Expected Utility Preferences versus Prospect Theory Preferences in Bargaining

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Abstract

Are individuals always better off when their preferences can be represented by expected utility? I study this question in a bargaining game where individuals bargain over a pie of fixed size, and I contrast the share received in the long-run by expected utility maximisers with the share they would receive if their preferences were described by prospect theory preferences instead when, in either case, they bargain with expected utility maximisers. I present a necessary and sufficient condition for individuals to obtain a higher share of the pie if their preferences obey prospect theory rather than expected utility. I decompose the effect that the three features that characterise prospect theory preferences – reference point dependence, loss-aversion and probability weighting – have on the bargaining outcome, and show that loss-aversion does not have any effect on the outcome of the bargaining process, reference-point dependent preference confers an unambiguous advantage and probability weighting is unambiguously disadvantageous. This ties in with the main result outlined earlier: if the upward pull of reference point dependence is relatively stronger than the downward push of probability weighting, then individuals are better off with prospect theory preferences than with expected utility preferences, and vice-versa.

JEL Classification: C73, C78, D01, D81, D83, D90.

Keywords: bargaining, expected utility prospect theory, reference point dependence, loss aversion, probability weighting

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1 Introduction

The expected utility framework of von Nuemann and Morgernstern (1944) has been the workhorse for analysing situations involving decision-making under risk and uncertainty. The primary appeal of the expected utility approach, apart from convenience of use and tractability, derives from its axiomatic foundations: individual preferences need only satisfy completeness, continuity and the independence axiom in order to have an expected utility representation. Furthermore, even though observed 'violations' of expected utility have cast doubt on its descriptive validity (for eg. Allais (1953) and Kahnemann and Tversky (1979)) and subsequently led to the development of alternative models of decision-making under risk – of which prospect theory (Kahnemann and Tversky (1979, 1992)) is arguably the most comprehensive competing alternative – it is argued that (individuals with) non-expected utility preferences could be persuaded to make a 'Dutch book' against himself i.e. if preferences of an individual do not satisfy expected utility, then a money pump can impoverish the individual, and hence he would be 'outperformed' and selected against by (individuals with) expected utility preferences. In this paper, I explore a question of this very nature: in a strategic situation of bargaining, are individuals necessarily better off with expected utility preferences than when they are endowed with a particular type of non-expected utility preferences, namely prospect theory preferences?

I consider a recurrent bargaining game, as in Young (1993b), where individuals from one population are randomly matched to individuals from another population to bargain over division of a pie of fixed size. The two randomly matched individuals in each pair simultaneously announce the share of the pie that they claim for themselves. They receive their respective claims only if the two demands are compatible with each other i.e. they sum up to no more than unity; otherwise, they receive nothing. In a bargaining situation such as this, it is reasonable that the history of play should influence the share that each individual claims for himself. Hence, I posit that each individual draws an independent random sample from the demands made by the other population's individuals in the last period, and he assumes that his randomly matched co-player's demand in this period is drawn from the distribution of demands expressed in the random sample. Then, in the current period, an individual almost always chooses an ex-ante 'optimal' demand (given his preferences) in response to the randomly drawn sample. I focus on the stochastically stable division of the pie, which is the mode of division that is expected to prevail most of the time in the long-run.

As a benchmark, I first consider the case where both populations comprise of individuals whose preferences have an expected utility representation. Here, decision making involves has two components: a von-Nuemann Morgenstern (vN-M) utility function that assigns a utility value to each outcome of the bargaining game, and the probability distribution of relevant outcomes. In context of the first component, I suppose that the vN-M utility functions of individuals in the two populations, namely A and B, are given by the increasing and concave functions u and v respectively, where the argument of each utility function is the share of the pie received. As for the second component, there are two relevant mutually exclusive and exhaustive events: either the claimed share is received, or it isn't, and the probability of either event is derived by each individual from his randomly drawn sample. That is, if he claims x fraction of the pie for himself, then he supposes that the probability of obtaining it is the relative frequency with which the individuals of the other population have demanded less than $1-x$ in his randomly drawn sample. Then, the expected utility of making a particular demand is the weighted sum of utilities of the two outcomes (where one outcome corresponds to receiving nothing, while the other outcome corresponds to receiving the claim made), with the weights being equal to the probability of occurrence of each event. Each individual almost always chooses a demand that maximises his expected utility. The limiting stochastically stable division of the bargaining game when individuals in both populations are expected utility maximisers coincides with the Nash bargaining solution (Nash (1950)), i.e. if x^* and $1-x^*$ is the share obtained by each individual in population A and B respectively in the limiting stochastically stable division, then x^* maximises $u(x)\,v(1-x)$ (see also Young (1993b)). A corollary is that if individuals in either population have the same vN-M utility function, then an equal division of the pie is the stochastically stable division.

Now, suppose that the preferences of individuals in population B are described by prospect theory instead, but individuals in population A continue to maximise expected utility. Prospect theory has two components - a value function, and a probability weighting function, and the distinguishing features of these two components are as follows:

(i) Each outcome is evaluated in terms of a gain or a loss about a reference point. Thus now, unlike expected utility preferences, where only the share of the pie that is received is germane, evaluation of an outcome depends on how the share of the pie compares with a particular share that serves as the reference point. Prospect theory assumes that the value function, the analogue of the utility function, is concave in the gains domain and convex in the losses domain, with a value of zero assigned to the outcome where the share received exactly equals the reference point. Importantly, prospect theory does not formalise the process of determination of the reference point but suggests that features such as recent history, norms or aspirations may affect the reference point. In this paper, the process of reference point formation is endogenous – since an individual supposes that his co-player's demand comes from the same distribution as the demands in the sample, I assume that an individual's reference point is obtained from the randomly drawn sample as well. Thus, similar in spirit to Koszegi and Rabin (2006), where expectations – which in this case arise from the random sample an individual draws – influences the reference point of an individual.

(ii) Individuals are loss-averse, i.e. a gain of any particular magnitude cannot compensate for a loss of equal magnitude.

In context of these two features, I assume that the value function of individuals of population B is $v(\cdot)$ in the gains domain (i.e. the vN-M utility function of population B when their preferences were represented by expected utility) and $-\lambda\, v(\cdot)$ in the losses domain. Here, $\lambda > 1$ indicates loss-aversion. This allows me to focus purely on the effect of change in preferences from the expected utility to prospect theory.

(iii) Objective probabilities are transformed by a probability weighting function $w(\cdot)$, the defining characteristic of which is the over-weighting of 'small' probabilities and under-weighting of large probabilities, i.e. for probability p 'sufficiently small', $w(p) > p$, and for probability p' 'sufficiently large', $w(p') < p'$.

The manner in which an individual with preferences described by prospect theory chooses his claim is as follows. He draws an independent random sample from the demands made by the population of expected utility maximisers in the previous period. This random sample determines the reference point for him in that particular period. He assumes that the demand that he will face in the current period is drawn from the demand distribution in the random sample. Each demand that an individual may make results in one of two events: it is either compatible with the demand of his random co-player, or it is not; in either case, the associated objective probability is obtained from the sample. For example, if he claims x fraction of the pie, then he supposes that the probability of obtaining it is the relative frequency with which the individuals of the other population have demanded less than $1-x$ in his randomly drawn sample, and the probability thus obtained is transformed by the weighting function. Thus, each demand gives rise to a prospect comprising of two events, namely obtaining the claim, and not obtaining it. Each of these two outcomes are then either coded as a gain or a loss, depending on how the outcome compares with the reference point. The value of the prospect induced by making a particular demand is simply the sum of the values of these two outcomes, weighted by the corresponding probability weight. The individual almost always chooses the demand that induces the prospect of highest value.

The above description highlights the difference in the decision making process of expected utility maximisers and prospect value maximisers. My objective is to compare the share of the pie received in the limiting stochastically stable division by population B individuals when they maximise expected utility, and when they maximise prospect value. Interestingly, I find that population B individuals receive a higher share of the pie when they are prospect value maximisers if and only if $\frac{v'(0)}{w'(0)} > v'(1-x^*)$, where $v'(0)$ is the marginal utility/value as the share approaches zero, $w'(0)$ is the slope of the probability weighting function as probability approaches zero, and $v'(1-x^*)$ is the marginal utility of the share received by them when they are expected utility maximisers. Thus, not only is it not possible to unequivocally state that expected utility maximising individuals (of a population) 'outperform' their counterfactual selves when they are endowed with non-expected utility preferences instead, but I also derive the necessary and sufficient conditions for the former to receive a lower share than in the latter case. The corollary of this is that prospect value maximising individuals also outperform the expected utility maximisers in the other population by receiving more half of the pie when the individuals in both populations have the same utility/value function (i.e. $u = v$) and the above condition holds.

As has been mentioned earlier, there are three principal differences between the prospect theory and expected utility approaches, namely, evaluation about a reference point, loss aversion, and probability weighting. In order to better understand the nature of the result outlined above, I decompose the effect of each of these three factors. For this purpose, I examine three alternative situations: (i) population B individuals' preferences are reference point dependent but they are not loss-averse and neither are probabilities transformed by the weighting function, (ii) population B individuals are loss-averse but they do not weight probabilities, and (iii) population B individuals transform the probabilities obtained from the sample by the same probability weighting function w, but their preferences do not exhibit reference point dependence, and hence no loss aversion either. (The last case corresponds to Quiggin's (1982) rank-dependent utility.) In each of these three cases, I analyse the limiting stochastically stable division of the pie when they bargain against the other population of expected utility maximisers, and ask, how does the share of the pie received by the three different types of individuals compare to the share they would have received had they been expected utility maximisers or prospect value maximisers instead.

