A Comment on "Transparency, Complementarity and Holdout"

Maurya, Amit Kumar

15 October 2018

A Comment on "Transparency, Complementarity and Holdout"

Amit Kumar Maurya*

October 15, 2018

Roy Chowdhury and Sengupta(2012) study a model of multilateral bargaining involving one buyer and $n \geq 2$ sellers. In odd periods buyer makes offers to all the sellers simultaneously, while in even periods all the sellers demand a price. If any of the seller accepts buyer's offer or if the buyer accepts any seller's demand, then the buyer pays him immediately and the seller leaves the game. Rest of the players continue the game in the same fashion. Authors show that when the buyer has no outside option, his maximum payoff is $\frac{1-\delta}{1+\delta}$, which approaches to zero as δ reaches 1(Roy Chowdhury and Sengupta 2012, Proposition 1). Note that the δ is the discount factor of the players. However, if the buyer has a positive outside option, however small, his payoff is $\frac{\delta}{1+\delta}$, which is almost half of the surplus when δ is very close to 1(Roy Chowdhury and Sengupta 2012, Proposition 2). **This drastic change in the maximum possible payoff to the buyer captures the impact of outside option on the same. This is one of the key results of the paper.**

The paper assumes common discount factor i.e. all player discount at equal rate. One main reason for employing such assumption is the ensuing mathematical comfort. We make following minor change in the model. Buyer discounts at the rate of $\delta_b \in (0,1)$ per period, while all the sellers discount at the rate of $\delta_s \in (0,1)$ per period, and $\delta_b \neq \delta_s$. **We show that under limiting conditions i.e.** $\lim \delta_b, \delta_s \to 1$**, even infinitesimal difference between** δ_b **and** δ_s**, can render an outside option ineffective in improving the buyer's payoff.** This result contradicts the key result of Roy Chowdhury and Sengupta(2012) described above.

Consider the Condition 1, which basically states that buyer's discount factor is more than that of the sellers by an amount which diminishes as players become more patient.

Condition 1: $\delta_b - \delta_s \geq \epsilon_k$, where $\epsilon_k = \frac{n\delta_s - n\delta_s^2 + k\delta_s^2 - k}{(n-k)\delta_s}$ and $k \in (0,1)$ is an arbitrarily chosen number.

*IGIDR, Goregaon(E), Mumbai, India - 400065; Email: amit@igidr.ac.in; Mobile: +91-98205 43796

Note that $\epsilon_k < 1 - \delta_s \ \forall \ \delta_s \in (0,1)$. This ensures that δ_b takes values in the range $(0,1)$ for all values of δ_s. Also, for sufficiently large n, $\frac{\partial \epsilon_k}{\partial \delta_s} < 0$. As δ_s approaches to 1, ϵ_k approaches to 0, and δ_b approaches 1 as well.

Proposition 1. *Under condition 1 and no outside option for the players, there exists an equilibrium such that the game ends in the first period and the buyer receives at least 1-k.*

Proof. Following equilibrium strategy profile proves the result.

For $n = 1$, the game is same as Rubinstein(1982), hence, Rubinstein(1982) strategies are followed. For $n > 1$, following strategies are followed. In odd periods, the buyer offers p to each of the sellers, and a seller rejects any offer below p. In even periods each seller demands 1, and the buyer rejects any demand more than $\delta_s p$. And, $p = \delta_s \frac{1-\delta_b}{1-\delta_b\delta_s}$.

As per the above equilibrium, all the sellers accept the offer of p in the first period. If any seller deviates and rejects the offer, he obtains the Rubinstein(1982) payoff, $\frac{1-\delta_b}{1-\delta_b\delta_s}$, in the next period. Given the value of p, this deviation is not profitable.

□

Since k is arbitrarily chosen, there exists an equilibrium such that the buyer obtains almost entire surplus even when he does not have any outside option. For example, let us fix $k = 0.001$. There exists an equilibrium such that the buyer obtains $1 - k = 0.999$. When $\lim \delta_s \to 1$, $\epsilon_{0.001} \to 0$, and hence $\delta_b - \delta_s \to 0^+$. Clearly, an outside option will be more or less ineffective in improving buyer's payoff. **Under limiting conditions, even an infinitesimal difference in the discouning factors of the buyer and the sellers is enough to render an outside option ineffective**. Hence, the result of Roy Chowdhury and Sengupta(2012) is extremely sensitive to even infinitesimal difference in the discount factors of the buyer and sellers under the limiting conditions.

References

[1] Roy Chowdhury, P. and Sengupta, K. (2012), "Transparency, Complementarity and Holdout", Games and Economic Behavior, 75, 598-612.
[2] Rubinstein, A. (1982), "Perfect Equilibrium in a Bargaining Model", Econometrica, 50, 97-109.