decisions relative to the 'finance channel', and (iv) a high responsiveness of investment to the profit share. Otherwise financialization has a contractionary effect on the capital accumulation.

Unlike Hein (2012a), we also find that the impact of an improvement in technological progress on the aggregate demand and on the equilibrium rate of capital accumulation is ambiguous and depends on the very regime the economy is in.

The main departure of our analysis from the earlier literature, however, is that in the long run, along with the rate of technological progress, we endogenize the financialization rate as well. In the long-run, we find richer dynamics than Hein (2012a). Unlike Hein (2012a) (where the equilibrium is unique), in our paper we find that multiple equilibria may arise. Because of the incorporation of the financialization dynamics, unlike Hein (2012a), we also find that the interaction between technological progress and financialization dynamics can lead to instability in the economy.

We find few other interesting results as well. First, for a fixed value of the financialization rate, the steady state rate of technological progress is stable. On the other hand, in the absence of technological change (or for a fixed value of the rate of technological progress), a contractionary effect of financialization on aggregate demand implies a stable steady state financialization rate. However in case of an expansionary effect of financialization on aggregate demand, an unstable equilibrium financialization rate is possible.

Second, for a sufficiently high and expansionary effect of financialization on the aggregate demand, under certain conditions, when the diffusion rate of technological innovations is critically low (because of stringent intellectual property rights and so on), the economy loses its stability and gives birth to a limit cycle. This very result of ours, in this context, is in contrast to Bhaduri (2006b). The conclusion in Bhaduri (2006b) is that faster diffusion rate of technological innovations is important for fueling the accumulation rate whereas it has potential to simultaneously destabilize the steady growth path. In our model under certain conditions, on the

contrary, higher speed of diffusion of technological innovations is not necessary for fueling growth but is important for stabilizing the economy itself. So, for ensuring stability in the economy, government (or institutional) intervention for weakening the stringent intellectual property rights are desirable. Similarly, under the above conditions, when the speed of deregulation of the financial market is very high the economy can lose its stability and a limit cycle can emerge. As a result, for ensuring stability in the economy more regulated financial markets are desirable.

Third, when the economy is in stagnation, intra-class conflict among firms plays an important role not only for achieving new equilibrium level of technological progress and financialization rate, but also for ensuring stability in the economy. In the period of stagnation, lower intra-class conflict among firms can shrink the stability corridor and for a sufficiently low level of intra-class conflict, ultimately a saddle point for the steady state of the economy can emerge.

Fourth, for a sufficiently high and expansionary effect of financialization on the aggregate demand, under certain conditions, a lower value of the responsiveness of the desired financialization rate to a unit change in the rate of technological progress can shrink the stability corridor of the steady state of the economy and can potentially lead to the emergence of saddle-node bifurcation (i.e. it ultimately can cause the transition between stability and instability of the equilibrium).

The analysis in this paper, however, has some limitations. First, in our model, households indebtedness has not been captured; this is important in the context of recent financial crisis, households borrowing plays a major role. Second, we have assumed the share of profit to be fixed. A more interesting, powerful, and realistic explanation is possible if we endogenize the share of profit as well. Finally, our model is based on closed economy without a role for government intervention. These issues are left for future research.

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A Proof of Proposition 10

Proof. Suppose $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > \lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0)$ and $\frac{\partial g^*}{\partial \Omega} < 0$. This ensures $N < 0$. For case 1.1 & 1.2 $P < 0$ (by assumption), $Q \gtrless 0$ (as $\Omega \gtrless \bar{\Omega}$) and $M > 0$. From above three types of different points we can conclude that $Det(J) > 0$ and $tr(J) < 0$ and so for all the possible cases the equilibrium points are stable.

Now suppose $\lambda \Big|_{\dot{\Omega}=0}^{\Omega=0} = \frac{\eta_0 - \eta_2 g(0)}{\eta_1} > \lambda \Big|_{\dot{\lambda}=0}^{\Omega=0} = \xi_0 + \xi_1 g(0) - \xi_3 \pi(0)$ and $0 < \frac{\partial g^*}{\partial \Omega} < \frac{1}{\eta_2}$. This ensures $N < 0$. For case 2.1 & 2.2 $P < 0$ (by assumption), $Q \gtrless 0$ (as $\Omega \gtrless \bar{\Omega}$) and $M > 0$. From above three types of different points we can conclude that $Det(J) > 0$ and $tr(J) < 0$ and so for all the possible cases the equilibrium points are stable. ■

