Free-on-board and uniform delivered pricing strategies in pure and mixed spatial duopolies: the strategic role of cooperatives

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Abstract

The present work analyzes free-on-board against uniform delivered strategic prices in pure and mixed duopolistic spatial markets with reference to the food sector. Along with investor owned firms (IOFs) that maximize profits, we introduce member welfare maximizing cooperatives (COOPs) and examine their impact on the strategic pricing choices. Demand is price responsive. We use a two stage game between two IOFs, between an IOF and a COOP, and between two COOPs. The findings indicate that the introduction of COOPs acts as a disciplinary factor regarding the pricing behavior of the IOFs. As competition in the spatial market escalates, we move from the quasi–collusive (FOB,FOB) Nash equilibrium, where there are only IOFs in the market, to the more aggressive (UD,UD) strategic pricing configuration where COOPs replace one or both IOFs in the market.

Keywords: oligopoly spatial competition; mixed; free-on-board; uniformly delivered

JEL classification: D40, Q13, L13, C72.

1 Introduction

The majority of firms operating in spatial markets use either free-on-board (FOB) or uniform delivered (UD) pricing (Fousekis, 2015; Zhang and Sexton, 2001). In the former, customers pay for the transportation costs incurred in the shipment of the product. In the latter, sellers bear all the costs of shipping, representing this way a form of price discrimination.

Pricing choices in spatial markets have been the subject of economic research for over seventy years (Anderson et al., 1989; Beckmann, 1973; Fetter, 1937; Hoover, 1937; Thisse and Vives, 1988; Tribl, 2009; Zhang and Sexton, 2001). The main characteristic of spatial competition is the distribution of buyers (or sellers) over the market, forcing this way economic exchange to incur costly transport. As a consequence, transportation costs give the dispersed firms market power over their local customers.[1]

Spatial pricing has been examined mostly for the case of monopoly (Beckmann, 1976; Beckmann and Thisse, 1987; Greenhut et al., 1987). Only a few studies have investigated a firm's pricing choices under competition. In one of them, Norman (1981) analyzed linear price distance functions under the assumption that firms maximize profits with a given market radius as well as under the assumption that price at the market boundary is fixed. Norman showed that under certain conditions, the degree of price discrimination increases as competition increases and can yield UD pricing in the limit.

When two or more firms operate in a market and under the assumption that market power lies with the sellers, two studies relate with the present work: Espinosa (1992) and Kats and Thisse (1993) investigate firms' choices between FOB and UD pricing policies in duopoly markets. Kats and Thisse used a two-stage game where in the first stage the spatial duopolists selected between FOB and UD

[1] In the case of oligopoly/oligopsony, dispersed firms must be in different locations. If firms are at the same location, space does not matter.

strategies and in the second stage they set prices, that were conditional on their first stage choices. The same behavior was repeated in Espinosa's infinite horizon model. Despite the differences in their theoretical approach, Espinosa (1992) and Kats and Thisse (1993) arrive at very similar conclusions: UD pricing is the equilibrium strategy in collusive as well as very competitive industries, whereas FOB pricing is the equilibrium strategy for intermediate market structures. The aforementioned studies assume that demand is perfectly inelastic: each consumers buys one unit subject to a reservation price. Furthermore, both studies assume that production costs are zero (an assumption that we adopt in our study as well).

Under the assumption that market power lies with the buyers, the studies by Zhang and Sexton (2001) and by Fousekis (2011a,b) relate to a great degree with the present work. Zhang and Sexton (2001) consider the FOB and UD pricing strategies between two profit maximizing IOFs in a market of a spatially dispersed duopsony. The authors extend the Kats and Thisse (1993) two stage model by accounting for the fact that in the food processing sector market power often lies with the buyers rather than with the sellers.[2] Zhang and Sexton employ a supply function with positive price elasticity, relaxing this way the assumption made by the studies of Espinosa (1992) and Kats and Thisse (1993) where the demand function was taken to be perfectly inelastic. The latter has been shown to bias the firms' pricing choices in favor of UD pricing. Zhang and Sexton's findings reveal that mutual FOB is the Nash equilibrium for intrinsically competitive market structures. Mixed FOB–UD pricing is the Nash equilibrium of the spatial game in less competitive markets. The strategic choice of mutual UD pricing emerges as the Nash equilibrium as spatial competition weakens. Following Zhang and Sexton (2001), the work by Fousekis (2011a) considers the FOB and UD strategic pricing choices but in a mixed duopsony market where spatial competition takes place between a profit maximizing

[2]Primary input bulkiness, perishability, high storage costs, limited mobility and access to alternative buyers as well as economies of scale in processing (Sexton, 1990; Tribl, 2009) are some of the reasons that can explain buyers' market power.

IOF and a welfare maximizing COOP. The results reveal that UD (FOB) pricing will be chosen by both competitors in markets where transportation costs are small (large) relative to the net value of the primary product. A mixed FOB (COOP)–UD (IOF) pricing is the Nash equilibrium of the game for intermediate market structures. In the third study, Fousekis (2011b) analyzes FOB and UD pricing policies in a pure duopsony spatial market where competition takes place between two welfare maximizing COOPs. COOPS are pricing according to the net average revenue product (NARP). According to the results, the aggressive (UD,UD) price configuration is the Nash equilibrium for high intensity of competition, whereas the quasi-collusive (FOB, FOB) is the Pareto superior Nash equilibrium as space gets more important.

Zhang and Sexton (2001) and Fousekis (2011a,b) use the food processing sector in order to justify buyers' market power and subsequently analyze spatial competition in pure and mixed duopsonies. In the agri-food system cooperatives very often compete alongside IOFs. In the US agricultural sector cooperatives account for 25 to 30 percent of the total farm marketing and supply expenditures (Drivas and Giannakas, 2010). In the European Union COOPs contribute more than 50% of the added value in the production, processing and commercialization of farm products (Fousekis, 2011a). Furthermore, cooperatives like the Dutch Cosun in sugar and the New Zealand Fontera in dairy products operate internationally (Fousekis, 2016). But, despite the prevalence of mixed markets, the relevant literature on spatial oligopolistic competition with the presence of cooperatives (IOF vs COOP and/or COOP vs COOP) has not received the appropriate attention. Furthermore, research results from the new empirical industrial organization on the existence of market power in the agri–food sector with the use of national or regional data, yields no or very little understanding on spatial imperfect competition. Given that FOB and UD are the main pricing strategies employed by IOFs and COOPs, the present study attempts to improve our understanding of spatial competition in the real world food (and not only) markets. FOB pricing is often the strategic choice in the real world market of cereals. On the other hand, UD pricing is usually the strategic choice in the real world markets of fruits, vegetables and raw milk.

In the light of the preceding, the objective of this work is to determine the equilibrium FOB/UD pricing strategies in pure (IOF vs IOF and COOP vs COOP) and mixed (IOF vs COOP) spatial duopolies. The IOF(s) and and the consumer COOP(s) provide the same physical good or service to consumers. We set up a two stage game similar to Kats and Thisse (1993) but we relax the assumption of perfectly inelastic demand since it can bias the results in favor of UD pricing. The present article assumes a linear demand function with a unitary (negative) slope. Furthermore, we borrow elements, very crucial to the analysis, from Zhang and Sexton (2001) and from Fousekis (2011a). We employ Hotelling-Smithies conjectures which is the spatial analogue of Bertrand-type competition.[3] Under this behavior, each firm assumes that the prices of the competitors are fixed. Lastly, in line with Espinosa (1992) and Kats and Thisse (1993), this work assumes that production costs are zero.

The case of a spatial competition with two IOFs is presented first and is also used as a benchmark for determining the economic ramifications of cooperative involvement in spatial competition. With the replacement of a profit maximizing IOF with a member welfare-maximizing COOP initially, and with the replacement of both IOFs with two COOPs later in the analysis, we attempt to shed light on the ability of COOPs to discipline private firms in the case of different combinations of pricing strategies. The cases where the IOF and the COOP are spatial monopolists are also reported.

In what follows, section 2 presents the IOF and the COOP as spatial monopolists. Section 3 analyzes pure and mixed spatial competition and the solution of the second-stage games. Section 4 presents the pure strategy Nash equilibria for each

[3]Capozza and Van Order (1978) use a linear demand function and Hotelling–Smithies conjectures.

game. Discussion of the results is offered in Section 5. Section 6 offers conclusions.

2 The IOF and the COOP as spatial monopolists

We assume that homogeneous consumers are continuously dispersed along a line with infinite length, according to the uniform density with D=1. The IOF (COOP) seller is located at the one end of the line. Average costs and marginal costs of production are constant, c.[4] Following Espinosa (1992) and Kats and Thisse (1993), we subsequently set the production costs c equal to zero. Each consumer has a linear demand function: $q = 1 - P$, where q is the quantity demanded and P is the selling price of the good. Following the literature, $m^I(m^C)$ is the mill price when the IOF (COOP) uses FOB pricing, and $u^I(u^C)$ is the mill price when the IOF (COOP) uses UD pricing. When the pricing policy is FOB, each consumer picks up the product, pays the mill price $m^I(m^C)$ and incurs the transportation costs from the firm's to the consumer's location.[5] Hence, under FOB pricing, the price that each consumer pays is: $P = m + \gamma r$, where γ is the transportation cost per unit of distance and r is the consumer's distance from the seller.[6] Under UD pricing strategy the firm or the cooperative organization delivers the product at the consumer's location. Thus, under UD pricing, the price that each consumer pays is: $P = u$.

[4]This study assumes zero fixed costs.

[5]The seller may also deliver the good to the buyer's location, as long as it charges mill price plus shipping costs.

[6]Following the relevant literature, the present article assumes that the transportation costs per unit of distance (γ) accounts also for the importance of space in the market. For lower values of γ, space is not important in the spatial market and competition between the agents is more intense. As γ is increasing, space gets more important, since the per unit shipping costs are higher, and the intensity of competition between the agents gets lower. Beyond certain values of γ, the agents act as spatial monopolists. The absolute importance of space in a market is defined as the product of the transportation cost per unit and the length of the market. Under the assumption of a unit length, the absolute importance of space is equal to γ.

