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Millan L. B. Mulraine

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# Real Exchange Rate Dynamics With Endogenous Distribution Costs\*

*(Job Market Paper)*

Millan L. B. Mulraine<sup>†</sup>

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## Abstract

The importance of distribution costs in generating the deviations from the law of one price has been well documented. In this paper we show that a two-country flexible price dynamic general equilibrium model driven by exogenous innovations to technology, and with a localized distribution services sector can replicate the key dynamic features of the real exchange rate. In doing so, the paper identifies the importance of two key channels for real exchange rate dynamics. That is, we show: (i) that shocks in the real sector are important contributors to movements in the real exchange rate, and (ii) that the endogenous wedge created by distribution costs of traded goods is a significant source of fluctuation for the real exchange rate, and the overall macro-economy as a whole. The evidence presented here demonstrates that this model - without any nominal rigidities, can account for up to 89% of the relative volatility in the real exchange rate.

**Keywords:** Distribution costs, Real exchange rate dynamics, Law of one price.

**JEL Classification:** E31, F31, F41

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<sup>†</sup>**Corresponding address:** Department of Economics, University of Toronto, 150 St. George Street, Toronto, ON M5S 3G7, Canada. Tel: 1 (416) 824 8069, Email: [millan.mulraine@utoronto.ca](mailto:millan.mulraine@utoronto.ca)

# 1 Introduction

In the international macroeconomics literature one puzzle remains as elusive today as it has been since its emergence thirty years ago - namely, the inability of theoretical economic models to replicate the key empirical features of the real exchange rate. That is, since the beginning of the post-Bretton Wood flexible exchange rate period of 1971, nominal and real exchange rates have exhibited significant volatility and persistence, and have been considerably more variable than the underlying economic fundamentals assumed to be associated with their determination. Following the publication of [Dornbusch \(1976\)](#) ‘overshooting’ model<sup>1</sup>, the quest of the new open economy macroeconomics literature has been to explain this puzzling behavior of bilateral real exchange rates by means of dynamic general equilibrium monetary models with nominal rigidities in the form of sticky prices, à la [Obstfeld and Rogoff \(1995\)](#).

This literature was further advanced by the monopolistic pricing-to-market model developed by [Betts and Devereux \(1996\)](#), which took account of the well established empirical observations on the pronounced deviations from the law of one price in traded goods - see [Lane \(2001\)](#) for a survey of the new open economy literature. This particular approach, however, is predicated on the assumption that movements in the deviation in the law of one price is due to rigidities in the pricing structure for these goods and not the wedge created by the distribution costs of these goods. Moreover, the fundamental premise of this strand of the literature is that the dynamic behavior of the exchange rate is due to the interaction between shocks to the money supply and sticky prices - thereby obviating the role of productivity innovations in determining the behavior of the real exchange rate, as postulated by the Balassa-Samuelson proposition<sup>2</sup>.

In an attempt to offer quantitative justifications for this approach, [Chari et al. \(2002\)](#) constructs a monetary dynamic general equilibrium (DGE) model to show that a model with

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<sup>1</sup> In this elegant theoretical exposition it was shown that the volatile behavior of the exchange rate was consistent with rational expectations in the presence of price rigidities. The overshooting of the nominal exchange rate beyond its long-run level, therefore, was shown to be the results of the interaction between monetary shocks and sluggish price adjustment.

<sup>2</sup>The Balassa-Samuelson hypothesis, due to the work of [Balassa \(1964\)](#) and [Samuelson \(1964\)](#), asserts that an increase in the productivity of tradables relative to non-tradables in the home country, if larger than the relative productivity of its trading partner, will cause an appreciation in the real exchange rate.

price stickiness for at least one period, and high risk aversion can replicate the fluctuations in the real exchange rate between the US and a European aggregate. They also show that adding real shocks (in the form total factor productivity (TFP) and government spending shocks) to their stylized model “change the model predictions little” - thereby concluding that real shocks are not important in explaining the dynamics of the real exchange rate.

The empirical evidence on the dynamic behavior of the real exchange rate over the post-Bretton Wood floating exchange rate period, however, casts doubts on the importance of shocks to the money supply in explaining the deviation in the real exchange rate. [Alexius \(2005\)](#) shows that between 60-90% of the volatility of the real exchange rate can be accounted for by relative productivity shocks across trading partners, thereby providing evidence that confirms the long held view of a Balassa-Samuelson effect on the real exchange rate. In addition to this study, [Carr and Floyd \(2002\)](#) shows that the dynamic behavior of the US-Canada real exchange rate, for example, can be explained by asymmetric real shocks, and that there appears to be no evidence to support the view that monetary shocks have had any significant impact on the dynamic behavior of the real exchange rate. Despite this evidence, the use of real quantitative general equilibrium models to explain the dynamic behavior of the real exchange rate has been largely absent from the international macroeconomics literature. In this paper, we present evidence that confirms the Balassa-Samuelson proposition by positing a real DGE model with a localized distribution services sector, and in so doing provide an avenue for this strand of the literature to be further advanced.

The importance of distribution costs in generating the deviation from the law of one price has been well documented. In a seminal piece, [Burstein et al. \(2003\)](#) shows that distribution costs are very large, and can account for over 40% of the final price of retail goods in the US and over 60% in Argentina. These costs are related to services such as transportation, insurance, and marketing and distribution services associated with bringing the internationally traded consumer good from the point of entry in the country to its final destination - the consumer. They are akin to the “iceberg costs” advocated by [Obstfeld and Rogoff \(2000\)](#). Consequently, to generate the deviations from the law of one price, this paper incorporates a localized distribution services sector. This technique departs fundamentally from the standard approach in the literature by treating the distribution services as a productive activity that combines domestic labor and capital inputs with the imported consumer good to create a new

domestically available consumer good. Thus, the model endogenously generates the wedge between the price of traded goods between the two countries, as observed in the data. The standard approach in the literature has been to assume that the consumption of a unit of imported consumer good requires a fixed amount of distribution services.

In this paper we develop a stylized two-country DGE model driven solely by exogenous innovations to real factors in the economy to demonstrate that this model can replicate the dynamic behavior observed in the real exchange rate between the US and a European aggregate. In particular, we specify a non-monetary DGE model driven by shocks to investment-specific technology (IST) and with a localized distribution services sector to show that a stylized model without nominal rigidities can match the stylized facts of the real exchange rate. The inclusion of a distribution service sector enables us to endogenously generate the deviations from the law of one price which has been a key feature of the real exchange rate and thus accounts for the contribution of the volatility in the real exchange rate made by movements in the relative price of traded goods.

The main exogenous propagation mechanism considered in this paper is shocks to the efficiency in the production of next period's capital goods - or investment-specific technology shocks. To take account of this unique measure of productivity innovation investment in machinery and equipment - which is a tradeable good - is directly affected by improved efficiency in their production of next period's capital stock. This form of innovation was shown by [Greenwood et al. \(1988, 2000\)](#) to account for over 30% of output fluctuation in the postwar US data. Further evidence on the impact of innovations to relative prices on the business cycle behavior of the US economy has been provided by [Fisher \(2003\)](#), who - using more recent data - finds that investment-specific technology shocks can accounts for over 50% of business cycle variation in hours worked, compared to only 6% accounted for by shocks to total factor productivity. [Boileau \(2002\)](#) also uses this propagation mechanism in a two-country framework to explore its ability to explain the cross-country correlation of output and the terms of trade.<sup>3</sup> As a comparison to the dynamics generated by the model propagated by investment-specific shocks, we shall also simulate the model using shocks to TFP.

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<sup>3</sup>[Letendre and Luo \(2005\)](#) and [Mulraine \(2004\)](#) also use this propagation mechanism in an open economy framework to study the dynamic behavior of the Canadian economy.

In more recent work, [Jin and Zeng \(2005\)](#) develops a flexible price model with distribution costs to show that with reasonably large distribution costs their model can generate highly volatile and persistent real exchange rate. This paper, however, differs from their work in a number of key dimensions. In this paper the innovation considered in this stylized economy comes from shocks to the efficiency of investment goods and not the Solow-neutral total factor productivity shocks considered in their work. Their model also incorporates innovations to the money supply. Secondly, and more importantly, in this paper we consider a distribution services sector instead of assuming that distribution costs are a fixed proportion of the price of traded consumer goods. And finally, in the model presented here, countries trade in not only consumption goods - as considered in their work, but also in the intermediate capital good.

The remainder of the paper proceeds as follows. In [Section 2](#) of this paper the model is presented with its dynamic features discussed in [Section 3](#). In [Section 4](#) the model solution is discussed and the simulation results from the calibration of the model to match the stylized facts of the real exchange rate presented. The paper then concludes in [Section 5](#).

## 2 The Model Environment

Consider a two-country, multi-sector general equilibrium open economy model in which each economy is comprised of three types of agents; a representative consumer, a representative consumer goods producer, and a representative distribution services firm. The two economies considered are structurally symmetric, as such it will suffice to describe the agents and their activities in the home country, with an asterisk (\*) denoting variables associated with the foreign country. These two economies are connected to each other through their trade in intermediate capital goods, consumer goods and financial assets.

### 2.1 The Representative Consumer

Each economy is populated by a large number of infinitely-lived identical agents of mass one who has taste for both domestic  $c_{1t}$  and foreign produced consumer goods  $c_{1t}^m$ . Each period the agent is endowed with a unit supply of labor which is divided among the two competing productive activities - the production of consumer goods  $l_{ct}$ , and the production of distribution

services  $l_{dt}$ . Labor is considered to be perfectly mobile across two productive sectors, but will be completely immobile across the two economies. The representative consumer chooses consumption  $c_t$  to maximize their expected discounted utility function expressed by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t), \quad 0 < \beta < 1 \quad (1)$$

where  $c_t$  is the consumption level of the composite consumer good which comprises of home and imported final consumption goods. The composite consumption good  $c_t$  is such that:

$$c_t = G(c_{1t}, c_{1t}^m) \quad (2)$$

where we aggregate home produced  $c_{1t}$  and imported  $c_{1t}^m$  final consumption goods using a constant elasticity of substitution (CES) aggregator function  $G(c_{1t}, c_{1t}^m)$  given by:

$$G(c_{1t}, c_{1t}^m) = [\omega_c (c_{1t})^{\eta_c} + (1 - \omega_c) (c_{1t}^m)^{\eta_c}]^{\frac{1}{\eta_c}} \quad (3)$$

This functional form of the CES aggregator enables us to characterize the respective preferences for home goods (or home bias) given by  $\omega_c$  relative to imported consumption goods, and the constant elasticity of substitution between home and foreign produced final goods,  $\sigma_c = \frac{1}{1-\eta_c}$ .

The representative household has access to a perfectly competitive international capital market where they can trade in international financial assets  $a_t$  at the endogenously determined risk-free world interest rate  $r_t$ . The accumulation of this type of financial asset will evolve according to the following process:

$$a_{t+1} = tb_t + (1 + r_t)a_t \quad (4)$$

where  $tb_t$  is the trade balance for the given period.

