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# Measuring performance and valuing firms: In search of the lost capital

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## **Abstract**

Residual income models are widely used for value-based management purposes as well as for firm valuation. Conventional residual income models depend on: (i) the capital invested, (ii) the rate of return, (iii) the opportunity cost of capital. This paper presents a new paradigm where a fourth element is added: the capital that investors lose when they infuse funds in the firm (or project). This “lost-capital” paradigm may be derived from several perspectives and has various economic, financial, accounting interpretations. It may be employed in real-life applications for incentive compensation and firm (project) valuation. A numerical example illustrates the implementation of the paradigm to the EVA model and the Edwards-Bell-Ohlson model. Also, a model is presented with the nice property of being aligned in sign with NPV.

**Keywords.** Performance measurement, value-based management, residual income, incentive compensation, lost capital, Economic Value Added, firm value.

**JEL codes.** G11, G12, G31, G34, M21, M41, M52.

# Measuring performance and valuing firms: In search of the lost capital

## Abstract

Residual income models are widely used for value-based management purposes as well as for firm valuation. Conventional residual income models depend on: (i) the capital invested, (ii) the rate of return, (iii) the opportunity cost of capital. This paper presents a new paradigm where a fourth element is added: the capital that investors lose when they infuse funds in the firm (or project). This “lost-capital” paradigm may be derived from several perspectives and has various economic, financial, accounting interpretations. It may be employed in real-life applications for incentive compensation and firm (project) valuation. A numerical example illustrates the implementation of the paradigm to the EVA model and the Edwards-Bell-Ohlson model. Also, a model is presented with the nice property of being aligned in sign with NPV.

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## 1 Introduction

Corporate finance and management accounting find a common terrain in the study of the notion of residual income, also called abnormal earning, which is formally computed as the difference between the actual income and the counterfactual income investors would receive if they invested their funds at the opportunity cost of capital. Coined by the General Electric Company, the term first appears in the literature in Solomons (1965, p. 63), although the same concept, differently labeled, was studied even earlier (e.g. Preinreich, 1936, 1938; Edwards and Bell, 1961, Bodenhorn, 1964). The contributions of Peasnell (1981, 1982), Peccati (1987, 1990, 1991) and Ohlson (1989, 1995), have caused a renewed interest in this notion in both corporate finance and management accounting, with particular regard to firm valuation, performance measurement, value-based management. A large number of theoretical and applied studies have appeared dealing with the subject (e.g. Stewart, 1991; Ohlson, 1995; Feltham and Ohlson, 1995; Rappaport, 1998; Lundholm and O’Keefe, 2001; Young and O’Byrne, 2001; Martin et al. 2003; Weaver and Weston, 2003; O’Byrne and Young, 2006) and a large number of textbooks and professional publications in corporate finance, managerial finance, management accounting directly deal with the topic (e.g. Brealey and Myers, 2000; Copeland et al., 2000; Palepu et al., 2000; Grinblatt and Titman, 2002; Revsine et al., 2005; Arnold, 2005).

An alternative paradigm of residual income has been recently introduced by Magni (2000a, 2000b, 2000c, 2001, 2003, 2005) which differs, conceptually and formally, from the standard paradigm used by academics,

analysts and practitioners. This paper aims at shedding light on this paradigm and provides some theoretical and practical results relevant for both valuation and incentive compensation. In particular, the standard paradigm may be seen as grounded on three elements: (i) the actual capital invested, (ii) the actual rate of return, (iii) the opportunity cost of capital (the foregone rate of return). The alternative paradigm takes into consideration an additional element: the lost capital, and is therefore here named *lost-capital* paradigm. As a paradigm, it generates several new metrics, in particular one for any existing metric in the standard paradigm. The new paradigm is presented in five autonomous though equivalent ways, in order to show its multifaceted significance and its sound economic meaning, and some differences and relations between the two paradigms are investigated. In particular, the new paradigm enables one to compute the project's (firm's) market value leaving out any consideration about timing: value is a function of book value and the sum of residual incomes: earnings aggregation, as opposed to discounting, applies.

A numerical example is also illustrated, where the paradigm is applied to the well-known *Economic Value Added* (Stewart, 1991) and to the so-called *Edwards-Bell-Ohlson* model (Ohlson, 1995). Furthermore, a third metric, namely Fernández's (2002) *Created Shareholder Value*, is transformed into the corresponding lost-capital metric. The conversion originates a metric that is consistent in sign with the Net Present Value. Therefore, this metric is particularly suited for managerial compensation, given that it directly ties performance to value creation.

The paper is structured as follows. Section 2 introduces the standard paradigm of residual income.<sup>1</sup> Section 3 presents the new paradigm from five different points of view: (i) a replicating cash-flow and its outstanding capital, (ii) the investor's wealth and its evolution through time, (iii) the construction of alternative depreciation plans (iv) the keynesian notion of user cost, (v) the lost-capital as an accumulation of past standard residual incomes. Section 4 investigates some relations between the two paradigms. Section 5 shows that the lost-capital paradigm is compatible with the Net Present Value (and the Market Value Added) and that value may be derived from lost-capital residual incomes by neglecting timing: only the sum of residual incomes is of concern for computing market value. Section 6 focuses on Economic Value Added and the Edwards-Bell-Ohlson model: first, they are derived as particular cases of the standard paradigm; then, the companion metrics are introduced in the lost-capital paradigm. Section 7 illustrates an example aiming at shedding light on the behavior of the two pairs of metrics and suggesting some possible implications for executive compensation, under the assumption that expectations are met. Section 8 shows that the lost-capital companion of Fernández's (2002) Created Shareholder Value is aligned in sign with the Net Present Value. Some concluding remarks end the paper.

For all notational conventions the reader should refer to Table 12 at the end of the paper.

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<sup>1</sup>The nouns 'profit', 'income', 'return', 'earning' will be used as synonyms, as well as the adjectives 'excess', 'residual', 'abnormal'.

## 2 The standard paradigm

Let  $\vec{a}=(-a_0, a_1, \dots, a_n)$  be an expected cash-flow stream released by project (firm)  $a$  in the span of  $n$  periods.

Let  $x_1, \dots, x_n$  be periodic rates of return such that

$$a_0 = \sum_{t=1}^n \frac{a_t}{\prod_{k=1}^t (1 + x_k)}.$$

For notational convenience we will often omit time subscripts, as long as ambiguity does not arise. Therefore, the above equation may be rewritten as

$$a_0 = \sum_{t=1}^n \frac{a_t}{(1 + x)^t}$$

where  $(1 + x)^t$  should be read as  $\prod_{k=1}^t (1 + x_k)$ . Thus, the symbol  $x$  represents either an internal rate of return or, rather, an internal discount function for project  $a$  that generalizes the notion of internal rate of return (see Weingartner, 1966; Peasnell, 1982, Franks and Hodges, 1984; Brief and Lawson, 1992; Peccati, 1990, 1991. See Pressacco and Stucchi, 1997, and Magni, 2004, 2005, for an internal rate of return based on the sign of the outstanding capital).

Let  $w_t(x)$ ,  $t = 1, 2, \dots, n$  be *arbitrary* numbers such that

$$w_t(x) = w_{t-1}(x)(1 + x) - a_t \quad t = 1, \dots, n \quad (1)$$

with  $w_0(x) := a_0$ . The above equation may conveniently be interpreted as the recursion formula for the project's outstanding capital. The undertaking of the project implies that, at the outset of each period, the capital  $w_{t-1}(x)$  is invested at the internal rate  $x$ , thus producing the interest  $xw_{t-1}(x)$ , which one may interpret as the profit of that period. Excess profit is profit above the profit that could be earned if the capital were invested in an alternative course of action (i.e. at an alternative rate of return). Letting  $i$  be the foregone rate of return (assumed constant for mere convenience),  $i \neq x$ , the foregone return in case of project rejection amounts to  $iw_{t-1}(x)$ . The latter is also known as opportunity cost.<sup>2</sup> The excess profit, or residual income, in the  $t$ -th period is therefore

$$RI_t^S = xw_{t-1}(x) - iw_{t-1}(x) = w_{t-1}(x)(x - i) \quad (2)$$

where  $xw_{t-1}(x)$  is the actual income. The formalization in eq. (2) is the classical one employed in the relevant literature (e.g. Edwards and Bell, 1961; Peasnell, 1981, 1982; Peccati, 1990, 1991; Ohlson, 1995; Pressacco and Stucchi, 1997; Martin and Petty, 2000; Arnold and Davies, 2000; Lundholm and O'Keefe, 2001; Martin et al. (2003)). This approach evidently rests on three basic elements: outstanding capital, internal rate of return, opportunity cost of capital. Different metrics are generated by this scheme, grounded on different notions of capital employed (asset side, equity side, economic, accounting, etc.), of cash flows employed (Free

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<sup>2</sup>Opportunity cost=foregone return, opportunity cost of capital=foregone rate of return.

Cash Flow, Equity Cash Flow, Capital Cash Flow<sup>3</sup>), of internal discount function employed (ROA, RONA, ROE, etc.).

*Remark 1.* It is worth noting that eq. (1) is consistent with the clean surplus concept (Brief and Peasnell, 1996). In business economics, this equation lies at the core of the notion of income (Lee, 1985; Hansen, 1972); in financial and actuarial mathematics, it represents the recursion formula for computing the balance (residual debt) in a loan contract (Kellison, 1991; Promislow, 2006; Werner and Sotnikov, 2006). The similarities between accounting and finance are here profound. Rewriting the equation as  $a_t = xw_{t-1}(x) - (w_{t-1}(x) - w_t(x))$  one may interpret the right-hand side either as the difference between income and change in book value or as the difference between interest and principal repayments: the former takes a management accounting perspective, the latter a financial one. However, to maintain consistency with Net Present Value,  $w_t(x)$  may be any number as long as  $x$  satisfies the equation: book value is therefore only one among many infinite possible choices.<sup>4</sup>

### 3 The *lost-capital* paradigm

The opportunity cost of investing in project  $a$  is that of renouncing to investing funds at the opportunity cost of capital  $i$ . This section presents a different way of interpreting the notion of foregone return, and therefore a different way of interpreting the notion of residual income. It has been originally introduced and investigated in Magni (2000a, 2000b, 2000c) and thoroughly investigated from several points of view in more recent years (2004, 2005, 2006). This section shows that it is possible to derive this notion from five different (but logically equivalent) arguments: an arbitrage-based argument using replicating cash-flows; an economic argument focussed on the investor's wealth; two accounting arguments involving the concept of depreciation and putting in place, respectively, depreciation schedules and the keynesian notion of user cost; a financial argument based on compounding interest on the standard residual income paradigm.

#### 3.1 The replicating cash-flow argument

As seen in the previous section, if the investor invests  $a_0$  in the project, his cash-flow is  $\bar{a}$  and the residual capital invested is  $w_{t-1}(x)$ , which is a dynamic system represented by

$$w_t(x) = w_{t-1}(x)(1 + x) - a_t. \quad (3)$$

Accepting the project the investor foregoes the opportunity of investing  $a_0$  in an alternative asset from which he could as well periodically withdraw the amounts  $a_t$ ,  $t = 1, \dots, n$ , so realizing the same pattern

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<sup>3</sup>For the notion of Capital Cash Flow, see Ruback (2002) and Fernández (2002).

<sup>4</sup>Admittedly,  $w_t$  itself may be labelled *book value*, given that book value is, in principle, arbitrary. In this view, for example, market value is only a particular choice of book value.

of cash flows as project  $a$ . Let  $w_t(i)$  be the outstanding balance at time  $t$  if the investor invests  $a_0$  in the alternative asset. In this case, the capital employed increases at the rate  $i$ , but falls by the amount  $a_t$ , which is withdrawn from the balance at the end of the period. This is described by the recurrence equation

$$w_t(i) = w_{t-1}(i)(1+i) - a_t \quad (4)$$

where, obviously,  $w_0(i) := a_0$ . Thus, if project is accepted, the outstanding balance in the  $t$ -th period is  $w_{t-1}(x)$ ; if, instead, the alternative asset is accepted, the outstanding balance is  $w_{t-1}(i)$ , which is here named the *lost capital*. The rate of return in the former case is  $x$ ; the rate of return in the latter case is  $i$ . Hence, the income in the former case is  $xw_{t-1}(x)$ , the income in the latter case is  $iw_{t-1}(i)$ . The residual income is therefore:

$$RI_t^L = xw_{t-1}(x) - iw_{t-1}(i). \quad (5)$$

The second addend is a lost return, obtained by multiplying the foregone return rate  $i$  by the lost capital  $w_{t-1}(i)$ .

This argument is evidently arbitrage-based: if  $a_0$  is invested in the project (firm), the payoff vector is  $(a_1, a_2, \dots, a_n)$ ; if instead  $a_0$  is invested at the cost of capital, the payoff stream is  $(a_1, a_2, \dots, a_n + w_n(i))$ . The terminal lost capital  $w_n(i)$  is the resulting arbitrage payoff generated by the replicating portfolio. If it is negative, project  $a$  is worth undertaking: a long position on the project and a short position on the alternative asset yield the arbitrage payoff vector  $(0, 0, 0, \dots, -w_n(i))$ ; if it is positive, the replicating cash-flow stream should be selected: a long position on the latter and a short position on  $a$  yield the arbitrage payoff vector  $(0, 0, 0, \dots, w_n(i))$ .

### 3.2 The wealth increase argument

Let us assume that an investor currently invests funds in a financial asset yielding a periodic return rate equal to  $i$  and let  $W_0$  be his net worth at time 0. If project  $a$  is not undertaken, the investor's wealth evolves according to the recursive equation

$$W_t(i) = W_{t-1}(i)(1+i) \quad (6)$$

so that  $W_t(i) = W_0(1+i)^t$ . If, instead, project  $a$  is undertaken, the investor, while renouncing to investing  $a_0$  at the rate  $i$ , receives the periodic sums  $a_t$ , which may be reinvested at the rate  $i$  in the financial asset.<sup>5</sup> In this case, the investor's wealth is a portfolio of two assets evolving at the rates  $x$  and  $i$  respectively. At time  $t$ , the investor's wealth amounts to

$$W_t(x, i) = w_t(x) + (W_{t-1}(x, i) - w_{t-1}(x))(1+i) + a_t \quad (7)$$

where  $w_t(x)$  is determined by eq. (3). Solving eq. (7) we find

$$W_t(x, i) = w_t(x) + (W_0 - a_0)(1+i)^t + \sum_{k=1}^t a_k(1+i)^{t-k}.$$

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<sup>5</sup>Note that this is just the standard assumption of the NPV rule.

This implies that wealth increase in case of project acceptance is

$$W_t(x, i) - W_{t-1}(x, i) = xw_{t-1}(x) + i \left( (W_0 - a_0)(1+i)^{t-1} + \sum_{k=1}^{t-1} a_k(1+i)^{t-1-k} \right),$$

whereas wealth increase in case of project rejection is

$$W_t(i) - W_{t-1}(i) = iW_0(1+i)^{t-1}.$$

Therefore, the excess increase in wealth is given by the difference of the alternative wealth increases:

$$\begin{aligned} \text{excess increase} &= (W_t(x, i) - W_{t-1}(x, i)) - (W_t(i) - W_{t-1}(i)) \\ &= xw_{t-1}(x) - ia_0(1+i)^{t-1} + i \sum_{k=1}^{t-1} a_k(1+i)^{t-1-k}. \end{aligned} \quad (8)$$

From eq. (4), we have  $w_{t-1}(i) = a_0(1+i)^{t-1} - \sum_{k=1}^{t-1} a_k(1+i)^{t-1-k}$ , so that eq. (8) becomes

$$\text{excess return} = xw_{t-1}(x) - iw_{t-1}(i) = \text{RI}_t^L \quad (9)$$

It is worth noting that we have found  $\text{RI}_t^L$  by making use of two alternative hypotheses about the evolution of the investor's wealth, namely the two dynamic systems in eq. (6) and eq. (7).

Note also that we may ideally part the investor's wealth into two assets in both cases:

$$W_{t-1}(x, i) = \overbrace{w_{t-1}(x)}^{\text{asset invested at rate } x} + \overbrace{(W_{t-1}(x, i) - w_{t-1}(x))}^{\text{asset invested at rate } i} \quad (10)$$

$$W_{t-1}(i) = \overbrace{(W_{t-1}(i) - W_{t-1}(x, i) + w_{t-1}(x))}^{\text{asset invested at rate } i} + \overbrace{(W_{t-1}(x, i) - w_{t-1}(x))}^{\text{asset invested at rate } i}. \quad (11)$$

The differential return between the two alternatives is not dependent on the second addends, which are shared by both alternatives; they may therefore be dismissed and, applying the corresponding rates of return, we find

$$\text{excess return} = xw_{t-1}(x) - i(W_{t-1}(i) - W_{t-1}(x, i) + w_{t-1}(x)).$$

Using the fact that  $W_{t-1}(i) - W_{t-1}(x, i) + w_{t-1}(x) = w_{t-1}(i)$  one finds back  $\text{RI}_t^L$ .

### 3.3 The depreciation argument

The lost-capital residual income may be ideally obtained by transforming the two alternative courses of action into two alternative depreciation schedules.

