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Donna, Javier and Espin-Sanchez, Jose

The Ohio State University, Northwestern

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Complements and Substitutes in Sequential Auctions: The Case of Water Auctions*

Javier D. Donna and José-Antonio Espín-Sánchez

The Ohio State University   Yale University

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Abstract

We use data on sequential water auctions to estimate demand when units are complements or substitutes. A sequential English auction model determines the estimating structural equations. When units are complements, one bidder wins all units by paying a high price for the first unit, thus deterring others from bidding on subsequent units. When units are substitutes, different bidders win the units with positive probability, paying prices similar in magnitude. We recover individual demand consistent with this stark pattern of outcomes and confirm it is not collusive, but consistent with non-cooperative behavior. Demand estimates are biased if one ignores these features.

JEL Codes: D44, C13, L10, L40.

Keywords: Auctions, Structural Demand Estimation, Market Structure, Competition, Collusion.

Javier D. Donna
The Ohio State University
Email: donna.1@osu.edu

José-Antonio Espín-Sánchez
Yale University
Email: jose-antonio.espin-sanchez@yale.edu

1 Introduction

There are many instances in the real world where several units of the same or similar goods are allocated sequentially or periodically using auctions. Examples include timber, procurement of public goods, electromagnetic spectrum, and treasury bills. The nature of the goods at auction and the firms bidding determine whether the goods are complements (increasing marginal returns or IMR) or substitutes (decreasing marginal returns or DMR). In many cases, IMR arise because firms incur fixed costs to realize the full value of purchased goods. This is the case of the machinery and workers needed to fell trees or build highways. Firms might experience DMR due to limited capacity to hire more workers or buy more machinery. DMR may also arise as a consequence of the downward sloping demand for the firms' final products, *e.g.*, once a firm has a valid spectrum for a given county, the value of another tranche of the spectrum decreases substantially. Firms would face IMR if the first effect dominates, DMR if the second effect dominates, and hill shaped marginal returns if both effects are important.

By affecting the valuation of subsequent units, fixed costs and decreasing returns determine bidder behavior and price dynamics. Price dynamics are central to connect observed bids to the underlying distributions that characterize individual demand, which is fundamental to discuss positive and normative questions. For instance, variation in prices caused by a high sunk cost will affect even relatively simple tasks such as measuring the dispersion in individuals' private valuations. Moreover, in such a case, a competitive environment could be incorrectly interpreted as collusive.

The existing literature on sequential auctions has provided little empirical evidence on the effect that complementarities *and* substitutabilities in the valuation of subsequent units have on price behavior within the same market. The main reason for this lack of evidence is the challenge of finding sufficient variation in the degree of complementarity within the same market. Our aim is to address this empirical gap. To that end, we examine a unique panel data set from sequential water auctions from a self-governed community of farmers in Mula, Spain.[1] Admittedly, few industrial organization economists will find water auctions from

[1]The four main fruit trees grown in the region are oranges, lemons, peaches, and apricots.

farmers in southeastern Spain interesting *per se*. We study this empirical setting because it allows us to exploit a unique scenario where a market exhibits periodic switches or reversions between regimes of complementarity and substitutability, *i.e.*, regimes where identical units may complement and substitute within the same market and for the same bidder. Weather conditions in Mula generate large changes in the degree of complementarity across seasons: variation in the importance of sunk costs relative to decreasing returns as described in section 3. We use this variation to: (1) analyze bidding behavior in sequential auctions in which buyers' preferences for multiple units exhibit both sunk costs and DMR, and (2) investigate its implications for price dynamics and price competition. This empirical setting allows to analyze a stark pattern of outcomes not previously documented in the literature. Sometimes, winning prices exhibit a standard competitive pattern. In this scenario, winning prices are similar in magnitude, regardless of whether the same or different bidders (farmers in our case) win the sequential units. Other times, one farmer wins all the units, pays a high price for the first unit, deters other farmers from entering subsequent auctions, and thus pays a very low price for the remaining units. We call this the *deterrence* effect. We show that this pattern of outcomes is consistent with a non-cooperative equilibrium, where the observed price dynamics are competitive, not collusive.

The data in this article come from all water auctions in Mula, Spain, from January 1954 through August 1966—when the last auction was run—and the allocation system switched to a bargaining system, as described in section 2. The data for our analysis consist of individual winning bids and auction covariates. These covariates include the amount of rainfall. The basic unit of sale is the right to use three hours of water (432,000 liters) for irrigation. For each weekday, eight units are sold for each schedule: *four* for daytime (7AM-7PM) and *four* for nighttime (7PM-7AM) irrigation. The auctioneer sells first the twenty units corresponding to nighttime and then the twenty units corresponding to daytime. This leaves ten *four-units* sets of auctions that are sold in order. (The ten sets of four-units auctions are: Monday-nighttime, Tuesday-nighttime, and so on, until Friday-daytime.) Thus, the relevant unit of analysis for investigating individuals' demand and the pattern of outcomes is four-units auctions. But units within each four-units set are not conditional-independent due to the presence of sunk costs. Observing the winner's identity allows us to estimate the model, as outlined in section 4. Local weather conditions determine the relevant agricultural irrigation technology and, hence, water demand. Additionally, as less rain falls in summer than in winter in southern Spain, the presence of seasonalities provides us with the variation in sunk costs relative to decreasing returns necessary to perform the empirical investigation.

We incorporate two features from our empirical setting. First, a sunk cost is incurred for the first unit bought because water flows through a channel dug into the ground. Some water is lost when the channel is dry (the first unit), but the loss is negligible for subsequent units. Engineers have estimated that 20% of the water of the first unit that travels through a dry channel was lost (Gómez-Espín, Gil-Meseguer, and García-Marín 2006). Second, DMR are present for subsequent units because the amount of irrigated land is fixed. We model the environment as a sequential ascending price English auction along the lines of von der Fehr (1994) in which bidders, by incurring a participation cost, decide whether to attend each sale.

The relative importance of sunk costs and DMR generates a trade-off, whereby buyers' bidding behavior depend on whether different units are complements or substitutes. When goods are complements, the same bidder wins all the objects paying a high price for the first unit equal to the valuation for the whole bundle (*four* times the second highest valuation for the first unit, adjusted for the complementarity effect and participation cost). By doing this, the winner of the first unit deters others from bidding on the remaining three units, allowing this bidder to pay very low prices (close to zero) for the remaining three units. The resulting price pattern, along with the same bidder winning all the units, may lead to an incorrect collusive interpretation. When goods are substitutes, different bidders win the objects with a positive probability and pay prices of similar magnitude, even when the same bidder wins all the objects. We provide empirical evidence for the key features of our model: participation and sunk costs.

The price patterns predicted by the model provide a straightforward method to determine the regime—complements or substitutes—being played. When goods are complements, very low prices are paid by the same winner (the winner of the first unit) for the second, third, and fourth units; thus, the difference between the price paid for the first and the remaining units is large. When goods are substitutes, the units might be bought by different bidders and the prices of all four units are similar; thus, the difference between the price paid for the first and the remaining units is negligible. This allows to determine the regime by looking at: (1) the identities of the winner (*i.e.* whether the same bidder bought all the four units), and (2) the difference between the price paid for the first and the remaining units (see section 4).

We estimate the distribution of private valuations by maximum likelihood using an exponential distribution and the English structure for the auction. To estimate sunk cost and DMR, we form moment conditions based on the structural equations of the model. We infer participation costs using data from auctions in which bidders were present, but no one placed bids. This method gives us bounds on participation costs.

Our empirical work establishes three main results. First, we recover individual demand—characterized by private valuations and the model's structural parameters—that is consistent with the described price patterns and the *deterrence* effect in particular. Second, the equilibrium price dynamics are consistent with competitive behavior. Non-cooperative behavior is not only consistent with the *deterrence* effect, but also predicts such price differentials. Incentives to deviate from a collusive strategy are higher in spring and summer, when water is more valuable. However, it is in spring and summer when we observe non-cooperative behavior more often. Finally, we show that estimates that ignore the importance of participation and sunk costs will be biased. We test whether price variations, conditional on covariates, are better explained by our proposed model or a standard English auction model without participation costs, using that the latter is encompassed by the former. The approach of Haile and Tamer (2003), that relies on two basic behavioral assumptions, provides a robust structural framework for inference. These minimal assumptions are not satisfied in the present context. We discuss how Haile and Tamer's structure can be interpreted in the current setting.

Section 2 discusses the auction system, the empirical regularities, and the modeling assumptions required in our context. Section 3 presents the model. Section 4 examines the estimation procedure. Section 5 presents the results, analyzes the importance of sunk costs, and the interpretation of complementarities. Section 6 concludes.

Contributions and Related Literature

We now describe the related literature and highlight how our article contributes to the current body of work. This article is closest to the empirical literature on sequential auctions with multi-unit demand. To the best of our knowledge, the price dynamics that we investigate (see section 2) have not been documented in the literature before. Most of the literature do not consider participation costs in their analysis. We show how participation costs affect equilibrium outcomes. We then use our model to partially identify participation costs and estimate informative bounds.

Numerous empirical studies have highlighted the importance of complementarities (Anton and Yao 1987; Gandal 1997; Wolfram 1998; Pesendorfer 2000; Marshall, Raiff, Richard, and Schulenberg 2006).[2] Substitutabilities are a major component in several industries such as sequential highway construction procurement auctions (Jofre-Bonet and Pesendorfer 2003), sequential timber auctions (List, Millimet, and Price 2004), or sequential cattle auctions (Zulehner 2009). Several authors have studied cases of either complements due to synergies among auctioned goods, or substitutes due to decreasing marginal utility (Black and De Meza 1992; Branco 1997; Liu 2011). Selling goods in a bundle increases a seller's revenue when goods are complements (Palfrey 1983; Levin 1997, Armstrong 2000). Our setting differs from these scenarios in that we consider sequential, instead of simultaneous, auctions. See Milgrom (2000) and Ausubel (2004) for recent contributions to this literature. Edelman, Ostrovsky, and Schwarz (2007) study the properties of a "generalized English auction" used to sell Internet advertisements and show their proposed mechanism has a unique equilibrium. Kagel and Levin (2005) experimentally investigate multi-unit demand auctions with synergies, and compare behavior in sealed-bid and ascending-bid uniform-price auctions. See Kagel (1995) for a survey on laboratory experimental auction markets.

Prior investigations of the relationship between sequential auctions and the complementarity or substitutability between identical units are more scarce (*e.g.* Jeitschko and Wolfstetter 2002; Jofre-Bonet and Pesendorfer 2012). Jeitschko and Wolfstetter 2002 analyze optimal sequential auctions in a binary-valuations case. They find that English auctions extract more

[2] Outside the auction literature, Gentzkow (2007) studies the value of new goods using a model encompassing the possibility of both complementarities and substitutabilities.

rent than first-price auctions. Our model differs as we consider the class of continuous valuation distributions. Jofre-Bonet and Pesendorfer (2012) allow for complementarities and substitutabilities in a model of sequential auctions. They find that although first-price auctions give greater revenue than second-price (English) auctions when the goods are substitutes, the opposite is true for complements. Both mechanisms are efficient in their model. Their predictions about price trends are consistent with previous findings. Contrary to our analysis with participation costs, where buyers are better informed than the seller, Jofre-Bonet and Pesendorfer (2012) examine the effect of capacity constraints on bidding behavior in procurement auctions using a two-period auction game where sellers have private information about their costs. Balat (2013) and Groeger (2014) analyze dynamic auctions in the highway procurement market. Balat (2013) extends the model from Jofre-Bonet and Pesendorfer (2012) by allowing endogenous participation and unobserved heterogeneity. Groeger (2014) analyzes bidder learning in the entry stage of an auction game. In contrast, we investigate a stark price dynamics not documented before (see section 2). In our empirical setting, the same identical units sometimes complement and other times substitute for the same bidder. We show that these price dynamics are not collusive, but consistent with non-cooperative behavior. In addition, we infer participation based on two simple assumptions that provide us informative bounds. Finally, Hendricks and Porter (1988) conducted an early and influential investigation on how interdependencies among auctioned objects affect the auction's outcome. They analyze auctions for drainage leases and show that better informed firms (which hold tracts neighboring the drainage tracts that were auctioned) earned higher rents than uninformed ones.

This article makes a methodological contribution by developing an empirical model of sequential English auctions with participation costs that allows units to complement and substitute for the *same* bidder depending on seasonalities. The model produces distinguishable price pattern predictions in each regime. This feature allows us to determine the regime under which the game is being played using the ratio between prices in different auctions. This allows us to weaken the behavioral assumptions, such as the specification of bidders' beliefs, that would be necessary to solve the whole game (see section 3). Similar to the work of Hendricks and Porter (1988) and Haile (2001), we show evidence inconsistent with the equilibrium predictions of standard models and supportive of a model that captures sunk costs, decreasing marginal returns, and participation costs. Not accounting for these features may lead to the incorrect interpretation of a competitive market as collusive.

We build upon the existing literature on participation costs and entry fees (McAfee and McMillan 1987; Engelbrecht-Wiggans 1993; von der Fehr 1994) by constructing a sequential English auction model similar to that of von der Fehr (1994). However, our set-up differs in that bidders are allowed to buy more than one unit of the good. von der Fehr (1994) considers the case when goods are independent and finds the same equilibrium as that of our complementarities case.

Although the auction literature has studied price dynamics and the relationship between sequentially auctioned goods (*e.g.* Weber 1983; McAfee and Vincent 1993; Benhardt and Scoones 1994; Engelbrecht-Wiggans 1994), to the best of our knowledge, we analyze a stark pattern of outcomes not investigated in the literature before. Sometimes, when goods are substitutes, winning prices exhibit a standard competitive pattern: regardless of whether the same or different bidders win the sequential units, winning prices are similar in magnitude. Other times, when goods are complements, the same bidder wins all units by paying a high price for the first unit, deterring others from bidding on subsequent units.[3] We show that this pattern of outcomes is consistent with a competitive market structure. We are not aware of any study where identical units may complement *and* substitute within the same market and for the *same* bidder. The literature in multi-unit auctions can be divided into sequential auctions, in which the auctioneer sells the units following a series of sequential steps using a single-unit auction each time, and simultaneous auctions, in which the auctioneer uses a complex mechanism to allocate all units simultaneously. For recent contributions see Kastl (2011), who investigates bidders submitting step functions as their bids in multi-unit treasury bills auctions, and Reguant (2013), who studies complementarity bidding mechanisms used in wholesale electricity auctions. Implementing a simultaneous auction requires a strong commitment from the auctioneer either to not renege in the promised mechanism, or to use the information elicited in the process to demand a higher price for the good. This also imposes

[3] Declining or downward price trends in sequential auctions, the results we describe in section 2, have been broadly documented (*e.g.* Ashenfelter 1989; Ashenfelter and Genesove 1992; McAfee and Vincent 1993).

technical difficulties in the way bidders frame their contingent bids (Cramton, Shoham, and Steinberg 2006). Neither of these conditions are satisfied in our setting. Hortaçsu (2011) discusses recent progress in the empirical study of multi-units auctions. See Kagel and Levin (2001) for an experimental investigation when bidders demand multiple units in sealed bid and ascending auctions. In addition to recovering the structural parameters that characterize individual demand and confirming it is consistent with non-cooperative behavior, which are of interest to the literature on empirical auctions, we collect a unique panel data set to examine a market institution that was active and stable for eight centuries in a self-governed community of farmers in southern Spain.[4] Understanding this strategic non-cooperative behavior of bidders in this stable market institution is of independent interest.

2 Background and Data

The data in this article come from all water auctions in Mula, Spain, from January 1954 through August 1966, when the last auction was run.[5] On August 1^{st}, 1966, the allocation system was modified from an auction to a two-sided bargaining system. In the bargaining system, the *Heredamiento de Aguas* (water-owners holding) and *Sindicato de Regantes* (land-owners association) arranged a fixed price for every *cuarta* of water (the smallest unit auctioned). Gradually, the *Sindicato de Regantes* bought shares in the *Heredamiento de Aguas* association until they finally merged in 1974. Thereafter, water was allocated to each farmer following a fixed quota with each piece of land entitled to some proportion of the water every year.[6]

The reasons for focusing on the period from 1954 to 1966 are, first, that it represents the final period of the auction allocating system in use for at least eight centuries in this region. Second, the government conducted a special agricultural census in 1954/55, providing detailed information about the farmers who bid in this period's auctions

[4]In the lead article of the first issue of the *American Economic Review,* Coman (1911) provides an early discussion of the same institution that is analyzed in detail in this article. For an extensive study of self-governed irrigation communities see Ostrom (1992).

[5]Data available in the historical archive of Mula go back to 1803.

[6]Donna and Espin-Sanchez (2016) shows that this institutional change—from auctions to quotas—was welfare improving in Mula.

The study of these sequential auctions introduces a unique circumstance for analyzing a stark pattern of outcomes not previously documented in the literature. Sometimes, winning prices exhibit a standard competitive pattern where, regardless of whether the same or different farmers win the sequential units, prices are similar in magnitude (Panel A in Figure 1). Other times, one farmer wins all sequential units: he pays a high price for the first unit, *deterring* other farmers from entering subsequent auctions, thus paying a very low price for the remaining units (Panel B in Figure 1).[7] This stark pattern of outcomes is consistent across the whole sample (see Table 1 and Figure 3 that we describe in section 2).

Water Auctions as an Allocation System

Although the process of allocating water in Mula has varied slightly over time, the basic structure has essentially remained unchanged since the 15^{th} century. Land in Mula is divided into *regadío* (irrigated land) and *secano* (dry land). Irrigation is permitted only in the former. A channel system directs water from the river to *regadío* lands.[8] *Regadío* are fertile lands close to rivers, and thus allow a more efficient use of the water in the region. Because it is forbidden to irrigate lands categorized as *secano*, only the farmers that own a piece of *regadío* land in Mula are allowed to buy water.