Each period, after observing the productivity innovation, the representative consumer devotes  $i_t$  of the total income in the form of investment which is used towards the creation of intermediate capital goods. This shock represents an exogenous innovation  $q_t$  to the efficiency of this investment good<sup>4</sup> in the creation of the total domestic stock of intermediate capital

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<sup>4</sup>Following the work of Greenwood et al. (2000), this paper espouses the use of *capital-embodied* technological changes as the main contributor to economic fluctuations in the economy considered. For an elaborate and exhaustive discussion on the issues related to the use of shocks to the efficiency of investment goods as a measure of technological innovation, see Fisher (1999).

goods given by  $m_t = q_t i_t$ . This stock of domestic intermediate goods is then divided between the portion used domestically  $k_{1t}$  for the production of next period capital stock, and the portion that is exported  $k_{1t}^*$  such that,  $m_t = k_{1t} + k_{1t}^*$ . The total domestic gross investment in the capital stock for the current period  $x_t$  involves a combination of domestic  $k_{1t}$  and foreign (imported) intermediate capital goods  $k_{2t}$ . Such that:

$$x_t = H(k_{1t}, k_{2t}) \quad (5)$$

where we aggregate home and foreign intermediate goods using a constant elasticity of substitution (CES) aggregator function  $H(k_{1t}, k_{2t})$ , such that:

$$H(k_{1t}, k_{2t}) = [\omega_k (k_{1t})^{\eta_k} + (1 - \omega_k) (k_{2t})^{\eta_k}]^{\frac{1}{\eta_k}} \quad (6)$$

Similar to the composite consumption goods outlined above, this functional for the intermediate good aggregator characterizes the respective preference for home goods (or home bias) given by  $\omega_k$  relative to foreign goods, and the constant elasticity of substitution between home and foreign produced final goods,  $\sigma_k = \frac{1}{1 - \eta_k}$ .

The representative consumer purchases the output of the consumer goods firm  $y_t$  and the output of the distribution goods firm  $c_{1t}^m$  from the available income for that period. In addition to trade in the intermediate capital goods, the consumer exports domestically produced consumer goods  $c_{1t}^*$  and trades in foreign assets  $a_t$ . As such, the budget constraint for the representative consumer is given by:

$$c_{1t} + p_{1t}^m c_{1t}^m + p_{kt} k_{1t} + p_{kt}^* k_{2t} + t b_t + \left( \frac{\phi_a}{2} \right) (a_{t+1} - \bar{a})^2 \leq w_t + r_t k_t \quad (7)$$

Subject to the following function which captures the evolution of the capital stock - net of the capital adjustment costs:

$$k_{t+1} = (1 - \delta) k_t + x_t - \left( \frac{\phi_k}{2} \right) (k_{t+1} - k_t)^2 \quad (8)$$

Where the variable  $x_t$  captures the stock of intermediate goods added to the non-depreciated closing stock of capital in period  $t$ . Capital adjustment costs are generally introduced in models of this type as a mechanism for smoothing the reallocation of wealth between physical capital and the foreign asset holdings in response to temporary differentials in the net rate of return between foreign and domestic assets. As a result, the adjustment cost can be seen



as the cost(s) associated with the installation of new machinery and equipment. This cost could be considered to include training, installation fees and the cost of disposing of the old stock of machinery. The essential thing to note here, however, is that this cost will act as a moderating force on the investment decision of domestic agents, thereby eliminating excessive responsiveness in investment decisions to small rate differentials and to ensure that any adjustment to the capital stock is gradual.

Following [Schmitt-Grohé and Uribe \(2003\)](#), we introduce a convex portfolio adjustment cost given by  $\left(\frac{\phi_a}{2}\right)(a_{t+1} - \bar{a})^2$  to prevent agents from playing Ponzi-type games, and consequently ensuring stationarity in the behavior of net foreign asset holdings. In this framework, given the symmetric nature of the two economies considered the steady-state value of net foreign asset is assume to be zero, that is  $(\bar{a} = 0)$ .

## 2.2 The Representative Final Goods Firm

The home country produces an internationally tradeable composite consumption commodity whose Cobb-Douglas production technology is given by:

$$y_t = z_t k_{ct}^\alpha l_{ct}^{1-\alpha} \quad (9)$$

Where  $z_t$ ,  $k_{ct}$  and  $l_{ct}$  represent the innovation to total factor productivity in the production of consumer goods, and the amount of capital and labor services which are allocated to this sector, respectively. Thus, the static profit maximization problem for this representative consumption good producer in the home country is given by:

$$\max \pi_t^c = y_t - w_t l_{ct} - r_t k_{ct} \quad (10)$$

Note that the constant returns to scale technology for this consumptions goods firm necessitates that it makes zero profit.

## 2.3 The Localized Distribution Sector Firm

As outlined above, the imported consumption good requires the services of the local distribution sector. We follow [Burstein et al. \(2003\)](#) in assuming that non-traded domestically produced commodities require no distribution services since the most important components

of this expenditure are housing, health and education services. This representative distribution sector firm, therefore, provides all the requisite services that are entangled in the movement of the imported consumption goods  $c_{2t}$  from its ‘point of entry’ in the home economy to the ‘point of consumption’  $c_{1t}^m$ . These services include transportation, advertising, insurance, warehousing and all other services associated with bringing the imported goods from the border to the retail outlet. In accordance with the findings of [Burstein et al. \(2004\)](#), we abstract from having traded investment goods requiring any distribution services, since they have shown that the share of distribution services in the retail prices of investment good is very low. As a result, the profit maximization problem for this sector is given by:

$$\max \pi_t^d = p_{1t}^m c_{1t}^m - w_t l_{dt} - r_t k_{dt} - p_{2t} c_{2t} \quad (11)$$

subject to a Cobb-Douglas production function given by:

$$c_{1t}^m = [\omega_d (d_t)^{\eta_d} + (1 - \omega_d) (c_{2t})^{\eta_d}]^{\frac{1}{\eta_d}} \quad (12)$$

In this regard, the distribution firm takes the imported consumer goods  $c_{2t}$  and augments it with distribution services  $d_t$  before making it available at the consumer outlet as  $c_{1t}^m$ . The production function for the distribution service is given by the following function:

$$d_t = z_t k_{dt}^\alpha l_{dt}^{1-\alpha} \quad (13)$$

One key feature of this framework is the fact that the distribution cost associated with the movement of the imported consumer good from the border to domestic consumers in this model  $\phi_t^d = p_{1t}^m - p_{2t}$  arises endogenously, a departure from the standard literature where these costs are considered to be fixed and exogenously. Note that the distribution firm pays  $p_{2t}$  at the border for the imported consumption good and sells it at  $p_{1t}^m$  to home consumers. The constant returns to scale technology for the representative distribution services firm ensures that profits are zero, and consequently the retail price of the imported consumer good includes no mark-up. As before, the functional for the imported good aggregator characterizes the respective share of the imported good accounted for by distribution services given by  $\omega_d$  relative to the imported good, and the constant elasticity of substitution between distributions services and foreign produced final goods,  $\sigma_d = \frac{1}{1-\eta_d}$ .

## 2.4 The Real Exchange Rate

To assess the dynamic properties of the real exchange rate of this stylized model we shall use two definitions of the real exchange rate. The first of which will be the real exchange associated with consumer prices. Recall that the representative consumer in both economies consumes two types of consumer goods - domestically produced consumer goods and the imported consumer good - given by  $c_{1t}$  and  $c_{1t}^m$ , respectively, for the domestic consumer, and  $c_{2t}$  and  $c_{2t}^m$  for the foreign consumer. The prices associated with these commodities are  $p_{1t}$  (normalized to 1) and  $p_{1t}^m$ , for the domestic consumer and  $p_{2t}$  and  $p_{2t}^m$  for the foreign consumer. As such, given that the CES functional form of the utility function, the CPI for the home country  $p_t^c$  and the foreign economy  $p_t^{c,*}$  are given by:

$$p_t^c = \left[ \omega_c^{\frac{1}{1-\eta_c}} + (1 - \omega_c)^{\frac{1}{1-\eta_c}} (p_{1t}^m)^{\frac{\eta_c}{\eta_c-1}} \right]^{\frac{\eta_c-1}{\eta_c}} \quad (14)$$

$$p_t^{c,*} = \left[ \omega_c^{\frac{1}{1-\eta_c}} (p_{2t})^{\frac{\eta_c}{\eta_c-1}} + (1 - \omega_c)^{\frac{1}{1-\eta_c}} (p_{2t}^m)^{\frac{\eta_c}{\eta_c-1}} \right]^{\frac{\eta_c-1}{\eta_c}} \quad (15)$$

If the law of one price for the traded consumer goods were to hold, then we would expect the consumer price of the traded consumer goods to be such that  $p_{2t} = p_{1t}^m$  and  $p_{1t} = p_{2t}^m = 1$ . Following [Betts and Kehoe \(2001\)](#), we obtain an expression for the real exchange rate associated with consumer prices as:

$$rer_t^c = \frac{p_t^{c,*}}{p_t^c} \quad (16)$$

Similarly, we can derive the the real exchange associated with the intermediate capital good - based on producer prices. Recall that the representative consumer in both economies uses two types of intermediate capital goods in the production of next period's capital goods - domestically produced investment good and the imported investment good - given by  $k_{1t}$  and  $k_{2t}$ , respectively for the home consumer, and  $k_{2t}^*$  and  $k_{1t}^*$  for the foreign consumer. The prices associated with these commodities are  $p_{kt}$  and  $p_{kt}^*$ , for the domestic consumer and  $p_{kt}^*$  and  $p_{kt}$  for the foreign consumer. As such, given that the CES functional form of investment, the PPI for the home country  $p_t^k$  and the foreign economy  $p_t^{k,*}$  are given by:

$$p_t^k = \left[ \omega_k^{\frac{1}{1-\eta_k}} (p_{kt})^{\frac{\eta_k}{\eta_k-1}} + (1 - \omega_k)^{\frac{1}{1-\eta_k}} (p_{kt}^*)^{\frac{\eta_k}{\eta_k-1}} \right]^{\frac{\eta_k-1}{\eta_k}} \quad (17)$$

$$p_t^{k,*} = \left[ \omega_k^{\frac{1}{1-\eta_k}} (p_{kt}^*)^{\frac{\eta_k}{\eta_k-1}} + (1 - \omega_k)^{\frac{1}{1-\eta_k}} (p_{kt})^{\frac{\eta_k}{\eta_k-1}} \right]^{\frac{\eta_k-1}{\eta_k}} \quad (18)$$

We obtain an expression for the real exchange rate associated with producer prices as:

$$rer_t^k = \frac{p_t^{k,*}}{p_t^k} \quad (19)$$

## 2.5 Stochastic Processes

To close the model setup the propagation mechanism for the shocks to IST and TFP are described by the following processes:

$$\mathbf{q}_{t+1} = \Gamma_q \mathbf{q}_t + \xi_t^q \quad (20)$$

and

$$\mathbf{z}_{t+1} = \Gamma_z \mathbf{z}_t + \xi_t^z \quad (21)$$

Where  $\mathbf{q} = [\ln(q), \ln(q^*)]'$  and  $\mathbf{z} = [\ln(z), \ln(z^*)]'$ . Here  $\Gamma_q$  and  $\Gamma_z$  are the matrices of coefficients, and  $\xi_q$  and  $\xi_z$  are the vectors of mean zero normal random variables with contemporaneous variance-covariance matrix given by  $\Sigma_q$  and  $\Sigma_z$ , corresponding to the innovations to IST and TFP, respectively.