2.1 The IOF as spatial monopolist

Given our assumptions, when the IOF employs FOB pricing it maximizes profits according to:

$$\Pi^I_{FOB} = (m^I - c)\int_0^{\hat{R}^I_{FOB}} (1 - m^I - \gamma\, r)\, \mathrm{d}r \overset{(c=0)}{=} m^I \int_0^{\hat{R}^I_{FOB}} (1 - m^I - \gamma\, r)\, \mathrm{d}r \quad (1)$$

The term inside the first parenthesis on the right hand side of equation 1 is the profit per unit produced. The parenthesis inside the integral is the total quantity demanded.

The market radius of the IOF is $\hat{R}^I_{FOB}$.[7] In order to find $\hat{R}^I_{FOB}$, which coincides with the location of the marginal (or indifferent) consumer, we set: $1 - m^I - \gamma\, \hat{R}^I_{FOB} = 0$. Solving for the location of the indifferent consumer we obtain: $\hat{R}^I_{FOB} = \frac{1-m^I}{\gamma}$.

Substituting in equation 1 for $\hat{R}^I_{FOB}$ and maximizing with respect to m^I we obtain: $m^I = \frac{1+2\,c}{3}$, $\hat{R}^I_{FOB} = \frac{2\,(1-c)}{3\,\gamma}$ and $\Pi^I_{FOB} = \frac{2\,(1-c)^3}{27\,\gamma}$. Setting the production costs equal to zero, namely $c = 0$, we get: $m^I = \frac{1}{3}$, $\hat{R}^I_{FOB} = \frac{2}{3\gamma}$ and $\Pi^I_{FOB} = \frac{2}{27\,\gamma}$.

Profit maximization when the IOF employs UD pricing is:

$$\Pi^I_{UD} = (1 - u^I)\int_0^{\hat{R}^I_{UD}} (u^I - c - \gamma\, r)\, \mathrm{d}r \overset{(c=0)}{=} (1 - u^I)\int_0^{\hat{R}^I_{UD}} (u^I - \gamma\, r)\, \mathrm{d}r \quad (2)$$

The term inside the first parenthesis on the right hand side of equation 2 is the profit per unit produced. The parenthesis inside the integral is the total quantity demanded. $\hat{R}^I_{UD}$ is the market radius. In order to find the location ($\hat{R}^I_{UD}$) of the indifferent consumer we set: $u - c - \gamma\, \hat{R}^I_{UD} = 0$. Solving for the location of the indifferent consumer we get: $\hat{R}^I_{UD} = \frac{u^I - c}{\gamma}$.

Substituting in equation 2 for $\hat{R}^I_{UD}$ and maximizing with respect to u^I we get: $u^I = \frac{2+c}{3}$, $\hat{R}^I_{UD} = \frac{2\,(1-c)}{3\,\gamma}$ and $\Pi^I_{FOB} = \frac{2\,(1-c)^3}{27\,\gamma}$. Setting $c = 0$, we obtain: $u^I = \frac{2}{3}$, $\hat{R}^I_{UD} = \frac{2}{3\,\gamma}$ and $\Pi^I_{UD} = \frac{2}{27\,\gamma}$.

[7] Monopolist faces demand up to the point where $q(P) \geqslant 0$.

2.2 The COOP as spatial monopolist

For the cooperative organizations the relevant literature uses a number of possible objective functions to be optimized. The present study maximizes the welfare of the COOP's members (Fousekis, 2011a).

Under FOB pricing the COOP maximizes:

$$W^C_{FOB} = (m^C - c) \int_0^{\hat{R}^C_{FOB}} (1 - m^C - \gamma r)\, \mathrm{d}r + \frac{1}{2} \int_0^{\hat{R}^C_{FOB}} (1 - m^C - \gamma r)^2\, \mathrm{d}r \qquad (3)$$

The term inside the first parenthesis on the right hand side of equation 3 is the profit per unit of production. The parenthesis inside the integral is the total quantity demanded. $\hat{R}^C_{FOB}$ is the market radius. In order to find the location ($\hat{R}^C_{FOB}$) of the marginal consumer we set: $1 - m^I - \gamma\, \hat{R}^I_{FOB} = 0$. Solving for the location of the indifferent consumer we obtain: $\hat{R}^I_{FOB} = \frac{1-m^I}{\gamma}$.

Substituting in equation 3 for $\hat{R}^I_{FOB}$ and maximizing with respect to m^C we get: $m^C = c$, $\hat{R}^C_{FOB} = \frac{(1-c)}{\gamma}$ and $W^C_{FOB} = \frac{(1-c)^3}{6\,\gamma}$. As we observe the FOB pricing COOP prices according to the average production cost (c). Setting the production costs equal to zero, we get: $m^C = 0$, $\hat{R}^C_{FOB} = \frac{1}{\gamma}$ and $W^C_{FOB} = \frac{1}{6\,\gamma}$.

Profit maximization when the COOP employs UD pricing is:

$$W^C_{UD} = (1 - u^C) \int_0^{\hat{R}^C_{UD}} (u^C - c - \gamma\, r)\, \mathrm{d}r + \frac{1}{2} \int_0^{\hat{R}^C_{UD}} (1 - u^C)^2\, \mathrm{d}r \qquad (4)$$

The term inside the first parenthesis on the right hand side of equation 4 is the profit per unit produced. The parenthesis inside the integral is the total quantity demanded. $\hat{R}^C_{UD}$ is the market radius. In order to find the location ($\hat{R}^I_{UD}$) of the indifferent consumer we set: $u^C - c - \gamma\, \hat{R}^C_{UD} = 0$. Solving for the location of the indifferent consumer we get $\hat{R}^C_{UD} = \frac{u^C - c}{\gamma}$.

Substituting in equation 4 for $\hat{R}^C_{UD}$ and maximizing with respect to u^C we get: $u^C = \dfrac{1 + 2\,c}{3}$, $\hat{R}^C_{UD} = \dfrac{2\,(1-c)}{3\,\gamma}$ and $W^C_{UD} = \dfrac{8\,(1-c)^3}{81\,\gamma}$. As derived in the Appendix, the total average production costs of the UD pricing COOP, when serving market area $\hat{R}$, are equal to: $AC = \frac{\gamma}{2}\,\hat{R} + c$. We also show (in the appendix) that the UD pricing COOP prices according to the total average production costs, as the FOB pricing COOP does. Setting c equal to zero we obtain: $u^C = \dfrac{1}{3}$, $\hat{R}^C_{UD} = \dfrac{2}{3\,\gamma}$ and $W^C_{UD} = \dfrac{8}{81\,\gamma}$.

Table 1 below summarizes all the findings.

Table 1: IOF and COOP as spatial monopolists

Pricing strategies	**IOF**	**COOP**
FOB:	$m^I = \frac{1}{3}$ $\hat{R}^I_{FOB} = \frac{2}{3\gamma}$ $\Pi^I_{FOB} = \frac{2}{27\gamma}$	$m^C = 0$ $\hat{R}^C_{FOB} = \frac{1}{\gamma}$ $W^C_{FOB} = \frac{1}{6\gamma}$
UD:	$u^I = \frac{2}{3}$ $\hat{R}^I_{UD} = \frac{2}{3\gamma}$ $\Pi^I_{UD} = \frac{2}{27\gamma}$	$u^C = \frac{1}{3}$ $\hat{R}^C_{UD} = \frac{2}{3\gamma}$ $W^C_{UD} = \frac{4}{27\gamma}$

3 Pure and mixed spatial competition

Maintaining the assumptions of section 2, we let the IOFs and/or the COOPS to be located at the two ends of a line with unit length. Firms hold Hotteling-Smithies conjectures when competing.

Tribl (2009) and Fousekis (2011a,b) assume that processing cooperatives price according to the net average revenue product (NARP). This behavior is consistent with the maximization of member welfare subject to break even constraint in processing (Cotterill, 1987). The present work adopts the same line of logic of the aforementioned studies. A behavior consistent with NARP pricing, when buying and processing the primary input, is pricing according to the average cost, when producing and selling the final product. Accordingly, the present work assumes that the FOB/UD pricing cooperative organizations price according to their average production costs. The equilibrium prices, of the FOB as well as the UD pricing COOPs when they operate as spatial monopolists (section 2), support the assumption of pricing according to the average cost of production.

All the equilibrium relationships are derived while setting the constant average and marginal costs of production (c) equal to zero (Espinosa, 1992; Kats and Thisse, 1993).

3.1 Pure duopoly: IOF vs IOF

3.1.1 Both IOFs use FOB pricing

Lets assume that firm A is located at the left end of the line and firm B is located at the right end of the line with unit length (Figure 1). Firm A(Firm B) sells at price $m_A(m_B)$ and consumers are responsible for the shipping costs. The market boundary between the two IOFs is determined as:

$$m_A + \gamma\,\hat{R} = m_B + \gamma\,(1-\hat{R}) \Rightarrow \hat{R} = \frac{m_B - m_A + \gamma}{2\,\gamma} \tag{5}$$

Firms are symmetric, which means that: $m_A = m_B = m$. Hence, each IOF chooses m to maximize:

$$\Pi_{FOB,FOB} = (m-c)\int_0^{\hat{R}} (1-m-\gamma\,r)\,dr \overset{(c=0)}{=} m\int_0^{\hat{R}} (1-m-\gamma\,r)\,dr \tag{6}$$

We substitute for $\hat{R}$ from equation 5 and maximize equation 6 with respect to m. The equilibrium mill price for each IOF is:

$$m_A = m_B = m = \frac{(2-5\,\gamma) + \sqrt{21\,\gamma^2 - 4\,\gamma + 4}}{4} \tag{7}$$

For $\gamma \leqslant \frac{4}{3}$, the two IOFs compete for the market boundary and each firm serves fifty percent of the market. For $\gamma > \frac{4}{3}$, competition ceases to exist and each FOB pricing firm operates as an isolated spatial monopolist (Table 1).