The mechanism to allocate water to those farmers was a sequential outcry ascending price (or English) auction. The auctioneer sold by auction each of the units sequentially and independently of each other. The auctioneer tracked the name of the buyer of every unit and the price paid by the winner. The farmers could not store water in their plots. Reselling water was forbidden. Although a farmer could steal water by opening the gate next to his own parcel, the technology for detection of this crime was effective as irrigation was done by flood irrigation (more on this in section 2). It was easy to determine who stole water just by identifying a flooded parcel from a farmer who did not buy water in the auction for that specific day-schedule (conditional on rainfall). The *Tribunal de los Hombres Buenos* (Council of Good Men), composed by elected members among the farmer

[7] In terms of purchasing power, one peseta from 1950 is approximately equivalent to 0.43 U.S. dollars from 2013 (for details see section A in the online appendix).

[8] The channel system was expanded from the 13^{th} to 15^{th} century as a response to the greater demand for land due to population increase. The *regadío* land structure has not changed since the 15^{th} century.

community, was responsible to adjudicate conflicts between the farmers. Conflicts mostly arose over unpermitted irrigation. We investigate this behavior in Donna and Espin-Sanchez (2015).

The basic selling unit is a *cuarta* (quarter), which is the right to use water that flows through the main channel for three hours. Water storage is done in the *De La Cierva* dam. Water flows from the dam through the channels at approximately 40 liters per second. As a result, one *cuarta* carries approximately 432,000 liters of water. Traditionally, auctions were held every 21 days to complete a *tanda* (quota), the basic aggregate unit of irrigation time. During our sample period, auctions were carried out every Friday.

During each session, 40 *cuartas* were auctioned: four *cuartas* for irrigation during the day (from 7:00 AM to 7:00 PM) and four *cuartas* for irrigation during the night (from 7:00 PM to 7:00 AM), for each weekday (Monday to Friday). The auctioneer first sold the 20 *cuartas* corresponding to the night-time, and then the 20 *cuartas* corresponding to the day-time. Within each day and night group, units were sold beginning with Monday's four *cuartas*, and finishing with Friday's.

Unit of Analysis

The most comprehensive independent unit of analysis that could be considered is the weekly auctions, encompassing all 40 units sold per week. This would be the relevant definition to answer questions related to demand fluctuations generated by supply shocks, such as no auctions due to drought or excessive rain, on an aggregate level. Alternatively, the narrowest possible unit purchased is a *cuarta* (1 of the 40 weekly units). As discussed below , the presence of SC and DMR indicate that *cuartas* within a day-schedule are not conditional-independent. Moreover, they are not the relevant unit of analysis to investigate individual farmers' demand, nor the price pattern described below.

Our original question is motivated by the price behavior caused by the *deterrence* effect. This particular behavior is observed within four-units auctions and is the relevant unit of analysis in the model. This is an implication of the way the auction is structured: twelve hours of water (subdivided into four *cuartas* of three hours each) during day-time and twelve hours of water during night-time, each weekday. The logic behind this structure is related to water requirements in the area.

First, water scarcity in the region made water accountability crucial. The standard unit used to measure surface area in Mula is called *tahúlla*. One *tahúlla* is, by definition, the surface area which can be irrigated in such a way that water level rises 1-foot high in 1 minute.[9] The surface area from one-*tahúlla* varies from one town to another, depending on soil conditions.[10] A four-consecutive units auction—half day, twelve hours of irrigation—is, in that sense, the amount of water that is absorbed by a regular *parcela* (individual piece of land). Water requirements could and actually do differ (a) across farmers depending on farming trees and land extension, and (b) for the same farmer over time depending on past rainfall.

Second, the irrigation technique used in Mula is flood irrigation. The farmer builds small embankments in his *parcela* and water is delivered to the land by the channel system that simply flows over the ground through the crop. Flood irrigation requires a minimum of water delivery that, for a regular *parcela*, is captured by one *tahúlla.*

Finally, a supply-side consideration also plays a role. The reason to supply water for 12 hours (during day-time and during night-time) is to guarantee a particular and homogenous quantity for each *cuarta* (which depends on water pressure because water units are defined in hours). Given that the *De La Cierva* dam is continuously filled with water from the river, spreading the supply provision across weekdays ensures the homogeneity of water units.

Our data confirm these three points, validating the relevant unit of analysis for individual demand as four-consecutive units. The most frequent quantity purchased by farmers is twelve hours of water (42% of sold units are 4CU). There are no observations where the same farmer buys more than four consecutive units, nor observations where the same farmer buys consecutive units across days (*e.g.* there are no observations where the same farmer buys the last units of a day-auction, and the first units of the night-auction).

[9] Although close in magnitude, the traditional Murcian measure of foot is not exactly the same as the foot measure used in the U.K. and the U.S. (Valiente 2001).

[10] The surface area of 1-*tahúlla* is 1,118 square meters in Murcia and 1,185 square meters in the old *Kingdom of Aragón*, except the region of *Pías Fundaciones*. For further details see Vera Nicolás (2004).

The Dataset

We combine data from four sources. The first is auction data, that we collected from the historical archive of Mula.[11] Based on bidding behavior and water availability, auction data can be divided into three categories: (i) Regular periods, when the name of the winner, price paid, date and time of the irrigation for each auction transaction was registered; (ii) No-supply periods, when no auctions were conducted due to water shortage in the river or damage to the dam or channels, usually due to intense rain; and finally (iii) No-demand periods, when auctions were held but no one bid, leaving the registration auction sheet blank. The sample for this study includes nearly 13 years of auction data spanning January 1954 to August 1966. Every week, 40 units (corresponding to 40 *cuartas*) were sold, with the exceptions being when no auction was run (no-supply) or no bids were observed (no-demand). A total of 17,195 auctions were run during the period under analysis.[12]

We link auction data to the data that we collected from the 1954/55 agricultural census from Spain, which provides information on individual characteristics of farmers' land.[13] The census was conducted by the Spanish government to enumerate all cultivated soil, production crops, and agricultural assets available in the country. Individual characteristics for the farmers' land (potential bidders which we link with the names in the auctions data) include the type of land and location, area, number of trees, production, and the price at which this production was sold in the census year. During the 13-year period under analysis, there were approximately 500 different bidders in our sample. The number of bidders who won auctions during a specific year was considerably lower—the mean for our sample is around 8 (see table 3, discussed in section 4)—and conditional on participation, each farmer won on average 22 units per year. This is consistent with the census data, where mean land extension is 5.5 *ha* with an average of 33 trees per *ha*. Average annual rainfall during the period is 320 mm. Recent irrigation studies on young citrus plantings have shown a water use of 2-5 megalitres per hectare annually (Chott and Bradley 1997). Water savings are possible if irrigation can be allocated to similar units of production, such as young trees or reworked sections of a

[11] From the section *Heredamiento de Aguas*, boxes No.: HA 167, HA 168, HA 169, and HA 170.

[12] Table A1 in the online appendix displays the frequency distribution of units in the auctions disaggregated by the units bought sequentially by the same farmer.

[13] From the section *Heredamiento de Aguas* in the historical archive of Mula, box No. 1,210.

property. In arid regions, like Murcia, water requirements are around 20% less and they are lower for mature trees. Some farmers that are part of water-owner holding use their own water instead of selling it through auctions. Although water stress during droughts affects the quality of production, trees would hardly die as a result. During a normal year without drought, trees could survive the whole year from rainfall alone. For further details see, *e.g.*, Chott and Bradley (1997), Wright (2000), and du Preez (2001). Finally, note that although the average number of trees per farmer is 161 (see Table A2 in subsection A in the online appendix), the average number of trees per hectare in our sample is 33, a lower number compared to the conventional agricultural standard spacing for citrus trees that is 100 trees per hectare.

We also link auction data to daily rainfall data for Mula and monthly price indices for Spain, which we obtain from the *Agencia Estatal de Metereología*, AEMET (the National Meteorological Agency), and the *Instituto Nacional de Estadística de España*, INE (the National Statistics Institute of Spain), respectively.

Preliminary Analysis

Mediterranean climate rainfall occurs mainly in spring and autumn. Peak water requirements for the products cultivated in the region are reached in spring and summer, between April and August. Soaring demand is reflected by the frequency of *auctions where the same farmer buys all four consecutive units* (4CU), which reaches its peak during these months (see Figure 9 and the discussion in section 5). The frequency of 4CU is not homogenous over time, but is related to seasonal rainfall, as can be seen in Figure 2. Overall, 42% of the units were sold in 4CU.[14] There are no observations where the same farmer buys more than four consecutive units, nor observations where the same farmer buys consecutive units across days (*e.g.* there are no observations where the same farmer buys the last units of a day-auction, and the first units of the night-auction).

We observe only the transaction price (winning bid) and the identity of the winner (name). (We do not observe all bids.) There is substantial price variation, both within and across

[14]Table A1 in section A in the online appendix displays the frequency distribution of units sold by number of units bought by the same farmer.

four-units auctions. Winning prices range from 0.05 *pesetas* (*ptas*) to 4,830 *ptas*, with a mean of 271.6 *ptas*. As expected, winning prices and the frequency distribution of 4CU are strongly correlated with past rainfall (Figure 2). Table 1 exhibits the distribution of winning prices by both the number of consecutive units bought by the same individual (1CU, 2CU, 3CU, or 4CU) and by sequential auction (first, second, third, or fourth). The greater variation that we observe for 4CU (with respect to non-4CU) has a well defined pattern. Although mean prices for the first auction in 4CU are considerably higher than for non-4CU (Table 1, Panel 2: 677.6 *ptas* for 4CU against 211.1 *ptas* for 1CU, 305 *ptas* for 2CU, or 410 *ptas* for 3CU), mean prices for fourth auctions in 4CU are the smallest (Table 1, Panel 5: 210.1 *ptas* for 4CU against 233.4 *ptas* for 1CU, 239.6 *ptas* for 2CU, or 311.6 *ptas* for 3CU). Median and maximum prices display similar patterns.

Figure 3 presents price variation by number of consecutive auctions won by the same individual (left panel) and by sequential auction (right panel). The figure shows that the stark pattern of outcomes from Panels A and B in Figure 1 is consistent across the whole sample. On the one hand, in the top panel of 3 we can see that price dispersion—as well as the mean and median price—is higher when the same farmer wins all four consecutive units (in the top panel, last vertical box labeled 4). On the other hand, in the bottom panel, where we further disaggregate each box from the top panel by unit (first unit, second unit, third unit, and fourth unit), we can see that the higher price dispersion for unit 4 in the top panel—as well as the higher mean and median—is generated by the greater variation in prices for first units in the lower panel (not by prices in second, third, and fourth units).

This particular pattern in prices is caused by the above mentioned *deterrence* effect whereby farmers exhibit different behavior based on seasonality and rainfall, *i.e.,* residual demand for water. During high demand and low rainfall months, the same farmer buys all four sequential units, paying a high price for the first unit (with respect to the median or average price conditional on rain) and very low prices for the remaining units. During months when demand is not high due to farming seasonalities or when rainfall is high, winning prices for all units are similar in magnitude, regardless of whether the same farmer wins all sequential units (4CU) or different farmers win subsequent units (1CU, 2CU or 3CU).

Aggregate prices over time display consistent trends with the ones found in the empirical literature on sequential auctions. Figure 4 shows that, on average, per unit prices decline by sequential unit (being the first unit of each day higher than second to fourth units), and by day of the week (prices decline from Monday to Friday). Figure 4 also shows that per unit prices are slightly higher during the day than during the night. High water requirements for citrus during summer causes prices to soar during those months (Panel A in Figure 5). As expected, prices are also higher during droughts, after conditioning on seasons (Figure 5).[15]

Table 2 shows that these correlations are robust after conditioning on past rain, unit, weekday, schedule, week-of-the-year, month, and individual fixed effects. The table displays the results obtained by regressing daily unit prices on a seven-day-rain moving average (*Rain MA7*), the rain on auction day, and the mentioned fixed effects. The estimated coefficients on *Rain MA7* have the expected sign and are statistically significant at the 1% level. From column 1, a 10 millimeter (*mm*) increase in average rain in the previous week is associated with a decrease of 40.5 *ptas* in the equilibrium price paid in the auction. The regression in column 2 adds unit, weekday, and schedule fixed effects. The estimated coefficient on *Rain MA7* increases in magnitude and also has the expected sign. This regression also shows that, as noticed in previous figures, price declines within day and across units (both for day-time and night-time auctions) and across schedules (price is on average 110 *ptas* lower for night-time auctions than for day-time auctions). The estimated coefficients show that equilibrium prices decline monotonically within the week (Figure 3). Columns 3 and 4 add, respectively, month seasonal dummies and individual fixed effects (we have 537 different individuals in our sample) to the specification in column 2. The estimated coefficient on *Rain MA7* in column 3, though smaller, again has the expected sign and is statistically different from zero. Similar qualitative results are obtained in column 4; however, the estimated coefficient on *Rain MA7* has increased. Note that the goodness of the fit in the last regression is 36%, indicating that average (or *ex-ante*) prices are explained relatively well by observables such as rain in the previous week and time of the allocation. This evidence supports the idea the observable (common knowledge) components of prices in drives four-units auctions. Although not reported, we performed an analogue analysis using average daily prices within schedule as a robustness exercise and obtained similar results.

[15]See the online appendix for a discussion on droughts.

3 The Model

As noted above, bidding behavior is a result of a complex decision process. There are three main features from the empirical setting that need to be accounted by the model: (i) sunk costs that farmers incur when they buy their first unit, (ii) decreasing marginal returns of subsequent units of water, and (iii) participation costs of farmers in this market.

Sunk Costs (SC). Water is allocated during the auction and is distributed on the specific day and time of the irrigation accordingly. Water stored in the dam is delivered to the farmer's plot on this date using the channel system. Except the main canal, all channels are dug into the ground (Figure 6). On the day of the irrigation, a guard opens the corresponding gates to allow the water to flow to the appropriate farmer's land. These channels are land-specific in the sense that different areas and lands have their own system of channels which carry water only when the corresponding gates are opened. A concern is that farmers whose lands lie next to each other may be buying different sequential units for the same auction. In this case, the SC would be incurred only by the first farmer for his first unit but not for the second farmer for his first unit. We use data on the specific location of the farmers that we match to auction winners to analyze these situations in section 5. There is a water loss that is incurred because water flows over a dry channel. Engineers have estimated this loss to be between 15% and 40% (20% on average) of the water carried by one *cuarta* when the channel is completely dry (see Gómez-Espín, Gil-Meseguer, and García-Marín 2006). This is the SC incurred by the bidder for his first unit. The SC is incurred only once, for the first unit, because water losses are associated with a wet channel are negligible. The channel dries out after approximately 12 hours without water (González-Castaño and Llamas-Ruiz 1991). In 1974 the system of sub-canals was made of concrete, instead of just dug in the ground, to prevent such losses (González-Castaño and Llamas-Ruiz 1991).

Decreasing Marginal Returns (DMR). The second feature refers to the decreasing marginal returns (DMR) effect. The classic textbook case for DMR is appropriate for our empirical application. Given that the amount of land owned by each farmer is fixed, marginal productivity of subsequent units of water is decreasing. When assessing the relative importance of DMR, the impact in summer would generally be greater than in autumn. More generally, one would expect DMR to be affected by season and rain. When water requirements are high, the slope of the marginal productivity function will be relatively flat; this is likely to occur in spring and summer. On the other hand, when water requirements are low, the slope of the marginal productivity function will be steeper; this is likely to happen in autumn or winter.

Participation Costs. There are several reasons why farmers face participation costs in this market. The main component of participation costs correspond to the hassle costs associated with active bidding. Only a fraction of the individuals who attended a Friday auction were actively engaged in the bidding for a particular sequential auction of water and not everyone who was present participated in every auction (Botía, Francisco, personal interview, Murcia, June 17, 2013).[16] As von der Fehr (1994) points out, a reasonable assumption for why only a portion of attendees participate may be that they consider it so unlikely to that they will win at a price below what they will be willing to pay, that they are not willing to bother to engage in bidding. We expect this type of costs to be very small but positive.[17]

Empirical evidence from our data is consistent with the assertion that farmers dislike participation, facing positive entry costs as they do. We observe multiple weeks per year when auctions were run, farmers showed up and bought the first units of water, but no one bid for the last units. As there was no reservation price and the minimum bid increment was cents, they could have potentially won all the remaining units bidding one cent. To the contrary, they decided not to bid and instead left the auction. For example, on January 22, 1954, units 1 to 16 were sold to seven different farmers but no one bid for units 17 to 20 (Figure 7). In 1954 we observe similar behavior for 14 weeks,[18] and this is consistent along the remaining years in our sample. To infer participation costs, we use $2,423$ auctions where some bidders where present and no one bid for the last units, *i.e.,* auctions similar to the one in Figure 7. Our interpretation is that the utility for all bidders is smaller than the

[16]A summary is available online in the online appendix at http://www.jdonna.org/water-auctions-web.

[17]Note that the results would be the same if participation costs were zero. However, the equilibrium when goods are complements would not be unique (see section 2).

[18]Weeks of January 22, February 5, April 5, May 1, May 8, May 15, May 22, May 29, June 5, June 12, July 3, July 10, November 26, and December 3.

participation cost, conditional on covariates. We use this information to partially identify participation costs (see page 30).

