An endogenous Goodwin-Keynes business cycle model: Evidence for Germany (1991-2007)

Konstantinos N. Konstantakis and Panayotis G. Michaelides and Theodore Mariolis

National Technical University of Athens, Greece, Panteion University, Greece

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Konstantinos N. Konstantakis
National Technical University of Athens, Greece

Panayotis G. Michaelides
National Technical University of Athens, Greece

Theodore Mariolis
Panteion University, Greece

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1. Introduction

Since its formulation, Goodwin's (1967) approach became a standard endogenous business cycle model. However, its shortcoming is its inability to capture empirically the economy's dynamics because no relevant framework was developed and thus all efforts to econometrically estimate the model did not end up in success (Mohun and Veneziani 2006). Here, we present a Goodwin model in a (post-) Keynesian spirit including an implementation framework which lends itself to empirical estimation with very satisfactory results for the German economy (1991-2007), in contrast to all previous efforts to estimate such a model (Flaschel et *al.* 2009).

We advance the literature on Goodwin type models as follows: first, we use a (post-)Keynesian accumulation function and present useful mathematical results; second, we model econometrically the German economy, the locomotive of EMU, in the period 1991-2007 using quarterly data; third, the total period is broken down based on structural breaks.

2. Theoretical Model

The rationale behind the model is that during booms, profit margin increases and unemployment decreases. This phase is followed by a recession phase since less unemployment reduces profits. Next, higher unemployment because of reduced profitability, leads to lower salaries, which increase profit margin leading to increased investment. Thus, a new boom starts, and so on (Moura and Ribeiro, 2013).

Consider a closed economy, with constant returns to scale and excess capacity of capital, producing commodities used for consumption and investment. Labour is homogenous and capital stock does not depreciate. There are two kinds of income: wages are paid at the end of the production period, whilst a fraction of profits, s ($0 < s \leq 1$), is saved. The degree of capacity utilization, u ($u > 0$), is given by the ratio of actual output to potential output, where the latter is proportional to the capital stock. The desired rate of capital accumulation is a strictly increasing function of the degree of capacity utilization and the share of profits in total income, h ($0 \leq h \leq 1$).

<u>Assumption 1:</u> g^S is determined by savings.

$$g^S = sr \tag{1}$$

<u>Assumption 2:</u> The accumulation function (Bhaduri & Marglin 1990) is:

$$g^I = F(u,h),\ F(\mathbf{0}) \geq 0,\ F_x \equiv (\partial F / \partial x) > 0,\ x = u,\ h \tag{2}$$

<u>Assumption 3</u>: The technological and effective demand determinants of the distributive variables are:

$$r = \pi_K hu \tag{3}$$

<u>Assumption 4:</u> The social determinants of the distributive variables are:

$$w = \pi_L (1-h) \tag{4}$$

<u>Assumption 5:</u> The short-run commodity market equilibrium is:

$$g^{\mathrm{I}} = g^{\mathrm{S}} \tag{5}$$

Assumption 6: Savings must increase by more than investment demand when u rises

$$g_u^{\mathrm{S}} - g_u^{\mathrm{I}} > 0 \text{ or } s\pi_K h > F_u \tag{6}$$

where: g^{S}, g^{I} denote the actual and the desired rates of capital accumulation, $F(\bullet)$ a continuous and twice differentiable function, r the profit rate, π_K the capital productivity, w the real wage rate, and π_L the labour productivity.

Assumption 7: Labour force, N, grows at the steady rate n.

$$\hat{N} = n \tag{7}$$

Assumption 8: The economy is characterised by a 'real wage Phillips curve'.

$$\hat{w} = \gamma E - \delta \tag{8}$$

where $E \equiv LN^{-1}$ denotes the employment rate, L the number employees, γ, δ are constants.

