A simple and accurate method to calculate domestic and foreign value-added in gross exports

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A series of papers have introduced an analysis of trade flows in value-added terms (Daudin et al., 2011; Johnson and Noguera, 2012; Koopman et al., 2014; Los et al., 2016). With the international fragmentation of production and vertical specialization in ‘global value chains’, gross trade flows can be misleading when a significant share of value-added comes from other countries than the exporting economy. For example, revealed comparative advantage (RCA) indices based on gross exports might not capture adequately the specialization of the exporting country (Brakman and Van Marrewijk, 2016). It is particularly the case when the exporter adds a small share of value-added through assembly tasks at the end of the value chain and is not the country leading the production process. The gravity model, which is broadly used in trade analysis, is also under scrutiny as it is generally estimated based on gross trade flows. Some authors have noted that it does not perform well when parts and components make a significant share of the value of gross exports (Baldwin and Taglioni, 2014).

Through a value-added decomposition of gross exports, one can identify the countries and industries that have provided the inputs used in exports and that ultimately benefit from trade. One can also remove any double counting which is problematic when assessing the factor content of trade or CO2 emissions embodied in trade. Such analysis can also change our understanding of trade imbalances, competitiveness or the impact of protectionism (Grossman, 2013; OECD, 2013; Johnson, 2018).

However, trade statistics are collected in gross terms and the decomposition of value-added in gross exports requires the use of input-output techniques and additional data from a global input-output table. Thanks to different international efforts, such as the World Input-Output Database (Timmer et al., 2015) or the OECD-WTO Trade in Value-Added initiative, international input-output information is now more readily available. But an important barrier remains for researchers and economists interested in measures
of trade in value-added terms. At this stage, there is no simple way of calculating the
domestic value-added, foreign value-added and double counting in gross exports. There
is also no consensus in the literature on such calculation beyond the domestic value-
added.
Initially, the literature has focused on the calculation of the import content of exports,
which provides an indication of all the imported intermediate inputs used in the
production of exports. Hummels et al. (2001) introduced the import content of exports
as an indicator of the share of vertical specialization in trade. But imported inputs are
not always made with foreign value-added. In a world of global value chains, the import
content of exports can include intermediate inputs originally produced in the exporting
economy (i.e. domestic) and embodied in foreign imported intermediates, thus creating
some domestic double counting.
Moreover, foreign inputs can also be involved in some ‘circular’ trade and come back
to the exporting economy at different stages of processing.\footnote{For example, Australia exports metal ores to China that are used to produce steel. Chinese steel is then imported by Australia to produce parts and components exported to China and embodied in exports of machinery. In this example, Australian value-added related to metal ores will appear several times in the gross exports of China.} Therefore, there is also
some double counting for foreign inputs in the import content of exports.
At the aggregate level, the double counting (domestic and foreign) is generally small
but for specific industries or specific pairs of countries, values can become non-
negligible (Koopman et al., 2014). Moreover, values are small because international
trade is costly. In a more integrated and less protectionist world, the level of double
counting would be higher (as observed at the country level when looking at the
difference between gross output and GDP).
Calculating domestic and foreign value-added in gross exports net of any double
counting (i.e. consistent with the GDP of countries participating in the value chain) is
not straightforward. But it is necessary when focusing on the factor content of trade or
when assessing the contribution of trade to GDP.\footnote{To circumvent the issue, authors have used in the past net measures of trade. By subtracting imports from exports, a net contribution of trade to GDP is obtained with no double counting. See for example Trefler and Zhu (2010). However, this approach cannot be used when one is interested in analyzing exports (or imports) instead of net trade.}
The first paper that has proposed a full decomposition of gross exports with domestic

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value-added (DVA), foreign value-added (FVA) and double counting terms (domestic and foreign) is Koopman et al. (2014), hereafter KWW. However, the authors start from an accounting identity and derive terms that are interpreted as DVA, FVA and double counting but without a proof that these terms actually measure what they are intended for. This is why Los et al. (2016) introduced in a comment on KWW a different method to derive DVA in gross exports. They rely on a hypothetical extraction method, a technique commonly found in the input-output literature. They first calculate the GDP of a country according to the Leontief model (Leontief, 1936) and then calculate a hypothetical GDP for the same country in a world where this country has no exports (by setting to zero exports in the inter-country input-output table). The difference between the two provides a measure of DVA in gross exports, which is consistent with GDP and anchored in theory. In addition, with this methodology, DVA is easier to calculate as compared to the 9-term decomposition of gross exports suggested by KWW. However, Los et al. (2016) do not extend their methodology to the calculation of FVA in gross exports. Focusing on DVA only, their methodology cannot provide either an expression for the double counting (i.e. the difference between gross exports and the sum of DVA and FVA).

