The inhibited (exhibited) spread of innovations

Amavilah, Voxi Heinrich

REEPS

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Voxi Heinrich Amavilah**
REEPS
PO Box 38061
Phoenix, AZ 85069, USA
vhsamavilah@gmail.com

This note makes general statements about standard models of the diffusion of innovations. Its premise is a familiar idea that innovations are socially-learned changes that spread like wildfires across diverse populations. However, the rate at which innovations spread is subject to the forces of exhibition and inhibition. Exhibitors promote the spread of innovations; inhibitors subjugate them. Hence, where the forces of subjugation are stronger than the forces of promotion, it is the slow spread, rather than the lack, of innovations which undermines the competitiveness of nations, and consequently frustrates economic performance. This suggests a need for a simple and more realistic model. Since the analytical components (basic equations and statistical inference) of the needed model are readily available, this note attempts a synthesis. Unfortunately in its current version the note is incomplete, and therefore makes only a tentative concluding remark. Even so, there is enough insight to warrant comment.

Keywords: innovation spread, inhibitors and exhibitors of innovation diffusion, inhibited or exhibited growth, logistic model, derivative Gompertz, diffusion of innovations

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1. Problem

The roots of economic progress, as G. L. Bach (1960) once put it, are resources, technology, specialization, and exchange. However, the recent literature makes it abundantly clear that the competitiveness of nations is mainly a matter of technology and technological capability (Charrtrand, 2002, 2006). The problem then is not so much that technology is scarce; nor is it the lack of technological innovations. The last two centuries alone have produced unprecedented technological innovations in superfluous quantities and qualities (Moykr, 200). The real difficulty is the uneven diffusion of innovations. For this reason "some countries prosper while other fall behind" (Fagerberg, Srholec, and Knell, 2007). Unfortunately, current models of the spread of innovations lack generalizable properties that would admit to the fact that not only do innovations, like wildfires, spread in unpredictable ways, the spread itself is often subject to systematic and systemic inhibitors and exhibitors for which simpler and more realistic models than those readily in use are needed. While

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*This note is preliminary and incomplete; it emphasizes the ideas, rather than the mathematics of those ideas. In any case, the author welcomes comments.

**Amavilah is an independent researcher, and an adjunct instructor of economics at Glendale College, Glendale, AZ, USA.

not promising to present a novel model of diffusion of innovations, this note contributes to the outline of how one may proceed to synthesize existing models to address the problem.

The modification is possible because the diffusion of innovations among human beings is not a fundamentally different process from what it is among other social beings like honey bees, for instance. It is a discrete and disjointed process that is not amenable to easy modeling despite what continuous-time mathematics suggests. First, one must have a well-functioning colony (community) of individuals with pressing needs and desires, and the willingness and ability to meet those needs and desires. When supported by the ability and willingness to satisfy them, needs compel different types of learning behavior as a matter of self-preservation. Learning leads discovery of innovations, which in turn enables adoption, and then diffusion.[1] The process is multi-phased, and each phase in the process requires learning. Learning occurs at different levels, and progresses at different rates.

Let's take a simple example from honey bees. Assume a bee colony of N_0 well-functioning members. The bees have to live (survive), and so they have to know how. The bees specialize to lower the cost of living, improve efficiency, the whole shebang. This means they have an effective "rule of law" and institutional mechanisms for monitoring and enforcing laws. To produce honey some n_0 field bees go out looking for flowers, a task they must be capable of performing. When field bees depart, they leave behind $N_1 = N_0 - n_0$ community members to do other things. Meanwhile dN_0 of N_0 and dn_0 of n_0 bees will die.[2] The death of dN_0 reduces the population to $N_1(1-d)$, leaving fewer mouths to feed, but also fewer bodies to store collected juices and to convert the juices into honey. The death of dn_0 leaves $n_0(1-d)$ field bees, which constrains the learning process and hence diffusion by depreciating knowledge to find or report the whereabouts of flowers.