Proposition 1: *Under normalized profit rate,* $\rho \equiv ru^{-1}$*, the elasticity of normalized profit rate with respect to real wage rate is:* $e_1 = -(1-h)h^{-1}$. (9)

Proposition 2: *Let* $u = f(h)$ ('IS – curve'). *The elasticity of u with respect to h is:* $e_2 = (F_h - s\pi_K u)(s\pi_K h - F_u)^{-1} hu^{-1}$. (10)

Theorem 1: *The motion equation that characterizes the economy, regarding the share of profits over income, is:* $\dfrac{\dot{h}}{h} = e_1\gamma E - e_1\delta$*, h≠ 0.* (11)

Theorem[1] 2: *The motion equation that characterizes the economy, regarding the employment rate, is:*
$\dfrac{\dot{E}}{E} = e_2\dfrac{\dot{h}}{h} + s\pi_K hf(h) - n$*, E≠ 0.* (12)

Stability:

The Jacobian matrix, $\mathbf{J} \equiv [J_{ij}]$, of (11) and (12) is:

$$J_{11} \equiv \partial\dot{h}/\partial h = (\gamma E - \delta) \tag{13a}$$

$$J_{12} \equiv \partial\dot{h}/\partial E = -\gamma(1-h) \tag{13b}$$

$$J_{21} \equiv \partial\dot{E}/\partial h = \{[(de_2/dh)e_1 + e_2 h^{-2}](\gamma E - \delta) + s\pi_K(1+e_2)f(h)\}E \tag{13c}$$

$$J_{22} \equiv \partial\dot{E}/\partial E = e_2 e_1(2\gamma E - \delta) + s\pi_K hf(h) - n \tag{13d}$$

Depending on the signs of $[J_{ij}]$, Tr**J** and Det**J** we characterize routinely the stability of the trivial $(h^* = 1, E^* = 0)$, and non-trivial $(h^{**} = (s\pi_K f(h^{**}))^{-1}n, E^{**} = \delta\gamma^{-1})$, equilibrium point(s) (h^{**}, E^{**}) (Zampieri and Gorni, 1992).

[1] All proofs are straightforward and are available upon request by the authors.

3. Estimation Method and Data

We substitute equation (11) into (12):

$$\frac{\dot{E}}{E} = e_2 e_1(\gamma E - \delta) + s\pi_\kappa h f(h) - n \qquad (14)$$

We are interested in the change of *u* as a result of a change in *h*, i.e. du/dh = z, where *z is a* real valued parameter and its estimated sign will determine the relationship (negative of positive) between *du* and *dh*. For simplicity, we assume that the relationship $u = f(h)$ is proportional, i.e. *u*=z*h,* yielding:

$$\frac{\dot{E}}{E} = e_2 e_1 \gamma E + z s\pi_\kappa h^2 + (-n - e_2 e_1 \delta) \qquad (15)$$

For given values of *E*, $\frac{\dot{E}}{E}$, and $s\pi_\kappa h^2$ this conforms to multivariate regression. The procedure provides us with direct estimates of z. Also, it provides us with an estimate of $e_2 e_1 \gamma$. Given, $e_1 = \frac{(\bar{h} - 1)}{\bar{h}}$, $e_2 = \frac{z\bar{h}}{\bar{u}}$ we obtain an estimate of γ where, in general, $\bar{x}$ denotes the average value of variable *x*. Similarly, given that n is exogenous and routinely calculated, from the estimate of the intercept in (15), we obtain the value of *δ,* since e_1 and e_2 are calculated as above.

Next, data on *h, E, s* and π_κ are needed. The productivity of capital (π_κ) is equal to the share of potential output over capital, where the potential output is obtained as the HP filtered GDP time series.

The data come from the US Federal Reserve Bank of St. Louis, in constant 2005 US dollars.

Our investigation stops in 2007 since at post-2007 era the dynamics of the traditional economic structures changed dramatically.

4. Empirical Analysis and Discussion

4.1.1 Periodicities

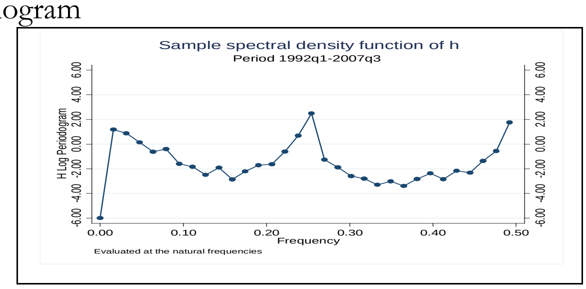
We estimate the average length of the cycles of *h*, based on the Fourier function (Rossi-Hansberg et al. 2009):

$$f(\omega) = \begin{cases} f(1-\omega), \text{ if } \quad \omega \in [0.5, 1] \\ 1/n \left| \sum_{t=1}^{n} x(t) e^{2\pi i(t-1)\omega} \right|, if \ \ \omega \in [0, 0.5) \end{cases}$$

where $\omega = 2\pi / n$ the natural frequency and *x(t)* the time series in time *t.*

Peaks in the periodogram represent dominant frequencies (Figure 1).