In addition to being difficult to implement empirically, several authors have expressed doubts about the KWW decomposition (Nagengast and Stehrer, 2016; Miroudot and Ye, 2017; Johnson, 2018). While its DVA terms are confirmed by Timmer et al. (2016), there is no such confirmation for the FVA terms and the double counting.

In this paper, we propose a hypothetical extraction that provides a theoretically-funded measure of FVA in gross exports and we provide full expressions for a decomposition of gross exports into four terms: DVA, domestic double counting (DDC), FVA and foreign double counting (FDC). As in Los et al. (2016), the fact that we calculate a hypothetical GDP without the exports from a given country is the basis for validating the interpretation of the FVA term. Our DVA term is the same as Los et al. (2016).

A difference with previous frameworks is that value-added is the exogenous variable in our decomposition instead of final demand. As such, it can be regarded as a supply side rather than demand side decomposition. An interesting outcome is that while we
confirm the KWW result for DVA and domestic double counting, our framework invalidates their formula for FVA and the foreign double counting. We provide an empirical illustration of how results differ at the end of the paper.

1. Data needs and basic equation to start the calculation

In order to calculate DVA, FVA and the double counting terms, one needs an inter-country input-output table (ICIO) with a matrix of intermediate consumption ($Z$), a matrix of final demand ($Y$), a vector of gross output ($X$) and a vector of value-added coefficients ($v$), i.e. a vector of value-added divided by the vector of gross output. The table is for a specific year and generally includes information by country and industry. In the rest of the analysis, we omit the subscripts for industries and focus on countries but the method works the same way for an analysis of gross exports in a given industry and the do-file we provide includes the industry dimension. The empirical results that illustrate the analysis are calculated with data from the WIOD database (Timmer et al., 2015) for the year 2014.

By dividing the matrix of intermediate consumption by the vector of output, one obtains the traditional input-output matrix $A$ with the technical coefficients indicating for each country and industry how many units of each input are required to produce one unit of output. In the basic input-output equation, gross output is expressed as:

$$X = AX + Y$$

(1)

2. Hypothetical extraction matrix

The hypothetical extraction method consists in ‘extracting’ industries or countries from the input-output structure by setting to zero their corresponding cells in the matrices and vectors involved (Miller and Lahr, 2001). The extraction matrix described by Los et al. (2016) and Johnson (2018) removes intermediate inputs from country 1 in the production of country 2, as they work with only two countries. It is expressed as

$$\begin{pmatrix}
    A_{11} & 0 \\
    A_{21} & A_{22}
\end{pmatrix}$$

In the case of Los et al. (2016), the number of countries in the extraction matrix does not matter, as they are only interested in DVA. However, for Johnson

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3 WIOD data can be downloaded on the website [www.wiod.org](http://www.wiod.org). We use the 2016 release.
(2018), it matters and the paper does not indicate how to extend the methodology to a higher number of countries. Another paper by Borin and Mancini (2017) suggests extending the extraction matrix to an arbitrary number of countries by setting to zero the coefficients that identify the requirement of inputs imported from country \( s \) within the input matrix. Their extraction matrix can be expressed as

\[
\begin{bmatrix}
A_{s1} & A_{s2} & \cdots & A_{sG} \\
\vdots & \ddots & \cdots & \vdots \\
0 & 0 & \cdots & 0 \\
A_{G1} & A_{G2} & \cdots & A_{GG}
\end{bmatrix}
\]

This expression should also correspond to the many-country case in an extension of Los et al. (2016).