Consider asset  $A$ , producing the cash-flow  $\vec{A} = (-a_0, a_1, a_2, \dots, a_n + s_n)$ , where  $s_n$  is the asset's scrap value, received at the end of its service life. Let  $v_t$  be the accounting value of this asset at time  $t$  (with  $v_0 := a_0$ ) and let  $\text{Dep}_t := v_{t-1} - v_t$  be the depreciation charge in the  $t$ -th period. While any depreciation such that  $\sum_{t=1}^n \text{Dep}_t = a_0$  is acceptable for accounting purposes (see Peasnell, 1982), there is one significant from



an economic point of view: the decline in the present value of asset  $A$ 's future cash flows; letting  $r$  be the discount rate, this asset's accounting value is

$$v_t = \sum_{k=t+1}^n \frac{a_k}{(1+r)^{k-t}} + \frac{s_n}{(1+r)^{n-t}} \quad (12)$$

and the accounting profit is therefore  $rv_{t-1}$ . From the usual accounting identity (clean surplus relation)

$$\text{cash flows} = \text{income} + \text{depreciation}$$

we find  $\text{Dep}_t(r) = a_t - rv_{t-1}$ , and, using eq. (12), we get to

$$\text{Dep}_t(r) = a_t - r \left( \sum_{k=t}^n \frac{a_k}{(1+r)^{k-(t-1)}} + \frac{s_n}{(1+r)^{n-(t-1)}} \right),$$

where  $s_n = s_n(r) = a_0(1+r)^n - \sum_{t=1}^n a_t(1+r)^{n-t}$ .

The decision of accepting or rejecting project  $a$  boils down, in this view, to a choice between different depreciation plans for asset  $A$ : the accountant may ideally select the depreciation schedule such that  $r=x$  or, alternatively, the one where  $r=i$ .<sup>6</sup> In the former case, the scrap value becomes  $s_n(x) = a_0(1+x)^n - \sum_{t=1}^n a_t(1+x)^{n-t}$ , which equals zero, given that  $x$  is the internal rate of return (discount function) of project  $a$ . In the latter case, the scrap value is  $s_n(i) = a_0(1+i)^n - \sum_{t=1}^n a_t(1+i)^{n-t}$ .<sup>7</sup>

From the point of view of periodic performance, we may say that if the depreciation charge is smaller with  $r=x$  than with  $r=i$  (i.e. if the value of asset  $A$  decreases less rapidly with acceptance of project  $a$ ), then performance is positive. In other words, the difference

$$\text{Dep}_t(i) - \text{Dep}_t(x)$$

formally translates the notion of residual income. It is easy to show that this difference is just the lost-capital residual income. We have

$$\begin{aligned} \text{Dep}_t(i) - \text{Dep}_t(x) &= \left( a_t - i \sum_{k=t}^n \frac{a_k}{(1+i)^{k-(t-1)}} - i \frac{s_n(i)}{(1+i)^{n-(t-1)}} \right) - \\ &\quad - \left( a_t - x \sum_{k=t}^n \frac{a_k}{(1+x)^{k-(t-1)}} - x \frac{s_n(x)}{(1+x)^{n-(t-1)}} \right). \quad (13) \end{aligned}$$

By definition of internal rate of return (discount function), we have

$$a_0(1+r)^n = \sum_{k=1}^n a_k(1+r)^{n-k} + s_n(r) = \sum_{k=1}^{t-1} a_k(1+r)^{n-k} + \sum_{k=t}^n a_k(1+r)^{n-k} + s_n(r) \quad r = x, i.$$

<sup>6</sup>In financial terms, this boils down to investing funds either at the rate  $x$  or at the rate  $i$ .

<sup>7</sup>That  $i$  is actually an internal rate of return for asset  $A$  is easily shown:

$$-a_0 + \sum_{t=1}^n \frac{a_t}{(1+i)^t} + \frac{s_n(i)}{(1+i)^n} = -a_0 + \sum_{t=1}^n \frac{a_t}{(1+i)^t} + \left( a_0 - \sum_{t=1}^n \frac{a_t}{(1+i)^t} \right) = 0.$$

Dividing by  $(1+r)^{n-t+1}$  we have

$$a_0(1+r)^{t-1} = \sum_{k=1}^{t-1} \frac{a_k}{(1+r)^{k-(t-1)}} + \sum_{k=t}^n \frac{a_k}{(1+r)^{k-(t-1)}} + \frac{s_n(r)}{(1+r)^{n-(t-1)}} \quad r = x, i$$

whence

$$a_0(1+r)^{t-1} - \sum_{k=1}^{t-1} \frac{a_k}{(1+r)^{k-(t-1)}} = \sum_{k=t}^n \frac{a_k}{(1+r)^{k-(t-1)}} + \frac{s_n(r)}{(1+r)^{n-(t-1)}} \quad r = x, i$$

From eqs. (3) and (4) we find

$$a_0(1+r)^{t-1} - \sum_{k=1}^{t-1} a_k(1+r)^{t-1-k} = w_{t-1}(r) \quad \text{and} \quad s_n(r) = w_n(r), \quad r = x, i$$

so that eq. (13) becomes

$$\begin{aligned} \text{Dep}_t(i) - \text{Dep}_t(x) &= (a_t - iw_{t-1}(i)) - (a_t - xw_{t-1}(x)) \\ &= xw_{t-1}(x) - iw_{t-1}(i) = \text{RI}_t^L. \end{aligned} \quad (14)$$

The lost-capital residual income may therefore be represented as an excess depreciation charge.

### 3.4 The keynesian user cost

A particular important case of the depreciation argument relates the notion of residual income to the keynesian notion of *user cost*. In his *General Theory of Employment Interest and Money* Keynes defines user cost, with reference to the entrepreneur, as the difference between “the value of his capital equipment at the end of the period ... and ... the value it might have had at the end of the period if he had refrained from using it” (Keynes, 1967, p. 66). Some years after, the same concept is investigated in Coase (1968), who relabels it *depreciation through use*, because it measures the decline in value due (not to time but) to a different use of the asset. To compute user cost we must therefore calculate “the present value of the net receipts ... by discounting them at a rate of interest” (Coase, 1968, p. 123). This “rate of discount coincides with that in the market” (Scott, 1953, p. 378). Using our symbols, to compute user cost one must discount the relevant expected cash flows. Reminding the arbitrage-based description in subsection 3.1 and supposing the investor does not undertake the project, his payoff vector is  $(-a_0, a_1, \dots, a_n + w_n(i))$ ; if, instead, project is undertaken, his payoff vector is  $(-a_0, a_1, \dots, a_n)$ . In the former case the discounted value of the cash-flow stream is, at time  $t$ ,  $\sum_{k=t+1}^n a_k(1+i)^{t-k} + w_n(i)(1+i)^{t-n}$ , whereas in the latter case the discounted value of the cash-flow stream is  $\sum_{k=t+1}^n a_k(1+i)^{t-k}$ . Therefore,

$$\begin{aligned} \text{user cost} &= \left[ \sum_{k=t+1}^n a_k(1+i)^{t-k} + w_n(i)(1+i)^{t-n} \right] - \sum_{k=t+1}^n a_k(1+i)^{t-k} \\ &= w_n(i)(1+i)^{t-n}. \end{aligned} \quad (15)$$

User cost is just the discounted value of the arbitrage payoff. This is implicitly acknowledged by Keynes himself, who recognizes the user cost as “the discounted value of the additional prospective yield which would be obtained at some later date” (Keynes, 1967, p. 70).

It is easy to show that user cost acts as a depreciation charge with respect to use rather than to time. Using eqs. (3) and (4), we easily find  $w_t(x) = \sum_{k=t+1}^n \frac{a_k}{(1+x)^{k-t}}$  and  $w_t(i) = \sum_{k=t+1}^n \frac{a_k}{(1+i)^{k-t}} + \frac{w_n(i)}{(1+i)^{n-t}}$ . Therefore,

$$w_t(i) - w_t(x) = \left[ \sum_{k=t+1}^n \frac{a_k}{(1+i)^{k-t}} + w_n(i)(1+i)^{t-n} \right] - \sum_{k=t+1}^n \frac{a_k}{(1+x)^{k-t}}. \quad (16)$$

If the market value of the asset is selected as the outstanding capital (i.e. if one sets  $w_t(x) := V_t$ ), eq. (16) just represents the user cost above computed: given that  $V_t = \sum_{k=t+1}^n \frac{a_k}{(1+i)^{k-t}}$ , eqs. (15) and (16) coincide.

Putting it differently, eq. (15) is a particular case of eq. (16); the latter provides a generalized notion of the keynesian user cost. It is worth noting that the lost-capital residual income may be expressed as the periodic variation of this (generalized) user cost: from eq. (14) and the usual recurrence equations we get to

$$\text{RI}_t^L = \overbrace{[w_{t-1}(i) - w_{t-1}(x)]}^{\text{user cost in } t} - \overbrace{[w_t(i) - w_t(x)]}^{\text{user cost in } t-1}.$$

### 3.5 The compounding argument

The lost-capital residual income may be generated with a compounding process that directly relates the two paradigms. To this end, the new paradigm is interpreted with the eye of a standard-minded evaluator.

The starting point is the standard residual income, which represents the periodic surplus accrued to the project. Let us focus on the  $t$ -th period and assume that the surpluses  $\text{RI}_1^S, \text{RI}_2^S, \dots, \text{RI}_{t-1}^S$  are reinvested, as they are generated, at the opportunity cost of capital  $i$ . At time  $t-1$  the accumulated surplus is  $\sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}$ . As a result, in the  $t$ -th period the investor receives the return  $xw_{t-1}(x)$  from the project and the return  $i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}$  from the accumulated surplus. Given that  $w_{t-1}(x)$  could be invested at the rate  $i$ , the investor foregoes the return  $iw_{t-1}(x)$ . Therefore,

$$\begin{aligned} \text{residual income} &= xw_{t-1}(x) + i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k} - iw_{t-1}(x) \\ &= \text{RI}_t^S + i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}. \end{aligned} \quad (17)$$

The above residual income is just  $\text{RI}_t^L$ . To show it, we remind that

$$w_{t-1}(i) = w_0(i)(1+i)^{t-1} - \sum_{k=1}^{t-1} a_k (1+i)^{t-1-k}$$

and  $a_k = w_{k-1}(x)(1+x) - w_k(x)$ , so that

$$w_{t-1}(i) = w_0(x)(1+i)^{t-1} - \sum_{k=1}^{t-1} (w_{k-1}(x)(1+x) - w_k(x))(1+i)^{t-1-k}.$$

Upon rearranging terms, we find

$$w_{t-1}(i) = w_{t-1}(x) - \sum_{k=1}^{t-1} w_k(x)(x-i)(1+i)^{t-1-k} \quad (18)$$

$$= w_{t-1}(x) - \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}. \quad (19)$$

Consequently, eq. (17) becomes the lost-capital residual income:

$$\text{residual income} = \text{RI}_t^S + i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k} \quad (20)$$

$$= \text{RI}_t^S + i (w_{t-1}(x) - w_{t-1}(i)) \quad (21)$$

$$= xw_{t-1}(x) - iw_{t-1}(i) = \text{RI}_t^L. \quad (22)$$

Focussing on the right-hand side of eq. (21), the second addend is the additional periodic return earned or given up by the investor in a period if he accepts the project. In such a case, he owns a capital greater or smaller by  $|w_{t-1}(i) - w_{t-1}(x)|$  than the capital he would own in the rejection case. On this differential amount he earns or foregoes a return rate of  $i$ . But eq. (19) tells us that

$$w_{t-1}(x) - w_{t-1}(i) = \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}, \quad (23)$$

i.e., the additional capital is just the compounded sum of all previous standard residual incomes. In other words, the accumulated surpluses of the past  $\text{RI}_k^S$ ,  $k = 1, 2, \dots, t-1$  represent the (additional or foregone) return “forgotten” by the standard paradigm.

As a result, the lost-capital paradigm may be seen as induced by a standard line of reasoning: it is just a standard residual income that keeps memory of the past (standard) residual incomes. (See Table 1 for a formal resume of the four arguments).

*Remark 2.* In the light of what we have seen in the previous subsection it is worthwhile noting that the accumulated standard residual incomes just represent the generalized user cost of eq. (16) (changed in sign). The user cost is therefore financially equivalent to the sum of compounded standard past residual incomes. This result is important for two reasons: first, user cost, which was defined by Keynes in a forward-looking perspective, is now expressed with a backward-looking perspective (past residual incomes); second, a relation linking firm value, lost capital and user cost is easily established: taking  $w_t(x)=V_t$  one finds, from eq. (23),

$$\begin{aligned} V_t &= w_t(i) + \sum_{k=1}^t \text{RI}_k^S (1+i)^{t-k} \\ &= \text{lost capital} + \text{user cost}. \end{aligned}$$

The market value of a firm (project) may therefore be expressed as the sum of the lost capital and the user cost.

## 4 Relations between paradigms

Both paradigms rest on the conceptual identity:

$$\text{Residual income} = \text{Actual income} - \text{Foregone income},$$

where the foregone income is the opportunity cost of investing in the project and acts as a capital charge:

$$\text{Residual income} = \text{Actual income} - \text{Capital charge}.$$

The foregone income is also interpreted as a normal income generated by a firm in the same class of risk, and residual income is therefore often called abnormal earning:

$$\text{Abnormal earning} = \text{Actual income} - \text{normal income}.$$

The differences between the two paradigms reside in the way the capital charge is calculated, and therefore in the notion of foregone income. The latter is the return the investor would have if he invested in the counterfactual alternative at the rate  $i$ . According to the standard paradigm (paradigm S), the investor could periodically invest the capital *actually* employed in the project ( $=w_t(x)$ ) at the return rate  $i$ . Conversely, the lost-capital paradigm (paradigm L) takes into consideration the fact that if the investor undertakes the project he loses the opportunity of owning a different capital ( $=w_t(i)$ ), which could be invested at the return rate  $i$ .<sup>8</sup> Therefore, in  $\text{RI}^S$  we have

$$\text{capital charge} = \text{actual capital} \cdot \text{foregone return rate},$$

whereas in  $\text{RI}^L$  we have

$$\text{capital charge} = \text{lost capital} \cdot \text{foregone return rate}.$$

While both paradigms measure the foregone return, they provide different (legitimate) interpretations of such a notion: in paradigm S foregoing return refers to “foregoing the return rate  $i$ ”, in paradigm L foregoing return refers not only to “foregoing the return rate  $i$ ” but also to “foregoing the capital  $w_{t-1}(i)$ ”

Using the replicating cash-flow argument above, the capital charge is arrived to by answering two different questions. The standard-minded investor asks:

*“What would income be in the  $t$ -th period if  $a_0$  were initially invested in the project and  $w_{t-1}(x)$  were invested at the rate  $i$ ?”*

---

<sup>8</sup>The capital  $w_t(i)$  is not simply foregone, but definitely lost; therefore in paradigm L the foregone income is a lost unrecoverable income. It is evident that the lost capital coincides with O’Hanlon and Peasnell’s (2002) *unrecovered* capital, though the authors use it in a classical framework, where the standard residual income is maintained. By contrast, Drukarczyk and Schueler’s (2000) *invested* capital, which also coincides with the lost capital, is used to provide a metric which genuinely belong to the lost-capital paradigm.

whereas the lost-capital-minded investor asks:

*“What would income be in the  $t$ -th period if the amount  $a_0$  were invested in a replicating cash-flow stream yielding return at the rate  $i$ ?”*

Looking at eq. (21), it is evident that  $RI^S$  and  $RI^L$  may differ not only in terms of absolute value but also in terms of sign. Therefore, there may be instances where a model signals positive performance whereas the other one signals negative performance: even if  $x_t > i$  (i. e.  $RI_t^S$  is positive),  $RI_t^L$  may still be negative if  $w_{t-1}(i)$  is sufficiently greater than  $w_{t-1}(x)$ . In other words, if the investor did not undertake the project, his wealth could be greater than the one produced by the project, enough to offset the smaller rate of return  $i$  yielded by the counterfactual alternative. Conversely, if a periodic rate of return  $x_t$  is smaller than the opportunity cost of capital, then paradigm S signals poor performance, but nonetheless  $w_{t-1}(i)$  may be so small with respect to  $w_{t-1}(x)$  as to more than compensate, leading to an overall positive excess profit in paradigm L. Even when the signs of  $RI_t^S$  and  $RI_t^L$  coincide, consistently indicating positive or negative performance, the magnitude is, in general, different. We actually have that  $RI_t^L \neq RI_t^S$  whenever  $w_{t-1}(i) \neq w_{t-1}(x)$ . In particular, as long as  $x_t > i$ , the lost-capital paradigm signals a poorer (respectively, better) performance if  $w_{t-1}(i) > w_{t-1}(x)$  (respectively,  $w_{t-1}(i) < w_{t-1}(x)$ ). The reason is evident: paradigm L takes account of the fact that if  $w_{t-1}(i) \neq w_{t-1}(x)$  an investor undertaking the project renounces in the  $t$ -th period to owning a capital greater (or smaller) by an amount of  $|w_{t-1}(i) - w_{t-1}(x)|$ . That is, he renounces to receiving a positive (respectively, negative) return on that amount at a rate  $i$ . This implies that paradigm L produces performance indexes that are sensitive to the counterfactual time evolution of the capital invested, whereas paradigm S erases all the counterfactual story keeping only the counterfactual rate  $i$ . In particular, by splitting  $RI^L$  into two addends, eq. (20) tells us that positive (negative) performances will positively (negatively) reverberate in the following periods tending to increase (lower)  $RI^L$  with respect to  $RI^S$ . If performance is good in one year, next-year residual income will be positively affected regardless of whether  $x_t$  is greater or smaller than  $i$ . For example, if it should happen that  $x_t < i$  in some period, then the residual income benefits from the second addend of eq. (20), which acts as an insurance bonus. If, instead,  $x_t > i$ , then the insurance part become an additional return. Evidently, the additional term works well if  $w_{t-1}(i) < w_{t-1}(x)$ . But this just depends on the past performances. If it occurs that  $w_{t-1}(i) > w_{t-1}(x)$ , the additional term is negative, which tends to lower residual income even if  $x_t > i$ . Again, this depends on the past performances.