And that is not the end of the constraints, because when $n_1 = n_0(1 - d)$ returns from the scouting mission to deliver news of the whereabouts of followers, now there are $N_2 = (N_0 - n_0)(1 - d) < N_1(1 - d) < N_0$ members. The next mission party will consist of surviving scouts (n_1), and pollen and juice collectors (n_3), i.e., $n_2 = n_1 + n_3 = n_0(1-d) + n_3$. However, for diffusion purposes only $n_4 = n_2(1-d) = (n_1 + n_3)(1-d)$ bees will return home to their colony of $N_3 = (N_0 - 2n_0)(1-d) - n_3$ members. The logical moral of the story is that there are forces that inhibit and/or exhibit learning, adoption, and hence the diffusion of innovations. Some inhibitions and exhibitions are *systematic* such as death; others are *systemic* such the organizational structure of a particular society which defines specialization, sets and enforces the rules of the game (cf. Taylor and Levitt, 2004). In the end the potential future sources and sinks of innovations, and subsequently its limit are generally $N_i(t+1) = \Delta N_i(t)\omega - n_i(t)$.

The diffusion of innovations among humans faces similar challenges and opportunities. How

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[1] I define diffusion as repeated and continuous adoption, with or without adaptation.

[2] I use the word "die" in the broadest possible sense to mean loss of optimal functionality, and it would include biological death as well as things like forgetting.

generalizable the bee example is to the human experience is the problem this note discusses. Section 2 takes a look at three related models of the diffusion of innovations. Section 3 introduces inhibitions that go beyond the conventional model, while Section 4 makes tentative remarks.

2.1. Learning and adopting from internally homogenous sources

$N(t)$ are members of a given economic community like a country at time t. Let $n(t) \in N(t)$ be the number of economic agents who have heard (learned) of an innovation at that time. For $t_1 < t_2, n(t_1) < n(t_2)$. Ignoring writing variable t for convenience, unless where necessary, the number of adopters (δn) is such that $\delta n / n = \Delta N / n, \Delta N = N - n.$ Then

$$\delta n = \alpha n \Delta N \delta t, \quad \alpha > 0. \tag{1}$$

Therefore, the rate of change in the number of adopters becomes

$$dn/dt = \alpha n \Delta N. \tag{2}$$

The solution to the differential, $dn = \alpha n \Delta N dt$, of (2) is

$$n = \frac{N e^{\alpha N t}}{(N-1) + e^{\alpha N t}}, \tag{3*}$$

which represents the cumulative number of adopters. Thus, (3*) assumes learning from internally homogeneous sources. It is the standard model of the diffusion of technological innovations used to describe the adoption of weed spray in Iowa and hybrid corn in Iowa, Kentucky, and Alabama in the 1950s, for instance (Rogers, 1983 [2003], Burghes and Borrie, 1981).

2.2. Learning and/or adopting from heterogenous sources

If learning is from heterogeneous sources as described in a number of recent papers by H. Peyton Young (2004, 2005, 2007), then one can express (1) as

$$\delta n = \alpha n \Delta N \delta t + \beta \Delta N \delta t = (\alpha n + \beta) \Delta N \delta t, \quad \beta \geq 0. \tag{4}$$

From (4) the rate of diffusion is

$$dn/dt = (\alpha n + \beta) \Delta N \Rightarrow dn = (\alpha n + \beta) \Delta N dt. \tag{5.1}$$

By variable separation,

$$\int \frac{dn}{(n+\theta)\Delta N} = \int \alpha dt, \ \ \theta = \beta / \alpha. \tag{5.2}$$

Upon partial differentiation

$$\frac{1}{n+\theta}\int [\frac{1}{n+\theta} + \frac{1}{\Delta N}] dn = A + \alpha t. \tag{5.3}$$

Assuming *n(0)* = 1, the cumulative number of adopters from more than one source is approximately

$$n = \frac{(\alpha+\beta)Ne^{(\alpha N+\beta)t} - \upsilon(N-1)}{(N-1)\alpha + (\alpha+\beta)e^{(\alpha N+\beta)t}}. \tag{6*}$$

