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Abstract

We argue that cooperation can become more fragile if (i) there are sufficiently many intermediate levels of cooperation and (ii) players cannot respond with large punishments to small deviations. Such disproportional punishments can be perceived as unreasonable or players can face external constraints—political checks, negative publicity, etc. Specifically, we show that regardless of how patient the players are, any prisoner's dilemma game can be extended with intermediate levels of cooperation in such a way that full conflict is the only equilibrium outcome of the extended game.

JEL classifications: D74, F51, C73.

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1 Introduction

In his seminal book, "The Strategy of Conflict," Thomas Schelling spoke of the difficulty of committing to a big retaliation in response to a small provocation. He was suggesting that in the face of this difficulty the possibility to wage a limited war, i.e. the availability of intermediate actions, might prevent the conflict from escalating: "if it [the threat] can be decomposed into a series of consecutive smaller threats, there is an opportunity to demonstrate on the first few transgressions that the threat will be carried out on the rest." More specifically, in the context of a repeated prisoner's dilemma situation, Schelling suggested that the presence of intermediate actions might actually allow for deescalation of conflict (see pp. 45–46). In this paper we argue that such intuition does not necessarily hold. In the absence of credible commitment to a full-out retaliation, the conflict, following a sequence of small deviations, can escalate fully. Specifically, we show that for any prisoner's dilemma game and any discount factor the cooperation can break down if there is a sufficient number of suitably chosen intermediate actions.

In other words, the very addition of intermediate actions coupled with a restriction of punishments to "reasonable" levels can break down the cooperation which was previously feasible. Thus, contrary to Schelling's (1980) intuition, intermediate actions can make cooperation more fragile, for example in situations where only symmetric punishments are feasible due to political constraints.

The underlying mechanism, that we study in detail further, can be demonstrated using the following prisoners' dilemma game as an example:

	C	D
C	$0, 0$	$-5, 2$
D	$2, -5$	$-3, -3$

The cooperative outcome (C, C) can be supported as an equilibrium in every round of a repeated game as long as $2 - 3\delta/(1-\delta) \leq 0$ or $\delta \geq 2/5$.

Now, suppose that the world has become more complex and an intermediate level of cooperation (or conflict) is available:

	C	D'	D
C	$0, 0$	$-2, 1$	$-5, 2$
D'	$1, -2$	$-1, -1$	$-4, 1$
D	$2, -5$	$1, -4$	$-3, -3$

Further, suppose that the players cannot punish deviations from C to D' by playing D. For example, if the Unites States were to respond with a large scale military engagement (D) following a single act of cyber attack against the US (D'), then the military action would need an authorization from the US Congress, which might consider such punishment excessive or unreasonable. Consequently, if a player considers a deviation to D', she can reasonably expect to be punished with the same action D' rather than with D. In this case, the deviation from (C, C) is profitable if $1 - \delta/(1-\delta) \geq 0$ or $\delta \leq 1/2$. If we suppose that (D', D') is played as a steady state, then a deviation to D is also profitable as long as $\delta \leq 1/2$. Thus, for any $\delta \in (2/5, 1/2)$ cooperation can be sustained as an equilibrium outcome in the smaller game (or in a game with strong punishments) but it is not sustainable in the extended game when punishments are limited. Moreover, full conflict is the only equilibrium outcome of the extended game.

Arguably, the world is substantially more interconnected today than it used to be decades or centuries ago. Nowadays, countries can engage in cooperation or conflict on various levels: from foreign direct investments to financial sanctions, from coordinated development of global IT networks to cyber warfare, from technological cooperation to espionage, from joint military exercises to locating strategic military installations closer to their opponents. Following decades of globalization and mutual integration our world is not necessarily a safer place to live in. In January 2018, the Bulletin of the Atomic Scientists moved the Doomsday Clock to two minutes before midnight. This is the clock's lowest value, matching that of 1953. The increasing threats of nuclear weapons and climate change and the inability of the world leaders to de-escalate the situation were stated as the reason. Yet at the time when mutual cooperation is paramount, major international powers keep provoking one another. The United States engage in cyber espionage and allegedly cyber warfare (e.g. Stuxnet). The Russian Federation allegedly does the same. At the moment of this writing, both countries are playing in a proxy war in Syria. Donald Trump in his telephone conversation with Vladimir Putin referred to the New START (Strategic Arms Reduction Treaty) as "one of several bad deals negotiated by the Obama administration." In July 2017, the first Chinese aircraft carrier was shown publicly in Hong Kong, at least one more carrier has been commissioned and will be built in the near future. Earlier in 2017, the Japanese prime minister, Shinzo Abe, expressed his intent to revise the Article 9 of the Japanese constitution—the rule restricting Japan from owning military forces with offensive potential—by 2020. In what might turn out to be another push towards Abe's cause, North Korea launched a ballistic missile that flew over Japan, a first since 1998. Should we be worried? Are such tensions and conflicts more likely to escalate in the 21^{st} century than they were back in the 20^{th} century?