In our case, we start from the input-output accounting equation (1) and re-organize its elements to separate out the exports of a given country:

\[
X = A^*X + Y^*u + E^-
\]

Taking country \( s \) as an example:

\[
\begin{bmatrix}
X_1 \\
\vdots \\
X_s \\
\vdots \\
X_G
\end{bmatrix}
= \begin{bmatrix}
A_{s1} & 0 & \cdots & A_{ss} & 0 \\
\vdots & \ddots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 \\
\vdots & \ddots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & A_{Gs} & A_{GG}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
\vdots \\
X_s \\
\vdots \\
X_G
\end{bmatrix}
+ \begin{bmatrix}
Y_{s1} & 0 & \cdots & Y_{ss} & 0 \\
\vdots & \ddots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 \\
\vdots & \ddots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & Y_{Gs} & Y_{GG}
\end{bmatrix}
\begin{bmatrix}
Y_1 \\
\vdots \\
Y_s \\
\vdots \\
Y_G
\end{bmatrix}
+ \begin{bmatrix}
E_{s1} \\
\vdots \\
E_s \\
\vdots \\
E_G
\end{bmatrix}
\]

The extraction matrix \( A^* \) includes the direct inputs for the production of goods in country \( s \) for domestic use and for exports. What is different in our extraction matrix is that we keep the diagonal block matrix and the column corresponding to inputs exported to country \( s \). The notation \( E^-_{j} \) means country \( j \)’s exports to all countries except country \( s \). In this equation, the extraction matrix fully accounts for the propagation of output (including for domestic use) and of exports in the ICIO.

Then, we re-arrange the above equation to calculate gross output:

\[
X = (I - A^*)^{-1}(Y^*u + E^-)
\]

We pre-multiply by the value-added ratios and map the global GDP distribution embodied in each term:
Here, matrix 

\[
B^* = (I - A^*)^{-1} = 
\begin{bmatrix}
(I - A_{ii})^{-1} & 0 & \cdots & (I - A_{1i})^{-1}A_{ii}(I - A_{ii})^{-1} & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & (I - A_{si})^{-1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & (I - A_{GG})^{-1}A_{Gsi}(I - A_{Gsi})^{-1} & \cdots & (I - A_{GG})^{-1}
\end{bmatrix}
\]  

(6)

3. Calculation of DVA and FVA

Following the same logic as Los et al. (2016), DVA for country \( s \) is the difference between the actual GDP of country \( s \) and a hypothetical GDP calculated without the exports of country \( s \):

\[
DVA_s = GDP_s - GDP_s^* = GDP_s - v_s(I - A_{ss})^{-1}Y_{ss} = v_s(I - A_{ss})^{-1}E_s
\]  

(7)

For the measurement of country \( s' \) FVA, we can first start with the decomposition of GDP of another country \( k \) (\( k \neq s \)) which is a trade partner of country \( s \), merging equation (5) and (6), we can obtain the decomposition expression of GDP as follows:

\[
GDP_k = v_k(I - A_{kk})^{-1}Y_{kk} + v_k(I - A_{kk})^{-1}A_{ks}(I - A_{ss})^{-1}E_s + v_k(I - A_{kk})^{-1}E_k \\
+ v_k(I - A_{kk})^{-1}Y_{sk} + v_k(I - A_{kk})^{-1}A_{sk}(I - A_{ss})^{-1}Y_{ss} \\
+ v_k(I - A_{kk})^{-1}Y_{ks} + v_k(I - A_{kk})^{-1}A_{ks}(I - A_{ss})^{-1}Y_{ks}
\]  

(8)

Equation (8) provides a full decomposition of GDP for country \( k \). The first term measures the share of GDP that just participates in the domestic propagation (i.e. not part of international trade). The second term measures country \( k \)'s GDP in exports of country \( s \), which can be regarded as the foreign value-added part for country \( s \) exports decomposition (the part that goes in FVA for country \( s \)).

The third term indicates country \( k \)'s GDP measured in other exports flows (i.e. in exports of other countries than country \( s \)), with two potential destinations: 1) absorbed by other countries as intermediate or final goods; 2) processed in other countries and re-exported -this part would be counted as FVA in the exports decomposition for other countries, and the expression would be consistent with the decomposition for country
The fourth and fifth terms correspond to country $k$’s GDP absorbed by country $s$ as intermediate or final goods (not part of the exports of country $s$).