*Remark 3.* To say that the lost-capital residual income depends on past performances makes sense only if one employs a standard line of reasoning: to a standard-minded evaluator paradigm L is just paradigm S with an added memory to recall the past. But to a lost-capital-minded evaluator, the comparison is just between two alternative incomes pertaining to the same period, and the residual income of one year does not reverberate on the following years. From this point of view, the additional term  $\sum_{k=1}^{t-1} RI_k^S (1+i)^{t-1-k}$  does

not represent accumulated (standard) residual incomes, but is just the additional capital that the investor could invest in the  $t$ -th period if he selected, at time 0, the counterfactual course of action. This is (again) consistent with the keynesian notion of user cost, seen as a depreciation due to different use of the funds (Coase, 1968).

## 5 Book values, market values, and income aggregation

An important issue is the relation paradigm L bears to a project's Net Present Value (firm's Market Value Added), and, therefore, to market values. If a residual-income paradigm is not consistent with the NPV, then it should be evidently dismissed. We now show that both paradigms are consistent with the NPV though with an opposite procedure:<sup>9</sup> paradigm S requires a *discount-then-sum* mechanism, while paradigm L requires a *sum-then-discount* approach. For the former we have, discounting and then summing:

$$\begin{aligned} \sum_{t=1}^n \frac{\text{RI}_t^S}{(1+i)^t} &= \sum_{t=1}^n \frac{w_{t-1}(x)(x-i)}{(1+i)^t} = \sum_{t=1}^n \frac{w_t(x) + a_t - w_{t-1}(x)(1+i)}{(1+i)^t} \\ &= \sum_{t=1}^n \frac{w_t(x) + a_t}{(1+i)^t} - \sum_{t=1}^n \frac{w_{t-1}(x)}{(1+i)^{t-1}} \\ &= \sum_{t=1}^n \frac{a_t}{(1+i)^t} - a_0 = \text{NPV} \end{aligned}$$

where we have used the equality  $a_0 + \sum_{t=1}^n w_t(x)(1+i)^{-t} = \sum_{t=1}^n w_{t-1}(x)(1+i)^{-(t-1)}$ .<sup>10</sup> As for paradigm L, if we first sum excess profits and then discount them back we obtain the NPV. To show it, just consider that, taking the sum in eq. (17) and rearranging terms, we have

$$\begin{aligned} \sum_{t=1}^n \text{RI}_t^L &= \sum_{t=1}^n \left( \text{RI}_t^S + i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k} \right) \\ &= \sum_{t=1}^n \text{RI}_t^S (1+i) \sum_{k=1}^t (1+i)^{n-k} \\ &= \sum_{t=1}^n \text{RI}_t^S (1+i)^{n-t} \end{aligned}$$

where the last equation is derived by induction. Discounting back,

$$\begin{aligned} \frac{1}{(1+i)^n} \sum_{t=1}^n \text{RI}_t^L &= \frac{1}{(1+i)^n} \sum_{t=1}^n \text{RI}_t^S (1+i)^{n-t} \\ &= \sum_{t=1}^n \text{RI}_t^S (1+i)^{-t} \\ &= \text{NPV}. \end{aligned} \tag{24}$$

<sup>9</sup>As for paradigm S, the result is well-known (e.g. Edwards and Bell, 1961, ch. 2; Peasnell, 1981, 1982; Ohlson, 1995; Martin and Petty, 2000, ch. 5; Lundholm and O'Keefe, 2001; Vélez-Pareja and Tham, 2003).

<sup>10</sup>We remind that  $w_n(x) = 0$ , because  $x$  is an internal rate of return (discount function) for project  $a$ .

The net terminal value is therefore obtained by a sum of un-compounded residual incomes:

$$\text{NPV}(1+i)^n = \sum_{t=1}^n \text{RI}_t^L. \quad (25)$$

The results in eqs. (24) and (25) have interesting theoretical and practical implications: they provide a strong link between accounting data and market values. Recalling that  $\text{NPV} = E_0 - a_0$  and letting  $w_t(x)$  be the equity book value we have, from eq. (24),

$$E_0 = a_0 + \frac{1}{(1+i)^n} \sum_{t=1}^n \text{RI}_t^L, \quad (26)$$

which says that the market value of equity is given by the book value plus the sum of future lost-capital residual incomes. The above equation highlights that time is not important: any permutation of the residual incomes will lead to the same value. Therefore, one does not have to worry about imputing each residual income to each date in which it will be generated. If expectations are correct in terms of value but incorrect in terms of timing, no error in valuation is made with the lost-capital paradigm.

## 6 Converting standard residual income into lost-capital residual income

The two paradigms generate several performance measures. In particular, for each such measure complying with paradigm S there corresponds a companion measure in paradigm L. Conversion is made by replacing the foregone income of paradigm S with the lost income of paradigm L. For illustrative purposes, we focus on Stewart's (1991) Economic Value Added (EVA) and on the Edwards-Bell-Ohlson (EBO) model (Edwards and Bell, 1961; Ohlson, 1995),<sup>11</sup> The two metrics belong to the set of standard residual income models, and are complementary: EVA adopts an entity (claimholders) approach; EBO adopts a proprietary (shareholder) approach.

### 6.1 EVA

Assume that (i) the book value of the firm's assets is taken as the outstanding capital, (ii) the free cash flows are taken as the relevant cash flows (iii) the RONA (Return On Net Assets) is taken as the periodic rate of return, and (iv) the WACC is taken as the opportunity cost of capital. Formally, this means  $w_t(x) := V_t^{bv}$ ,  $a_t := \text{FCF}$ ,  $x := \text{RONA}$ ,  $i := \text{WACC}$ . Therefore, eq. (3) becomes

$$V_t^{bv} = V_{t-1}^{bv} \cdot (1 + \text{RONA}) - \text{FCF}$$

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<sup>11</sup>Abusing notation, we will henceforth use the acronym EBO to refer to the corresponding residual income as well.



for  $t > 0$ , and  $V_0^{bv} := a_0$ . Reminding that  $V_{t-1}^{bv} \cdot \text{RONA} = \text{NOPAT}$  and applying eq. (2), the standard performance measure becomes

$$\text{RI}^S = \text{NOPAT} - \text{WACC} \cdot V_{t-1}^{bv}. \quad (27)$$

If, instead, paradigm L is applied, letting  $w_t(i) := \mathbb{V}_t$  be the lost capital and using eq. (4) one finds

$$\mathbb{V}_t = \mathbb{V}_{t-1} \cdot (1 + \text{WACC}) - \text{FCF}$$

for  $t > 0$ , with  $\mathbb{V}_0 := a_0$ . Thus, the lost-capital measure (eq. (5)) results in

$$\text{RI}^L = \text{NOPAT} - \text{WACC} \cdot \mathbb{V}_{t-1}. \quad (28)$$

The measures in eqs. (27) and (28) represent the original Economic Value Added and its lost-capital companion, respectively.

## 6.2 EBO

A different metric is generated when (i) the book value of equity is taken as the outstanding capital, (ii) the equity cash flows are taken as the relevant cash flows, (iii) the ROE (Return On Equity) is taken as the periodic rate of return, and (iv) the cost of equity  $k_e$  is taken as the opportunity cost of capital. Formally,  $w_t(x) := E_t^{bv}$ ,  $a_t := \text{ECF}$ ,  $x := \text{ROE}$ ,  $i := k_e$ , so that

$$E_t^{bv} = E_{t-1}^{bv} \cdot (1 + \text{ROE}) - \text{ECF}$$

for  $t > 0$ , with  $E_0^{bv} := a_0$ . Therefore, reminding that  $E_{t-1}^{bv} \cdot \text{ROE} = \text{PAT}$ , the standard measure becomes

$$\text{RI}^F = \text{PAT} - k_e \cdot E_{t-1}^{bv}. \quad (29)$$

If one applies paradigm L to this measure and let  $w_t(i) := \mathbb{E}_t$  be the lost equity, one has

$$\mathbb{E}_t = \mathbb{E}_{t-1} \cdot (1 + k_e) - \text{ECF}$$

for  $t > 0$ , with  $\mathbb{E}_0 := a_0$ . Thus, the lost-capital measure results in

$$\text{RI}^S = \text{PAT} - k_e \cdot \mathbb{E}_{t-1}. \quad (30)$$

The measures in eqs. (29) and (30) represent EBO as originally conceived and its lost-capital companion, respectively. To sum up, the standard paradigm depends on the threesome (RONA, WACC,  $V^{bv}$ ),<sup>12</sup> whereas the lost-capital paradigm depends on the foursome (RONA, WACC,  $V^{bv}$ ,  $\mathbb{V}$ ) (see Table 2).<sup>13</sup>

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<sup>12</sup>Or (ROE,  $k_e$ ,  $E^{bv}$ ) for EBO.

<sup>13</sup>Or (ROE,  $k_e$ ,  $E^{bv}$ ,  $\mathbb{E}$ ) for EBO.

## 7 An example

This section applies the two paradigms to a firm created to undertake a project that requires an initial investment of 13 800, of which 12 000 are spent in fixed assets and 1 800 in working capital requirements. Straight-line depreciation is assumed for the fixed assets. It is also assumed that the required return on assets is 12% and that the book value of debt equals the market value of debt (i.e. debt rate=required return to debt). Other input data are collected in Table 3; Table 4 gives the firm's accounting statements and the resulting cash flows, and Table 5 focuses on equity and firm valuation. The market value of equity is first found by using three different discounted-cash-flow methods: the Adjusted Present Value (APV) method, introduced by Myers (1974), the ECF- $k_e$  method (equity approach), and the FCF-WACC method (entity approach). Logically, they all give the same result (see Fernández, 2002). Afterwards, a residual-income perspective is used to obtain the market value: Tables 6-7 show the application of the two paradigms to the EVA model and the EBO model. Obviously, both paradigms supply the same market values as the discounted-cash-flow technique's.<sup>14</sup>

The examples show a situation of positive EVAs and EBOs in each period. First of all, note that in the first period the two paradigms give the same answer, because the outstanding capitals coincide ( $w_0(x) = w_0(i)$ ). In the next periods, the lost-capital measures are constantly greater than the standard measures. Also, the periodic variation in the lost-capital measures are greater. For example, in Table 6 the standard EVA's variations are given by (281, 282, 283, 286), the lost-capital EVA's variations are (282, 313, 347, 376). In Table 7 we have, consistently, that the EBO's variations are (296, 298, 306, 372) and (302, 350, 427, 811), respectively.

As anticipated, the lost-capital has an insurance component for negative situations, which is just the user cost previously introduced. Suppose the fourth-year sales amount to 8 000 instead of 10 000 (Table 8), other things equal. Both paradigms report negative performance in the fourth year.<sup>15</sup> Yet, the lost-capital paradigm smoothes the negativeness, because it takes account of the fact that the past year's results were better, which implies that the lost capital at the beginning of the fourth year is smaller than the actual capital employed:  $V_3^{bv} > V_3$  and  $E_3^{bv} > E_3$ . It is easy to see that if the fourth-year sales are equal to 8 600 instead of 10 000 (other things unvaried), the corresponding standard measures become negative, whereas the lost-capital measures keep positive (Table 9). In this case, while the RONA (respectively, ROE) is indeed smaller than the WACC (respectively,  $k_e$ ) in the fourth year, the bonus given by the additional amount  $WACC_4 \cdot (V_3^{bv} - V_3) = 96$  (respectively,  $k_{e4} \cdot (E_3^{bv} - E_3) = 185$ ) is so high as to more than compensate the negative standard EVA (respectively, EBO): we have  $16 = -80 + 96$ , and  $164 = -21 + 185$ .

<sup>14</sup>As previously shown, the time ordering of residual incomes is immaterial in paradigm L, if the objective is firm valuation. However, if the objective of the analysis is incentive compensation, time is obviously relevant in this paradigm as well.

<sup>15</sup>The reader should not be discomforted by the fact that *each* period's residual income changes. If one period's sales change, the corresponding ECF and FCF change, so that the market value of equity is changed in every year, which implies that both  $k_e$  and WACC change in every year, which in turn induces a change in the capital charge of every period.

Evidently, the bonus may symmetrically act a penalty role if past performance is negative. For example, consider the case where in the third year sales amount to 8 000 (other things unvaried). This makes the third-year residual incomes negative for both paradigms (Table 10). Due to insurance bonus for positive past performances, the lost-capital residual incomes are less negative than the standard ones. Yet, the third-year negative performance penalizes the fourth-year performance, which is smaller than that reported by the standard residual incomes. Note that in the fifth year, performance recorded by the lost-capital paradigm is again higher than the standard one's, due to the renewed recent positive performance of the fourth year. In other words, as compared to the standard metric, performance is amplified in negative and in positive sense (bonus and penalty roles).<sup>16</sup>

It is also worth noting that the dependence of a lost-capital measure on the past is not an easy one (the measure does not merely depends on the previous period's  $RI_{t-1}^S$ , but on  $RI_1^S, RI_2^S, \dots, RI_{t-2}^S$ , and therefore on all the previous rates of return and all the previous opportunity costs of capital). It may be conjectured that managers willing to pursue personal objective may refrain from gaming the measure, given that they hardly will be able to assess the consequences on the following years' indexes. What they are aware of is that their performance is measured on the ground of past residual incomes as well as the current one. Whether these elements tend to reduce agency problems and whether managers rewarded through a lost-capital residual income are more inclined to behave optimally is not a trivial issue and deserves a thorough investigation. The efficacy of the paradigm also depends on the type of compensation plan selected. For example there are at least three ways of using a metric: the historical use, according to which the manager's bonus is a share of the RI:

$$\text{bonus} = x\% \text{ RI};$$

the XY compensation plan, according to which bonus is tied to RI variation:

$$\text{bonus} = x\% \text{ RI} + y\% \Delta \text{ RI};$$

and the excess RI improvement plan, according to which the expected RI improvement (EI) plays a major role:

$$\text{bonus} = \text{target bonus} + y\% (\Delta \text{ RI} - \text{EI})$$

(see Young and O'Byrne, 2001). For positive-RI companies using either the historical plan or the XY plan, we can say that the manager's bonuses computed with the lost-capital paradigm are greater than the ones computed in the standard paradigm, because in the former both RI and  $\Delta$  RI are greater than the

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<sup>16</sup>It is worth stressing again that the memory-dependent interpretation is a useful one for comparing the two paradigms, but it presupposes a standard-minded point of view. The memory-dependent feature of the lost-capital metrics just means that if money were invested at the opportunity cost of capital, the investor would have, in each period, a different (greater or smaller) capital. This appreciation or depreciation, equal to the keynesian user cost, would imply, in that very period, an additional or foregone interest. Such an interest is a penalty if positive, a bonus if negative.

corresponding ones in the latter (proof is straightforward using eq. (20)). However, things are complicated by the fact that comparisons may be made along two dimensions: the type of metric selected and the paradigm chosen. That is, a metric in a paradigm may be compared with the same metric in the alternative paradigm, or with an alternative metric in the same paradigm, or with an alternative metric in the alternative paradigm. Having two paradigms and a wide set of metrics it may be the case that a metric in one paradigm is more incentive than a different metric in the alternative paradigm. Given that firms may use many different plans to compute managers' bonuses, the impact on performance measurement and incentive compensation depends on (at least) three factors:

- the paradigm
- the metric
- the compensation plan

## 8 Aligning performance measures with value creation

A mystifying problem in value-based management is that residual income does not measure value creation in the period considered, so that either some adjustments are made to residual income itself or compensation plans are devised so as to tie residual income to value creation, in order to align managers' behaviors to shareholders' objectives (Ehrbar, 1998; Stewart, 1991; O'Hanlon and Peasnell, 2000; Young and O'Byrne, 2001; Martin et al., 2003). Grinyer (1985, 1987) proposes an index labelled *Earned Economic Income*, which has the nice property of being aligned with the Net Present Value. However, beside the fact that "the relationship between EEI and RI appears not to be well understood" (Peasnell, 1995, p. 235), his metric is equal in sign to the NPV only if the project's cash flows are all of the same sign (Martin et al., 2003, Peasnell, 1995, Grinyer, 1995). This section shows that converting Fernández's (2002) Created Shareholder Value into the corresponding lost-capital metric, one obtains a metric that is perfectly aligned with the Net Present Value, irrespective of the sign of the cash flows (i.e. even if some cash flows are opposite in sign).

