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ABSTRACT: This paper studies a Ricardian model of international trade with a continuum of products in a general equilibrium model in which firms engage in oligopolistic competition. It provides a bridge between trade models based on perfect competition and models based on imperfect competition. Compared with a model based on perfect competition, the incorporation of fixed cost leads to the result that an increase of domestic labor may increase the relative wage of the domestic country.

KEYWORDS: Comparative advantage, Ricardian model, oligopolistic competition, increasing returns to scale, trade policy

1. Introduction

For Ricardian models, the source of international trade is that countries have different technologies. In Ricardian models, the number of products is usually specified as an integer number. With an integer number of products, equilibrium specialization pattern may change discontinuously with exogenous parameters. In a pioneering paper, Dornbusch et al. (1977) have studied a Ricardian model with a continuum of products. With this specification of a continuum of products, the number of products produced in an economy becomes a continuous variable. As continuity is conducive to Riemann integration and differentiation, a time-honored result is that this assumption of a continuum of goods is very useful in conducting comparative static studies. It is not strange that the framework of Dornbusch et al. (1977) has been frequently used to generated useful insights. For some examples, the continuum assumption is used in Matsuyama (2002) and Cheng et al. (2005).

The existence of fixed costs of production is a salient feature of modern production (Chandler, 1990). First, it is commonly believed that long-run growth depends on technological advances which rely on research and development expenditure (Romer, 1990). Research and development expenditure are fixed costs. Second, machines and buildings are also fixed costs of production. Finally, there are also other types of fixed costs of production, such as advertising expenditure. With the existence of significant fixed costs, market structure will be featured by imperfect competition. In Dornbusch et al. (1977), the type of market structure is perfect competition. One issue not addressed in their paper is whether results derived under perfect
competition are robust to the alternative assumption of imperfect competition. Imperfect competition has been proposed by various scholars. Blanchard and Kiyotaki (1987) argue that imperfect competition is a better characterization of many industries than perfect competition. For example, the oil industry is dominated by a couple of colossal firms. Chandler (1990) also argues that many significant industries in the United States such as the steel industry has been characterized by imperfect competition since early twentieth century. Thus a Ricardian model incorporating imperfect competition is an interesting topic to study.

In this paper, we study international trade based on differences in technologies. One contribution of this paper lies in studying a Ricardian model of international trade in a general equilibrium framework in which firms engage in oligopolistic competition. There are two countries: home and foreign. Following Dornbusch et al. (1977), there is a continuum of products. To produce each product, a fixed cost and a marginal cost are needed. Countries may have different marginal costs in the production of each product. We show that the following results valid under perfect competition are robust under the alternative assumption of oligopolistic competition. First, a decrease of foreign productivity increases the range of products produced in the home country and increases the relative wage rate in the home country. Second, with positive transportation costs, a transfer from the foreign to the home country increases the relative wage rate of the home country. Finally, a small tariff imposed by a country increases the relative wage rate of this country.

We show that an increase of domestic population increases the range of products produced domestically. However, it may also increase domestic wage rate. This is different from the result based on perfect competition in which an increase of domestic population always decreases relative wage in the home country. The reason is that there is an additional effect in this model: a higher domestic labor force increases the degree of competition in the product market and decreases the average cost and make domestic firms relatively more productive. This higher productivity effect acts as a counter effect to the effect under perfect competition. If the higher productivity effect dominates, an increase of domestic population is accompanied by an increase of relative wage rate of the home country. We also show that a decrease of fixed costs across countries can also have income distribution effects.

The model’s incorporation of fixed costs of production leads to result fits nicely with reality. With perfect competition, a country’s relative wage rate is negatively related to its labor
endowment. Consistent with this model incorporating fixed costs of production, casual observation shows that there is no monotonic relationship between a country’s relative wage rate and its size. First, for Canada and US, the two countries are similar in terms of their geographical sizes and culture. The population in Canada is about one ninth of that of US. However, the wage rate in Canada is lower than that in the US. Second, China and India have the largest population in the world, but their wage rates are lower than the wage rate in the US.

This paper also contributes to the literature by providing a bridge between trade models based on perfect competition and models based on imperfect competition. With zero fixed costs, the model degenerates to a model of perfect competition as studied in Dornbusch et al. (1977). When countries have the same marginal costs in the production of all products, the model degenerates to a model of trade based on increasing returns to scale and oligopolistic competition, such as in Brander (1981), Markusen (1981), Horstmann and Markusen (1986), Lahiri and Ono (2004), and Zhou (2007a, 2007b).