We can also derive FVA following the logic of the hypothetical extraction. The value-added of foreign country (country $k$) in country $s$’ exports is equal to the difference between the actual GDP of foreign country and a hypothetical GDP calculated without the exports of country $s$. The hypothetical GDP can be obtained by setting country $s$’ exports to zero ($E_s=0$) in equation 5. We have:

$$FVA_s = \sum_{k \neq s} \left( GDP_k - GDP_k^H \right)$$

$$= \sum_{k \neq s} \left\{ GDP_k - [v_k (I - A_k)^{-1} Y_k + v_k (I - A_k)^{-1} E_s + v_k (I - A_k)^{-1} A_k (I - A_s)^{-1} Y_s] \right\}$$

$$= \sum_{k \neq s} v_k (I - A_k)^{-1} A_k (I - A_s)^{-1} E_s$$

(9)

4. The definition and identification of double counting

The key to calculate FVA in gross exports is to properly identify what is foreign value-added for a specific country and how it differs from ‘double counting’. In the framework developed by KWW, double counting is defined as the value-added that crosses international borders more than once. Therefore, all the foreign value-added in this framework is already double counted. The authors are interested in removing double counting from aggregate world trade statistics and in this case any foreign value-added in exports is by definition domestic value-added in the exports of another country and double counted. In order to decompose the gross exports of a specific country and to introduce a FVA term, the authors then refer to a ‘pure’ double counting, which is the difference between gross exports and the sum of DVA and FVA. But their definition remains convoluted when implemented in the decomposition.

Pointing out this issue, Borin and Mancini (2017) propose a different definition for the double counting. From the point of view of a specific exporting country, double counting corresponds to the value-added that has crossed the country’s border more than once. It is a better starting point but the issue with a definition of double counting based on the number of border crossings is that the input-output framework cannot tell
us how many times value-added has crossed borders. The input-output matrix identifies international and domestic transactions but there are many paths through which value-added can reach final consumers and these paths are not known. They are summarized in a single input-output matrix that has collapsed the different production stages (Los and Timmer, 2018). Borin and Mancini (2017) want to measure foreign value-added when it enters a specific country for the “first time”. But before entering a specific country, this value-added has already crossed all possible borders according to the input-output table.

To circumvent this difficulty, we have to start from an extraction matrix that can identify inputs used in exports coming back to the exporting economy, but without being dependent on the number of border crossings. With the hypothetical extraction method, DVA is the difference between the actual GDP of the exporting country and a hypothetical GDP calculated without the exports to a given economy. This difference does not depend on the number of border crossings. We can similarly calculate FVA based on the difference between the actual GDP and hypothetical GDP of the foreign economies that export inputs without having to refer to border crossings. The double counting is then just the residual that remains after subtracting DVA and FVA from gross exports. Conceptually, the domestic double counting (DDC) is the residual that remains after subtracting DVA from the domestic content of exports ($v_s B_{ss} E_s$) and the foreign double counting (FDC) is the residual that remains after subtracting FVA from the foreign content of exports ($\sum_{k \neq s} v_s B_{ks} E_s$). This definition is also consistent with the measurement of GDP in the exporting country and in other countries and does not depend on the number of border crossings. Double counting is the value-added generated in one country and already measured in the GDP of this country that appears twice or more in the gross exports of a given economy. It simplifies the decomposition of gross exports by not trying to cover all the possible cases of inputs crossing domestic borders.

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4 See Miroudot and Ye (2018) for a similar definition but starting from the supply-side input-output model (Ghosh model). The fact that double counting for the exporting economy starts when the same value-added is crossing the country’s border more than can be explained based on the input rounds in the generation of value-added in exported goods.
and foreign borders multiple times, but by mapping global GDP across countries.