The Created Shareholder Value (CSV) belongs to the class of standard residual income models. It is computed by picking  $a_t = \text{ECF}$ ,  $w_t(x) = E_t$  for every  $t \geq 1$  and  $i = k_e$ . Reminding the initial condition  $w_0(x) := a_0$ , the residual income in this model is

$$\text{CSV}_t = \begin{cases} a_0(x_t - k_e) & \text{if } t = 1 \\ E_{t-1}(x_t - k_e) & \text{if } t > 1 \end{cases} \quad (31)$$

where  $x_t = (E_t + a_t - a_{t-1})/a_{t-1}$  if  $t=1$  (see Fernández, 2002, p. 281),  $x_t = k_e$  otherwise (this implies  $\text{CSV}_t = 0$  for all  $t > 1$  if expectations are met). In order to convert the standard CSV into its lost-capital companion,

the capital charge  $k_e E_{t-1}$  must be replaced by  $k_e \mathbb{E}_{t-1}$  so that residual income becomes

$$\text{lost-capital CSV}_t = \begin{cases} a_0(x_t - k_e) & \text{if } t = 1 \\ k_e(E_{t-1} - \mathbb{E}_{t-1}) & \text{if } t > 1. \end{cases} \quad (32)$$

As for the  $t=1$ , we have

$$\text{CSV}_1 = a_0 \left( \frac{E_1 + a_1 - a_0}{a_0} - k_e \right) = \left( \frac{E_1 + a_1}{1 + k_e} - a_0 \right) (1 + k_e) = \text{NPV}(1 + k_e).$$

As for  $t > 1$ , note that the difference  $(E_{t-1} - \mathbb{E}_{t-1})$  is exactly the keynesian user cost of eq. (15) in an equity approach, because  $E_t = \sum_{k=t+1}^n a_k (1 + k_e)^{-(k-t)}$  and  $\mathbb{E}_t = \sum_{k=t+1}^n a_k (1 + k_e)^{-(k-t)} + \mathbb{E}_n (1 + k_e)^{-(n-t)}$ . Therefore,

$$E_{t-1} - \mathbb{E}_{t-1} = \mathbb{E}_n (1 + k_e)^{-(n-t+1)}.$$

But  $\mathbb{E}_n = a_0 (1 + k_e)^n - \sum_{k=1}^n a_k (1 + k_e)^{n-k} = -\text{NPV}(1 + k_e)^n$ , so that one may write

$$E_{t-1} - \mathbb{E}_{t-1} = \text{NPV}(1 + k_e)^{t-1}$$

whence

$$\text{CSV}_t = k_e (E_{t-1} - \mathbb{E}_{t-1}) = k_e \text{NPV}(1 + k_e)^{t-1}.$$

As a result, the lost-capital companion of CSV is always aligned in sign with the NPV. Indeed, it measures the increase of Net Present Value period by period:

$$k_e (E_{t-1} - \mathbb{E}_{t-1}) = \text{NPV}(1 + k_e)^t - \text{NPV}(1 + k_e)^{t-1} = \Delta \text{NPV}.$$

Referring to the example of section 7, Table 11 supplies the value of CSV in the two paradigms.<sup>17</sup>

## 9 Conclusions

This paper presents a new paradigm of residual income aimed at appraising projects (firms) as well as measuring periodic performance. Originally introduced with the name with the name of *Systemic Value Added* (Magni, 2000a, 2000b, 2001, 2003), the new paradigm translates the notion of opportunity cost differently from the classical paradigm. The new paradigm takes account of the capital foregone by the investor as well as the foregone return rate. In other words, if the investor invested in the alternative asset he would have, at the beginning of each period, a different capital than the actual one. This capital, which is definitely lost if project is undertaken, would generate income at the opportunity cost of capital. Hence, the paradigm is here relabeled “lost-capital paradigm”. This paper presents five different but equivalent ways of conceptualizing the lost-capital paradigm: (i) the project’s (firm’s) cash-flow stream may be replicated by

<sup>17</sup>If an entity approach is taken, rather than a proprietary approach, the the lost-capital companion of CSV becomes Drukarczyk and Schueler’s (2000) Net Economic Income.

investing funds at the cost of capital: the lost-capital RI is given by the difference between the alternative incomes; (ii) the dynamic system representing the investors' wealth manifests a different increase if funds are invested funds at the cost of capital: the lost-capital RI is given by the difference between alternative wealth increases; (iii) alternative depreciation plans are considered: the lost-capital RI is obtained as difference between alternative depreciation charges; (iv) the keynesian notion of user cost (depreciation through use) is retrieved and the lost-capital RI is computed as the periodic variation of the (generalized) user cost (v) the standard RI may be ideally banked to earn a rate of return equal to the cost of capital: the lost-capital RI is calculated by summing the standard RI and the interest earned on the accumulated past standard RIs.

The five arguments bear strong relations one another. In particular, user-cost is equal to the discounted value of the arbitrage payoff derived from the replicating portfolio (subsection 3.1) and equal to the accumulated past (standard) residual incomes (subsection 3.5). Also, the wealth increase argument is equivalent to the depreciation argument: in this case depreciation is computed on the entire net worth, taking account of the entire net worth derived from reinvestment of the cash flows at the cost of capital (see eq. (8)).

With the aid of the fifth argument, which relates the standard paradigm to the lost-capital paradigm, this paper shows that consistency with the NPV (MVA) is guaranteed in the lost-capital paradigm as well: however, the standard paradigm uses a *discount-then-sum* procedure, whereas the lost-capital paradigm uses a *sum-then-discount* mechanism; what differs is the distribution of the NPV across periods. While such a distribution is relevant for the standard paradigm to get the market value, it is not for the lost-capital paradigm: forecasting each (lost-capital) residual income and the corresponding period in which it is generated is unnecessary. Only the total sum of residual incomes is needed. Thus, the aggregation property of the lost-capital paradigm makes the residual income models particularly attractive as opposed to the discounted-cash-flow models and, in addition, offers an improvement with respect to the standard paradigm, given that the latter supplies the correct equity market value (and net terminal value) only if each and every residual income is forecasted for each and every year.

As for value-based management, the lost-capital paradigm tends to amplify results with respect to the standard paradigm, both in positive and negative sense. This and other features are worth investigating in order to ascertain which conditions make the new paradigm more incentive a tool for managers. Evidently, this also depends on the kind of compensation scheme used by the company to reward managers. For example, if the XY compensation plan is used (where  $\text{bonus} = x\% \text{RI} + y\% \Delta \text{RI}$ ), the lost-capital paradigm is more incentive for positive-RI companies, because both RI and  $\Delta \text{RI}$  are greater in the lost-capital paradigm than in the standard one. A further element that deserves a detailed analysis is how residual incomes change when expectations change and/or are not met, with particular concern to relation with value creation.

A particular model that deserves a thorough investigation is the lost-capital companion of Fernández's (2002) Created Shareholder Value. It is shown that the metric obtained from its conversion into the lost-capital paradigm is perfectly aligned in sign with the Net Present Value. In this particular case, residual income does measure value creation and this kind of metric could be fruitfully used for management com-

pensation.

The new paradigm may be an opportunity to search for new theoretical insights in management accounting and managerial finance as regards the relations among accounting values, market values, value creation. Also, it adds some irons in the fire of the value-based management debate. This, far from being a problem, should be seen as an opportunity for developing new measures and finding intriguing relations across metrics and across paradigms. It may be of some interest to find out whether the differences in the standard metrics are mirrored by the differences in the corresponding lost-capital metrics, or whether pros and cons of either measure change as the paradigm adopted is changed. It may be the case that the search for a satisfying compensation plan will lead to an index based on multiple metrics, possibly involving the use of both paradigms.

As a final consideration, it is evident that the aggregation property of the lost-capital paradigm establishes a powerful argument for management accounting to play a major role in project and firm valuation, as well as in capital budgeting decision problems. On one side, Ohlson's breakthrough result says that, under suitable assumptions on the stochastic process of abnormal earnings and with  $n$  sufficiently large, the *future* market value is *approximated* by a function of earnings; on the other side, the lost-capital result says that, under no particular assumption on stochastic processes and whatever the value of  $n$ , the *current* market value is *equal to* a function of earnings. These two results are conducive to a reinstatement of fundamental analysis as an important tool for either valuation and capital budgeting purposes.

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**Table 1. The five arguments arguments**

cash-flow replicated	return from project $xw_{t-1}(x)$	lost return $iw_{t-1}(i)$	<b>Lost-capital residual income</b>
wealth increases	wealth increase $W_t(x, i) - W_{t-1}(x, i)$	lost wealth increase $W_t(i) - W_{t-1}(i)$	
depreciation charges	asset's depreciation $Dep_t(x)$	lost depreciation $Dep_t(i)$	
user cost	user cost in $t - 1$ $[w_{t-1}(i) - w_{t-1}(x)]$	user cost in $t$ $[w_t(i) - w_t(x)]$ .	
standard RI capitalized	standard RI $RI_t^S$	foregotten return $i \sum_{k=1}^{t-1} RI_k^S (1+i)^{t-1-k}$	

**Table 2. EVA and EBO variables in the two paradigms**

	$x$	$i$	$w_t(x)$	$w_t(i)$	$\implies$	capital charge
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$
<i>Standard Paradigm</i>						
EVA	RONA	WACC	$V^{bv}$		$\implies$	$WACC \cdot V^{bv}$
EBO	ROE	$k_e$	$E^{bv}$		$\implies$	$k_e \cdot E^{bv}$
<i>Lost-capital Paradigm</i>						
EVA	RONA	WACC	$V^{bv}$	$\mathbb{V}$	$\implies$	$WACC \cdot \mathbb{V}$
EBO	ROE	$k_e$	$E^{bv}$	$\mathbb{E}$	$\implies$	$k_e \cdot \mathbb{E}$

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**Table 3. Input data**

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Investment	13 800	Depreciation rate	20%
Gross Fixed Assets	12 000	Corporate tax rate	33%
WCR	1 800	Required return on assets	12%
Sales	10 000	Debt rate	7%
Cost of Sales	3 670	Required return on debt ( $k_D$ )	7%
Gen. & Admin. Expenses	1 600		
Debt	4 000		

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**Table 4. Balance Sheet, Income Statement, Cash Flows**

time	0	1	2	3	4	5
<i>BALANCE SHEET</i>						
Gross fixed assets	12 000	12 000	12 000	12 000	12 000	12 000
–cumulative depreciation	0	–2 400	–4 800	–7 200	–9 600	–12 000
Net fixed assets	12 000	9 600	7 200	4 800	2 400	0
WCR	1 800	1 800	1 800	1 800	1 800	0
<b>NET ASSETS</b>	13 800	11 400	9 000	6 600	4 200	0
Debt	4 000	4 000	4 000	4 000	4 000	0
Equity (book value)	9 800	7 400	5 000	2 600	200	0
<b>NET WORTH &amp; LIABILITIES</b>	13 800	11 400	9 000	6 600	4 200	0
<i>INCOME STATEMENT</i>						
Sales		10 000	10 000	10 000	10 000	10 000
Cost of sales		3 670	3 670	3 670	3 670	3 670
Gen. & Adm. expenses		1 600	1 600	1 600	1 600	1 600
Depreciation		2 400	2 400	2 400	2 400	2 400
EBIT		2 330	2 330	2 330	2 330	2 330
Interest		280	280	280	280	280
PBT		2 050	2 050	2 050	2 050	2 050
Taxes		677	677	677	677	677
PAT		1 374	1 374	1 374	1 374	1 374
+Depreciation		2 400	2 400	2 400	2 400	2 400
+Δ Debt		0	0	0	0	–4 000
–Δ WCR		0	0	0	0	1 800
ECF		3 774	3 774	3 774	3 774	1 574
FCF <sup>18</sup>		3 961	3 961	3 961	3 961	5 761

<sup>18</sup>FCF=ECF–ΔD +  $k_D D \cdot (1 - T)$ .

**Table 5. Valuation**

time	0	1	2	3	4	5
$k_U$		12%	12%	12%	12%	12%
$V_U = PV[\text{FCF}; k_U]$	15 300	13 175	10 795	8 129	5 144	0
$DVTS = PV[T \cdot k_D \cdot D; k_D]^{(a)}$	379	313	242	167	86	0
$V = V_U + DVTS$	15 679	13 488	11 038	8 296	5 230	0
$\mathbf{E} = V_U + DVTS - D$	<b>11 679</b>	9 488	7 038	4 296	1 230	0
$k_e$		13.550%	13.943%	14.670%	16.461%	27.907%
$\mathbf{E} = PV[\text{ECF}; k_e]$	<b>11 679</b>	9 488	7 038	4 296	1 230	0
WACC		11.290%	11.199%	11.053%	10.786%	10.151%
$V = PV[\text{FCF}; \text{WACC}]$	15 679	13 488	11 038	8 296	5 230	0
$\mathbf{E} = V - D$	<b>11 679</b>	9 488	7 038	4 296	1 230	0
$\mathbf{MVA} = NPV = E - E^{bv}$	1 879					

<sup>(a)</sup>We use  $k_D$  to discount tax shields. However, it is worth noting that there is a lively debate in the literature on which is the the correct discount rate for discounting tax shields. While this issue is not relevant to this paper, the reader may be willing to turn to the contributions of Myers (1974), Tham and Vélez-Pareja (2001), Arzac and Glostén (2005), Fernández (2005), Cooper and Nyborg (2006). (To bypass the issue, the reader may well dismiss the first five rows of the Table and consider  $k_e$  as exogenously given).

**Table 6. EVA in the two paradigms**

time	0	1	2	3	4	5
NOPAT= $EBIT \cdot (1-T)$		1 561	1 561	1 561	1 561	1 561
$V^{bv}=D+E^{bv}$	13 800	11 400	9 000	6 600	4 200	0
$V$ (lost capital)	13 800	11 397	8 712	5 714	2 369	-3 151
<i>Standard Paradigm</i>						
capital charge (opportunity cost)		1 558	1 277	995	712	426
<b>EVA</b>		<b>3</b>	<b>284</b>	<b>566</b>	<b>849</b>	<b>1 135</b>
MVA (=discount and sum)	1 879					
$E=E^{bv}+MVA$	11 679					
<i>Lost-capital Paradigm</i>						
capital charge (opportunity cost)		1 558	1 276	963	616	240
<b>EVA</b>		<b>3</b>	<b>285</b>	<b>598</b>	<b>945</b>	<b>1 321</b>
MVA (=sum and discount)	1 879					
$E=E^{bv}+MVA$	11 679					



**Table 7. EBO in the two paradigms**

time	0	1	2	3	4	5
PAT		1 374	1 374	1 374	1 374	1 374
$E^{bv}$	9 800	7 400	5 000	2 600	200	0
$\mathbb{E}$ (lost equity capital)	9 800	7 354	4 606	1 509	-2 017	-4 153
<i>Standard Paradigm</i>						
capital charge (opportunity cost)		1 328	1 032	733	428	56
<b>EBO</b>		<b>46</b>	<b>342</b>	<b>640</b>	<b>946</b>	<b>1 318</b>
MVA (=discount and sum)	1 879					
$E=E^{bv}+MVA$	11 679					
<i>Lost-capital Paradigm</i>						
capital charge (opportunity cost)		1 328	1 025	676	248	-563
<b>EBO</b>		<b>46</b>	<b>348</b>	<b>698</b>	<b>1 125</b>	<b>1 936</b>
MVA (=sum and discount)	1 879					
$E=E^{bv}+MVA$	11 679					

**Table 8. Fourth-year sales equal to 8 000**

year	1	2	3	4	5
<b>EVA</b>					
Standard Paradigm	<b>9</b>	<b>291</b>	<b>575</b>	<b>-477</b>	<b>1 135</b>
Lost-capital paradigm	<b>9</b>	<b>292</b>	<b>608</b>	<b>-381</b>	<b>1 188</b>
<b>EBO</b>					
Standard Paradigm	<b>34</b>	<b>326</b>	<b>616</b>	<b>-439</b>	<b>1 318</b>
Lost-capital paradigm	<b>34</b>	<b>330</b>	<b>671</b>	<b>-251</b>	<b>1 537</b>

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**Table 9. Fourth-year sales equal to 8 600**

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year	1	2	3	4	5
<b>EVA</b>					
Standard Paradigm	7	289	573	-80	1 135
Lost-capital paradigm	7	290	605	16	1 228
<b>EBO</b>					
Standard Paradigm	37	331	624	-21	1 318
Lost-capital paradigm	37	336	680	164	1 658

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**Table 10. Third-year sales equal to 8 000**

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year	1	2	3	4	5
<b>EVA</b>					
Standard Paradigm	9	292	-763	849	1 135
Lost-capital paradigm	9	293	-730	803	1 173
<b>EBO</b>					
Standard Paradigm	32	323	-727	946	1 318
Lost-capital paradigm	32	328	-673	894	1 480

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**Table 11. CSV in the two paradigms**

time	0	1	2	3	4	5
outstanding capital	9 800	9 488	7 038	4 296	1 230	0
lost equity capital	9 800	7 354	4 606	1 509	-2 017	-4 153
<i>Standard Paradigm</i>						
<b>CSV</b>		<b>2 134</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
MVA (=discount and sum)	1 879					
$E=E^{bv}+MVA$	11 679					
<i>Lost-capital Paradigm</i>						
<b>CSV</b>		<b>2 134</b>	<b>298</b>	<b>357</b>	<b>459</b>	<b>906</b>
MVA (=sum and discount)	1 879					
$E=E^{bv}+MVA$	11 679					

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**Table 12. Notational Conventions**

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$\vec{a}$	cash-flow (vector)
$a_t$	cash flow available at time $t$
$a$	project
$x_t, x$	(periodic) internal rate of return
$w_t(x)$	actual capital employed
$i$	opportunity cost of capital
$RI^S$	residual income in the standard paradigm
ROA, RONA, ROE	Return On Assets, Return On Net Assets, Return On Equity
$w_t(i)$	lost capital
$RI^L$	residual income in the lost-capital paradigm
$W_0$	investor's wealth at time 0
$W_t(i)$	investor's wealth at time $t$ in case of project rejection
$W_t(x, i)$	investor's wealth at time $t$ in case of project acceptance
NPV	Net Present Value
$A$	asset
$\vec{A}$	cash-flow (vector)
$s_n$	scrap value
$v_t$	asset $A$ 's accounting value at time $t$
$Dep_t$	depreciation charge
$r$	asset $A$ 's internal rate of return
$V_t$	market value of the project (firm)
$E_t$	equity (market value)
EVA	Economic Value Added
WACC	Weighted Average Cost of Capital
$V^{bv}$	total capital (book value)
FCF	Free Cash Flow
NOPAT	Net Operating Profit After Taxes

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(The Table is continued on the next page)

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**Table 12. (continued) Notational Conventions**

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$V_t$	total lost capital (equity+debt)
EBO	Edwards-Bell-Ohlson
$k_e$	cost of equity
$E^{bv}$	equity (book value)
ECF	Equity Cash Flow
PAT	Profit After Taxes
$\mathbb{E}_t$	lost equity capital
RI	Residual income
$\Delta$	variation
EI	expected RI improvement
CSV	Created Shareholder Value
MVA	Market Value Added
WCR	Working Capital Requirements
$k_D$	required return on debt (=debt rate)
EBIT	Earnings Before Interest and Taxes
PBT	Profit Before Taxes
D	debt (market value=book value)
T	corporate tax rate
$k_U$	required return on assets
$V_U$	value of the unlevered firm
PV[A; B]	$\sum_{t=1}^n \frac{A_t}{\prod_{k=1}^t (1+B_k)}$
DVTS	discounted value of tax shields

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# Measuring performance and valuing firms: In search of the lost capital

## Abstract

Residual income models are widely used for value-based management purposes as well as for firm valuation. Conventional residual income models depend on: (i) the capital invested, (ii) the rate of return, (iii) the opportunity cost of capital. This paper presents a new paradigm where a fourth element is added: the capital that investors lose when they infuse funds in the firm (or project). This “lost-capital” paradigm may be derived from several perspectives and has various economic, financial, accounting interpretations. It may be employed in real-life applications for incentive compensation and firm (project) valuation. A numerical example illustrates the implementation of the paradigm to the EVA model and the Edwards-Bell-Ohlson model. Also, a model is presented with the nice property of being aligned in sign with NPV.