## 2.6 Model Behavior

In order to provide intuition into the model's behavior we shall briefly analyze the main channels of operation present. Recall that the dynamic behavior of this model hinges on two key features incorporated in this framework, namely: (i) the distribution services sector, and (ii) trade in both consumer and intermediate capital goods. The first of these features provides the natural wedge between the domestic and foreign price of traded consumption goods - or the deviation from the law of one price in traded goods. This endogenous wedge arises from the fact that imported consumption goods must first be augmented with domestic distribution services before they are consumed by agents in the model.

The role of distribution services in the model is quite simple, but significant. Its existence means that the price of the same commodity across the two countries will differ by the cost of providing the distribution services. As a consequence of this difference in the price across the two countries for traded consumption goods, movements in the price of a unit of distribution service will result in deviation in the relative price of traded consumer goods across the two countries, and hence deviations in the real exchange rate via a difference in the aggregate

price index of the respective economies. The endogenous nature of this sector means that this wedge will vary with relative prices, and will not be constant as is the standard in the literature. Moreover, because distribution services require the input of labor and capital, there is an inevitable trade-off between the production of domestic consumer goods and distribution services.

The trade in both producer and consumer goods - with only consumer goods requiring distribution services, provides another key conduit through which any propagation can have divergent impacts across the two economies. Foremost among these will be its impact on the consumption and investment decision of agents during the period in which the shock occurs. Because the final consumption  $c_t$  and investment  $x_t$  good requires a mix of domestic and foreign produced goods, and with the elasticity of substitution between these goods being constant, there will be opportunities for technology and wealth sharing across the two economies as a result of asymmetric shocks.

For example, consider a unit shock to investment efficiency in the domestic economy relative to the foreign economy. The first-period wealth effect from this innovation will result in increased consumption and investment for domestic agents, while the substitution effect will favor accumulation of investment goods relative to consumption goods. The decrease in consumption will favor the importation of the relatively cheaper foreign-produced consumer goods while the increase in capital will result in the creation of more domestic intermediate capital goods. To finance these additional expenditure, domestic agents will inevitably borrow against the higher rate of return from capital in the future. Clearly, the ramifications of these two effects cannot be fully analyzed in this stochastic model as they will affect not only the quantities traded, but also the prices of these commodities traded and consequently, the aggregate price level. As a result we shall now turn to the numerical solution to the model.

### 3 Recursive Problem for the Home Economy

Let  $V(k, k^*, a, q, q^*, z, z^*)$  be the value function for the representative agent whose assets at the beginning of the period are given by the holding of foreign assets  $a$ , and the stock of capital  $k$ , and who faces the equilibrium wage and rental rate of capital are given by  $w$  and  $r$ , respectively. Let the aggregate state of the world facing this representative consumer be

described by  $\lambda(k, k^*, a, q, q^*, z, z^*)$ , where  $A(\lambda)$  and  $K(\lambda)$  are the aggregate holding of assets, and  $P(\lambda)$ ,  $R(\lambda)$ , and  $W(\lambda)$  are the aggregate vector of prices, and the equilibrium rental price for capital and labor, respectively. The dynamic problem facing the representative household of the home economy<sup>5</sup> who takes the aggregate state of the world as given can be represented by the following Bellman equation:

$$V(k, k^*, a, q, q^*, z, z^*) = \max_{a', c_1, c_1^m, c_2, k_1, k_2, k', k_c, l_c} \{\log(c) + \beta E[V(k', k^*, a', q', q^*, z, z^*)]\} \quad (22)$$

subject to:

$$c_1 + p_2 c_2 + p_k k_1 + p_k^* k_2 + tb + \left(\frac{\phi_a}{2}\right)(a' - \bar{a})^2 \leq y + d \quad (23)$$

$$tb = a' - (1 + r)a \quad (24)$$

$$l_c + l_d \leq 1 \quad (25)$$

$$k_c + k_d \leq k \quad (26)$$

$$k' = (1 - \delta)k + x - \left(\frac{\phi_k}{2}\right)(k' - k)^2 \quad (27)$$

$$x = [\omega_k (k_1)^{\eta_k} + (1 - \omega_k)(k_2)^{\eta_k}]^{\frac{1}{\eta_k}} \quad (28)$$

$$c = [\omega_c (c_1)^{\eta_c} + (1 - \omega_c)(c_1^m)^{\eta_c}]^{\frac{1}{\eta_c}} \quad (29)$$

$$c_1^m = [\omega_d (d)^{\eta_d} + (1 - \omega_d)(c_2)^{\eta_d}]^{\frac{1}{\eta_d}} \quad (30)$$

$$y = zk_c^\alpha l_c^{1-\alpha} \quad (31)$$

$$d = zk_d^\alpha l_d^{1-\alpha} \quad (32)$$

$$q' = \rho_q q + \rho_{q, q^*} q^* + \varepsilon_q \quad (33)$$

and

$$z' = \rho_z z + \rho_{z, z^*} z^* + \varepsilon_z \quad (34)$$

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<sup>5</sup>A symmetric recursive problem exists for the foreign country.

### 3.1 Definition of the Competitive Equilibrium

A competitive equilibrium for the decentralized world economy is a vector of prices  $P(p_1^m, p_2^m, p_2, p_k, p_k^*, r, w, w^*)^6$  and allocations  $(a', k', c_1, c_1^m, c_2, k_1, k_2, k_c, k_d, l_c, l_d)$  for the home country, and  $(k^{*'}, c_1^*, c_2^m, c_2^*, k_1^*, k_2^*, k_c^*, k_d^*, l_c^*, l_d^*)$  for the foreign country such that:

1. The representative households in the home and foreign country solve their problems taking the aggregate state of the world  $\lambda(\cdot)$  and the vector of aggregate prices  $P(\cdot)$  as given, with the optimal allocations to their problem satisfying  $a' = A'(\lambda)$ ,  $c_1 = C_1(\lambda)$ ,  $c_1^* = C_1^*(\lambda)$ ,  $c_1^m = C_1^m(\lambda)$ ,  $c_2^m = C_2^m(\lambda)$ ,  $k_1 = K_1(\lambda)$ ,  $k_1^* = K_1^*(\lambda)$ ,  $k_2 = K_2(\lambda)$ ,  $k_2^* = K_2^*(\lambda)$ ,  $k' = K'(\lambda)$  and  $k^{*'} = K^{*'}(\lambda)$ ;
2. Given prices, the allocations  $l_c = L_c(\lambda)$ ,  $l_c^* = L_c^*(\lambda)$ ,  $k_c = K_c(\lambda)$  and  $k_c^* = K_c^*(\lambda)$ , solve the profit maximization problems for the representative consumer goods producers in the home and foreign economy.
3. Given prices, the quantities  $c_2 = C_2(\lambda)$ ,  $c_2^* = C_2^*(\lambda)$ ,  $l_d = L_d(\lambda)$ ,  $l_d^* = L_d^*(\lambda)$ ,  $k_d = K_d(\lambda)$  and  $k_d^* = K_d^*(\lambda)$  solves the profit maximization problems for the representative distribution services firms in the home and foreign economy.
4. All factors and goods markets clear.
5. The global goods and asset markets<sup>7</sup> clear. Such that:

$$tb + tb^* = 0 \tag{35}$$

## 4 Quantitative Assessment

Having fully specified the model, we shall now turn to the quantitative assessment of its properties. Note that due to the stochastic nature of the model specification, the competitive equilibrium allocations do not have closed-form analytical solutions. Thus, the model must be solved numerically. This will be done by finding an approximate solution to the log-linear

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<sup>6</sup>All prices considered in this model economy are expressed relative to the price of the domestically produced consumer good  $p_1$  - which has been normalized to 1. As such, the relative price of the traded intermediate goods  $k_1$  for the home country and  $k_2$  for the foreign country, are given by  $p_k = \frac{1}{q}$  and  $p_k^* = \frac{p_2^*}{q^*}$ , respectively. Note that these prices correspond to the relative price of investment goods in the respective economies.

<sup>7</sup>The trade balance (or net exports) for the home economy is given by  $tb = c_1^* + p_k k_1^* - p_2 c_2 - p_k^* k_1$ . As such, the balance of payment can be expressed by  $tb + ra - (a' - a) = 0$ .

approximation of the first order conditions about their steady-state values. The approach used here is akin to the method advocated by [King et al. \(1988\)](#), and is similar to that used by [Schmitt-Grohé and Uribe \(2003\)](#). A complete discussion of the solution to this model, and the derivation of its steady-state properties is provided in [Appendix A.1 - A.3](#) of the paper. The moments used in this analysis are the relative percentage standard deviation - that is, the standard deviations of the macroeconomics variables of interest relative to the standard deviation of GDP, the first-order autocorrelation of each variable of interest, and their respective cross-country correlation.

#### 4.1 Parameter Calibration

Since the two economies considered in this framework are symmetric in all respects, all the parameters used have the same value in both economies. See [Table 3](#) for a complete summary of all parameter values. Following [Backus et al. \(1994\)](#), the constant discount factor is set at  $\beta = 0.96$ , corresponding to an annual steady-state interest rate of  $r^* = 0.04$ , and the constant depreciation rate for capital was set at  $\delta = 0.10$ . For the benchmark model the value of the elasticity of substitution between home and foreign consumer goods, distribution services and imported consumer goods, and home and foreign machinery and equipment were set to  $\sigma_c = \sigma_d = \sigma_k = 2.5$ , such that  $\eta_c = \eta_d = \eta_k = 3/5$ , respectively. To determine the share parameters for the domestic consumption goods, investment goods and distribution services given by,  $\omega_c$ ,  $\omega_k$  and  $\omega_d$ , respectively, we follow the calibration procedure of [Boileau \(2002\)](#) who set the steady-state output share of trade in final goods and trade in equipment to 20% and 10%, respectively. In addition to these output shares, following the findings of [Burstein et al. \(2003\)](#), we also set the steady-state distribution margin for imported consumer goods equal to 42% - equivalent to the distribution margin for consumer prices in the US. As a result of this calibration procedure, we obtain the respective shares given by  $\omega_c = 0.575$ ,  $\omega_k = 0.635$  and  $\omega_d = 0.320$ .