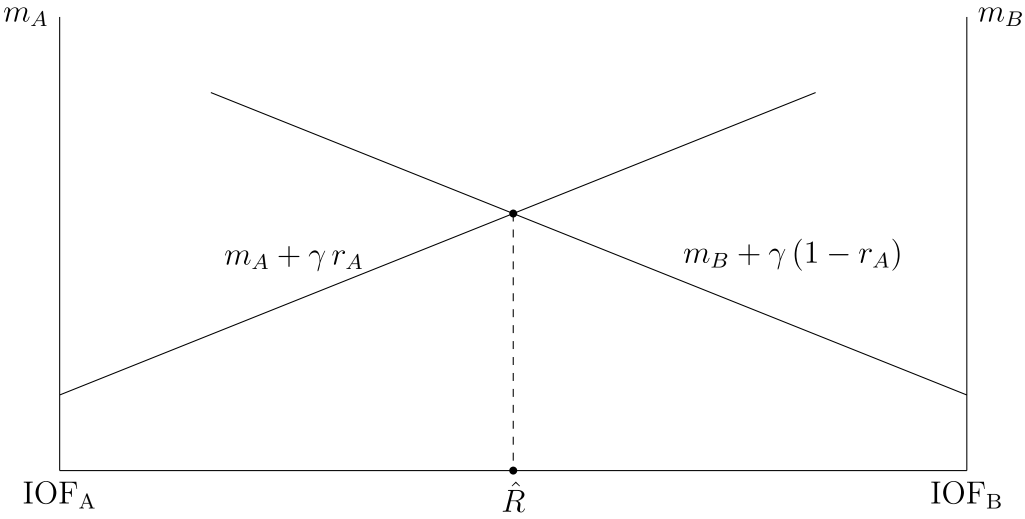

Figure 1: Competition when both IOFs use FOB pricing.

3.1.2 One IOF uses FOB pricing and the other uses UD pricing

Lets assume that the FOB pricing firm is located at the left end and the UD pricing firm is located at the right end of the line with unit length. Three subcases are relevant. In the first one, for lower values of γ, the market boundary is defined by the IOFs' gross prices. In the second subcase, for intermediate values of γ, the boundary is determined by the UD pricing IOF's monopoly market area. In the third one, the IOFs operate as isolated spatial monopolists.

For small values of γ, the market boundary between the two IOFs is determined as:

$$m + \gamma\,\hat{R} = u \Rightarrow \hat{R} = \frac{u - m}{\gamma} \tag{8}$$

The FOB pricing IOF picks m to maximize:

$$\Pi_{FOB} = (m - c)\int_0^{\hat{R}} (1 - m - \gamma\, r)\, dr \overset{(c=0)}{=} m\int_0^{\hat{R}} (1 - m - \gamma\, r)\, dr \tag{9}$$

Maximizing equation 9 with respect to m, for a given price u by the UD pricing IOF, we get the reaction function for the FOB pricing IOF:

$$m = \frac{2 - \sqrt{3\,u^2 - 6\,u + 4}}{3} \tag{10}$$

The UD pricing IOF maximizes:

$$\Pi_{UD} = (1-u)\int_{\hat{R}}^{1} (u - c - \gamma\,(1-r))\,dr \overset{(c=0)}{=} (1-u)\int_{\hat{R}}^{1} (u - \gamma\,(1-r))\,dr \tag{11}$$

Maximizing equation 11 with respect to u, given the FOB pricing IOF's behavior, we obtain the reaction function of the UD pricing IOF:

$$u = \frac{(4\,m + 4\,\gamma + 3) - \sqrt{7\,m^2 + (14\,\gamma - 12)\,m + 7\,\gamma^2 - 12\,\gamma + 9}}{9} \tag{12}$$

Solving equations 10 and 12 simultaneously we obtain the equilibrium values for m and u. This mode of interaction continues until $\gamma = 1.0516$.[8]

In the second subcase, for intermediate values of γ, the boundary is determined by the UD pricing IOF's monopoly market area. The monopoly price for the UD pricing IOF is $u = \frac{2}{3}$ and the market area is $\hat{R}_{UD} = \frac{2}{3\,\gamma}$. As Figure 2 illustrates, consumers in the interval $(R^E, 1 - \hat{R}_{UD},)$ will bear the transportation/shipping costs and purchase the product from the UD pricing IOF, at a lower gross price. Accordingly, the effective market boundary of the UD pricing IOF is R^E.

The effective boundary R^E is calculated as:

$$m + \gamma\,R^E = u + \gamma\,(1 - \hat{R}_{UD} - R^E) \overset{(u=\frac{2}{3},\hat{R}_{UD}=\frac{2}{3\,\gamma})}{\Rightarrow} R^E = \frac{\gamma - m}{2\,\gamma} \tag{13}$$

The term $\gamma\,(1-\hat{R}_{UD}-R^E)$, after the plus sigh on the right hand side of equation 13, represents the transportation costs involved when consumers travel between R^E and

[8]If we set the equilibrium value u (equation 12) equal to $\frac{2}{3}$ (the monopoly price for the UD pricing IOF), we find $\gamma = 1.0516$.

$(1 - \hat{R}_{UD})$, in order to purchase the good from the UD pricing IOF. As equation 13 demonstrates, the effective boundary of the UD pricing IOF equals to: $R^E = \frac{\gamma - m}{2\gamma}$.

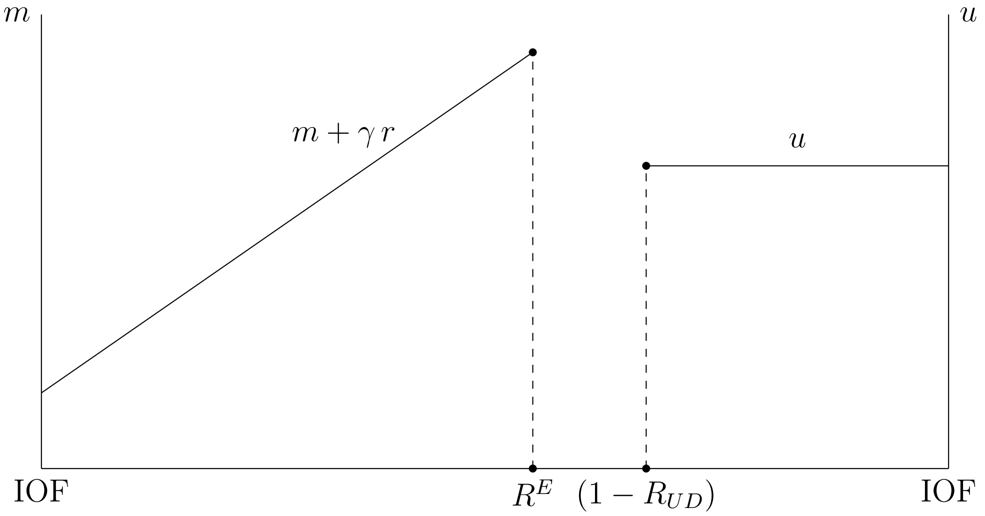

Figure 2: Competition when one IOF uses FOB and the other uses UD pricing (the *effective* boundary).

Given the behavior of the UD pricing IOF, the FOB pricing IOF acts as a spatial monopolist, with a restricted market radius from zero to R^E.

Accordingly, profits for the FOB pricing IOF are:

$$\Pi_{FOB} = (m - c) \int_0^{R^E} (1 - m - \gamma r)\, dr \overset{(c=0)}{=} m \int_0^{\frac{\gamma - m}{2\gamma}} (1 - m - \gamma r)\, dr \tag{14}$$

Maximizing equation 14 with respect to m, we obtain the equilibrium mill price:

$$m = \frac{2\sqrt{21\,g^4 - 42\,g^3 + 15\,g^2 + 6\,g + 4} + (12\,g^2 - 12\,g + 4)}{15} \tag{15}$$

For $\gamma \leqslant 1.0516$, the market boundary is determined by the firms' gross prices, as in equation 8. For $1.0516 < \gamma \leqslant \frac{4}{3}$, the market boundary is the UD–pricing IOF's effective (R^E) market area. For $\gamma > \frac{4}{3}$, the two IOFs operate as spatial monopolists and competition ceases to exist.

3.1.3 Both IOFs use UD pricing

Pure strategy equilibria, when symmetric UD pricing IOFs face competition from each other, do not exist (Schuler and Hobbs, 1982; Zhang and Sexton, 2001). An equilibrium in pure strategies exists only for those values of γ that enables firms to behave as isolated spatial monopolists. However a symmetric, mixed strategy Nash equilibrium exists.[9]

Given a price for firm B, we analyze firm A's profitability under two alternative pricing policies: i) Firm A undercuts firm B's price, and ii) Firm A overbids firm B's price. Firm A does not exactly match Firm B's price because this is never the best response on behalf of firm A (Zhang and Sexton, 2001).

When Firm A undercuts firm B's higher price ($u_A < u_B$), the maximum market radius of the undercutting IOF (+) is: $\hat{R}_A^+ = \frac{u_A^+}{\gamma}$. Beyond $\hat{R}_A^+$ firm A faces no demand.

Profits (Π_A^+) for firm A are given by:

$$\Pi_A^+ = (1-u_A^+)\int_{\hat{0}}^{\hat{R}_A^+}(u_A^+-c-\gamma\,r)\,dr \overset{(c=0)}{=} (1-u_A^+)\int_{\hat{0}}^{\frac{u_A^+}{\gamma}}(u_A^+-\gamma\,r)\,dr = (u_A^+)^2\frac{(1-u_A^+)}{2\,\gamma} \tag{16}$$

When Firm A concedes to firm B's lower price ($u_A > u_B$), the market radius of the conceding IOF ($-$) is: $(1-\hat{R}_A) = (1-\frac{u_A^-}{\gamma})$.

Profits (Π_A^-) for firm A are given by:

$$\begin{aligned}\Pi_A^- &= (1-u_A^-)\int_{\hat{0}}^{\hat{R}_A}(u_A^- - c - \gamma\,r)\,dr \overset{(c=0)}{=} (1-u_A^-)\int_{\hat{0}}^{(1-\frac{u_A^-}{\gamma})}(u_A^- - \gamma\,r)\,dr \\ &= (4\,\gamma\,u_A^- - 3\,(u_A^-)^2-\gamma^2)\frac{(1-u_A^-)}{2\,\gamma}\end{aligned} \tag{17}$$

Firm A's profits when undercutting (equation 16), and Firm A's profits when conceding (equation 17), are discontinuous in u_A. A violation of continuity and

[9] We follow the logic of Zhang and Sexton (2001). Their analysis relies on the prior studies by Beckmann (1973) and Shilony (1977).

quasi–concavity in payoffs leads to the non–existence of Nash equilibrium in pure strategies (Dasgupta and Maskin, 1986). We follow Zhang and Sexton (2001) in order to obtain a symmetric Nash equilibrium in mixed strategy with the employment of the cumulative distribution function $G(u)$. If G is continuous in u_A, then $G(u_A)$ is the optimal mixed strategy for firm A.