**Keywords.** Performance measurement, value-based management, residual income, incentive compensation, lost capital, Economic Value Added, firm value.

## 1 Introduction

Corporate finance and management accounting find a common terrain in the study of the notion of residual income, also called abnormal earning, which is formally computed as the difference between the actual income and the counterfactual income investors would receive if they invested their funds at the opportunity cost of capital. Coined by the General Electric Company, the term first appears in the literature in Solomons (1965, p. 63), although the same concept, differently labeled, was studied even earlier (e.g. Preinreich, 1936, 1938; Edwards and Bell, 1961, Bodenhorn, 1964). The contributions of Peasnell (1981, 1982), Peccati (1987, 1990, 1991) and Ohlson (1989, 1995), have caused a renewed interest in this notion in both corporate finance and management accounting, with particular regard to firm valuation, performance measurement, value-based management. A large number of theoretical and applied studies have appeared dealing with the subject (e.g. Stewart, 1991; Ohlson, 1995; Feltham and Ohlson, 1995; Rappaport, 1998; Lundholm and O’Keefe, 2001; Young and O’Byrne, 2001; Martin et al. 2003; Weaver and Weston, 2003; O’Byrne and Young, 2006) and a large number of textbooks and professional publications in corporate finance, managerial finance, management accounting directly deal with the topic (e.g. Brealey and Myers, 2000; Copeland et al., 2000; Palepu et al., 2000; Grinblatt and Titman, 2002; Revsine et al., 2005; Arnold, 2005).

An alternative paradigm of residual income has been recently introduced by Magni (2000a, 2000b, 2000c, 2001, 2003, 2005) which differs, conceptually and formally, from the standard paradigm used by academics,

analysts and practitioners. This paper aims at shedding light on this paradigm and provides some theoretical and practical results relevant for both valuation and incentive compensation. In particular, the standard paradigm may be seen as grounded on three elements: (i) the actual capital invested, (ii) the actual rate of return, (iii) the opportunity cost of capital (the foregone rate of return). The alternative paradigm takes into consideration an additional element: the lost capital, and is therefore here named *lost-capital* paradigm. As a paradigm, it generates several new metrics, in particular one for any existing metric in the standard paradigm. The new paradigm is presented in five autonomous though equivalent ways, in order to show its multifaceted significance and its sound economic meaning, and some differences and relations between the two paradigms are investigated. In particular, the new paradigm enables one to compute the project's (firm's) market value leaving out any consideration about timing: value is a function of book value and the sum of residual incomes: earnings aggregation, as opposed to discounting, applies.

A numerical example is also illustrated, where the paradigm is applied to the well-known *Economic Value Added* (Stewart, 1991) and to the so-called *Edwards-Bell-Ohlson* model (Ohlson, 1995). Furthermore, a third metric, namely Fernández's (2002) *Created Shareholder Value*, is transformed into the corresponding lost-capital metric. The conversion originates a metric that is consistent in sign with the Net Present Value. Therefore, this metric is particularly suited for managerial compensation, given that it directly ties performance to value creation.

The paper is structured as follows. Section 2 introduces the standard paradigm of residual income.<sup>1</sup> Section 3 presents the new paradigm from five different points of view: (i) a replicating cash-flow and its outstanding capital, (ii) the investor's wealth and its evolution through time, (iii) the construction of alternative depreciation plans (iv) the keynesian notion of user cost, (v) the lost-capital as an accumulation of past standard residual incomes. Section 4 investigates some relations between the two paradigms. Section 5 shows that the lost-capital paradigm is compatible with the Net Present Value (and the Market Value Added) and that value may be derived from lost-capital residual incomes by neglecting timing: only the sum of residual incomes is of concern for computing market value. Section 6 focuses on Economic Value Added and the Edwards-Bell-Ohlson model: first, they are derived as particular cases of the standard paradigm; then, the companion metrics are introduced in the lost-capital paradigm. Section 7 illustrates an example aiming at shedding light on the behavior of the two pairs of metrics and suggesting some possible implications for executive compensation, under the assumption that expectations are met. Section 8 shows that the lost-capital companion of Fernández's (2002) Created Shareholder Value is aligned in sign with the Net Present Value. Some concluding remarks end the paper.

For all notational conventions the reader should refer to Table 12 at the end of the paper.

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<sup>1</sup>The nouns 'profit', 'income', 'return', 'earning' will be used as synonyms, as well as the adjectives 'excess', 'residual', 'abnormal'.

## 2 The standard paradigm

Let  $\vec{a}=(-a_0, a_1, \dots, a_n)$  be an expected cash-flow stream released by project (firm)  $a$  in the span of  $n$  periods.

Let  $x_1, \dots, x_n$  be periodic rates of return such that

$$a_0 = \sum_{t=1}^n \frac{a_t}{\prod_{k=1}^t (1 + x_k)}.$$

For notational convenience we will often omit time subscripts, as long as ambiguity does not arise. Therefore, the above equation may be rewritten as

$$a_0 = \sum_{t=1}^n \frac{a_t}{(1 + x)^t}$$

where  $(1 + x)^t$  should be read as  $\prod_{k=1}^t (1 + x_k)$ . Thus, the symbol  $x$  represents either an internal rate of return or, rather, an internal discount function for project  $a$  that generalizes the notion of internal rate of return (see Weingartner, 1966; Peasnell, 1982, Franks and Hodges, 1984; Brief and Lawson, 1992; Peccati, 1990, 1991. See Pressacco and Stucchi, 1997, and Magni, 2004, 2005, for an internal rate of return based on the sign of the outstanding capital).

Let  $w_t(x)$ ,  $t = 1, 2, \dots, n$  be *arbitrary* numbers such that

$$w_t(x) = w_{t-1}(x)(1 + x) - a_t \quad t = 1, \dots, n \quad (1)$$

with  $w_0(x) := a_0$ . The above equation may conveniently be interpreted as the recursion formula for the project's outstanding capital. The undertaking of the project implies that, at the outset of each period, the capital  $w_{t-1}(x)$  is invested at the internal rate  $x$ , thus producing the interest  $xw_{t-1}(x)$ , which one may interpret as the profit of that period. Excess profit is profit above the profit that could be earned if the capital were invested in an alternative course of action (i.e. at an alternative rate of return). Letting  $i$  be the foregone rate of return (assumed constant for mere convenience),  $i \neq x$ , the foregone return in case of project rejection amounts to  $iw_{t-1}(x)$ . The latter is also known as opportunity cost.<sup>2</sup> The excess profit, or residual income, in the  $t$ -th period is therefore

$$\text{RI}_t^S = xw_{t-1}(x) - iw_{t-1}(x) = w_{t-1}(x)(x - i) \quad (2)$$

where  $xw_{t-1}(x)$  is the actual income. The formalization in eq. (2) is the classical one employed in the relevant literature (e.g. Edwards and Bell, 1961; Peasnell, 1981, 1982; Peccati, 1990, 1991; Ohlson, 1995; Pressacco and Stucchi, 1997; Martin and Petty, 2000; Arnold and Davies, 2000; Lundholm and O'Keefe, 2001; Martin et al. (2003)). This approach evidently rests on three basic elements: outstanding capital, internal rate of return, opportunity cost of capital. Different metrics are generated by this scheme, grounded on different notions of capital employed (asset side, equity side, economic, accounting, etc.), of cash flows employed (Free

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<sup>2</sup>Opportunity cost=foregone return, opportunity cost of capital=foregone rate of return.



Cash Flow, Equity Cash Flow, Capital Cash Flow<sup>3</sup>), of internal discount function employed (ROA, RONA, ROE, etc.).

*Remark 1.* It is worth noting that eq. (1) is consistent with the clean surplus concept (Brief and Peasnell, 1996). In business economics, this equation lies at the core of the notion of income (Lee, 1985; Hansen, 1972); in financial and actuarial mathematics, it represents the recursion formula for computing the balance (residual debt) in a loan contract (Kellison, 1991; Promislow, 2006; Werner and Sotnikov, 2006). The similarities between accounting and finance are here profound. Rewriting the equation as  $a_t = xw_{t-1}(x) - (w_{t-1}(x) - w_t(x))$  one may interpret the right-hand side either as the difference between income and change in book value or as the difference between interest and principal repayments: the former takes a management accounting perspective, the latter a financial one. However, to maintain consistency with Net Present Value,  $w_t(x)$  may be any number as long as  $x$  satisfies the equation: book value is therefore only one among many infinite possible choices.<sup>4</sup>

### 3 The *lost-capital* paradigm

The opportunity cost of investing in project  $a$  is that of renouncing to investing funds at the opportunity cost of capital  $i$ . This section presents a different way of interpreting the notion of foregone return, and therefore a different way of interpreting the notion of residual income. It has been originally introduced and investigated in Magni (2000a, 2000b, 2000c) and thoroughly investigated from several points of view in more recent years (2004, 2005, 2006). This section shows that it is possible to derive this notion from five different (but logically equivalent) arguments: an arbitrage-based argument using replicating cash-flows; an economic argument focussed on the investor's wealth; two accounting arguments involving the concept of depreciation and putting in place, respectively, depreciation schedules and the keynesian notion of user cost; a financial argument based on compounding interest on the standard residual income paradigm.

#### 3.1 The replicating cash-flow argument

As seen in the previous section, if the investor invests  $a_0$  in the project, his cash-flow is  $\bar{a}$  and the residual capital invested is  $w_{t-1}(x)$ , which is a dynamic system represented by

$$w_t(x) = w_{t-1}(x)(1 + x) - a_t. \quad (3)$$

Accepting the project the investor foregoes the opportunity of investing  $a_0$  in an alternative asset from which he could as well periodically withdraw the amounts  $a_t$ ,  $t = 1, \dots, n$ , so realizing the same pattern

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<sup>3</sup>For the notion of Capital Cash Flow, see Ruback (2002) and Fernández (2002).

<sup>4</sup>Admittedly,  $w_t$  itself may be labelled *book value*, given that book value is, in principle, arbitrary. In this view, for example, market value is only a particular choice of book value.

of cash flows as project  $a$ . Let  $w_t(i)$  be the outstanding balance at time  $t$  if the investor invests  $a_0$  in the alternative asset. In this case, the capital employed increases at the rate  $i$ , but falls by the amount  $a_t$ , which is withdrawn from the balance at the end of the period. This is described by the recurrence equation

$$w_t(i) = w_{t-1}(i)(1+i) - a_t \quad (4)$$

where, obviously,  $w_0(i) := a_0$ . Thus, if project is accepted, the outstanding balance in the  $t$ -th period is  $w_{t-1}(x)$ ; if, instead, the alternative asset is accepted, the outstanding balance is  $w_{t-1}(i)$ , which is here named the *lost capital*. The rate of return in the former case is  $x$ ; the rate of return in the latter case is  $i$ . Hence, the income in the former case is  $xw_{t-1}(x)$ , the income in the latter case is  $iw_{t-1}(i)$ . The residual income is therefore:

$$RI_t^L = xw_{t-1}(x) - iw_{t-1}(i). \quad (5)$$

The second addend is a lost return, obtained by multiplying the foregone return rate  $i$  by the lost capital  $w_{t-1}(i)$ .

This argument is evidently arbitrage-based: if  $a_0$  is invested in the project (firm), the payoff vector is  $(a_1, a_2, \dots, a_n)$ ; if instead  $a_0$  is invested at the cost of capital, the payoff stream is  $(a_1, a_2, \dots, a_n + w_n(i))$ . The terminal lost capital  $w_n(i)$  is the resulting arbitrage payoff generated by the replicating portfolio. If it is negative, project  $a$  is worth undertaking: a long position on the project and a short position on the alternative asset yield the arbitrage payoff vector  $(0, 0, 0, \dots, -w_n(i))$ ; if it is positive, the replicating cash-flow stream should be selected: a long position on the latter and a short position on  $a$  yield the arbitrage payoff vector  $(0, 0, 0, \dots, w_n(i))$ .

### 3.2 The wealth increase argument

Let us assume that an investor currently invests funds in a financial asset yielding a periodic return rate equal to  $i$  and let  $W_0$  be his net worth at time 0. If project  $a$  is not undertaken, the investor's wealth evolves according to the recursive equation

$$W_t(i) = W_{t-1}(i)(1+i) \quad (6)$$

so that  $W_t(i) = W_0(1+i)^t$ . If, instead, project  $a$  is undertaken, the investor, while renouncing to investing  $a_0$  at the rate  $i$ , receives the periodic sums  $a_t$ , which may be reinvested at the rate  $i$  in the financial asset.<sup>5</sup> In this case, the investor's wealth is a portfolio of two assets evolving at the rates  $x$  and  $i$  respectively. At time  $t$ , the investor's wealth amounts to

$$W_t(x, i) = w_t(x) + (W_{t-1}(x, i) - w_{t-1}(x))(1+i) + a_t \quad (7)$$

where  $w_t(x)$  is determined by eq. (3). Solving eq. (7) we find

$$W_t(x, i) = w_t(x) + (W_0 - a_0)(1+i)^t + \sum_{k=1}^t a_k(1+i)^{t-k}.$$

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<sup>5</sup>Note that this is just the standard assumption of the NPV rule.

This implies that wealth increase in case of project acceptance is

$$W_t(x, i) - W_{t-1}(x, i) = xw_{t-1}(x) + i \left( (W_0 - a_0)(1+i)^{t-1} + \sum_{k=1}^{t-1} a_k(1+i)^{t-1-k} \right),$$

whereas wealth increase in case of project rejection is

$$W_t(i) - W_{t-1}(i) = iW_0(1+i)^{t-1}.$$

Therefore, the excess increase in wealth is given by the difference of the alternative wealth increases:

$$\begin{aligned} \text{excess increase} &= (W_t(x, i) - W_{t-1}(x, i)) - (W_t(i) - W_{t-1}(i)) \\ &= xw_{t-1}(x) - ia_0(1+i)^{t-1} + i \sum_{k=1}^{t-1} a_k(1+i)^{t-1-k}. \end{aligned} \quad (8)$$

From eq. (4), we have  $w_{t-1}(i) = a_0(1+i)^{t-1} - \sum_{k=1}^{t-1} a_k(1+i)^{t-1-k}$ , so that eq. (8) becomes

$$\text{excess return} = xw_{t-1}(x) - iw_{t-1}(i) = \text{RI}_t^L \quad (9)$$

It is worth noting that we have found  $\text{RI}_t^L$  by making use of two alternative hypotheses about the evolution of the investor's wealth, namely the two dynamic systems in eq. (6) and eq. (7).

Note also that we may ideally part the investor's wealth into two assets in both cases:

$$W_{t-1}(x, i) = \overbrace{w_{t-1}(x)}^{\text{asset invested at rate } x} + \overbrace{(W_{t-1}(x, i) - w_{t-1}(x))}^{\text{asset invested at rate } i} \quad (10)$$

$$W_{t-1}(i) = \overbrace{(W_{t-1}(i) - W_{t-1}(x, i) + w_{t-1}(x))}^{\text{asset invested at rate } i} + \overbrace{(W_{t-1}(x, i) - w_{t-1}(x))}^{\text{asset invested at rate } i}. \quad (11)$$

The differential return between the two alternatives is not dependent on the second addends, which are shared by both alternatives; they may therefore be dismissed and, applying the corresponding rates of return, we find

$$\text{excess return} = xw_{t-1}(x) - i(W_{t-1}(i) - W_{t-1}(x, i) + w_{t-1}(x)).$$

Using the fact that  $W_{t-1}(i) - W_{t-1}(x, i) + w_{t-1}(x) = w_{t-1}(i)$  one finds back  $\text{RI}_t^L$ .

### 3.3 The depreciation argument

The lost-capital residual income may be ideally obtained by transforming the two alternative courses of action into two alternative depreciation schedules.

Consider asset  $A$ , producing the cash-flow  $\vec{A} = (-a_0, a_1, a_2, \dots, a_n + s_n)$ , where  $s_n$  is the asset's scrap value, received at the end of its service life. Let  $v_t$  be the accounting value of this asset at time  $t$  (with  $v_0 := a_0$ ) and let  $\text{Dep}_t := v_{t-1} - v_t$  be the depreciation charge in the  $t$ -th period. While any depreciation such that  $\sum_{t=1}^n \text{Dep}_t = a_0$  is acceptable for accounting purposes (see Peasnell, 1982), there is one significant from

an economic point of view: the decline in the present value of asset  $A$ 's future cash flows; letting  $r$  be the discount rate, this asset's accounting value is

$$v_t = \sum_{k=t+1}^n \frac{a_k}{(1+r)^{k-t}} + \frac{s_n}{(1+r)^{n-t}} \quad (12)$$

and the accounting profit is therefore  $rv_{t-1}$ . From the usual accounting identity (clean surplus relation)

$$\text{cash flows} = \text{income} + \text{depreciation}$$

we find  $\text{Dep}_t(r) = a_t - rv_{t-1}$ , and, using eq. (12), we get to

$$\text{Dep}_t(r) = a_t - r \left( \sum_{k=t}^n \frac{a_k}{(1+r)^{k-(t-1)}} + \frac{s_n}{(1+r)^{n-(t-1)}} \right),$$

where  $s_n = s_n(r) = a_0(1+r)^n - \sum_{t=1}^n a_t(1+r)^{n-t}$ .

