Bargaining order in multilateral bargaining with imperfect compliments

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Abstract

Using a sequential model of multilateral bargaining involving one buyer and two sellers, who are selling objects which are imperfect compliments for the buyer, we analyse buyer's preferred bargaining order i.e. whether the buyer prefers to buy higher valuation object first or second. For a narrow range of parameters, where players are patient enough and objects exhibit high degree of complimentarity, multiple equilibria exist such that both the bargaining orders are preferred. For rest of the range, which is relatively much larger, buyer prefers to buy higher valuation object first.

Key words: Multilateral bargaining; Bargaining order; Imperfect compliments

JEL codes: C72, C78

1 Introduction

We analyse a model of multilateral bargaining in which one buyer negotiates with two sellers, each of whom own an unit object. These objects are imperfect complements ie each object is valuable to the buyer, however owning both the objects is the most desirable. We consider a general case in which one object is more valuable to the buyer than the other. Using a sequential bargaining model we attempt to know which bargaining order buyer prefers ie would he like to buy from the seller holding more valuable object first or from the seller holding low value object first. We characterize the conditions under which buyer prefers one order over the other. For most of the range of parameters, buyer prefers to buy higher valuation object first. However, for a narrow range of parameters, multiple equilibria exist such that some support one order, while the rest support the other order. Our main results are captured in the propositions 6 and 7.

While most of the literature on multilateral bargaining concerns with inefficiency and multiplicity of equilibria[1], Xiao(2010), Shubhro(2016), and Krastiv and Yildrim (2009) study bargaining order. Using different models, Xiao(2010) and Shubhro(2016) characterize the conditions under which buyer prefers one seller over another. Note that the bargaining order in these papers is based on the sellers' valuation of their objects, while in ours it is based on the buyer's valuation of the objects. Hence, in our model the objects cannot be perfect compliments, while they are in Xiao(2010) and Shubhro(2016). Krastiv and Yildrim (2009) study buyer's preferred bargaining order based on the bargaining power of the sellers which is measured as their chance of being the proposer in a one shot bargaining with the buyer. Key features of our model are sequential bargaining of infinite periods and binding offers. Both these features are quiet the norm in multilateral bargaining literature.[2] In next section we describe the model, which we analyse in section 3. We conclude in the section 4.

2 Model

One buyer negotiates with two sellers, denoted by s_1 and s_2, to buy each of the objects owned by them. Both objects are imperfect complements. Buyer's utility from owning the objects is the following. v(1)=u $<$ 1, v(2)=0 and v(1,2)=1, where v(1), v(2) and v(1,2) are buyer's utility when

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[1] See Cai(2000, 2003), Krishna and Serrano(1996), Roy Chaudhury and Sengupta(2012).

[2] See Cai(2000, 2003), Roy Chaudhury and Sengupta(2012), Xiao(2010) and Shubhro(2016).

he owns s_1's object, s_2's object and both the objects respectively. The bargaining game consists of infinite rounds. Each round has a maximum of two periods, in first of which the buyer makes an offer to a seller and the seller responds with acceptance or rejection of the same. In the second period of the round, the seller demands a price (counter-offer) and the buyer responds. The buyer switches to the other seller in the next round. Hence, the game may possibly continue for infinite rounds (and periods). If an offer or counter-offer is accepted, the buyer makes the payment to the respective seller, who in turn leaves the game, and the game enters the next round. Game ends when the buyer buys both the objects. All players are risk neutral and discount at the rate of $\delta \in (0,1)$ per period. We use Markov Subgame Perfect Nash Equilibrium, henceforth equilibrium, as the solution concept.

3 Analysis

Following two lemmas are the minor results.

Lemma 1. *Perpetual disagreement is not possible in equilibrium.*

Proof. See Cai (2000, lemma 4). □

Lemma 2. *In every equilibrium, the buyer buys both the objects in consecutive periods.*

Proof. Consider an equilibrium. The buyer must acquire an object, say in period t. This follows from the lemma 1. The subsequent subgame is a Rubinstein(1982) game with immediate outcome. Hence, the second object is obtained by the buyer in the period t+1. □

We now analyse the game in which the buyer negotiates with the seller 1 in the first period. Following five propositions characterize all the equilibria of the game. The proofs follow same methodology as employed in Shubhro(2016), from which we borrow the following notations as well.