When firm A undercuts firm B's price:

$$G(u_A) = \int_{\hat{0}}^{u_A} dG(u_B) = prob(0 < u_A \leqslant u_B) \tag{18}$$

and when firm A concedes:

$$1 - G(u_A) = \int_{u_A}^{1} dG(u_B) = prob(u_A > u_B) \tag{19}$$

The expected payoff of firm A, when it charges price u_A and firm B employs G, is:

$$\begin{aligned} E[\Pi_A(u_A, u_B)] &= \int_{\hat{0}}^{u_A} \Pi_A^+ \, dG(u_B) + \int_{u_A}^{1} \Pi_A^- \, dG(u_B) \\ &= \Pi_A^+ \; G(u_A) + \Pi_A^- \; (1 - G(u_A)) \end{aligned} \tag{20}$$

If u_{A1} is the higher price that firm A can charge, then $G(u_{A1}) = 0$, namely the probability that Firm A will charge a price higher than u_{A1} is zero. This implies that, when charging u_{A1}, firm A concedes to B's lower price. Profits for firm A, when conceding, are given by equation 17:

$$V = \Pi_A^- = (4\,\gamma\,u_{A1} - 3\,(u_{A1})^2 - \gamma^2)\frac{(1 - u_{A1})}{2\,\gamma}, \tag{21}$$

where V is the value of the game. Maximizing V with respect to u_{A1} we get:

$$u_{A1} = \frac{(4\,\gamma + 3) - \sqrt{7\,\gamma^2 - 12\,\gamma + 9}}{9}, \tag{22}$$

Equation 22 provides us with u_{A1}, which is the higher UD price that firm A will charge in mixed strategy.

Substituting the optimal value u_{A1} (equation 22) in equation 21, we obtain the maximum value (V^*)of the game:

$$\begin{aligned} V^* &= \frac{1}{2\gamma}\{(1-\frac{4\gamma+3-\sqrt{7\gamma^2-12\gamma+9}}{9})\,[4\gamma\,(\frac{4\gamma+3-\sqrt{7\gamma^2-12\gamma+9}}{9})- \\ &\quad -\frac{(4\gamma+3-\sqrt{7\gamma^2-12\gamma+9})^2}{27}-\gamma^2]\} \end{aligned} \tag{23}$$

If u_{A2} is the lower price that firm A will ever charge, then $G(u_{A2}) = 1$, namely the probability that Firm A will charge a price higher than u_{A2} is one. This implies that, under u_{A2}, firm A is adopting the undercutting strategy. Profits for firm A, when undercutting firm B's price, are given by equation 16:

$$V = \Pi_A^+ = \frac{(u_{A2})^2}{2\gamma}\,(1-u_{A2}) \tag{24}$$

where V is the value of the noncooperative game as in equation 21. Substituting the optimal value of the game (V^*), as given in equation 23, in equation 24, we can also obtain u^*_{A2}, which is the lower UD price that firm A will charge in mixed strategy.

For $\gamma \leqslant \frac{4}{3}$, the equilibrium is in mixed strategies and the payoff of each symmetric IOF is given by the optimal value function (V^*) of the game. For $\gamma > \frac{4}{3}$, each UD pricing IOF operates as an isolated spatial monopolist.

3.2 Mixed duopoly: COOP vs IOF

3.2.1 Both the COOP and the IOF use FOB pricing

Figure 3 presents the location of the two agents.[10] The COOP is located at the left end of the line (point 0) and prices according to the average cost of production, c.

[10]In the mixed duopoly subcase, the present work assumes that the COOP (IOF) is always located at the left (right) end of the line with unit length.

Hence, $m^C = c = 0$. The IOF is located at the right end of the line (point 1). The market boundary between the IOF and the COOP is determined as:

$$m^C + \gamma\,\hat{R} = m^I + \gamma\,(1 - \hat{R}) \overset{(m^C=c)}{\Rightarrow} c + \gamma\,\hat{R} = m^I + \gamma\,(1 - \hat{R}) \overset{(c=0)}{\Rightarrow} \hat{R} = \frac{1 - m^I + \gamma}{2\,\gamma} \quad (25)$$

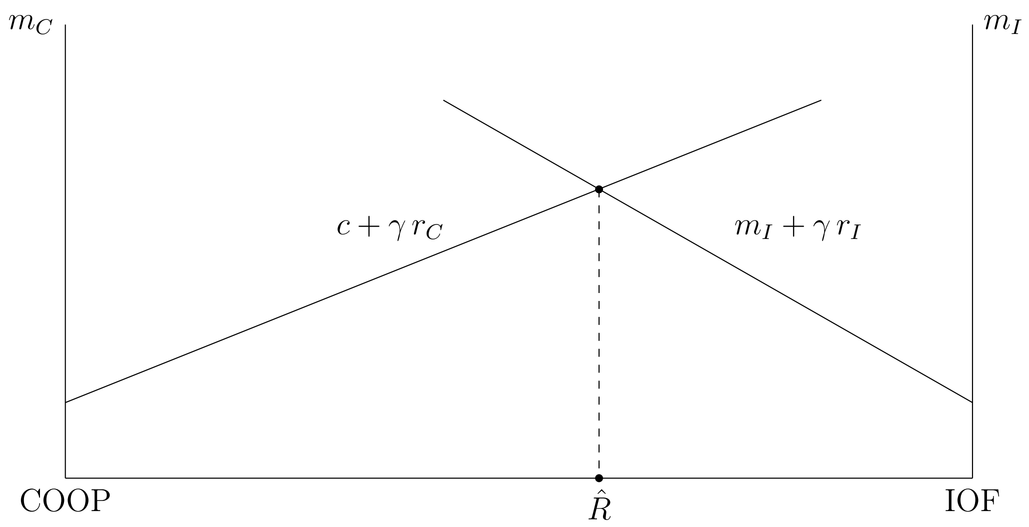

Figure 3: Competition when both the COOP and the IOF use FOB pricing.

The IOF picks m^I to maximize:

$$\Pi^I_{FOB,FOB} = (m^I - c) \int_{\hat{R}}^{1} (1 - m^I - \gamma\,(1 - r))\,dr \overset{(c=0)}{=} m^I \int_{\hat{R}}^{1} (1 - m^I - \gamma\,(1 - r))\,dr \quad (26)$$

We substitute for $\hat{R}$ from equation 25 and maximize equation 26 with respect to m^I. The equilibrium mill price for the IOF is:

$$m^I = \frac{(2\,\gamma + 4) - \sqrt{13\,\gamma^2 - 20\,\gamma + 16}}{9} \quad (27)$$

For $\gamma < \frac{5}{3}$, the IOF and the COOP compete for the market boundary. For $\gamma = \frac{5}{3}$,the COOP serves sixty percent of the market leaving the remaining forty percent for the IOF. The equilibrium mill prices are $m^C = c = 0$ and $m^I = \frac{1}{3}$. For $\gamma > \frac{5}{3}$, the two sellers become isolated spatial monopolists and there is no competition between the COOP and the IOF.

3.2.2 The COOP uses FOB pricing and the IOF uses UD pricing

The FOB pricing COOP sells according to the average cost of production, c. Hence, $m^C = c = 0$. For small values of γ, the market boundary is determined by the agents' gross prices:

$$m^C + \gamma\,\hat{R} = u^I \overset{(m^C=c)}{\Rightarrow} c + \gamma\,\hat{R} = u^I \overset{(c=0)}{\Rightarrow} \hat{R} = \frac{u^I}{\gamma} \tag{28}$$

The IOF picks u^I to maximize:

$$\Pi^I_{FOB,UD} = (1-u^I)\int_{\hat{R}}^{1}(u^I - c - \gamma\,(1-r))\,dr \overset{(c=0)}{=} (1-u^I)\int_{\hat{R}}^{1}(u^I - \gamma\,(1-r))\,dr \tag{29}$$

Substituting for $\hat{R}$ from equation 28 and maximizing equation 29 with respect to u^I, we obtain the equilibrium UD price for the IOF:

$$u^I = \frac{(4\,\gamma + 3) - \sqrt{7\,\gamma^2 - 12\,\gamma + 9}}{9} \tag{30}$$

For intermediate values of γ, the market boundary coincides with the IOF's effective boundary, which is located at R^E (figure 4). The COOP cannot extend its market boundary further than R^E. Consumers in the interval $(R^E, 1 - R^I_{UD})$ will bear the transportation/shipping costs and purchase the product from the IOF, at a lower gross price (Fousekis, 2011a). Accordingly, the effective market boundary of the IOF is R^E. At the effective boundary R^E it holds:

$$c + \gamma\,\hat{R^E} = u^I + \gamma\,(1 - R^I_{UD} - R^E) \overset{(c=0)}{\Rightarrow} \gamma\,\hat{R^E} = u^I + \gamma\,(1 - \frac{2}{3\,\gamma} - R^E) \tag{31}$$

The term $\gamma\,(1 - \frac{2}{3\,\gamma} - R^E)$, after the plus sigh on the right hand side of equation 31, represents the transportation/shipping costs involved when consumers travel from R^E to the IOF's monopoly boundary, in order to purchase the good from the IOF. Solving for the effective boundary of the IOF from equation 31 we obtain $R^E = \frac{1}{2}$.

For $\gamma < \frac{4}{3}$, the market boundary is determined by the relationship in equation 28. For $\frac{4}{3} \leqslant \gamma < 2$, the market boundary between the COOP and the IOF is defined by the IOF's effective boundary. For $\gamma = 2$, the monopolistic boundary of the IOF is located at 0.67 whereas its effective boundary at 0.5. For $\gamma > 2$, the two sellers operate as isolated spatial monopolists since competition stops existing between the COOP and the IOF.