Set Up

We use the three main specific features from the empirical setting to build our model. A SC is incurred only for the first unit bought whereas DMR are present for second to fourth units. The relative importance of the SC and DMR generate a trade-off, whereby bidders play one strategy of another based on whether different units are complements or substitutes. A simple way to show this intuition is by assuming that the initial SC is proportional to the value of water, and DMR are linear in the number of units bought. We parametrize the SC effect by $(1-\rho_1)\, v_i$. The interpretation of SC is the percentage of water loss from the first unit bought because water is flowing through a dry channel (hence, the SC is proportional to the valuation of the bidder for the unit of water). When the SC is zero, $\rho_1 = 1$. In this case the bidder obtains the complete first unit (*i.e.* 100 percent of the first unit). When the SC is positive, $\rho_1 < 1$, a percentage of the water is lost and the bidder obtains ρ_1 of the first unit (*i.e.* $\rho_1 \times 100$ percent of the first unit). One would expect that, conditional on rain, water loss would be constant within season and relatively higher (lower ρ_1) in summer.[19] We parametrize the DMR of unit k by ρ_k for $k = 1, 2, \ldots, K$. Let ρ be the vector of parameters that characterizes marginal utilities, *i.e.* $\rho \equiv (\rho_1, \rho_2, \ldots, \rho_K)$. Then, the marginal utility for bidder i for each unit k is:

$$MU_{ki} = \rho_k \cdot v_i,$$

where v_i, known only by bidder i, is a scalar that captures the valuation that the bidder assigns to their first (complete) unit of water, *i.e.,* when $\rho_1 = 0$ we have $MU_{1i} = v_i$.

We consider v_i to be independent and identically distributed on the interval $\mathbb{R}_+$, according to the cumulative distribution function $F(v_i)$, for all bidders $i = 1, \ldots, N$. We assume that $F(v_i)$ admits a continuous density $f(v_i) > 0$ and has full support. It is assumed that $E[v_i] < \infty$. Let μ be the parameter that fully characterizes the distribution $F(v_i)$. The

[19] We discuss variation of SC across auctions (conditional on covariates) in page 27 in the article.

private valuation, v_i, is known only by bidder i, and it is learned before entering the first auction. We do not consider the case where farmers might have different valuations for different units of water. The reason for this is that the units are identical and we condition on observables that may affect the price of water in the econometric specification (see section 4). We obtained similar results by allowing the valuation for subsequent units to be different draws from the same distribution. The exposition of the model, however, becomes more cumbersome.

Farmers are price takers in both the input and the output markets. Farmers compete in the output market and sell their production in the international market (*e.g.* international markets for apricot, oranges, lemon, *etc.*). The market share of the output produced by Mula's farmers is negligible in the international market, thus the prices of the output in the international market are not affected by Mula's production. The farmers who compete in the auction are only those with irrigable land, and they make a small fraction of all farmers in Mula. They are price takers in the market for inputs, such as labor and manure. Thus, there are no common components to the valuation of the bidders, conditional on the observables (*e.g.* rainfall, day of the week, time of the day, *etc.*).

The seller wants to allocate K identical units. These units are auctioned off sequentially by the seller using an English ascending price auction for every unit. All participating bidders observe the total number of individuals who participate in the auction, N. After every auction, each participant observes both the price paid by the winner and the winner's identity. The seller continues to run subsequent auctions sequentially until all the units are allocated. The primitives of the model, (K, N, μ, ρ), are common knowledge. We assume that all bidders share the same utility function, $U(\cdot)$. We think this is reasonable in the present setting for the following reasons. First, all plots are located within a small area (less than 4 km^2) and have the same land quality and weather. Second, although farmers may harvest different crops, they all have similar water needs per tree across the year. Idiosyncratic differences in water needs in a specific week are captured in our econometric specification by the farmer's type, v_i. Finally, the system of canals is such that all the plots are located within a few hundred meters from the mail canal. Sub-canals coming from the main canal have all about the same length and are dug into the ground.

The strategy set for every bidder is the vector $\sigma \equiv \left(y_i^k, b_i^k\right)_{i=1,\ldots,N}^{k=1,\ldots,K}$, where $y_i^k \in \{0,1\}$, $y_i^k = 1$ indicates that bidder i participates in the auction for unit k ($y_i^k = 0$ if bidder i does not participate in the auction for unit k), and b_i^k is the maximum amount that bidder i is willing to pay for unit k. Bidders play sequentially, or stage by stage. This means that they choose $\sigma_i^k = \left(y_i^k, b_i^k\right)$ after learning the outcome of the previous $(k-1)$ auctions. Bidders participating in auction k observe the price at which each bidder is no longer active (bids are observable) except for the winning bid. The information transmission is consistent with the auction being an English (or ascending price) auction rather than a second price auction. We model the game as in a button auction. Each bidder holds a button while the price continuously rises. A bid for bidder i is the value at which bidder i stops holding the button. When there are only two bidders active (holding the button) and one of them releases the button, the auction ends. The active bidder wins the object and pays the price at which the runner up stopped.[20]

The seller allocates the unit to the highest bidder: $x_j^k \in \{0,1\}$ and $x_j^k = 1$ when $j = argmax_i\left(b_i^k\right)$ (and 0 otherwise), at a price equal to the second highest bid: $p^k = b_l^k$, where $l = argmax_{i \neq j}\left(b_i^k\right)$. Let $X_i^k \equiv \sum_{j=1}^{j=k} x_i^j$.be the number of units that bidder i has won before participating in auction k. If only one bidder participates in a specific auction this bidder obtains the object for free. Each object is either allocated to one of the N bidders, or it is lost if none of the bidders decide to participate in the auction.

Participation decisions in each auction are done simultaneously by all bidders. To take part in every auction each bidder incurs a participation cost, $c > 0$, at the beginning of the period. If only one bidder participates, this bidder obtains the object for free but it bears the participation cost, c, nonetheless. The process is then repeated in every period.[21] As discussed in previous subsection, the assumption of positive entry costs is consistent with the data in our empirical setting, where we observe no demand for some of the units, even though the reservation price is zero. The interpretation is that, in those situations where no-demand is observed, the value that bidders assign to that unit is smaller than the participation cost,

[20] See Cassady (1967) and Milgrom and Weber (1982) for details.

[21] Bidders enter the auction if, and only if, the expected utility they obtain from the game is positive. See von der Fehr (1994) for a discussion of entry when the goods are complements or the conditions needed for entry when the entry cost in the first auction is positive.

c.[22]

The *ex-post* utility for a bidder who buys l units and participates in m auctions is:

$$U_i\left(l, m, v_i; \rho, c\right) = \sum_{k=1}^{l} \rho_k \cdot v_i - \sum_{k=1}^{K} y_i^k \cdot c = \sum_{k=1}^{l} \rho_k \cdot v_i - m \cdot c.$$

In the remainder of the article we refer to $v_{N:N}$ as the highest realization of the random variables $v_1, \ldots, v_n$ drawn independently from CDFs $F_1, \ldots, F_N$ (one draw from each distribution), and $v_{N-1:N}$, as the second highest realization. More generally, $v_{j:N}$ is the j^{th} order statistic for a sample of size N from the distribution $F\left(v_i\right)$.

We follow von der Fehr (1994) and assume that the entry cost is small enough so that the expected payoff of entering is greater than zero for all bidders. This assumption has no impact on the bidding behavior of subsequent auctions because no information is revealed before entering the first auction, but it greatly simplifies the notation and the analysis. We summarize this in the next assumption $A0$.

Assumption 0 [A0]: $\mathbb{E}\left[U_i\left(l, m, v_i; \rho, c\right) \geq 0\right]$. If $A0$ is not satisfied, then some bidders may not enter the first auction. In the article we consider only *symmetric equilibria*. Thus, in the first stage all bidders play the same entry strategy.[23] This strategy is a threshold strategy, *i.e.*, each bidder enters the game if, and only if, its own valuation, v_i, is greater than some common threshold, v^*. This threshold value depends on the expected value of participating in the game. In particular, v^* is the valuation that would make a player indifferent between entering the game or not, because its expected utility of participating would be zero. The expected utility would be a function of the parameters of the game and the number of players.

It is straightforward to see that, when considering *symmetric equilibria*, the expected utility of entering the game after the first auction (second, third, or fourth auctions) is always lower than that of entering the game in the first auction. In other words, individuals whose valuation was below v^* and did not enter the first auction, never find it optimal to enter subsequent stages. The previous result implies that we observe only bids from individuals

[22]We later use this information to partially identify participation costs (see page 30). See above in this section for justification of this assumption in our specific empirical setting.

[23]By restricting attention to symmetric equilibria we rule out equilibria where some bidders do not enter the first auction to learn about the types of its opponents, and then decide whether to enter the remaining auctions.

that enter the first stage. In the data, we do not observe whether individuals bid in previous auctions, we observe only winning bids. Hence, we cannot recover the unconditional underlying distribution of bids, $F(v_i)$, but rather the underlying distribution of bids *conditional* on entering the first auction, *i.e.*, $F^*(v_i) = F(v_i|v_i > v^*)$. Note that when c goes to zero, v^* goes to zero and the two distributions coincide. Given that our estimates for c are very small (less than 14 cents of a *peseta*), the difference between both distributions is small. For a discussion on selection due to entry costs see, *e.g.*, Li and Zheng (2009, 2012) and Marmer, Shneyerov, and Xu (2013).

Four-Units Auctions.

Our goal in this theoretical section is to characterize only the results that summarize the equilibrium price behavior used for the structural estimation, rather than a full characterization of the equilibrium of the model.[24] The next two assumptions allow us to determine the regime under which the game is being played.

Assumption 1 [A1]: $\rho_1 \leq \rho_4$.

Assumption 2 [A2]: $\rho_1 + \rho_2 \leq \rho_3 + \rho_4$.

When $K = 4$, we call it a *strict complements regime* when A1 and A2 holds. We call a *weak substitutes regime* when neither A1 nor A2 holds. The following results summarize equilibrium winning price behavior as a function of the model's primitives (valuations, SC, DMR, and participation costs). We later use these results for the estimation. We consider only pure strategy symmetric Perfect Bayesian Equilibrium (PBE).[25] All proofs and extensions are in section B in the online appendix.

[24]We provide a full characterization of the game in the case where good are substitutes and there are two units to be sold in Appendix B. As shown there, the equilibrium strategies when goods are substitutes may involve pooling at several intervals. Using such information for the structural estimation would require imposing additional assumptions on the behavior of the farmers (*e.g.* how farmers form their beliefs in such cases). However, we can still estimate the parameters of interest without imposing these additional restrictions nor solving the complete game.

[25]When $K = 2$, cases where $\rho_2 \leq 0$ and $\rho_1 = \rho_2$ are equivalent to von der Fehr (1994), in subsections 3.2 and 3.4, respectively. Uniqueness, however, is not proved by von der Fehr in any of those cases.

Proposition 1. *In a* strict complements regime *(i.e., when A1 and A2 hold) the pure strategy symmetric PBE is:*

- *First auction:*
  - *Participation: bidder i will always participate in the first auction,* i.e. $y_i^1 = 1$.
  - *Bidding Strategy:* $b_i^1(v_i) = \sum_{k=1}^{4} \rho_k \cdot v_i - 4c$.
- *Second, third, and fourth auctions:*
  - *Participation: bidder i participates in each auction if, and only if, she won the first auction,* i.e. $y_i^k = 1$ *if, and only if,* $x_i^1 = 1$.
  - *Bidding Strategy: If bidder i participates in each auction ($y_i^k = 1$ for $k = 2, 3, 4$), she will continue bidding until the price reaches its own valuation for all remaining units,* $b_i^l(v_i) = \sum_{k=l}^{4} \rho_k \cdot v_i - (4 - l)\, c$.

Corollary 1. *In a* strict complements *regime (i.e., when A1 and A2 hold) the* ex-post *utility of the winner satisfies:*

$$\sum_{k=1}^{4} p^k = \sum_{k=1}^{4} \rho_k \cdot v_{N-1:N} - 4c. \tag{1}$$

Lemma 1. *In a* weak substitutes *regime (i.e., when neither A1 nor A2 holds) the probability that a bidder different from the winner enters the last auction is decreasing in the participation cost, c. Moreover, this probability goes to* 1 *when c goes to zero,* i.e.*:*

$$\underset{c \to 0}{Lim} \left\{ Pr\left(y_i^K = 1 \mid x_j^1 = 1, i \neq j \right) \right\} = 1.$$

Corollary 2. *In a* weak substitutes *regime (i.e., when neither A1 nor A2 holds) the* ex-post *marginal utility of the winner in the last auction, depending on how many units the winner won, satisfies:*

If the winner won all four units:

$$\rho_4 \cdot v_{N:N} \geq p^4 = \rho_1 \cdot v_{N-1:N} - c. \tag{2}$$

If the winner won three units, two out of the first three, and the last one:

$$\rho_3 \cdot v_{N:N} \geq p^4 = \rho_2 \cdot v_{N-1:N} - c. \tag{3}$$

In equation 2 the winner wins all the units, so its marginal utility is $\rho_4 \cdot v_{N:N}$. Because the winner won all units, the runner up did not win any unit and its marginal utility is $\rho_1 \cdot v_{N-1:N}$. In equation 3 the winner wins three units, so its marginal utility is $\rho_3 \cdot v_{N:N}$. The runner-up won one of the previous units and its marginal utility is: $\rho_2 \cdot v_{N-1:N}$.

4 Estimation

Econometric Specification

We estimate the model via maximum likelihood using an exponential distribution for the individual valuations. In this subsection we describe how the likelihood is formed and how we account for rain expectations and auction heterogeneity.

Regime Determination. The predicted price pattern by our model for each each regime (strict complements and weak substitutes) provides us with a straightforward method to determine the regime being played. When goods are strict complements, very low prices—or, according to the auctioneer who ran the auctions, symbolic prices that are "close" to zero (Botía, interview, available in the online appendix)—are paid, by the winner of the first unit, for the second, third, and fourth units (Panel B in Figure 1). Thus, the difference between the price paid for the first and the remaining units is large. When goods are substitutes, the units might be bought by different bidders and the prices of all four units are similar (Panel A in Figure 1). Thus, the difference between the price paid for the first and the remaining units is negligible. This allows to separate the data into four categories by looking at the identities of the winner (*i.e.* whether the same bidder bought all the four units), and the difference between the price paid for the first and the remaining units:[26] In principle one could estimate a simultaneous equation switching regression model along the lines of Porter

[26] See section C in the online appendix for details.

(1983) and Ellison (1994). In such model, the parameters that characterize demand would be estimated as in this section conditional on the regime classification; the regime classification would be unknown, and the parameters governing the distribution of the regime classification would be estimated by, *e.g.*, an adaptation of the E-M algorithm (Kiefer 1980). However, in our case we can determine the regime classification directly by examining the data, from the observed prices. The first-order distinguishing feature of the regime are the predicted price patterns by the theoretical model. Panels A and B in Figure 1 show one instance of how these patterns look throughout the data. This is the most straightforward and, thus, our preferred approach.

a) Same bidder wins all four units and goods are in a strict complements regime (*i.e.* when A1 and A2 hold),

b) Same bidder wins all four units and goods are in a weak substitutes regime (*i.e.* when neither A1 nor A2 holds),

c) Last winner also bought two out of the first three units, three units in total, and goods are in a weak substitutes regime (*i.e.*, when neither A1 nor A2 holds),

d) Otherwise.

Categories a, b, and c define three separate pricing strategies. In region a, winning prices are determined by equation 1. In region b, winning prices are determined by equation 2. In region c, winning prices are determined by equation 3. Let D^a be an indicator variable that equals 1 if the winning price is in region a, and 0 otherwise. Define analogously D^b, D^c for regions b, and c, respectively. See section 4 for a discussion about the regions of the likelihood and the covariates. Note that we do not use the data in region d. In order to use the data in region d we would need to make additional assumptions: (i) regarding the parameter space, (ii) regarding the distribution of types, and (iii) to fully solve the model when neither A1 or A2 holds. This is beyond the scope of this article. As we discussed above, we are able to recover the parameters of interest using only the data in regions a, b, and c.

Identification. For the case of an English auction, the conditional distribution of private valuations is non-parametrically identified when the transaction price and the number of bidders are observable (Athey and Haile 2002). This result is immediately useful in our sequential English auction model where bids are conditional-independent draws from a distribution $F_V(.)$ and the equilibrium (observed) transaction price is a function of the second highest valuation, $v_{N-1:N}$. Consider the strict complements regime. Winning prices are determined by equation 1. The distribution of valuations is identified up to the multiplying constant, $\sum_{k=1}^{4} \rho_k$, using equation 1 and the result from Athey and Haile (2002). Identification of the remaining parameters, ρ_k, would require four additional independent restrictions (in addition to equation 1). Two additional restrictions are provided by the model from corollary 2 (equations 2 and 3). But we observe only winning bids in the data. Then, two of the four ρ_k, $k = 1, \ldots, 4$, are not identified without further structure. We use a specification with linear DMR due to the mentioned data limitation. Linear DMR impose two additional restrictions. First, we define $\rho_1 = 1 - \alpha$ and $\rho_2 = 1 - \beta$. (Note that these are not restrictions on the parameter space.) We then restrict the parameter space by assuming that $\rho_3 = 1 - 2\beta$ and $\rho_4 = 1 - 3\beta$ (*i.e.* linear DMR). Hence, we have three independent restrictions (equations 1, 2, and 3) and three parameters to estimate (μ, α, β), where μ is a parameter that fully characterizes the distribution of valuations.[27] With observability of all bids (not just the winning bids as in our empirical setting), ρ_3 and ρ_4 would be identified and we would not need to impose the linearity assumption on marginal returns.[28]

Equation 2 establishes the relation between ρ_1 and μ (α and μ). Equation 3 establishes the relation between ρ_2 and μ (β and μ). Finally, equation 1 establishes the relation between ρ_1, ρ_2, ρ_3, ρ_4 and μ (α, β and μ). If DMR are not linear, then equations 2 and 3 would still hold. That is, $\rho_1 = 1 - \alpha$ measures the value of first unit bought, net of the sunk cost and $\rho_2 = 1 - \beta$ measures the value of the second unit bought, including a decline in the value due to DMR. However, equation 1 would not hold. In particular, the term multiplying $v_{N-1:N}$ would no longer be equal to $\sum_{k=1}^{4} \rho_k = \left[4 - \alpha - 6\beta_t^C\right]$, but would still be a function of α and β_t^C. That means that the estimation method proposed here could be applied to other specific (non-linear) functional forms of DMR just by changing equation 1. In general, we can characterize the DMR function as $\rho_k = k(\beta)$, while fixing $\rho_1 = 1 - \alpha$ and $\rho_2 = 1 - \beta$ without loss of generality. In the linear case we let $\rho_k = k(\beta) = 1 - (k-1)\beta$, but the

[27] Note, however, that the distribution of private valuations is non-parametrically identified from the result from Athey and Haile 2002.

