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Abstract: Since its original formulation, Goodwin's (1967) approach became a standard endogenous business cycles model. However, despite its elegant mathematical formulation, the empirical estimation of Goodwin-type models has not always ended up in success. The present paper uses the so-called Bhaduri-Marglin accumulation function in Goodwin's original growth cycle model. Based on its derived equations of motion and dynamic properties, we econometrically estimate the proposed model for the case of the US economy in the time period 1960-2012, using structural breaks. The empirical estimation is very satisfactory and, in general terms, consistent with economic theory and the findings by other researchers on the US economy. The results of this work suggest that the proposed approach is an appropriate vehicle for expanding and improving traditional Goodwin-type models.

Keywords: Bhaduri-Marglin accumulation function, Goodwin type models, US economy

JEL classification: B51, C62, C67, E32

1. Introduction

A number of models that are based on Goodwin's (1967) class struggle approach have emerged in the over the last period (see, among others, Barbosa-Filho and Taylor 2006). However, despite their elegant mathematical formulation, their empirical estimation does not always end up in success.

As we know, Goodwin's system, which constitutes an economic equivalent of the Lotka-Volterra predator-prey system[1], is 'structurally unstable', i.e. sensitive to perturbations in its functional structure. [2] Several scholars have argued that Goodwin's (1967) approach neglects altogether any effective demand issues, and this has been generally recognized as a fundamental weakness of the model.[3] In fact, Marglin and Bhaduri (1988) have shown, by means of a static (post-)Keynesian model, that income redistribution between profits and wages has ambiguous effects on the equilibrium rates of capacity utilization, profits and accumulation (see also Bhaduri and Marglin, 1990, and Kurz, 1990).

This paper incorporates the Bhaduri-Marglin accumulation function in Goodwin's (1967) model[4] and explores its dynamics and econometric performance for the case of the US economy, in the period 1960-2007. Our investigation stops in 2007 since, at post-2007 era, the dynamics of the traditional economic structures changed dramatically, both in the USA and globally, as the relevant econometric tests show. The theoretical model of this paper has been presented in Mariolis (2013), while a version of it has been econometrically estimated for the case of the German economy (1991-2007) in Konstantakis *et al.* (2014).

In comparison to previous contributions, the present work advances the research conducted on Goodwin's growth cycles model by using the post-Keynesian Bhaduri-Marglin accumulation function in Goodwin's (1967) original model and by modelling econometrically, based on the proposed approach, the largest economy in the world, namely the US economy in the time period 1960-2007.

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1 Another economic equivalent is Palomba's model of investment-consumption conflict. See further Gandolfo (2008).

2 Similarly, the absence of inherent 'structural stability' in Goodwin's model is not a reason for rejecting it *a priori* (See, e.g. Sportelli, 1995, Vercelli, 1984, and Veneziani and Mohun, 2006).

3 See, however, the subsequent contributions by Goodwin (1986), and Goodwin and Punzo (1987, ch. 4), which also allow for heterogeneous capital commodities.

4 See also Canry (2005), Barbosa-Filho and Taylor (2006), Flaschel *et al.* (2008), Flaschel and Luchtenberg (2012, ch. 4), Nikiforos and Foley (2012) and Sasaki (2013) who use the Bhaduri-Marglin accumulation function.

Also, we extend the estimation provided by Konstantakis *et al.* (2014) in the following ways: first, we generalize the *IS*–curve representation used, by introducing a linear representation instead of a mere proportional one; second, we investigate – instead of *a priori* assuming it – whether the saving rate out of profits and the potential output-capital ratio should be considered as being constant throughout the econometric analysis or time varying; third, we offer detailed technical proofs for the stated propositions in Konstantakis et *al.* (2014).

2. Background Literature

As is well known, the seminal work of Lotka (1925) and Volterra ([1926] 1931) on the so-called predator-prey model, where two species interact in a struggle on the survival of the fittest, was introduced in *economics* by Goodwin (1967). For Goodwin (1967) the predator-prey model was a distributive conflict between the two classes that are incorporated in the model, namely capitalists and workers. An early attempt to empirically investigate Goodwin's original model was made by Atkinson (1969). Desai (1973), in an influential paper, managed to incorporate in the model both actual and anticipated price inflation, as well as excess capacity. The proposed model shed new light on the dynamics between the key macroeconomic variables representing the share of labour in national income and the proportion of the labour force employed. Sah and Desai (1981) showed that, in the presence of technical change within Goodwin's framework, the perpetual conflict cycles of the model are replaced by trajectories that converge to equilibrium in either a monotonic or cyclical manner. Ploeg (1981) also came to similar conclusions.

The mathematical properties of the Goodwin model have been thoroughly investigated by a number of economists. For instance, the possibility of chaotic behavior in a Goodwin class of models has been extensively examined by Pohjola (1981), while the stability dynamics of the model have been thoroughly investigated by Velupillai (1979) and Flaschel (1984). Again, Ploeg (1987) introduced differential savings and technical change in Goodwin's model and studied the effect of productivity on wage bargain. The results suggested that, in the presence of technical change, the model's structural stability conditions change.

Nevertheless, the key variable of wages in Goodwin's model, which is related to the demand side of an economy, has not been fully explored yet. In a prominent paper, Bhaduri and Marglin (1990) developed a framework in a closed economy set up that could incorporate the notion of exogenous real wage variations in a closed economy context. Chatterji and Sparks (1991) examined the equilibrium unemployment that arises in a class of models where the workers' utility was a quasi-linear function of both their wage and effort. The dynamics of their model shed light on the efficiency wage model developed by Shapiro and Stiglitz (1984), implying that productivity shocks directly affect real worker's wage and effort. Sportelli (1993), following Kolmogorov's (1936) approach, introduced an investment function that involved profits expectations along with an extended Phillip's curve into Volterra's equations. According to this work, the proposed model overcame Goodwin's instability. Choi (1995) managed to reconcile the work of Chatterji and Sparks (1991) and Shapiro and Stiglitz (1984) with that of Goodwin (1967), by examining the robustness of Goodwin's growth cycle when the effort level of the workers depends on the level of real wage. The results of this investigation suggest that, in the presence of a proportional relationship between the effort level and the real wage, the stationary equilibrium that arises is stable. In a seminal paper, Franke and Asada (1994) reconciled Goodwin's model with a dynamic IS-LM approach. According to their model, local (in)stability is characterized by (high) low interest elasticities of money demand.

Gordon (1995) studied the profit-led character of the US economy. More precisely, the paper examined the impact of profit rate as a distributional parameter on capacity utilization, observing a positive relationship, using both closed and open economy set ups. Goldstein (1996) was among the first to investigate, empirically, the role of the cyclical profit squeeze in post-War II US era, giving credit to Goodwin's model. Again, Goldstein (1999) empirically tested an extended Goodwin model, using a VAR specification and US data for the period 1949-1995. The results gave some extra credit to the profit squeeze hypothesis, in the sub-periods 1949-1970 and 1970-1985.