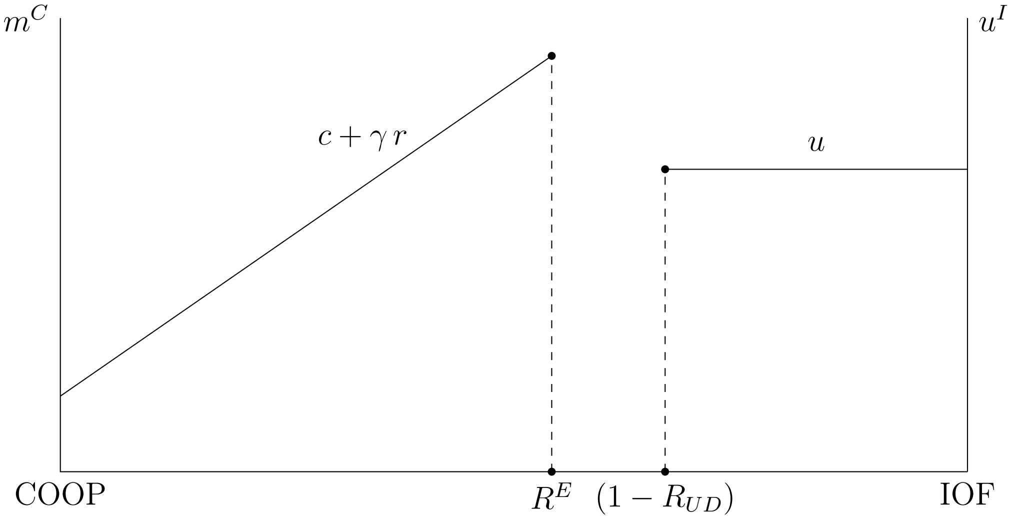

Figure 4: Competition when the COOP uses FOB and the IOF uses UD pricing (the IOF's *effective* boundary).

3.2.3 The COOP uses UD pricing and the IOF uses FOB pricing

Three subcases are relevant. In the first one, for small values of γ, the market boundary is defined by the IOF's and the COOP's gross prices. In the second subcase, for intermediate values of γ, the boundary is determined by the COOP's monopoly market radius. In the third one, the IOF and the COOP operate as isolated spatial monopolists.

The uniform delivered pricing COOP prices according to the average cost of production: $u^C = \frac{\gamma}{2}\,\hat{R} + c$. Hence, in the first subcase, the market boundary between the IOF and the COOP is determined by the COOP's and the IOF's gross prices:

$$u^C = m^I + \gamma\,(1-\hat{R}) \overset{(u^C = \frac{\gamma}{2}\hat{R}+c)}{\Rightarrow} \frac{\gamma}{2}\,\hat{R} + c = m^I + \gamma\,(1-\hat{R}) \overset{(c=0)}{\Rightarrow} \hat{R} = \frac{2\,(m^I+\gamma)}{3\,\gamma} \quad (32)$$

The IOF picks m^I to maximize:

$$\Pi^I_{UD,FOB} = (m^I - c)\int_{\hat{R}}^{1}(1-m^I-\gamma\,(1-r))\,dr \overset{(c=0)}{=} m^I \int_{\hat{R}}^{1}(1-m^I-\gamma\,(1-r))\,dr \quad (33)$$

We substitute for $\hat{R}$ from equation 32 and maximize equation 33 with respect to m^I. The equilibrium FOB price for the IOF is:

$$m^I = \frac{(\gamma+6) - \sqrt{7\,\gamma^2 - 24\,\gamma + 36}}{12} \quad (34)$$

In the second subcase, the COOP operates as a spatial monopolist. The monopoly price for the UD pricing COOP is $u^C = \frac{1}{3}$ and the market area is $\hat{R}^C_{UD} = \frac{2}{3\gamma}$. Beyond $\hat{R}^C_{UD} = \frac{2}{3\gamma}$, the COOP does not serve its customers since there is a violation of the even-breaking constraint.

Given the COOP's behavior, the FOB pricing IOF acts as a spatial monopolist with a restricted market radius: $(1 - \hat{R}^C_{UD}) = (1 - \frac{2}{3\gamma})$.

The IOF chooses m^I to maximize:

$$\Pi^I_{UD,FOB} = (m^I - c)\int_{\frac{2}{3\gamma}}^{1}(1-m^I-\gamma\,(1-r))\,dr \overset{(c=0)}{=} m^I \int_{\frac{2}{3\gamma}}^{1}(1-m^I-\gamma\,(1-r))\,dr \quad (35)$$

The IOF's equilibrium mill price is:

$$m^I = \frac{8 - 3\,\gamma}{12} \quad (36)$$

For $\gamma \leqslant 0.8148$, the market boundary is determined by the firms' gross prices. For $0.8148 < \gamma \leqslant \frac{4}{3}$, the market boundary is the COOP's monopoly boundary. For $\gamma > \frac{4}{3}$, the two sellers become isolated spatial monopolists and competition ceases to exist between the COOP and the IOF.

3.2.4 Both the COOP and the IOF use UD pricing

When symmetric UD pricing IOFs face competition from each other, equilibrium in pure strategies exists only for those values of γ that allows firms to behave as isolated spatial monopolists.do not exist (Schuler and Hobbs, 1982; Zhang and Sexton, 2001).

We follow Fousekis (2011a) in order to determine whether, for the game between the UD pricing IOF and COOP examined here, the equilibrium is in pure or mixed strategies. The first step is to obtain the objective functions for the IOF and the COOP, under undercutting and conceding. Subsequently, we will check if they are continuous or discontinuous.[11]

When the COOP concedes ($-$) to IOF's lower price ($u_C > u_I$), the effective boundary of the undercutting IOF satisfies the relationship:

$$u_I^+ = u_C^- - \gamma\,(1 - R_I^+ - R^E) \tag{37}$$

The IOF by undercutting operates as a spatial monopolist and sets:

$$u_I^+ - c - \gamma\,R_I^+ = 0 \overset{(c=0)}{\Rightarrow} R_I^+ = \frac{u_I^+}{\gamma} \tag{38}$$

Substituting for R_I^+ (equation 38) in equation 37, we obtain:

$$u_C^- = \gamma - \gamma\,R^E \tag{39}$$

The COOP prices according to the average cost of production:

$$u_C^- = \frac{\gamma}{2}\,R^E \tag{40}$$

[11]Dasgupta and Maskin (1986) showed that a violation of continuity and quasi–concavity in payoffs leads to the non–existence of Nash equilibrium in pure strategies.

Equating relationships 39 and 40 we obtain:

$$\gamma - \gamma\, R^{E} = \frac{\gamma}{2}\, R^{E} \Rightarrow R^{E} = \frac{2}{3} \tag{41}$$

Substituting for $\hat{R}^{E}$ (equation 41) into equation 39 or equation 40 we obtain:

$$u_{C}^{-} = \gamma - \gamma\,(\frac{2}{3}) \Rightarrow \;\; u_{C}^{-} = \frac{\gamma}{3} \tag{42}$$

When the COOP concedes (-), the welfare of the members is:

$$W_{C}^{-} = \frac{1}{2}\int_{0}^{R^{E}} (1 - u_{C}^{-})^{2}\, dr = \frac{1}{2}\,(1 - u_{C}^{-})^{2}\, R^{E} = \frac{1}{3}(1 - \frac{\gamma}{3})^{2} \tag{43}$$

On the other hand, the undercutting COOP (+) has a market area of: $R_{C}^{+} = \frac{2\, u_{C}^{+}}{\gamma}$ (since it prices according to the average cost of production). The welfare of the members is:

$$W_{C}^{+} = \frac{1}{2}\int_{0}^{R_{C}^{+}} (1 - u_{C}^{+})^{2}\, dr = \frac{1}{2}\,(1 - u_{C}^{+})^{2}\, R_{C}^{+} = (1 - u_{C}^{+})^{2}\, \frac{u_{C}^{+}}{\gamma} \tag{44}$$

When $u_{C}^{+} = \frac{\gamma}{3} = u_{C}^{-}$, member welfare from conceding and from undercutting are equal:

$$W_{A}^{-} = W_{A}^{+} = \frac{1}{3}(1 - \frac{\gamma}{3})^{2} \tag{45}$$

Hence, for those values of γ where the COOP and the IOF compete for the market area (and there is total coverage of market area), member welfare from undercutting and from conceding is identical. This means that the welfare function of the COOP is not discontinuous. As a consequence, the cooperative does not randomize between undercutting and conceding. Given that for small values of γ the UD pricing COOP does not randomize between conceding and undercutting, it wouldn't make sense for the IOF to randomize (Fousekis, 2011a). As an outcome, we can proceed to find the Nash equilibrium in pure strategies.

For small values of γ, the COOP sets $u^C = \frac{\gamma}{3}$ and serves a market area of $\hat{R}^C = \frac{2}{3}$. The boundary for the conceding IOF is located at $\hat{R}^C = \frac{2}{3}$ and its restricted market radius is: $1 - \hat{R}^C = \frac{1}{3}$.

Accordingly, the IOF chooses u^I to maximize:

$$\Pi^I_{UD,UD} = (1-u^I) \int_{\frac{2}{3\gamma}}^{1} (u^I - c - \gamma\,(1-r))\, dr \overset{(c=0)}{=} (1-u^I) \int_{\frac{2}{3\gamma}}^{1} (u^I - \gamma\,(1-r))\, dr \quad (46)$$

The IOF's equilibrium price is:

$$u^I = \frac{1}{2} + \frac{\gamma}{4}(1 - \frac{2}{3\gamma}) \quad (47)$$

For intermediate values of γ, the COOP operates as an isolated spatial monopolist and sets: $u^C = \frac{1}{3}$. The market area that the COOP serves is: $\hat{R}^C_{UD} = \frac{2}{3\gamma}$. Given the COOP's behavior, the restricted market area to be served by the conceding IOF is: $(1 - \hat{R}^C) = (1 - \frac{2}{3\gamma})$.

Accordingly, the conceding IOF chooses u^I to maximize:

$$\Pi^I_{UD,UD} = (1-u^I) \int_{\frac{2}{3\gamma}}^{1} (u^I - c - \gamma\,(1-r))\, dr \overset{(c=0)}{=} (1-u^I) \int_{\frac{2}{3\gamma}}^{1} (u^I - \gamma\,(1-r))\, dr \quad (48)$$

The IOF's equilibrium price is:

$$u^I = \frac{1}{2} + \frac{\gamma}{4}(1 - \frac{2}{3\gamma}) \quad (49)$$

Competition for the market area continues until $\gamma = \frac{4}{3}$.