Eq. (6*) is a logistic model that accounts for heterogenous sources of learning, and hence adoption of innovations (Urban and Hauser, 1981). It is the equivalent of H. Peyton Young's (2004) Equation 14 (p. 13) - a big improvement on (3*). However, it is too orderly in structure, which pre-ordains its predictions (Amavilah, 2007). In a creative re-interpretation of Rogers (1983 [2003]), Barbara Wejnert (2002) provides a conceptual integration of three components of comprehensive models of the diffusion of innovations that stand in the way of such model orderliness: "characteristics of innovations"; "characteristics of innovators"; and the "environmental context".

The first component concerns mainly financial and economic factors; the second focuses on the socioeconomy of innovations, and the third on the geography of innovations. All three components are potential sources and sinks of inhibitions and exhibitions of innovations. For example, when innovations have larger private benefits than costs, they are likely to be adopted quickly, but spread slowly given such things as the protection of property rights. Pure public innovations are nonrival and nonexcludable; their rates of adaption and spread are largely a function of the political process, and its institutions. For instance, in a traditional economy the adoption and spread of innovations that increase productivity but reduce social security (perhaps by reducing social status and risking social disorder, real or perceived), will be slow. The influence of geography on the diffusion of innovations is a common subject of analysis among economists of growth and change. Not too long ago the stress was on the (in)appropriateness to developing nations of intermediate technologies - a subject that, then as now, has strained the relations between developing countries on one hand, and multinational corporations in developed countries on the other hand. Today similar emphasis goes into the analysis of the externalities of clubs, trading blocs, networks, neighborhoods, and the like. This all indicates that Wejnert's framework is consistent with Fagerberg, Srholec, and Knell's (2007) argument that the spread of innovations is a function of many variables including the technical capacity of adopters (cf. Drucker, 1985). Technical capacity depends largely on economic factors like demand and price, in Fagerberg, Srholec, and Knell's analysis. Thus, current diffusion models are deficient to the extent that most of them compress the learning, and adoption dimensions of diffusion into one dimension whose success is measured in revealed use. Paradoxically, in real life success is seldom a good guide to progress. Mathematicians Springer, Herlihy, Mall, and Beggs (1968,) point out that "... *there is nothing very secret about success*. Failures are the real secrets. As individuals we learn a lot more from our disappointments and failures than from success. ... Learning seems to come from successive elimination of the causes for failure. From this point of view, *nothing fails like success*" (pp. 272-273, italics original).

2.3. Innovations spread like wildfire

It is true that we do not know learning has taken place until adoption is complete. We do not know adoption is complete until we see the innovation in use. However, Marx-Schumpeter's "gale of creative destruction" guarantees that what is in use is sooner or later discarded. In reality diffusion takes place in economic time determined by market forces and distributed in discrete units and over discrete time. There are lags involved; one may learn in year t, but adopt in year t+j >> t. One also may adopt in year t+j, and still chose not to "share", or even quit that same year. In situations where successful use is preceded by many failures, the rate of spread of innovations is surely underestimated. This all suggests the need to modify (6*) to allow for social learning behavior of the type described in Young (ib.), as well as the wildfire-like phenomenon outlined in Amavilah (2007). In such cases the number of spreaders, adjusted for wildfire-like unpredictable behavior is

$$n^{*}=R(t)n,\ \ R(t)=(\Omega+\Omega\psi)/\mu, \tag{7}$$

where R(t) is Rothermel's parameter of the spread of wildfire (Amavilah, 2007, p. 5). While (6*) is logistic, in (7) R(t) may be distributed differently. Let's assume R(t) is a Gompertz so that (7) becomes

$$n^{*}=[e^{\lambda-\zeta\gamma^{t}}]n\ =[e^{\lambda-\zeta\gamma^{t}}]\frac{(\alpha+\beta)Ne^{(\alpha N+\beta)t}-\upsilon(N-1)}{(N-1)\alpha+(\alpha+\beta)e^{(\alpha N+\beta)t}} \tag{8}$$

Since $n=n^{*}/R$, (8) = (6) only if R = 1. R = 1 requires e^{0}, which in turn requires $\lambda-\zeta\gamma^{t}=0$ - not possible. Thus, according to (8) innovations spread like wildfire in that R is subject to the local forces of mutual propagation and inhibition.