[28] Note that assumptions A1 and A2 are equivalent to assume that $\alpha \geq 4\beta$.

estimation method could incorporate other non-linear functional forms, which would change only equation 1.

Specification of Decreasing Marginal Returns (DMR). For our estimation we allow DMR, β_t, to vary across auctions holding fixed SC, α (more about this below). We allow β_t to vary with farmers' expectations of rain in each auction $t = 1, \ldots, T$. We proxy these expectations by actual (*i.e.* observed) future rain, so $\beta_t = \beta_0 + \beta_1 R_t^F$, where R_t^F is a dummy variable (defined next) that is linked to expectations about future rain in t, and β_0 and β_1 are parameters. $R_t^F = 1$ if farmers expect that rain is going to be positive (for the day for which they are buying water) and zero otherwise. We further let β_t have different intercepts in each regime:

$$\begin{aligned} \beta_t^S &= \beta_0^S + \beta_1^S R_t^F \\ \beta_t^C &= \beta_0^C + \beta_1^C R_t^F \end{aligned} . \tag{4}$$

Table 4 provides an heuristic argument to understand the reasons behind this equation. The table presents probit regressions of a dummy variable identifying the regime (strict complements *vs.* weak substitutes) on future rain and other covariates. We interpret future rain in these regressions as a proxy for aggregate expected future rain for the farmers. Table 4 shows that low expected rain and high demand months (May to August) significantly increase the likelihood of being in a strict complements regime. The interpretation is that farmers have some information (expectations) about future rain. Although the idiosyncratic component of this information is captured by their type, v_i, the common component is captured by β_t. When farmers expect, on aggregate, no rain in a given day, they will play the strategies corresponding to the strict complements regime. Seasonality also affects the demand for water and affects the position of a farmer in the production curve. The results in Table 4 show that it is the slope on the marginal return effect that drives the change of regime, holding fixed SC.

In our parametrization we fix α across auctions and season but we allow β_t to vary. We expect α to vary across auctions and seasons as well. But this variation is not separately identified from the variation on β_t because it is the relative magnitude of the effects that matters. The rationale for why we let β_t vary (instead of α) is that a regime switch is driven by the (residual) demand for water by the farmers, as determined by rain and seasonal effects. In other words, we expect the changes in β_t to be more important in magnitude than the changes in α. Therefore, the estimated changes in β_t should be interpreted relative to changes with respect to α.[29]

The Likelihood. The econometric problem consists of finding the parameter that characterizes the common distribution of valuations F and the structural parameters that best rationalize the bidding data. As discussed in the previous section, the bid levels at which bidders drop out of the auctions are not observed, except the bidder with the second-highest valuation. We estimate the model *via* maximum likelihood assuming that farmers draw independent and private valuations from an exponential distribution at each four-units auction, conditional on observed auction-specific covariates. (We discuss the assumptions below in section 4.)

Our model and the context of the market under analysis provide insight on how the characteristics of farmers and auctions should affect private values, but it offers little guidance on the functional form of this distribution. We assume that farmers' valuations, v_i, follow an an exponential distribution for each four-units auction.[30] In section 5 we report the results from a Kolmogorov-Smirnov test where the null hypothesis that the distribution of private valuations are draws from an exponential distribution cannot be rejected.

Let $v_i \sim F(v;\mu)$, where $F(v;\mu) = (1-e^{-\mu v})\,\mathbf{1}\{v \geq 0\}$ is the CDF of an exponential distribution that is characterized by the scalar $\mu > 0$. Equations 1, 2, 3, and 4 jointly identify the parameter vector $\left(\mu, \alpha, \beta_0^C, \beta_0^S, \beta_1^C, \beta_1^S\right)$, conditional on the regime and exogenous covariates, R_t^F. Note that the third equation in the system is, actually, $p_b^4 = Max\left\{(1-\alpha)v_{N-1:N}, (1-\beta)\,v_{N-2:N}\right\}$, because we do not know whether the runner-up in the last auction was the bidder who already won one unit or a bidder without previous purchases. However, when N is large, $(1-\alpha)v_{N-1:N} < (1-\beta)\,v_{N-2:N}$ if $\beta \simeq \alpha$. But, in the case that $\beta \simeq \alpha$, the same bidder will not win three out of four units. That is, in an auction where

[29] We obtained similar results to the ones on Tables 5 and 6 fixing β and allowing α_t to vary in each auction $t = 1, \ldots, T$.

[30] In our earlier working paper Donna and Espin-Sanchez (2012) we used an Exponentiated Gamma (EG) distribution, which gives a closed-form solution for the PDF of the j^{th} order statistic.

N is large and the same bidder wins three out of four units, we expect β to be significantly greater than α. Therefore, the equation can be simplified to $p_b^4 = (1-\alpha)v_{N-1:N}$. The full system of equations is given by:

$$\begin{aligned}
\sum_{k=1}^{4} p_a^k &= \left[4-\alpha-6\beta_t^C\right] v_{N-1:N} - 4c \\
p_b^4 &= (1-\alpha)v_{N-1:N} - c \\
p_c^4 &= (1-\beta_t^S)v_{N-1:N} - c \\
\beta_t^S &= \beta_0^S + \beta_1^S R_t^F \\
\beta_t^C &= \beta_0^C + \beta_1^C R_t^F.
\end{aligned} \tag{5}$$

Let $\theta \equiv (\alpha, \beta_0^C, \beta_0^S, \beta_1^C, \beta_1^S)$ and let v_i be a conditional-independent draw from $F(v_i; \mu|\theta, R_t^F, D_t^j)$. Then, the likelihood function is given by:

$$\begin{aligned}
L(v_i; \mu \mid \theta, R_t^F, D_t^j) = \prod_{t=1}^{T} f_{N-1:N}\left(\frac{\sum_{k=1}^4 p_t^k + 4c}{4-\alpha-6(\beta_0^C+\beta_1^C R_t^F)}; \mu \mid \theta, R_t^F\right)^{D_t^a} \quad \times \\
f_{N-1:N}\left(\frac{p_t^4 + c}{1-\alpha}; \mu \mid \theta, R_t^F\right)^{D_t^b} \quad \times \quad f_{N-1:N}\left(\frac{p_t^4 + c}{1-\beta_0^S+\beta_S^C R_t^F}; \mu \mid \theta, R_t^F\right)^{D_t^c},
\end{aligned} \tag{6}$$

where $f_{N-1:N}(v;\mu)$ is the probability density function (PDF) of the $(N-1)^{th}$ order statistic from a sample of N from the exponential distribution of valuations F, and D_t^a, D_t^b, D_t^c are, respectively, indicator variables for cases a, b, and c, as defined above at the beginning of this subsection .

Auction Heterogeneity. We allow the mean of the distribution of valuations to depend on various characteristics that are drawn from the data. We assume that observed prices follow a linear function of the following exogenous variables and estimate all parameters using the likelihood function:[31]

[31] Laffont, Ossard, and Vuong (1995) assume that private values follow a log-normal distribution and let the mean of the logarithm of the valuations be a linear function of exogenous characteristics. Haile and Tamer (2003) condition on covariates by constructing the conditional empirical distribution functions using Gaussian kernels.

$$\mu = \mathbb{E}\left(v_t^i\right) = Z_t'\gamma = \gamma_0 + \gamma_1 R_t^P + \gamma_2 \left(R_t^P\right)^2 + \gamma_3 Night_t + \sum_{k=2}^{5} \gamma_{2+k} Day_t^k + \sum_{k=2}^{12} \gamma_{6+k} Month_t^k. \quad (7)$$

The first exogenous variable, R_t^P, refers to *Past Rain*, a moving average of the daily rain beginning seven days prior to the date of the auction; we include a quadratic term to allow for non linearities in past rain. The second variable is a dummy variable that equals one if the water was bought for night use. The next four variables are a set of dummy variables for each weekday. Finally, the last eleven variables are a complete set of monthly dummy variables to condition on seasonality. Water prices soar in this market during the dry summer and drop in winter. We accommodate these shocks to demand with seasonal monthly dummy variables. We estimate the parameter vector γ by maximum likelihood using the equation in 7 to model the mean distribution of valuations, μ. See sections E and F in the online appendix for details about the estimation procedure.

Identification and Estimation of Participation Costs. Although, throughout the previous estimation procedure, participation costs, c, have been fixed at an arbitrary small magnitude, we recover them from our data. We use our model and data where auctions were run, no bids observed and farmers were present, along with the structural estimates. Participation costs are identified by the necessary condition for a bidder to bid in the first auction that is given by:

$$(1-\alpha)v_{N:N} < c.$$

More generally, a condition that additionally involves second, third, and fourth marginal utilities for the case where the bidder also enters the individual auctions for two, three or four units should be considered. In these cases, participation costs are also greater than the average marginal utility for second, third, and fourth units. Formally:

$$Max\left\{1-\alpha, \frac{2-\alpha-\beta_t}{2}, \frac{3-\alpha-2\beta_t}{3}, \frac{4-\alpha-3\beta_t}{4}\right\} v_{N:N} < c. \quad (8)$$

Note that, when $\alpha < \beta_t$, the former condition is sufficient, implying the latter. In our econometric specification the structural parameter α is fixed whereas the parameter β_t varies according to the farmers' expectations of (exogenous) future rain. One would expect to observe auctions without bids when farmers' expectations for rain, as captured by actual future rain, are high (which in the model is represented by a relatively high β_t). Therefore, absence of bids will occur only when $\alpha < \beta_t$, thus, the former identification restriction is sufficient.

Analogously, using the model and the remaining data not used in the structural estimation, we obtain an upper bound using that participation cost are lower than the minimum registered price (conditional on covariates, sunk cost, and DMR).

Discussion

Conditional Independent Private Valuations (CIPV). For the estimation we assume that farmers have independent and private valuations at each four-units auction, conditional on observed auction-specific covariates. The first justification for CIPV is that each bidding farmer (who may or may not be a water-owner) has his own land extension, and his own mixture of trees and crops. This eliminates a strict common value scenario. In addition, in the econometric specification we account for observables that affect all farmers in a similar way such as (past and future) rainfall, schedule of the auction, day of the week, weather seasonality, etc. (see section 4 for details). Second, the products sold are units of water. Assuming that farmers have private information from other farmers about the characteristics of this product is not in line with the homogeneous nature of water units. Finally, the conditional-independence assumption is the most credible in our context, given the varying nature of farming products and soil conditions across farmers. To understand why, recall that sellers in the water market are a holding formed by the water owners and buyers are farmers that own fertile land. Around 500 different farmers are observed to win auctions in our sample. Not all of these farmers show up at every auction or decide to participate if they are present. Farming products cultivated in the area are mainly fruit and citrus trees (lemon, orange, peach, mandarin, and apricot), and vegetables (tomato, lettuce, and onion).

The amount of water required by the trees depends on the time of the year and type of crop (citrus trees should not be irrigated daily). Moreover, and given that we condition on seasonality, water requirements vary across products. For example, water needs for grapefruit and lemons are about 20% higher than those for oranges, whereas water requirements for mandarins are about 10% less. Ground conditions (which also vary across areas where different farmers have their land) also affect water necessity.[32] The variations across farmers generated by these factors provide support for the fact that the conditional-independence assumption seems satisfied, given that each day the market is quite specific and because we work with data for four-consecutive auctions as a unit of analysis (sequential auctions). Our justification of the CIPV paradigm is in line with the literature on empirical auctions. For first price descending auctions see, *e.g.*, Laffont, Ossard, and Vuong (1995) in an application to agricultural products (greenhouse eggplants in Marmande, France) where the number of bidders vary between 11 and 18. For English auctions, Haile and Tamer (2003) apply their limited structure model to U.S. Forest Service timber auctions, where the number of bidders vary from 2 to 12.

Auction Heterogeneity. Observed heterogeneity across auctions arises due to seasonal effects, rain, and the day and time of the week when the auction occurs. This means that the distribution of private values for the t^{th} auction, $F_t(v_i)$ is not constant across auctions. In our estimation, we recover the family of distributions $F(v_i|Z_t, \gamma)$. That is, we assume for every four-units auction that $F_t(v_i) = F(v_i|Z_t, \gamma)$, where $\gamma \in R^k$ is a parameter vector and Z_t is a vector of fully observed characteristics describing the environment of the t^{th} auction. We described the inclusion of these covariates above.

Number of Potential Bidders. The number of potential bidders in each auction, N_t, is not observed. Moreover, it is not identified (Athey and Haile 2002). We assume that it is constant for every four-units auction, $N_t = N$. Table 3 displays the timing structure for different bidders in our sample. For our estimation, we let the number of potential bidders in each auction be the yearly average of different farmers who won auctions in our sample.

[32] Table A3 in the online appendix displays appropriate intervals for watering citrus.

The agricultural products that are cultivated in the area are mainly citrus trees, which are harvested once per year. The number of different bidders who bought at least one unit during a specific year constitutes a good approximation of the number of farmers who were actively bidding in each four-units auction during that year. The monthly average of different bidders who bought water in the sample (years 1954 to 1966) is 8.31 (Table 3). We estimate the model using different values of N for robustness.[33]

Unobserved Heterogeneity. Throughout, we have assumed that the vector Z_t of covariates is fully observed by the econometrician. In our environment, unobserved heterogeneity implies that the distribution of bids may not be conditional-independent across t. All farmers may, for example, observe some factor unobservable by the researcher that shifts the location of the distribution values. This unobserved heterogeneity could lead to correlation among bidders' valuations, causing an identification problem and inconsistent estimates to arise. From the agricultural census data we observe individual characteristics of the farmers which we are able to link to the winning bids. Given the structure of the agricultural water market we are modeling, it does not appear to be an important concern once we consider the homogeneity of the selling good and the observed characteristics we introduce in our estimations (seasonality, past and future rain, among others). Modeling unobserved heterogeneity may require additional assumptions on the behavior of unobservables, such as independence, separability, strict monotonicity, and is beyond the scope of this article. For a discussion on this issue see, among others, Athey, Levin, and Seira (2011) for an application to timber auctions, and Krasnokutskaya (2011) for a semi-parametric approach to Michigan highway procurement contracts. Roberts (2009) uses information contained in reserve prices to allow bidders' private signals to depend on the realization of the unobserved heterogeneity. Balat (2013) allows for unobserved heterogeneity using dynamic auctions in the highway procurement market.

[33] In Table 5 we present the results for $N \in \{8, 10\}$. We have performed a sensitivity analysis to different values of N_t that are consistent with the pattern observed in Table 3 and the evidence described in section 2. In addition, we broke the sample into four periods and performed the estimation independently in each period allowing the mean value of N_t to vary by period. We obtained similar results to the ones reported in Table 5.

Dynamic Strategic Considerations. The way in which the auction system is carried out every week raises the question of the importance of dynamic strategic considerations between four-units auctions both among days (Monday to Friday for a specific schedule) and between schedules (day-time vs night-time for a specific day). Tables 1 and 2 show that winning prices decline across days (for a given schedule) and at night (for a given day), which is consistent with the literature on empirical sequential auctions. These dynamic strategic considerations are outside the scope of the present investigation, and we abstract from them in the model. We account for these issues with a set of dummy variables. Our reduced-form regressions in Table 2 show correlations between average winning prices "within four-units blocks" and a set of covariates. These covariates (which include "schedule" dummy variables for day *vs.* night irrigation, a set of dummy variables for the day of the week of the irrigation, and a "seasonal" set of monthly dummy variables) account for these dynamics, in that the residuals after including these covariates (*i.e.*, both the residuals from the reduced-form regressions in Table 2, and the residuals from the structural model in Tables 5 and 6) are uncorrelated with any of the observables in our data. Thus, we believe that our structural empirical analysis does not suffer from an omitted variable bias, and that the dummy variables effectively account for those dynamic issues.

In particular we can distinguish three types of dynamics in this market: a) Within four-units blocks; b) across four-units *blocks* (within week); c) across weeks. Incorporating all these dynamics into the structural model would be too cumbersome. We are addressing a) in the current article, and we address c) in Donna and Espin-Sanchez (2016). In this article we use "schedule" dummy variables to account for b), and seasonal effects to account for c). Moreover, in this article we could have assumed that each bidder observes a vector of ten values at the beginning of the week, each value corresponding to the valuation, v_i, for each of the ten four-units *cuartas*. In this case units across four-units block would be *ex-ante* equivalent, and the dynamics across four-units block would be modeled as in McAfee and Vincent (1993). Alternatively we could assume that bidders begin the auction on Friday knowing only their valuations for the Monday-night cuartas, and only after that auction is finished they learn their valuation for the next set of four cuartas (in this case, Tuesday-night), and so on. In that case the proper model would be Engelbrecht-Wiggans (1994), and the objects would be stochastically equivalent.