The rest of the paper is organized as follows. First, we set up the model and establish equilibrium conditions. Second, we prove the existence of a unique equilibrium and conduct comparative static studies. Third, we incorporate transportation costs into the framework and address the impact of an international transfer. Fourth, we examine the impact of a tariff. Finally, we identify some avenues for future research and conclude.

2. The Model

There are two countries: home and foreign. Variables corresponding to the foreign country are denoted by asterisk marks. Foreign consumers are assumed to have the same preferences as domestic consumers. In this section, there is no transportation cost between the two countries. Markets in the two countries are integrated. For each product produced by firms, trade leads to equal price in the two countries.

The number of consumers in the home country is \( L \). Each consumer supplies one unit of labor inelastically. Labor is the only factor of production. It is assumed that there is a continuum of products indexed by a number \( z \in [0,1] \). To produce each product, there is a fixed cost and a marginal cost of production. In terms of labor units, all products have the same level of fixed cost \( f \). This fixed cost is the same in both countries. However, products may differ in terms of their marginal costs of production. For the same product, the two countries may have different marginal costs.
costs of production. For product \( z \), the marginal costs in terms of labor units in the two countries are denoted by \( \beta(z) \) and \( \beta^*(z) \) respectively. Products are indexed so that marginal costs are ranked in order of decreasing home country comparative advantage. Relative marginal cost between the two countries is defined by

\[
A(z) = \frac{\beta^*(z)}{\beta(z)}, \quad A'(z) < 0.
\]

In the above specification, like Dornbusch et al. (1977), it is assumed that \( A \) is a continuous function of \( z \).

A consumer’s consumption of product \( z \) is \( c(z) \). A consumer’s utility function is specified as \( \int_0^1 \ln c(z)dz \).\(^1\) As firms earn a profit of zero in equilibrium, the only source of income is labor income. The wage rate in the home country is \( w \) and the price level of product \( z \) is \( p(z) \). A consumer’s budget constraint is \( \int_0^1 p(z)c(z)dz = w \). A consumer takes the wage rate and prices of products as given and chooses the quantities of consumption to maximize utility. With this type of utility function, utility maximization requires that a consumer spends a fixed percentage of income on each product:

\[
p(z)c(z) = w. \quad (1)
\]

Similarly, for foreign consumers, utility maximization leads to

\[
p(z)c^*(z) = w*. \quad (2)
\]

Firms producing the same product are identical. Like Zhou (2004, 2007a, 2007b), firms producing the same product are assumed to engage in Cournot competition.\(^2\) The level of output for a firm producing product \( z \) is \( x(z) \). A firm’s total revenue is \( px \) and its total cost is \( (f + \beta x)w \). Thus a firm’s profit is \( px - (f + \beta x)w \). For each firm, it takes the wage rate as given and chooses the level of output to maximize its profit. Optimal choice of output requires that

---

\(^1\) Yu (2005) has studied a trade model with a general constant elasticity of substitution utility function. To address North-South trade, Flam and Helpman (1987) and Matsuyama (2002) have studied trade models in which preferences are nonhomothetic.

\(^2\) For some recent models of international trade based on monopolistic competition, see Schmitt and Yu (2001), Yu (2005), and Ederington and McCalman (2008).
\[ p + x \frac{\partial p}{\partial x} = \beta w. \]
The number of firms producing product \( z \) is \( m(z) \).

Combination of results from a consumer’s utility maximization with a firm’s optimal choice of output leads to

\[ p(z) \left(1 - \frac{1}{m(z)}\right) = \beta w. \]

(3)

Equation (3) shows that a firm’s price is a markup over its marginal cost of production \( \beta w \). The markup factor decreases with the number of firms producing the same product.