The value of the elasticity of substitution chosen accords to the “large elasticity” case considered by [Backus et al. \(1994\)](#). This value was chosen as the benchmark case since the higher elasticity has been shown by them to be associated with a lower correlation between the trade balance and the terms of trade. As such, with a higher elasticity of substitution we



would expect the price deviation across countries to be lower, given the inverse relationship between the elasticity of substitution and the terms of trade. Consequently, this high value will provide us with the lower limit on the volatility of the real exchange rate in this framework.<sup>8</sup>

Following [Boileau \(2002\)](#), the productivity innovation to investment efficiency has a standard deviation of 0.013 and a cross-country correlation of 0.345. The exogenous investment-specific shock process has a autocorrelation of 0.553 and a cross-country feedback of 0.027. As such the matrices for  $\Sigma_q$  and  $\Gamma_q$  are given by.

$$\Sigma_q = 10^{-4} \begin{pmatrix} 1.687 & 0.582 \\ 0.582 & 1.687 \end{pmatrix}, \quad \Gamma_q = \begin{pmatrix} 0.553 & 0.027 \\ 0.027 & 0.553 \end{pmatrix}$$

For the properties of the innovation to TFP we follow [Kehoe and Perri \(2002\)](#) and [Chari et al. \(2002\)](#), in which case the standard deviation is set at 0.007 and a cross-country correlation of 0. The zero cross-country spillover follows the findings of [Baxter \(1995\)](#) and [Kollmann \(1996\)](#) who find very little evidence of technology spillover. The exogenous investment-specific shock process has a autocorrelation of 0.95 and a cross-country feedback of 0.025. As such the matrices for  $\Sigma_z$  and  $\Gamma_z$  are given by.

$$\Sigma_z = 10^{-4} \begin{pmatrix} 0.4900 & 0.1125 \\ 0.1225 & 0.4900 \end{pmatrix}, \quad \Gamma_z = \begin{pmatrix} 0.95 & 0 \\ 0 & 0.95 \end{pmatrix}$$

Given the parameter values for the stochastic processes described above, the only remaining parameter to be determined in this model will be  $\phi_i$ ,  $i \in \{a, k\}$  - the adjustment costs parameter. These parameter are particularly important since they determine (to a large extent) the investment and net foreign assets responses to innovations to the exogenous process considered. Despite their significance, however, these parameters cannot be determined ex-ante as they have no empirical counterparts. As such, we set their values in a manner that will limit the volatility in investment, while simultaneously ensuring stationarity in all macroeconomic variables. In particular, we set them such that the volatility of consumption is equal to its empirical counterpart and to ensure stationarity in the holding of net foreign assets. To this end, we use a value of  $\phi_a = 0.36$  and  $\phi_k = 1.60$ . This value of the capital adjustment cost is close to the lower limit of the range of 1.50 and 2.32 advocated by [Greenwood et al.](#)

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<sup>8</sup>As a sensitivity check we provide the moments for two other values  $\sigma_i = 1.5$  and  $\sigma_i = 2.0$ .

(2000). For the TFP shocks model we follow [Kehoe and Perri \(2002\)](#) by setting the capital adjustment cost parameter  $\phi_k = 0.60$ .

## 4.2 Simulation Results

This section is primarily aimed at comparing the performance of the model presented in matching the dynamic properties of the data. To do this, particular attention will be paid to the ability of the model to match key moments in the data. That is, we shall examine the percentage standard deviation (relative volatility)<sup>9</sup>, first-order autocorrelation and cross-country correlation of total consumption, total investment, net export, the CPI-based real exchange rate, and the PPI-based real exchange rate. Further analysis will be conducted by examining the associated dynamics of these variables of interest by studying the impulse responses generated from a 1% shocks to technology.

## 4.3 Investment-Specific Technology (IST) Shocks Model

The requisite unconditional moments generated from the simulation exercises are presented in [Table 1](#) below. Column 2 of [Table 1](#) refers to the associated moments obtained from [Chari et al. \(2002\)](#). In Columns 5 we present the unconditional moments for the macroeconomic aggregates for the benchmark model, in which case the elasticity of substitution between traded goods are set at  $\sigma_i = 2.5$ . As seen in this column, with the model simulated to match the relative standard deviation of consumption, the model can explain as much as 32% of the CPI-based real exchange rate, with the relative volatility of the PPI-based real exchange rate being twice that of its CPI-based counterpart. The model also provides autocorrelation statistic for the CPI-real exchange rate that are relatively close to those observed in the data.

The benchmark model matches quite reasonably the persistence of consumption and GDP, though it does a better job at matching the high persistence of consumption than it does in matching that of GDP. Despite the appealing performance in matching the relative volatility and persistence of the real exchange rate, consumption and GDP, the model performs very poorly in matching the empirical behavior of investment. As is evident for the table, the model overestimates the relative volatility of investment by a factor of 2 and underestimates

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<sup>9</sup>We do not directly report the volatility of the variables of interest but instead report their volatility relative to the volatility of total output.

its persistence by a factor of 2. This excessive volatility emerges as a consequence of the high elasticity of substitution considered and is a direct consequence of the nature of the productivity innovation considered, similar to the evidence documented by [Boileau \(2002\)](#).

In terms of the cross-correlation of the macroeconomic aggregates, the benchmark model has been able to replicate the positive cross-correlation of GDP, consumption and investment - consistent with the data. This outcome is important since it highlights the fact that this model with trade in equipment avoids the counterfactual negative cross-country correlation in investment and almost-perfect cross-country correlation in consumption - two outcomes that have been the feature of standard international business cycle models. In essence, by incorporating trade in equipment, the innovation to investment technology in the home country is exported to the foreign country which will invariably benefit from this new technology, consistent with the empirical observations. This feature mitigates against the impact of risk-sharing in form of financial asset flows to the country with the favorable productivity innovation, thereby eliminating the negative cross-country investment correlation and the associated high cross-country correlation of consumption.

In addition to these key features, the model is also able to correctly replicate the ranking of the cross-country correlation of output and consumption, compared to the standard predictions in the literature. In general, stochastic DGE models have provided estimates of the cross-country correlation of consumption that have been higher than that of output, opposite to the observed ranking in the data where  $\rho_{y,y^*} > \rho_{c,c^*}$ . This anomaly has been called the “quantity puzzle” by [Backus et al. \(1992\)](#). This progress, however, must be tempered by the fact that these statistics are over-estimated by a factor of two, relative to the moments in the data in the case of consumption.

The dynamic behavior of the variables of interest in the stylized model following a 1% shock to investment-efficiency - with the elasticity of substitution set to 2.5 - are presented in [Figure 1 - Figure 3](#). Here we can ascertain the response of home ([Figure 1](#)) and foreign ([Figure 2](#)) macroeconomic aggregates, and prices, foreign assets and the trade balance ([Figure 3](#)). From these, it becomes apparent that the shocks to productivity in domestic investment goods along with trade in these intermediate goods result in an immediate accumulation of investment in capital goods in both countries, though the level of accumulation in the home country where the shock occurred was twice that of its trading partner. The magnitude of the rise in

investment in the two countries, however, differs as the home country experiences a considerably higher increase in investment than its trading partner. Consistent with economic theory, the home country substitutes the foreign intermediate goods for the more efficient domestic counterpart, while in the foreign country the local intermediate goods was substituted by the relatively cheaper imported investment goods.

The behavior of total consumption in these two economies is also instructive. As is evident from the graph total consumption falls significantly in the first period before rising above its steady-state level in both countries. Similar to the evidence on the behavior of investment accumulation, the level of consumption falls by a larger amount in the foreign country than in the home economy. In the home country the substitution effect dominated the wealth effect from the higher productivity in the home country as agents substitutes consumption in the initial period of the shock for investment goods which was used to financed the higher consumption (relative to its steady-state level) in subsequent periods. Note that in the home country the greater increase in investment along with a smaller reduction in consumption relative to the foreign country was financed by the accumulation of foreign debts.

Figure 3 provides dynamics for the various prices and the domestic holdings of net foreign assets. As seen, the home country accumulates foreign debts (negative trade balance) which is used to financed its increased investment and consumption over the horizon of the analysis. The dynamic movement of the price indices considered and their associated real exchange rate is consistent the Balassa-Samuelson, the productivity shock resulted in a depreciation in the real exchange rate.

#### 4.4 The Role of the Distribution Sector

To ascertain the role of distribution services in generating the dynamics of the real exchange rate in the benchmark model, we posit a more parsimonious model with a modified localized distribution services sector. That is, we assume that the quantity of distribution services required to augment the foreign-produced imported consumer goods  $c_{2t}$  in the production of the locally consumed imported good  $c_{1t}^m$  is a fixed proportion of domestic output of the

tradeable consumer goods<sup>10</sup>. That is:

$$d_t = \phi_d y_t \tag{36}$$

Where  $\phi_d$  was set equal to 0.02 to reflect the fact that in the steady-state of the benchmark model distribution services accounted for 2% of GDP. Note that in this framework the capital stock and the unitary supply of labor will be devoted entirely to the production of the tradeable consumer good. Moreover, for the sake of comparative purposes we maintained the parameterization used in the benchmark model. The results from this simulation exercise are presented in Column 6 of [Table 1](#) below.

From the results presented in [Table 1](#) it becomes evident that the modified model generates far less volatility than the benchmark model. In the case of the CPI-based real exchange rate we have that the relative volatility is almost ten times smaller than in the benchmark model, while for the PPI-based model the reduction in the volatility is by a factor of 3. The majority of this reduction arises from the reduced volatility in the domestic price of the traded consumer good  $p_{1t}^m$ <sup>11</sup> - a direct consequence of the reduced volatility of the distribution costs. The disproportionate reduction in the relative volatility of the CPI-based real exchange rate compared to the relative volatility of the PPI-based real exchange rate results from the definition of the the CPI-based real exchange rate given by [Equation 14 - 16](#), in which case it becomes evident that the endogenous wedge created by the distribution costs becomes less volatile thereby reducing the volatility of the domestic price of the traded consumer good  $p_{1t}^m$ .

The modified model configuration also resulted in reduced relative volatility in investment and the trade balance - with a modest increase in the relative volatility of in consumption. More generally, this model also correctly replicates the ranking of the cross-country correlation of output and consumption, consistent with the stylized fact. We also obtained, but do not report here, the impulse responses of the main macroeconomic variables of interest. These impulse responses are similar in their direction to the benchmark model with the main difference being that the magnitude of the responses were less than in the benchmark model - as expected.

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<sup>10</sup>All the other features of the benchmark model were maintained.

<sup>11</sup>Recall that the domestic price of the imported consumer good  $p_{1t}^m = p_{2t} + \phi_t^d$ . Where  $\phi_t^d$  is the distribution cost of producing  $c_{1t}^m$ .

## 4.5 Sensitivity Analysis

Following [Backus et al. \(1994\)](#), we vary the value of the elasticity of substitution as a sensitivity check of the behavior of the benchmark model. In Column 3 of [Table 1](#) we provide unconditional moments for the macroeconomic aggregates for an elasticity of substitution equal given by  $\sigma_i = 1.5$ , and in Column 4 we provide moments for  $\sigma_i = 2.0$ .<sup>12</sup> The values for the adjustment cost parameters were maintained at the same level as in the benchmark model. Note also that to maintain the import shares outlined above, the CES shares given by the  $\omega_i$ 's must also be re-calibrated. The re-calibrated values are provided in [Table 3](#) below.