5. In a nutshell: simple and accurate formulas for DVA, FVA and double counting terms

Based on the above analysis and the results from hypothetical extractions discussed in Section 3, we propose the following formula for a 4-term decomposition of gross exports:

\[
E_s = v_s(I - A_{ss})^{-1}E_s + v_s[B_{ss} - (I - A_{ss})^{-1}]E_s \\
+ \sum_{k \neq s} v_k(I - A_{sk})^{-1}A_{ks}(I - A_{ss})^{-1}E_s + \sum_{k \neq s} v_k[B_{ks} - (I - A_{sk})^{-1}A_{ks}(I - A_{ss})^{-1}]E_s
\]  

Equation (10) provides a full decomposition of gross exports with four terms that are respectively: domestic value-added net of any double counting (DVA), domestic double counting (DDC), foreign value-added net of any double counting (FVA) and foreign double counting (FDC) in country \( s \)'s exports. The do-file that can be downloaded with this paper implements the formula for exports of all countries (and all industries) in the ICIO, creating 4 vectors with DVA, DDC, FVA and FDC in the country-industry dimension.

6. Results using the WIOD database

As an illustration of how the methodology can be used, we decompose gross exports for all the countries included in the WIOD database. The first columns of Table 1 include the 4 terms calculated with our formulas: DVA, DDC, FVA and FDC. For a comparison with other decomposition frameworks, we have included in additional columns results for the same four terms based on KWW, as well as DVA as calculated in Los et al. (2016) - RES being the difference between gross exports and DVA since these authors only provide a formula for DVA.

<table>
<thead>
<tr>
<th>Country</th>
<th>Our framework</th>
<th>Los et al. (2016)</th>
<th>KWW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gross exports (million USD)</td>
<td>DVA</td>
<td>DDC</td>
</tr>
<tr>
<td>AUS</td>
<td>287,161</td>
<td>85.83</td>
<td>0.14</td>
</tr>
<tr>
<td>AUT</td>
<td>210,995</td>
<td>63.86</td>
<td>0.29</td>
</tr>
<tr>
<td>BEL</td>
<td>383,013</td>
<td>53.96</td>
<td>0.39</td>
</tr>
</tbody>
</table>
Table 1 confirms that there is a consensus for the calculation of DVA and that both our
framework and KWW provide the same DDC. There is however a difference when it comes to FVA and FDC.

It should be noted that, while often small, both the domestic and foreign double counting terms are interesting from an analytical point of view. Domestic double counting identifies inputs that are exported and come back to the domestic economy embodied in foreign inputs that are then used in exports. It can be an indicator of participation in global value chains at different stages of production. From a trade policy perspective, a country should avoid creating barriers to imports of inputs that incorporate its own domestic value-added. The domestic double counting can be an indication of how prevalent this case is.

A high level of foreign double counting is also an indicator of integration in GVCs for the exporting country. Partners that provide inputs at different stages of production and have inputs that come back to the domestic economy might be interesting to identify. First, these partners might be important in terms of economic integration in supply chains and there might be economic gains in further facilitating this integration. Second, there might be policies or specific barriers that prevent inputs from being domestically processed instead of being shipped back and forth at different stages of processing. The foreign double counting can also be interesting to measure and to further investigate.

7. Concluding remarks

In this paper, we have provided a new methodology to calculate domestic and foreign value-added in gross exports, net of any double counting. We have also provided expressions for domestic and foreign double counting terms in order to have a full decomposition of gross exports. The formulas are simple enough to allow a calculation of trade flows in value-added terms using existing inter-country input-output tables. While the paper does not go into the details of the calculation at the industry or bilateral level, the do-file we provide includes such dimensions. The industry level is not especially complicated but requires some clarity about the industry dimension, which is the industry of gross exports in our framework. Other decompositions are possible based on the industry of origin or industry of the final product but it requires a different framework.
At the bilateral level, there are further complications as explained by Nagengast and Stehrer (2016). In particular, because one has to decide in which bilateral exports is the value-added that ‘travels’ across different countries before being incorporated in final products. Consequently, one should not expect the sum of bilateral values to match the aggregate figure (with the world as partner), as pointed out by Los and Timmer (2018). The implementation we provide in the do-file for bilateral flows follows our definition of double counting and GDP decomposition approach. The value-added is identified in the exports of the country in which it was initially generated.5

We hope that with a simple and accurate method to calculate domestic and foreign value-added in gross exports, further analysis of trade in value-added terms will shed new light on international trade issues.

References


5 Our approach is ‘source-based’ in the terminology of Nagengast and Stehrer.