Figure 1: Periodogram



The periodogram suggests the existence of a dominant cycle with a period of 5.5 years (Schirwitz 2006).

4.1.2 Stationarity

We check for stationarity using the augmented Dickey and Fuller (1979) (ADF) methodology, whose general representation is:

$$\Delta Y_t = \alpha + bt + \rho Y_{t-1} + \sum_{i=1}^{m} \gamma_t \Delta Y_{t-1} + \varepsilon_\tau$$

where Δ the first difference operator and *t* time.

All the variables that enter the model were found stationary.

4.1.3 Outliers

We test for outliers using Hadi's (1992, 1994) test which is based on the optimal formation of two distinct sample subsets using a four-step algorithm according to the distance: $D_i(C_R, S_R) = \sqrt{(x_i - C_R)^T S_R^{-1}(x_i - C_R)}$, where: i=1,..*n* the number of observations, x_i the observations, C_R the robust location estimator and S_R the robust covariance matrix estimator. See Table 1.

Table 1: Hadi's test results

Variables	$\dot{E}/E$	E	$\pi_k sh^2$
Initially accepted	2	2	2
Expand to (n+k+1)	41	41	41
Expand, p-value=0.01	74	79	81
Outliers	6	2	0
Years excluded	1991(Q1), 1991(Q2), 1991(Q3), 1991(Q4), 1994(Q1), 2006(Q2)	1991(Q2),1991(Q3)	

The first observations, as well as 1994(Q1) and 2006(Q1) should be excluded.

4.1.4 Heteroscedasticity

In the presence of heteroskedasticity, using White's (1980) standard errors we obtain B.L.U.E. estimators. The White estimator transforms the variance matrix as follows:

$$\Sigma^* = \begin{pmatrix} \widehat{u_1^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \widehat{u_n^2} \end{pmatrix}$$

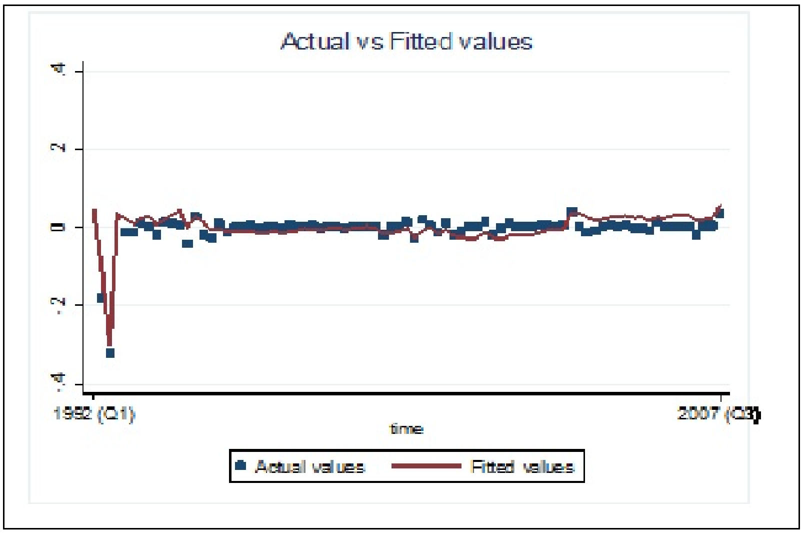
where $\widehat{u_i^2}$, i=1,..n are the standard errors obtained by OLS multiplied by (N/(N-K-1)) where N is the sample size and K is the number of regressors. The estimator is: $Var(\widehat{b}) = (X'X)^{-1} X'\Sigma^* X(X'X)^{-1}$ which is BLUE (Greene 2012). See Table 2.

Table 2: Estimation results (1992-2007)

Variables	Coef	t-stat	p-value
E	0.52	2.71	0.01
$s\pi_{\kappa}h^2$	-0.11	-2.25	0.03
Intercept	-0.26	-2.68	0.01
R-sq=0.18, F-stat=3.95			

Actual versus fitted values are presented in Figure 2.

Figure 2: Actual vs Fitted values



4.1.5 Structural Breaks

We test for the existence of a structural break in 2000 (EMU formation). We use three different methodologies:

(a) the Chow (1960) test which tests whether one single period regression $y_t = \alpha_0 + \alpha_1 x_t + u_t$ is more suitable than two separate regressions splitting the data into two sub-periods at the break point *t*, expressed as: $\begin{matrix} y_t = \beta_1 + \beta_2 x_t + u_{1t} \\ y_t = \delta_1 + \delta_2 x_t + u_{2t} \end{matrix}$.