The decision of accepting or rejecting project  $a$  boils down, in this view, to a choice between different depreciation plans for asset  $A$ : the accountant may ideally select the depreciation schedule such that  $r=x$  or, alternatively, the one where  $r=i$ .<sup>6</sup> In the former case, the scrap value becomes  $s_n(x) = a_0(1+x)^n - \sum_{t=1}^n a_t(1+x)^{n-t}$ , which equals zero, given that  $x$  is the internal rate of return (discount function) of project  $a$ . In the latter case, the scrap value is  $s_n(i) = a_0(1+i)^n - \sum_{t=1}^n a_t(1+i)^{n-t}$ .<sup>7</sup>

From the point of view of periodic performance, we may say that if the depreciation charge is smaller with  $r=x$  than with  $r=i$  (i.e. if the value of asset  $A$  decreases less rapidly with acceptance of project  $a$ ), then performance is positive. In other words, the difference

$$\text{Dep}_t(i) - \text{Dep}_t(x)$$

formally translates the notion of residual income. It is easy to show that this difference is just the lost-capital residual income. We have

$$\begin{aligned} \text{Dep}_t(i) - \text{Dep}_t(x) &= \left( a_t - i \sum_{k=t}^n \frac{a_k}{(1+i)^{k-(t-1)}} - i \frac{s_n(i)}{(1+i)^{n-(t-1)}} \right) - \\ &\quad - \left( a_t - x \sum_{k=t}^n \frac{a_k}{(1+x)^{k-(t-1)}} - x \frac{s_n(x)}{(1+x)^{n-(t-1)}} \right). \quad (13) \end{aligned}$$

By definition of internal rate of return (discount function), we have

$$a_0(1+r)^n = \sum_{k=1}^n a_k(1+r)^{n-k} + s_n(r) = \sum_{k=1}^{t-1} a_k(1+r)^{n-k} + \sum_{k=t}^n a_k(1+r)^{n-k} + s_n(r) \quad r = x, i.$$

<sup>6</sup>In financial terms, this boils down to investing funds either at the rate  $x$  or at the rate  $i$ .

<sup>7</sup>That  $i$  is actually an internal rate of return for asset  $A$  is easily shown:

$$-a_0 + \sum_{t=1}^n \frac{a_t}{(1+i)^t} + \frac{s_n(i)}{(1+i)^n} = -a_0 + \sum_{t=1}^n \frac{a_t}{(1+i)^t} + \left( a_0 - \sum_{t=1}^n \frac{a_t}{(1+i)^t} \right) = 0.$$

Dividing by  $(1+r)^{n-t+1}$  we have

$$a_0(1+r)^{t-1} = \sum_{k=1}^{t-1} \frac{a_k}{(1+r)^{k-(t-1)}} + \sum_{k=t}^n \frac{a_k}{(1+r)^{k-(t-1)}} + \frac{s_n(r)}{(1+r)^{n-(t-1)}} \quad r = x, i$$

whence

$$a_0(1+r)^{t-1} - \sum_{k=1}^{t-1} \frac{a_k}{(1+r)^{k-(t-1)}} = \sum_{k=t}^n \frac{a_k}{(1+r)^{k-(t-1)}} + \frac{s_n(r)}{(1+r)^{n-(t-1)}} \quad r = x, i$$

From eqs. (3) and (4) we find

$$a_0(1+r)^{t-1} - \sum_{k=1}^{t-1} a_k(1+r)^{t-1-k} = w_{t-1}(r) \quad \text{and} \quad s_n(r) = w_n(r), \quad r = x, i$$

so that eq. (13) becomes

$$\begin{aligned} \text{Dep}_t(i) - \text{Dep}_t(x) &= (a_t - iw_{t-1}(i)) - (a_t - xw_{t-1}(x)) \\ &= xw_{t-1}(x) - iw_{t-1}(i) = \text{RI}_t^L. \end{aligned} \quad (14)$$

The lost-capital residual income may therefore be represented as an excess depreciation charge.

### 3.4 The keynesian user cost

A particular important case of the depreciation argument relates the notion of residual income to the keynesian notion of *user cost*. In his *General Theory of Employment Interest and Money* Keynes defines user cost, with reference to the entrepreneur, as the difference between “the value of his capital equipment at the end of the period ... and ... the value it might have had at the end of the period if he had refrained from using it” (Keynes, 1967, p. 66). Some years after, the same concept is investigated in Coase (1968), who relabels it *depreciation through use*, because it measures the decline in value due (not to time but) to a different use of the asset. To compute user cost we must therefore calculate “the present value of the net receipts ... by discounting them at a rate of interest” (Coase, 1968, p. 123). This “rate of discount coincides with that in the market” (Scott, 1953, p. 378). Using our symbols, to compute user cost one must discount the relevant expected cash flows. Reminding the arbitrage-based description in subsection 3.1 and supposing the investor does not undertake the project, his payoff vector is  $(-a_0, a_1, \dots, a_n + w_n(i))$ ; if, instead, project is undertaken, his payoff vector is  $(-a_0, a_1, \dots, a_n)$ . In the former case the discounted value of the cash-flow stream is, at time  $t$ ,  $\sum_{k=t+1}^n a_k(1+i)^{t-k} + w_n(i)(1+i)^{t-n}$ , whereas in the latter case the discounted value of the cash-flow stream is  $\sum_{k=t+1}^n a_k(1+i)^{t-k}$ . Therefore,

$$\begin{aligned} \text{user cost} &= \left[ \sum_{k=t+1}^n a_k(1+i)^{t-k} + w_n(i)(1+i)^{t-n} \right] - \sum_{k=t+1}^n a_k(1+i)^{t-k} \\ &= w_n(i)(1+i)^{t-n}. \end{aligned} \quad (15)$$

User cost is just the discounted value of the arbitrage payoff. This is implicitly acknowledged by Keynes himself, who recognizes the user cost as “the discounted value of the additional prospective yield which would be obtained at some later date” (Keynes, 1967, p. 70).

It is easy to show that user cost acts as a depreciation charge with respect to use rather than to time. Using eqs. (3) and (4), we easily find  $w_t(x) = \sum_{k=t+1}^n \frac{a_k}{(1+x)^{k-t}}$  and  $w_t(i) = \sum_{k=t+1}^n \frac{a_k}{(1+i)^{k-t}} + \frac{w_n(i)}{(1+i)^{n-t}}$ . Therefore,

$$w_t(i) - w_t(x) = \left[ \sum_{k=t+1}^n \frac{a_k}{(1+i)^{k-t}} + w_n(i)(1+i)^{t-n} \right] - \sum_{k=t+1}^n \frac{a_k}{(1+x)^{k-t}}. \quad (16)$$

If the market value of the asset is selected as the outstanding capital (i.e. if one sets  $w_t(x) := V_t$ ), eq. (16) just represents the user cost above computed: given that  $V_t = \sum_{k=t+1}^n \frac{a_k}{(1+i)^{k-t}}$ , eqs. (15) and (16) coincide.

Putting it differently, eq. (15) is a particular case of eq. (16); the latter provides a generalized notion of the keynesian user cost. It is worth noting that the lost-capital residual income may be expressed as the periodic variation of this (generalized) user cost: from eq. (14) and the usual recurrence equations we get to

$$\text{RI}_t^L = \overbrace{[w_{t-1}(i) - w_{t-1}(x)]}^{\text{user cost in } t} - \overbrace{[w_t(i) - w_t(x)]}^{\text{user cost in } t-1}.$$

### 3.5 The compounding argument

The lost-capital residual income may be generated with a compounding process that directly relates the two paradigms. To this end, the new paradigm is interpreted with the eye of a standard-minded evaluator.

The starting point is the standard residual income, which represents the periodic surplus accrued to the project. Let us focus on the  $t$ -th period and assume that the surpluses  $\text{RI}_1^S, \text{RI}_2^S, \dots, \text{RI}_{t-1}^S$  are reinvested, as they are generated, at the opportunity cost of capital  $i$ . At time  $t-1$  the accumulated surplus is  $\sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}$ . As a result, in the  $t$ -th period the investor receives the return  $xw_{t-1}(x)$  from the project and the return  $i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}$  from the accumulated surplus. Given that  $w_{t-1}(x)$  could be invested at the rate  $i$ , the investor foregoes the return  $iw_{t-1}(x)$ . Therefore,

$$\begin{aligned} \text{residual income} &= xw_{t-1}(x) + i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k} - iw_{t-1}(x) \\ &= \text{RI}_t^S + i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}. \end{aligned} \quad (17)$$

The above residual income is just  $\text{RI}_t^L$ . To show it, we remind that

$$w_{t-1}(i) = w_0(i)(1+i)^{t-1} - \sum_{k=1}^{t-1} a_k (1+i)^{t-1-k}$$

and  $a_k = w_{k-1}(x)(1+x) - w_k(x)$ , so that

$$w_{t-1}(i) = w_0(x)(1+i)^{t-1} - \sum_{k=1}^{t-1} (w_{k-1}(x)(1+x) - w_k(x))(1+i)^{t-1-k}.$$

Upon rearranging terms, we find

$$w_{t-1}(i) = w_{t-1}(x) - \sum_{k=1}^{t-1} w_k(x)(x-i)(1+i)^{t-1-k} \quad (18)$$

$$= w_{t-1}(x) - \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}. \quad (19)$$

Consequently, eq. (17) becomes the lost-capital residual income:

$$\text{residual income} = \text{RI}_t^S + i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k} \quad (20)$$

$$= \text{RI}_t^S + i (w_{t-1}(x) - w_{t-1}(i)) \quad (21)$$

$$= xw_{t-1}(x) - iw_{t-1}(i) = \text{RI}_t^L. \quad (22)$$

Focussing on the right-hand side of eq. (21), the second addend is the additional periodic return earned or given up by the investor in a period if he accepts the project. In such a case, he owns a capital greater or smaller by  $|w_{t-1}(i) - w_{t-1}(x)|$  than the capital he would own in the rejection case. On this differential amount he earns or foregoes a return rate of  $i$ . But eq. (19) tells us that

$$w_{t-1}(x) - w_{t-1}(i) = \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k}, \quad (23)$$

i.e., the additional capital is just the compounded sum of all previous standard residual incomes. In other words, the accumulated surpluses of the past  $\text{RI}_k^S$ ,  $k = 1, 2, \dots, t-1$  represent the (additional or foregone) return “forgotten” by the standard paradigm.

As a result, the lost-capital paradigm may be seen as induced by a standard line of reasoning: it is just a standard residual income that keeps memory of the past (standard) residual incomes. (See Table 1 for a formal resume of the four arguments).

*Remark 2.* In the light of what we have seen in the previous subsection it is worthwhile noting that the accumulated standard residual incomes just represent the generalized user cost of eq. (16) (changed in sign). The user cost is therefore financially equivalent to the sum of compounded standard past residual incomes. This result is important for two reasons: first, user cost, which was defined by Keynes in a forward-looking perspective, is now expressed with a backward-looking perspective (past residual incomes); second, a relation linking firm value, lost capital and user cost is easily established: taking  $w_t(x)=V_t$  one finds, from eq. (23),

$$\begin{aligned} V_t &= w_t(i) + \sum_{k=1}^t \text{RI}_k^S (1+i)^{t-k} \\ &= \text{lost capital} + \text{user cost}. \end{aligned}$$

The market value of a firm (project) may therefore be expressed as the sum of the lost capital and the user cost.

## 4 Relations between paradigms

Both paradigms rest on the conceptual identity:

$$\text{Residual income} = \text{Actual income} - \text{Foregone income},$$

where the foregone income is the opportunity cost of investing in the project and acts as a capital charge:

$$\text{Residual income} = \text{Actual income} - \text{Capital charge}.$$

The foregone income is also interpreted as a normal income generated by a firm in the same class of risk, and residual income is therefore often called abnormal earning:

$$\text{Abnormal earning} = \text{Actual income} - \text{normal income}.$$

The differences between the two paradigms reside in the way the capital charge is calculated, and therefore in the notion of foregone income. The latter is the return the investor would have if he invested in the counterfactual alternative at the rate  $i$ . According to the standard paradigm (paradigm S), the investor could periodically invest the capital *actually* employed in the project ( $=w_t(x)$ ) at the return rate  $i$ . Conversely, the lost-capital paradigm (paradigm L) takes into consideration the fact that if the investor undertakes the project he loses the opportunity of owning a different capital ( $=w_t(i)$ ), which could be invested at the return rate  $i$ .<sup>8</sup> Therefore, in  $\text{RI}^S$  we have

$$\text{capital charge} = \text{actual capital} \cdot \text{foregone return rate},$$

whereas in  $\text{RI}^L$  we have

$$\text{capital charge} = \text{lost capital} \cdot \text{foregone return rate}.$$

While both paradigms measure the foregone return, they provide different (legitimate) interpretations of such a notion: in paradigm S foregoing return refers to “foregoing the return rate  $i$ ”, in paradigm L foregoing return refers not only to “foregoing the return rate  $i$ ” but also to “foregoing the capital  $w_{t-1}(i)$ ”

Using the replicating cash-flow argument above, the capital charge is arrived to by answering two different questions. The standard-minded investor asks:

*“What would income be in the  $t$ -th period if  $a_0$  were initially invested in the project and  $w_{t-1}(x)$  were invested at the rate  $i$ ?”*

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<sup>8</sup>The capital  $w_t(i)$  is not simply foregone, but definitely lost; therefore in paradigm L the foregone income is a lost unrecoverable income. It is evident that the lost capital coincides with O’Hanlon and Peasnell’s (2002) *unrecovered* capital, though the authors use it in a classical framework, where the standard residual income is maintained. By contrast, Drukarczyk and Schueler’s (2000) *invested* capital, which also coincides with the lost capital, is used to provide a metric which genuinely belong to the lost-capital paradigm.



whereas the lost-capital-minded investor asks:

*“What would income be in the  $t$ -th period if the amount  $a_0$  were invested in a replicating cash-flow stream yielding return at the rate  $i$ ?”*

Looking at eq. (21), it is evident that  $RI^S$  and  $RI^L$  may differ not only in terms of absolute value but also in terms of sign. Therefore, there may be instances where a model signals positive performance whereas the other one signals negative performance: even if  $x_t > i$  (i. e.  $RI_t^S$  is positive),  $RI_t^L$  may still be negative if  $w_{t-1}(i)$  is sufficiently greater than  $w_{t-1}(x)$ . In other words, if the investor did not undertake the project, his wealth could be greater than the one produced by the project, enough to offset the smaller rate of return  $i$  yielded by the counterfactual alternative. Conversely, if a periodic rate of return  $x_t$  is smaller than the opportunity cost of capital, then paradigm S signals poor performance, but nonetheless  $w_{t-1}(i)$  may be so small with respect to  $w_{t-1}(x)$  as to more than compensate, leading to an overall positive excess profit in paradigm L. Even when the signs of  $RI_t^S$  and  $RI_t^L$  coincide, consistently indicating positive or negative performance, the magnitude is, in general, different. We actually have that  $RI_t^L \neq RI_t^S$  whenever  $w_{t-1}(i) \neq w_{t-1}(x)$ . In particular, as long as  $x_t > i$ , the lost-capital paradigm signals a poorer (respectively, better) performance if  $w_{t-1}(i) > w_{t-1}(x)$  (respectively,  $w_{t-1}(i) < w_{t-1}(x)$ ). The reason is evident: paradigm L takes account of the fact that if  $w_{t-1}(i) \neq w_{t-1}(x)$  an investor undertaking the project renounces in the  $t$ -th period to owning a capital greater (or smaller) by an amount of  $|w_{t-1}(i) - w_{t-1}(x)|$ . That is, he renounces to receiving a positive (respectively, negative) return on that amount at a rate  $i$ . This implies that paradigm L produces performance indexes that are sensitive to the counterfactual time evolution of the capital invested, whereas paradigm S erases all the counterfactual story keeping only the counterfactual rate  $i$ . In particular, by splitting  $RI^L$  into two addends, eq. (20) tells us that positive (negative) performances will positively (negatively) reverberate in the following periods tending to increase (lower)  $RI^L$  with respect to  $RI^S$ . If performance is good in one year, next-year residual income will be positively affected regardless of whether  $x_t$  is greater or smaller than  $i$ . For example, if it should happen that  $x_t < i$  in some period, then the residual income benefits from the second addend of eq. (20), which acts as an insurance bonus. If, instead,  $x_t > i$ , then the insurance part become an additional return. Evidently, the additional term works well if  $w_{t-1}(i) < w_{t-1}(x)$ . But this just depends on the past performances. If it occurs that  $w_{t-1}(i) > w_{t-1}(x)$ , the additional term is negative, which tends to lower residual income even if  $x_t > i$ . Again, this depends on the past performances.

*Remark 3.* To say that the lost-capital residual income depends on past performances makes sense only if one employs a standard line of reasoning: to a standard-minded evaluator paradigm L is just paradigm S with an added memory to recall the past. But to a lost-capital-minded evaluator, the comparison is just between two alternative incomes pertaining to the same period, and the residual income of one year does not reverberate on the following years. From this point of view, the additional term  $\sum_{k=1}^{t-1} RI_k^S (1+i)^{t-1-k}$  does

not represent accumulated (standard) residual incomes, but is just the additional capital that the investor could invest in the  $t$ -th period if he selected, at time 0, the counterfactual course of action. This is (again) consistent with the keynesian notion of user cost, seen as a depreciation due to different use of the funds (Coase, 1968).