However, it is important to note that, even if present, dynamic behavior considerations do not invalidate the model's assumptions. As emphasized above, the conditional-independent units of analysis are four-units auctions (not day-auctions of eight units or week-auctions of 40 units) which, conditional on covariates, are homogeneous goods. As can be seen from the correlations presented in Table 2, previous patterns are consistent along the whole sample and robust to the inclusion of a whole set of fixed effects and covariates. The principal difference between prices in these four-units auctions is related to the uncertainty of future rain. As it is explained above, we include covariates for schedule, day-of-the-week, and past rain in our structural estimation that capture technological or strategic effects. Future rain, on the other hand, is also included as a proxy for farmers' beliefs to account for these possible strategic behaviors unaccounted by previous covariates. In that sense, our estimates should be interpreted as four-units day-schedule specific auctions, conditional on past rain and seasonality. It seems implausible that after accounting for these observables and unobservables,[34] and given that the relevant unit of analysis is the four-units auction, dynamic behavior would affect our results concerning individual demand. Once we condition on these covariates, the concern that a bidder's outside option would vary according to the day of the week (or schedule) is addressed by redefining the idiosyncratic individual valuation in such a way that the new one be the original valuation net of the outside option. By normalizing the outside option of Friday-night to zero the model's assumptions remain valid.

Regions of the Likelihood and Covariates. Another concern may be selection in the regions of the likelihood. As emphasized in section 4, categories a, b, and c define three separate pricing strategies. (In region a, winning prices are determined by equation 1. In region b, winning prices are determined by equation 2. In region c, winning prices are determined by equation 3.) Table A4 in the online appendix displays a comparison of the covariates in the three regions of the likelihood. As expected, prices are higher in the strict complements regime (region a) relative to the weak substitutes regime (regions b and c). This is because the amount of rainfall is lower under the strict complements regime (region

[34]Although farmers use their reasonable good predictions in their decisions, we use actual future rainfall in our estimation.

a) relative to the weak substitutes regime (regions b and c). Rainfall is lower in region a) (relative to regions b and c) due to weather seasonalities: the percentage of observations in Apr-May (when the agricultural products need the water the most) is substantially higher in region a) (strict complements) relative to regions b) and c) (weak substitutes). The opposite is true during the low demand season (Jan-Mar and Oct-Dec). Finally, note that there is no substantial variation (between the strict complement and weak substitutes regimes) in terms of the percentage of observations by Schedule (day or night) and Weekday (Mo, Tu, We, Th, and Fr).

5 Estimation Results, Counterfactuals, and Discussion

Estimation Results

In this section we present the estimation results under various econometric specifications. We let private valuations for each four-units auction follow an exponential distribution, and follow the described estimation procedure. As discussed above, the number of bidders, N, is determined by the monthly average of different bidders who bought water in the sample (years 1954 to 1966). In this 13-year sample, the average is slightly above 8. Each of these farmers regularly won auctions. It is reasonable to assume that they attended the auctions. Tables 5 and 6 present our estimation results. Columns 1, 3, and 5 present the estimates for $N = 8$, whereas columns 2, 4, and 6 do it for $N = 10$. In their simulated Non Linear Least Squares (NLLS) estimation, Laffont, Ossard, and Vuong (1995) search for the best value of N by minimizing a lack-of-fit criterion (proposition 4). Note that, as discussed in section 4, identification of the distribution of valuations and structural parameters of our model requires observation of the total number of bidders. The rationale for this is straightforward: whether second highest realization of the random variable v_i is from a sample of size $N = 10$, or from a sample of size $N = 100$, it is crucial to interpret the second highest bid (observable in our data). Although observation of an additional order statistic can eliminate this requirement (Song 2004), this would require imposing further structure on the distribution of beliefs in our model (to interpret auctions where, for example, three different farmers win auctions), which is outside the scope of this investigation. Moreover, we observe only winning bids in the data (see section 2). For each specification, we present the estimates of the model's structural parameters in Table 5 and the estimates of the covariates in Table 6. Table 6 is the continuation of Table 5. That is, for each specification (column) in Table 5, Table 6 displays the estimates of the covariates in that specification.

All parameters have the expected signs. We use the estimate of the parameter γ (that characterizes the distribution of private valuations), to compute the mean valuation of the first complete unit of water. In the case of column 3, the value of the first complete unit of water is 155.78 *pesetas*. As expected, in the specification in column 4 (with 10 different bidders), the mean value of the first complete unit of water is slightly lower, 135.20 *pesetas*.

The parameter β_1^R, $R \in \{C, S\}$ captures the effect of future rain. As farmers' expectations of future rain increase, DMR are more severe ($\beta_1^R > 0$, $R \in \{C, S\}$). This increases farmers' likelihood of playing the strategies of a not-strict complements regime (see Table 4) and thus reduces their valuation of subsequent units of water ($\frac{\partial p_t^i}{\partial R_t^F} < 0$). Predicted DMR are obtained by adding the estimates of intercepts, $\hat{\beta}_0^R$, $R \in \{C, S\}$, to the estimates of the slope, $\hat{\beta}_1^R$, $R \in \{C, S\}$, conditional on the rain on the day of the auction. When evaluated at the average future rain from each regime, the following null hypothesis (joint test) that overall DMR are lower in the strict complements regime (as predicted by the model) cannot be rejected (*p-value* above 10%). $H_0: \ \hat{\beta}_0^S + \hat{\beta}_1^S \hat{\mathbb{E}}_s(R_t^F) > \hat{\beta}_0^C + \hat{\beta}_1^C \hat{\mathbb{E}}_c(R_t^F)$, where $\hat{\mathbb{E}}_s(R_t^F) = \frac{1}{T_s} \sum_{t: D_t^a = 0} R_t^F$, $\hat{\mathbb{E}}_c(R_t^F) = \frac{1}{T_c} \sum_{t: D_t^a = 1} R_t^F$, T_s, and T_c are the number of auctions in not-strict complements and strict complements regimes, respectively.

The estimates of the SC parameter, α, are statistically significant in all specifications. Given the choice of parametrization for sunk costs, the parameter estimates can be interpreted as the percentage loss in terms of a complete unit of water (section 3). For our estimate in column 3 this represents a loss of 4.75 *pesetas* (using the mean value of 155.78 *pesetas* for a complete unit).

The estimated coefficients for covariates have the expected sign. For specification 3, for instance, prices in August (February) are significantly 234.08 *pesetas* higher (11.30 *ptas* lower) than on January. This is consistent with the conventional wisdom that water is more (less) valuable during these months because of high (low) water demand. Also as expected, past rain decreases observed prices in the data. For specification in column 3, an increase in the average rainfall by 1 *mm* from the previous week (with respect to the day of irrigation), decreases average conditional price of a unit of water by 1.61 *pesetas*.

Participation cost are recovered using data where auctions were run with farmers present, but no bids were placed, along with the identifying restriction that holds in such cases.[35] Out of the $3,203$ auctions where no bids were placed, we use the $2,423$ where some bidders where present (auctions similar to the one in Figure 7). We obtain the following interval estimate using specification 3: $0.0082 < \hat{c} < 0.1431$. That is, participation costs are positive but small (less than 14 cents of a *peseta*). This is in line with the intuition from the model: hassle or opportunity costs because farmers value their time.

Counterfactuals

In this subsection we use the estimated model to compare the welfare under the three scenarios described below: myopic farmers, large units, and paved channels. In each counterfactual scenario we compare the welfare relative to the benchmark case in specification 3 from tables 5 and 6.

Welfare Scenarios

Myopic Farmers

In our setting we define a farmer as "myopic" if the farmer does not internalize the effect of the sunk cost (SC). A myopic farmer would "absorb" the SC, in that it would bid without accounting for the resulting complementarity effect. The myopic farmer, however, would still be affected by the SC. Thus, using the notation from our model, myopic farmers incorrectly believe that $\alpha = 0$, and bid according to standard English auction model. Myopic farmers correctly account for the decreasing marginal returns effect (DMR).

To compute the counterfactual we use the estimated demand system—characterized by the estimated private valuations and the model's structural parameters—from specification 3 from tables 5 and 6, and simulate the bidding behavior assuming that farmers bid according

[35] See page 30.

to a standard English auction model with $\alpha = 0$ and the estimated DMR effect, $\hat{\beta} > 0$ (from specification 3 from tables 5 and 6). Then we compute the actual SC absorbed by the farmers using the actual demand's estimates (*i.e.* using the estimated distribution of valuations and sunk cost, $\hat{\alpha}$, from specification 3 from tables 5 and 6). We use the actual setting (*i.e.* when famers are not myopic and behave as observed in the data) as a benchmark and report the efficiency gains/losses relative to this benchmark.

Large Units

We consider an alternative selling mechanism, whereby the cartel would sell "large units." We define these large units as a single ascending price English auction per four-units auction. Thus, farmers are only able to bid for the large four-units auction (*i.e.* the right to use 12 hours of water for irrigation), not for the individual units (*i.e.* they cannot buy single units with the right to use 3 hours of water for irrigation as in the benchmark case).

To compute the counterfactual we use the estimated demand system from section 5 and simulate the bidding behavior assuming that farmers can bid only for large units. Again, we use the actual setting (*i.e.* with non-myopic famers who can buy single units to use 3 hours of water for irrigation) as a benchmark, and report the efficiency gains/losses relative to this benchmark.

Paved Channels

Finally, we consider the counterfactual scenario, where the irrigation channels are paved. In Mula the channels were pavemented in the 1970s(Gómez-Espín, Gil-Meseguer, and García-Marín (2006)). Paved channels imply that farmers do not incur a SC for the first unit. Under this counterfactual scenario farmers incur only DMR, not SC. Thus, $\alpha = 0$ and $\beta > 0$.

To compute the counterfactual we use the estimated demand system from section 5, and simulate bidding behavior assuming that $\alpha = 0$ and $\beta > 0$. As in the previous cases, we use the actual setting (*i.e.* with non-myopic famers, who can buy single units to use 3 hours of water for irrigation, and where $\alpha > 0$ and $\beta > 0$) as a benchmark, and report the efficiency gains/losses relative to this benchmark.

Welfare Measures

We compute two sets of welfare measures:

Efficiency Gains/Losses per Unit. This measure represents the efficiency gain/loss per unit (mean across units and farmers), and is measured in *pesetas*. We also report the standard deviation, and the largest and smallest efficiency gain/loss.

Total Efficiency Gains/Losses. This measure represents the total efficiency gain/loss in each counterfactual scenario. The total efficiency measure is the sum (across all units and farmers) of the gains/losses per unit during the 13 years sample period in our data, and is also measured pesetas.

For each set of welfare measures we report the efficiency gains/losses due to SC, DMR, and the total. The latter is defined as the sum of the efficiency gains/losses due to SC plus DMR.

Welfare Results

Table 7 reports the results from the counterfactual analysis. Myopic consumers incur substantial losses in terms of absorbing additional SC. They continue to buy the same (expected) amount of water as in the benchmark scenario. But their bidding behavior ignores the complementarity effect caused by the SC. Each time they buy a unit, they incur an additional SC and save the corresponding DMR, if they buy only one unit. Panel A shows that, on average, there is a 28.56 *pesetas* efficiency loss per unit due to ignoring the presence of SC. Strong seasonalities indicate that there is substantial heterogeneity in terms of the mean efficiency loss per unit, with a minimum value of 6 cents and a maximum value of 60.63 *pesetas*. As expected, not accounting for the SC generates savings in terms of the DMR because the latter are incurred only when more than one unit is bought sequentially. Myopic farmers are very unlikely to buy more than one unit. Panel B shows that the total efficiency loss due to the farmers ignoring the presence of SC are substantial: $42,497.36$ *pesetas* during the 13 years sample period in our data. The savings resulting from incurring less DMR partially compensates for this loss. The total net loss due to myopic farmers is $1,846.59$ *pesetas*.

The second counterfactual scenario investigates an alternative selling mechanism, whereby the cartel would only auction bundles of large four-units. Panels A and B show that this mechanism would exacerbate the costs arising due to DMR. The mean lost due to DMR is only 3.27 *pesetas*. This is because, in the benchmark, when farmers need the water the most, they buy bundles of four-units (even when they can buy individual units) due to the complementarity effect generated by the SC. Again, there is substantial heterogeneity in terms of the mean efficiency loss per unit due to seasonalities: the largest loss due to the DMR is 615.29 *pesetas* per unit. As expected, large four-units bundles generate SC savings that are relatively small (on average 20 cents per unit). Panel B shows that the counterfactual selling mechanism of large units would generate a net total efficiency loss of $4,563.74$ *pesetas* during the 13 years sample period in our data.

The final counterfactual scenario shows what happened in Mula in the 1970s, when the channels were paved. Paved channels imply that farmers do not incur a SC for the first unit. This results in a mean efficiency gain of 12.91 *pesetas* per unit relative to the benchmark scenario, as reported in Panel A. Farmers continue to incur the DMR and internalize them; thus there is no change in efficiency due to the DMR. The total net efficiency gain due to paved channels is large: 19.208.09 *pesetas* during the 13 years sample period in our data, as reported in Panel B.

Discussion

Robustness and Goodness of the Fit. In comparing columns 1-2 and 3-4, it is clear that the model with covariates outperforms the model without, as shown by the significance of past rain and seasonal dummy estimates, the increase in the likelihood function, and the improvement in the goodness of the fit. The main reason is the dependence of prices on seasonal factors, which we capture in our specification with seasonal dummy variables. From the residual analysis we find no evidence that the increase in the log likelihood function is due to the parametric misspecification of the value distribution itself. Our specification survives the Kolmogorov-Smirnov test, so that the exponential distribution of private valuations cannot be rejected (for the specification in column 3 the *p-value* of the test is 41%).[36]

As regards the goodness-of-fit, our specification in column 3 performs quite well. The $pseudo - R^2 = 58\%$ is obtained by computing predicted prices by our model: $pseudo - R^2 = 1 - \frac{\sum_{t=1}^{T}(p_t - \hat{p}_t)^2}{\sum_{t=1}^{T}(p_t - \bar{p})^2}$, where $\hat{p}_t$ are prices predicted by the model and $\bar{p}$ is the mean of prices. These results are in line with the R^2 obtained in the reduced-form regressions. Although not directly comparable given the distribution assumptions in the structural approach, the $R^2 = 23\%$ in the reduced-form specification with all covariates (column 3 in Table 2) can be heuristically interpreted as the proportion of variability in the data set that is accounted for by the covariates. The proportion accounted for by the model without covariates displayed in column 1 in Table 5 is $R^2 = 32\%$.[37] As can be seen in Figure 8, our model allows us to follow winning prices accurately.[38] The figure displays real prices against predicted prices using three different models: (i) our structural model (specification 3 in Table 5), (ii) a standard English auction model (specification 5 in Table 5, that we discuss in the next subsection), and (iii) a reduced-form model (specification 4 from Table 2 that includes as regressors *Past Rain* and multiple fixed effects, including individual fixed effects).

Understanding the Importance of the Model. We proceed now to analyze our model's implications with respect to the importance of SC and DMR. Suppose that the researcher neglects the dynamics that arise from the model and, instead, estimates a standard English auction model. Suppose, for instance, that we are in the strict complements regime and that valuations follow a distribution with mean, μ_v, and standard deviation, σ_v. Then, the estimated mean of the distribution of valuations using the standard model will be underestimated: $\mathbb{E}\left(\hat{v}_i\right)^{SM} < \mathbb{E}\left(v_i\right) = \mu_v$, where SM stands for standard model. Similarly, the estimation of the standard deviation of valuations will be overestimated: $\mathbb{V}\left(\hat{v}_i\right)^{SM} > \mathbb{V}\left(v_i\right) = \sigma_v$.[39] The same is true in the weak substitutes case.

[36] We perform the nonparametric test to evaluate the equality of two distributions of valuations: our sample of private values with a reference from an exponential distribution.

[37] If we additionally add individual fixed effects to the reduced-form specification, the R^2 just increases from 23% to 36% (column 4 in Table 2).

[38] We describe how we compute the predicted prices in section D in the online appendix. See also section D in the online appendix for a high definition version of this figure.

[39] In the strict complements case, and given a fixed number of potential bidders N, the (true) mean and variance of the $N-1$ order statistic will be greater than the estimated using the standard model because the (true) price paid will be $[4 - \alpha - 6\beta_{t,c}]\, v_{N-1:N} - 4c$ and not $4v_{N-1:N}$ (predicted by the standard model).

Overestimation of the variance of the distribution is caused by attributing the variation in prices (among different units) to a relatively more dispersed underlying distribution. The farmer actually pays for the whole bundle in the first unit, thus deterring the entrance of other bidders in the remaining three auctions. The mean is underestimated when the common SC and DMR are not accounted for in the estimation. In the case of the exponential distribution used in our specifications, this failure translates into an underestimation of the parameter μ.

Columns 5 and 6 in Table 5 present the estimates from a standard English (button) auction. Aside from the mentioned bias in the parameter that characterizes the distribution, the results in these columns indicate that taking SC and DMR into account significantly contributes to the model's explanatory power. Figure 8 shows predicted prices from the standard (button) English auction model (specification 5 in Table 5), and compares them with actual prices and with those from our structural model (specification 3 in Table 5).[40] Consistent with these results, the *p-value* for the null hypothesis that $\hat{\alpha} = \hat{\beta}_0^C = \hat{\beta}_0^S = \hat{\beta}_1^C = \hat{\beta}_1^S = \hat{c} = 0$ is less than 10^{-4}.