International relationships are rich in detail and we do not claim that our theoretical explorations are directly applicable to the aforementioned conflicts. However, we hope to contribute to the understanding of one specific aspect of international relationships, or other conflicting situation. Namely, does the emergence of intermediate levels of conflict, cyber warfare being one such example, make the uncooperative outcomes in prisoner's dilemma type games more likely?

To this end, we consider a general prisoner's dilemma game with an arbitrary number of intermediate levels of deviation (or cooperation). We further assume that grim strategies cannot be used credibly, i.e. a player cannot respond with a punishment that is stronger than the original deviation. For instance, if we consider the case of international relationships, then the Doctrine of Proportionality is one of the fundamental principles of international law. "According to the doctrine, a state is legally allowed to unilaterally defend itself and right a wrong provided the response is proportional to the injury suffered".[1] Even in cases when international law is not directly applicable, countries follow the principle of proportionality. For example, such tit-for-tat strategies are often used when diplomats get expelled. Given our assumption on proportional punishments, we show that no matter how patient the players are, there are intermediate levels of deviation such that no cooperative equilibria exist. Whether emerging intermediate actions in real life games are of such nature is a different question, but we show that the risk is present.

The literature on prisoner's dilemma with intermediate actions is relatively scarce. Snidal (1985) shows that new strategic difficulties arise in such games, e.g., having multiple Pareto-efficient outcomes instead of just one. He does not, however, speak of the possibility of conflict escalation. In an independent work, Langlois (1989) explicitly allows for conflict escalation. He considers a repeated prisoner's dilemma game with a continuum of intermediate actions and with linear payoffs, and he shows that there exists a Markov equilibrium in linear strategies that can sustain full cooperation. In comparison with our work, Langlois does not impose any restrictions on the degree of punishment, whereas such restrictions are the main focus of our discussion.

In a later work, Friedman and Samuelson (1990) analyze repeated games with continuous payoffs, thus similar to Langlois (1989), but the authors restrict the punishment to be proportional to the deviation. If the deviation approaches zero, so does the punishment. Friedman and Samuelson consider reference dependent strategies, and show that that if the discount factor is large enough, then with these strategies it is possible to construct a deescalating equilibrium despite having limited punishment. In contrast, we consider the limiting behaviour of discrete games and simpler tit-for-tat strategies. We arrive at the opposite

[1] See https://www.cfr.org/backgrounder/israel-and-doctrine-proportionality, accessed on 9^{th} of August 2018.

conclusion: for every value of the discount factor there are games with sufficiently many intermediate actions where escalation cannot be precluded. The difference between our result and that of Friedman and Samuelson (1990) arises due to different interpretations of what a small punishment is: a tit-for-tat action in our case and a punishment proportional to gains in payoffs in their case. We discuss this crucial difference in more detail at the end of the paper. In their later work, Friedman and Samuelson (1994) showed that the Folk Theorem of Fudenberg and Maskin (1986) can be extended to games with continuous reaction functions.

A paper with a setup most similar to ours is McGinnis (1991). McGinnis models intermediate levels of cooperation as a sequence of overlapping prisoner's dilemma games. Importantly, he argues for a specific log-linear payoff function as most suitable for the study of international conflict. Given his specific payoff function, he shows that the equilibrium will likely be sustained at one of the intermediate levels of cooperation and not at full cooperation. In contrast with McGinnis, we study a general payoff structure and show that a more extreme outcome—namely, no cooperation—is always a possibility.