## 5 Book values, market values, and income aggregation

An important issue is the relation paradigm L bears to a project's Net Present Value (firm's Market Value Added), and, therefore, to market values. If a residual-income paradigm is not consistent with the NPV, then it should be evidently dismissed. We now show that both paradigms are consistent with the NPV though with an opposite procedure:<sup>9</sup> paradigm S requires a *discount-then-sum* mechanism, while paradigm L requires a *sum-then-discount* approach. For the former we have, discounting and then summing:

$$\begin{aligned} \sum_{t=1}^n \frac{\text{RI}_t^S}{(1+i)^t} &= \sum_{t=1}^n \frac{w_{t-1}(x)(x-i)}{(1+i)^t} = \sum_{t=1}^n \frac{w_t(x) + a_t - w_{t-1}(x)(1+i)}{(1+i)^t} \\ &= \sum_{t=1}^n \frac{w_t(x) + a_t}{(1+i)^t} - \sum_{t=1}^n \frac{w_{t-1}(x)}{(1+i)^{t-1}} \\ &= \sum_{t=1}^n \frac{a_t}{(1+i)^t} - a_0 = \text{NPV} \end{aligned}$$

where we have used the equality  $a_0 + \sum_{t=1}^n w_t(x)(1+i)^{-t} = \sum_{t=1}^n w_{t-1}(x)(1+i)^{-(t-1)}$ .<sup>10</sup> As for paradigm L, if we first sum excess profits and then discount them back we obtain the NPV. To show it, just consider that, taking the sum in eq. (17) and rearranging terms, we have

$$\begin{aligned} \sum_{t=1}^n \text{RI}_t^L &= \sum_{t=1}^n \left( \text{RI}_t^S + i \sum_{k=1}^{t-1} \text{RI}_k^S (1+i)^{t-1-k} \right) \\ &= \sum_{t=1}^n \text{RI}_t^S (1+i)^t - \sum_{k=1}^{n-1} \text{RI}_k^S (1+i)^{n-k} \\ &= \sum_{t=1}^n \text{RI}_t^S (1+i)^{n-t} \end{aligned}$$

where the last equation is derived by induction. Discounting back,

$$\begin{aligned} \frac{1}{(1+i)^n} \sum_{t=1}^n \text{RI}_t^L &= \frac{1}{(1+i)^n} \sum_{t=1}^n \text{RI}_t^S (1+i)^{n-t} \\ &= \sum_{t=1}^n \text{RI}_t^S (1+i)^{-t} \\ &= \text{NPV}. \end{aligned} \tag{24}$$

<sup>9</sup>As for paradigm S, the result is well-known (e.g. Edwards and Bell, 1961, ch. 2; Peasnell, 1981, 1982; Ohlson, 1995; Martin and Petty, 2000, ch. 5; Lundholm and O'Keefe, 2001; Vélez-Pareja and Tham, 2003).

<sup>10</sup>We remind that  $w_n(x) = 0$ , because  $x$  is an internal rate of return (discount function) for project  $a$ .

The net terminal value is therefore obtained by a sum of un-compounded residual incomes:

$$\text{NPV}(1+i)^n = \sum_{t=1}^n \text{RI}_t^L. \quad (25)$$

The results in eqs. (24) and (25) have interesting theoretical and practical implications: they provide a strong link between accounting data and market values. Recalling that  $\text{NPV} = E_0 - a_0$  and letting  $w_t(x)$  be the equity book value we have, from eq. (24),

$$E_0 = a_0 + \frac{1}{(1+i)^n} \sum_{t=1}^n \text{RI}_t^L, \quad (26)$$

which says that the market value of equity is given by the book value plus the sum of future lost-capital residual incomes. The above equation highlights that time is not important: any permutation of the residual incomes will lead to the same value. Therefore, one does not have to worry about imputing each residual income to each date in which it will be generated. If expectations are correct in terms of value but incorrect in terms of timing, no error in valuation is made with the lost-capital paradigm.

## 6 Converting standard residual income into lost-capital residual income

The two paradigms generate several performance measures. In particular, for each such measure complying with paradigm S there corresponds a companion measure in paradigm L. Conversion is made by replacing the foregone income of paradigm S with the lost income of paradigm L. For illustrative purposes, we focus on Stewart's (1991) Economic Value Added (EVA) and on the Edwards-Bell-Ohlson (EBO) model (Edwards and Bell, 1961; Ohlson, 1995),<sup>11</sup> The two metrics belong to the set of standard residual income models, and are complementary: EVA adopts an entity (claimholders) approach; EBO adopts a proprietary (shareholder) approach.

### 6.1 EVA

Assume that (i) the book value of the firm's assets is taken as the outstanding capital, (ii) the free cash flows are taken as the relevant cash flows (iii) the RONA (Return On Net Assets) is taken as the periodic rate of return, and (iv) the WACC is taken as the opportunity cost of capital. Formally, this means  $w_t(x) := V_t^{bv}$ ,  $a_t := \text{FCF}$ ,  $x := \text{RONA}$ ,  $i := \text{WACC}$ . Therefore, eq. (3) becomes

$$V_t^{bv} = V_{t-1}^{bv} \cdot (1 + \text{RONA}) - \text{FCF}$$

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<sup>11</sup>Abusing notation, we will henceforth use the acronym EBO to refer to the corresponding residual income as well.

for  $t > 0$ , and  $V_0^{bv} := a_0$ . Reminding that  $V_{t-1}^{bv} \cdot \text{RONA} = \text{NOPAT}$  and applying eq. (2), the standard performance measure becomes

$$\text{RI}^S = \text{NOPAT} - \text{WACC} \cdot V_{t-1}^{bv}. \quad (27)$$

If, instead, paradigm L is applied, letting  $w_t(i) := \mathbb{V}_t$  be the lost capital and using eq. (4) one finds

$$\mathbb{V}_t = \mathbb{V}_{t-1} \cdot (1 + \text{WACC}) - \text{FCF}$$

for  $t > 0$ , with  $\mathbb{V}_0 := a_0$ . Thus, the lost-capital measure (eq. (5)) results in

$$\text{RI}^L = \text{NOPAT} - \text{WACC} \cdot \mathbb{V}_{t-1}. \quad (28)$$

The measures in eqs. (27) and (28) represent the original Economic Value Added and its lost-capital companion, respectively.

## 6.2 EBO

A different metric is generated when (i) the book value of equity is taken as the outstanding capital, (ii) the equity cash flows are taken as the relevant cash flows, (iii) the ROE (Return On Equity) is taken as the periodic rate of return, and (iv) the cost of equity  $k_e$  is taken as the opportunity cost of capital. Formally,  $w_t(x) := E_t^{bv}$ ,  $a_t := \text{ECF}$ ,  $x := \text{ROE}$ ,  $i := k_e$ , so that

$$E_t^{bv} = E_{t-1}^{bv} \cdot (1 + \text{ROE}) - \text{ECF}$$

for  $t > 0$ , with  $E_0^{bv} := a_0$ . Therefore, reminding that  $E_{t-1}^{bv} \cdot \text{ROE} = \text{PAT}$ , the standard measure becomes

$$\text{RI}^F = \text{PAT} - k_e \cdot E_{t-1}^{bv}. \quad (29)$$

If one applies paradigm L to this measure and let  $w_t(i) := \mathbb{E}_t$  be the lost equity, one has

$$\mathbb{E}_t = \mathbb{E}_{t-1} \cdot (1 + k_e) - \text{ECF}$$

for  $t > 0$ , with  $\mathbb{E}_0 := a_0$ . Thus, the lost-capital measure results in

$$\text{RI}^S = \text{PAT} - k_e \cdot \mathbb{E}_{t-1}. \quad (30)$$

The measures in eqs. (29) and (30) represent EBO as originally conceived and its lost-capital companion, respectively. To sum up, the standard paradigm depends on the threesome (RONA, WACC,  $V^{bv}$ ),<sup>12</sup> whereas the lost-capital paradigm depends on the foursome (RONA, WACC,  $V^{bv}$ ,  $\mathbb{V}$ ) (see Table 2).<sup>13</sup>

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<sup>12</sup>Or (ROE,  $k_e$ ,  $E^{bv}$ ) for EBO.

<sup>13</sup>Or (ROE,  $k_e$ ,  $E^{bv}$ ,  $\mathbb{E}$ ) for EBO.

## 7 An example

This section applies the two paradigms to a firm created to undertake a project that requires an initial investment of 13 800, of which 12 000 are spent in fixed assets and 1 800 in working capital requirements. Straight-line depreciation is assumed for the fixed assets. It is also assumed that the required return on assets is 12% and that the book value of debt equals the market value of debt (i.e. debt rate=required return to debt). Other input data are collected in Table 3; Table 4 gives the firm's accounting statements and the resulting cash flows, and Table 5 focuses on equity and firm valuation. The market value of equity is first found by using three different discounted-cash-flow methods: the Adjusted Present Value (APV) method, introduced by Myers (1974), the ECF- $k_e$  method (equity approach), and the FCF-WACC method (entity approach). Logically, they all give the same result (see Fernández, 2002). Afterwards, a residual-income perspective is used to obtain the market value: Tables 6-7 show the application of the two paradigms to the EVA model and the EBO model. Obviously, both paradigms supply the same market values as the discounted-cash-flow technique's.<sup>14</sup>

The examples show a situation of positive EVAs and EBOs in each period. First of all, note that in the first period the two paradigms give the same answer, because the outstanding capitals coincide ( $w_0(x) = w_0(i)$ ). In the next periods, the lost-capital measures are constantly greater than the standard measures. Also, the periodic variation in the lost-capital measures are greater. For example, in Table 6 the standard EVA's variations are given by (281, 282, 283, 286), the lost-capital EVA's variations are (282, 313, 347, 376). In Table 7 we have, consistently, that the EBO's variations are (296, 298, 306, 372) and (302, 350, 427, 811), respectively.

As anticipated, the lost-capital has an insurance component for negative situations, which is just the user cost previously introduced. Suppose the fourth-year sales amount to 8 000 instead of 10 000 (Table 8), other things equal. Both paradigms report negative performance in the fourth year.<sup>15</sup> Yet, the lost-capital paradigm smoothes the negativeness, because it takes account of the fact that the past year's results were better, which implies that the lost capital at the beginning of the fourth year is smaller than the actual capital employed:  $V_3^{bv} > V_3$  and  $E_3^{bv} > E_3$ . It is easy to see that if the fourth-year sales are equal to 8 600 instead of 10 000 (other things unvaried), the corresponding standard measures become negative, whereas the lost-capital measures keep positive (Table 9). In this case, while the RONA (respectively, ROE) is indeed smaller than the WACC (respectively,  $k_e$ ) in the fourth year, the bonus given by the additional amount  $WACC_4 \cdot (V_3^{bv} - V_3) = 96$  (respectively,  $k_{e_4} \cdot (E_3^{bv} - E_3) = 185$ ) is so high as to more than compensate the negative standard EVA (respectively, EBO): we have  $16 = -80 + 96$ , and  $164 = -21 + 185$ .

<sup>14</sup>As previously shown, the time ordering of residual incomes is immaterial in paradigm L, if the objective is firm valuation. However, if the objective of the analysis is incentive compensation, time is obviously relevant in this paradigm as well.

<sup>15</sup>The reader should not be discomforted by the fact that *each* period's residual income changes. If one period's sales change, the corresponding ECF and FCF change, so that the market value of equity is changed in every year, which implies that both  $k_e$  and WACC change in every year, which in turn induces a change in the capital charge of every period.

Evidently, the bonus may symmetrically act a penalty role if past performance is negative. For example, consider the case where in the third year sales amount to 8 000 (other things unvaried). This makes the third-year residual incomes negative for both paradigms (Table 10). Due to insurance bonus for positive past performances, the lost-capital residual incomes are less negative than the standard ones. Yet, the third-year negative performance penalizes the fourth-year performance, which is smaller than that reported by the standard residual incomes. Note that in the fifth year, performance recorded by the lost-capital paradigm is again higher than the standard one's, due to the renewed recent positive performance of the fourth year. In other words, as compared to the standard metric, performance is amplified in negative and in positive sense (bonus and penalty roles).<sup>16</sup>

It is also worth noting that the dependence of a lost-capital measure on the past is not an easy one (the measure does not merely depends on the previous period's  $RI_{t-1}^S$ , but on  $RI_1^S, RI_2^S, \dots, RI_{t-2}^S$ , and therefore on all the previous rates of return and all the previous opportunity costs of capital). It may be conjectured that managers willing to pursue personal objective may refrain from gaming the measure, given that they hardly will be able to assess the consequences on the following years' indexes. What they are aware of is that their performance is measured on the ground of past residual incomes as well as the current one. Whether these elements tend to reduce agency problems and whether managers rewarded through a lost-capital residual income are more inclined to behave optimally is not a trivial issue and deserves a thorough investigation. The efficacy of the paradigm also depends on the type of compensation plan selected. For example there are at least three ways of using a metric: the historical use, according to which the manager's bonus is a share of the RI:

$$\text{bonus} = x\% \text{ RI};$$

the XY compensation plan, according to which bonus is tied to RI variation:

$$\text{bonus} = x\% \text{ RI} + y\% \Delta \text{ RI};$$

and the excess RI improvement plan, according to which the expected RI improvement (EI) plays a major role:

$$\text{bonus} = \text{target bonus} + y\% (\Delta \text{ RI} - \text{EI})$$

(see Young and O'Byrne, 2001). For positive-RI companies using either the historical plan or the XY plan, we can say that the manager's bonuses computed with the lost-capital paradigm are greater than the ones computed in the standard paradigm, because in the former both RI and  $\Delta$  RI are greater than the

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<sup>16</sup>It is worth stressing again that the memory-dependent interpretation is a useful one for comparing the two paradigms, but it presupposes a standard-minded point of view. The memory-dependent feature of the lost-capital metrics just means that if money were invested at the opportunity cost of capital, the investor would have, in each period, a different (greater or smaller) capital. This appreciation or depreciation, equal to the keynesian user cost, would imply, in that very period, an additional or foregone interest. Such an interest is a penalty if positive, a bonus if negative.

corresponding ones in the latter (proof is straightforward using eq. (20)). However, things are complicated by the fact that comparisons may be made along two dimensions: the type of metric selected and the paradigm chosen. That is, a metric in a paradigm may be compared with the same metric in the alternative paradigm, or with an alternative metric in the same paradigm, or with an alternative metric in the alternative paradigm. Having two paradigms and a wide set of metrics it may be the case that a metric in one paradigm is more incentive than a different metric in the alternative paradigm. Given that firms may use many different plans to compute managers' bonuses, the impact on performance measurement and incentive compensation depends on (at least) three factors:

- the paradigm
- the metric
- the compensation plan

## 8 Aligning performance measures with value creation

A mystifying problem in value-based management is that residual income does not measure value creation in the period considered, so that either some adjustments are made to residual income itself or compensation plans are devised so as to tie residual income to value creation, in order to align managers' behaviors to shareholders' objectives (Ehrbar, 1998; Stewart, 1991; O'Hanlon and Peasnell, 2000; Young and O'Byrne, 2001; Martin et al., 2003). Grinyer (1985, 1987) proposes an index labelled *Earned Economic Income*, which has the nice property of being aligned with the Net Present Value. However, beside the fact that "the relationship between EEI and RI appears not to be well understood" (Peasnell, 1995, p. 235), his metric is equal in sign to the NPV only if the project's cash flows are all of the same sign (Martin et al., 2003, Peasnell, 1995, Grinyer, 1995). This section shows that converting Fernández's (2002) Created Shareholder Value into the corresponding lost-capital metric, one obtains a metric that is perfectly aligned with the Net Present Value, irrespective of the sign of the cash flows (i.e. even if some cash flows are opposite in sign).