I use the following notation to denote the share of the pie received by population B individuals in the limiting stochastically stable division when they have the different types of preferences mentioned above, and when they bargain with the expected utility maximising individuals in population A:

(i) x_{EU}: share of the pie received by population B individuals in the limiting stochastically stable division when they are expected utility maximisers as well

(ii) x_{PT}: share of the pie received by population B individuals in the limiting stochastically stable division when their preferences are described by prospect theory

(iii) x_{RPD}: share of the pie received by population B individuals in the limiting stochastically stable division when their preference exhibits reference-point dependence, but not loss aversion and neither are probabilities transformed by the weighting function

(iv) $x_{RPD,LA}$: share of the pie received by population B individuals in the limiting stochastically stable division when their preference exhibits reference-point dependence and loss aversion, but probabilities are not transformed by the weighting function

(v) x_{RD}: share of the pie received by population B individuals in the limiting stochastically stable division when they have rank dependent utility, i.e. their preference exhibit neither reference-point dependence nor loss aversion, but probabilities are transformed by the weighting function

I then show that $x_{RD} < x_{EU} < x_{RPD} = x_{RPD,LA}$ and $x_{RD} < x_{PT} < x_{RPD} = x_{RPD,LA}$ Hence, individuals of population B receive the highest and lowest share when they have reference point dependent preferences and probability weighted preferences respectively, and the presence/absence of loss-aversion has no additional effect. Interestingly, individuals with reference point dependent preferences always outperform their counterfactual expected utility maximising selves. Combining this with the previous result, I obtain $x_{RD} < x_{EU} < x_{PT} < x_{RPD} = x_{RPD,LA}$ if $\frac{v'(0)}{w'(0)} > v'(x_{EU})$, and $x_{RD} < x_{PT} < x_{EU} < x_{RPD} = x_{RPD,LA}$ otherwise.

These results show that the a priori indeterminacy about whether individuals receive a higher share of the pie if they are expected utility maximisers or prospect value maximisers can be ascribed to the relative strengths of the downward push of probability weighting and the upward pull of reference point dependence. In fact, this interpretation comes not only from the above inequalities but also be seen analytically in the inequality $\frac{v'(0)}{w'(0)} > v'(x_{EU})$. I show that the term $v'(0)$ originates from pure reference point dependence of preferences and causes $x_{EU} < x_{RPD}$, while the term $w'(0)$ arises purely from probability weighting and causes $x_{EU} < x_{RD}$. I interpret $v'(0)$ and $w'(0)$ as the strengths of reference point dependence and probability weighting respectively; if the former outweighes the latter, i.e. $\frac{v'(0)}{w'(0)} > v'(x_{EU})$, then $x_{PT} > x_{EU}$ while if the latter is more dominant, i.e. $\frac{v'(0)}{w'(0)} < v'(x_{EU})$, then $x_{PT} < x_{EU}$.

In summary, the expected utility hypothesis of von Nuemann and Morgenstern (1944) has been widely, and almost all pervasively, used to analyse situations of risk and uncertainty. However, the descriptive validity of expected utility has been called into question due to observed violations of the axioms of expected utility, the most prominent examples of this being in Allais (1953) and Kahnemann and Tversky (1979). Even then, an argument is favour of expected utility has been that if individuals display non-expected utility, then they would voluntarily agree to subject themselves to a money pump, or make a Dutch book against themselves; this leads one to surmise that individuals with non-expected utility would be exploited and driven away by individuals with expected utility preferences. Machina (1989) discusses some of these issues, and Green (1987) formalises the conditions under which an individual with non-expected utility preferences can be persuaded to "make book against himself". Here, I use a bargaining model, as in Young (1993b), to analyse if individuals would always prefer to maximise expected utility – the choice of this strategic situation is motivated by the fact that it allows direct interaction and confrontation between individuals with expected utility preferences and prospect theory preferences. I report necessary and sufficient conditions under which individuals would actually prefer to have prospect theory preferences over expected utility preferences.

2 Model, analysis and results

A and B represent two equally-sized non-empty finite populations of n individuals. Time is discrete and in each (time) period, each individual in one population is randomly matched to exactly one individual from the other population. I remain agnostic about the specifics of the matching process; hence, particular matches may be more likely than others due to reasons of social/geographical proximity. The randomly matched individuals bargain over a surplus/pie, the size of which is normalised to unity, by simultaneously announcing the fraction of the surplus that they claim for themselves. The claims that can be made come from a finite set of feasible demands $D(\delta) = \{\delta, 2\,\delta, \ldots, 1-\delta\}$, where $\delta > 0$ is the precision of demand. An individual receives his claim if his demand is compatible with that of his randomly matched co-player, i.e. the sum of the claims in the pair does not exceed unity; otherwise, they receive nothing. This signifies the end of the period, and in the next period, the same events recur in this recurrent model of bargaining: the individuals are randomly matched, they simultaneously demand a share of the surplus for themselves, and they receive their demand if it is compatible with the demand of the co-player. $\omega_A(t) = (x_A^1(t), \ldots, x_A^n(t))$ and $\omega_B(t) = (x_B^1(t), \ldots, x_B^n(t))$ represent the n demands made in period t by individuals in population A and B respectively. $\omega(t) = (\omega_A(t), \omega_B(t))$ represents the state in the beginning of period $t+1$ (when demands have not yet been made in period $t+1$), with the state space being $\Omega = D(\delta)^n \times D(\delta)^n$.

I now describe the process by which individuals choose the demand that they make in each period in this 'unperturbed' process of bargaining. With strictly positive probability less than unity, that may vary across individuals, an individual exhibits inertia and makes the same claim that he had made in the previous period. With the strictly positive complementary probability, each individual gathers information by independently drawing a random sample of size s without replacement from the n demands made by individuals of the other population in the previous periods, with $s \leq n$. When the sampling is incomplete (i.e. $s < n$), I assume that every feasible sample has strictly positive probability, but not necessarily equal probability, of being drawn by each individual. This gives a probability distribution of demands made by (individuals of) the other population. Each individual assumes that the demand made by his randomly chosen co-player in this period will be drawn from this distribution, and so, in response, he chooses the ex-ante optimal demand (given his preferences). This preferences of the individuals are described next.

I assume that preferences of individuals in population A lends itself to an expected utility representation. $u : \mathbb{R}_+ \to \mathbb{R}_+$ is the increasing and concave von Nuemann-Morgenstern (vN-M) utility function, and is normalised so that $u(0) = 0$. The argument of the vN-M utility function is the share of the pie received by the individual. The implication of u being increasing is that individuals prefer a higher share of the pie, and concavity of u implies diminishing marginal utility and risk-aversion. Individuals in population A maximise expected utility with respect to the cumulative probability distribution of demands expressed by the randomly drawn sample. That is, for any claim x than an individual makes, the probability of obtaining the claim $p(x)$ is the relative frequency with which demands no greater than $1 - x$ appear in the sample drawn from the other population's demands; thus, the claim of x gives an expected utility of $p(x)\, u(x) + (1 - p(x))\, u(0) = p(x)\, u(x)$. Each individual (from population A) chooses the demand that maximises this expected utility i.e. he chooses a demand from the set $argmax_x p(x)\, u(x)$, where $x \in D(\delta)$.

In the benchmark case, I assume that individuals in population B are also expected utility maximisers, and hence they choose their claim in a similar manner; the only difference is that their vN-M utility function is $v : \mathbb{R}_+ \to \mathbb{R}_+$. I will compare this to the situation where the preferences of individuals in population B are described by Cumulative Prospect Theory (Kahnemann and Tversky (1992)). Representation of preferences under Cumulative Prospect Theory differs from expected utility representation in three crucial aspects:

(i) Outcomes are not evaluated at their absolute levels (as in the case of expected utility maximisers). The 'value function' (Prospect Theory's analogue of the vN-M utility function) evaluates outcomes as a gain or a loss about a reference point, and is concave (or risk-averse) in the gains domain and convex (or risk-seeking) in the losses domain. Let $v_+ : \mathbb{R}_+ \to \mathbb{R}$ and $v_- : \mathbb{R}_- \to \mathbb{R}$ be the value function in the gains and losses domain respectively, with v_+ being increasing and concave, v_- being increasing and convex, and $v_+(0) = v_-(0) = 0$. If x_i is the share of the pie received by an individual i and $r_i \in [0, 1]$ denotes his reference point, then the argument of the value function is $x_i - r_i$; so the corresponding value is $v_+(x_i - r_i)$ if $x_i - r_i > 0$ or $v_-(x_i - r_i)$ otherwise.

(ii) Individuals are loss-averse: the marginal effect a loss of a particular magnitude outweighs the marginal effect of a gain of the same magnitude, i.e. $v'_-(-x) > v'_+(x)$.

(iii) Individuals transform objective probabilities to weight the value of an outcome. For any claim x than an individual makes, there are two mutually exclusive and exhaustive outcomes: the claim is obtained (when it is comptiable with the demand of the co-player) or it is not obtained. The probability of obtaining the claim (similarly, not obtaining the claim) $p(x)$ (similarly, $1-p(x)$) is the relative frequency with which demands no greater than (similarly, strictly greater than) than $1 - x$ appear in the sample drawn from the other population's demands. Next, it is not the probability of occurrence of an event that is transformed (as in the original version of Prospect Theory in Kahnemann and Tversky (1979)), but the transformation is via a cumulative functional w that operates on the probability distribution (Kahnemann and Tversky (1992)). In this paper, I assume that the same cumulative functional operates on all outcomes (i.e. gains as well as losses); this implies that any two mutually exclusive and exhaustive outcomes with probabilities p and $1 - p$ receive a weight of $w(p)$ and $w(1 - p)$

respectively, with the additional property that $w(p) + w(1-p) = 1$.[1] Importantly, 'small' probabilities are over-weighted while 'large' probabilities are under-weighted – if probability $p > 0$ is sufficiently small, then $w(p) > p$, and if probability $p < 1$ is sufficiently large, then $w(p) < p$. Crucially, under-weighting of small probabilities implies $w'(0) > 1$, i.e. the slope of the weighting function around 0 is greater than unity. Furthermore, $w(1) = 1$, and $w(0) = 0$.