3. Inhibited (exhibited) spread of innovations

If *n(t)* is inhibited (exhibited) by *I* - the ratio of the rate of resource consumption in the process of diffusion (learning, adoption, adaptation), to the capacity utilization rate - that is,

$$\frac{\textit{resource consumption rate}}{\textit{capacity utilization rate}} \equiv I = \frac{\eta n^* + \rho n_1^*}{n^*}, \quad n_1^* = \frac{dn*}{dt}, \tag{9}$$

then the rate of the spread of innovation would be

$$\dot{n}^{**} = \xi(1 - (\frac{\eta n^* + \rho n_1^*}{n^*}) = \xi(1 - \eta - \frac{\rho n_1^*}{Rn}). \tag{10}$$

Adjusting (8) for inhibitions (exhibitions) as well as wildfire-like behavior yields

$$n = \frac{\xi \rho n_1^*}{\xi(1-\eta)R} = \frac{\xi \rho n_1^*}{\xi(1-\eta)e^{\lambda - \zeta\gamma^t}}. \tag{11*}$$

This means that (8) simplifies to

$$n^* = \frac{\xi \rho n_1^*}{\xi(1-\eta)}. \tag{12}$$

Eqs. (11) and (12) are economically informative. To see how, from (11) define $\tau = \xi\rho$ and $\pi = \xi(1-\eta)$ so that

$$n = \frac{\tau n_1^*}{\pi e^{\lambda - \zeta\gamma^t}}. \tag{13}$$

Then

$$\frac{dn}{dt} = \frac{\tau dn_1^*/dt - \zeta\gamma^t \ln\gamma\tau}{\pi e^{\lambda - \zeta\gamma^t}}. \tag{14}$$

Making necessary substitutions we have

$$\frac{dn}{dt}=\frac{\xi\rho(n_1^{*}-\zeta\gamma^{t}\ln\gamma)}{\xi(1-\eta)e^{\lambda-\zeta\gamma^{t}}}. \tag{15}$$

But since from (9) $n_1^{*}=dn^{*}/dt$, then in (14)-(15) $dn_1^{*}/dt=d^2n^{*}/dt^2$, so that

$$\frac{dn}{dt}=\frac{\xi\rho(1-\zeta\gamma^{t}\ln\gamma)}{\xi(1-\eta)e^{\lambda-\zeta\gamma^{t}}}. \tag{16}$$

Obviously the differential of (16) is

$$dn=\frac{\xi\rho(1-\zeta\gamma^{t}\ln\gamma)}{\xi(1-\eta)e^{\lambda-\zeta\gamma^{t}}}dt. \tag{17}$$

Solving (18) for the cumulative number of adopters yields

$$n=\int dn\ =\int\frac{\xi\rho(1-\zeta\gamma^{t}\ln\gamma)}{\xi(1-\eta)e^{\lambda-\zeta\gamma^{t}}}dt. \tag{18*}$$

This shows that (3*), (6*), (11*), and (18*) cannot be the same. Each expression predicts a different n, dn/dt, and dn. However, (18*) is a preferable generalization of which the other three are special cases. Table 1 summarizes the differences among the models, but leaves to a separate effort the explanation of conditions under which the three expressions can be equal.

4. Tentative concluding remark

Traditional models of the spread of innovations such as (3*) assume a homogenous source of learning about innovations so that diffusion follows a neat logistic curve. Although such a rationalization is possible, it is highly improbably in a multi-media world. Adopters of innovations may not even know (remember) from which of the many sources they learned an innovation from. Since learning itself is a cumulative process anyway, the last straw alone does not break the camel’s back.