Similarly, for products produced in the foreign country, the relationship between price and marginal cost is

\[ p^*(z) \left(1 - \frac{1}{m^*(z)}\right) = \beta^* w^*. \]

(4)

Free entry and exit lead to zero profit for each firm. The zero-profit condition for a domestic firm requires that

\[ p(z)x(z) - [f + \beta x(z)]w = 0. \]

(5)

Similarly, zero profit for a foreign firm requires that

\[ p^*(z)x^*(z) - [f + \beta^* x^*(z)]w^* = 0. \]

(6)

Each country specializes in the production of products it has comparative advantage. Let \( z_b \) denote the borderline product for which the price in the two countries is equal. The home country will specialize in the range of products \( 0 \leq z \leq z_b \) and the foreign country will specialize in the range of products \( z_b \leq z \leq 1 \). This borderline product is defined by

\[ p(z_b) = p^*(z_b). \]

(7)

For each product with \( z \in [0, z_b] \), the supply of this product is \( m(z)x(z) \). The demand for this product is the sum of domestic demand \( Lc \) and foreign demand \( L^* c^* \), which is equal to \( Lc + L^* c^* \). Clearance of product market requires that supply equals demand for domestically produced goods:

\[ m(z)x(z) = Lc(z) + L^* c^*(z), \text{ for } z \in [0, z_b]. \]

(8)

---

\(^3\) In this paper, the number of firms is a real number rather than restricted to be an integer number. The number of firms is determined by the zero-profit condition. Sections 3.7 and 4.5 of Brander (1995) provide survey of models of oligopolistic competition with fixed costs and free entry.

\(^4\) Zhou (2004, 2007a, 2007b) has a detailed illustration of the derivation of this type of formula.
Product market equilibrium for foreign produced goods requires that supply equals demand for foreign produced goods:

$$m^*(z)x^*(z) = Lc(z) + L^*c^*(z), \text{ for } z \in [z_h, 1]$$  \hspace{1cm} (9)

In the home country, the total demand for labor is $$\int_0^{z_h} m(f + \beta x)dz$$. The total supply of labor is equal to $$L$$. Labor market equilibrium in the home country requires that labor demand equals labor supply:

$$\int_0^{z_h} m(f + \beta x)dz = L.$$  \hspace{1cm} (10)

Similarly, labor market equilibrium in the foreign country requires that

$$\int_{z_h}^1 m^*(f + \beta^* x^*)dz = L^*.$$  \hspace{1cm} (11)

Equations (1)-(11) form a system of 11 equations defining 11 variables $$w, w^*, c, c^*, m, m^*, x, x^*, p, p^*, \text{ and } z_h$$. An equilibrium is a tuple $$(w, w^*, c, c^*, m, m^*, x, x^*, p, p^*, z_h)$$ satisfying equations (1)-(11).

3. Comparative Statics

In this section, we first establish the existence of a unique equilibrium and then perform comparative static studies to explore properties of the equilibrium.

By plugging the value of $$m$$ from (3) and the value of $$x$$ from (5) into (10), we get the following equation:

$$wL = z_h(wL + w^*L^*).$$  \hspace{1cm} (12)

Equation (12) can also be interpreted as that equilibrium in the market for products produced in the home country requires that domestic labor income $$wL$$ equals world spending on domestically produced goods.

From equations (1), (3), (5), (7), and (8), the price of a product is

$$p(z) = \frac{\beta(z)w}{1 - \sqrt{fw}}.$$  \hspace{1cm} (13)

5 Strictly speaking, $$w, w^*, c, c^*, m, m^*, x, x^*, p, \text{ and } p^*$$ are functions of $$z$$.
Equation (13) shows that the price of a product increases monotonically with its marginal cost of production.

Let the wage rate in the foreign country be normalized to one: \( w^* \equiv 1 \). With this normalization, the domestic wage rate also denotes the wage ratio between the two countries.

Rearrangement of equation (12) leads to
\[
\Lambda_1 \equiv (1 - z_b) w L - z_b L^* = 0 .
\] (14)

Equations (7) and (13) lead to
\[
\Lambda_2 \equiv 1 - \sqrt{\frac{f}{wL + L^*}} - \frac{A(z_b)}{w} \left(1 - \sqrt{\frac{fw}{wL + L^*}}\right) = 0 .
\] (15)

The effect of a uniform change of the technology of the foreign country can be captured by a change of \( A \). This type of change may be caused by the adoption of a general-purpose technology in the foreign country. A decrease of \( A \) means a uniform proportional reduction of foreign marginal cost. Equations (14) and (15) form a system of two equations defining \( w \) and \( z_b \) as functions of exogenous parameters. When the level of fixed cost \( f \) equals zero, this system degenerates to the case of perfect competition, as studied in Dornbusch et al. (1977). From equation (14), \( \partial \Lambda_1 / \partial z_b < 0 \), and \( \partial \Lambda_1 / \partial w > 0 \). Thus, \( \Lambda_1 \) shows a positive relationship between \( z_b \) and \( w \). From equation (15), \( \partial \Lambda_2 / \partial z_b > 0 \), and \( \partial \Lambda_2 / \partial w > 0 \). Thus, \( \Lambda_2 \) shows a negative relationship between \( z_b \) and \( w \).