It follows from the above that a claim of x_i by individual i induces a prospect that gives a share of x_i with probability $p(x_i)$, and a share of 0 with probability $1 - p(x_i)$. The value of this prospect is $w(p(x_i))\, v_{+/-}(x_i - r_i) + (1 - w(p(x_i)))\, v_-(0 - r_i)$, $r_i \in [0,1]$ being his reference point, and $v_{+/-} = v_+$ if $x_i - r_i > 0$ and $v_{+/-} = v_-(x_i - r_i)$ otherwise. The individual chooses the claim that induces the prospect of the highest value, i.e. he chooses a demand from the set $argmax_{x_i}\, w(p(x_i))\, v_{+/-}(x_i - r_i) + (1 - w(p(x_i)))\, v_-(0 - r_i)$, with $x_i \in D(\delta)$.

The only aspect that needs further elaboration is the determination of the reference point, which is the share of the pie an individual 'expects' to receive, in the sense that losses or gains are anchored around this point. I posit that the reference point is a function of the random sample drawn by the individual. For the moment, I only assume that the if any two individuals from the same population draw a sample that has the same cumulative distribution of demands, then their reference point is the same, and that if the sample drawn by an individual comprises of only one demand x, then the reference point of the individual is $1 - x$.

The bargaining process is in a convention at time period t if $(x_A^1(t-1)\ldots, x_A^n(t-1)) = (\bar{x}, \ldots, \bar{x})$ and $(x_B^1(t-1)\ldots, x_B^n(t-m)) = (1-\bar{x}, \ldots, 1-\bar{x})$ for some $x \in D(\delta)$. The convention in which population A demands and obtains fraction $\bar{x}$ is denoted by $\omega_{\bar{x}}$. Once a convention $\omega_{\bar{x}}$ is attained, any individual from population A (similarly, population B) continues to demand $\bar{x}$ (similarly, $1 - \bar{x}$) in all time periods to follow, irrespective of whether the individual is an expected utility maximiser or prospect value maximiser. This is because:

(i) Any sample that can be drawn by an individual in population A comprises only of $1 - \bar{x}$; hence,the individual assigns probability one to the event that his randomly chosen co-player will demand $1 - \bar{x}$; consequently, $\bar{x}$ is the expected utility maximising demand for that individual from population A. Similarly, for any individual in population B, any sample that can be drawn results in the reference point being $1 - \bar{x}$; this individual assigns probability one to the event that his randomly chosen co-player will demand $1 - \bar{x}$, and this event receives a weight of $w(1) = 1$; consequently, $1 - \bar{x}$ is the demand that maximises the value of the induced prospect.

(ii) Inertia, which is the other component of the decision-making process of the individuals, would also lead individuals to make the same choice.

However, I note that while the maximised value of the prospect induced by demanding a share

[1] The method of using a cumulative functional on the probability distribution to transform probabilities was first proposed in Quiggin (1982), and resulted in the formulation of rank-dependent utility.

in accordance with the established convention is exactly equal to zero (since the individual receives a share that is exactly equal to the reference point), the expected utility of demanding a share in accordance with the established convention is strictly positive.

In the proposition to follow (proof in the appendix), I show that the bargaining process converges almost surely to a convention i.e. the probability of a transition to a convention from any initial state is strictly positive, irrespective of whether population B individuals maximise expected utility or prospect value.

Proposition 1. *The unperturbed bargaining process converges almost surely to a convention, and stays locked into the convention, irrespective of whether population B individuals maximise expected utility or prospect value.*

The description of the bargaining game thus far assumes that individuals always make a demand that is ex-ante optimal given the preferences. I now introduce the possibility that individuals make mistakes/experiment with another mode of behaviour. With independent probability $\varepsilon > 0$, each individual makes a demand that neither maximises his expected utility (in case of expected utility maximisers) or value of the induced prospect (in case of prospect value maximisers) nor is it due to inertia. This makes the transition from one convention to another possible. Starting from a particular convention, if enough number of mistakes/experimentations (in the demands made by the other population) accumulate in the sample drawn by an individual, then the set of demands that maximises his expected utility/prospect value may contain a demand that does not conform to the established convention; this opens the path of transition to another convention.

This perturbed process of bargaining can be represented by a Markov process $P(\varepsilon)$ on the state space Ω. This Markov process is ergodic: for each $\varepsilon > 0$, there exists a unique stationary distribution $\mu(\varepsilon)$ given by $\mu(\varepsilon)P(\varepsilon) = \mu(\varepsilon)$. Young (1993a) and Kandori, Mailath and Rob (1993) define the stochastically stable set as the set of states that receive positive weight in the limiting stationary distribution $\mu^* = lim_{\varepsilon\downarrow 0}\mu(\varepsilon)$, the importance of it being that states in this set are expected to be observed most of the time in the long-run. The identification of this set is determined by the relative difficulty/ease of transition from one set of conventions to the complementary set of conventions. The radius-modified co-radius theorem in Ellison (2000) demonstrates that if the radius of a convention ω_x, which is the minimum number of experimentations needed to transit from the basin of attraction of the convention, is greater than its modified co-radius, which is the maximum of the minimum number of experimentations needed to transit between any two adjoining conventions along a minimal resistance path to ω_x from any other convention, then ω_x is the stochastically stable state.

The limiting stochastically stable division is defined as the division of the pie $(x, 1-x)$ in the stochastically stable state as the precision of demand $\delta \to 0$. (In the division $(x, 1-x)$,

x refers to the share obtained by individuals of population A while $1-x$ refers to the share of the pie received by individuals of population B.) In the benchmark case where both populations comprise of expected utility maximisers, the limiting stochastically stable division coincides with the Nash bargaining solution, where the Nash bargaining solution is the division $(x_n, 1-x_n)$, where $x_n = argmax_x u(x)\, v(1-x)$ (Young (Theorem 3,1993b)). This result is presented in the proposition below (proof in the appendix).

Proposition 2. *The limiting stochastically stable division in the bargaining game when both populations A and B comprise only of expected utility maximisers, with vN-M utility functions u and v respectively, coincides with the Nash bargaining solution.*

Corollary 1. *Each individual receives exactly half of the pie in the limiting stochastically stable division of the bargaining game when both populations comprise of expected utility maximisers and all individuals have the same vN-M utility function.*

In context of the above proposition, I denote the share of the pie received by population B individuals in the limiting stochastically stable division by $x_{EU} = 1 - x_n$. The objective is to compare x_{EU} with the share of the pie received by individuals in population B when the expected utility preferences are replaced by prospect theory obeying preferences. If, in the latter case, the population B individuals receive less than x_{EU} in the limiting stochastically stable division, then these individuals are better off with preferences that can be represented by expected utility than with preferences that are described by prospect theory. In order to focus purely on the effect that arises because expected utility representation of preferences is replaced by prospect theory obeying preferences, I keep everything else the same by assuming that the value function $v_+(x) = v(x)$ and $v_-(x) = -\lambda\, v(x)$, where $\lambda > 1$ indicates loss-aversion.

Proposition 1 shows that the unperturbed bargaining process attains a convention in finite time when population B individuals maximise prospect value. In order to analyse the stochastically stable, I examine the relative ease of transition between conventions in the presence of mistakes/experimentations. One important consideration in this analysis involves the adaptation of the reference point. When the state is described by a convention, there is no ambiguity about the reference point: by the assumption made earlier, an individual's reference point is simply the share received by him in the convention. However, in the presence of experimentations, it is possible that the reference point, which depends on the drawn sample, adapts or changes. On one extreme, it is quite probable that reference point adaption is sluggish or conservative: for example, it is equal to the share received by the corresponding individual in the established convention, and it adapts only when another convention supplants the established convention. On the other extreme, it is also probable that reference point adaptation is radical, or shows excess momentum, by changing as soon as a single mistake/experimentation is observed. The relevance of reference point adaptation comes from the fact that the reference point not only determines whether an outcome is coded as a loss or a gain, but also the magnitude of a gain or a loss, and therefore the value of the prospect corresponding to each demand that the individual might make. In other words, depending on the reference point, the same prospect may be associated with different values. Thus, reference point adaptation has a bearing on the share of the pie that maximises the induced prospect value, and consequently on the ease of transition between conventions, and hence, on stochastically stable division as well.

I now make an assumption about how the reference point of individuals in population B adapts. If the state of the bargaining game is described by the convention ω_x where the pie is divided in the manner $(x, 1-x)$, then any sample that may be drawn by individuals of population B comprises only of x; then, their reference point as a function of the sample is denoted by $r(x)$ and is equal to $1-x$. Now suppose that some individuals in population A experiment/make a mistake by announcing a claim of $x' \neq x$, some instances of which appear in the sample of a population B individual, the other demands in the sample being equal to x. Let the reference point corresponding to this sample be $r(f, x, x')$, where $f \in [0,1]$ is the fraction of demands in the sample that equals x. Without loss of generality, suppose $x' > x$. The second assumption on the reference point function is that it is non-increasing in the sample vector, and so, $r(x) \geq r(f, x, x')$; however, since $r(0, x, x') = 1 - x'$ and the reference point function is non-increasing, $r(f, x, x') - r(x) \in [x - x', 0]$. The third assumption is that if $x'' > x'$, then $r(f, x, x'') - r(f, x, x') \in [x' - x'', 0]$. That is, if the experimentation/mistake comprised of demands of x'' (instead of x') and if the same proportion of mistakes (equal to $1-f$) appear in the sample, then (i) the reference point cannot increase since it is non-increasing in the sample vector, and (ii) the decrease in the reference point cannot be more than $x'' - x'$. If, on the other hand, $x'' < x'$, then $r(f, x, x'') - r(f, x, x') \in [0, x'' - x']$.