More recently, Harvie (2000), in a seminal work in the field, provided an empirical investigation of Goodwin's model, using OECD data for the time period 1949-1994. However, his results implied that the model was not perfectly able to explain business cycles in the US economy, a fact which was attributed (Tarasow 2010) to the fact that Harvie used the original variables without inducing stationartity as modern econometric theory dictates. In another prominent paper, Hein and Ochsen (2003) investigated the impact of exogenous variations in the interest rate on the equilibrium position in a Kaleckian effective demand model. Their results showed that a negative relation between interest rate and equilibrium rates of capacity utilization, accumulation and profits only exists under certain circumstances. Their model was empirically estimated using OECD data for a panel of selected countries.

In an empirical work, Flaschel *et al.* (2005) estimated an augmented Goodwin model, using data for the US economy in the time period 1955-2004. According to their findings, on the basis of a price and nominal wage Philips curve and a type of interest rate reaction function, Goodwin's model is satisfactory. Also, Asada (2006) established a framework, based on Goodwin and Keynes that incorporated debt accumulation.

Mohun and Veneziani (2006), in another prominent work, investigated Goodwin's model for the US economy taking into consideration only the private sector of the US economy. Their data accounted for the period 1948-2002 and their results exhibited cyclical patterns as well as structural breaks in the trend relationships between the employment rate and the wage share. Harvie et *al.* (2007), in an influential paper, extended the Goodwin model in a way that made it able to generate asymmetric growth cycles as an explicit solution. Hein and Vogel (2007) estimated the relationship between distributional income and economic growth based on the Bhaduri-Marglin (1990) framework, using data on UK, USA, Netherlands, Austria, France and Germany, for the time period 1960-2005. Their results confirmed, partly, the model's hypothesis that wage-led growth becomes more feasible when the effects of distribution on foreign trade are taken into account.

Recently, Tarassow (2010) made an attempt to investigate the validity of Goodwin's model for the USA, in the time period 1948-2006. The paper's findings gave credit to the view that income distribution is driven by labour market dynamics. Lastly, Moura and Ribeiro (2013), in a very recent work, investigated the validity of Goodwin's model using Brazilian data for the period 1981-2009. In their investigation they assumed that the individual income distribution in Brazil is described by a Compertz-Pareto distribution. Their findings partly confirmed, both quantitatively and qualitatively, the implications of the model.

In what follows, we will briefly set out the mathematical formulation of the proposed model and provide some useful technical results, formally.

3. The Theoretical Model

Consider a closed capitalist economy, with constant returns to scale and excess capacity of capital, producing only one commodity which can be used for consumption and investment. Homogeneous labour is the only primary input, capital stock does not depreciate, and competitive conditions are close to free competition, which implies that the underutilization of productive capacity is caused essentially by an insufficient effective demand.[5]. There are only two classes, workers, employed in proportion to the level of production (i.e. there is no supplementary labour) and capitalists, and two kinds of income, wages and profits. Wages are paid at the end of the production period and there are no savings out of this income, whilst a given and constant fraction of profits, s ($0 < s \leq 1$), is saved. The degree of capacity utilization, u (u>0), is given by the ratio of actual output to potential output, where the latter is taken to be proportional to the capital stock in existence. The desired rate of capital accumulation is a strictly increasing function of both the degree of capacity utilization and the share of profits in total income, h ($0 \leq h \leq 1$). Finally, technological change, fiscal and monetary considerations are ignored.[6]

On the basis of these assumptions, we write the following system of equations (see further Bhaduri and Marglin, 1990, and Kurz, 1990):[7]

Assumption 1: g^{S} is determined by the amount of savings.

$$g^{S} = sr \tag{1}$$

Assumption 2: The accumulation function is defined as follows:

$$g^{I} = F(u,h),\ F(\mathbf{0}) \geq 0,\ F_x \equiv (\partial F / \partial x) > 0,\ x = u,\ h \tag{2}$$

[5] See also Kurz (1994, Sections 3 and 6).

[6] As Kurz (1990, pp. 232-233) stresses, "within the framework of the present model the choice of technique problem cannot generally be considered to be decided in terms of the technical conditions of production alone: the degree of capacity utilization matters too. The latter, however, reflects a multiplicity of influences, such as the state of income distribution and savings and investment behavior […]. In particular, there is the possibility that, assessed in terms of the degree of utilization associated with the existing technique, a new technique proves superior, while in terms of its own characteristic steady-state degree of utilization it turns out to be inferior.". For fiscal and monetary considerations, see You and Dutt (1996) and Hein (2008, Part II), respectively, and the references therein.

[7] A 'dot' above a variable denotes time derivative, whereas a 'hat' denotes logarithmic derivative with respect to time, respectively.

Assumptions (3) and (4): The technological, effective demand and social determinants of the distributive variables are, respectively, given as follows:

$$r = \pi_K h u \tag{3}$$

$$w = \pi_L (1-h) \tag{4}$$

Assumption 5: The short-run commodity market equilibrium is defined as:

$$g^{\mathrm{I}} = g^{\mathrm{S}} \tag{5}$$

Assumption 6: Savings must increase by more than investment demand when u rises

$$g_u^{\mathrm{S}} - g_u^{\mathrm{I}} > 0 \text{ or } s\pi_K h > F_u \tag{6}$$

Note: Assumption (6), gives the stability condition (Marglin and Bhaduri, 1988, and Bhaduri, 2007), the so-called 'Keynesian Stability Condition'.
where:
g^{S}, g^{I} denote the actual and the desired rates of capital accumulation, respectively, $F(\bullet)$ a continuous and twice differentiable function, r the profit rate, π_K the capacity-capital ratio (or capital productivity), w the real wage rate, and π_L the labour productivity.

Proposition 1: *Under normalized profit rate,* $\rho \equiv r u^{-1}$*, the elasticity of normalized profit rate with respect to real wages is given by the expression* $e_1 = -(1-h)h^{-1}$.

Proof

See Mathematical Appendix.

Lemma 1: *Given equations (1), (2), (3) and (5), the non-Hicksian IS-curve is defined as:* $F(u,h) = s\pi_K h u$.

Proof

See Mathematical Appendix.

Proposition 2: *Let* $u = f(h)$. *The elasticity of u with respect to h, is given by the expression* $e_2 = (F_h - s\pi_K u)(s\pi_K h - F_u)^{-1} h u^{-1}$.

Proof

See Mathematical Appendix.

Note: From relation (6), the term $(sh\pi_K - F_u)^{-1}$ is positive, so $F_h > s\pi_K u$ implies that $e_2 > 0$, and *vice versa.*

Lemma 2: *An elastic, negatively sloped IS–curve necessarily implies that* $dr/dh < 0$.

Proof

See Mathematical Appendix.

Note: The system is able to generate three alternative sets of steady-state equilibria (according to Kurz' s, 1990, pp. 222-226, terminology):

(i) A "regime of over-accumulation", characterised by $du/dh < 0$ and $dr/dh > 0$, prevails when : $f(h)F_u < hF_h < s\pi_K hf(h)$

(ii) A "regime of underconsumption", characterised by $du/dh < 0$ and $dr/dh < 0$, prevails when : $hF_h < f(h)F_u$

(iii) A "Keynesian regime", characterised by $du/dh > 0$ and $dr/dh > 0$, prevails when: $s\pi_K f(h) < F_h$

Following the original Goodwin (1967) model, we further assume that:

(i) The labour force, N, grows at the steady rate n, i.e.

$$\hat{N} = n \tag{13}$$

Steady-state growth at full employment (Harrod-Domar-Kaldor growth path) requires that the 'natural' rate of growth, n, must be less than the actual rate of capital accumulation corresponding to the maximum feasible value of the profit share, $h = 1$, and to any actual value of the degree of capacity utilization, $u = \overline{u}$, i.e.

$$n < s\pi_K \overline{u} \tag{13a}$$

(see equations (1) and (3)).[8]

(ii) The economy is characterised by a 'real wage Phillips curve', i.e.