Lastly, there is a group of papers that study prisoner's dilemma games with intermediate actions in an evolutionary setting: To (1988), Frean (1996), Wahl and Nowak (1999), Darwen and Yao (2002). To (1988) considers a fixed population of strategies and shows that the strategies where the punishment does not exceed the deviation are most profitable. In our paper we specifically focus on such strategies, and To's findings corroborate our setup. Frean (1996) assumes a payoff structure that effectively corresponds to a zero discount rate and, not surprisingly, he finds that his strategies evolve towards full cooperation. Both Wahl and Nowak (1999) and Darwen and Yao (2002) arrive at the result that full cooperation is less evolutionary likely when intermediate actions are present. None of these papers documents escalating dynamics, and all these papers assume either linear or restricted quadratic payoffs. In contrast, we study whether there are payoff structures, not necessarily linear, that can lead to escalating dynamics.

2 General Analysis and Discussion

In this section we present and discuss our general result. Consider an arbitrary prisoners' dilemma game:

$$\Gamma = \begin{array}{c|cc} A\backslash B & C & D \\ \hline C & R,R & S,T \\ D & T,S & P,P \end{array}$$

where $T > R > P > S$ and $2R > T + S$.

If this game is played once, then the only Nash equilibrium is (D, D). If we consider a repeated version of this game, then (C, C) can be sustained in an equilibrium if and only if the discount factor $\delta \geq \frac{T-R}{T-P}$.

Game Γ has two levels of cooperation: full cooperation and full defection. Broadly speaking, we want to ask the following question: what happens with the cooperative equilibrium if we add intermediate levels of conflict to game Γ? To make this question precise, we need to define what we mean by a game with intermediate levels of conflict or cooperation; we also need to define the class of strategies that we plan to study.

For any $N > 2$ we define class $\mathcal{G}_N$ of games with N levels of cooperation as follows. Each element $\Gamma_N \in \mathcal{G}_N$ is a game between two players, A and B, where each player can choose an action $a \in \{1, \dots, N\}$. Choosing $a = 1$ means full cooperation, choosing $a = N$ means full defection. For each action choice (a, b) the payoff for player A is $u_A(a, b)$ and for player B it is $u_B(a, b)$. We consider symmetric games, namely $u_A(a, b) = u_B(b, a) = u(a, b)$. We further impose that for any a, b, and c such that $1 \leq a < b \leq N$ and $1 \leq c \leq N$ the following restrictions hold for the payoff matrix:

1. $u(1, 1) = R$, $u(N, 1) = T$, $u(1, N) = S$, $u(N, N) = P$,

2. $u(b, c) > u(a, c)$, $u(c, b) < u(c, a)$,

3. $u(a, a) > u(b, b)$, and $2u(a, a) > u(a, b) + u(b, a)$.

Condition 1 (consistency) means that full cooperation and full conflict lead to the same outcomes as in the original game Γ. Condition 2 (monotonicity) guarantees that intermediate actions generate intermediate payoffs. Finally, condition 3 (prisoners' dilemma) means that every 2×2 principal submatrix of the payoff matrix can itself be viewed as a prisoners' dilemma game. We impose Condition 3 to avoid local changes in the strategic nature of the game when intermediate actions are added. For example, this condition helps us to exclude games where local escalation is mutually profitable. Note that the games in a given $\mathcal{G}_N$ are characterised by the same set of players and actions but differ in their payoff functions, which, however, must be compatible with conditions 1–3.

We consider an infinitely repeated game, where each stage game is some $\Gamma_N \in \mathcal{G}_N$. We assume that both players discount their payoffs with the same discount factor δ.

Finally, for a given N we limit our attention to the class of strategies Σ_N, where each element $\sigma \in \Sigma_N$ is defined as follows:

1. start play with some action $a_0 \in \{1, \ldots, N-1\}$,
2. in any round t play $a_t = \max\{a_{t-1}, b_{t-1}\}$, where a_{t-1}, b_{t-1} are actions played in the previous round.[2]

In other words, in Σ_N the punishment never exceeds the deviation. We therefore call such strategies Markovian strategies with limited punishment.

Note that elements in Σ_N differ only in their starting points, namely action a_0. Further, as every $\Gamma_N \in \mathcal{G}_N$ has the same set of players and actions, Σ_N is well-defined for any $\Gamma_N \in \mathcal{G}_N$.