The evidence provided in [Table 1](#) shows that in the case of the IST shocks model, the lower the elasticity of substitution the better the performance of the model in matching the relative volatility of the CPI-based real exchange rate. In the case of the “low elasticity” case the model matches over 90% of the relative volatility of the CPI-based real exchange rate. The model also improves on matching the empirical properties of investment. In addition to slight improvements in the volatility of investment<sup>13</sup>, the model correctly replicates the stylized facts on the cross-country correlation of GDP, consumption and investment. That is, the evidence provided shows that a model with trade in equipment avoids the counterfactual negative cross-country correlation in investment and almost-perfect cross-country correlation in consumption obtained by the standard two-country real business cycles models. Moreover, the model was also able to achieve the cross-country correlation ranking of consumption and output observed in the data, thereby providing an avenue for solving the “quantity puzzle”.

## 4.6 Total Factor Productivity (TFP) Shocks Model

In [Table 2](#) we present the simulated values for the unconditional moments of the stylized model propagated by innovations to TFP<sup>14</sup>. It is evident from this table that the standard TFP shocks model performs very poorly in replicating the stylized features of the real exchange

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<sup>12</sup>The associated impulse responses for these values of the elasticity of substitution are not shown.

<sup>13</sup>Note that the volatility of investment generated by the model remains three times the size observed in the data.

<sup>14</sup>The features of this TFP shocks model is identical to the IST shocks model considered above with two exceptions. Firstly, the properties of the innovations to TFP were taken from [Kehoe and Perri \(2002\)](#) and [Chari et al. \(2002\)](#). Secondly, the value for the capital adjustment costs parameter in this model was set equal to 0.6, as in [Kehoe and Perri \(2002\)](#) - compared to a value of 1.6 in the IST shocks model.

rate. As seen, this form of productivity innovation accounts for at most 35% of the relative volatility of the CPI-based real exchange rate, compared to the corresponding values of 89% from the IST shocks model. This outcome concurs with the findings of Chari et al. (2002) who show that adding real shocks (in the form TFP and government spending shocks) to their stylized model “change the model predictions little”. On the other hand, this model outperform the IST shocks model in matching the relative volatility of investment by obtaining estimate that are not as excessively volatile as in the IST shocks model.

The model also does poorly in matching the observed cross-country correlations of the macroeconomics variables of interest. In particular, the estimates of the benchmark model  $\sigma_i = 2.5$  exhibit negative cross-country correlations. This outcomes is the result of the assumptions of zero spill-over in the TFP innovations across the two countries and the high elasticity of substitution between the goods. This feature of the model is in sync with similar estimates in the literature. The signs of the estimates, however, reverses when the elasticity of substitution is reduced - with the exception of consumption which remained negative in the low-elasticity case. In terms of the serial correlation of the variables, the estimates were reasonably close to their empirical counterpart with the serial correlation of investment being particularly close.

As a way of understanding the discrepancies in the behavior of the TFP shocks model we present impulse responses of some variables of interest to a 1% shock to total factor productivity in the home country in Figure 4 - Figure 6, with the elasticity of substitution set to 2.5. Here the response of home (Figure 4) and foreign (Figure 5) macroeconomic aggregates, and prices, foreign assets and the trade balance (Figure 6) are displayed. Note that in this model the persistence and cross-country feedback of the unit shock to TFP are  $\rho_z = 0.95$  and  $\rho_{z,z^*} = 0$ , compared to the corresponding values of  $\rho_q = 0.55$  and  $\rho_{q,q^*} = 0.027$  in the IST shocks model.

The first observation that arises from the graph is that in the home country the wealth effect dominates the substitution effect. That is, with the relative productivity shock and zero spill-over to the foreign country, the home country engages in the accumulation of foreign debts by lending to the foreign country, compared to the IST shocks model in which the home country borrowed from the foreign country to take advantage of the improved efficiency in investment goods. In addition to this observation, the prices indices reported and their

associated real exchange rates moved in opposite behavior to their counterparts in the IST shocks model with the asset accumulation resulting in the appreciate of the two measures of the real exchange rate.

## 5 Conclusion and Extensions

In this paper we show that a model propagated by shocks to investment-specific technology (IST), and with a localized distribution services sector can account for up to 79% of the relative volatility in the real exchange rate. These findings accord with the Balassa-Samuelson proposition, and provide evidence to show that real innovations in the economy can account for a significant portion of the empirical volatility of the real exchange rate, consistent with the findings of [Alexius \(2005\)](#). Moreover, by incorporating the localized distribution services sector, the model has been able to endogenously generate the deviation in the prices of traded goods in the two economies considered. In addition to generating dynamics consistent with the stylized facts on the real exchange rate, we have been able to show that a model with trade in intermediate capital goods can provide a resolution to the longstanding “quantity puzzle” of [Backus et al. \(1992\)](#).

The model specification considered is motivated by the evidence on the importance of the distribution costs in generating the deviation in the law of one price, and the long held view that productivity innovations play an important role in real exchange rate dynamics. It is aimed at providing an avenue for considering the contribution of innovation to productivity in explaining the real exchange rate - a facet of the literature that has been largely missing. In particular, the model departs from the use of the standard TFP shock model which has been shown by [Chari et al. \(2002\)](#) to be ineffective in generating the requisite volatility in the real exchange rate.

The model specification offers a number of avenues for further extension. An interesting extension to the framework considered here will be to augment the IST shocks model with innovations to government spending, and to compare the performance of a model with these two real innovations with those of a model driven by innovations to the money supply. It also affords us with the ability to directly examine the role of distribution services in real exchange rate dynamic and to compare this stylized behavior to that in the data.



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Table 1: Observed and Simulated Moments for IST Shocks Model

	Data:	$\sigma_i = 1.5:$	$\sigma_i = 2.0:$	$\sigma_i = 2.5:$	$\sigma_i = 2.5^*:$
<b>Relative Volatilities:</b>					
Consumption	0.8300	1.3040	1.0885	0.8396	1.0033
Investment	2.7800	6.0709	6.6812	7.7531	4.1997
Trade Balance	0.1100	0.0026	0.0031	0.0034	0.0012
RER (CPI)	4.3600	3.6119	1.9969	1.4021	0.1575
RER (PPI)	-	4.8260	2.8582	3.3757	1.3854
<b>Serial Correlations:</b>					
Output	0.8800	0.9790	0.9734	0.9623	0.9953
Consumption	0.8900	0.8878	0.9570	0.9160	0.9441
Investment	0.9100	0.6040	0.5453	0.4858	0.7722
Trade Balance	0.8200	0.1968	0.1581	0.1463	-0.1027
RER (CPI)	0.8300	0.6883	0.9549	0.7460	0.8936
RER (PPI)	-	0.5933	0.7702	0.5994	0.7059
<b>Cross-Country Correlation:</b>					
Output	0.6000	0.9193	0.8657	0.8725	0.8622
Consumption	0.3800	0.6631	0.7468	0.6631	0.5286
Investment	0.3300	0.8866	0.6606	0.7093	0.7379

**Notes:** The first column refers to data that have been logged and detrended using the HP filter and obtained from [Chari et al. \(2002\)](#). With the exception of the trade balance, the relative volatility statistics refer to the ratio of the standard deviation of the variables of interest divided by the standard deviation of GDP. The standard deviation of the trade balance is simply the standard deviation of the ratio of the trade balance to GDP.

\* Model without distribution service sector.