An alternative approach is to ask how the incomplete model from Haile and Tamer (2003) can be adapted to the present case.[41] This alternative approach relies on two basic assumptions with intuitive appeal: (i) bidders do not bid more than they are willing to pay for a unit, and (ii) bidders do not allow an opponent to win at a price they are willing to beat. In our case, with SC and DMR, these two simple assumptions are violated. In the strict complements regime, bidders bid according to $b_i^1(v_i) = [4 - \alpha - 6\beta]\, v_i - 4c > v_i$, violating (i), and no bidder (except the highest type) participates in the second to fourth unit auctions, violating (ii). In the non weak substitutes regime, both assumptions are also violated, though the intuition is different. In this case, the equilibrium is only partially revealing: bidders' strategies are step functions, so the equilibrium is semi-pooling. When α is greater but close to β, bidders bid above their valuations to intimidate other bidders and deter entry in the second auction, thus (i) is violated. Additionally, the same argument as in Black and De Meza (1992) and Liu (2011) applies when goods are substitutes. The winner of the first

[40] See footnote 38.

[41] Larsen (2013) uses a similar approach to Haile and Tamer to obtain bounds about the primitives in an auction model followed by dynamic bargaining with two-sided incomplete information without solving for the equilibrium of the game.

auction imposes a negative externality on himself. His willingness to pay for the second unit is lower than it was for the first unit, making him a weaker bidder in such situations. Given that all bidders will internalize this effect, some will bid below their marginal utility for the object in the first auction. The greater are DMR, β, the greater this underbidding effect will be.

Applying these assumptions to the four-units bundle would not to produce informative bounds because marginal valuations of the units differ according to the regime and the number of different winners per four-units auction. Bundling the four-units or even applying Haile and Tamer's approach separately for each regime, requires the model in section 3 as an interpretation of the underlying behavior.[42]

Complementarities are not Collusion. An alternative hypothesis of farmers' behavior in the strict complements regime is that bidders might be playing some collusive (non-competitive) strategy. As emphasized in section 2, the demand side of this market for water is composed of as many as hundreds of farmers (Table 3). Even when farmers attend the auction and do not bid, the observed number of different winners is relatively high (Figure 7). (Note that all auctions were run in weeks similar to the one in Figure 7, so water for units 17-20 was available in the dam to sell.) Farmers compete for water that will ultimately determine the quality and quantity of their crop, and in some cases, even the survival of their trees (for example, in drought years). It is unlikely that farmers can make credible collusive commitments in such a situation. Contemporaries emphasized the opposite situation: farmers competed aggressively for water, especially during droughts, although water owners were reluctant to lower the price of the water to meet the needs of the poorest farmers.[43]

The high number of non-collusive auctions provides evidence farmers did not collude. Farmers met every week, hence the discount rate from one week to the next one was close to 1. If we focus on two consecutive four-units auctions, the discount rate is virtually 1. Thus, any collusive agreement would be easy to sustain and we would observe no "price-wars", or deviations from collusive strategies. If the collusion hypothesis were true, all auctions would

[42] Note also that failure to consider the effect of the structural parameters (SC and DMR) explicitly introduces difficulties.

[43] These opinions, along with a qualitative analysis can be found in Vera Nicolás (2004).

look collusive except, perhaps, during certain periods where we would observe price-wars. We observe in many cases, however, that both regimes are present during the same week. Unlike Baldwin, Marshall, and Richard (1997) this is not a formal test.[44]

Nevertheless, taking the analysis one step further, if the collusion hypothesis were true, we would expect more collusion in autumn-winter and less collusion in spring-summer. Incentives to deviate from the collusion strategy are higher in spring-summer because the value of the water is higher due to seasonalities (Figures 2 and Panel A in Figure 3). Punishment is about the same in any season. The maximum punishment would be to play the competitive equilibrium forever. Future discounted earnings in this case are similar in summer and in winter. Hence, deviating from the collusive strategy is more profitable in summer than in winter. However, the data show the opposite pattern. Figure 9 displays the distribution of auctions in the complementarities regime by month. Complementarities are more likely to be observed in summer than in winter, when water requirements (and hence equilibrium prices) soar. This is in line with our interpretation according to the model with sunk and entry costs.

A "competitive" collusion? We have implicitly assumed that farmers' plots were spaced sufficiently far apart from each other. Specifically, we assumed that no other farmer could use the same sub-channel just used by his neighbor. In reality, this assumption is not true for all cases. Because the cost of watering the sub-channel is sunk, if the plots of two farmers are located next to each other and they share the same sub-channel, then one farmer could free-ride and outbid the first winner in the second auction. Knowing this, the first winner would bid lower in the first auction. This situation would reduce the revenue of the auction and create inefficiencies. Because farmers might not internalize this free-riding effect, they would take into account the equilibrium outcome for the remaining auctions, and lower their bid in the first auction. They would then will try to outbid their neighbors in later auctions.

In a situation such as this, it would be relatively easy to sustain a collusive agreement among neighboring farmers. The number of members of the coalition would be small (say,

[44]Collusion in repeated auctions has been analyzed conditional (Hopenhayn and Skrzypacz 2004) and unconditional (Porter and Zona 1999 and Pesendorfer 2000) on the history of the game. A discussion on how to detect collusion can be found in Porter (2005).

three or four farmers), and because they are neighbors, they would know each other well and might even share animals or machinery for agricultural purposes. Each farmer in the coalition would compete in the auction for the first unit, but would not enter the remaining auctions if one member of the coalition won the first unit. With this agreement they would achieve efficiency by solving the free riding problem. With the resulting increase in efficiency, the revenue of the auction would also increase, and the auctioneer would not be opposed to the "collusion." This situation would not affect our results unless farmers coordinated bidding rings to not outbid neighboring farmers in the first auction. It will affect only the outcome when both the bidder with the highest valuation and the bidder with the second highest valuation belong to the same ring, but the bidder with the third highest valuation belongs to a different ring. In this case, our model predicts that the observed price is the valuation of the second highest bidder, but it actually corresponds to the valuation of the third highest bidder. This is unlikely in our empirical setting because the nearly 500 farmers would form around 150 rings (based on the geographical locations that we obtained from the census data). The probability that the two bidders with the highest valuation belong to the same ring is virtually zero. Moreover, the difference between the second highest and the third highest valuation will be small in any case. [45]

Efficiency. The model displayed in section 3 assumes that it is costly for the bidders to enter the auction. In order to compare the sequential ascending price auction with other mechanisms, this entry cost has to be taken into account. In this context, and following Stegeman (1996), we interpret entry cost as the cost farmers incur when they send a message to the auctioneer, or to some other farmers. Here, the notions of *ex-ante* and *ex-post* efficiency are no longer equivalent. Although it may be *ex-ante* efficient that more than one player sends a message, it is always *ex-post* efficient that at most one player sends a message.

For this case, where it is costly to send messages to the planner, Stegeman (1996) shows that the ascending price auction has an equilibrium that is *ex-ante* efficient. In contrast, the first-price auction may have no efficient equilibrium, and the author considers only the

[45] There is an extensive literature on the theory of bidding cartels (*e.g.* Graham and Marshall 1987; Hendricks, Porter, and Guofu (2008); Hopenhayn and Skrzypacz 2004; and McAfee and McMillan 1992). For English auctions, Asker 2010 empirically investigates a bidding cartel of collectable stamps. See Harrington 2008 for a survey.

single-unit case. In our sequential unit case, we have shown that when goods are strict complements the analysis is identical to the single unit case. Hence, the result applies here as well. However, when goods are weak substitutes, the result applies only to the last auction. Although outside the scope of this article, further work to investigate whether a sequential ascending price auction is *ex-post* efficient when the planner has to allocate several objects to players that face SC, DMR, and costly messages, would be a useful extension.

6 Conclusions

By affecting bidders' behavior in sequential auctions, sunk costs and decreasing marginal returns (DMR) in the presence of participation costs generate very different price dynamics within the same market. This difference in price dynamics is attributable to the varying extent to which the value of sequential goods complements or falls relative to previous units. The *deterrence* effect, whereby the same bidder pays a high price for the first unit (deterring others from entering subsequent auctions), and a low price for the remaining units, arises when sunk costs are relatively high compared to the DMR, thus creating complementarities among the goods. Substitutability arises due to decreasing returns when sunk costs are relatively small. In this case, equilibrium prices are similar in magnitude, regardless of whether the same or different bidders win the objects. Careful consideration of these features is fundamental to demand characterization, a cornerstone of many positive and normative questions in economics.

Using a novel data set from a decentralized market institution that operated privately for eight centuries in southern Spain, we document these price dynamics and develop a model to recover the underlying structural parameters and distribution of valuations. Although the bidders are better informed than the sellers in our model, the latter know that the sequential English auction allocates water (*ex-ante*) efficiently. Not requiring farmers to reveal their marginal valuations is an advantage of the mechanism, whose simplicity reduces costs associated with its implementation and helps explain its stability. We address three main questions. Are water units complements or substitutes, and why? Is the *deterrence* effect consistent with a competitive market structure or a consequence of collusive behavior among farmers? What would happen to the estimates in this setting if the researcher, by ignoring the importance of participation and sunk costs, failed to account for the complementarity feature of the sequential goods?

First we document that during the period under study both complementarities and substitutabilities are observed in the data, generating different price dynamics. Seasonality, related to the water requirements of the crops and the expected rainfall, affects the relative importance of sunk costs and decreasing returns, causing bidders to behave accordingly in these regimes. Second, the apparent collusive behavior, when the same bidder wins all the goods, paying very low prices for all the units following the first unit, is actually competitive (or non-cooperative). Contrary to the collusion hypothesis, this behavior is caused by complementarities, and is observed when the value of water (as well as the average price paid per unit and, thus, the incentive to deviate from a collusion strategy) increases relative to the standard competitive pattern registered in the weak substitutes regime. This shows the importance of interpreting the data through the economic model. Finally, by estimating our model, we confirm the relevance of participation and sunk costs in our empirical environment. By testing the performance of our model relative to a standard English auction model without participation costs, we confirm that estimations using the latter are not accurate. Aside from the bias generated by ignoring sunk costs and decreasing returns, price dynamics play an important role, as it is not appropriate to attribute the variation in prices among sequential units (when the goods are complements) to a relatively more disperse underlying distribution of valuations.

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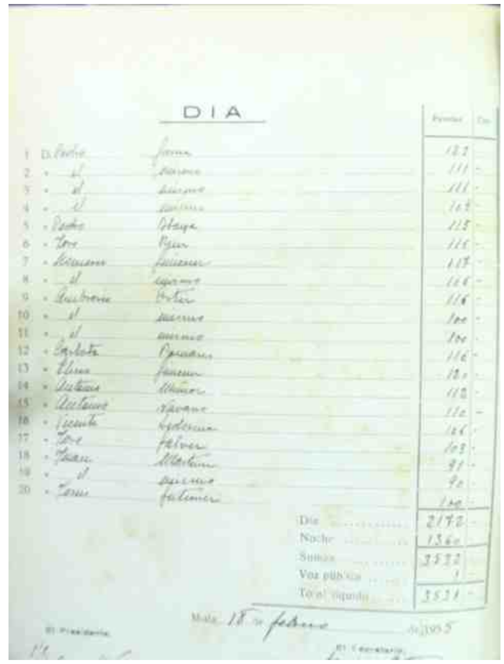
Figure 1: Auction Samples.

Panel A: Goods Are Substitutes.

Auction #	Name	Price	Day
1	Pedro Fernández	123	Mo
2	Pedro Fernández	111	
3	Pedro Fernández	111	
4	Pedro Fernández	109	
5	Pedro Blaya	115	Tu
6	Jose Ruiz	116	
7	Mauricio Gutiérrez	117	
8	Mauricio Gutiérrez	106	
9	Ambrosio Ortíz	116	We
10	Ambrosio Ortíz	100	
11	Ambrosio Ortíz	100	
12	Carlota Pomares	116	
13	Eliseo Gutiérrez	120	Th
14	Antonio Muñoz	112	
15	Antonio Navarro	110	
16	Vicente Ledesma	106	
17	Jose Gálvez	103	Fr
18	Juan Martínez	91	
19	Juan Martínez	90	
20	Jesus Gutiérrez	100	


DIA

Día

Noche

Sumas

Voz pública

Total líquido

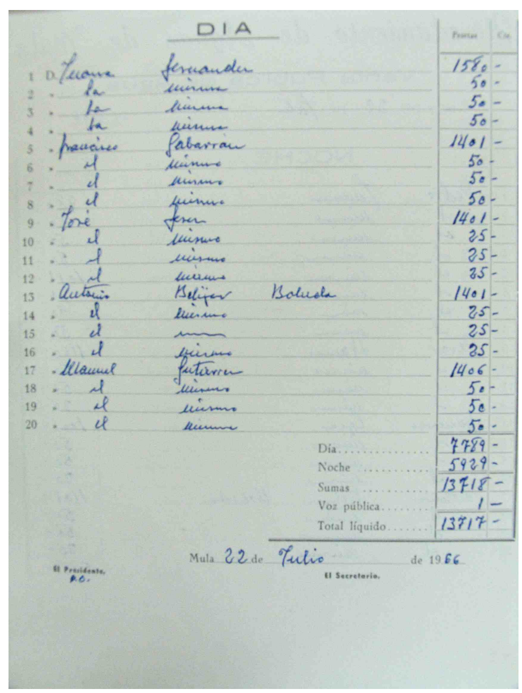
Mula 18 de febrero de 1955

El Presidente

El Secretario

Panel B: Goods are Complements.

Auction #	Name	Price	Day
1	Juana Fernández	1580	Mo
2	Juana Fernández	50	
3	Juana Fernández	50	
4	Juana Fernández	50	
5	Francisco Gabarrón	1401	Tu
6	Francisco Gabarrón	50	
7	Francisco Gabarrón	50	
8	Francisco Gabarrón	50	
9	Jose Fernández	1401	We
10	Jose Fernández	25	
11	Jose Fernández	25	
12	Jose Fernández	25	
13	Antonio Belijar Boluda	1401	Th
14	Antonio Belijar Boluda	25	
15	Antonio Belijar Boluda	25	
16	Antonio Belijar Boluda	25	
17	Manuel Gutiérrez	1406	Fr
18	Manuel Gutiérrez	50	
19	Manuel Gutiérrez	50	
20	Manuel Gutiérrez	50	


DIA

Día 7789

Noche 5929

Sumas 13718

Voz pública 1

Total líquido 13717

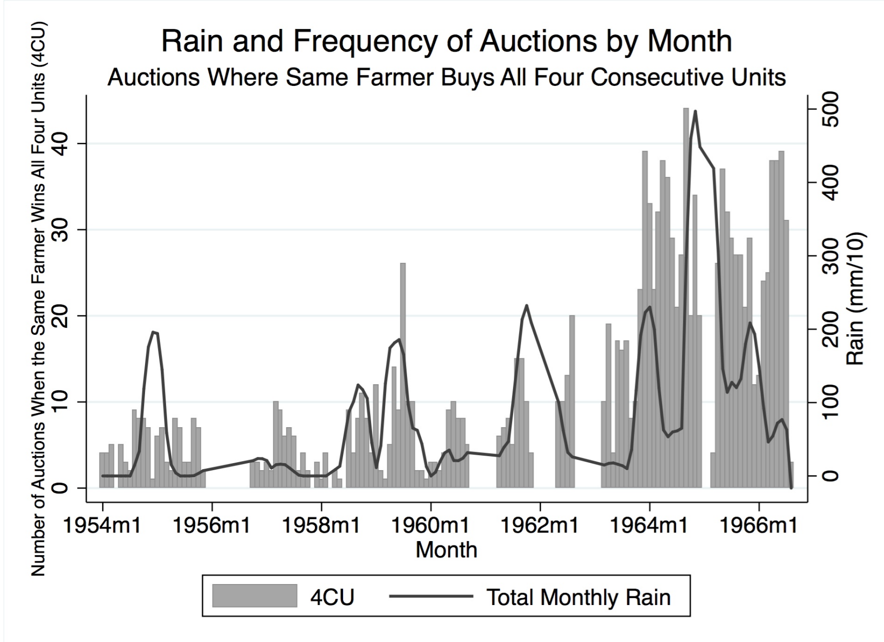
Mula 22 de Julio de 1966

El Presidente

El Secretario

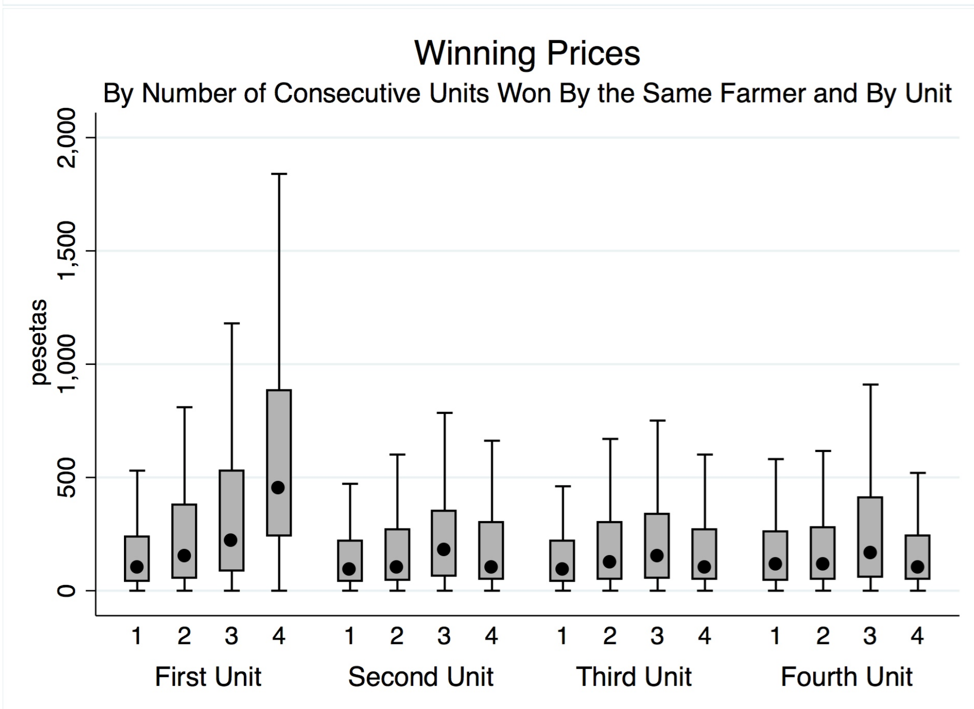
Notes: Units 1 to 4 are the units bought on Monday (Mo) during day (unit 1 corresponds to right to irrigate from 7AM to 10AM, unit 2 from 10AM to 1PM, unit 3 from 1PM to 4PM, and unit 4 from 4PM to 7PM). Similarly, units 5 to 8 are the units bought on Tuesday (Tu) during day; units 9 to 12 are the units on Wednesday (We) during day; units 13 to 16 are the units on Thursday (Th) during day; and units 17 to 20 are the units on Friday (Fr) during day. Panel A: Sample from original data obtained from the historical archive: Goods Are Substitutes (Winter - February 18, 1955, Day). Panel B: Sample from original data obtained from the historical archive: Goods Are Complements (Summer - July 22, 1966, Day).