I re-iterate that I do not impose any restriction on the reference point function when the support of the random sample that determines it comprises of more than two demands. Further, the assumption about the reference point function is consistent with the two extremes of reference point adaptation: (i) $r_B(\frac{1}{s}, x, x') = 1 - x'$ i.e. the reference point adaptation shows excess momentum by moving from $1-x$ when the convention is ω_x to $1-x'$ as soon as a single mistake/experimentation appears in the sample of demands from the other population, and (ii) $r_B(\frac{s-1}{s}, x, x') = 1 - x$ i.e. the reference point adaptation shows inertia by not changing from $1-x$ until the entire sample comprises of demands of the other population comprises only of x'. An example of a reference-point function that satisfies this property is where it is a weighted average of demands in the sample, where the weights assigned to the different demands may vary according to the initial convention, and need not be equal to the frequency of occurrence of each demand.

The next proposition (proof in the appendix) illustrates how individuals fare when they have prospect theory preferences instead of expected utility preferences.

Proposition 3. *In the limiting stochastically stable division, individuals with preferences that are described by prospect theory receive a higher share of the pie than they would receive if their preferences had an expected utility representation instead if and only if* $\frac{v'(0)}{w'(0)} > v'(x_{EU})$*.*

The proposition above provides the necessary and sufficient condition for a population of individuals with prospect theory preference to receive a higher share than it would receive if its preferences could instead be represented by expected utility. The expression on the left hand side is the ratio of the marginal value at zero, and the marginal probability weight at $p = 0$, while the right hand side is the marginal value of the share of the Nash bargaining solution. Thus, the direction of the inequality determines if the individuals are better off with prospect theory preferences. A corollary of the proposition above is that in the particular case that $u(\cdot) = v(\cdot)$, the population of individuals with prospect theory preferences receive more than half of the pie, and the population of expected utility maximisers correspondingly receive less than half of the pie, if and only if $\frac{u'(0)}{w'(0)} = \frac{v'(0)}{w'(0)} > u'(1 - x_n) = v'(1 - x_n)$. Thus, in this case, the individuals with prospect theory preferences not only outperform their counterfactual expected utility maximising selves but also outperform the actual expected utility maximisers in the other population.

Corollary 2. *Suppose that the utility function of individuals in population A is the same as the value function of individuals in population B, and let it be the function $u(\cdot)$. Then, the population of individuals with preferences that can be described by prospect theory obtain more than half of the pie while the population of expected utility individuals obtain less than half of the pie if and only if* $\frac{u'(0)}{w'(0)} > u'(1 - x_n)$*.*

As mentioned earlier, prospect theory differs from expected utility representation of preferences in three principal ways: (i) evaluation of outcomes about a reference point, (ii) loss aversion, and (iii) transformation of objective probabilities to probability weights. I will now de-construct the effect that each of these features by isolating the individual effect that they have on the stochastically stable division. The proof of the above proposition shows that the value of the parameter λ does not play a role in the result; consequently, loss aversion (i.e. when $\lambda > 1$) or loss-seeking (when $\lambda < 1$) does not play a role either. This gives rise to the next proposition.

Proposition 4. *Loss aversion does not play a role in the determination of the limiting stochastically stable division. Hence Proposition* 3 *and Corollary* 4 *hold irrespective of the presence or absence of loss-aversion or loss-seeking behaviour on part of the individuals of population B.*

I now examine the effect that the features of reference-point dependence and probability weighting have on the stochastically stable division. Firstly, suppose that the preferences of the individuals in population B are not reference-point dependent, i.e. the utility of an outcome depends only on the absolute value of the share received rather than in terms of gains or losses about the reference point. However, the objective probability associated with each outcome is transformed by the weighting function, and the weight placed on the utility of each outcome is the probability weight associated with that particular outcome. In other words, the preferences of the individuals in population B have rank-dependent utility representation in the sense of Quiggin (1982). Then, the next proposition (proof in the appendix) shows that in the limiting stochastically stable division, this population of individual receives a lower share of the pie than they would have if their preferences could instead be represented either by expected utility or by prospect theory.

Proposition 5. *In the limiting stochastically stable division, the population of individuals whose preferences are represented by rank-dependent utility receive a lower share compared to the share of the pie they would receive if their preferences could instead be represented by either expected utility or prospect theory.*

Corollary 3. *Suppose that the utility function of individuals in population A is the same as the value function of individuals in population B. Then, the population of individuals with rank-dependent utility obtain less than half of the pie when they bargain either against a population of expected utility maximisers or against a population of individuals whose preferences are described by prospect theory.*

Finally, I examine the stochastically stable division when the individuals in population B have 'reference-dependent' preferences, i.e. the preferences are described by prospect theory but for the feature that the objective probabilities associated with each outcome are not distorted by the probability weighting function. That is, outcomes are evaluated as gains or losses about a reference point, and each possible outcome is weighted by the objective probability associated with it. The analysis of this situation is the same as in Proposition 3, where the individuals of population B have preferences but with the weighting function $w(p) = p$. This leads to the next proposition (proof in the appendix).

Proposition 6. *In the limiting stochastically stable division, the population of individuals whose preferences are reference point dependent receive a higher share of the pie compared to the share they would receive if their preferences could instead be represented by either expected utility, prospect theory, or rank-dependent utility.*

Corollary 4. *Suppose that the utility function of individuals in population A is the same as the value function of individuals in population B. Then, the population of individuals with* *reference-point dependent preferences obtain more than half of the pie when they bargain either against a population of individuals whose preferences can be represented either by expected utility maximisers, rank-dependent utility or prospect theory.*

The results presented demonstrate the effect that the each of the components of loss-aversion, reference-point dependence and probability weighting have on the limiting stochastically stable division. While loss-aversion is neutral, in the sense of not having any bearing on the determination of the limiting stochastically stable state, probability weighting and reference-point dependence give rise to opposing forces. In comparison to preferences which have an expected utility representation, reference-point dependence confers an advantage on the individuals in the sense of increasing the share of the pie they would have received otherwise, while probability weighting exerts a downward pressure on the same. These two opposing forces express themselves in the condition in Proposition 3, i.e. $x_{PT} \lesseqgtr x_{EU}$ if and only if $\frac{v'(0)}{w'(0)} \lesseqgtr v'(x_{EU})$, that determines if individuals are better off with prospect theory preferences or with expected utility preferences. This condition reflects that whether x_{PT} is greater or smaller than x_{EU} depends on the relative strengths of $v'(0)$ and $w'(0)$, where the terms $w'(0)$ and $v'(0)$ comes from probability weighting (see proof of Proposition 5) and reference-point dependence of preferences (see proof of Proposition 6) respectively. Thus if the influence of $v'(0)$ is stronger than that of $w'(0)$, then the upward pull of reference point dependence outweighs the downward push of probability weighting, and vice-versa.

3 Conclusion

Expected utility is used pervasively in the economics literature to represent preferences of individuals under situations of risk and uncertainty. The widespread use of expected utility owes to the fact that it is founded by a-priori reasonable axioms and that it is very tractable analytically. Yet another justification that is often forwarded is that individuals with non-expected utility would be outperformed by expected utility maximising individuals. These are the issues which motivate this paper. Are individuals always better off when their preferences can be represented by expected utility? I examine this question in a bargaining environment, where individuals are engaged in direct interaction and confrontation with one another. I contrast the share of the pie received in the long-run by (a population of) expected utility maximisers with the share received by them if their preferences were described by prospect theory preferences instead when they bargain with (a population of) expected utility maximisers. The choice of prospect theory as the alternative to expected utility is motivated by the fact that it is arguably the leading preference structure in terms of descriptive validity.

I demonstrate that it is not necessarily true that individuals are better of when they are endowed with expected utility preferences. In fact, I present a necessary and sufficient condition that indicates when individuals obtain a higher share of the pie their preferences are in line with prospect theory (rather than expected utility). In order to understand the nature of this result more deeply, I isolate the effect of the three main components that go to describe prospect theory preferences, namely, reference point dependence, loss-aversion and probability weighting. I show that loss-aversion does not have an effect on the outcome of the bargaining process. However, reference-point dependent preference confers an unambiguous advantage while probability weighting is unambiguously disadvantageous. These two opposing forces provide deeper insight into the earlier result. If the upward pull of reference point dependence is relatively stronger than the downward push of probability weighting, then individuals are better off with prospect theory preferences than with expected utility preferences, and vice-versa.

Appendix

Proof of Proposition 1

Proof. Let $\omega(t) = (\omega_A(t), \omega_B(t))$ denote the current state. Then, with positive probability, all individuals in population B display inertia so that $\omega_B(t) = \omega_B(t+1)$. Now, with positive probability, each individual in population A draws the same sample from $\omega_B(t)$, and responds with the same expected utility maximising demand x_A. Then, with positive probability, in period $t+2$, all individuals in population A display inertia and choose the same demand x_A so that $\omega_A(t+1) = \omega_B(t+2) = \{x_A, \ldots, x_A\}$. Further, in period $t+2$, with positive probability, each individual in population B responds to the previous period's demands of population A; the support of any sample drawn from $\omega_A(t+2)$ is $\{x_A\}$. So, all individuals in population B demand $1 - x_A$, and the state of the bargaining game in period $t+2$ is the convention ω_{x_A}. Thus, from any initial state, the bargaining process reaches a convention with positive probability; by the earlier observation, once a convention is reached, the process stays locked into that convention. ■

Proof of Proposition 2

Proof. *Step 1.* Suppose that the current convention is ω_x, where population A individuals demand (and receive) x and population B individuals demand (and receive) $1-x$, for some $x \in D(\delta)$. Now, the minimum number of experimentations/mistakes needed to move from ω_x to another convention $\omega_{x'}$ is given by $s\,min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{u(x)}{u(1-\delta)}, \frac{v(1-x)}{v(1-\delta)}\}$ (Lemma 1, Young (1993b)). The terms in the above expression come from consideration of the following:

(i) Suppose that some population B individuals, by mistake or because of experimentation, demand a higher share than is dictated by the current convention. In particular, suppose these individuals demand $1-x+\delta$. With positive probability, all individuals in population A

draw a sample that contains all such mistakes. Then, population A individuals demand $x-\delta$ instead of x in response when sufficiently many population B individuals demand $1-x+\delta$. If i is the number of mistakes, all of which appear in the sample of size s with positive probability, then the expected utility received by a population A individual from demanding x is $\frac{i}{s}\,u(0)+(1-\frac{i}{s})\,u(x)=(1-\frac{i}{s})\,u(x)$ while the expected utility from demanding $x-\delta$ is $u(x-\delta)$. So, individuals in population A demand $x-\delta$ in response when $(1-\frac{i}{s})\,u(x)\leq u(x-\delta)$. Then, the minimum number of mistakes is $i^{*}=s\,(1-\frac{u(x-\delta)}{u(x)})$.
It is easily verified that amongst all mistakes by population B individuals with demands greater than $1-x$, the minimum number of mistakes needed in order for population A individuals to demand a share other than x is obtained when the mistake is with $1-x+\delta$: hence, amongst all mistakes with a demand greater than $1-x$, it is sufficient to consider the case when population B individuals demand $1-x+\delta$ by mistake.