Any original game Γ has an equilibrium in Σ_2 strategies if δ is large enough. (We label C as 1 and D as 2.) This is the equilibrium where each player starts with $a = 1$, and $(1, 1)$

[2]This strategy space is limited, because it does not support de-escalating strategies. We could have defined the set of possible actions after imposing a restriction on "reasonable" punishments as $a_t \leq \max\{a_{t-1}, b_{t-1}\}$. However, with our choice of intermediate payoffs we still would not have been able to obtain a symmetric equilibrium that leads to any outcome other than full conflict.

remains a steady state from there on. However, as the number of actions increases, strategies with limited punishment might fail to deliver an equilibrium in Γ_N. This is formally captured by the following proposition.

Proposition 1. *For any $\delta < 1$ there is N large enough and a game $\Gamma_N \in \mathcal{G}_N$ such that no pair of strategies (σ, σ), with $\sigma \in \Sigma_N$, constitutes an equilibrium in Γ_N.*

Proof. See Appendix. □

Our proof is constructive. We explicitly build games Γ_N using a discretization of a suitably chosen continuous payoff function. Our construction is by no means unique and many other examples that lead to full conflict can be made. The essential requirement for any such construction is that there are sufficient incentives for deviation around cooperative outcomes, while the payoffs remain sufficiently bounded so as to satisfy conditions 1-3. Not every game with a large number of intermediate actions leads to a break-down of cooperation. For example, if there is a big "gap" in the payoffs somewhere along the main diagonal of the game, the conflict escalation naturally stops at that gap, provided that the discount factor is sufficiently large. Or, if the game is extended "uniformly", i.e. with payoffs defined linearly along the main diagonal, linearly above it (triangle $R-S-P$), and linearly below it (triangle $R-T-P$), then all the incentives of the original game are preserved, the critical value of the discount factor remains the same, and the additional intermediate actions do not result in escalation.[3] Nonetheless, introduction of intermediate actions typically reduces the scope for cooperation and requires players to be more patient, as our example in the introduction suggests.

If no equilibrium in our class of strategies exists, then the only steady-state symmetric equilibrium that remains is the full conflict equilibrium (N, N). We do not focus on the precise mechanics of escalation when the players find themselves in a cooperative outcome that cannot be supported as an equilibrium. It is conceivable that similarly to institutional

[3]It is straightforward to show that this "uniform" extension belongs to class $\mathcal{G}_N$.

restrictions on large punishments there might be institutional restrictions on large deviations. In such cases the conflict would escalate in a sequence of small deviations and gradually approach the most non-cooperative outcome.

Our result is robust to the assumption that punishment must be symmetric. Any moderate degree of asymmetry, in a sense that a deviation to action a can be punished by at most action $a+k$, where k is a fixed number, still leads to the same conclusion, but probably requires a higher N. It is critical, however, that the loss in the payoff from the strongest feasible punishment is not too high. This can be guaranteed by a construction algorithm similar to the one we use in our proof as long as k does not depend on N.

Our finding contrasts to that in Schelling (1980), who argued that intermediate levels of conflict and punishment make cooperation more stable. His logic was based on the idea that small threats are more credible and therefore act a sufficient deterrence device. Our analysis suggests that, although being credible, these small punishments might not be sufficiently grim, which results in step-by-step escalation of conflict.

The relation between our paper and Friedman and Samuelson (1990) is of particular interest. Friedman and Samuelson show that cooperative outcomes can be achieved in games with continuous strategies, where small deviations are met with small punishments. We consider similar strategy profiles but we focus on discrete rather than continuous games, and we show that in discrete games cooperation can break down. Let us elaborate on these seemingly contradictory conclusions.

The crucial difference between our papers lies in how we define "small punishments". In Friedman and Samuelson (1990), a punishment is considered "small" if it is proportional to the gains of a deviator, while in our case a punishment is considered "small" if it is proportional to the deviation distance in the action space. If we were to introduce Friedman and Samuelson's concept into our discrete games, then punishments could be strong enough to prevent any escalation. In particular, our Proposition 1 would not hold. Similarly, if our concept of small punishments was introduced into Friedman and Samuelson, then full escalation might happen for some choices of the payoff function.[4] Thus, our concept of a small, proportional punishment makes our results differ both from the results found in the usual repeated games literature and from those by Friedman and Samuelson.