[8] According to Gandolfo (1997, p. 461, footnote 14), the validity of condition (13a) in Goodwin's (1967) model (where $s = 1$ and $\overline{u} = 1$) is confirmed by empirical evidence (our symbols): "0.20 can be taken as a safe lower limit for π_K, and 0.12 as a safe upper limit for the productivity-augmented n". It might be considered, however, that the validity of such conditions in growth cycle models should be postulated (also see Weber, 2005, and Desai *et al.*, 2006).

$$\hat{w} = \gamma E - \delta \qquad (14)$$

where $E \equiv LN^{-1}$ denotes the employment rate, L the number employees, and $\gamma, \delta > 0$ are positive constants.

<u>Theorem 1</u>: *Assuming h ≠ 0, the motion equation that characterizes the economic system, regarding the share of profits over income, is given by the expression* $\frac{\dot{h}}{h} = e_1(\gamma E - \delta)$

Proof

See Mathematical Appendix.

<u>Theorem 2:</u> *Assuming E ≠ 0, the motion equation that characterizes the economic system, regarding the employment rate, is given by the expression* $\frac{\dot{E}}{E} = e_2\frac{\dot{h}}{h} + s\pi_K h f(h) - n$

<u>Proof:</u>

See Mathematical Appendix.

Consequently, the model reduces to the non-linear equations (15) and (18), that has two equilibria with $\dot{h} = \dot{E} = 0$, namely:

$$h^* = 1,\ E^* = 0 \qquad (19)$$

and

$$h^{**} = (s\pi_K f(h^{**}))^{-1} n,\ E^{**} = \delta\gamma^{-1} \qquad (19a)$$

where the latter is economically meaningful ($0 < h^{**}, E^{**} < 1$) when

$$n < s\pi_K f(h^{**}) \text{ and } \delta < \gamma \qquad (19b)$$

To relations (19a-b) there corresponds a unique value for g^S ($= n$), and may correspond - when $e_2^{**} < 0$ - more than one economically meaningful value(s) for h and, therefore, for u and w.[9]

<u>Stability conditions:</u>

The Jacobian matrix, $\mathbf{J} \equiv [J_{ij}]$, of equations (15) and (18) is:

$$J_{11} \equiv \partial\dot{h} / \partial h = (\gamma E - \delta) \qquad (20a)$$

[9] Consider, for instance, the case of a linear accumulation function, which necessarily implies that $e_2 < 0$.

$$J_{12} \equiv \partial\dot{h} / \partial E = -\gamma(1-h) \tag{20b}$$

$$J_{21} \equiv \partial\dot{E} / \partial h = \{[(de_2 / dh)e_1 + e_2 h^{-2}](\gamma E - \delta) + s\pi_K (1+e_2) f(h)\}E \tag{20c}$$

$$J_{22} \equiv \partial\dot{E} / \partial E = e_2 e_1 (2\gamma E - \delta) + s\pi_K hf(h) - n \tag{20d}$$

a) At the trivial fixed point (h^*, E^*), $e_1^* = 0$ and $\mathbf{J}^*$ is diagonal, with $J_{11}^* < 0$ and $J_{22}^* > 0$ (take into account relation (13a)); therefore, it is a saddle point, precisely like in Goodwin's (1967) model.

b) Next, consider the non-trivial fixed point(s), (h^{**}, E^{**}). Then $J_{11}^{**} = 0$, $J_{12}^{**} < 0$, and there are the following cases:

(i) When $e_2^{**} > 0$, it follows that $\mathrm{Tr}\,\mathbf{J}^{**} < 0$ and $\mathrm{Det}\,\mathbf{J}^{**} > 0$: locally stable.

(ii) When $e_2^{**} = 0$, it follows that $\mathrm{Tr}\,\mathbf{J}^{**} = 0$ and $\mathrm{Det}\,\mathbf{J}^{**} > 0$: centre precisely like in Goodwin's (1967) model. Hence, (h^{**}, E^{**}) is either a focus (stable or unstable) or a centre, depending on the precise form of $f(h)$ (see, e.g., Andronov et *al.*, 1987, pp. 278-280).

(iii) When $-1 < e_2^{**} < 0$, it follows that $\mathrm{Tr}\,\mathbf{J}^{**} > 0$ and $\mathrm{Det}\,\mathbf{J}^{**} > 0$: unstable.

(iv) When $e_2^{**} = -1$, it follows that $\mathrm{Tr}\,\mathbf{J}^{**} > 0$ and $\mathrm{Det}\,\mathbf{J}^{**} = 0$ ($J_{21}^{**} = 0$): unstable.

(v) When $e_2^{**} < -1$, it follows that $\mathrm{Tr}\,\mathbf{J}^{**} > 0$ and $\mathrm{Det}\,\mathbf{J}^{**} < 0$: saddle point.[10]

It is then concluded that the local dynamic behaviour of the system depends on the elasticity of the *IS*–curve, which, in its turn, depends on the form of the accumulation function. This elasticity determines the effect of a rising profit share on the volume of employment, and may be conceived as a 'friction coefficient' (also consider Samuelson, 1971, pp. 982-983) that alters the conservative dynamics of Goodwin's (1967) system: The equilibrium in the Keynesian regime ($e_2^{**} > 0$: positive friction) is locally stable, whilst that in the overaccumulation regime ($-1 < e_2^{**} < 0$: negative friction) is unstable. And in the border between these two regimes ($e_2^{**} = 0$), the possible existence of cyclic

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[10] Since $\partial(\dot{E}E^{-1}) / \partial(1-h) < 0$ does not necessarily hold true (see equation (20c)), this system does not correspond to Kolmogorov's 'predator ($1-h$) – prey (E) model' (see, e.g. May, 1972, p. 901). When $\partial(\dot{E}E^{-1}) / \partial(1-h) > 0$, the "two species are in symbiosis" (see Hirsch and Smale, 1974, p. 273). It is also noted that a 'U-shaped' *IS* – curve (see also Marglin and Bhaduri, 1988, pp. 22-23, and Bhaduri and Marglin, 1990, pp. 392-393) may generate a Hopf bifurcation of periodic solutions (see Mariolis, 2013, Appendix).

paths cannot be excluded. Finally, the equilibrium in the underconsumption regime ($e_2^{**} < -1$), where $\text{Det}\,\mathbf{J}^{**}$ switches from positive to negative, is saddle-path stable.

4. Econometric Model and Data

No doubt, the proposed model should be confronted with data in order to allow formal statistical estimation of parameters and functions of interest.