(ii) The term $s\,(1-\frac{v(1-x-\delta)}{v(1-x)})$ has a similar explanation: this is the minimum number of experimentations/mistakes by population A individuals in demanding a share greater than x that causes the population B individuals to deliberately demand a share other $1-x$ in response.

(iii) Another type of experimentation/mistake is when some population B individuals demand a share that is lower than $1-x$. In particular, suppose they demand δ. Again, with positive probability, all the mistakes appear in the sample drawn by each population A individuals. If i represent the number of such mistakes, then population A individuals demand $1-\delta$ instead of x when $\frac{i}{s}\,u(1-\delta)+(1-\frac{i}{s})\,u(0)\geq u(x)$. So, the minimum number of mistakes is $i^{*}=s\,\frac{u(x)}{u(1-\delta)}$.
It is again easily verified that amongst all mistakes with demands less than $1-x$, the minimum number of mistakes needed in order for population A individuals to demand a share other than x is obtained when the mistake is with δ: hence, amongst all mistakes with a demand less than $1-x$, it is sufficient to consider the case when population B individuals demand δ by mistake.

(iv) Similarly, the term $s\,\frac{v(1-x)}{v(1-\delta)}$ is minimum number of mistakes made by population A individuals with a demand less than x that causes population B individuals to deliberately demand a share other than $1-x$ in response.

I now make the observation that once $s\,min\{1-\frac{u(x-\delta)}{u(x)},1-\frac{v(1-x-\delta)}{v(1-x)},\frac{u(x)}{u(1-\delta)},\frac{u(1-x)}{u(1-\delta)}\}$ mistakes occur, a transition from the convention ω_x to another convention $\omega_{x'}$ happens with positive probability. To see this, suppose, without loss of generality, that the minimum number of mistakes given by the expression above are made by the population A individuals in period t. Then, with positive probability, all these mistakes appear in the sample drawn by each population B individual in the next period $t{+}1$, and they respond by demanding $1-x'\neq 1-x$; also, with positive probability, all population A individuals display inertia so that $\omega_A(t) = \omega_A(t+1)$. In period $t+2$, with positive probability, all population B individuals display inertia so that $\omega_B(t+1) = \omega_B(t+2) = (1-x', \ldots, 1-x')$, while all population A individuals draw a sample from $\omega_B(t+1) = (1-x', \ldots, 1-x')$, and respond optimally by demanding x'. Then, in period $t+2$, the convention $\omega_{x'}$ is reached. Thus, $s\, min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{u(x)}{u(1-\delta)}, \frac{u(1-x)}{u(1-\delta)}\}$ is the minimum number of experimentations that can cause a transition from a convention ω_x to another convention $\omega_{x'}$.

Step 2. The convention ω_{x^*} is stochastically stable if and only if x^* maximises $s\, min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{u(1-x)}{u(1-\delta)}\}$ on $D(\delta)$ (Lemma 2, Young (1993b)). The reason for this is:

(i) Concavity of the utility functions imply $\frac{u(x)}{u(1-\delta)} \geq \frac{v(1-x)}{v(1-\delta)}$. Hence, $s\, min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{u(x)}{u(1-\delta)}, \frac{u(1-x)}{u(1-\delta)}\}$$=s\, min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{v(1-x)}{v(1-\delta)}\}$

(ii) In the above expression, the terms $1-\frac{u(x-\delta)}{u(x)}$ and $\frac{v(1-x)}{v(1-\delta)}$ are monotonically decreasing in x while $1-\frac{v(1-x-\delta)}{v(1-x)}$ is monotonically increasing in x. Further, $1-\frac{v(1-x-\delta)}{v(1-x)} < min\{1-\frac{u(x-\delta)}{u(x)}, \frac{v(1-x)}{v(1-\delta)}\}$ when $x=\delta$, and $1-\frac{v(1-x-\delta)}{v(1-x)} > min\{1-\frac{u(x-\delta)}{u(x)}, \frac{v(1-x)}{v(1-\delta)}\}$ when $x = 1-\delta$. Hence the function $s\, min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{v(1-x)}{v(1-\delta)}\}$ is first strictly increasing in x, and then strictly decreasing in x. This implies that the expression either uniquely attains its maximum at some x^*, or attains its maximum at two adjacent values x^* and $x^*+\delta$.
For the moment, I assume that x^* is the unique maximising value – uniqueness will actually be argued for in Step 3 below. Then, $s\, min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{v(1-x)}{v(1-\delta)}\}$ equals $s\,(1-\frac{v(1-x-\delta)}{v(1-x)})$ when $x \leq x^*$ and $s\, min\{1-\frac{u(x-\delta)}{u(x)}, \frac{v(1-x)}{v(1-\delta)}\}$ when $x > x^*$. In other words, the minimum resistance path out of a convention ω_x with $x \leq x^*$ involves $s\,(1-\frac{v(1-x-\delta)}{v(1-x)})$ experimentations and leads to the convention $\omega_{x+\delta}$ – recall from (ii) in Step 1 above that $s\,(1-\frac{u(1-x-\delta)}{u(1-x)})$ is the minimum number of mistakes made by population A with the demand $x+\delta$ that lead to population B individuals demanding $1-x-\delta$. Similarly, the minimum resistance path out of a convention ω_x with $x > x^*$ involves a transition to either the convention $\omega_{x-\delta}$ when $s\, min\{1-\frac{u(x-\delta)}{u(x)}, \frac{v(1-x)}{v(1-\delta)}\} = s\,(1-\frac{u(x-\delta)}{u(x)})$ (recall part (i) in Step 1 above), or ω_δ when $s\, min\{1-\frac{u(x-\delta)}{u(x)}, \frac{v(1-x)}{v(1-\delta)}\} = s\,\frac{v(1-x)}{v(1-\delta)}$ (recall part (iv) in Step 1 above).
Now, by definition, $s\, min\{1-\frac{u(x^*-\delta)}{u(x^*)}, 1-\frac{v(1-x^*-\delta)}{v(1-x^*)}, \frac{v(1-x^*)}{v(1-\delta)}\}$ is the radius of the convention ω_{x^*}, and $s\, min\{1-\frac{u(x^*-\delta)}{u(x^*)}, 1-\frac{v(1-x^*-\delta)}{v(1-x^*)}, \frac{v(1-x^*)}{v(1-\delta)}\} > s\, min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{v(1-x)}{v(1-\delta)}\}$ for all $x \neq x^*$. In addition, the modified co-radius of ω_{x^*} is strictly less than the radius because:
(i) ω_{x^*} can be reached from any convention ω_y, $y < x^*$ by the path $\omega_y, \omega_{y+\delta}, \ldots, \omega_{x^*-\delta}, \omega_{x^*}$, and the minimum resistance between any two adjoining conventions along this path equals $s\,(1-\frac{v(x-\delta)}{v(x)})$ (recall the discussion in the paragraph above), which is lower than the radius of ω_{x^*}, and,
(ii) ω_{x^*} can be reached from any convention ω_y, $y > x^*$ either by the path $\omega_y, \omega_{y-\delta}, \ldots, \omega_{x^*+\delta}, \omega_{x^*}$

when $s\,min\{1-\frac{u(x-\delta)}{u(x)}, \frac{v(1-x)}{v(1-\delta)}\} = s\,(1-\frac{u(x-\delta)}{u(x)})$ (recall the discussion in the above paragraph), or when $s\,min\{1-\frac{u(x-\delta)}{u(x)}, \frac{v(1-x)}{v(1-\delta)}\} = s\,\frac{v(1-x)}{v(1-\delta)}$, by the path ω_y, ω_δ (recall discussion in the earlier paragraph), and then using (i) above, via the path $\omega_\delta, \omega_{2\,\delta}, \ldots, \omega_{x^*-\delta}, \omega_{x^*}$. In either case, the minimum resistance between any two adjoining conventions along this path equals $s\,min\{1-\frac{u(x-\delta)}{u(x)}, \frac{v(1-x)}{v(1-\delta)}\}$, which is lower than the radius of ω_{x^*}.

By the radius-modified co-radius theorem in Ellison (2000), since the radius of ω_{x^*} is higher than the modified co-radius of ω_{x^*}, ω_{x^*} is the stochastically stable state.