Figure 2: Rain and Frequency Distribution of 4CU Over the Sample Period



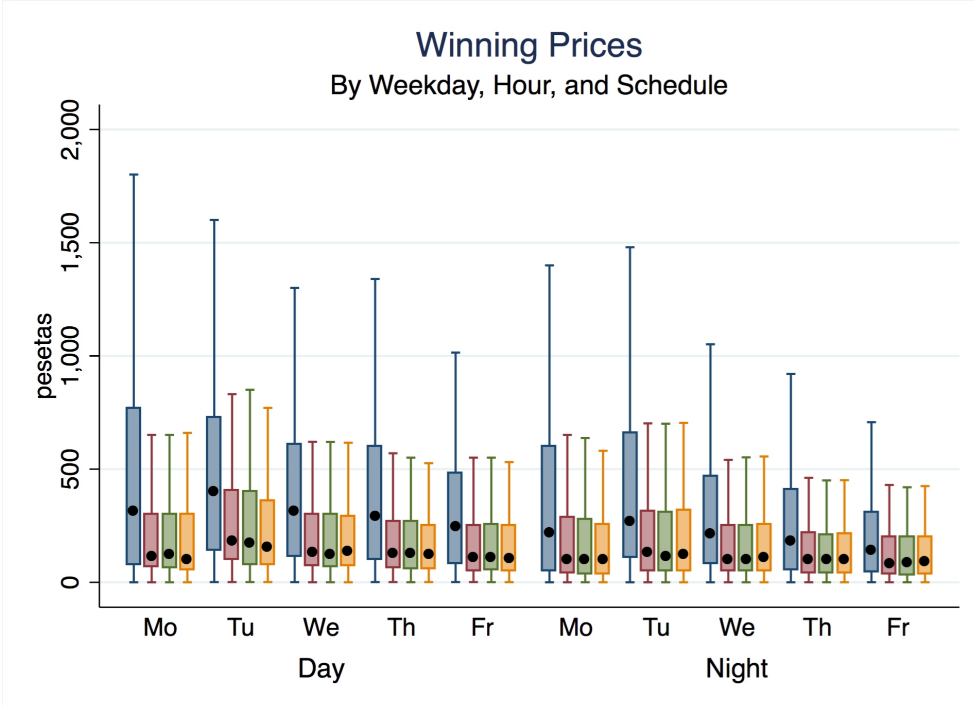
Notes: The figure displays for each month: i) the number of auctions where the same farmer wins all four consecutive units, and ii) total rain using a Nadaraya-Watson kernel regression (of 'total rain' on 'month of the year') with an Epanechnikov kernel with bandwidth selected by cross validation.

Figure 3: Winning Prices: by Number of Consecutive Units Bought by the Same Farmer and by Unit



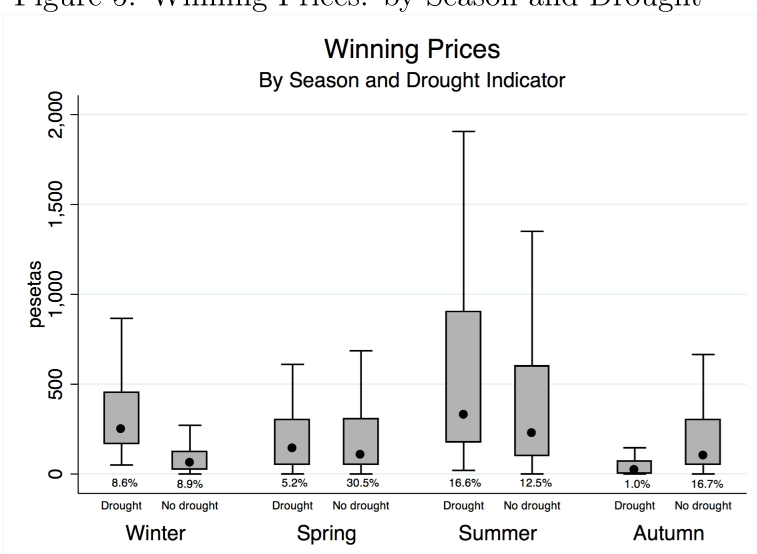
Notes: The figure displays price variation from the raw data (for the whole sample) disaggregated by:
i) Top Panel: Number of consecutive units won by the same farmer (1, 2, 3, and 4; note that we called them 1CU, 2CU, 3CU, and 4CU in Tables A1 in the online and appendix and in table1).
ii) Bottom Panel: further disaggregating each vertical box from the Top Panel by unit (First Unit, Second Unit, Third unit, and Fourth unit). Note that 1, 2, 3, and 4 indicate the number of consecutive units won by the same farmer (same as in the graph in the top).
Each vertical box (unit) displays the maximum price (upper adjacent value), 75th percentile (upper hinge), median (black circle marker), 25th percentile (lower hinge), and minimum price (lower adjacent value).

Figure 4: Winning Prices: by Weekday, Hour and Schedule



Notes: The figure displays the distribution of winning prices by: i) Unit (First Unit in Blue, Second Unit in Red, Third Unit in Green, and Fourth Unit in Orange); Weekday (Mo=Monday, Tu=Tuesday, We=Wednesday, Th=Thursday, and Fr=Friday); and Schedule (Day=Day-Time and Night=Night-Time). Thus, the figure displays the distribution of prices of each of the 40 units auctioned per week for the whole sample (disaggregated by Unit, Weekday, and Schedule). Each vertical box (unit) displays the maximum price (upper adjacent value), 75th percentile (upper hinge), median (black circle marker), 25th percentile (lower hinge), and minimum price (lower adjacent value).

Figure 5: Winning Prices: by Season and Drought



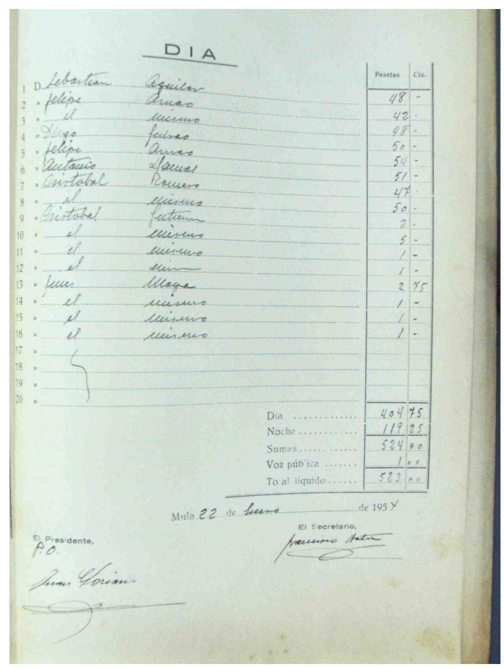
Notes: The figure displays the distribution of winning prices by: i) Season and Drought Indicator. Each vertical box displays the maximum price (upper adjacent value), 75th percentile (upper hinge), median, 25th percentile (lower hinge), and minimum price (lower adjacent value). We define a drought as an indicator that equals one when average monthly rain during the specific year is below a consensus threshold defined in the literature in terms of the historic annual average (following Gil Olcina 1994 we use a threshold of 40%). The numbers below each box correspond to the percentage (in terms of the whole sample) of observations in each box (*i.e.* al these numbers sum up to 100%).

Figure 6: The Channel System in Mula and the Sunk Cost of Initiating the Irrigation

Notes: The main canal (left panel) was made of concrete. The individual sub-channels (right panel) were dug into the ground. Thus, in these sub-channels, a water loss is incurred because water flows over a dry sub-channel (some water is absorbed by the ground).

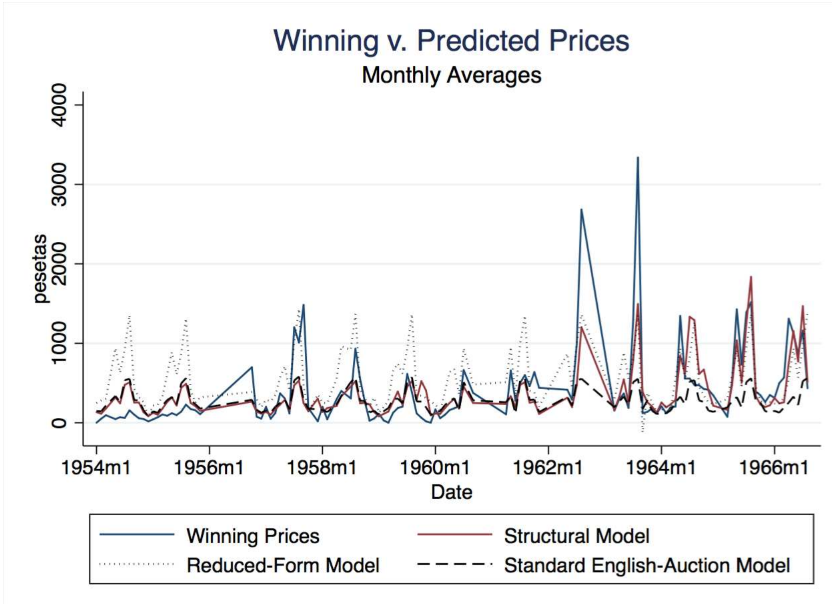
Figure 7: Auction Sample: Auction where Farmers Are Present and No Bids Are Placed

Auction #	Name	Price	Day
1	Sebastian Aguilar	48	Mo
2	Felipe Amaro	42	
3	Felipe Amaro	48	
4	Diego Guirao	50	
5	Felipe Amaro	54	Tu
6	Antonio Llamas	51	
7	Cristóbal Romero	47	
8	Cristóbal Romero	50	
9	Cristóbal Gutiérrez	2	We
10	Cristóbal Gutiérrez	5	
11	Cristóbal Gutiérrez	1	
12	Cristóbal Gutiérrez	1	
13	Luis Moya	2.75	Th
14	Luis Moya	1	
15	Luis Moya	1	
16	Luis Moya	1	
17			Fr
18			
19			
20			



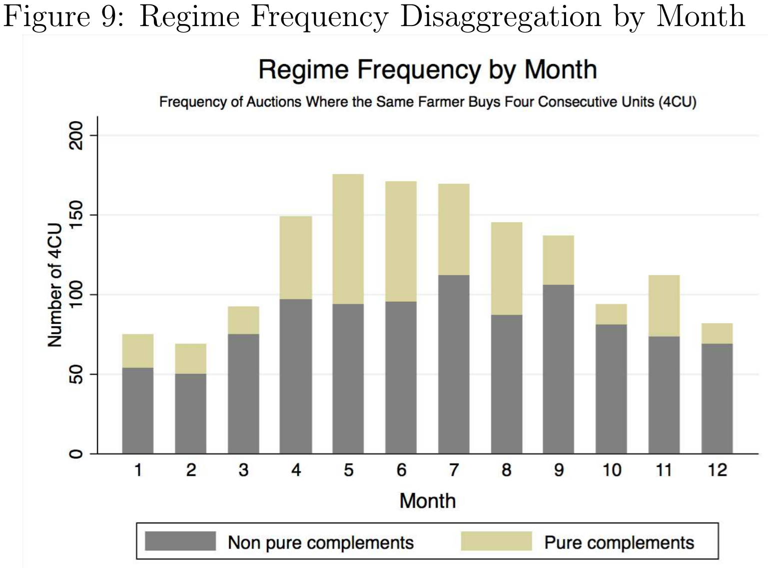
Notes: Sample from original data obtained from the historical archive: Auction where farmers are present and no bids are placed (Winter - January 22, 1954, Day).

Figure 8: Winning and Estimated Prices



Notes: The figure displays real prices against predicted prices using three different models: (i) our structural model (specification 3 in Table 5), (ii) a standard (button) English auction model (specification 5 in Table 5), and (iii) a reduced-form model for the sample using as regressors: *Past Rain*, unit (3 dummy variables), weekday (4 dummy variables), schedule (1 dummy variable), month (11 dummy variables), year (12 dummy variables), and individual fixed effects, in addition to a constant (for details about the reduced-form specification see Table 2 discussed in section 2). The graph shows the mean monthly averages of the prices. Similar results are obtained using a spline (available in our earlier working article Donna and Espin-Sanchez 2012). See section D in the online appendix for a high definition version of this figure.

Figure 9: Regime Frequency Disaggregation by Month



Notes: The figure depicts the frequency of auctions where the same farmer buys all four consecutive units (4CU), by regime (see section 2) and month. (Note that the sum of 4CU over months and regimes—the vertical lines in the graph—is equal to 1470 = 5880/4. See Table 1 in the article and Table A1 in the online appendix.) It can be seen that complementarities are more likely to be observed in summer than in winter, where water requirements (and, hence, equilibrium prices) soar. We interpret this as evidence in favor of the competition hypothesis (according to our model with entry and sunk costs) and against the collusion hypothesis.

Table 1: Distribution of Winning Prices: by Number of CU and Sequential Auction

Panel 1: Price distribution by number of consecutive winning bids: All Auctions

	Median	Mean	SD	Max	Min	Obs
1CU	101	218.2	327.9	3000	0.05	3530
2CU	123	256.7	364.6	2700	0.05	2866
3CU	190	320.0	415.5	4050	0.05	1716
4CU	182	339.9	470.2	4830	0.05	5880

Panel 2: Price distribution by number of consecutive winning bids: First Auction

	Median	Mean	SD	Max	Min	Obs
1CU	100	211.1	304.1	2921	0.05	977
2CU	150	305.0	427.8	2700	0.05	673
3CU	220.5	410.0	512.5	4050	0.05	382
4CU	451	677.6	689.5	4830	0.05	1470

Panel 3: Price distribution by number of consecutive winning bids: Second Auction

	Median	Mean	SD	Max	Min	Obs
1CU	93.25	219.8	373.0	3000	0.10	624
2CU	103.5	230.2	328.0	2685	0.05	867
3CU	181	294.9	364.7	2850	0.05	539
4CU	101	242.7	309.3	2605	0.05	1470

Panel 4: Price distribution by number of consecutive winning bids: Third Auction

	Median	Mean	SD	Max	Min	Obs
1CU	94	200.8	312.3	2357	0.10	715
2CU	126.5	256.5	353.9	2601	0.10	778
3CU	151.5	285.55	379.0	2801	0.05	536
4CU	100	229.2	294.2	2701	0.05	1470

Panel 5: Price distribution by number of consecutive winning bids: Fourth Auction

	Median	Mean	SD	Max	Min	Obs
1CU	114.5	233.4	330.2	2601	0.05	1214
2CU	113.5	239.6	344.8	2601	0.10	548
3CU	167	311.6	411.6	2630	0.05	259
4CU	100	210.1	272.6	2935	0.05	1470

Panel 6: Price distribution for 4CU

Auction	Median	Mean	SD	Max	Min	Obs
1st to 4th	182	339.9	470.2	4830	0.05	5880
1st	451	677.6	689.5	4830	0.05	1470
2nd	101	242.7	309.3	2605	0.05	1470
3rd	100	229.2	294.2	2701	0.05	1470
4th	100	210.1	272.6	2935	0.05	1470
1st and 2nd	253	460.2	576.9	4830	0.05	2940
2nd and 3rd	101.0	235.9	301.9	3001	0.05	2940
3rd and 4th	100.0	219.7	283.7	2935	0.05	2940
1st to 3rd	200.0	383.2	512.4	4830	0.05	4410
2nd to 4th	100.0	227.3	292.7	3001	0.05	4410

Notes: The table displays the Distribution of Winning Prices. Panels 1 to 5 presents the Distribution of Prices disaggregated by cases where the same farmer buys one, two, three, or four consecutive units (1CU, 2CU, 3CU, or 4CU, respectively). Panel 1 presents the Distribution of Prices for All Auctions (*i.e.* First, Second, Third, and Fourth Auctions). Panel 2 presents the Distribution of Prices for First Auctions. Panel 3 presents the Distribution of Prices for Second Auctions. Panel 4 presents the Distribution of Prices for Third Auctions. Panel 5 presents the Distribution of Prices for Fourth Auctions. Finally, Panel 6 presents Distribution of Prices just for 4CU (*i.e.* for the subsample of 5880 auctions where the same farmer won all four consecutive units). Note that the first line in Panel 6 (1st to 4th) displays the same information as the last line in Panel 1 (4CU). The second line in Panel 6 (1st) displays the same information as the last line in Panel 2 (4CU). The third line in Panel 6 (2nd) displays the same information as the last line in Panel 3 (4CU). The fourth line in Panel 6 (3rd) displays the same information as the last line in Panel 4 (4CU). The fifth line in Panel 6 (4th) displays the same information as the last line in Panel 5 (4CU).