Table 1 - Three general solutions to three general models of the diffusion of innovations

Solutions	**Model 1**	**Model 2**	**Model 3**
Rate of change	$dn/dt=\alpha n(N-n))$	$dn/dt=(\alpha n+\beta)[N-n]$	$\frac{dn}{dt}=\frac{\xi\rho(1-\zeta\gamma^t\ln\gamma)}{\xi(1-\eta)e^{\lambda-\zeta\gamma^t}}$
Differential	$dn=\alpha n(N-n)dt$	$dn=(\alpha n+\beta)[N-n]dt$	$dn=\frac{\xi\rho(1-\zeta\gamma^t\ln\gamma)}{\xi(1-\eta)e^{\lambda-\zeta\gamma^t}}dt$
Cumulative number of adopters	$n=\frac{Ne^{\alpha Nt}}{(N-1)+e^{\alpha Nt}}$	$n=\frac{(\alpha+\beta)Ne^{(\alpha N+\beta)t}-\upsilon(N-1)}{(N-1)\alpha+(\alpha+\beta)e^{(\alpha N+\beta)t}}$	$n=\int dn=\frac{\xi\rho(1-\zeta\gamma^t\ln\gamma)}{\xi(1-\eta)e^{\lambda-\zeta\gamma^t}}dt$

Nota bene: Model 1 is conventional model with one source of learning. Model 2 is Model 1 adjusted for multiple sources of learning. Model 3 adjusts for multiple sources of learning, wildfire-like behavior, and inhibitions.

Adjusting (3*) for social learning from heterogenous sources adds a considerable degree of dynamism and realism to the process - resulting in (6*). If the social and private payoffs are equal, the equilibrium rate of social learning drives the equilibrium rates of adoption and diffusion. Away from this equilibrium, nothing obvious suggests why (6*) should be logistic empirically. For one, innovations are always associated with externalities. If the marginal social benefits of an innovation outweigh the marginal private benefits, private markets of innovations will under-produce innovations. This raises the need to produce innovations by public means (subsidize supply or voucher demand) or live with the inefficient cost of under-production. Second, if the marginal social cost is higher than the private marginal cost of an innovation, over-production is the consequence, with further implications for the correction of negative externalities.

In either case solutions away from the equilibrium are ambiguous. More likely than not innovations would spread in unpredictable ways, even across homogenous sources, justifying the need to modify the social-learning model for unpredictable behavior. One can accommodate unpredictable behavior with a simple hypothesis that innovations spread like wildfires, and the result is (7) and (8). For example, assuming that R(t) is a well-behaved (differentiable) function with a solution $0 \leq R^*(t) \leq +1$, then (6*) is logistic only in the neighborhood of +1. Above and below R*(t), not only does it scale (12) up or down, it makes (12) fluctuate wildly like fire. This occurs not so much because the population is heterogenous, but more so because of the specific and diverse characteristics as Barbara Wejnert (2002) suggests (cf. Bulliet, 2002).

An additional complication is that, while *n* may be logistic, *R* may not be. Assuming that R(t) is a Gompertz, for instance, gives (12). It is true that the derivative of a Gompertz belongs to the same family of modified expontial functions as the logistic model (Banerjee, 2007, Harris, 1992, Thompson, 1992). However, that does not mean the two are the same - not all twins are identical twins.

Like wildfires innovations spread in unpredictable ways. Underlying this behavior are systematic and systemic inhibitions and exhibitions. In that case, the final solution to the diffusion process would depend on the interactions between the forces that promote and those that subjugate the spread of innovations. As Wejnert (2002) put it, "... the broad array of diffusion variables ... exert their effects on the process of diffusion interactively. The interaction between variables can be either potentiating or mitigating, and the relative weight of each variable may change according to the circumstances characterizing the innovation and its context" (p.318). Under such conditions it is no longer clear whether or not the logistic model is still an accurate description of the diffusion process. What is clear is that when the forces of promotion are stronger than the forces of subjugation, the capacity of the population to adopt and diffuse an innovation increases; when the opposite prevails, as the bee example illustrates, the innovation process is seriously complicated. In such situations understanding of the individual characteristics of the entire diffusion process is essential.

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