Step 3. It only remains to argue that as $\delta \to 0$, the stochastically stable state is the Nash bargaining solution $(x_n, 1-x_n)$, where $x_n = argmax_x u(x)\,v(1-x)$ (Lemma 3, Young (1993b)). The x^* that maximises $s\,min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{v(1-x)}{v(1-\delta)}\}$ also maximises $\frac{1}{\delta}\,s\,min\{1-\frac{u(x-\delta)}{u(x)}, 1-\frac{v(1-x-\delta)}{v(1-x)}, \frac{v(1-x)}{v(1-\delta)}\}$. Now, as $\delta \to 0$, $\frac{1}{\delta}\,(1-\frac{u(x-\delta)}{u(x)}) \to \frac{u'(x)}{u(x)}$, and $\frac{1}{\delta}\,(1-\frac{v(1-x-\delta)}{v(1-x)}) \to \frac{v'(1-x)}{v(1-x)}$, both of which are bounded, while $\frac{1}{\delta}\,\frac{v(1-x)}{v(1-\delta)}$ becomes unbounded. Hence, as $\delta \to 0$, the above expression is maximised when $\frac{u'(x)}{u(x)} = \frac{v'(1-x)}{v(1-x)}$. This is a necessary and sufficient condition for x to maximise $ln(u(x)) + ln(v(1-x)) = ln(u(x)\,v(1-x))$. Thus, as $\delta \to 0$, the stochastically stable division is $(x_n, 1-x_n)$, where $x_n = argmax_x u(x)\,v(1-x)$ – the Nash bargaining solution. ■

Proof of Proposition 3

Proof. *Step 1.* Suppose that the current convention is ω_x, where population A individuals demand x and population B individuals demand $1-x$, for some $x \in D(\delta)$. The minimum number of experimentations/mistakes needed to move from a convention ω_x to another convention $\omega_{x'}$ is $s\,min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(\frac{v_-(\alpha-\delta)-v_+(\alpha)}{v_-(x+\alpha-1)-v_+(\alpha)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v_-(\alpha-\delta)-v_-(x-1-\alpha)}{v_-(x-\delta+\alpha)-v_-(x-1-\alpha)})\}$. The terms in the above expression come from consideration of the following:

(i) $s\,(1-\frac{u(x-\delta)}{u(x)})$: this is the minimum number of individuals in population B who make a mistake/experiment by demanding a share greater than x that results in the population A individuals to deliberately demand a share other than $1-x$ in response (see Part (i) in Step 1 for detailed explanation).

(ii) Suppose that some population A individuals, by mistake or experimentation, demand a share x' that is higher than x, the share that is dictated by the current convention. Then, population B individuals demand $\tilde{x}$ instead of $1-x$ (i.e. $\tilde{x} \neq 1-x$) if the value of the prospect induced by demanding $\tilde{x}$ is at least as much as the value of the prospect induced by demanding the $1-x$. Let i be the number of population A individuals who make this mistake. With positive probability, each individual in population B draws a sample that contains all these i instances of x'. Since all individuals in population B draw the same sample, let $r = 1-x-\alpha$ denote reference point corresponding to this sample, where $\alpha \in [0, x'-x]$ reflects the extent to which the reference point adapts from that of the established convention ω_x. If $\alpha = 0$,

then $r = 1 - x$ and the reference point adaptation displays inertia, while if $\alpha = x' - x$, then $r = 1 - x'$ and the reference point adaptation displays excess momentum.

If the randomly drawn sample has $\frac{i}{s}$ proportion of x' and $1 - \frac{i}{s}$ proportion of x, then the value of the prospect induced by a population B individual demanding $1-x$ is $w(\frac{i}{s})\, v_{-}(0-(1-x-\alpha)) + w(1-\frac{i}{s})\, v_{+}(1-x-(1-x-\alpha))$=$w(\frac{i}{s})\, v_{-}(x+\alpha-1) + w(1-\frac{i}{s})\, v_{+}(\alpha)$ while the value of the prospect induced by demanding $\tilde{x} = 1-x'$ is $w(1)v_{-}(1-x'-(1-x-\alpha)) = v_{-}(\alpha+x-x')$. (It is easily verified that the value of the prospect induced by any demand other than $1-x$ or $1-x'$ is strictly dominated by the prospect that is induced by $1-x$ or by $1-x'$, and hence any other demand is not relevant to the calculus.) So, a population B individual will choose $1-x'$ over $1-x$ when $v_{-}(\alpha+x-x') \geq w(\frac{i}{s})\, v_{-}(x+\alpha-1) + w(1-\frac{i}{s})\, v_{+}(\alpha)$. Because the RHS is decreasing in i and because the LHS is negative and does not depend on i, for minimum number of experimentations, the inequality above will hold with equality i.e. $v_{-}(\alpha+x-x') = w(\frac{i}{s})\, v_{-}(x+\alpha-1) + w(1-\frac{i}{s})\, v_{+}(\alpha)$.

Importantly, it can be seen from the equality above that $\frac{i}{s}$ is increasing in x' for the following reason. Suppose the equality holds at a particular set of values of x, x' and α. Now, let x' increase by $\Delta x' > 0$. As result of this, let α change by $\Delta\alpha \in [0, \Delta x']$. Then, the LHS, $v_{-}(\alpha+x-x')$, becomes even more negative while the terms on the RHS, $v_{-}(x+\alpha-1)$ and $v_{+}(\alpha)$, do not reduce in value; hence for the equality to hold, $\frac{i}{s}$ has to increase. Thus, $\frac{i}{s}$ is increasing in x'.

Since $\frac{i}{s}$ is increasing in x', it follows that for the minimum number of experimentations, $x' = x + \delta$. As a result, the equality that defines i can be re-written as $v_{-}(\alpha-\delta) = w(\frac{i}{s})\, v_{-}(x+\alpha-1) + w(1-\frac{i}{s})\, v_{+}(\alpha)$. It can be now be seen that i is increasing in x – I will make use of this in Step 2. Also, using $w(\frac{i}{s}) + w(1-\frac{i}{s}) = 1$, the minimum number of experimentations that can result in a transition from the convention is given by $i = s\, w^{-1}(\frac{v_{-}(\alpha-\delta)-v_{+}(\alpha)}{v_{-}(x+\alpha-1)-v_{+}(\alpha)})$, where $w^{-1}(\cdot)$ is the inverse of $w(\cdot)$.

(iii) $s\, \frac{u(x)}{u(1-\delta)}$ is minimum number of individuals in population B who make mistakes/experiment by demanding less than x that causes population A individuals to deliberately demand a share other than $1-x$ in response (see Part (iii) in Step 1 for detailed explanation).

(iv) Suppose that some population A individuals, by mistake or experimentation, demand a lower share $x' < x$ than is dictated by the current convention. Then, population B individuals demand $\tilde{x}$ instead of $1-x$ (i.e. $\tilde{x} \neq 1-x$) if the value of the prospect induced by demanding $\tilde{x}$ is at least as much as the value of the prospect induced by demanding $1-x$. Let i be the number of population A individuals who demand x'. With positive probability, each population B individual draws a sample that contains all these i instances of x'. Because all these individuals draw a sample that contains the same information, let $r = 1-x+\alpha$ denote their reference point, where $\alpha \in [0, x-x']$ reflects the extent to which the reference point

adapts from that of the established convention ω_x.

If the randomly drawn sample has $\frac{i}{s}$ proportion of x' and $1-\frac{i}{s}$ proportion of x, then the value of the prospect induced by a population B individual from demanding $1-x$ is $w(1)\,v_-(1-x-(1-x+\alpha)) = v_-(-\alpha)$, and the value of the prospect from demanding $\tilde{x} = 1-x'$ is $w(\frac{i}{s})\,v_+(1-x'-(1-x+\alpha)) + w(1-\frac{i}{s})\,v_-(0-(1-x+\alpha)) = w(\frac{i}{s})\,v_+(x-x'-\alpha) + w(1-\frac{i}{s})\,v_-(x-1-\alpha)$. (It is easily verified that the value of the prospect induced by any demand other than $1-x$ or $1-x'$ is strictly dominated by the prospect that is induced by $1-x$ or by $1-x'$, and hence these other demands are not relevant to the calculus). So, a population B individual demands $1-x'$ if $v_-(-\alpha) \leq w(\frac{i}{s})\,v_+(x-x'-\alpha) + w(1-\frac{i}{s})\,v_-(x-1-\alpha)$. Because the RHS is increasing in i and the LHS does not depend on i, to obtain the minimum number of mistakes, the inequality above will hold with equality, i.e. $v_-(-\alpha) = w(\frac{i}{s})\,v_+(x-x'-\alpha) + w(1-\frac{i}{s})\,v_-(x-1-\alpha)$.

Importantly, it can be seen from the equality above that $\frac{i}{s}$ is increasing in x'. The reason is as follows. Suppose the equality holds at a particular set of values of x, x' and α. Now, let x' decrease by $\Delta x' > 0$ to $x' - \Delta x' < x'$, and as a consequence of this, let α increase by $\Delta\alpha \in [0, \Delta x']$. As a result, both the LHS (i.e. $v_-(-\alpha)$) and the second term of the RHS (i.e. $v_-(x-1-\alpha)$) decrease in value and become even more negative while the first term of the RHS (i.e. $v_+(x-x'-\alpha)$) increases. Because of the convexity of v_-, and because $x-1-\alpha < -\alpha$, the same change in α by $\Delta\alpha$ results in a smaller decrease in the value of $v_-(x-1-\alpha)$ compared to the decrease in the value of $v_-(-\alpha)$, and this is further compounded by the fact that $v_-(x-1-\alpha)$ is weighted by $w(1-\frac{i}{s}) \leq 1$. Hence, following the change in $\Delta x'$, the RHS now is greater than the LHS, and to restore the equality, i must decrease. Then, for the minimum number of experimentations out of the convention, x' is set equal to δ, the above equality re-written as $v_-(-\alpha) = w(\frac{i}{s})\,v_+(x-\delta+\alpha) + w(1-\frac{i}{s})\,v_-(x-1-\alpha)$, with $\alpha \in [0, x-\delta]$. It can be now be seen that i is decreasing in x – I will make use of this in Step 2. Using $w(\frac{i}{s}) + w(1-\frac{i}{s}) = 1$, the minimum number of experimentations that can result in a transition from the convention is given by $i = s\,w^{-1}(\frac{v_-(-\alpha)-v_-(x-1-\alpha)}{v_+(x-\delta+\alpha)-v_-(x-1-\alpha)})$, where $w^{-1}(\cdot)$ is the inverse of $w(\cdot)$.