The null hypothesis Ho is that $\beta_1 = \delta_1, \beta_2 = \delta_2$ and it is routinely tested against the critical values in the F-test tables with F(k,n-2k) degrees of freedom using the statistic

$$F = \frac{RSS_c - (RSS_1 + RSS_2)/k}{RSS_1 + RSS_2 / n - 2k}.$$

(b) Following Andrews (1993), the SupW of the Wald test which allows for heteroscedasticity is taken over all break dates in the region [t1, t2] which contains candidate break dates. We use the rule t1 = [0.15n] and t2 = [0.85n], and calculate the SupW. The results indicate the existence of a structural break (Table 3).

Table 3: Structural Break test

H0: No Structural Change				
Break Periods: 1st Break Period: 1992-2000, 2nd Break Period: 2000-2007				
	Chow-test	**Wald Test**	**LR-test**	**LM test**
t-stat	3.02	9.81	9.26	8.74
p-value	0.04	0.01	0.01	0.01

(c) The structural break test of Zivot and Andrews (1992) is used. The selection criterion for the break date is based on the t-statistic from an ADF test and a minimum value of t-statistic is the indication of the break date. See Table 4.

Table 4: Zivot-Andrews (1992) test

ADF test		
Variable	**Period**	**t-stat**
$\dot{E}/E$	1992(Q1)-1999(Q4)	-30.18
	1992(Q1)-2000(Q1)	-30.68
	1992(Q1)-2000(Q2)	-30.33

The results of the tests show that a structural break takes place in 2000. We use the simple rule of splitting the sample at the estimated break, following Pesaran and Timmermann (2007), yielding two sub-periods (1992-2000, 2000-2007).

All the variables are checked for stationarity in the sub-periods and are stationary. The estimates are in Table 5.

Table 5: Robust Model Estimations

Period 1992-2000			
Variables	**Coef**	**t-stat**	**p-value**
E	0.47	2.21	0.04
$s\pi_{\kappa}h^2$	-0.10	-1.98	0.06
Intercept	-0.24	-2.18	0.04
R-sq=0.14, F-stat=2.95			

Period 2000-2007			
Variables	**Coef**	**t-stat**	**p-value**
E	0.59	2.07	0.04
$s\pi_{\kappa}h^2$	0.15	1.64	0.10
Intercept	-0.32	-2.11	0.04
R-sq=0.22, F-stat=2.85			

Assuming that e_1, e_2 are equal to their average values $\bar{e}_1, \bar{e}_2$, the values of the coefficients γ and δ are revealed. The estimated values are presented in Table 6.

Table 6: Estimated parameters

Period	**z**	**γ**	**δ**
1992-2007	-0.10	-0.34	-0.17
1992-2000	-0.10	-0.32	-0.17
2000-2007	0.15	-0.36	-0.20

From the estimated parameters z, γ, δ and the calculated parameters $\bar{e}_1, \bar{e}_2, \bar{h}, \bar{n}$ we infer that the estimates are consistent with economic theory, and e_1<0. Also, all estimated results are statistically significant, while the equations explain a considerable part of the variability of the dependent variable. The results are very satisfactory given the various imperfections in this sort of data (Mankiw et *al.* 1992).

Regarding stability (Table 7), in the total period examined, in the first sub-period and in the second sub-period, the non-trivial equilibrium point of the German economy is in a saddle path, meaning that the equilibrium point is unstable, except for the initial conditions that led to this eigenvector (Hamburg et *al.* 2008). Thus, the equilibrium point is "vulnerable" to shocks.

Table 7: Model's stability

Period	$\bar{e}_1$	$\bar{e}_2$	**Tr J****	**Det J****	**Stability**
1992-2007	<0	<0	<0	<0	Saddle path
1992-2000	<0	<0	<0	<0	Saddle path
1999-2007	<0	>0	>0	<0	Saddle path

5. Conclusion

The standard Goodwin model suffers from an inability to capture empirically the economy's dynamics. Here, we set out an extended Keynes-Goodwin model including a relevant econometric framework. Our main finding is that the equilibrium points for the largest EMU economy, Germany, are fragile and vulnerable to shocks. Future research focusing on monetary variables would be of great interest.

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