Appendix

Proof of Proposition 1. We prove the proposition as follows. First, we choose a continuous payoff function such that any uniform discretization of this function satisfies conditions 1, 2, and 3 (consistency, monotonicity, and prisoners' dilemma). Second, we show that for any $\delta < 1$ there is a discretization that is sufficiently fine so that the corresponding game does not have an equilibrium in Σ_N.

Choose K to be the smallest integer such that

$$K > \max\left\{\frac{T-R}{P-S}, \frac{T-R}{T-2R+P}, 1\right\} + 1.$$

Note that $K \geq 3$. For $a \in \{1, \ldots, N\}$ and $b \in \{1, \ldots, N\}$ let

$$u(a,b) = f\left(\frac{a-1}{N-1}, \frac{b-1}{N-1}\right), \tag{1}$$

where

$$f(x,y) = \begin{cases} (T-R)(x-y)^{1-1/K} + R - (R-P)y & \text{if} \quad x \geq y, \\ (S-P)(y-x)^{1/K} + R - (R-P)y & \text{if} \quad x < y, \end{cases} \tag{2}$$

is a continuous function defined on $[0,1] \times [0,1]$. Note that K and f do not depend on N.

We verify conditions 1, 2, and 3 now. We have

$$u(1,1) = f(0,0) = R, \quad u(N,1) = f(1,0) = T,$$

$$u(1,N) = f(0,1) = S, \quad u(N,N) = f(1,1) = P.$$

[4]Our payoff function (2) is one such example. Note that it is not Lipschitz-continuous along the main diagonal of the action space and therefore violates the conditions of Theorem 1 in Friedman and Samuelson (1990).

So, Condition 1 is satisfied. Next, we have

$$\frac{\partial}{\partial x} f(x,y) = \begin{cases} \dfrac{K-1}{K}(T-R)(x-y)^{-1/K} > 0 & \text{if} \quad x > y, \\ \dfrac{1}{K}(P-S)(y-x)^{1/K-1} > 0 & \text{if} \quad x < y, \end{cases}$$

$$\frac{\partial}{\partial y} f(x,y) = \begin{cases} -\dfrac{K-1}{K}(T-R)(x-y)^{-1/K} - (R-P) < 0 & \text{if} \quad x > y, \\ -\dfrac{1}{K}(P-S)(y-x)^{1/K-1} - (R-P) < 0 & \text{if} \quad x < y. \end{cases}$$

Therefore $f(x,y)$ is strictly increasing in x and strictly decreasing in y. Consequently, Condition 2 is satisfied.

Note that

$$\frac{\partial}{\partial x} f(x,x) = -(R-P) < 0,$$

hence f is strictly decreasing along its main diagonal and the first part of Condition 3 is satisfied. The second part of the condition requires that $2u(a,a) > u(a,b) + u(b,a)$ for any integer a, b such that $1 \leq a < b \leq N$. Using our definition of u and rearranging terms, we obtain

$$(R-P)\frac{b-a}{N-1} - (T-R)\left(\frac{b-a}{N-1}\right)^{1-1/K} + (P-S)\left(\frac{b-a}{N-1}\right)^{1/K} > 0$$

or, equivalently,

$$(R-P)\left(\frac{b-a}{N-1}\right)^{1-1/K} - (T-R)\left(\frac{b-a}{N-1}\right)^{1-2/K} + (P-S) > 0. \tag{3}$$

Let

$$g(\phi) = (R-P)\phi^{1-1/K} - (T-R)\phi^{1-2/K} + (P-S).$$

Then to show that (3) holds for any integer a, b such that $1 \leq a < b \leq N$ it suffices to show that $g(\phi) > 0$ for all $\phi \in [0,1]$.

Given that $K \geq 3$, we have that $g(\phi)$ is continuous and bounded on $[0,1]$. Hence, we only need to check the sign of g at its boundary and inflection points. We have $g(0) = P - S > 0$ and $g(1) = 2R - (T+S) > 0$ (convexity of the original game).