Hence, for $\gamma \leqslant 1$, the COOP sets $u^C = \frac{\gamma}{3}$ and the IOF sets $u^I_{-} = \frac{1}{2} + \frac{\gamma}{4}(1 - \frac{2}{3\gamma})$. The market areas that the COOP and the IOF serve are, two thirds $(\frac{2}{3})$ and one third $(\frac{1}{3})$, respectively. For $1 < \gamma \leqslant \frac{4}{3}$, the COOP and the conceding IOF set $u^C = \frac{1}{3}$ and $u^I_{-} = \frac{1}{2} + \frac{\gamma}{4}(1 - \frac{2}{3\gamma})$, respectively. The market area that the COOP serves is $\hat{R}^C = \frac{2}{3\gamma}$, whereas the market radius for the IOF equals to $(1 - \hat{R}^C) = (1 - \frac{2}{3\gamma})$. For values of $\gamma > \frac{4}{3}$, the COOP and the IOF operate as isolated monopolists.

3.3 Pure duopoly: COOP vs COOP

3.3.1 Both COOPs use FOB pricing

COOP A is located at the left end of the line (point 0) and COOP B is located at the right end of the line (point 1).[12] COOP A (COOP B) sells at price $m_A(m_B)$ and customers are responsible for the transportation costs. Each COOP prices according to the average cost of production, c.

The market boundary is determined as:

$$m_A + \gamma\,\hat{R}_A = m_B + \gamma\,(1 - \hat{R}_A) \Rightarrow \hat{R}_A = \frac{m_B - m_A + \gamma}{2\,\gamma} \tag{50}$$

Due to symmetry: $m_A = m_B = m$. Furthermore, since each COOP prices according to the average cost of production: $m = c = 0$. In equilibrium, when the COOPs compete for the market area, $m = c = 0$ and $\hat{R}_A = \hat{R}_B = \frac{1}{2}$. This mode of interaction continues until $\gamma = 2$. Hence, for $\gamma \leqslant 2$, the two COOPs compete for the market boundary and each one of them serves half of the market. For $\gamma > 2$, competition ceases to exist and the two COOPs operate as isolated spatial monopolists.

3.3.2 One COOP uses FOB pricing and the other uses UD pricing

The UD pricing COOP is located at the left end of the line (point 0) and the FOB pricing COOP is located at the right end of the unit length line (point 1). $\hat{R}_{UD}(\hat{R}_{FOB})$ is the market area served by the UD (FOB) pricing COOP. When the two COOPs compete for the market radius, $\hat{R}_{FOB} = (1 - \hat{R}_{UD})$. Both COOPs price according to the average production costs: $m = c = 0$ and $u = c + \frac{\gamma}{2}\,\hat{R}_{UD} = \frac{\gamma}{2}\,\hat{R}_{UD}$.

[12]The same line of figures apply to this spatial game as well. The only difference is that two COOPs are at the ends of the line competing.

The market boundary i.s determined as:

$$m+\gamma\,(1-\hat{R}_{UD}) = u \overset{(m=c=0)}{\Rightarrow} \gamma\,(1-\hat{R}_{UD}) = u \overset{(u=\frac{\gamma}{2}\,\hat{R}_{UD})}{\Rightarrow} \gamma\,(1-\hat{R}_{UD}) = \frac{\gamma}{2}\,\hat{R}_{UD} \quad (51)$$

Solving equation 51 we obtain: $\hat{R}_{UD} = \frac{2}{3}$. Accordingly, $\hat{R}_{FOB} = \frac{1}{3}$ and $u = \frac{\gamma}{3}$. As we can observe, the more aggressive UD pricing COOP serves a larger market area.

The two COOPs compete for the market area until the gross consumer prices are equal:

$$\gamma\,(1-\frac{2}{3\,\gamma}) = \frac{1}{3} \;\Rightarrow \gamma = 1$$

Hence, for $\gamma \leq 1$, the two COOPs compete for the market boundary: the UD pricing COOP serves two thirds of the market and the FOB pricing COOP serves the remaining one third of the market.

For intermediate values of γ, the UD pricing COOP acts as an isolated monopolist. The FOB pricing COOP acts as an isolated monopolist as well, but with a restricted market area: $\hat{R}_{FOB} = (1-\frac{2}{3\,\gamma})$.

This mode of interaction continues until the market area the monopolistic FOB pricing COOP uses and the area that the monopolistic UD pricing COOP does not serve are equal to each other:

$$\frac{1}{\gamma} = (1-\frac{2}{3\,\gamma}) \;\Rightarrow \gamma = \frac{5}{3}$$

Accordingly, for $1 < \gamma \leqslant \frac{5}{3}$, the UD pricing COOP behaves as a spatial monopolist and the market area that serves is $\hat{R}_{UD} = \frac{2}{3\,\gamma}$. The FOB pricing COOP operates as a spatial monopolist with a constrained market radius $(1-\frac{2}{3\,\gamma})$. For $\gamma > \frac{5}{3}$, competition ceases to exist and each COOP behaves as an isolated spatial monopolist.

3.3.3 Both COOPs use UD pricing

We follow Fousekis (2011a,b) in order to determine whether, for the game between the IOF and COOP examined here, the equilibrium is in pure or mixed strategies. We are going to examine whether the welfare functions for the IOF and the COOP and verify their continuity (or discontinuity).

When COOP A undercuts (+) COOP B's price ($u_A < u_B$), member welfare is:

$$W_A^{+} = \frac{1}{2} \int_0^{R_A^+} (1 - u_A)^2 \, dr = \frac{1}{2} (1 - u_A)^2 \, \frac{2\, u_A}{\gamma} \; = (1 - u_A)^2 \, \frac{u_A}{\gamma} \tag{52}$$

When COOP A concedes (−) to COOP B's lower price ($u_A > u_B$), then member welfare is:

$$W_A^{-} = \frac{1}{2} \int_0^{R_A^-} (1 - u_A)^2 \, dr = \frac{1}{2} (1 - u_A)^2 \, (1 - \frac{2\, u_B}{\gamma}) \tag{53}$$

When the symmetric UD pricing COOPs compete for the market area, they set $u_A = u_B = (c + \frac{\gamma}{2} R) =$ total average production cost. Since $c = 0$, $u_A = u_B = \frac{\gamma}{2} R$. Under competition, and symmetry, $\hat{R}_A = \hat{R}_B = \frac{1}{2}$. Accordingly, $u_A = u_B = \frac{\gamma}{4}$.

Substituting the value of $\frac{\gamma}{4}$ (for u_A and u_B) in equations 52 and 53, and evaluating the welfare of the members when COOP A undercuts and when it concedes we get:

$$W_A^{+} = W_A^{-} = \frac{1}{4} (1 - \frac{\gamma}{4})^2 \tag{54}$$

Hence, for those values of γ where the symmetric COOPs compete for the market area (and there is total coverage of market area), welfare from undercutting and from conceding is identical. This means that the welfare function is not discontinuous and the cooperatives do not randomize between undercutting and conceding. As an outcome, we have the existence of a Nash equilibrium in pure strategies. Competition continues until $\gamma = \frac{4}{3}$. Thus, for $\gamma \leqslant \frac{4}{3}$, the UD pricing COOPs interact with each other and compete for the market area. For values of $\gamma > \frac{4}{3}$, the two COOPs operate as isolated monopolists.

4 Nash equilibria in pure strategies

In the first stage of the game the agents choose their pricing strategies (FOB or UD). In the second stage of the game the agents price according to the pricing rules as derived in Section 3. Figures 5 to 8 summarize the findings from the Nash equilibria of the second stage of the game. The pure strategy Nash equilibria, for the three different duopolistic spatial games examined in this article, are presented in what follows.

4.1 IOF vs IOF

Panel A of Figure 5 shows the IOF's profits, under FOB and UD pricing, when the rival IOF employs FOB pricing. For $\gamma \leqslant 0.494$, FOB pricing entails higher profits than UD pricing. For $0.494 < \gamma \leqslant 1.315$, UD pricing entails higher profits than FOB pricing. For $\gamma > 1.315$, FOB and UD pricing present the IOF with the same amount of profits.

Panel B of Figure 5 presents the IOF's profits, when employing FOB and UD pricing, given that the rival IOF uses UD pricing. For $\gamma \leqslant 0.607$, FOB pricing entails higher profits than UD pricing. For $0.607 < \gamma \leqslant 1.667$, UD pricing entails higher profits than FOB pricing. For $\gamma > 1.667$, FOB and UD pricing present the IOF with the same profits.

For $\gamma \leqslant 0.494$, FOB pricing is a strictly dominant strategy for both IOFs. For $0.494 < \gamma \leqslant 0.607$, the (FOB, UD) and (UD, FOB) are the pure strategy Nash equilibria. The (UD, UD) emerges as a Nash equilibrium for $0.607 < \gamma \leqslant 1.315$. The (UD, UD) is also the pure strategy Nash equilibrium for $1.315 < \gamma \leqslant 1.667$. For $\gamma > 1.667$, the IOFs act as isolated monopolists and competition is not present under any price strategy combination. Hence, and any combination of the pricing policies constitutes a pure strategy Nash equilibrium.

As the importance of space is decreasing and competition between the two IOFs is more intense (low values of γ), the quasi–collusive (FOB,FOB) pricing configuration is the pure strategy Nash equilibrium of the game. As space gets more important (higher costs of transportation), the more aggressive UD pricing is the strategic choice for both IOFs.

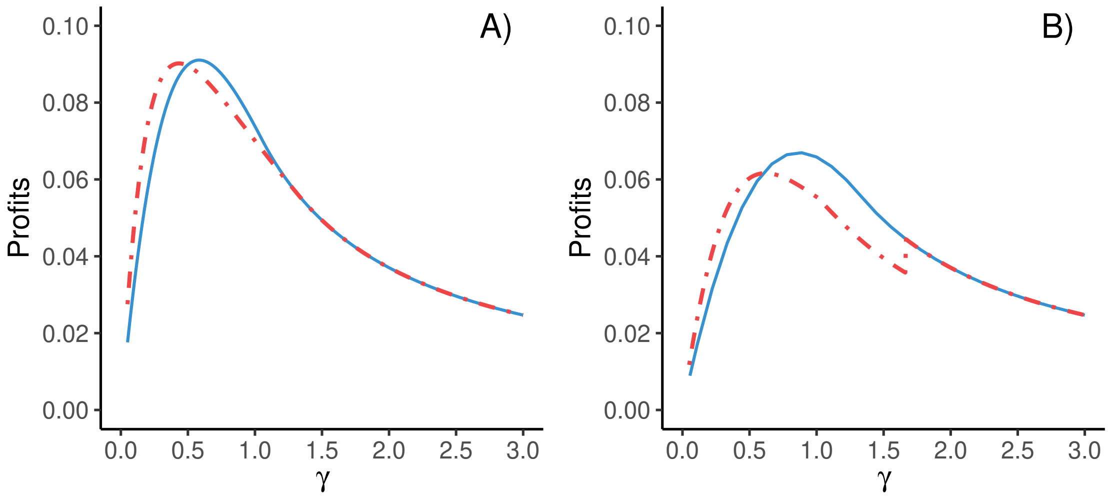

Figure 5: Profits for firm i given: A) firm j uses FOB pricing, and B) firm j uses UD pricing. Continuous blue (dotdashed red) lines represent UD (FOB) pricing strategy for firm i.