The proposed model reduces to the non-linear equations (15) and (18):

$$\frac{\dot{h}}{h} = e_1(\gamma E - \delta) \qquad (15)$$

$$\frac{\dot{E}}{E} = e_2\frac{\dot{h}}{h} + s\pi_\kappa h f(h) - n \qquad (18)$$

where the latter is economically meaningful ($0 < h^{**}, E^{**} < 1$) when

$$n < s\pi_K f(h^{**}) \text{ and } \delta < \gamma$$

Next, we have to transform this into an estimable form, before we can proceed with formal estimation. So, we start by substituting equation (15) directly into (18) to obtain the following form:

$$\frac{\dot{E}}{E} = e_2 e_1(\gamma E - \delta) + s\pi_\kappa h f(h) - n \qquad (21)$$

We are interested in the change of *u* as a result of a change in *h*, i.e. *du*/*dh* = z, where *z is a* real valued parameter and its estimated sign will determine the relationship (negative of positive) between *du* and *dh*. We assume that the *IS*–curve $u = f(h)$ is linear, implying that *u* = *zh* + *θ*, where *z* and *θ* are real valued parameters. This yields:

$$\frac{\dot{E}}{E} = e_2 e_1 \gamma E + \theta s\pi_\kappa h + z s\pi_\kappa h^2 + (-n - e_2 e_1 \delta) \qquad (22)$$

For given *E*, $\frac{\dot{E}}{E}$, $s\pi_\kappa h$ and $s\pi_\kappa h^2$ based on the economy's aggregate data, this conforms to the general form of multiple linear regression and its estimation is straightforward.

This procedure will provide us with direct estimates of z and θ. Also, it will provide us with an estimate of $e_2 e_1 \gamma$. Given that $e_1 = \frac{\bar{h}-1}{\bar{h}}$ and $e_2 = \frac{z\bar{h}}{z\bar{h}+\theta}$ we obtain an estimate of *γ*, where, in general, $\bar{x}$ denotes the average value of variable *x*. Similarly, given that n is exogenous and can be calculated routinely based on the available data, from the estimate of the intercept in (22), we obtain the value of *δ,* since e_1 and e_2 are calculated as above.

In order to proceed with formal estimation, data, on *h, E, s* and π_κ are needed. Our investigation starts in 1960 and stops in 2012.

The variables used are: Employment and Population in number of persons; Capital, GDP and Labour Cost in 2000 constant prices in millions of US dollars. The data come from OECD's AMECO database. The variable of savings comes from the US Federal Reserve Bank of St. Louis, also in constant 2000 prices in millions of US dollars. The profits are calculated based on the methodology used, among others, in Wolff (2003). In addition, the productivity of capital (π_κ) is equal to the share of potential output over capital, where the potential output is, typically, obtained as the HP filtered GDP time series.

5. Empirical Analysis: Total Period

Before proceeding to the estimation of our model we will test for the existence of potential outliers in our dataset as well as for possible structural breaks, as econometric theory dictates. In this context, based on economic intuition about the recent US economic history we will make an attempt to break down the 1960-2012 time period into relevant sub-periods.

Following the relevant literature regarding the US economy in the time period 1960-2012, we examine the existence of outliers in the 60's. In fact, according to Dumenil and Levy (2001), in the mid 60's the profit margin in the US economy has altered significantly, expressing the end of what is now conceived as the Golden Age of US Capitalism. Also, this implies that a structural break might be relevant at around the end of the 60's when the period of stagflation made its appearance, followed by the oil crises. To this end, we employ the Hadi (1992, 1994) outlier test for all the variables that enter our model.

Outliers

Initially, we rearrange the *n* observations of the sample in an ascending sort using the distance: $D_i(C_R, S_R) = \sqrt{(x_i - C_R)^T S_R^{-1}(x_i - C_R)}$ (a), where: i=1,..*n* is the number of observations, x_i denotes the observations, C_R denotes the robust location estimator and S_R denotes the robust covariance matrix estimator. Next, we divide the observations in two subsets according to the distance function where the first subset (*basic subset*) contains p+1 observations and the second subset (*non-basic subset*) contains *n*-*p*-1 observations. If the basic subset is of full rank, we compute the distance of observations: $D_i(C_b, S_b) = \sqrt{(x_i - C_b)^T S_b^{-1}(x_i - C_b)}$ (a), where b denotes the basic subset. If the basic subset is not of full rank, then we compute the eigenvalues of S_b, $\lambda_1 \geq .. \geq \lambda_p = 0$ as well as the matrix containing the corresponding set of normalized eigenvectors V_b, and then we compute the distance of observation: $D_i(C_b, S_b) = \sqrt{(x_i - C_b)^T V_b W_b V_b^T (x_i - C_b)}$ (b), where W_b denotes the diagonal matrix whose *j*-th element is $w_j = \dfrac{1}{\max(\lambda_j, \lambda_s)}$, $j = 1,..n$ and λ_s is the smallest non-zero eigenvalue of S_b.

Furthermore, we rearrange the observations according to the distance (a) or (b) depending on the matrix rank of the basic subset. We divide again the observations in two subsets and augment the basic subset by one observation. Finally, we repeat this procedure until the following criterion is met $\Pr\{\min(D_i(C_b, S_b),\ \text{i} \in \text{ non-basic subset}) \geq c_a :\ \text{the sample X contains no outliers}\} = 1 - a$, where a is the level of significance chosen. Lastly, we compute the robust distances given by: $D_i(C_b, S_b) = \sqrt{(x_i - C_b)^T (c_b S_b)^{-1}(x_i - C_b)}$, where $c_b = \dfrac{c_{nrp}}{\chi^2_{p.05}}$ is a correction factor to obtain consistency when the data come from multivariate normal distribution and $c_{nrp} = (\dfrac{1+r}{n-p})^2$ where r is the number of observations in the final basic subset *b*.

The results of the Hadi (1992, 1994) test are presented in Table 1.

Table 1: Hadi's test for outliers (1960-2012)

Hadi test for Outliers				
Variable s	$\dot{E}/E$	**E**	$s\pi_\kappa h$	$s\pi_\kappa h^2$
Number of obs	52	52	52	52
Initially accepted	2	2	2	2
Expand to (n+k+1)	25	25	26	26
Expand, p-value=0.01	52	52	49	49
Outliers	0	0	3	3
Years excluded as outliers				1961, 1962, 1963

As expected, we find that the first observations of our dataset i.e. the first years of the 1960s, should be excluded from the analysis.

Structural Breaks

Now, based on economic intuition we proceed by testing for the existence of a structural break around the early 1980's, which marks an upward phase in the US profit rate (see, among others, Dumenil and Levy, 2001, and Goldstein, 1996) followed by the second oil crisis that is said to have ended in the early 1980's, as well as around 2006 which marks the first signs of the US subprime crisis. To this end, we use three different methodologies, to test for structural breaks.