The following proposition establishes the existence and uniqueness of an equilibrium.

**Proposition 1** There exists a unique equilibrium.

Proof: Existence: Plug the value of \( w \) from (14) into (15) leads to the following equation defining \( z_b \) implicitly:
\[
\Lambda \equiv 1 - \sqrt{\frac{(1 - z_b)f}{L^*}} - \frac{A(z_b)(1 - z_b)L}{z_bL^*} \left(1 - \sqrt{\frac{fz_b}{L}}\right) = 0 .
\]

For \( z_b = 0 \), \( \Lambda < 0 \). For \( z_b = 1 \), \( \Lambda > 0 \). Since \( \Lambda \) is a continuous function of \( z_b \), there exists at least one value of \( z_b \) such that \( \Lambda = 0 \).

Uniqueness: Since \( \Lambda_1 \) depicts a positive relationship and \( \Lambda_2 \) depicts a negative relationship between \( z_b \) and \( w \), \( \Lambda_1 \) and \( \Lambda_2 \) will intersect only once. ■
We now explore the properties of the equilibrium. Total differentiation of $\Lambda_1$ and $\Lambda_2$ with respect to $z_b$, $w$, $f$, $L$, and $A$ leads to

$$
\begin{align*}
\left(\begin{array}{c}
\frac{\partial \Lambda_1}{\partial z_b} & \frac{\partial \Lambda_1}{\partial w} \\
\frac{\partial \Lambda_2}{\partial z_b} & \frac{\partial \Lambda_2}{\partial w}
\end{array}\right)
\begin{pmatrix} dz_b \\ dw \end{pmatrix}
&= 
\begin{pmatrix} 0 \\ -\frac{\partial \Lambda_2}{\partial f} \end{pmatrix} df + 
\begin{pmatrix} -\frac{\partial \Lambda_1}{\partial L} \\ -\frac{\partial \Lambda_2}{\partial L} \end{pmatrix} dL + 
\begin{pmatrix} 0 \\ -\frac{\partial \Lambda_2}{\partial A} \end{pmatrix} dA.
\end{align*}
$$

(16)

Let $\Delta$ denote the determinant of the coefficient matrix. Since $\partial \Lambda_1 / \partial z_b < 0$, $\partial \Lambda_1 / \partial w > 0$, $\partial \Lambda_2 / \partial z_b > 0$, and $\partial \Lambda_2 / \partial w > 0$, it is clear that $\Delta < 0$.

The following proposition studies the impact of a uniform change of foreign productivity.

**Proposition 2** A decrease in foreign productivity increases the range of products produced in the home country and increases the relative wage of the home country.

**Proof:** Application of Cramer’s rule to (16) leads to

$$
\begin{align*}
\frac{dz_b}{dA} &= \frac{\partial \Lambda_1}{\partial L} \frac{\partial \Lambda_2}{\partial w} / \Delta > 0, \\
\frac{dw}{dA} &= -\frac{\partial \Lambda_1}{\partial L} \frac{\partial \Lambda_2}{\partial z_b} / \Delta > 0.
\end{align*}
$$

Results in Proposition 2 are consistent with intuition. A higher marginal cost in the foreign country means a lower productivity in the foreign country. As a result, the range of products produced by the foreign country decreases and the relative wage of the foreign country decreases.

**Proposition 3** $\frac{dz_b}{df} < 0$ and $\frac{dw}{df} < 0$ if and only if $A(z_b) > \sqrt{w}$.

**Proof:** From (15), it can be shown that $\partial \Lambda_2 / \partial f > 0$ if and only if $A(z_b) > \sqrt{w}$.

Application of Cramer’s rule to (16) leads to

$$
\begin{align*}
\frac{dz_b}{df} &= \frac{\partial \Lambda_1}{\partial w} \frac{\partial \Lambda_2}{\partial f} / \Delta, \text{ and } \\
\frac{dw}{df} &= -\frac{\partial \Lambda_1}{\partial z_b} \frac{\partial \Lambda_2}{\partial f} / \Delta.
\end{align*}
$$

Thus

$$
\begin{align*}
\frac{dz_b}{df} < 0 \text{ and } \frac{dw}{df} < 0 \text{ if and only if } A(z_b) > \sqrt{w}.
\end{align*}
$$


The intuition behind Proposition 3 is as follows. A decrease of fixed cost decreases price of products produced in both countries. If $A(z_b) > \sqrt{w}$, the foreign country’s wage rate is relatively high. A decrease of fixed cost has a larger impact on the home country. As a result, the range of products produced in the home country and the wage rate increases.