An equilibrium profile is described by 8 parameters- $o_1, \hat{o_1}, co_1, c\hat{o}_1, o_2, \hat{o_2}, co_2,$ **and** $c\hat{o}_2$. Buyer offers o_1 to seller 1 in the first period. $\hat{o_1}$ is the minimum offer from the buyer which the seller 1 is willing to accept. In the second period, seller 1 counter-offers co_1 to himself. The buyer rejects any counter-offer more than $c\hat{o}_1$ from the seller 1. Similarly, in third period, buyer offers o_2 to seller 2, who accepts any offer at least as much as $\hat{o_2}$. The seller 2 counter-offers co_2 to himself in the fourth period. The buyer rejects any counter-offer more than $c\hat{o}_2$ from the seller 2.

Equilibrium payoff is denoted by $\{p_b, p_1, p_2, t\}$, where the first three terms are the payoffs to the buyer, the sellers s_1 and s_2 respectively as seen from period t. We chose the period of the first purchase as t. Note that the counting of the periods start with 1 (not 0).

Proposition 1. *For* $u(\delta^5+\delta^4+\delta^3+\delta^2-1) \geq \delta^5+\delta^4+\delta^3-1$*, following strategy profile constitutes an equilibrium. We denote this equilibrium by* E_1*.*

$$
\begin{array}{ll}
o_1 = \hat{o_1} & o_2 = \hat{o_2} \\
\hat{o_1} = \delta c\hat{o}_1 & \hat{o_2} = \delta c\hat{o}_2 \\
co_1 = c\hat{o}_1 & co_2 = c\hat{o}_2 \\
 & \\
c\hat{o}_1 = \frac{u(1+\delta+\delta^2)+\delta(1+\delta^2)}{(1+\delta)^2(1+\delta^2)} & c\hat{o}_2 = \frac{\delta+\delta^3-\delta u}{(1+\delta)^2(1+\delta^2)}
\end{array}
$$

Equilibrium payoff is $\{\frac{u+\delta}{1+\delta} - o_1, o_1, \frac{\delta^2}{1+\delta}(1-u), 1\}$*.*

Proof. See the appendix. □

Proposition 2. *For* $u(\delta^2+\delta^3-\delta-\delta^7) \leq \delta^2+\delta^4-\delta-\delta^7 \cup u(\delta+\delta^7-\delta^2-\delta^3) \leq 1-\delta^2-\delta^4+\delta^7 \cup u(1-\delta^5) \geq \delta^2+\delta^3-\delta-\delta^5$*, following strategy profile constitutes an equilibrium. We denote this equilibrium by* E_2*.*

$$
\begin{array}{ll}
o_1 = \hat{o_1} & o_2 = \hat{o_2} \\
\hat{o_1} = \delta c\hat{o}_1 & \hat{o_2} = \frac{\delta^4}{1+\delta}(1-u) \\
co_1 = c\hat{o}_1 & co_2 > c\hat{o}_2 \\
 & \\
c\hat{o}_1 = \frac{u(1-\delta^5)+\delta+\delta^5-\delta^2}{1+\delta} & c\hat{o}_2 = \frac{\delta}{1+\delta}(u(\delta-\delta^6-1)+1-\delta-\delta^3+\delta^2+\delta^6)
\end{array}
$$

Equilibrium payoff is $\{\frac{u+\delta}{1+\delta} - o_1, o_1, \frac{\delta^2}{1+\delta}(1-u), 1\}$.

Proof. See the appendix. □

Proposition 3. *For $u(1-\delta^5) \leq \delta^2+\delta^3-\delta-\delta^5$, following strategy profile constitutes an equilibrium. We denote this equilibrium by E_3.*

$$\begin{array}{ll} o_1 = \hat{o_1} & o_2 = \hat{o_2} \\ \hat{o_1} = \frac{\delta^4}{1+\delta} & \hat{o_2} = \frac{\delta^4}{1+\delta}(1-u) \\ co_1 > c\hat{o}_1 & co_2 > c\hat{o}_2 \end{array}$$

$$c\hat{o}_1 = \frac{u+\delta}{1+\delta} - \frac{\delta^2}{1+\delta} + \frac{\delta^5}{1+\delta}(1-u) \qquad c\hat{o}2 = \frac{\delta}{1+\delta}(1-u-\delta) + \frac{\delta^5}{1+\delta}$$

Equilibrium payoff is $\{\frac{u+\delta}{1+\delta} - o_1, o_1, \frac{\delta^2}{1+\delta}(1-u), 1\}$.