We first use the Chow (1960) test, which tests whether one single period regression $y_t = \alpha_0 + \alpha_1 x_t + u_t$ is more suitable than two separate regressions, splitting the data into two sub-periods at the break point *t,* expressed as:

$$\begin{aligned} y_t &= \beta_1 + \beta_2 x_t + u_{1t} \\ y_t &= \delta_1 + \delta_2 x_t + u_{2t} \end{aligned}$$

The null hypothesis Ho is that there is no structural break, i.e. $\beta_1 = \delta_1, \beta_2 = \delta_2$ and is routinely tested against the critical values in the F-test tables with F(*k*,*n*-2*k*) degrees of freedom using the following statistic $F - \dfrac{RSS_c - (RSS_1 + RSS_2)/k}{RSS_1 + RSS_2 / n - 2k}$.

Next, following Andrews (1993), the SupW is taken over all break dates in the region [*t*1, *t*2] where *t*1 > 1 and *t*2 < *n*, and *n* is the number of observations. The region [*t*1, *t*2] contains candidate break dates. We avoid the proposed break (early 1980's) to be too near the beginning (1964) or end of sample (2007), because the estimates and tests will be misleading. We have used the rule *t*1 = [0.15*n*] and *t*2 = [0.85*n*], and we have calculated the SupW (of the Wald test) in this interval, because the SupF (of the F test) assumes homoscedasticity. The results of the structural break test indicated the existence of a structural break in year 1982 as well as a structural break in the year 2007 and are presented, for brevity's sake, compactly in Table 2 and Table 3.

Table 2: Structural Break tests

Structural Break test for 1982 in the period 1970-2007				
H0: No Structural Change				
Break Periods: 1st Break Period: 1970-1981, 2nd Break Period: 1982-2007				
	Chow-test	**Wald Test**	**LR-test**	**LM test**
t-stat	3.09	19.40	16.34	13.89
p-value	0.02	0	0	0.01

Table 3: Structural Break tests

Structural Break test for 2007 in the period 1983-2012				
H0: No Structural Change				
Break Periods: 1st Break Period: 1983-2007, 2nd Break Period: 2007-2012				
	Chow test	**Wald Test**	**LR-test**	**LM-test**
t-stat	2.97	5.39	4.32	6.21
p-value	0.02	0	0	0

Furthermore, the endogenous structural break test of Zivot and Andrews (1992), that utilizes the full sample by using a different dummy variable for each possible break date, has also been conducted. The selection criterion for the break date is based on the t-statistic from an ADF test and a minimum (i.e. most negative) value of t-statistic will be the indication of the break date.

The results of the ADF test for the dependent variable of our model around the years 1982 and 2007 are presented, for brevity's sake, compactly in Table 4[11] and Table 5 and confirm our previous finding of the existence of a structural break in the years 1982 and 2007, respectively.

Table 4: Zivot-Andrews (1992) test around 1981-1983

ADF test		
Variable	**Period**	**t-stat**
	1970-1981	-2.66
	1970-1982	-2.75
$\dot{E}/E$	1970-1983	-2.42

Table 5: Zivot-Andrews (2007) test around 2006-2008

ADF test		
Variable	**Period**	**t-stat**
	1983-2006	-2.91
	1983-2007	-2.99
$\dot{E}/E$	1983-2008	-2.90

The results of the aforementioned tests clearly show that a structural break takes place in 1982 and in 2007. This fact, combined with the existence of outliers in the period 1960-1963, implies that the period 1960-2007 is broken down into two sub-periods starting in 1964 and breaking in 1982 and 2007. We use the simple rule of splitting the sample at the estimated break, following, among others, Pesaran and Timmermann (2007). This provides us with two sub-periods, namely 1964-1982, 1984-

[11] Detailed yearly calculations are available upon request regarding all three tests, i.e. Chow (1960), Andrews (1993) and Zivot and Andrews (1992).

2007 and 2008-2012. Since the remaining observations, after the structural break observed in 2007, are too few (5), any formal statistical inference in this period would be meaningless. After all, as we know, at post-2007 era, the dynamics of the traditional economic structures changed dramatically, both in the USA and globally.

Periodicities

Since Goodwin type models are characterized by (endogenous) cycles in the fundamental variables, we start our investigation by examining the periodicities of the fundamental variables that enter the proposed model.

To this end, using spectral analysis, we investigate the periodicities of business cycles, meaning the average length of the cycles of profits over income (*h*) and the employment rate (*E*), based on the Fourier-transformed function of the cycle, which has often been used in the relevant literature (e.g. Iacobucci, 2003, and Owens and Sarte, 2005). The periodogram is a graph of the spectral density function of a time series as a function in the natural frequency domain. The representation has the following form:

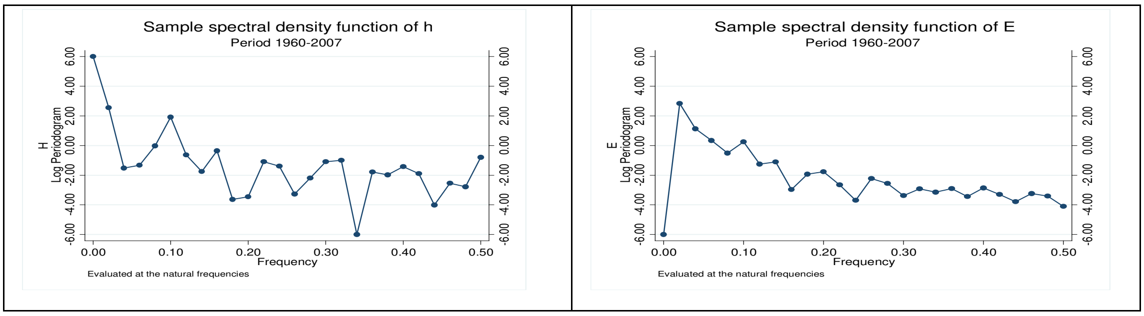
$$f(\omega)=\begin{cases} f(1-\omega), \text{ if } \quad \omega\in[0.5,1] \\ 1/n\left|\sum_{t=1}^{n} x(t)e^{2\pi i(t-1)\omega}\right|, if \ \ \omega\in[0,0.5) \end{cases}$$

where $\omega = 2\pi / n$ is the natural frequency and *x(t)* is the time series in time *t*.

The rationale of the above Fourier transformation is that it first standardizes the amplitude of the density by the sample variance of the time series, and then plots the logarithm of that standardized density. Peaks in the periodogram represent the dominant frequencies (cycles) in the data.

The periodograms of the aforementioned variables are presented in Figure 1.

Figure 1: Periodograms of *h* and *E* (1960-2007)



The periodogram of the share of profits over income (*h*) suggests the existence of a medium-term cycle with a period of approximately 6-8 years, whereas a longer cycle of 15-17 years is also present. Furthermore, the periodogram of the employment rate (*E*) suggests the existence of a short-term cycle with a period of approximately 2-3 years.