The following proposition studies the impact of a change of domestic population.

**Proposition 4** (i) $dz_b / dL > 0$; (ii) A sufficient condition for $dw / dL < 0$ is that $\sqrt{w} > A(z_b)$.

Proof: Application of Cramer’s rule to (16) leads to

$$\frac{dz_b}{dL} = \left( \frac{\partial A_1}{\partial w} \frac{\partial A_2}{\partial L} - \frac{\partial A_1}{\partial L} \frac{\partial A_2}{\partial w} \right) / \Delta > 0,$$

$$\frac{dw}{dL} = \left( \frac{\partial A_1}{\partial L} \frac{\partial A_2}{\partial z_b} - \frac{\partial A_1}{\partial z_b} \frac{\partial A_2}{\partial L} \right) / \Delta.$$

Partial differentiation of (15) yields

$$\frac{dA_2}{dL} = \frac{1}{2} f^{1/2} (wL + L^*)^{-3/2} w \left( 1 - A(z_b) / \sqrt{w} \right).$$

From the above two equations, a sufficient condition for $dw / dL < 0$ is that $\sqrt{w} > A(z_b)$. ■

Proposition 4 shows that an increase of the domestic population increases the range of products produced by the home country. This result is consistent with Dornbusch et al. (1977). Proposition 4 also shows that an increase of domestic population may increase the relative wage rate of the home country. This is different from Dornbusch et al. (1977) in which a perfect competition framework is used. In Dornbusch et al. (1977), with an increase of labor force in the home country, at the initial wage rate, the home country will have a trade deficit. A decrease of wage rate will eliminate this trade deficit. Under perfect competition, an increase of domestic labor always decreases the relative wage rate in the home country. In this paper, there is one additional effect. Under oligopolistic competition, a firm’s price is a markup over its marginal cost. The markup factor depends on the degree of competition in the product market. The higher the degree of competition, the lower the degree of markup. With an increase of domestic labor
force, more firms will produce the same product in the home country. This decreases the price. As firms earn profits of zero, the price is equal to the average cost of production. If the decrease of average cost is sufficiently strong, the real wage rate in the home country increases with the domestic labor force. A necessary condition for this is that the wage ratio is less than the relative productivity at the cutoff level of product.

When countries have the same marginal costs in the production of all products, the model degenerates to models of trade based on increasing returns to scale and oligopolistic competition, such as Brander (1981), Markusen (1981), Horstmann and Markusen (1986), Lahiri and Ono (2004), and Zhou (2007a, 2007b). Like models based on increasing returns, as countries have the same technology and labor is the only factor of production, which country exports which product is undetermined.

In the case of perfect competition as in Dornbusch et al. (1977), the only source of the gains from trade is that each country specializes in the production of products with comparative advantage. In this model, there is an additional source of the gains from trade. With trade, the size of the market is the world market which is larger than each national market. As shown in Brander (1981) and Markusen (1981), with imperfect competition, the opening of trade leads to a lower level of monopoly distortion and a lower price. Thus the opening of trade is always beneficial.

4. Transportation Cost and Impact of An International Transfer

In this section, transportation costs are incorporated into the model and the impact of an international transfer is studied. As discussed in Caves et al. (2007, Chapter 4), the transfer problem is relevant in various situations, such as war reparations and debt relief.