Proof. See the appendix.

□

Proposition 4. *For $u(\delta+\delta^7-\delta^2-\delta^3) \geq 1-\delta^2-\delta^4+\delta^7$, following strategy profile constitutes an equilibrium. We denote this equilibrium by E_4.*

$$\begin{array}{ll} o_1 = \hat{o_1} & o_2 < \hat{o_2} \\ \hat{o_1} = \delta c\hat{o}_1 & \hat{o_2} = \frac{\delta^4}{1+\delta}(1-u) \\ co_1 = c\hat{o}_1 & co_2 > c\hat{o}_2 \end{array}$$

$$c\hat{o}_1 = (\frac{u+\delta}{1+\delta})(\frac{1-\delta^3}{1-\delta^4}) \qquad c\hat{o}_2 = \frac{\delta}{1+\delta}(1-u-\delta) + \delta^2(\frac{u+\delta}{1+\delta})(\frac{1-\delta^3}{1-\delta^4})$$

Equilibrium payoff is $\{\frac{u+\delta}{1+\delta} - o_1, o_1, \frac{\delta^2}{1+\delta}(1-u), 1\}$.

Proof. See the appendix. □

Proposition 5. *For $u(1-\delta_4) \leq \delta_3+\delta_5-\delta-\delta_8$, following strategy profile constitutes an equilibrium. We denote this equilibrium by E_5.*

$$\begin{array}{ll} o_1 < \hat{o_1} & o_2 = \hat{o_2} \\ \hat{o_1} = \frac{\delta^4}{1+\delta} & \hat{o_2} = \delta c\hat{o}_2 \\ co_1 > c\hat{o}_1 & co_2 = c\hat{o}_2 \end{array}$$

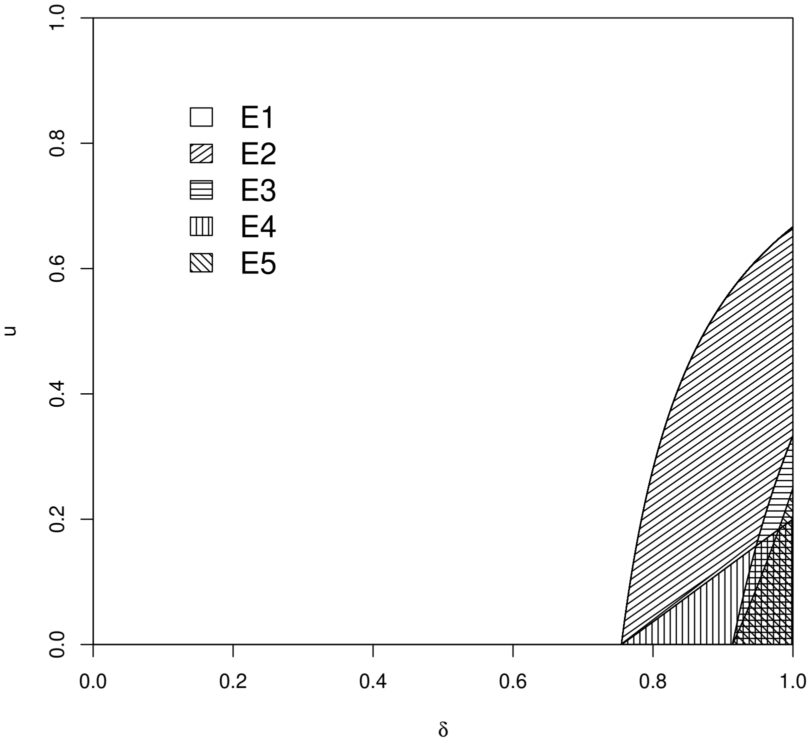
$$c\hat{o}_1 = \frac{u(1-\delta^4)+\delta(1-\delta)(1-\delta^4)+\delta^3(1-\delta^3)}{(1+\delta)(1-\delta^4)} \qquad c\hat{o}_2 = (\frac{\delta}{1+\delta})(\frac{1-\delta^3}{1-\delta^4})$$

Equilibrium payoff is $\{\frac{\delta}{1+\delta} - o_2, \frac{\delta^2}{1+\delta}, o_2, 3\}$.