Stationarity

We check the stationarity properties of the various time series in the period investigated. If the results suggest that a time series is non-stationary in the original variables, then first differencing is highly recommended. As we know, there are several ways to test for stationarity. In this paper, we use the Augmented Dickey-Fuller (ADF) methodology (Dickey and Fuller, 1979) because of its widespread acceptance in the literature. The ADF test is based on the following model (Kaskarelis, 1993):

$$\Delta \mathrm{Y}_t = \alpha + bt + \rho Y_{t-1} + \sum_{i=1}^{m} \gamma_t \Delta \mathrm{Y}_{t-1} + \varepsilon_\tau$$

where Δ is the first difference operator, *t* the time and ε the error term:

(a) if b≠0 and -1<ϱ<0 implies a trend stationary model;

(b) if b=0 and -1<ϱ<0 implies an ARMA Box/Jenkins class of models;

(c) if b= 0 and ϱ= 0 implies a difference stationary model where *Y* variable is integrated of degree one I(1). If we assume that the cyclical component is stationary, the secular component has a unit root and *Y* follows a random walk process, i.e. it revolves around the zero value in a random way (Heyman and Sobel, 2004, p. 263); furthermore, if $a \neq 0$ Y follows a random walk process with a drift.

All variables that enter the proposed model are checked for stationarity (see Table 1, left part). All the non-stationary variables of the model have been first differenced so as to induce stationarity, and the first differenced variables are now found to be stationary (see Econometric Appendix: Table 1B, right part). Our model estimation will include only stationary variables in accordance with modern econometric theory and practice.

Furthermore, using stationary variables in the Goodwin model is also consistent, among others, with the seminal work by Barbosa-Filho and Taylor (2006).

Following early work in the field such as Harvie (2000), a single estimate of π_K was calculated by calculating its mean, i.e. $\pi_K = \overline{\pi_K}$. Also, a given and constant fraction of profits (s) equal to unity is assumed, s = 1. See Table 6 (left part). Next, the simplifying assumptions of a (i) constant potential output-capital ratio; (ii) given and constant fraction of profits saved are relaxed. See Table 6 (right part).

Heteroscedasticity

Given the presence of heteroskedasticity in both models, we make use of White (1980) and Huber (1967) standard errors in our regression in order to obtain BLUE estimators. More precisely, the White-Huber estimator transforms the Variance matrix Σ that is obtained from standard OLS as follows:

$$\Sigma^* = \begin{pmatrix} \widehat{u_1^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \widehat{u_n^2} \end{pmatrix}$$

where $\widehat{u_i^2}$, i=1,..n is the standard errors obtained by OLS multiplied by (N/(N-K-1)) where N is the sample size and K is number of regressors entering the OLS equation. Thus, the variance estimator of robust OLS is: $Var(\widehat{b}) = (X'X)^{-1} X'\Sigma^* X(X'X)^{-1}$. This estimator, in the presence of heteroskedasticity, is known to be BLUE (e.g. Greene, 2010). The results of our estimation are presented in Table 6.

Table 6: Estimation results (1960-2007)

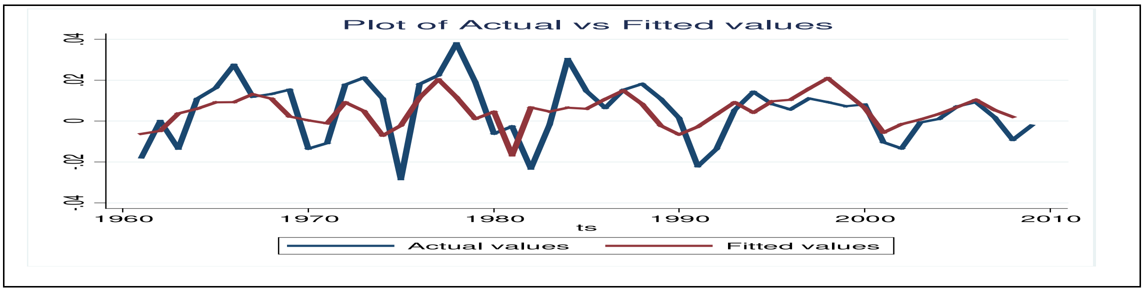
Estimates 1960-2007 (s=1 $\pi_\kappa = \bar{\pi}_\kappa$)				**Estimates 1960-2007 ($s \neq 1, \pi_\kappa \neq \bar{\pi}_\kappa$)**			
Variables	**Coef**	**t-stat**	**p-value**	**Variables**	**Coef**	**t-stat**	**p-value**
E	-0.76	-2.31	0.03	E	-0.78	-2.26	0.03
$s\pi_\kappa h$	20.21	0.76	0.45	$s\pi_\kappa h$	6.64	1.67	0.10
$s\pi_\kappa h^2$	-37.51	-0.76	0.44	$s\pi_\kappa h^2$	-23.75	-1.68	0.10
Intercept	-0.98			*Intercept*	0.002		
R-sq=0.18, F-stat=3.57				**R-sq=0.27, F-stat=4.21**			

From Table 6 we can infer that our choice of relaxing the two most commonly used simplifying assumptions mentioned above renders all the variables of the model statistically significant and increases its overall fitting performance, in contrast to the alternative choice.

Now, there are no signs of serious violation of the basic assumptions concerning the residuals, as was easily confirmed with the aid of the relevant procedures: specifically, the normality of the errors was assessed through the formal examination of the frequency distribution of the residuals. Also, as for the assumption that the residuals are independent of each other, investigation of the scatter plot of the standardized residuals against the time variable did not provide serious evidence of possible dependence between successive values, i.e. an autocorrelation effect. Also, the Durbin–Watson statistic indicated that the hypothesis that the residuals are autocorrelated cannot be accepted in our investigation.

Based on the clearly superior performance of the model when the two most commonly used simplifying assumptions are relaxed, we continue our investigation. The figure of the actual versus the fitted values of the model when $s \neq 1, \pi_{\kappa} \neq \bar{\pi}_{\kappa}$ are presented in Figure 2.

Figure 2: Actual vs Fitted values for the period (1960-2007)



6. Empirical Analysis: Sub-periods

All variables that enter our proposed model are checked anew for stationarity (see Table 7, left part), for the specific sub-periods examined. All non-stationary variables of the model have been first differenced so as to induce stationarity, and the first differenced variables are now found to be stationary (see Econometric Appendix: Table 2B, right part). Note that, as expected, the stationarity characteristics of the sub-periods are different to those of the total period.

In order to econometrically estimate the model in the two sub-periods, we excluded - following common practice - the values directly around the break point in order to avoid obtaining misleading results.[12] The exclusion of five observations (1979-1983) around the break point has resulted in the following two sub-periods (1964-1978, 1984-2007), which present the most statistically significant results. Given the presence of heteroscedasticity, the econometric implementation has been adjusted in accordance with the procedure set out earlier (Section 5). The estimation results for the two sub-periods are depicted in Table 8.