B Proof of Proposition 11

Proof. Differentiating the trace of the Jacobian matrix with respect to θ and then evaluating it at $\theta = 0$ we get,

$$\frac{\partial(trJ)}{\partial \theta} = \left[\xi_1 \frac{\partial g}{\partial \lambda} - 1 \right] < 0$$

So the trace is smooth, differentiable and monotonically decreasing in the speed of adjustment, θ . The trace disappears at $\theta = \hat{\theta}$ where $\hat{\theta} = \frac{-\phi \left[\eta_2 \frac{\partial g}{\partial \Omega} - 1 \right]}{\xi_1 \frac{\partial g}{\partial \lambda} - 1} > 0$. So the characteristic equation has purely imaginary roots at $\theta = \hat{\theta}$ and the transversality condition is satisfied. Hence, $\theta = \hat{\theta}$ provides a point of Andronov-Hopf bifurcation. ■

C Proof of Proposition 12

Proof. Differentiating the trace of the Jacobian matrix with respect to ϕ and then evaluating it at $\phi = 0$ we get,

$$\frac{\partial(trJ)}{\partial \phi} = \left[\eta_2 \frac{\partial g}{\partial \Omega} - 1 \right] > 0$$

So the trace is smooth, differentiable and monotonically increasing in the speed of adjustment, ϕ . The trace disappears at $\phi = \hat{\phi}$ where $\hat{\phi} = \frac{-\theta[\xi_1 \frac{\partial g}{\partial \lambda} - 1]}{\eta_2 \frac{\partial g}{\partial \Omega} - 1} > 0$. So the characteristic equation has purely imaginary roots at $\phi = \hat{\phi}$ and the transversality condition is satisfied. Hence, $\phi = \hat{\phi}$ provides a point of Andronov-Hopf bifurcation. ■

D Proof of Proposition 13

Proof. At point G , the slope of the $\dot{\Omega} = 0$ curve is equal to the slope of the $\dot{\lambda} = 0$ curve, i.e.

$$\begin{aligned} \left. \frac{d\lambda}{d\Omega} \right|_{\dot{\lambda}=0} &= \left. \frac{d\lambda}{d\Omega} \right|_{\dot{\Omega}=0} \\ \Rightarrow \theta\phi(PN - QM) &= 0 \end{aligned}$$

So the determinant of the Jacobian matrix $\det J = 0$. So one of the two eigenvalues is zero.

$$\begin{aligned} NP &= MQ \\ \Rightarrow \eta_1 = \hat{\eta}_1 &= \left\{ \frac{(\xi_1 \frac{\partial g}{\partial \lambda} - 1)(\eta_2 \frac{\partial g}{\partial \Omega} - 1)}{\xi_1 \frac{\partial g}{\partial \Omega} + \xi_2(1 - 2\Omega) - \xi_3 \frac{\partial \pi}{\partial \Omega}} \right\} - \left(\eta_2 \frac{\partial g}{\partial \lambda} \right) \end{aligned} \quad (\text{D.1})$$

Above equation must be evaluated at point G .

Differentiating $\dot{\lambda}$ (of equation 3.2) w.r.t. η_1 we get

$$\left. \frac{\partial \dot{\lambda}}{\partial \eta_1} \right|_{(\lambda^*, \Omega^*) \text{ evaluated at point G}} = 0 \quad (\text{D.2})$$

Differentiating $\dot{\Omega}$ (of equation 3.10) w.r.t. η_1 we get

$$\left. \frac{\partial \dot{\Omega}}{\partial \eta_1} \right|_{(\lambda^*, \Omega^*) \text{ evaluated at point G}} = \phi\lambda$$

Above equation when evaluated at $(\lambda^*, \Omega^*; \eta_1)$ at point G provides non-zero value.

So the vector $\left(\begin{array}{c} \frac{\partial \dot{\lambda}}{\partial \eta_1} \\ \frac{\partial \dot{\Omega}}{\partial \eta_1} \end{array} \right)_{(\lambda^*, \Omega^*) \text{ evaluated at point G}} \neq \mathbf{0}$. So the non-hyperbolicity and transversality conditions both are satisfied. Thus when η_1 decreases to $\hat{\eta}_1$, the saddle-node bifurcation occurs. ■