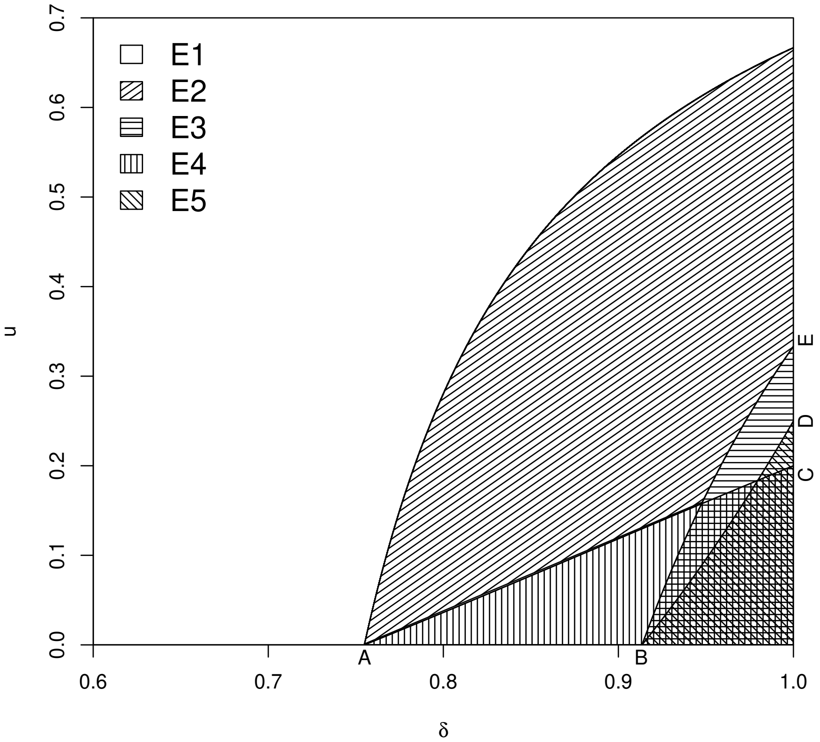
Proof. See the appendix. □

Figure 1 summarizes the above propositions (equilibria).
Note that in the equilibrium E_1 all the offers and counter offers are accepted. The equilibrium E_1 exists for low values of δ or high values of u. When players are impatient, they would not like to wait, and they would be eager to make deals as soon as possible. Hence, E_1 exists for low values of δ. Whereas E_2 to E_5 exist for high value of δ, and consequently some of the offers and counter offers are rejected. In E_2, all the offers and counter-offers are accepted except for co_2, which is rejected. In E_3, both the offers are accepted, while both the counter-offers are rejected. In E_4, o_1 and co_1 are accepted, while o_2 and co_2 are rejected. In E_5, o_1 and co_1 are rejected, while o_2 and co_2 are accepted.

Note that the above propositions (1 to 5) also characterize the set of equilibria of the game in which buyer starts the negotiations with the seller s_2. Only difference is that of the payoffs. By comparing these payoffs, we can evaluate buyer's preferred bargaining order. Following two propositions present the corresponding results, which are the main findings of this paper.



(a)



(b)

Figure 1: (a) All the 5 equilibria and the respective range of parameter values for which they are applicable. (b) Magnified view of the right hand side and bottom corner of figure (a). Coordinates of points A, B, C, D and E are (0.755, 0), (0.913, 0), (1, 0.20), (1, 0.25) and (1, 0.33) respectively.

Proposition 6. *For For $u(1-\delta_4) \geq \delta_3 + \delta_5 - \delta - \delta_8$, the buyer prefers to buy from s_1, the seller with higher valuation object, first.*

Proof. In the parameter range defined above, equilibrium E_1, E_2, E_3 and E_4 are applicable. See figure 1. It can be verified that buyer obtains higher payoff if he buys from s1 first in all these equilibria. See the appendix for calculations. □

Proposition 7. *For For $u(1-\delta_4) \leq \delta_3 + \delta_5 - \delta - \delta_8$, there exist equilibria such that both the bargaining orders are supported.*

Proof. In the parameter range defined above, equilibria E_3, E_4 and E_5 overlap. See figure 1. In E_3 and E_4, the buyer obtains higher payoff if he buys from s1 first. In E_5, the buyer obtains higher payoff if he buys from s_2 first. See the appendix for calculations. □