Step 2. The convention ω_{x^*} is stochastically stable if and only if x^* maximises $s\,min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(\frac{v_-(\alpha-\delta)-v_+(\alpha)}{v_-(x+\alpha-1)-v_+(\alpha)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v_-(-\alpha)-v_-(x-1-\alpha)}{v_+(x-\delta+\alpha)-v_-(x-1-\alpha)})\}$ on $D(\delta)$ (see Step 2 in the proof sketch of Proposition 2 for a detailed explanation).

Here, I only note that the terms $1-\frac{u(x-\delta)}{u(x)}$ and $w^{-1}(\frac{v_-(-\alpha)-v_-(x-1-\alpha)}{v_+(x-\delta+\alpha)-v_-(x-1-\alpha)})$ are monotonically decreasing in x while the terms $w^{-1}(\frac{v_-(\alpha-\delta)-v_+(\alpha)}{v_-(x+\alpha-1)-v_+(\alpha)})$ and $\frac{u(x)}{u(1-\delta)}$ are monotonically increasing in x. Further, $\min\{w^{-1}(\frac{v_-(\alpha-\delta)-v_+(\alpha)}{v_-(x+\alpha-1)-v_+(\alpha)}), \frac{u(x)}{u(1-\delta)}\} < min\{w^{-1}(\frac{v_-(-\alpha)-v_-(x-1-\alpha)}{v_+(x-\delta+\alpha)-v_-(x-1-\alpha)}),$ $1-\frac{u(x-\delta)}{u(x)}\}$ when $x=\delta$ and $\min\{w^{-1}(\frac{v_-(\alpha-\delta)-v_+(\alpha)}{v_-(x+\alpha-1)-v_+(\alpha)}), \frac{u(x)}{u(1-\delta)}\} > min\{w^{-1}(\frac{v_-(-\alpha)-v_-(x-1-\alpha)}{v_+(x-\delta+\alpha)-v_-(x-1-\alpha)}),$

$1-\frac{u(x-\delta)}{u(x)}\}$ when $x=1-\delta$.

Thus, $s\,min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(\frac{v_-(\alpha-\delta)-v_+(\alpha)}{v_-(x+\alpha-1)-v_+(\alpha)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v_-(-\alpha)-v_-(x-1-\alpha)}{v_+(x-\delta+\alpha)-v_-(x-1-\alpha)})\}$ is initially increasing in x and then decreasing in x. As argued in Step 2, Proposition 2, by the radius-modified co-radius theorem, a state ω_x is stochastically stable if and only x maximises the above expression. Since the above expression is initially increasing in x and then decreasing in x, it follows that either there is unique stable division $(x, 1-x)$, or there are at most two adjoining stable divisions $(x, 1-x)$ and $(x+\delta, 1-x-\delta)$ that are stochastically stable.

Step 3. If x^* maximises $s\,min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(\frac{v_-(\alpha-\delta)-v_+(\alpha)}{v_-(x+\alpha-1)-v_+(\alpha)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v_-(-\alpha)-v_-(x-1-\alpha)}{v_+(x-\delta+\alpha)-v_-(x-1-\alpha)})\}$, then it also maximises $\frac{1}{\delta}\,s\,min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(\frac{v_-(\alpha-\delta)-v_+(\alpha)}{v_-(x+\alpha-1)-v_+(\alpha)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v_-(-\alpha)-v_-(x-1-\alpha)}{v_+(x-\delta+\alpha)-v_-(x-1-\alpha)})\}$. Now, as $\delta \to 0$, $\frac{1}{\delta}(1-\frac{u(x-\delta)}{u(x)}) \to \frac{u'(x)}{u(x)}$, and $\frac{1}{\delta}\,w^{-1}(\frac{v_-(\alpha-\delta)-v_+(\alpha)}{v_-(x+\alpha-1)-v_+(\alpha)}) \to \frac{1}{w'(0)}\frac{-v'_-(0)}{v_-(x-1)}$, while the terms $\frac{1}{\delta}\frac{u(x)}{u(1-\delta)}$ and $\frac{1}{\delta}\,w^{-1}(\frac{v_-(\alpha-\delta)-v_-(x-1-\alpha)}{v_+(x-\delta+\alpha)-v_-(x-1-\alpha)})$ become unbounded. Hence, as $\delta \to 0$, the above expression is maximised when $\frac{u'(x)}{u(x)} = \frac{1}{w'(0)}\frac{-v'_-(0)}{v_-(x-1)}$. Thus the stochastically stable division is $(x^*, 1-x^*)$ where x^* is the solution of $\frac{u'(x)}{u(x)} - \frac{1}{w'(0)}\frac{-v'_-(0)}{v_-(x-1)} = 0$.

The concluding step involves a comparison of x^* above with x_n. Since $\frac{u'(x_n)}{u(x_n)} - \frac{v'(1-x_n)}{v(1-x_n)} = 0$, $\frac{1}{w'(0)}\frac{-v'_-(0)}{v_-(x_n-1)} \lesseqqgtr \frac{v'(1-x_n)}{v(1-x_n)} \Leftrightarrow \frac{u'(x_n)}{u(x_n)} - \frac{1}{w'(0)}\frac{-v'_-(0)}{v_-(x_n-1)} \gtreqqless 0 \Leftrightarrow x^* \gtreqqless x_n$ (since the LHS of the second inequality is decreasing in x). Using $v_-(-x) = -\lambda\, v(x)$, $x^* \gtreqqless x_n$ if and only if $\frac{v'(0)}{w'(0)} \lesseqqgtr v'(1-x_n)$. Denoting the share of the pie received by the population of individuals with prospect theory preferences by x_{PT}, $x_{PT} = 1-x^* \lesseqqgtr x_{EU} = 1-x_n$ if and only if $\frac{v'(0)}{w'(0)} \lesseqqgtr v'(1-x_n) = v'(x_{EU})$.

Thus, if I compare the stochastically stable division when population B individuals have preferences that can be represented by expected utility to the stochastically stable division when they have preferences that can be described by prospect theory, I find that the share of the pie received by the latter is the same if $\frac{v'(0)}{w'(0)} = v'(x_{EU})$, smaller if $\frac{v'(0)}{w'(0)} < v'(x_{EU})$ and greater if $\frac{v'(0)}{w'(0)} > v'(x_{EU})$. ■

Proof of Proposition 5

Proof. *Step 1.* Suppose that the current convention is ω_x, where population A individuals demand x and population B individuals demand $1-x$, for some $x \in D(\delta)$. The minimum number of experimentations/mistakes needed to move from ω_x to another convention $\omega_{x'}$ is given by $s\,min\{1-\frac{u(x-\delta)}{u(x)}, 1-w^{-1}(\frac{v(1-x-\delta)}{v(1-x)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v(1-x)}{v(1-\delta)})\}$. The terms in the above expression come from consideration of the following:

(i) The term $s\,(1-\frac{u(x-\delta)}{u(x)})$ is the minimum number of mistakes made by population B individuals in demanding a share greater than x that causes the population B individuals to deliberately demand a lower share than $1-x$ in response (see Part (i) in Step 1 for detailed

explanation).

(ii) Suppose instead that population A individuals, by mistake or experimentation, demand a share x' that is higher than x, the share dictated by the current convention. Let i be the number of population A individuals who make this mistake. With positive probability, each individual in population B draws a sample that contains all these i instances of x'. Then, population B individuals demand $\tilde{x}$ instead of $1-x$ (where $\tilde{x} \neq 1-x$) if the rank dependent utility from demanding $\tilde{x}$ is at least as much as the rank-dependent utility from demanding the $1-x$. If the randomly drawn sample has $\frac{i}{s}$ proportion of x' and $1-\frac{i}{s}$ proportion of x, then the rank-dependent utility of a population B individual demanding $1-x$ is $w(\frac{i}{s})\, v(0) + w(1-\frac{i}{s})\, v(1-x)$, while the rank-dependent utility due to demanding $\tilde{x} = 1-x'$ is $v(1-x')$. (It is easily verified that the utility arising from any demand other than $1-x$ or $1-x'$ is strictly dominated by the utility obtained from demanding $1-x$ or by $1-x'$, and hence these other demands are not relevant to the calculus). Then, a population B individual will choose $1-x'$ over $1-x$ when $w(\frac{i}{s})\, v(0) + w(1-\frac{i}{s})\, v(1-x) \leq v(1-x')$; since the LHS is decreasing in i, the minimum number of experimentations/mistakes will be obtained when the above condition is satisfies with equality, i.e. $i = s\, w^{-1}(1-\frac{v(1-x')}{v(1-x)})$. Now, in the above equality, i is increasing in x'. Hence, x' is set equal to $x+\delta$ to obtain the minimum number of experimentations/mistakes that make a transition from the convention possible. This gives $i = s\, w^{-1}(1-\frac{v(1-x-\delta)}{v(1-x)})$.

(iii) The term $s\,\frac{u(x)}{u(1-\delta)}$ is minimum number of mistakes made by population B individuals with a demand less than x that causes population B individuals to deliberately demand a share other than $1-x$ in response (see Part (iii) in Step 1 for detailed explanation).

(iv) Suppose instead that population A individuals, by mistake or experimentation, demand a share x' that is lower than the share x that is dictated by the current convention. Let i be the number of population A individuals who make this mistake. With positive probability, each individual in population B draws a sample that contains all these i instances of x'. If the randomly drawn sample has $\frac{i}{s}$ proportion of x' and $1-\frac{i}{s}$ proportion of x, then the rank-dependent utility of a population B individual demanding $1-x$ is $v(1-x)$, while the rank-dependent utility due to demanding $\tilde{x} = 1-x'$ is $w(\frac{i}{s})\, v(1-x') + w(1-\frac{i}{s})\, v(0)$. Then, a population B individual will choose $1-x'$ over $1-x$ if $v(1-x) \leq w(\frac{i}{s})\, v(1-x') + w(1-\frac{i}{s})\, v(0)$; since the RHS is increasing in i, the minimum number of experimentations/mistakes will be obtained when the above condition is satisfies with equality, i.e. $i = s\, w^{-1}(\frac{v(1-x)}{v(1-x')})$. Now, in the above equality, i is increasing in x'. Hence, x' is set equal to δ to obtain the minimum number of experimentations/mistakes that make a transition from the convention possible. This gives $i = s\, w^{-1}(\frac{v(1-x)}{v(1-\delta)})$.