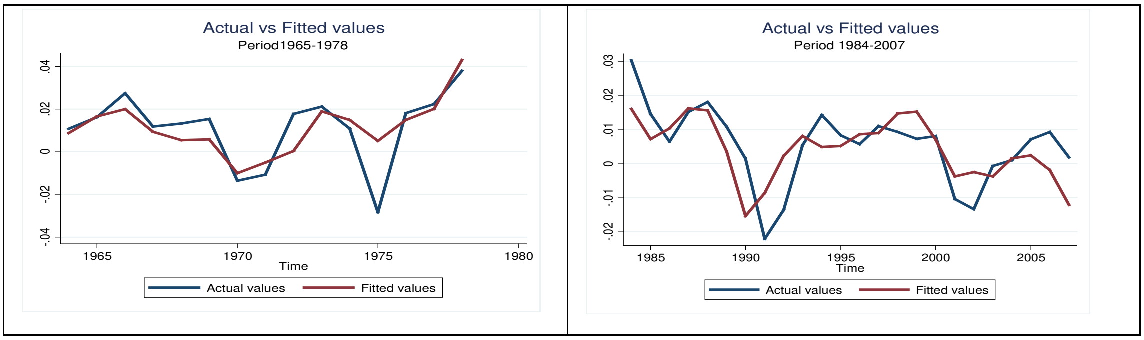
Table 7: Model Estimations (1964-1978, 1984-2007)

Estimation for the period 1964-1978			
Variables	**Coef**	**t-stat**	**p-value**
E	1.04	3.04	0.02
$s\pi_{\kappa}h$	87.39	7.21	0.00
$s\pi_{\kappa}h^2$	-288.68	-6.86	0.00
Intercept	-0.03		
R-sq=0.91, F-stat=31.38			

Estimation for the period 1984-2007			
Variables	**Coef**	**t-stat**	**p-value**
E	-1.57	-2.64	0.02
$s\pi_{\kappa}h$	-8.36	-2.06	0.05
$s\pi_{\kappa}h^2$	34.83	2.27	0.04
Intercept	-0.0004		
R-sq=0.50, F-stat=3.58			

The Actual versus Fitted values plots are presented in Figure 3.

Figure 3: Actual vs Fitted values plot (1964-1978 and 1984-2007)



As set out in Section 3, the estimates of θ and z come straightforward from the estimation of the model. Furthermore, under the assumption that e_1, e_2 are assumed to

[12] See also Pesaran and Timmermann (2007).

be equal with their respective average values $\bar{e}_1, \bar{e}_2$, the underlying values of the coefficients *γ* and *δ*, are revealed. The estimated values of the coefficients by period are presented in Table 8.

Table 8: Estimated Model parameters by period

Period	$\bar{e}_1$	$\bar{e}_2$	**θ**	**Z**	**γ**	**δ**
1960-2007	-2.13	7.92	6.64	-23.75	0.050	0.001
1964-1978	-2.33	-110.18	87.39	-288.68	0.040	4E(-5)
1984-2007	-1.94	3.4	-8.36	34.83	0.240	0.002

Now, from equations (20), using the estimates of the coefficients *γ, δ, θ, z* computed earlier, and the average values of *E, h,* $s\pi_K$, for each period examined, we obtain the Jacobian matrix **J,** for each period examined.

Table 9 summarizes the regimes and stability results of the estimated models, by period, based on the criteria presented earlier (Section 3)

Table 9: Regimes and Stability of each period

Period	$\bar{e}_2$	**Tr J****	**Det J****	**Regime**	**Stability**
1960-2007	7.92 > 0	<0	>0	Keynesian	Locally stable
1964-1978	-110.18 <-1	>0	<0	Under-consumption	Saddle point
1984-2007	3.4 >0	<0	>0	Keynesian	Locally stable

7. Summary and Discussion

Our empirical analysis started with the investigation of the cyclical character of the two fundamental variables of all Goodwin-type models, namely the share of profits over income (*h*) and the employment rate (*E*). The periodogram of the share of profits over income (*h*) suggests the existence of a medium-term cycle with a period of approximately 6-8 years, whereas a longer cycle of 15-17 years is also present. Our results are consistent, among others, with the works by Harvie (2000) and Flaschel and Groh (1995).

Taking into consideration the dynamics of the US economy, the long-run cycle could be attributed to the shift in emphasis of macroeconomic policy in the USA in the late 70's toward combating inflation rather than maintaining full employment (Argitis and Pitelis, 2001). The dominant medium-term cycle could be attributed to differences in the monetary policy implemented by the US Federal Reserve Bank. The periodogram for the employment rate suggests the existence of a short-term cycle with a period of approximately 2-3 years, which are traditionally attributed to US inventory cycles.

Next, we estimated our model using data on the US economy. From Table 2, we can infer that our choice of relaxing the two most commonly used simplifying assumptions (i.e. that of a constant capital to [potential] output ratio and the one of a given and constant fraction of profits to be saved) rendered all the variables of the model statistically significant and increased its overall fitting performance, in contrast to the alternative choice, given that the simplifying assumptions were not expected to approximate reality with any given accuracy.

Following the relevant literature regarding the US economy in the time period 1960-2012, we tested for the existence of outliers since, according to Dumenil and Levy (2001), in the mid 60's the profit rate in the US economy changed dramatically, signifying the end of what is now characterized as the Golden Age of US Capitalism, coinciding with a profit squeeze and a rise in savings. To this end, we conducted the Hadi (1992, 1994) outlier test for all the variables that enter our model. Our main finding, i.e. that the first years of the 1960s act as outliers and should be excluded from the analysis is expected and consistent, in general terms, with the relevant literature arguing that the 1960s is a decade when the first phase of the Golden Age of US economy ended.

Now, based on economic intuition we proceeded by testing for the existence of a structural break around the early 1980's, which marks an upward phase in the US profit rate (see, among others, Dumenil and Levy, 2001 and Goldstein 1996) followed by the second oil crisis that is said to have ended in the early 1980's. We used three different methodologies to test for structural breaks.

The results of the aforementioned tests clearly showed that a structural break took place in 1982. This fact, combined with the existence of outliers in the period 1960-1963, implies that the period 1960-2007 is broken down into two sub-periods starting in 1964, breaking in 1982 and ending in 2007 approximately. Following Pesaran and Timmermann (2007), we used the simple rule of splitting the sample at the estimated break and this provided us with two sub-periods, namely 1964-1982 and 1984-2007. Next, following common practice, we excluded the values directly around the break point in order to avoid obtaining misleading results.

In brief, from the estimation results, we can see that the signs of the estimated coefficients are consistent with the stated hypotheses and economic theory, namely: γ, $\delta > 0$ such that $\delta < \gamma$ and $e_1 < 0$. Also, the estimated results are statistically significant for the independent variables, while the equation explains a considerable part of the variability of the dependent variable. The results should be assessed as satisfactory given the various imperfections in this sort of country data (Mankiw, Romer and Weil, 1992: 408), as well as given the crisis period and the various shocks that the US economy faced in the period examined.

As far as the stability conditions and the regimes that the US economy exhibits, according to our model we have that the US economy, regarding the period 1960-2007, is characterized by a Keyensian regime or an "exhilarationist" regime (Bhaduri and Marglin 1990) meaning that the economy is profit-led (Bowles and Boyer 1988, and Gordon 1993), and seems to be in a stable path. More specifically, we have cooperation between capital and labour, since $e_2 e_1 < -1$ and, thus $d[(1-h)u]/dh > 0$. In another formulation "a given increase in the profit share stimulates the level of demand and capacity utilisation sufficiently to increase aggregate employment and the wage *bill*" (Bhaduri and Marglin (1990, p. 384). The same picture is in force for the US economy for the sub-period of 1984-2007, just after the second oil crisis with the emergence of new technology trends that attracted the majority of investment activity in the USA (Dumenil and Levy 2001). Nevertheless, the period of 1964-1978 is characterized by an under-consumption regime, meaning that an increase in the real wage rate implies higher profit and growth rates because the positive effect of demand is greater than the negative effect of higher costs ('paradox of costs'), while the economy is at a saddle path. This could, in turn, be attributed to the stagflation that the US economy faced (Dumenil and Levy 2001) which was accompanied by the profit squeeze that followed the Golden era of capitalism.