Clearly, buyer prefers to buy high utility object first, except when players are sufficiently patient and objects are sufficiently closer to being perfect compliments (i.e. low u), in which case there exists equilibria such that both the bargaining orders are supported. The intuition behind the result is following. In the zone where both the bargaining orders are supported in equilibrium, there exist two equilibria such that in each of them, one of the sellers hold out ie unwilling to sell first. Hence each equilibrium supports one of the bargaining orders. The reason a seller holds out is the lucrative Rubinstein(1982) payoff available to the seller who sells last. For low values of δ, the Rubinstein payoff is poor, hence the sellers would not like to wait or hold out. Also, for high value of u, both sellers have low incentive to hold out. If the seller s_1 sells first, the bargaining is over a surplus of $u + \frac{\delta(1-u)}{1+\delta} = \frac{u+\delta}{1+\delta}$. If he waits and sells second, the bargaining is over a surplus of unity. Clearly, for high enough u, the waiting is unlikely to be lucrative. If s_2 sells first, he is bargaining over a surplus of $\frac{\delta}{1+\delta}$, which is discounted value of the payoff available to the buyer in the Rubinstein bargaining with s_1. If he holds out and goes second, he is bargaining over a surplus of $1-u$. Clearly, for high u, selling second is not lucrative for s_2. Hence, only when δ is high and u is low enough, the sellers have incentive to hold out and obtain the Rubinstein payoff. Hence the multiple equilibria supporting both the bargaining orders exist in the corresponding zone.

4 Conclusion

We study a model of multilateral bargaining in which a buyer attempts to buy objects from two sellers. The objects are imperfect compliments for the buyer. We enquire buyer's preferred bargaining order ie whether buyer prefers to buy the higher valuation object first or the lower valuation object first. For most of the range of the parameters, buyer prefers to buy higher valuation object first. However, for a narrow range of parameters which involves high discount factor and high degree of complimentarity, multiple equilibria exist such that both the bargaining orders are supported.

5 Appendix

Proof of proposition 1

It is an infinite game, in which the game repeats after every 4 periods. Since we focus on Markov SPNE only, the strategies repeat after every 4 periods. Hence, we need to describe equilibrium strategies for first 4 periods only. In the first 4 periods a total of 4 offers and counter-offers are made. Hence, depending upon various combinations of offers and counter-offers being accepted or rejected, the total number of possible outcomes is $2^4 = 16$. Out of these 16, only 5 outcomes (E_1 to E_5) are possible in equilibrium. The remaining 11 outcomes are not possible in equilibrium. **Let us construct equilibrium E_1(described in proposition 1) in which all the 4 offers and counter-offers are accepted**. We use following 8 steps (a to h) to construct the equilibrium. These 8 steps give us values of 8 parameters and 4 inequalities. These 8 parameters are $o_1, \hat{o_1}, co_1, c\hat{o}_1, o_2, \hat{o_2}, co_2$, and $c\hat{o}_2$ (see section 3 for the description of these parameters). The 4 inequalitites are the constraints to which the equilibrium is subjected. When these 4 inequalities together represent an area or a line or a point, the equilibrium exists, otherwise the corresponding "equilibrium" does not exist. This is how it can be shown that 5 out of 16 outcomes exist as equilibria, and the rest of 11 outcomes do not form equilibria.

a) Consider period 4. Let $\hat{co}_2$ be the seller 2's counteroffer for which the buyer is indifferent between accepting and rejecting it. What is the buyer's payoff when he accepts $\hat{co}_2$? Buyer pays $\hat{co}_2$ in the current period and receives the Rubinstein(1982) payoff of $\frac{1}{1+\delta}$ in the next period. Hence, his net payoff is $x = \frac{\delta}{1+\delta} - \hat{co}_2$. What is the buyer's payoff when he rejects $\hat{co}_2$? Current period is wasted since the buyer rejects the counter-offer. In the next period the buyer pays o_1 to seller 1 (as all 4 offers and counter-offers are accepted in E_1) and receives u. In the next period, the buyer receives the Rubinstein payoff of $\frac{1-u}{1+\delta}$. Hence, his net payoff is $y = \frac{\delta^2(1-u)}{1+\delta} + \delta u - \delta o_1$. By equating x and y, we get an equation for $\hat{co}_2$.
$x = y \implies$