Samuelson (1952, 1954) has studied the impact of a transfer on the paying country’s terms of trade when there are two products and the market structure is perfect competition. Following Samuelson (1954), transportation cost is specified as the iceberg type: for each unit of product sent out, only $g$ percent arrives, and $0 < g \leq 1$. This percentage is assumed to be the same for all products. The incorporation of transportation costs leads to the existence of nontraded goods. For goods in the range $[0, z_h]$, they are produced by the home country only. For goods in the range $(z_h, z_f)$, they are nontraded and produced by both countries. For goods in the range $[z_f, 1]$, they are produced by the foreign country only.
For goods in the range \([0, z_h]\) produced in the home country, product market equilibrium requires that supply equals demand:

\[
x_h = Lc + \frac{1}{g} L^*c^*.
\]                      (17)

For goods in the range \([z_f, 1]\) produced in the foreign country, product market equilibrium requires that supply equals demand:

\[
x_f^* = \frac{1}{g} Lc + L^*c^*.
\]                      (18)

Let the level of transfer from the foreign country to the home country in terms of labor units in the foreign country be denoted by \(T\). A domestic resident’s disposable income is the sum of the wage rate and the transfer. Each consumer in the home country receives a transfer of \(T/L\). The budget constraint for a domestic resident is

\[
\int_0^1 p c dz = w + \frac{T}{L}.
\]                      (19)

A foreign resident’s disposable income is the difference between the foreign wage rate and the transfer. The budget constraint for a foreign resident is

\[
\int_0^1 p^*c^* dz = w^* - \frac{T}{L^*}.
\]                      (20)

For product \(z_h\), the price of this product in the home country plus the transportation cost equals its price in the foreign country:

\[
g\ \ p(z_h) = p^*(z_h).
\]                      (21)

For product \(z_f\), the price of this product in the foreign country plus the transportation cost equals its price in the home country:

\[
p(z_f) = g\ p^*(z_f).
\]                      (22)

Labor market equilibrium in the home country requires labor demand equals labor supply:

\[
\int_0^{z_h} m(f + \beta x)dx + \int_{z_h}^{z_f} m(f + \beta x)dx = L.
\]                      (23)

Labor market equilibrium in the foreign country requires labor demand equals labor supply:

\[
\int_{z_h}^{z_f} m(f + \beta x)dx + \int_{z_f}^1 m^*(f + \beta^* x^*)dx = L^*.
\]                      (24)
With transportation costs, equations (3)-(6) are still valid. Together with equations (17)-(24), they form a system of twelve equations defining twelve variables \( w, w^*, c, c^*, m, m^*, x, x^*, p, p^*, z_h, \) and \( z_f \). This system of equations can be simplified to the following system of equations:

\[
\Phi_1 \equiv (wL + T)(1 - z_f) - (L^* - T)z_h = 0, \quad (25)
\]

\[
\Phi_2 \equiv 1 - \frac{f}{wL + L^*} \left(1 - \frac{fw}{g w} \right) = 0, \quad (26)
\]

\[
\Phi_3 \equiv 1 - \frac{f}{wL + L^*} \left(1 - \frac{fw}{g w} \right) = 0. \quad (27)
\]

When there is no transportation cost \( (g = 1), z_h = z_f \). In this case, the system of equations (25)-(27) degenerates to the system of equations (14)-(15). Since the level of transfer \( T \) drops out of the system when there is no transportation cost, we come to the well-known result (Samuelson, 1952; Dorbusch et al., 1977) that the relative wage rate is unaffected by the level of transfer when transportation cost is zero. The following proposition studies the impact of a transfer when the transportation cost is positive.

**Proposition 5** If transportation cost is positive, a transfer from the foreign country to the home country increases the relative wage of the home country, decreases the range of products exported from the home country and increases the range of products exported from the foreign country.

**Proof:** Differentiation of the system of equations (25)-(27) leads to

\[
\begin{bmatrix}
\frac{\partial \Phi_1}{\partial z_h} & \frac{\partial \Phi_1}{\partial z_f} & \frac{\partial \Phi_1}{\partial w} \\
\frac{\partial \Phi_2}{\partial z_h} & 0 & \frac{\partial \Phi_2}{\partial w} \\
0 & \frac{\partial \Phi_3}{\partial z_f} & \frac{\partial \Phi_3}{\partial w}
\end{bmatrix}
\begin{bmatrix}
dz_h \\
dz_f \\
dw
\end{bmatrix}
=
\begin{bmatrix}
-\frac{\partial \Phi_1}{\partial T} \\
0 \\
0
\end{bmatrix}
dT.
\]

\[\text{6 The derivation of (25) is similar to that of (14).}\]
Let $\Delta_\Phi$ denote the determinant of the coefficient matrix. From (25)-(27), it can be shown that $\Delta_\Phi > 0$. Also, it can be shown that $\frac{\partial \Phi_1}{\partial T} > 0$, $\frac{\partial \Phi_2}{\partial z_h} > 0$, $\frac{\partial \Phi_2}{\partial w} > 0$, $\frac{\partial \Phi_3}{\partial z_f} > 0$, and $\frac{\partial \Phi_3}{\partial w} > 0$.