To sum up, the overall empirical investigation of the proposed extended Goodwin model that has incorporated the Bhaduri-Marglin accumulation function is able to adequately capture the behavior of the US economy, in the period 1960-2007. In addition, our model was able to shed light in two distinct sub-periods of the US economy that experience different dynamics, regarding both the regimes and the stability of the economic system.

8. Conclusion

We have used a Goodwin type model that incorporates the Bhaduri-Marglin (1990) accumulation function, in order to study empirically the US economy in the time period 1960-2007, right before the outburst of the US sub-prime crisis and the subsequent global recession. Our investigation stops in 2007 since, at post-2007 era, the dynamics of the traditional economic structures changed dramatically, both in the USA and globally.

In comparison to other contributions: the present work uses the Bhaduri-Marglin accumulation function, presents formally some useful mathematical results and econometrically estimates the model for the largest economy in the world, namely the USA, in the period 1960-2007. Meanwhile, the simplifying assumptions of a constant capital - potential output ratio and of a given and constant fraction of profits saved are relaxed, a choice which proves to be empirically justified and improves significantly the performance of our proposed model. Also, the total period is broken down into two sub-periods based on the relevant structural break tests conducted.

Undoubtedly, future and more extended research on the subject seems to be necessary focusing on additional variables (e.g. monetary), which have often proved to be relevant. Similarly, the proposed approach could be routinely extended empirically to include other economies in the world that could help further explain global imbalances. We believe that both ideas are of great interest and constitute good examples for future work in the field.

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MATHEMATICAL APPENDIX

Proof (Proposition 1):

The profit rate is defined as $r = \pi_K hu$, while real wages are defined as $w = \pi_L(1-h)$, which in turn implies that $h = \frac{\pi_L - w}{\pi_L}$ (7), assuming $\pi_K \neq 0$. So, the linear frontier curve of ϱ-w is given by the following expression $\rho = \pi_K(1-\pi_L^{-1}w)$ (8).

Therefore, the elasticity of the normalized rate of profits with respect to real wages is

$$e_1 \equiv \frac{d\log\rho}{d\log w} \Leftrightarrow e_1 = \frac{d\log(\pi_K(1-\pi_L^{-1}w))}{d\log w} \Leftrightarrow$$
$$e_1 = -\pi_L^{-1}w(1-\pi_L^{-1}w)^{-1} \Leftrightarrow$$
$$e_1 = -(1-h)h^{-1} \quad (9)$$

Proof (Lemma 1):

Equation (5), by substitution of equations (1) and (2), yields:

$F(u,h) = sr$

And by substituting the profit rate (r) from equation (3), we obtain:

$F(u,h) = s\pi_K hu$

which is a relationship between profit share and degree of capacity utilization $u = f(h)$.

Proof (Proposition 2):

By differentiating *F* with respect to both variables and substituting into the equation defining the elasticity of *u* with respect to *h*, we obtain trivially that:

$$e_2 = (F_h - s\pi_K u)(s\pi_K h - F_u)^{-1} hu^{-1} \qquad (10)$$

Proof (Lemma 2):

Let $u = f(h)$. Then, equation (3) implies $r = \pi_K hf(h)$ (11)

By differentiation of equation (11) with respect to *h* we obtain:

$$dr/dh = (1+e_2)\pi_K f(h) \qquad (12)$$

Proof (Theorem 1):

Equations (4) and (7) imply that:

$$w = -\pi_L e_1 h$$

Now, using equation (14), we obtain:

$$\dot{h} = (\delta - \gamma E)(1-h) = e_1(\gamma E - \delta)h \Rightarrow$$

$$\frac{\dot{h}}{h} = (\delta - \gamma E)(1-h) = e_1(\gamma E - \delta) \Rightarrow$$

$$\frac{\dot{h}}{h} = e_1(\gamma E - \delta) \quad (15)$$

<u>Proof (Thorem 2):</u>

Since $L = \pi_L^{-1}\pi_K uK$, where K denotes the capital stock in existence, and $g^S \equiv \hat{K}$, it follows that $\hat{L} = \hat{u} + g^S$ or, recalling equations (1), (3) and (10),

$$\hat{L} = e_2\hat{h} + s\pi_K hf(h) \tag{16}$$

Substituting equations (12) and (16) in $\hat{E} = \hat{L} - \hat{N}$ yields:

$$\dot{E} = [e_2\frac{\dot{h}}{h} + s\pi_K hf(h) - n]E \Longrightarrow \tag{17}$$

$$\frac{\dot{E}}{E} = e_2\frac{\dot{h}}{h} + s\pi_K hf(h) - n \tag{18}$$

ECONOMETRIC APPENDIX

Table 1B: ADF test (1960-2007)

ADF test Original Variables (1960-2007)			
Variables	t-stat	p-stat	Stationary
$\dot{E}/E$	-4.91	0	Yes
E	-1.22	0.71	No
H	-2.06	0.26	No
$s\pi_K h$ **(s=1, $\pi_K = \overline{\pi_K}$)**	-2.06	0.26	No
$s\pi_K h^2$ **(s=1, $\pi_K = \overline{\pi_K}$)**	-2.04	0.25	No
$s\pi_K h$	-4.07	0	Yes
$s\pi_K h^2$	-3.97	0	Yes

ADF test first differenced variables (1960-2007)			
Variables	t-stat	p-stat	Stationary
E	-4.78	0	Yes
H	-7.21	0	Yes
$s\pi_K h$ **(s=1, $\pi_K = \overline{\pi_K}$)**	-6.72	0	Yes
(s=1, $\pi_K = \overline{\pi_K}$)	-6.84	0	Yes

Table 2B: ADF test (1979-1982, 1983-2007)

ADF test, original variables (1964-1982)			
Variables	t-stat	p-value	Stationarity
$\dot{E}/E$	-2.49	0.05	Yes
E	-0.37	0.91	No

ADF test, first-differenced variables (1964-1982)			
Variables	t-stat	p-value	Stationarity
E	2.98	0.04	Yes

$s\pi_{\kappa}h$	-0.66	0.86	No
$s\pi_{\kappa}h^2$	-0.73	0.84	No

$s\pi_{\kappa}h$	-3.24	0.02	Yes
$s\pi_{\kappa}h^2$	-3.23	0.02	Yes

ADF test original variables (1983-2007)			
Variables	**t-stat**	**p-value**	**Stationarity**
$\dot{E}/E$	-2.48	0.05	Yes
E	-0.99	0.75	No
$s\pi_{\kappa}h$	-1.91	0.33	No
$s\pi_{\kappa}h^2$	-1.91	0.33	No

ADF test first-differenced variables (1983-2007)			
V□□ iable s	**t-stat**	**p-value**	**Stationarity**
E	2.99	0.04	Yes
$s\pi_{\kappa}h$	-4.16	0	Yes
$s\pi_{\kappa}h^2$	-4.29	0	Yes