$$\hat{co}_2 = \frac{\delta}{1+\delta} - \frac{\delta^2(1-u)}{1+\delta} - \delta u + \delta o_1 \tag{1}$$

b) Consider period 4. Since all 4 offers and counter-offers are accepted in E_1, co_2 must be accepted. Therefore, the seller s_2 must counter-offer the maximum the buyer accepts. This implies,

$$co_2 = \hat{co}_2 \tag{2}$$

What is the s_2's payoff if he counter-offers more than $\hat{co}_2$? The current period is wasted since the counter-offer is rejected by the buyer. In the next period the buyer pays o1 to seller 1 (as all 4 offers and counter-offers are accepted in E_1). In the next period, the s_2 receives the Rubinstein payoff of $x = \frac{\delta(1-u)}{1+\delta}$. Since the seller counter-offers himself $\hat{co}_2$, following inequality must hold.
$\hat{co}_2 \geq \delta^2 x \implies$

$$\hat{co}_2 \geq \frac{\delta^3(1-u)}{1+\delta} \tag{I.1}$$

c) Consider period 3. Since s2 receives $\hat{co}_2$ in period 4, he will reject any offer below $\delta\hat{co}_2$ in period 3. Therefore,

$$\hat{o}_2 = \delta\hat{co}_2 \tag{3}$$

d) Consider period 3. Since o_2 is accepted in E_1, we have,

$$o_2 = \hat{o}_2 \tag{4}$$

What is the buyer's payoff when he offers $\hat{o}_2$? Buyer pays $\hat{o}_2 = \delta\hat{co}_2$ in the current period and obtains Rubinstein payoff of $\frac{1}{1+\delta}$ in the next period. His net payoff is $x = \frac{\delta}{1+\delta} - \delta\hat{co}_2$. What is the buyer's payoff if he offers less than $\hat{o}_2$? In that case, his offer is rejected. In the next period, he pays $\hat{co}_2$ to s2. In the subsequent period he obtains the Rubinstein payoff of $\frac{1}{1+\delta}$. His net payoff is $y = \frac{\delta^2}{1+\delta} - \delta\hat{co}_2$. Since the buyer offers $\hat{o}_2$, following inequality must hold.
$x \geq y$
$\implies \frac{\delta}{1+\delta} - \delta\hat{co}_2 \geq \frac{\delta^2}{1+\delta} - \delta\hat{co}_2$
$\implies \frac{\delta}{1+\delta} \geq \frac{\delta^2}{1+\delta}$
$\implies$

$$1 \geq \delta \tag{I.2}$$

Above inequality is true for all values of δ.

e) Consider period 2. Let $\hat{co}_1$ be the seller 1's counteroffer for which the buyer is indifferent between accepting and rejecting it. What is the buyer's payoff when he accepts $\hat{co}_1$? Buyer pays $\hat{co}_1$ in the current period and receives u. Also, he receives the Rubinstein(1982) payoff of $\frac{1-u}{1+\delta}$ in the next period. Hence, his net payoff is $x = u + \frac{\delta(1-u)}{1+\delta} - \hat{co}_1$. What is the buyer's payoff when he rejects $\hat{co}_1$? Current period is wasted since the buyer rejects the counter-offer. In the next period the buyer pays o_2 to seller 2 (as all 4 offers and counter-offers are accepted in E_1). In the next period, the buyer receives the Rubinstein payoff of $\frac{1}{1+\delta}$. Hence, his net payoff is $y = \frac{\delta^2}{1+\delta} - \delta o_2$. By equating x and y, we get an equation for $\hat{co}_1$.
$x = y \implies$

$$\hat{co}_1 = u + \frac{\delta(1-u)}{1+\delta} - \frac{\delta^2}{1+\delta} + \delta o_2 \tag{5}$$

f) Consider period 2. Since all 4 offers and counter-offers are accepted in E_1, co_1 must be accepted. Therefore, the seller s_1 must counter-offer the maximum the buyer accepts. This implies,

$$co_1 = c\hat{o}_1 \tag{6}$$

What is the seller's payoff if he counter-offers more than $c\hat{o}_1$? The current period is wasted since the counter-offer is rejected by the buyer. In the next period the buyer pays o_2 to seller 2 (as all 4 offers and counter-offers are accepted in E_1). In the next period, the s_1 receives the Rubinstein payoff of $x = \frac{\delta}{1+\delta}$. Since the seller counter-offers himself $c\hat{o}_1$, following inequality must hold.
$c\hat{o}_1 \geq \delta^2 x \implies$