Application of Cramer’s rule leads to

$$\frac{dw}{dT} = \left( \frac{\partial \Phi_1}{\partial T} \right) / \Delta_\Phi > 0,$$

$$\frac{dz_h}{dT} = \left( \frac{\partial \Phi_1}{\partial T} \frac{\partial \Phi_2}{\partial w} \right) / \Delta_\Phi < 0,$$

$$\frac{dz_f}{dT} = \left( \frac{\partial \Phi_1}{\partial T} \frac{\partial \Phi_3}{\partial w} \right) / \Delta_\Phi < 0.$$

The intuition behind Proposition 5 is similar to that under perfect competition, as illustrated in Dornbusch et al. (1977) and Rogoff and Obstfeld (1996: chapter 4). With a transfer from foreign to home, the demand for nontraded goods in the home country increases. This leads to an increase of prices and the wage rate in the home country. The increase of the wage rate in the home country decreases the competitiveness of products produced in the home country. As a result, the range of products exported by the home country shrinks and the range of products exported by the foreign country expands.

5. Impact of a Tariff

In this section, the impact of a tariff is studied. The level of transportation cost is set to zero in this section. Suppose the home country charges a uniform tariff over all products it imports. It is assumed that the tariff revenue is distributed to domestic consumers in a lump-sum form. The tariff rate in the foreign country is equal to zero.

The tariff rate is $t$. With the existence of tariff, some products will be produced in both countries. Similar to the previous section, for goods in the range $[0, z_h]$, they are produced by the home country only. For goods in the range $(z_h, z_f)$, they are nontraded and produced by both countries. For goods in the range $[z_f, 1]$, they are produced by the foreign country only.

For product $z_h$, the prices in the two countries should be the same:
\[ p(z_h) = p^*(z_h). \] (28)

For product \( z_f \), the price in the foreign country plus the tariff equal the price in the home country:
\[ p(z_f) = (1 + t)p^*(z_f). \] (29)

A domestic consumer’s income is the sum of the wage income and the tariff revenue. A domestic consumer’s budget constraint is
\[ \int_0^1 pcdz = w + \int_{z_f}^1 tp^*cdz. \] (30)

For goods in the range \([0, z_h]\), domestic firms produce for both domestic and foreign consumers. Product market equilibrium in the world market requires that
\[ m_x = Lc + L^*c^*. \] (31)

For goods in the \([z_f, 1]\), foreign firms produce for both domestic and foreign consumers. Product market equilibrium in the world market requires that
\[ m^*x^* = Lc + L^*c^*. \] (32)

With the existence of tariff, equations (2), (3), (4), (5), (6), (23), and (24) are still valid. Together with equations (28)-(32), they form a system of twelve equations defining twelve variables \( w, w^*, c, c^*, m, m^*, x, x^*, p, p^*, z_h, \) and \( z_f \). Like Section 4, the above system of equations can be simplified to the following system of equations:

\[ \Gamma_1 \equiv \frac{(1 - z_f)wL}{1 + z_f t} - L^*z_h = 0, \] (33)

\[ \Gamma_2 \equiv 1 - \frac{f(1 + z_f t)}{wL + L^*(1 + z_f t)} - \frac{A(z_h)}{w} \left(1 - \frac{f(1 + z_f t)w}{(1 + t)wL + L^*(1 + z_f t)}\right) = 0, \] (34)

\[ \Gamma_3 \equiv 1 - \frac{f(1 + z_f t)}{wL + L^*(1 + z_f t)} - \frac{(1 + t)A(z_f)}{w} \left(1 - \frac{f(1 + z_f t)w}{(1 + t)wL + L^*(1 + z_f t)}\right) = 0. \] (35)

Equations (33)-(35) form a system of three equations defining \( z_h, z_f, \) and \( w \) as functions of exogenous variables. When the level of tariff \( t = 0 \), \( z_h = z_f \). In this case, the system of
equations (33)-(35) degenerates to the system of equations (14)-(15). The following proposition studies the impact of a small tariff imposed by the home country.\textsuperscript{7}