Solving $g'(\phi) = 0$ we obtain that g has a unique inflection point on $(0, \infty)$ given by

$$\phi_0 = \left(\frac{T-R}{R-P} \, \frac{K-2}{K-1} \right)^K.$$

Suppose that $\frac{T-R}{R-P} > 1$. We have required that $K > \frac{T-R}{T-2R+P} + 1$. From the first inequality if follows that $T - 2R + P > 0$, and therefore the second inequality yields $\frac{K-2}{K-1} > \frac{R-P}{T-R}$. Consequently, $\phi_0 > 1$. There are thus no inflection points on $[0, 1]$, and so $g(\phi) > 0$ for all $\phi \in [0, 1]$.

Suppose that $\frac{T-R}{R-P} \leq 1$. Then $\phi_0 < 1$. Evaluating g at ϕ_0 and rearranging, we get

$$g(\phi_0) = (P - S) - \frac{T-R}{K-1} \phi_0^{1-2/K} > (P - S) - \frac{T-R}{K-1}.$$

We have required that $K > \frac{T-R}{P-S} + 1$. It immediately follows that $g(\phi_0) > 0$. So, g is strictly positive at its boundary points as well as at its unique interior inflection point. Hence, $g(\phi) > 0$ for all $\phi \in [0, 1]$. Summarizng, we shown that the second part of Condition 3 holds.

Having payoffs u as defined in (1), we proceed to show that given any $\delta < 1$ there exists a sufficiently large N so that no symmetric pair of strategies from Σ_N forms an equilibrium.

Consider a pair of strategies (σ, σ), with $\sigma \in \Sigma_N$. A necessary condition for these strategies to form an equilibrium is that the first player does not have an incentive to deviate from some steady state (a, a) to $(a + 1, a)$, or

$$\frac{1}{1-\delta} u(a, a) \geq u(a + 1, a) + \frac{\delta}{1-\delta} u(a + 1, a + 1). \tag{4}$$

Conversely, if this condition is not satisfied, then no such pair of strategies forms an equilibrium. Expanding u in (4) and rearranging terms we obtain

$$\frac{\delta}{1-\delta} \, \frac{R-P}{N-1} \geq \frac{T-R}{(N-1)^{1-1/K}}$$

and therefore for

$$N > \left(\frac{\delta}{1-\delta} \, \frac{R-P}{T-R} \right)^K + 1$$

the equilibrium does not exist. Thus, given any $\delta < 1$ we can choose N sufficiently large so that (4) does not hold. For such an N, no pair of strategies (σ, σ), with $\sigma \in \Sigma_N$, constitutes an equilibrium. $\square$

References

Darwen, P. J. and Yao, X. (2002). Co-evolution in iterated prisoner's dilemma with intermediate levels of cooperation: application to missile defense. *International Journal of Computational Intelligence and Applications*, 2(1):83–107.

Frean, M. (1996). The evolution of degrees of cooperation. *Journal of Theoretical Biology*, 182(4):549–559.

Friedman, J. W. and Samuelson, L. (1990). Subgame perfect equilibrium with continuous reaction functions. *Games and Economic Behavior*, 2:304–324.

Friedman, J. W. and Samuelson, L. (1994). An extension of the "folk theorem" with continuous reaction functions. *Games and Economic Behavior*, 6(1):83–96.

Fudenberg, D. and Maskin, E. (1986). The folk theorem in repeated games with discounting or with incomplete information. *Econometrica*, 54(3):533–554.

Langlois, J.-P. P. (1989). Modeling deterrence and international crises. *Journal of Conflict Resolution*, 33(1):67–83.

McGinnis, M. (1991). Limits to cooperation: Iterated graduated games and the arms race. *International Interactions*, 16(4):271–293.

Schelling, T. C. (1980). *The strategy of conflict*. Harvard university press.

Snidal, D. (1985). Coordination versus prisoners' dilemma: Implications for international cooperation and regimes. *The American Political Science Review*, 79(4):923–942.

To, T. (1988). More realism in the prisoner's dilemma. *Journal of Conflict Resolution*, 32(2):402–408.

Wahl, L. M. and Nowak, M. A. (1999). The continuous prisoner's dilemma: I. linear reactive strategies. *Journal of Theoretical Biology*, 200(3):307–321.