4.2 IOF vs COOP

Panel A of Figure 6 shows the payoff to the IOF, under FOB and UD pricing, when the rival COOP employs FOB pricing. UD pricing entails higher profits than FOB pricing for $\gamma \leqslant 2$. For $\gamma > 2$ the FOB and UD pricing policies present the IOF with the same amount of profits. Panel B of Figure 6 presents the profits to the IOF, under FOB and UD pricing, given that the COOP uses UD pricing. For $\gamma \leqslant 1$, UD pricing presents the IOF with higher profits than FOB pricing. Profits for the IOF are the same, under FOB and UD pricing, for $\gamma > 1$.

Panel A of Figure 7 shows the welfare of the COOP's members, under FOB and UD pricing, given that the rival IOF employs FOB pricing. For $\gamma \leqslant 0.734$, UD pricing entails higher member welfare than FOB pricing. For $\gamma > 0.734$, FOB pricing presents the COOP's member with higher welfare than UD pricing. Panel B of Figure 7 shows the COOP's member welfare, under FOB and UD pricing, given that the rival IOF uses UD pricing. UD pricing entails higher member welfare than FOB pricing for $\gamma \leqslant 0.874$ as well as for $\gamma > 2$. FOB pricing entails higher member welfare than UD pricing for $0.874 < \gamma \leqslant 2$.

For $\gamma \leqslant 0.734$, the UD pricing is a strictly dominant strategy for both the IOF and the COOP. For $0.734 < \gamma \leqslant 0.874$, the pricing strategy (UD,UD) is the still the pure strategy Nash equilibrium (but not strictly dominant). For $0.874 < \gamma \leqslant 1$, the (FOB,UD) is the weakly dominant strategy equilibrium, namely the COOP employing FOB pricing and the IOF using UD pricing. For $1 < \gamma \leqslant 2$, FOB pricing is a strictly dominant strategy for the COOP and UD pricing is a weakly dominant strategy for the IOF. For $\gamma > 2$, the (FOB,FOB) pricing configuration is the (pareto superior) pure strategy Nash equiblrium.

As space gets less important and competition between the two agents is more intense (low values of γ), UD pricing is the strictly dominant strategy for both the COOP and the IOF. Hence, the more aggresive (UD,UD) pricing configuration is the (striclty dominant) pure strategy Nash equilibrium of the game. As the cost of shipping is increasing (higher values of γ), the FOB pricing becomes part of the strategic choice for both agents. When acting as isolated spatial monopolists ($\gamma > 2$), (FOB,FOB) is the pure strategy Nash equilibrium of the game.

4.3 COOP vs COOP

Panel A of Figure 8 depicts the COOP's member welfare under FOB and UD pricing, when the rival COOP uses FOB pricing. The UD pricing entails higher welfare than FOB pricing for $\gamma \leqslant 1.035$. For $\gamma > 1.035$, the FOB pricing entails higher member welfare than UD pricing. Panel B of Figure 4 presents the COOP's member welfare,

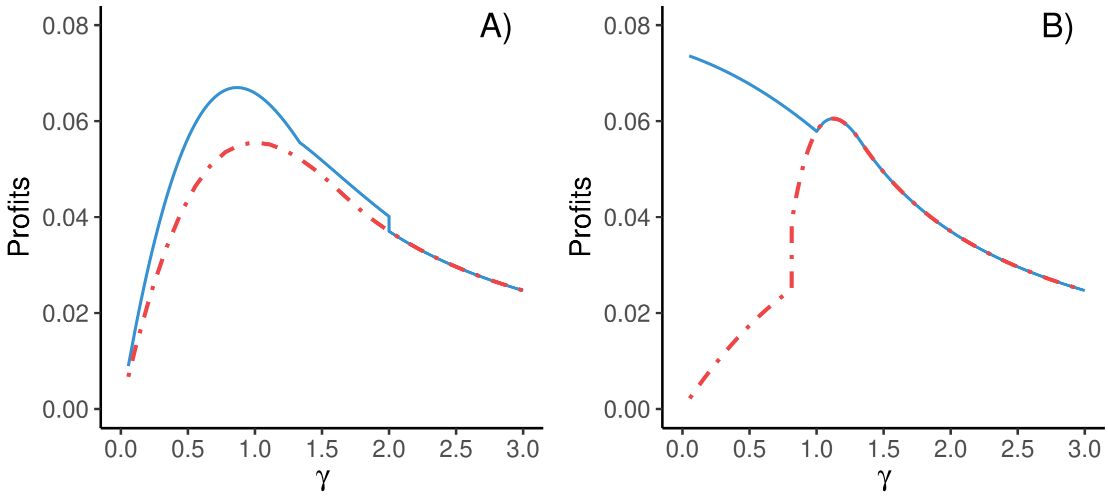

Figure 6: Profits for IOF given: A) COOP uses FOB pricing, and B) COOP uses UD pricing. Continuous blue (dotdashed red) lines represent UD (FOB) pricing strategy for the IOF.

under FOB and UD pricing, given that the rival COOP employs UD pricing. The UD pricing entails higher member welfare than the FOB for $\gamma \leqslant 1.175$. The FOB pricing entails higher welfare, for the members of the COOP, than UD pricing for $\gamma > 1.175$.

For $\gamma \leqslant 1.035$, the UD is a strictly dominant strategy for both COOPs. For $1.035 < \gamma \leqslant 1.175$, there are the two pure strategy Nash equilibria: (FOB, FOB) and (UD, UD). The former is pareto superior to the latter. Hence, for intermediate values of γ, both COOPs will be better off if they coordinate their actions and employ FOB pricing strategies (i.e. tacit collusion). For $\gamma > 1.175$, the FOB is a strictly dominant strategy for both COOPs.

According to the results, the aggressive (UD,UD) strategy is the Nash equilibrium for high intensity of competition, whereas the quasi-collusive (FOB, FOB) is the Pareto superior Nash equilibrium for intermediate and low intensity of competition (higher shipping costs).

Table 2 summarizes all the Nash equilibria of the second stage of the game, for each of the three spatial games.

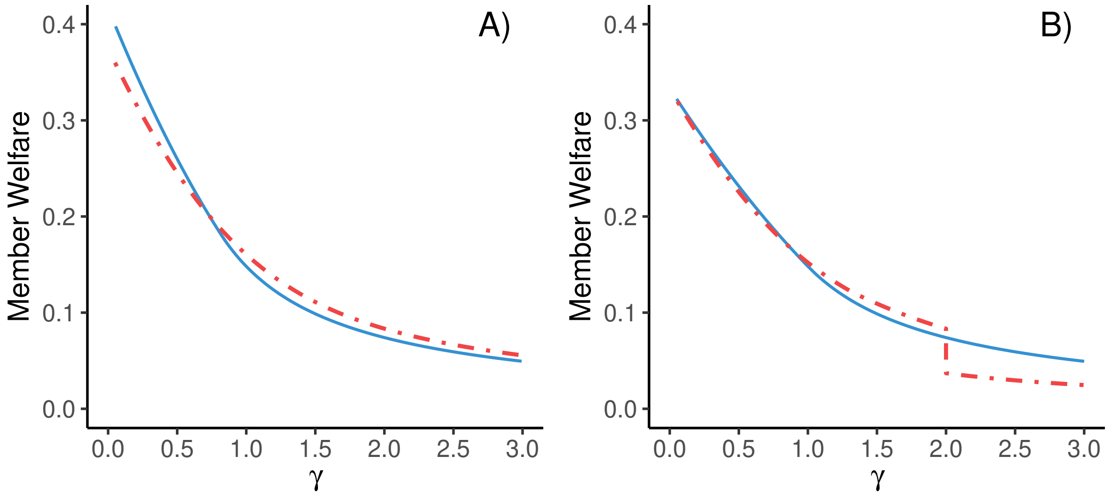

Figure 7: Member welfare for COOP given: A) IOF uses FOB pricing, and B) IOF uses UD pricing. Continuous blue (dotdashed red) lines represent UD (FOB) pricing strategy for the COOP.

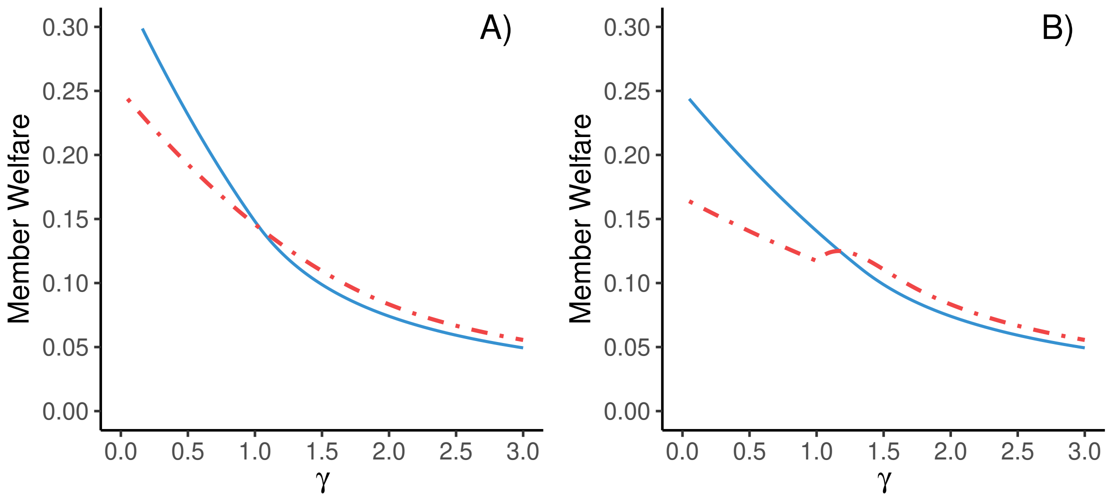

Figure 8: Member welfare for COOP i given: A) COOP j uses FOB pricing, and B) COOP j uses UD pricing. Continuous blue (dotdashed red) lines represent UD (FOB) pricing strategy for COOP i.