**Proposition 6** When the initial tariff rate is zero, a small increase of tariff by the home country increases the relative wage rate of the home country.\textsuperscript{8}

Proof: Differentiation of equations (33)-(35) with respect to $z_h$, $z_f$, $w$, and $t$ leads to

$$
\begin{pmatrix}
\frac{\partial \Gamma_1}{\partial z_h} & \frac{\partial \Gamma_1}{\partial z_f} & \frac{\partial \Gamma_1}{\partial w} \\
\frac{\partial \Gamma_2}{\partial z_h} & \frac{\partial \Gamma_2}{\partial z_f} & \frac{\partial \Gamma_2}{\partial w} \\
0 & \frac{\partial \Gamma_3}{\partial z_f} & \frac{\partial \Gamma_3}{\partial w}
\end{pmatrix}
\begin{pmatrix}
dz_h \\
dz_f \\
dw
\end{pmatrix}
= 
\begin{pmatrix}
-\frac{\partial \Gamma_1}{\partial t} \\
-\frac{\partial \Gamma_2}{\partial t} \\
-\frac{\partial \Gamma_3}{\partial t}
\end{pmatrix}
dt.
$$

Let $\Delta_r$ denote the determinant of the above coefficient matrix. For $t = 0$, it can be shown that $\frac{\partial \Gamma_2}{\partial z_f} = 0$, and $\frac{\partial \Gamma_3}{\partial z_f} > 0$. Thus $\Delta_r > 0$ when $t = 0$. For $t = 0$, application of Cramer’s rule leads to

$$
\frac{dw}{dt} = \left(\frac{\partial \Gamma_1}{\partial z_h} \frac{\partial \Gamma_2}{\partial z_f} \frac{\partial \Gamma_3}{\partial t} + \frac{\partial \Gamma_1}{\partial z_f} \frac{\partial \Gamma_2}{\partial z_h} \frac{\partial \Gamma_3}{\partial t} - \frac{\partial \Gamma_1}{\partial t} \frac{\partial \Gamma_2}{\partial z_h} \frac{\partial \Gamma_3}{\partial z_f}\right)/\Delta_r.
$$

For $t = 0$, the nominator of the right-hand side is positive. Thus $dw/dt > 0$ when $t = 0$.\textsuperscript{9}

The intuition behind Proposition 6 is similar to the familiar terms of trade effect of a small tariff as illustrated in Feenstra (2004: chapter 7). A tariff by the home country decreases the demand for foreign produced goods. The reduced demand for foreign goods leads to a decrease of export prices of foreign produced goods. This again leads to a decrease of foreign wage. As a result, the relative wage of the home country increases.

\textsuperscript{7} If the foreign country also imposes a tariff, the welfare for each country may be lower than that when each country engages in free trade. As shown by Johnson (1953), a country may gain from a tariff in a Nash equilibrium in which both countries set tariffs optimally.

\textsuperscript{8} Like the proof of Proposition 6, it can be shown that the impact of a tariff on the range of domestically exported and foreign exported goods is ambiguous.
6. Conclusion

In this paper, we have studied a Ricardian model of international trade in a general equilibrium framework in which firms engage in oligopolistic competition and there is a continuum of products. The model provides a bridge between perfect competition and imperfect competition. First, when the level of fixed cost is zero, the model degenerates to the perfect competition case. Second, when countries have the same marginal costs in the production of all products, the model degenerates to a model of trade based on increasing returns to scale and oligopolistic competition. We show that the following results valid under perfect competition are robust under the alternative assumption of oligopolistic competition. First, a decrease of foreign productivity increases the range of products produced in the home country and increases the relative wage in the home country. Second, with positive transportation costs, a transfer from the foreign to the home country increases the relative wage rate of the home country. Finally, a small tariff imposed by a country increases the relative wage rate of this country. Different from the case of perfect competition, we show that an increase of domestic population can increase domestic wage rate. We also show that a decrease of fixed costs across countries can also have income distribution effects.

There are some interesting avenues for future research. First, in this model, preference is homothetic. To study North-South trade between countries with different income levels, nonhomothetic preferences may be incorporated into the model. Second, in this model, technologies are exogenously given. To address the endogenous determination of technologies, the model may be extended to a dynamic framework in which technologies are results of research and development expenditure. Finally, in this model, various are real. To study the impact of monetary policy and related issues, money and nominal rigidities may be incorporated into the model.

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References