$$c\hat{o}_1 \geq \frac{\delta^3}{1+\delta} \tag{I.3}$$

g) Consider period 1. Since s1 receives $c\hat{o}_1$ in period 2, he will reject any offer below $\delta c\hat{o}_1$ in period 1. Therefore,

$$\hat{o_1} = \delta c\hat{o}_1 \tag{7}$$

h) Consider period 1. Since o_1 is accepted in E_1, we have,

$$o_1 = \hat{o_1} \tag{8}$$

What is the buyer's payoff when he offers $\hat{o_1}$? Buyer pays $\hat{o_1} = \delta c\hat{o}_1$ and receives u in the current period and obtains Rubinstein payoff of $\frac{1-u}{1+\delta}$ in the next period. His net payoff is $x = \frac{\delta(1-u)}{1+\delta} + u - \delta c\hat{o}_1$. What is buyer's payoff if he offers less than $\hat{o_1}$? In that case, his offer is rejected. In the next period, he pays $c\hat{o}_1$ to s1 and receives u. In the subsequent period he obtains the Rubinstein payoff of $\frac{1-u}{1+\delta}$. His net payoff is $y = \frac{\delta^2(1-u)}{1+\delta} + \delta u - \delta c\hat{o}_1$. Since the buyer offers $\hat{o_1}$, following inequality must hold.
$x \geq y$
$\implies \frac{\delta(1-u)}{1+\delta} + u - \delta c\hat{o}_1 \geq \frac{\delta^2(1-u)}{1+\delta} + \delta u - \delta c\hat{o}_1$
$\implies \frac{\delta(1-u)}{1+\delta} + u \geq \frac{\delta^2(1-u)}{1+\delta} + \delta u$
$\implies$

$$\frac{\delta(1-u)(1-\delta)}{1+\delta} \geq u(\delta - 1) \tag{I.4}$$

The above inequality is true for all values of δ and u.

We have 8 equations, numbered 1 to 8, in 8 variables, which upon solving gives us the values of $o_1, \hat{o_1}, co_1, c\hat{o}_1, o_2, \hat{o_2}, co_2$, and $c\hat{o}_2$. Also, we have 4 inequalities, labeled I.1 to I.4. Inequalities I.2 and I.4 are true for entire range of parameter values of u and δ. Hence, these inequalities can be ignored. After substituting the values of variables, the inequalities I.1 and I.3 become - $u(\delta^3 + \delta^4 + \delta^5 + \delta^6 - \delta) \geq \delta^4 + \delta^5 + \delta^6 - \delta$ and $u(1 + \delta + \delta^2) \geq \delta^4 + \delta^5 + \delta^6 - \delta$ respectively. It can be verified that the first inequality makes the second inequality redundant. Hence, E_1 is confined to the area represented by the first inequality.

Proof of propositions 2 to 5

Using arguments similar to those used in the steps a to h in the proof of the proposition 1, propositions 2 to 5 can be proved. Using same methodology it can be shown that the rest of the 11 outcomes out of 16 possibilities cannot be supported in equilibrium.

Proof of propositions 6 and 7

Bargaining order 1 refers to the game in which s_1 is the first seller to negotiate with the buyer. Similarly, bargaining order 2 refers to the game in which s_2 is the first seller to negotiate with the buyer. Let $B_{1,i}$ and $B_{2,i}$ denote buyer's payoff in bargaining order 1 and 2 respectively in the equilibrium E_i. Now we compare buyer's payoffs in both the bargaining orders in each of the equilibria.