The Created Shareholder Value (CSV) belongs to the class of standard residual income models. It is computed by picking  $a_t = \text{ECF}$ ,  $w_t(x) = E_t$  for every  $t \geq 1$  and  $i = k_e$ . Reminding the initial condition  $w_0(x) := a_0$ , the residual income in this model is

$$\text{CSV}_t = \begin{cases} a_0(x_t - k_e) & \text{if } t = 1 \\ E_{t-1}(x_t - k_e) & \text{if } t > 1 \end{cases} \quad (31)$$

where  $x_t = (E_t + a_t - a_{t-1})/a_{t-1}$  if  $t=1$  (see Fernández, 2002, p. 281),  $x_t = k_e$  otherwise (this implies  $\text{CSV}_t = 0$  for all  $t > 1$  if expectations are met). In order to convert the standard CSV into its lost-capital companion,

the capital charge  $k_e E_{t-1}$  must be replaced by  $k_e \mathbb{E}_{t-1}$  so that residual income becomes

$$\text{lost-capital CSV}_t = \begin{cases} a_0(x_t - k_e) & \text{if } t = 1 \\ k_e(E_{t-1} - \mathbb{E}_{t-1}) & \text{if } t > 1. \end{cases} \quad (32)$$

As for the  $t=1$ , we have

$$\text{CSV}_1 = a_0 \left( \frac{E_1 + a_1 - a_0}{a_0} - k_e \right) = \left( \frac{E_1 + a_1}{1 + k_e} - a_0 \right) (1 + k_e) = \text{NPV}(1 + k_e).$$

As for  $t > 1$ , note that the difference  $(E_{t-1} - \mathbb{E}_{t-1})$  is exactly the keynesian user cost of eq. (15) in an equity approach, because  $E_t = \sum_{k=t+1}^n a_k (1 + k_e)^{-(k-t)}$  and  $\mathbb{E}_t = \sum_{k=t+1}^n a_k (1 + k_e)^{-(k-t)} + \mathbb{E}_n (1 + k_e)^{-(n-t)}$ . Therefore,

$$E_{t-1} - \mathbb{E}_{t-1} = \mathbb{E}_n (1 + k_e)^{-(n-t+1)}.$$

But  $\mathbb{E}_n = a_0 (1 + k_e)^n - \sum_{k=1}^n a_k (1 + k_e)^{n-k} = -\text{NPV}(1 + k_e)^n$ , so that one may write

$$E_{t-1} - \mathbb{E}_{t-1} = \text{NPV}(1 + k_e)^{t-1}$$

whence

$$\text{CSV}_t = k_e (E_{t-1} - \mathbb{E}_{t-1}) = k_e \text{NPV}(1 + k_e)^{t-1}.$$

As a result, the lost-capital companion of CSV is always aligned in sign with the NPV. Indeed, it measures the increase of Net Present Value period by period:

$$k_e (E_{t-1} - \mathbb{E}_{t-1}) = \text{NPV}(1 + k_e)^t - \text{NPV}(1 + k_e)^{t-1} = \Delta \text{NPV}.$$

Referring to the example of section 7, Table 11 supplies the value of CSV in the two paradigms.<sup>17</sup>

## 9 Conclusions

This paper presents a new paradigm of residual income aimed at appraising projects (firms) as well as measuring periodic performance. Originally introduced with the name with the name of *Systemic Value Added* (Magni, 2000a, 2000b, 2001, 2003), the new paradigm translates the notion of opportunity cost differently from the classical paradigm. The new paradigm takes account of the capital foregone by the investor as well as the foregone return rate. In other words, if the investor invested in the alternative asset he would have, at the beginning of each period, a different capital than the actual one. This capital, which is definitely lost if project is undertaken, would generate income at the opportunity cost of capital. Hence, the paradigm is here relabeled “lost-capital paradigm”. This paper presents five different but equivalent ways of conceptualizing the lost-capital paradigm: (i) the project’s (firm’s) cash-flow stream may be replicated by

<sup>17</sup>If an entity approach is taken, rather than a proprietary approach, the the lost-capital companion of CSV becomes Drukarczyk and Schueler’s (2000) Net Economic Income.



investing funds at the cost of capital: the lost-capital RI is given by the difference between the alternative incomes; (ii) the dynamic system representing the investors' wealth manifests a different increase if funds are invested funds at the cost of capital: the lost-capital RI is given by the difference between alternative wealth increases; (iii) alternative depreciation plans are considered: the lost-capital RI is obtained as difference between alternative depreciation charges; (iv) the keynesian notion of user cost (depreciation through use) is retrieved and the lost-capital RI is computed as the periodic variation of the (generalized) user cost (v) the standard RI may be ideally banked to earn a rate of return equal to the cost of capital: the lost-capital RI is calculated by summing the standard RI and the interest earned on the accumulated past standard RIs.

The five arguments bear strong relations one another. In particular, user-cost is equal to the discounted value of the arbitrage payoff derived from the replicating portfolio (subsection 3.1) and equal to the accumulated past (standard) residual incomes (subsection 3.5). Also, the wealth increase argument is equivalent to the depreciation argument: in this case depreciation is computed on the entire net worth, taking account of the entire net worth derived from reinvestment of the cash flows at the cost of capital (see eq. (8)).

With the aid of the fifth argument, which relates the standard paradigm to the lost-capital paradigm, this paper shows that consistency with the NPV (MVA) is guaranteed in the lost-capital paradigm as well: however, the standard paradigm uses a *discount-then-sum* procedure, whereas the lost-capital paradigm uses a *sum-then-discount* mechanism; what differs is the distribution of the NPV across periods. While such a distribution is relevant for the standard paradigm to get the market value, it is not for the lost-capital paradigm: forecasting each (lost-capital) residual income and the corresponding period in which it is generated is unnecessary. Only the total sum of residual incomes is needed. Thus, the aggregation property of the lost-capital paradigm makes the residual income models particularly attractive as opposed to the discounted-cash-flow models and, in addition, offers an improvement with respect to the standard paradigm, given that the latter supplies the correct equity market value (and net terminal value) only if each and every residual income is forecasted for each and every year.

As for value-based management, the lost-capital paradigm tends to amplify results with respect to the standard paradigm, both in positive and negative sense. This and other features are worth investigating in order to ascertain which conditions make the new paradigm more incentive a tool for managers. Evidently, this also depends on the kind of compensation scheme used by the company to reward managers. For example, if the XY compensation plan is used (where  $\text{bonus} = x\% \text{RI} + y\% \Delta \text{RI}$ ), the lost-capital paradigm is more incentive for positive-RI companies, because both RI and  $\Delta \text{RI}$  are greater in the lost-capital paradigm than in the standard one. A further element that deserves a detailed analysis is how residual incomes change when expectations change and/or are not met, with particular concern to relation with value creation.

A particular model that deserves a thorough investigation is the lost-capital companion of Fernández's (2002) Created Shareholder Value. It is shown that the metric obtained from its conversion into the lost-capital paradigm is perfectly aligned in sign with the Net Present Value. In this particular case, residual income does measure value creation and this kind of metric could be fruitfully used for management com-

pensation.

The new paradigm may be an opportunity to search for new theoretical insights in management accounting and managerial finance as regards the relations among accounting values, market values, value creation. Also, it adds some irons in the fire of the value-based management debate. This, far from being a problem, should be seen as an opportunity for developing new measures and finding intriguing relations across metrics and across paradigms. It may be of some interest to find out whether the differences in the standard metrics are mirrored by the differences in the corresponding lost-capital metrics, or whether pros and cons of either measure change as the paradigm adopted is changed. It may be the case that the search for a satisfying compensation plan will lead to an index based on multiple metrics, possibly involving the use of both paradigms.

As a final consideration, it is evident that the aggregation property of the lost-capital paradigm establishes a powerful argument for management accounting to play a major role in project and firm valuation, as well as in capital budgeting decision problems. On one side, Ohlson's breakthrough result says that, under suitable assumptions on the stochastic process of abnormal earnings and with  $n$  sufficiently large, the *future* market value is *approximated* by a function of earnings; on the other side, the lost-capital result says that, under no particular assumption on stochastic processes and whatever the value of  $n$ , the *current* market value is *equal to* a function of earnings. These two results are conducive to a reinstatement of fundamental analysis as an important tool for either valuation and capital budgeting purposes.

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**Table 1. The five arguments**

cash-flow replicated	return from project $xw_{t-1}(x)$	lost return $iw_{t-1}(i)$	<b>Lost-capital residual income</b>
wealth increases	wealth increase $W_t(x, i) - W_{t-1}(x, i)$	lost wealth increase $W_t(i) - W_{t-1}(i)$	
depreciation charges	asset's depreciation $Dep_t(x)$	lost depreciation $Dep_t(i)$	
user cost	user cost in $t - 1$ $[w_{t-1}(i) - w_{t-1}(x)]$	user cost in $t$ $[w_t(i) - w_t(x)]$	
standard RI capitalized	standard RI $RI_t^S$	foregotten return $i \sum_{k=1}^{t-1} RI_k^S (1+i)^{t-1-k}$	

**Table 2. EVA and EBO variables in the two paradigms**

	$x$	$i$	$w_t(x)$	$w_t(i)$	$\implies$	capital charge
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$
<i>Standard Paradigm</i>						
EVA	RONA	WACC	$V^{bv}$		$\implies$	$WACC \cdot V^{bv}$
EBO	ROE	$k_e$	$E^{bv}$		$\implies$	$k_e \cdot E^{bv}$
<i>Lost-capital Paradigm</i>						
EVA	RONA	WACC	$V^{bv}$	$\mathbb{V}$	$\implies$	$WACC \cdot \mathbb{V}$
EBO	ROE	$k_e$	$E^{bv}$	$\mathbb{E}$	$\implies$	$k_e \cdot \mathbb{E}$

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**Table 3. Input data**

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Investment	13 800	Depreciation rate	20%
Gross Fixed Assets	12 000	Corporate tax rate	33%
WCR	1 800	Required return on assets	12%
Sales	10 000	Debt rate	7%
Cost of Sales	3 670	Required return on debt ( $k_D$ )	7%
Gen. & Admin. Expenses	1 600		
Debt	4 000		

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**Table 4. Balance Sheet, Income Statement, Cash Flows**

time	0	1	2	3	4	5
<i>BALANCE SHEET</i>						
Gross fixed assets	12 000	12 000	12 000	12 000	12 000	12 000
–cumulative depreciation	0	–2 400	–4 800	–7 200	–9 600	–12 000
Net fixed assets	12 000	9 600	7 200	4 800	2 400	0
WCR	1 800	1 800	1 800	1 800	1 800	0
<b>NET ASSETS</b>	13 800	11 400	9 000	6 600	4 200	0
Debt	4 000	4 000	4 000	4 000	4 000	0
Equity (book value)	9 800	7 400	5 000	2 600	200	0
<b>NET WORTH &amp; LIABILITIES</b>	13 800	11 400	9 000	6 600	4 200	0
<i>INCOME STATEMENT</i>						
Sales		10 000	10 000	10 000	10 000	10 000
Cost of sales		3 670	3 670	3 670	3 670	3 670
Gen. & Adm. expenses		1 600	1 600	1 600	1 600	1 600
Depreciation		2 400	2 400	2 400	2 400	2 400
EBIT		2 330	2 330	2 330	2 330	2 330
Interest		280	280	280	280	280
PBT		2 050	2 050	2 050	2 050	2 050
Taxes		677	677	677	677	677
PAT		1 374	1 374	1 374	1 374	1 374
+Depreciation		2 400	2 400	2 400	2 400	2 400
+Δ Debt		0	0	0	0	–4 000
–Δ WCR		0	0	0	0	1 800
ECF		3 774	3 774	3 774	3 774	1 574
FCF <sup>18</sup>		3 961	3 961	3 961	3 961	5 761

<sup>18</sup>FCF=ECF–ΔD +  $k_D D \cdot (1 - T)$ .

**Table 5. Valuation**

time	0	1	2	3	4	5
$k_U$		12%	12%	12%	12%	12%
$V_U = PV[\text{FCF}; k_U]$	15 300	13 175	10 795	8 129	5 144	0
$DVTS = PV[T \cdot k_D \cdot D; k_D]^{(a)}$	379	313	242	167	86	0
$V = V_U + DVTS$	15 679	13 488	11 038	8 296	5 230	0
$\mathbf{E} = V_U + DVTS - D$	<b>11 679</b>	9 488	7 038	4 296	1 230	0
$k_e$		13.550%	13.943%	14.670%	16.461%	27.907%
$\mathbf{E} = PV[\text{ECF}; k_e]$	<b>11 679</b>	9 488	7 038	4 296	1 230	0
WACC		11.290%	11.199%	11.053%	10.786%	10.151%
$V = PV[\text{FCF}; \text{WACC}]$	15 679	13 488	11 038	8 296	5 230	0
$\mathbf{E} = V - D$	<b>11 679</b>	9 488	7 038	4 296	1 230	0
$\mathbf{MVA} = NPV = E - E^{bv}$	1 879					

<sup>(a)</sup>We use  $k_D$  to discount tax shields. However, it is worth noting that there is a lively debate in the literature on which is the the correct discount rate for discounting tax shields. While this issue is not relevant to this paper, the reader may be willing to turn to the contributions of Myers (1974), Tham and Vélez-Pareja (2001), Arzac and Glostén (2005), Fernández (2005), Cooper and Nyborg (2006). (To bypass the issue, the reader may well dismiss the first five rows of the Table and consider  $k_e$  as exogenously given).

**Table 6. EVA in the two paradigms**

time	0	1	2	3	4	5
NOPAT= $EBIT \cdot (1-T)$		1 561	1 561	1 561	1 561	1 561
$V^{bv}=D+E^{bv}$	13 800	11 400	9 000	6 600	4 200	0
V (lost capital)	13 800	11 397	8 712	5 714	2 369	-3 151
<i>Standard Paradigm</i>						
capital charge (opportunity cost)		1 558	1 277	995	712	426
<b>EVA</b>		<b>3</b>	<b>284</b>	<b>566</b>	<b>849</b>	<b>1 135</b>
MVA (=discount and sum)	1 879					
$E=E^{bv}+MVA$	11 679					
<i>Lost-capital Paradigm</i>						
capital charge (opportunity cost)		1 558	1 276	963	616	240
<b>EVA</b>		<b>3</b>	<b>285</b>	<b>598</b>	<b>945</b>	<b>1 321</b>
MVA (=sum and discount)	1 879					
$E=E^{bv}+MVA$	11 679					

**Table 7. EBO in the two paradigms**

time	0	1	2	3	4	5
PAT		1 374	1 374	1 374	1 374	1 374
$E^{bv}$	9 800	7 400	5 000	2 600	200	0
$\mathbb{E}$ (lost equity capital)	9 800	7 354	4 606	1 509	-2 017	-4 153
<i>Standard Paradigm</i>						
capital charge (opportunity cost)		1 328	1 032	733	428	56
<b>EBO</b>		<b>46</b>	<b>342</b>	<b>640</b>	<b>946</b>	<b>1 318</b>
MVA (=discount and sum)	1 879					
$E=E^{bv}+MVA$	11 679					
<i>Lost-capital Paradigm</i>						
capital charge (opportunity cost)		1 328	1 025	676	248	-563
<b>EBO</b>		<b>46</b>	<b>348</b>	<b>698</b>	<b>1 125</b>	<b>1 936</b>
MVA (=sum and discount)	1 879					
$E=E^{bv}+MVA$	11 679					

**Table 8. Fourth-year sales equal to 8 000**

year	1	2	3	4	5
<b>EVA</b>					
Standard Paradigm	<b>9</b>	<b>291</b>	<b>575</b>	<b>-477</b>	<b>1 135</b>
Lost-capital paradigm	<b>9</b>	<b>292</b>	<b>608</b>	<b>-381</b>	<b>1 188</b>
<b>EBO</b>					
Standard Paradigm	<b>34</b>	<b>326</b>	<b>616</b>	<b>-439</b>	<b>1 318</b>
Lost-capital paradigm	<b>34</b>	<b>330</b>	<b>671</b>	<b>-251</b>	<b>1 537</b>

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**Table 9. Fourth-year sales equal to 8 600**

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year	1	2	3	4	5
<b>EVA</b>					
Standard Paradigm	7	289	573	-80	1 135
Lost-capital paradigm	7	290	605	16	1 228
<b>EBO</b>					
Standard Paradigm	37	331	624	-21	1 318
Lost-capital paradigm	37	336	680	164	1 658

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**Table 10. Third-year sales equal to 8 000**

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year	1	2	3	4	5
<b>EVA</b>					
Standard Paradigm	9	292	-763	849	1 135
Lost-capital paradigm	9	293	-730	803	1 173
<b>EBO</b>					
Standard Paradigm	32	323	-727	946	1 318
Lost-capital paradigm	32	328	-673	894	1 480

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**Table 11. CSV in the two paradigms**

time	0	1	2	3	4	5
outstanding capital	9 800	9 488	7 038	4 296	1 230	0
lost equity capital	9 800	7 354	4 606	1 509	-2 017	-4 153
<i>Standard Paradigm</i>						
<b>CSV</b>		<b>2 134</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
MVA (=discount and sum)	1 879					
$E=E^{bv}+MVA$	11 679					
<i>Lost-capital Paradigm</i>						
<b>CSV</b>		<b>2 134</b>	<b>298</b>	<b>357</b>	<b>459</b>	<b>906</b>
MVA (=sum and discount)	1 879					
$E=E^{bv}+MVA$	11 679					

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**Table 12. Notational Conventions**

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$\vec{a}$	cash-flow (vector)
$a_t$	cash flow available at time $t$
$a$	project
$x_t, x$	(periodic) internal rate of return
$w_t(x)$	actual capital employed
$i$	opportunity cost of capital
$RI^S$	residual income in the standard paradigm
ROA, RONA, ROE	Return On Assets, Return On Net Assets, Return On Equity
$w_t(i)$	lost capital
$RI^L$	residual income in the lost-capital paradigm
$W_0$	investor's wealth at time 0
$W_t(i)$	investor's wealth at time $t$ in case of project rejection
$W_t(x, i)$	investor's wealth at time $t$ in case of project acceptance
NPV	Net Present Value
$A$	asset
$\vec{A}$	cash-flow (vector)
$s_n$	scrap value
$v_t$	asset $A$ 's accounting value at time $t$
$Dep_t$	depreciation charge
$r$	asset $A$ 's internal rate of return
$V_t$	market value of the project (firm)
$E_t$	equity (market value)
EVA	Economic Value Added
WACC	Weighted Average Cost of Capital
$V^{bv}$	total capital (book value)
FCF	Free Cash Flow
NOPAT	Net Operating Profit After Taxes

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(The Table is continued on the next page)

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**Table 12. (continued) Notational Conventions**

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$V_t$	total lost capital (equity+debt)
EBO	Edwards-Bell-Ohlson
$k_e$	cost of equity
$E^{bv}$	equity (book value)
ECF	Equity Cash Flow
PAT	Profit After Taxes
$\mathbb{E}_t$	lost equity capital
RI	Residual income
$\Delta$	variation
EI	expected RI improvement
CSV	Created Shareholder Value
MVA	Market Value Added
WCR	Working Capital Requirements
$k_D$	required return on debt (=debt rate)
EBIT	Earnings Before Interest and Taxes
PBT	Profit Before Taxes
D	debt (market value=book value)
T	corporate tax rate
$k_U$	required return on assets
$V_U$	value of the unlevered firm
PV[A; B]	$\sum_{t=1}^n \frac{A_t}{\prod_{k=1}^t (1+B_k)}$
DVTS	discounted value of tax shields

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