Figure 1: Home Country's Response to a 1% IST Shock,  $\sigma_i = 2.5$

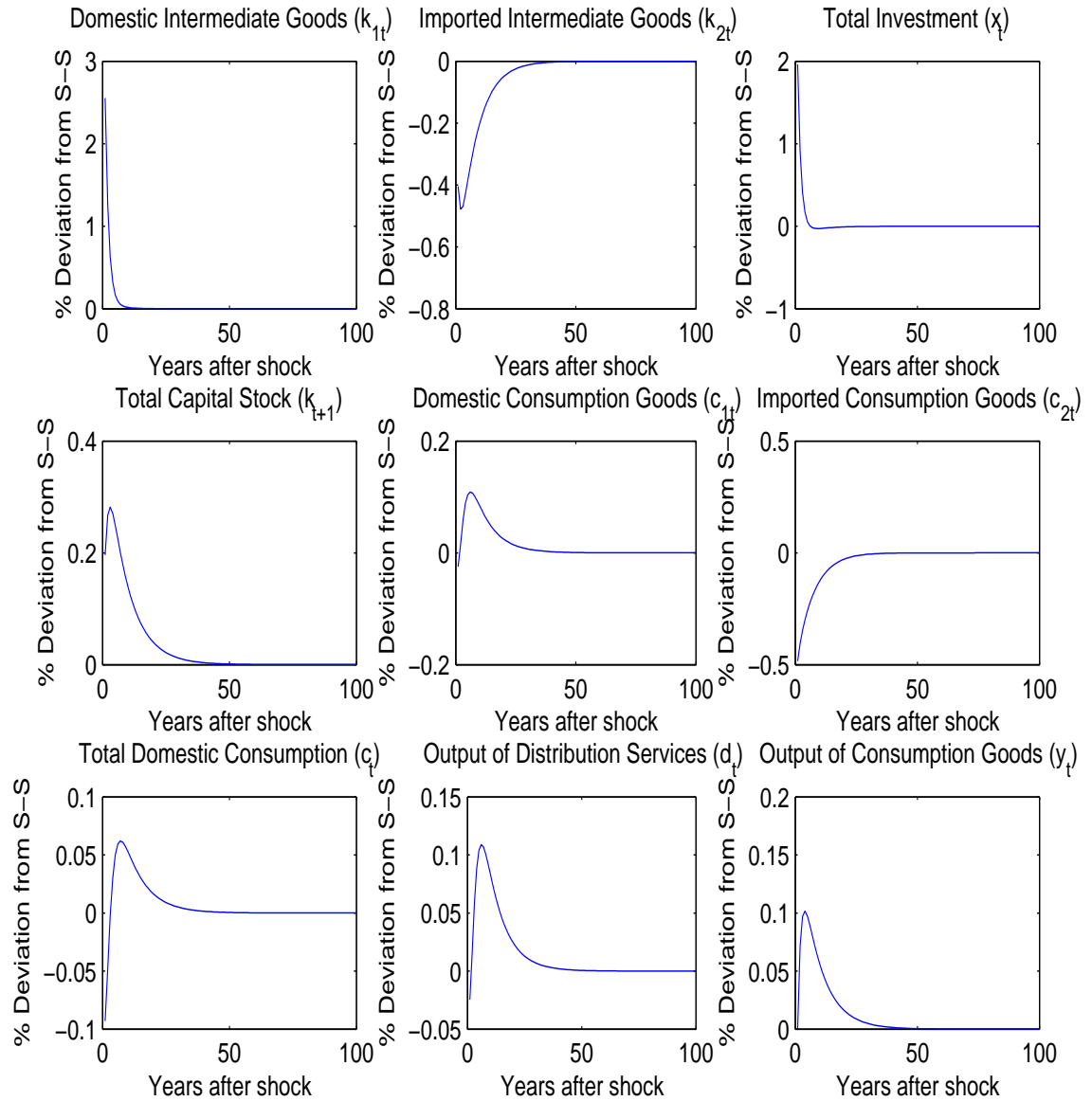


Figure 2: Foreign Country's Response to a 1% IST Shock,  $\sigma_i = 2.5$

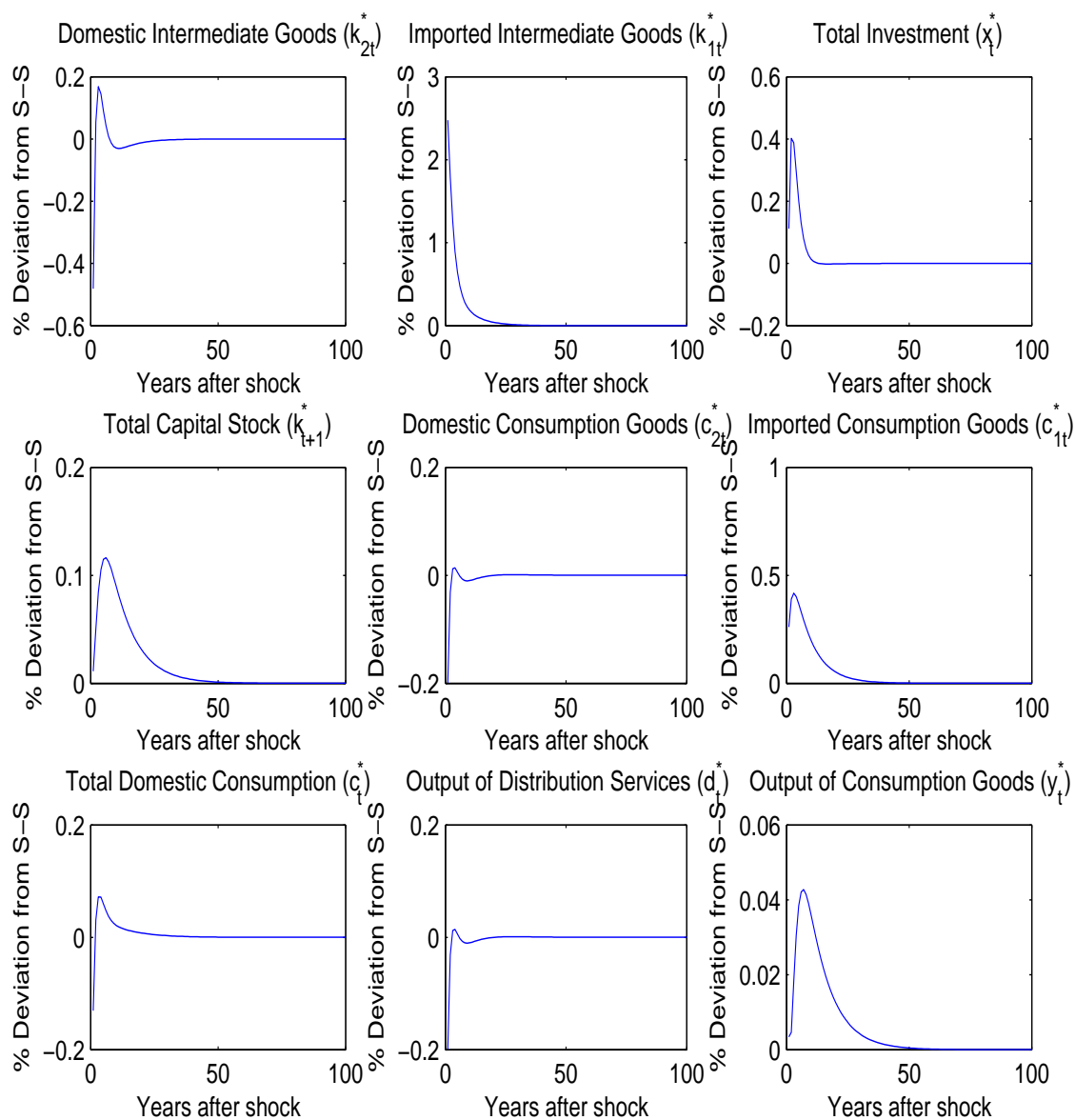


Figure 3: Response of Prices to a 1% IST Shock,  $\sigma_i = 2.5$

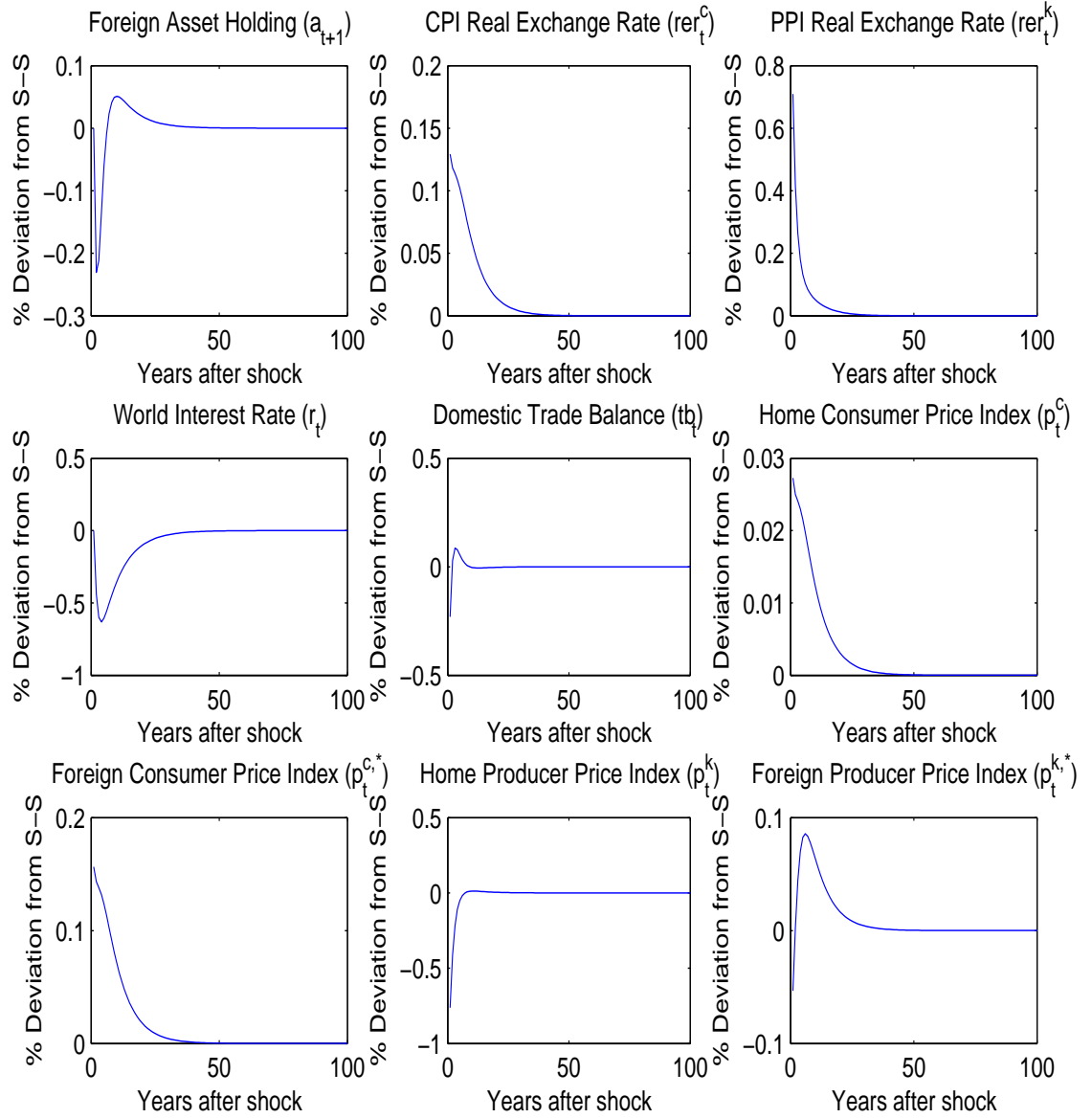


Table 2: Observed and Simulated Moments for TFP Shocks Model

	Data:	$\sigma_i = 1.5:$	$\sigma_i = 2.0:$	$\sigma_i = 2.5:$
<b>Relative Volatilities:</b>				
Consumption	0.8300	0.5262	0.6658	0.9148
Investment	2.7800	0.9391	1.0847	1.5921
Trade Balance	0.1100	0.0024	0.0027	0.0019
RER (CPI)	4.3600	0.8826	0.4946	1.5150
RER (PPI)	-	0.7333	0.4140	1.2482
<b>Serial Correlations:</b>				
Output	0.8800	0.9700	0.9736	0.9812
Consumption	0.8900	0.9905	0.9945	0.9919
Investment	0.9100	0.9212	0.9696	0.9314
Trade Balance	0.8200	0.6183	0.5072	0.8739
RER (CPI)	0.8300	0.9940	0.9732	0.9709
RER (PPI)	-	0.9940	0.9732	0.9709
<b>Cross-Country Correlation:</b>				
Output	0.6000	0.3024	0.3199	-0.0161
Consumption	0.3800	-0.1068	0.1131	-0.4974
Investment	0.3300	0.4057	0.4372	-0.3174

**Notes:** The first column refers to data that have been logged and detrended using the HP filter and obtained from Chari et al. (2002). With the exception of the trade balance, the relative volatility statistics refer to the ratio of the standard deviation of the variables of interest divided by the standard deviation of GDP. The standard deviation of the trade balance is simply the standard deviation of the ratio of the trade balance to GDP.