Step 2. The convention ω_{x^*} is stochastically stable if and only if x^* maximises

$s\,min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(1-\frac{v(1-x-\delta)}{v(1-x)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v(1-x)}{v(1-\delta)})\}$ on $D(\delta)$ (see Step 2 in the proof sketch of Proposition 2 for a detailed explanation).

Here, I only note that the terms $1-\frac{u(x-\delta)}{u(x)}$ and $w^{-1}(\frac{v(1-x)}{v(1-\delta)})$ are monotonically decreasing in x while the terms $w^{-1}(1-\frac{v(1-x-\delta)}{v(1-x)})$ and $\frac{u(x)}{u(1-\delta)}$ are monotonically increasing in x. Further, $\min\{w^{-1}(1-\frac{v(1-x-\delta)}{v(1-x)}), \frac{u(x)}{u(1-\delta)}\} < min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(\frac{v(1-x)}{v(1-\delta)})\}$ when $x=\delta$ and $\min\{w^{-1}(1-\frac{v(1-x-\delta)}{v(1-x)}), \frac{u(x)}{u(1-\delta)}\} > min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(\frac{v(1-x)}{v(1-\delta)})\}$ when $x=1-\delta$.

Thus, $s\,min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(1-\frac{v(1-x-\delta)}{v(1-x)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v(1-x)}{v(1-\delta)})\}$ is initially increasing in x and then decreasing in x. As argued in Step 2, Proposition 2, by the radius-modified co-radius theorem, a state ω_x is stochastically stable if and only x maximises the above expression. Since the above expression is initially increasing in x and then decreasing in x, it follows that either there is unique stable division $(x, 1-x)$, or there are at most two adjoining stable divisions $(x, 1-x)$ and $(x+\delta, 1-x-\delta)$ that are stochastically stable.

Step 3. The x^* that maximises $s\,min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(1-\frac{v(1-x-\delta)}{v(1-x)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v(1-x)}{v(1-\delta)})\}$ also maximises $\frac{1}{\delta}\,s\,min\{1-\frac{u(x-\delta)}{u(x)}, w^{-1}(1-\frac{v(1-x-\delta)}{v(1-x)}), \frac{u(x)}{u(1-\delta)}, w^{-1}(\frac{v(1-x)}{v(1-\delta)})\}$. Now, as $\delta \to 0$, $\frac{1}{\delta}(1-\frac{u(x-\delta)}{u(x)}) \to \frac{u'(x)}{u(x)}$, and $w^{-1}(1-\frac{v(1-x-\delta)}{v(1-x)}) \to w^{-1\prime}(0)\,\frac{v'(1-x)}{v(1-x)} = \frac{1}{w'(0)}\,\frac{v'(1-x)}{v(1-x)}$, while the terms $\frac{1}{\delta}\,\frac{u(x)}{u(1-\delta)}$ and $\frac{1}{\delta}\,w^{-1}(\frac{v(1-x)}{v(1-\delta)})$ become unbounded. Hence, as $\delta \to 0$, the above expression is maximised when $\frac{u'(x)}{u(x)} = \frac{1}{w'(0)}\,\frac{v'(1-x)}{v(1-x)}$. Thus the stochastically stable division is $(\tilde{x}, 1-\tilde{x})$ where $\tilde{x}$ is the solution of $\frac{u'(x)}{u(x)} - \frac{1}{w'(0)}\,\frac{v'(1-x)}{v(1-x)} = 0$.

The concluding step involves a comparison of $\tilde{x}$ above with x_n, where $\frac{u'(x_n)}{u(x_n)} = \frac{v'(1-x_n)}{v(1-x_n)}$, and x^*, where $\frac{u'(x^*)}{u(x^*)} = \frac{1}{w'(0)}\,\frac{v'(0)}{v(1-x)}$.

(i) Firstly, since $w'(0) > 1$, $\frac{1}{w'(0)}\,\frac{v'(1-x)}{v(1-x)} < \frac{v'(1-x)}{v(1-x)}$, for all $x \in D(\delta)$. So, $\frac{u'(x_n)}{u(x_n)} - \frac{1}{w'(0)}\,\frac{v'(1-x_n)}{v(1-x_n)} > \frac{u'(x_n)}{u(x_n)} - \frac{v'(1-x_n)}{v(1-x_n)} = 0$. As a result, $\frac{u'(x_n)}{u(x_n)} - \frac{1}{w'(0)}\,\frac{v'(1-x_n)}{v(1-x_n)}$ is decreasing in x, $\tilde{x} > x_n$.

(ii) Secondly, by concavity of v, $\frac{1}{w'(0)}\,\frac{v'(1-x)}{v(1-x)} < \frac{1}{w'(0)}\,\frac{v'(0)}{v(1-x)}$ for all $x \in D(\delta)$. So, $0 = \frac{u'(\tilde{x})}{u(\tilde{x})} - \frac{1}{w'(0)}\,\frac{v'(1-\tilde{x})}{v(1-\tilde{x})} > \frac{u'(\tilde{x})}{u(\tilde{x})} - \frac{1}{w'(0)}\,\frac{v'(0)}{v(1-\tilde{x})}$. Since $\frac{u'(\tilde{x})}{u(\tilde{x})} - \frac{1}{w'(0)}\,\frac{v'(0)}{v(1-\tilde{x})}$ is decreasing in x, $\tilde{x} > x^*$.

Denoting the share of the pie received by the population of individuals with rank-dependent utility by x_{RD}, $x_{RD} = 1-\tilde{x} < 1-x_n = x_{EU}$ and $x_{RD} = 1-\tilde{x} < 1-x^* = x_{PT}$. Thus, in the stochastically stable division, population B individuals receive a lower share of the pie when their preferences are represented by rank-dependent utility than when their preferences are described either by expected utility or by prospect theory. ■

Proof of Proposition 6

Proof. It is sufficient to use $w(p) = p$ in the proof of Proposition 3. Thus, the limiting stochastically stable division is $(x^*, 1-x^*)$ where x^* is the solution of $\frac{u'(x)}{u(x)} - \frac{-v'_{-}(0)}{v_{-}(x-1)} = 0$. So now, it only remains to compare $\hat{x}$ with (i) x_n, where $\frac{u'(x_n)}{u(x_n)} = \frac{v'(1-x_n)}{v(1-x_n)}$, (ii) x^*, where

$\frac{u'(x^*)}{u(x^*)} = \frac{1}{w'(0)} \frac{v'(0)}{v(1-x^*)}$, and (iii) $\tilde{x}$, where $\frac{u'(\tilde{x})}{u(\tilde{x})} = \frac{1}{w'(0)} \frac{v'(1-\tilde{x})}{v(1-\tilde{x})}$.

(i) Firstly, by concavity of v, $\frac{v'(0)}{v(1-x)} > \frac{v'(1-x)}{v(1-x)}$, for all $x \in D(\delta)$. As a result, $\frac{u'(x_n)}{u(x_n)} - \frac{v'(0)}{v(1-x_n)} < \frac{u'(x_n)}{u(x_n)} - \frac{v'(1-x_n)}{v(1-x_n)} = 0$. Since $\frac{u'(x)}{u(x)} - \frac{v'(0)}{v(1-x)}$ is decreasing in x, $\hat{x} < x_n$.

(ii) Secondly, since $w'(0) > 1$, $\frac{v'(0)}{v(1-x)} > \frac{1}{w'(0)} \frac{v'(0)}{v(1-x)}$ for all $x \in D(\delta)$. As a result, $\frac{u'(x^*)}{u(x^*)} - \frac{v'(0)}{v(1-x^*)} < \frac{u'(x^*)}{u(x^*)} - \frac{1}{w'(0)} \frac{v'(0)}{v(1-x^*)} = 0$. Since $\frac{u'(x)}{u(x)} - \frac{v'(0)}{v(1-x)}$ is decreasing in x, $\hat{x} < x^*$.

(iii) Finally, by concavity of v, $\frac{v'(1-x)}{v'(0)} < 1$, while $w'(0) > 1$, i.e. $w'(0) > \frac{v'(1-x)}{v'(0)}$. This implies $\frac{1}{w'(0)} \frac{v'(1-x)}{v(1-x)} < \frac{v'(0)}{v'(1-x)}$. As a result, $\frac{u'(\hat{x})}{u(\hat{x})} - \frac{1}{w'(0)} \frac{v'(1-\hat{x})}{v(1-\hat{x})} > \frac{u'(\hat{x})}{u(\hat{x})} - \frac{v'(0)}{v'(1-\hat{x})} = 0$. Since $\frac{u'(x)}{u(x)} - \frac{1}{w'(0)} \frac{v'(1-x)}{v(1-x)}$ is decreasing in x, $\hat{x} < \tilde{x}$.

Denoting the share of the pie received by the population of individuals with reference-point dependent preferences by x_{RPD}, I find $x_{RPD} = 1 - \hat{x} > 1 - x_n = x_{EU}$, $x_{RPD} = 1 - \hat{x} > 1 - x^* = x_{PT}$ and $x_{RPD} = 1 - \hat{x} > 1 - \tilde{x} = x_{RD}$. Thus, in the stochastically stable division, population B individuals receive a higher share of the pie when their preferences are reference-point dependent than when their preferences are described either by expected utility, prospect theory, or rank-dependent utility. ■

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