(i) **Claim:** $B_{1,1} > B_{2,1}$.
$B_{1,1} = \frac{u+\delta}{1+\delta} - o_1$
$B_{2,1} = \frac{\delta}{1+\delta} - o_2$
Let us assume $B_{1,1} > B_{2,1}$ is false.
$B_{1,1} > B_{2,1} \implies \frac{u+\delta}{1+\delta} - o_1 > \frac{\delta}{1+\delta} - o_2$
$\implies$

$$\frac{u}{1+\delta} > o_1 - o_2 \tag{9}$$

$c\hat{o}_1 - c\hat{o}_2 = \frac{u}{1+\delta} + \frac{\delta u}{1+\delta} + \delta(o_2 - o_1)$
$\iff c\hat{o}_1 - c\hat{o}_2 = u - \delta^2(c\hat{o}_1 - c\hat{o}_2)$
$\iff c\hat{o}_1 - c\hat{o}_2 = \frac{u}{1+\delta^2}$
$\iff o_1 - o_2 = \frac{\delta u}{1+\delta^2}$
Substituting the value of $o_1 - o_2$ in the equation 9, we get,
$\frac{u}{1+\delta} > \frac{\delta u}{1+\delta^2}$
$\implies 1 > \delta$, which must be false because of our assumption. This is a contradiction since we know that the statement is true for all values of δ. Hence our assumption must be false, and the claim must be true.

(ii) **Claim:** $B_{1,2} > B_{2,2}$.
$B_{1,2} = \frac{u+\delta}{1+\delta} - o_1$
$B_{2,2} = \frac{\delta}{1+\delta} - o_2$
Let us assume $B_{1,2} > B_{2,2}$ is false.
$B_{1,2} > B_{2,2} \implies \frac{u}{1+\delta} > o_1 - o_2 = \delta c\hat{o}_1 - o_2$
$\implies \frac{u}{1+\delta} > \delta \frac{u(1-\delta^5)+\delta+\delta^5-\delta^2}{1+\delta} - \frac{\delta^4(1-u)}{1+\delta}$
$\implies$

$$u(1+\delta^6-\delta-\delta^4) > \delta^2+\delta^6-\delta^3-\delta^4 \tag{10}$$

Because of our assumption, the above inequality, equation 10, must not hold over the entire range of parameter values for which E_2 is applicable.
Let δ_0 be the solution of the equation $\delta^2 + \delta^3 - 1 = 0$. This implies $\delta_0 \approx 0.755$. Let δ_1 be the solution of the equation $x = 1 + \delta^6 - \delta - \delta^4 = 0$. This implies $\delta_1 \approx 0.857$.
Statement A: For $\delta > \delta_1, x < 0 \implies u < \frac{\delta^2+\delta^6-\delta^3-\delta^4}{1+\delta^6-\delta-\delta^4} = y$, say.
Statement B: For $\delta < \delta_1, x > 0 \implies u > \frac{\delta^2+\delta^6-\delta^3-\delta^4}{1+\delta^6-\delta-\delta^4}$.
Statement C: It can be verified that $y > 1$ for $\delta \in [\delta_1, 1)$, and $y < 0$ for $\delta \in (\delta_0, \delta_1)$. Also, it can be verified that E2 exists for $\delta \in (\delta_0, 1)$.
Statement A, B and C together proves that the inequality (equation 10) holds for the entire range of the parameter values for which E_2 is applicable. This proves the claim by contradiction.

(iii) **Claim:** $B_{1,3} > B_{2,3}$.
$B_{1,3} = \frac{u+\delta}{1+\delta} - o_1 = \frac{u+\delta}{1+\delta} - \frac{\delta^4}{1+\delta}$
$B_{2,3} = \frac{\delta}{1+\delta} - o_2 = \frac{\delta}{1+\delta} - \frac{\delta^4(1-u)}{1+\delta}$
Let us assume that $B_{1,3} > B_{2,3}$ is false. This implies that $1 > \delta^4$ is false, which is a contradiction. This contradiction means that our assumption is false. Hence, the claim is proved by contradiction.

(iv) **Claim:** $B_{1,4} > B_{2,4}$.
$B_{1,4} = \frac{u+\delta}{1+\delta} - o_1$
$B_{2,4} = \delta^2(\frac{u+\delta}{1+\delta} - o_1)$
Let us assume that $B_{1,4} > B_{2,4}$ is false. This implies that $1 > \delta^2$ is false, which is a contradiction. This contradiction means that our assumption is false. Hence, the claim is proved by contradiction.

(v) **Claim:** $B_{2,5} > B_{2,5}$.
$B_{1,5} = \delta^2(\frac{\delta}{1+\delta} - o_2)$
$B_{2,5} = \frac{\delta}{1+\delta} - o2$
Let us assume that $B_{2,5} > B_{1,5}$ is false. This implies that $1 > \delta^2$ is false, which is a contradiction. This contradiction means that our assumption is false. Hence, the claim is proved by contradiction.

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