Figure 4: Home Country's Response to a 1% IST Shoc,  $\sigma_i = 2.5$

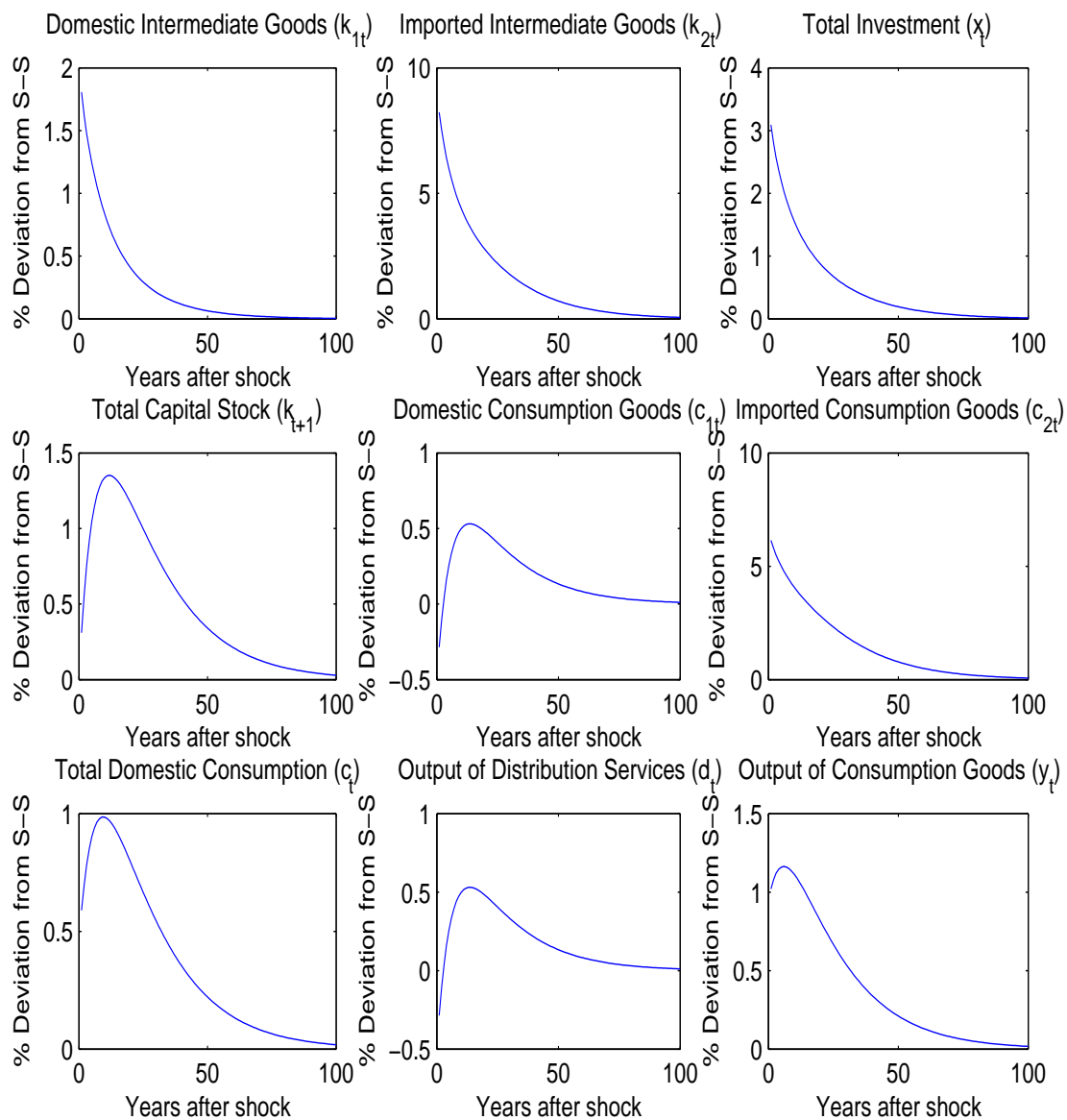


Figure 5: Foreign Country's Response to a 1% TFP Shock,  $\sigma_i = 2.5$

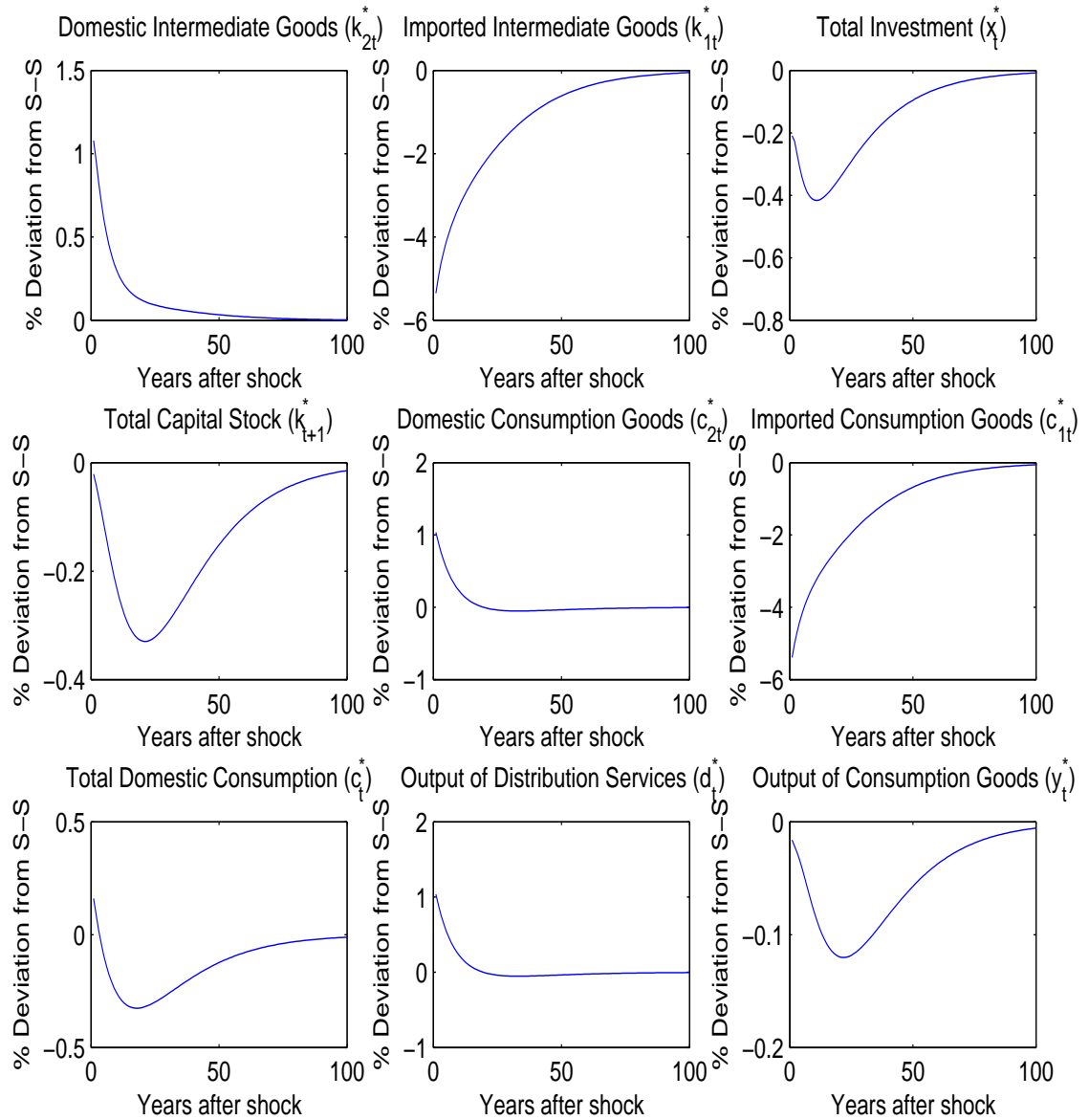


Figure 6: Foreign Country's Response to a 1% TFP Shock,  $\sigma_i = 2.5$

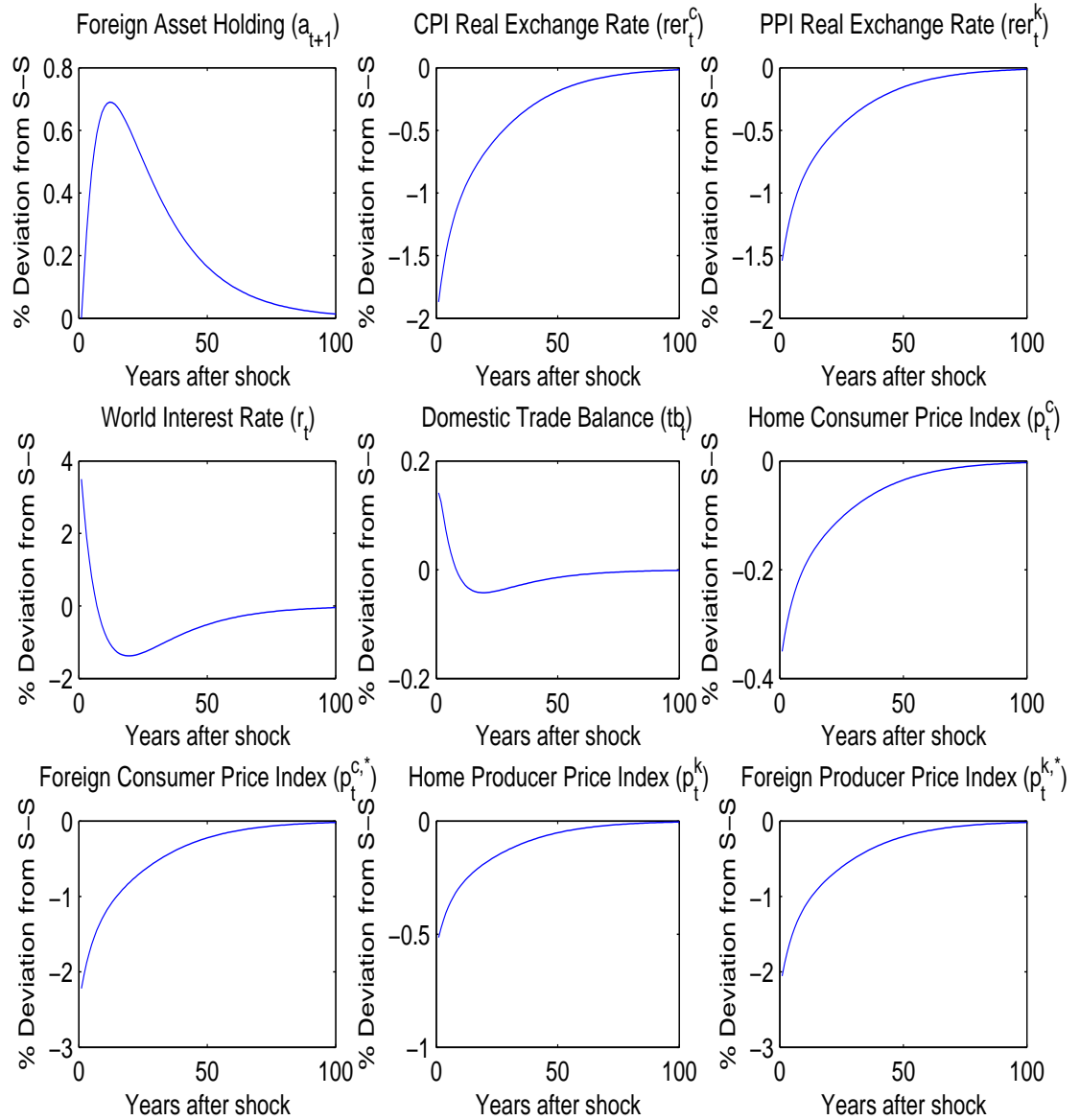


Table 3: Steady-State Parameter Values:

Parameters:	Definitions:
<b>Benchmark:</b>	
$\beta = 0.96$	Constant annual discount factor
$r_t^* = 0.04$	Steady-state annual world interest rate
$\delta = 0.10$	Constant depreciation rate for Capital stock
$\alpha = 0.36$	Share of capital services in production
$\sigma_c = 2.5$	Elasticity of sub. between domestic and imported consumer goods
$\sigma_k = 2.5$	Elasticity of sub. between domestic and foreign intermediate goods
$\sigma_d = 2.5$	Elasticity of sub. between dist. svcs and foreign consumer goods
$\omega_c = 0.575$	Share of domestic consumer goods
$\omega_d = 0.320$	Share of distribution services
$\omega_k = 0.635$	Share of domestic intermediate goods
$\phi_a = 0.36$	Foreign asset adjustment costs
$\phi_k = 1.6$	Capital adjustment costs (IST Shocks Model)
$\phi_k = 0.6$	Capital adjustment costs (TFP Shocks Model)
<b>Low Elasticity:</b>	
$\sigma_i = 1.5$	Elasticity of sub. between goods
$\omega_c = 0.820$	Share of domestic consumer goods
$\omega_d = 0.240$	Share of distribution services
$\omega_k = 0.800$	Share of domestic intermediate goods
<b>Mid-level Elasticity:</b>	
$\sigma_i = 2.0$	Elasticity of sub. between goods
$\omega_c = 0.635$	Share of domestic consumer goods
$\omega_d = 0.300$	Share of distribution services
$\omega_k = 0.670$	Share of domestic intermediate goods

## APPENDIX 1

### A Derivations and Proofs

#### A.1 The Problem for the Decentralized Domestic Economy

$$\max_{\{a_{t+1}, c_{1t}, c_{2t}, k_{1t}, k_{2t}, k_{t+1}, k_{ct}, k_{dt}, l_{ct}, l_{dt}, c_{1t}^m, y_t, d_t\}} E_t \sum_{t=0}^{\infty} \beta^t \log(c_t), \quad 0 < \beta < 1 \quad (37)$$

s.t.

$$c_{1t} + p_{2t}c_{2t} + p_{kt}k_{1t} + p_{kt}^*k_{2t} + tb_t \leq y_t - \left(\frac{\phi_a}{2}\right)(a_{t+1} - \bar{a})^2 \quad (38)$$

$$tb_t = a_{t+1} - (1 + r_t)a_t \quad (39)$$

$$l_{ct} + l_{dt} \leq 1 \quad (40)$$

$$k_{ct} + k_{dt} \leq k_t \quad (41)$$

$$k_{t+1} = x_t + (1 - \delta)k_t - \left(\frac{\phi_k}{2}\right)(k_{t+1} - k_t)^2 \quad (42)$$

$$x_t = \left[ \omega_k (k_{1t})^{\eta_k} + (1 - \omega_k)(k_{2t})^{\eta_k} \right]^{\frac{1}{\eta_k}} \quad (43)$$

$$c_t = \left[ \omega_c (c_{1t})^{\eta_c} + (1 - \omega_c)(c_{1t}^m)^{\eta_c} \right]^{\frac{1}{\eta_c}} \quad (44)$$

$$c_{1t}^m = \left[ \omega_d (d)^{\eta_d} + (1 - \omega_d)(c_{2t})^{\eta_d} \right]^{\frac{1}{\eta_d}} \quad (45)$$

$$y_{ct} = z_t k_{ct}^{\alpha} l_{ct}^{(1-\alpha)} \quad (46)$$

$$d_t = z_t k_{dt}^{\alpha} l_{dt}^{(1-\alpha)} \quad (47)$$

$$q_{t+1} = \rho_q q_t + \rho_{q,q^*} q_t^* + \varepsilon_t^q \quad (48)$$

$$z_{t+1} = \rho_z z_t + \rho_{z,z^*} z_t^* + \varepsilon_t^z \quad (49)$$

#### A.2 The FOCs

Using  $\lambda_t^c$ ,  $\lambda_t^l$ ,  $\lambda_t^k$ ,  $\lambda_t^x$ ,  $\lambda_t^m$  and  $\lambda_t^d$  as the multipliers associated with Equation 38, 40, 41, 42, 45 and 47 we have the following first order condition with respect to  $a_{t+1}$ ,  $c_{1t}$ ,  $c_{1t}^m$ ,  $c_{2t}$ ,  $d_t$ ,  $k_{t+1}$ ,  $k_{1t}$ ,  $k_{2t}$ ,  $k_{ct}$ ,  $k_{dt}$ ,  $l_{ct}$  and  $l_{dt}$ :

$$\lambda_t^c \left[ 1 + \phi_a (a_{t+1} - \bar{a}) \right] = \beta E_t \lambda_{t+1}^c (1 + r_{t+1}) \quad (50)$$

$$\lambda_t^c = \omega_c (c_{1t})^{\eta_c - 1} \left[ \omega_c (c_{1t})^{\eta_c} + (1 - \omega_c)(c_{1t}^m)^{\eta_c} \right]^{-1} \quad (51)$$

$$\lambda_t^m = (1 - \omega_c)(c_{1t}^m)^{\eta_c - 1} \left[ \omega_c (c_{1t})^{\eta_c} + (1 - \omega_c)(c_{1t}^m)^{\eta_c} \right]^{-1} \quad (52)$$

$$\lambda_t^c p_{2t} = \lambda_t^m (1 - \omega_d) (c_{2t})^{\eta_d - 1} \left[ \omega_d (d_t)^{\eta_d} + (1 - \omega_d) (c_{2t})^{\eta_d} \right]^{\frac{1 - \eta_d}{\eta_d}} \quad (53)$$

$$\lambda_t^d = \lambda_t^m \omega_d (d_t)^{\eta_d - 1} \left[ \omega_d (d_t)^{\eta_d} + (1 - \omega_d) (c_{2t})^{\eta_d} \right]^{\frac{1 - \eta_d}{\eta_d}} \quad (54)$$

$$\lambda_t^x [1 + \phi_k (k_{t+1} - k_t)] = \beta E_t \left\{ \lambda_{t+1}^x [(1 - \delta) + \phi_k (k_{t+2} - k_{t+1})] + \lambda_{t+1}^k \right\} \quad (55)$$

$$\lambda_t^c p_{kt} = \lambda_t^x \omega_k (k_{1t})^{\eta_k - 1} \left[ \omega_k (k_{1t})^{\eta_k} + (1 - \omega_k) (k_{2t})^{\eta_k} \right]^{\frac{1 - \eta_k}{\eta_k}} \quad (56)$$

$$\lambda_t^c p_{kt}^* = \lambda_t^x (1 - \omega_k) (k_{2t})^{\eta_k - 1} \left[ \omega_k (k_{1t})^{\eta_k} + (1 - \omega_k) (k_{2t})^{\eta_k} \right]^{\frac{1 - \eta_k}{\eta_k}} \quad (57)$$

$$\lambda_t^k = \lambda_t^c \alpha \frac{y_t}{k_{ct}} \quad (58)$$

$$\lambda_t^k = \lambda_t^d \alpha \frac{d_t}{k_{ct}} \quad (59)$$

$$\lambda_t^l = \lambda_t^c (1 - \alpha) \frac{y_t}{l_{ct}} \quad (60)$$

$$\lambda_t^l = \lambda_t^d (1 - \alpha) \frac{d_t}{l_{ct}} \quad (61)$$

### A.3 Deterministic Steady State Conditions

Symmetry in the two economies ensures that  $p_{2t} = p_{1t} = 1$ , similarly, given that  $q_t = q_t^* = 1$  and  $z_t = z_t^* = 1$  in steady-state, we must also have  $tb_t = 0$  and  $a_t = a_{t+1} = 0$ . In steady-state we have  $\lambda_t^i = \lambda_{t+1}^i = \lambda^i \forall i$ , which implies that  $\beta(1 + r) = 1$ .

From Equation 50, 55, 56 and 58:

$$r + \delta = \alpha \frac{y}{k_c} \omega_k (k_1)^{\eta_k - 1} \left[ \omega_k (k_1)^{\eta_k} + (1 - \omega_k) (k_2)^{\eta_k} \right]^{\frac{1 - \eta_k}{\eta_k}} \quad (62)$$

From Equation 50, 53, 54, 55, 56 and 60:

$$r + \delta = \alpha \frac{d}{k_d} \left( \frac{\omega_d}{1 - \omega_d} \right) \left( \frac{c_2}{d} \right)^{\eta_d - 1} \omega_k (k_1)^{\eta_k - 1} \left[ \omega_k (k_1)^{\eta_k} + (1 - \omega_k) (k_2)^{\eta_k} \right]^{\frac{1 - \eta_k}{\eta_k}} \quad (63)$$

From Equation 53, 54, 60 and 61:

$$\left( \frac{d}{y} \right) \left( \frac{l_c}{l_d} \right) = \left( \frac{1 - \omega_d}{\omega_d} \right) \left( \frac{d}{c_2} \right)^{1 - \eta_d} \quad (64)$$

From Equation 56 and 57:

$$\frac{k_1}{k_2} = \left[ \frac{\omega_k}{1 - \omega_k} \right]^{\frac{1}{1 - \eta_k}} \quad (65)$$

From Equation 51, Eq. 52 and 53:

$$\frac{c_1}{c_1^m} = \left[ \left( \frac{1 - \omega_c}{\omega_c} \right) (1 - \omega_d) \left[ \omega_d \left( \frac{d}{c_2} \right)^{\eta_d} + (1 - \omega_d) \right]^{\frac{1 - \omega_d}{\eta_d}} \right]^{\frac{1}{\eta_c - 1}} \quad (66)$$

To complete the deterministic steady-state conditions we have the following constraints:

$$l_c + l_d = 1 \quad (67)$$

$$k_c + k_d = k \quad (68)$$

$$\omega_k \left( \frac{k_1}{k_2} \right)^{\eta_k} + (1 - \omega_k) = \left( \frac{\delta k}{k_2} \right)^{\eta_k} \quad (69)$$

$$\omega_d \left( \frac{d}{c_2} \right)^{\eta_d} + (1 - \omega_d) = \left( \frac{c_1^m}{c_2} \right)^{\eta_d} \quad (70)$$

$$\frac{d}{k_d} = \left( \frac{k_d}{l_d} \right)^{\alpha-1} \quad (71)$$

$$\frac{y}{k_c} = \left( \frac{k_c}{l_c} \right)^{\alpha-1} \quad (72)$$

$$y = c_1 + c_2 + k_1 + k_2 \quad (73)$$

To fully calibrate the steady-state we have a system of 12 equations and 12 unknown. The unknowns we need to solve for are:  $y, k_c, k_d, l_c, l_d, k, k_1, k_2, d, c_1, c_2$ , and  $c_1^m$ . We use the factor price equalization condition in both the labor and capital markets to obtain  $\frac{d}{c_2} = \left( \frac{\omega_d}{1-\omega_d} \right)^{\frac{1}{1-\eta_d}}$  and  $\frac{k_d}{l_d} = k = \frac{k_c}{l_c}$ . These two conditions, along with the value of  $\frac{k_1}{k_2}$  can then be used to obtain the remaining variables of interest.