Table 2: Nash equilibria in strategic pricing choices

IOF vs IOF		**COOP vs IOF**		**COOP vs COOP**	
Range of γ	*Nash equilibria*	*Range of* γ	*Nash equilibria*	*Range of* γ	*Nash equilibria*
$\gamma \leqslant 0.494$	(FOB,FOB)***	$\gamma \leqslant 0.734$	(UD,UD)***	$\gamma \leqslant 1.035$	(UD,UD)***
$0.494 < \gamma \leqslant 0.607$	(FOB,UD) or (UD,FOB)	$0.734 < \gamma \leqslant 0.874$	(UD,UD)	$1.035 < \gamma \leqslant 1.175$	(FOB,FOB) or (UD,UD)
$0.607 < \gamma \leqslant 1.315$	(UD,UD)***	$0.874 < \gamma \leqslant 1$	(FOB,UD)***	$\gamma > 1.175$	(FOB,FOB)***
$1.315 < \gamma \leqslant 1.667$	(FOB,FOB),(UD,UD)	$1 < \gamma \leqslant 2$	(FOB,UD)**		
$\gamma > 1.667$	Any combination	$\gamma > 2$	(FOB,FOB)		

*** Strictly dominant strategy equilibrium.

** Weakly dominant strategy equilibrium.

5 Discussion

The results of the present study indicate that, in pure and mixed spatial duopoly games, the behavioral assumptions of the agents, along with the costs of transportation, are the most important determinants of the Nash equilibrium strategies. These findings are in line with the findings in the relevant literature (Fousekis, 2011a,b; Zhang and Sexton, 2001).

The investor owned firms maximize profits, which involves pricing according to the marginal revenue product and charging the highest possible price to the consumers of the product. On the other hand, the cooperative organizations maximize member welfare and price according to the average cost of production (break even in the production of the good). The latter translates into charging the lowest possible price to the members of the the cooperative organizations. As a consequence, COOPs behave very aggressively. Their introduction in the market intensifies the pricing competition in the spatial duopoly market and eliminates quasi–collusive Nash equilibria. Fousekis (2011a,b) arrives at the same conclusion when COOPs are also introduced in the market.

The findings of the present work reveal that, when only the IOFs are competing in the market, the quasi–collusive (FOB,FOB) pricing configuration is the Nash equilibrium of the game as competition gets more intense (low values of γ). The introduction of a COOP in the market (instead of one of the IOFs) has an impact on the strategic outcome of the game. The IOF now has to compete against a rival who prices very aggressively for all the values of γ, under which the two rivals have to compete for the market area. In this case, the lower values of γ, instead of fostering a quasi–collusive Nash equilibrium raise the level of aggressive pricing from both agents. As an outcome, both the IOF and the COOP employ the more aggressive UD pricing as their strategic choice, as competition increases. When introducing a second COOP in the market, UD is the strictly dominant pricing strategy for each COOP as competition gets more vigorous. The cooperative organizations can be more aggressive because it is relatively easy to satisfy the break even constraint, even with lower prices offered to the members. For intermediate or higher per unit shipping costs, namely as space gets more important, FOB becomes part of agents' strategic choice, in all three spatial games. Under FOB, consumers have to pay for the social costs of transportation. Hence, the introduction of the COOP(s) in the market eliminates the quasi–collusive Nash equilibria and intensifies the level of price competition.

It is worth noting that under a price responsive (linear) demand function, the FOB pricing choice is the strictly dominant strategy when COOPs are introduced in the market. Previous works (Espinosa, 1992; Kats and Thisse, 1993) had revealed that, when IOFs are competing in a spatial duopoly game with a perfectly inelastic demand function, results can be biased in favor of UD pricing. This is attributed to the fact that under a perfectly inelastic demand function, UD pricing is a firm's strategic choice because it enables the firm to capture the entire consumer's surplus. As a consequence, we can not tell whether the strategic choice of UD pricing policy is due to the competitive advantage of UD relative to FOB pricing or to its superiority at exploiting a perfectly inelastic demand function. Our findings indicate that, under a price responsive demand function, FOB pricing becomes part of the strategic choice of the agents as COOPs are introduced in the market and competition gets less intense. In the spatial game where only COOPs are present, FOB pricing is the strictly dominant strategy as space gets more important. In that case, the bias (if any) should be in favor of FOB rather than UD pricing.

The findings of this article are comparable to the results by Zhang and Sexton (2001), Fousekis (2011a), and Fousekis (2011b). In all three aforementioned studies, a spatial duopsonsy game is examined under a linear supply function. The first one considers competition between two IOFs, the second one investigates competition between a COOP and an IOF, and the third one examines competition between two COOPs. COOPs are member welfare maximizers that price according to the NARP (net average revenue product), which means breaking even in processing. When the shipping costs are relatively low and competition escalates, the more aggressive UD pricing becomes the dominant strategy equilibrium as COOP(s) are introduced in the market.

On a final note, UD pricing emerges as the strategic choice in oligopolistic industries with high shipping costs (Zhang and Sexton, 2001). On the other hand, in markets where shipping costs are relatively low, FOB is the strategic pricing policy: consumers purchase the product and haul it home. According to the findings of the present study, the presence of a COOP in an oligopolistic spatial market will benefit the consumers. Buyers of the product will not have to bear the costs of transportation, even when the hauling/shipping costs are relatively low, due to the fact that UD pricing is the strictly dominant strategy for the competing agents.

6 Conclusions

Firms competing in markets where space matters typically employ either FOB or UD pricing policies (Zhang and Sexton, 2001). Most prior studies have employed duopoly models in order to analyze the effect of the competitiveness of the spatial market on the pricing decisions of profit maximizing IOFs. But, especially in the markets of primary commodities (agricultural/food markets), the agents can be cooperative organizations which may compete against investor owned firms or against each other. In the European Union as well as in the United States, agricultural – consumers' and producers' – cooperatives contribute substantial parts of the added value in the production, processing and marketing of farm products. However, despite the increasing importance of COOPs, there have been only a handful of studies on spatial pricing choices where cooperative organizations are involved. Furthermore, most of them emphasize on the buyer's rather than the seller's side (Fousekis, 2011a,b; Tribl, 2009; Zhang and Sexton, 2001).

The outcome of spatial price competition depends to a great degree on the firms' objective functions, the firms' conjectures regarding the behavior of their rivals and the firms' pricing strategies. Additionally, the literature has revealed that the presence of shipping/transportation costs has a significant impact on the behavior of the agents and subsequently on the competitiveness of the spatial market.

The objective of the present study is to gradually introduce welfare member maximizing COOPs in a duopolistic spatial market and examine their impact on the choice of the pricing strategies as well as on the mode of competition in the market. Unlike the majority of previous works, where consumers' demand was taken to be perfectly inelastic, the present work assumes that the demand function is price responsive. Under the assumption of Hotelling–Smithies conjectures, three separate spatial games were examined: (i) an IOF competing against an IOF, (ii) a COOP competing against an IOF, and (iii) a COOP competing against a COOP. In each case, a two stage game between the agents was employed. In the first stage the competing agents choose between FOB or UD pricing policies and in the second stage they employ pricing rules given the choices in the first stage.

According to the findings, the introduction of COOP(s) in the market intensifies the pricing competition in the duopolistic spatial market by eliminating quasi–collusive Nash equilibria. This is mainly attributed to the fact that the cooperative organizations can be more aggressive because it is easier to satisfy the break even constraint, even with lower prices offered to their members.

The findings of previous works in the relevant literature (Espinosa, 1992; Kats and Thisse, 1993) indicated that, when only IOFs are competing in a spatial duopoly game, a perfectly inelastic demand function can bias the results in favor of UD pricing (Zhang and Sexton, 2001). Our results indicate that, under a price responsive demand function, FOB pricing becomes part of the strategic choice of the agents as COOPs are introduced in the market and competition gets less intense. In the spatial game where only COOPs are the competing agents, FOB pricing is the strictly dominant strategy as space gets more important. Hence, the bias should be in favor of FOB rather than UD pricing.

This work has examined the strategic pricing choices in a duopoly spatial market, between agents with similar or different objective functions (IOF vs COOP), under symmetric production costs. The literature has often examined cases where one agent is more cost effective than the other. Hence, a potential avenue for future research can be how asymmetries in the costs of production between the agents – especially when an IOF is competing against a COOP – might have (or not) an impact on the Nash equilibria of the game and on the competitive dimensions of the spatial market.

Appendix

The total average costs of production for the UD pricing COOP, when serving market area equal to R^C_{UD}, are:

$$AC = \frac{TC}{Q} = \frac{(1-u^C)\int_0^{R^C_{UD}}(c+\gamma\, r)\, dr}{(1-u^C)\, R^C_{UD}} = \frac{(1-u^C)\, R^C_{UD}\,(c+\frac{\gamma}{2}\, R^C_{UD})}{(1-u^C)\, R^C_{UD}} \tag{1}$$

The term on the numerator is the total costs of production, and the term on the denominator represents the total quantity produced, when the COOP serves a market area of R^C_{UD}.

Solving equation 1 we obtain:

$$AC = c + \frac{\gamma}{2}\, R^C_{UD} \tag{2}$$

The market area that the monopoly UD pricing COOP serves is: $\hat{R}^C_{UD} = \dfrac{2\,(1-c)}{3\,\gamma}$. If we substitute the expression for $\hat{R}^C_{UD}$ in equation 2 we obtain:

$$AC = \frac{1+2\,c}{3} \tag{3}$$

As derived in section 2, the optimal UD price for the spatial monopolist COOP is $u^C = \dfrac{1+2\,c}{3}$. Hence, the monopoly UD pricing COOP prices according to the average production costs.

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