Table 2: Correlation Between Winning Prices and Covariates

Variables	(1)	(2)	(3)	(4)
Rain MA7	-4.054***	-4.112***	-2.991***	-3.174***
	(0.674)	(0.689)	(0.558)	(0.623)
Rain Day Bought	-0.235	-0.185	0.052	0.178
	(0.143)	(0.142)	(0.156)	(0.153)
Unit 2 Day		-167.95***	-167.82***	-180.61***
		(19.47)	(19.45)	(21.89)
Unit 3 Day		-173.03***	-172.91***	-188.05***
		(19.93)	(19.92)	(22.75)
Unit 4 Day		-176.54***	-176.54***	-190.83***
		(20.44)	(20.44)	(23.10)
Unit 2 Night		-237.58***	-237.86***	-249.33***
		(24.90)	(24.94)	(27.35)
Unit 3 Night		-243.32***	-243.52***	-257.65***
		(25.45)	(25.49)	(28.31)
Unit 4 Night		-254.84***	-255.19***	-266.41***
		(25.73)	(25.78)	(28.91)
Tuesday		26.02***	32.19***	10.06
		(7.49)	(8.24)	(12.48)
Wednesday		-34.58***	-29.39**	-31.73**
		(10.63)	(11.97)	(15.47)
Thursday		-59.65***	-55.41***	-42.92***
		(10.77)	(12.35)	(15.04)
Friday		-94.95***	-95.54***	-76.37***
		(12.71)	(14.50)	(17.29)
Night		-110.09***	-111.04***	-102.68***
		(11.36)	(10.95)	(13.43)
Unit FE	No	Yes	Yes	Yes
Weekday FE	No	Yes	Yes	Yes
Schedule FE	No	Yes	Yes	Yes
Month FE	No	No	Yes	Yes
Individual FE	No	No	No	Yes
R^2	0.016	0.083	0.230	0.359
Observations	13,801	13,801	13,801	13,801

Notes: All columns are OLS regressions. Dependent variable is the winning price in each auction (one *cuarta*). Robust standard errors in parentheses. FE stands for *Fixed Effects*. *Individual FE* refers to a set of dummy variables identifying different winners (names) in our sample. We obtain similar results including *Week FE* (a set of dummy variables identifying 52 or 53 weeks of the corresponding year). *** p<0.01, ** p<0.05, * p<0.1. Sample restricted to auctions with positive bids during the period January 1954 to August 1966.

Table 3: Timing Structure of Different Winners: Estimation Sample

Month	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	Total
1	4	6	0	3	1	11	3	0	0	0	28	0	11	61
2	4	4	0	2	4	2	3	0	0	0	19	0	21	57
3	5	3	0	9	0	1	2	0	0	10	29	8	23	79
4	0	2	0	6	2	5	4	6	0	17	28	38	28	121
5	5	7	0	6	1	13	9	6	9	4	32	30	31	130
6	3	7	0	7	0	8	10	7	10	14	23	25	29	119
7	2	3	0	6	9	26	8	5	13	15	17	21	23	117
8	9	3	0	3	4	10	7	14	18	15	21	16	3	102
9	8	8	0	3	8	10	5	13	0	8	35	19	0	97
10	8	7	3	2	11	2	0	9	0	10	16	19	0	78
11	7	2	3	0	8	2	0	4	0	21	29	23	0	82
12	1	0	2	2	3	1	0	0	0	36	18	12	0	69
Total	48	43	8	43	48	80	47	54	44	106	179	147	128	537

Notes: Total, in the last row, refers to the total number of *different winners* for the *specific year* (column). Given that, within a year, the same bidders win multiple units in several months, this number is below the sum over months, by year. Similarly for the last column, where *Total* is the number of *different bidders* for the *specific month* (row) during the 13-year sample. Finally, 537, refers to the total number of different bidders in the whole sample. The monthly average of different bidders who bought water in the sample (years 1954 to 1966) is 8.31.

Table 4: Rain Expectations and Regime Coordination

Variables	(1)	(2)	(3)
Future Rain	-1.6e-03***	-1.5e-03***	-1.6e-03***
	(0.5e-03)	(0.5e-03)	(0.6e-03)
Weekday FE	NO	YES	YES
Schedule FE	NO	YES	YES
Month FE	NO	NO	YES

Notes: Sample restricted to the one used in the structural estimation in Table 5. Almost identical results are obtained using the whole sample. All specifications are probit regressions. Marginal effects are reported. Robust standard errors in parenthesis. Dependent variable is a dummy variable equal to one if the regime is strict complements. *Future Rain* is a moving average of rain in Mula for seven days after the corresponding date of the auction (*Future Rain* is a proxy variable for farmers' rain expectations for the day where they are buying water). *Past Rain* (a moving average of rain in Mula for seven days before the corresponding date of the auction) and *Actual Rain* (the amount of rain in Mula in the day of the auction) are not statistically significant in any of the above regressions. *** p<0.01, ** p<0.05, * p<0.1.

Table 5: Structural Estimation

Structural parameters	Specifications					
	Sequential Auction Model				Standard Model	
	(1)	(2)	(3)	(4)	(5)	(6)
Mean Valuation $[\mathbb{E}(\hat{V})]$	150.343	130.012	155.782	135.201	166.991	142.863
	(12.814)	(11.047)	(17.608)	(14.674)	(87.420)	(81.089)
Sunk Cost ($\hat{\alpha}$)	0.030	0.0301	0.031	0.032		
	(6.9e-03)	(7.6e-04)	(2.1e-03)	(4.9e-03)		
$\hat{\beta}_0^c$	1.03e-02	1.01e-02	1.01e-02	1.02e-02		
	(2.6e-05)	(7.3e-05)	(5.0e-03)	(7.4e-04)		
$\hat{\beta}_0^s$	1.00e-02	1.02e-02	1.03e-02	1.49e-02		
	(0.0036)	(0.0019)	(4.3e-03)	(1.5e-03)		
Future Rain						
$\hat{\beta}_1^c$	-2.41e-14	-2.23e-17	-1.73e-12	1.82e-14		
	(7.1e-02)	(3.4e-03)	(8.0e-10)	(3.29e-10)		
$\hat{\beta}_1^s$	0.213	0.1828	0.2128	0.1828		
	(0.003)	(0.004)	(0.017)	(0.079)		
Mean $\hat{\rho}$						
- *Strict Complements*	0.239	0.205	0.237	0.219		
- *Weak Substitutes*	-0.332	-0.286	-0.322	-0.286		
N	8	10	8	10	8	10
Past Rain Polynomial	No	No	Yes	Yes	Yes	Yes
Schedule Dummy	No	No	Yes	Yes	Yes	Yes
Weekday Dummy Variables	No	No	Yes	Yes	Yes	Yes
Month Dummy Variables	No	No	Yes	Yes	Yes	Yes
Pseudo R^2	0.324	0.316	0.576	0.571	0.142	0.133
Log likelihood	-12,206	-13,068	-10,755	-11,298	-50,931	-54,957
# of Auctions	5,952	5,952	5,952	5,952	5,952	5,952

Notes: Bootstrapped standard errors (with $B = 1,000$ bootstrap repetitions) are reported in parenthesis (for the Mean Valuation it corresponds to the bootstrapped standard error corresponding to $Z_t'\gamma$). Estimates in columns 1 to 4 (sequential auction model) are obtained using the estimation procedure described in section 4 (see sections E and F in the online appendix for details). For the distribution of private values and inclusion of covariates, we use an exponential distribution. Estimates in specifications 5 and 6 (standard model) are MLE obtained by maximizing the likelihood function from a standard English auction model allowing the mean of the distribution of valuations depend on the same characteristics as in the other specifications as indicated in equation 7, without fixed costs nor decreasing marginal returns (the sample is the same as the one in columns 1 to 4, including in this case all sequential prices in the estimation). Number of years in the sample is 13. Number of months in the sample is 119. The number of different winners (across all 13 years) is 537. The complementarity parameter, ρ, is computed as detailed in the in section 3. When the goods are strict complements is given by $\rho_t^C = \frac{\alpha - 3\beta_t^C}{1-\alpha} = \frac{\alpha - 3(\beta_0^C + \beta_1^C R_t^F)}{1-\alpha}$. The table reports, for each specification of the sequential auction model in columns 1 to 4: $\hat{\bar{\rho}}^C = \frac{\hat{\alpha} - 3\left(\hat{\beta_0^C} + \hat{\beta_1^C} \frac{1}{T}(\sum_{t=1}^T D_{a,t} R_t^F)\right)}{1-\hat{\alpha}}$. Similarly, when the goods are weak substitutes, the table reports, for each specification: $\hat{\bar{\rho}}^S = \frac{\hat{\alpha} - 3\left(\hat{\beta_0^S} + \hat{\beta_1^S} \frac{1}{T}(\sum_{t=1}^T (D_{b,t} R_t^F + D_{c,t} R_t^F))\right)}{1-\hat{\alpha}}$.

Table 6: Structural Estimation (continued)

Structural parameters	Specifications					
	Sequential Auction Model				Standard Model	
	(1)	(2)	(3)	(4)	(5)	(6)
Covariates						
Past Rain ($\hat{\gamma}_1$)			-1.615	-1.415	-1.426	-1.227
			(0.238)	(0.605)	(0.436)	(0.334)
(Past Rain)^2 ($\hat{\gamma}_2$)			0.007	0.006	0.004	0.003
			(0.018)	(0.041)	(0.177)	(0.353)
Night ($\hat{\gamma}_3$)			-24.835	-22.3778	-30.014	-26.115
			(6.719)	(8.793)	(2.192)	(4.956)
Tuesday ($\hat{\gamma}_4$)			-1.416	-1.070	-2.642	-2.539
			(0.574)	(7.615)	(1.519)	(1.909)
Wednesday ($\hat{\gamma}_5$)			-2.228	-1.867	-5.858	-5.203
			(0.282)	(0.708)	(2.431)	(8.280)
Thursday ($\hat{\gamma}_6$)			-12.225	-10.437	-15.645	-13.584
			(0.623)	(0.710)	(5.330)	(6.755)
Friday ($\hat{\gamma}_7$)			-18.562	-15.679	-28.025	-24.461
			(7.610)	(7.723)	(11.27)	(1.979)
Feb. ($\hat{\gamma}_8$)			-11.304	-10.846	-4.955	-4.587
			(31.036)	(25.299)	(2.529)	(4.851)
Mar. ($\hat{\gamma}_9$)			25.717	22.328	34.976	30.358
			(12.459)	(8.809)	(9.847)	(7.257)
Apr. ($\hat{\gamma}_8$)			74.336	64.410	79.014	67.498
			(20.376)	(13.843)	(21.234)	(30.691)
May. ($\hat{\gamma}_{10}$)			122.740	106.571	114.391	96.805
			(20.835)	(41.612)	(46.658)	(26.490)
Jun. ($\hat{\gamma}_{11}$)			59.182	51.351	57.539	48.619
			(16.416)	(10.303)	(7.277)	(13.711)
Jul. ($\hat{\gamma}_{12}$)			205.034	178.187	225.191	191.960
			(48.903)	(17.205)	(39.410)	(32.087)
Aug. ($\hat{\gamma}_{13}$)			234.08	202.33	247.37	209.96
			(25.007)	(36.652)	(74.393)	(76.830)
Sep. ($\hat{\gamma}_{14}$)			77.844	62.152	87.986	75.459
			(30.352)	(27.768)	(39.766)	(16.519)
Oct. ($\hat{\gamma}_{15}$)			78.009	66.774	80.636	70.440
			(28.334)	(30.935)	(30.903)	(26.283)
Nov. ($\hat{\gamma}_{16}$)			6.115	4.937	13.049	11.044
			(2.410)	(1.122)	(4.605)	(6.827)
Dec. ($\hat{\gamma}_{17}$)			1.855	1.404	2.817	2.493
			(1.734)	(2.565)	(2.296)	(2.678)
Intercept ($\hat{\gamma}_0$)	150.343	130.012	90.994	79.948	101.771	87.825
	(12.814)	(11.047)	(23.757)	(31.541)	(54.609)	(82.320)
N	8	10	8	10	8	10
Past Rain Polynomial	No	No	Yes	Yes	Yes	Yes
Schedule Dummy	No	No	Yes	Yes	Yes	Yes
Weekday Dummy Variables	No	No	Yes	Yes	Yes	Yes
Month Dummy Variables	No	No	Yes	Yes	Yes	Yes
Pseudo R^2	0.324	0.316	0.576	0.571	0.142	0.133
Log likelihood	-12,206	-13,068	-10,755	-11,298	-50,931	-54,957
# of Auctions	5,952	5,952	5,952	5,952	5,952	5,952

Notes: See notes in Table 5.

Table 7: Counterfactuals

Panel A: Efficiency gains/losses per unit relative to the benchmark

	Myopic Farmers (1)	Large Units (2)	Paved Channels (3)
Sunk Costs	-28.56	0.20	12.91
	(14.97)	(1.37)	(6.82)
	[-60.63, -0.06]	[0,15.49]	[0.02,39.97]
Deacreasing Marginal Returns	27.31	-3.27	0
	(27.71)	(33.99)	-
	[0.05,479.22]	[-615.29,0]	-
Total	-1.25	-3.06	12.91
	(35.72)	(24.12)	(6.82)
	[-60.63,479.22]	[-615.29, 15.49]	[0,39.97]

Panel B: Total efficiency gains/losses relative to the benchmark

	Myopic Farmers (1)	Large Units (2)	Paved Channels (3)
Sunk Costs	-42,497.36	303.94	19,208.09
Deacreasing Marginal Returns	40,651.43	-4,867.68	0
Total	-1,846.59	-4,563.74	19,208.09

Notes: The table displays the welfare measures computed in the three counterfactual scenarios described in section 5. All numbers are in *pesetas*, and report comparisons of the welfare relative to the benchmark case in specification 3 from Tables 5 and 6 (*i.e.* the benchmark case has: non-myopic farmers, single units with right to use 3 hours of water for irrigation, and non-paved channels). Positive numbers indicate efficiency gains relative to the benchmark; negative numbers indicate efficiency losses relative to the benchmark. Panel A displays the efficiency gain/loss per unit (mean across units and farmers). Standard deviation is reported in parenthesis. The largest and smallest values are reported in squared brackets. Panel B displays the total efficiency gain/loss in each counterfactual scenario. The total efficiency measure is the sum (across all units and farmers) gain/loss per unit during the 13 years sample period in our data. In both panels, for each set of welfare measures, we report the efficiency gains/losses due to sunk cost, decreasing marginal returns, and the total change. The latter is defined as the sum of the efficiency gains/losses due to sunk cost plus decreasing marginal returns.

Appendix to "Complements and Substitutes in Sequential Auctions: The Case of Water Auctions"

(For Online Publication)

Javier D. Donna and José-Antonio Espín-Sánchez

The Ohio State University   Yale University

August 1, 2017

Javier D. Donna
Department of Economics
The Ohio State University
1945 N High St, 425 Arps Hall
Columbus, OH 43210
Phone: 614-688-0364
Email: donna.1@osu.edu

José-Antonio Espín-Sánchez
Department of Economics
Yale University
27 Hillhouse Ave
New Haven, CT 06511
Phone: 203-432-0890
Email: jose-antonio.espin-sanchez@yale.edu

Appendix.

This is the online appendix for "Complements and Substitutes in Sequential Auctions: The Case of Water Auctions" by Javier D. Donna and Jose-Antonio Espín-Sánchez.

A Data Description.

In this section we provide a detailed description of the auction allocating system and the data.

To get a sense of the industry context during the period under analysis, we present a brief description for the region's demographics, agriculture production and weather.[1] Murcia's population share in Spain was around 3% during the period. As a municipality, Mula comprised 2% of Murcia in 1954, ranking Mula 20th in terms of population. The three main citrus fruits produced in the area are apricot, lemon, and peach trees. Murcia's share of these crops was 50% (2.3 million), 44% (1.5 million), and 42% (4.3 million), respectively, in terms of Spain's total production of these fruits for the year 1962. *Regadío* land in Murcia constitutes 4% (70,000 ha) of Spain's.

In terms of purchasing power, one peseta from 1950 is approximately equivalent to 0.43 USD from 2013. Note that $\frac{43.06}{166.39} \times 1.18 \times 1.40 = 0.43$ where 43.06 is the purchasing power of one peseta from 1950 in 1999,[2] 166.39 is the exchange rate of pesetas per euro in January 1999 (obtained from the European Central Bank), 1.18 is the exchange rate of U.S. dollars per euro in January 1999 (obtained from the European Central Bank), and 1.40 is the purchasing power of one dollar from 1999 in 2013.[3]

Mediterranean climate rainfall occurs mainly in spring and autumn. Peak water requirements for the products cultivated in the region are reached in spring and summer, between April and August. During this period more frequent irrigation is advisable because citrus trees are more sensitive (in terms of quality of production) to water deficits.

Weather is important for our analysis as it is a determinant of seasonality. The coastal strip of southeast Spain is the most arid region of all continental Europe due to the *foehn* effect, and because of its location to the west of the mountain chain *Sistema Penibético*, which includes the *Mulhacen* (the second highest mountain in Europe). Although annual average rainfall is 320 mm, rainfall frequency distribution is skewed, making the majority of years dryer than this annual average. The number of days when torrential rain occurs is not particularly high, but when such rain occurs it is substantial. As an example, on October 10^{th} 1943, 681 mm of rain water were measured in Mula, more than twice the yearly average for our sample.

It is important to identify cases in the four-unit auctions where the same farmer won four or three sequential units, so as to avoid imposing further structure on the dynamic strategic considerations of the bidders, which are outside the scope of this paper. A complete characterization of the equilibrium when goods are strict substitutes is outside the scope of this paper. It would require further structure on the primitives of the model as the equilibrium depends on the believes about other player's types, and the strategies, that each bidder have. We analyze these dynamics effects in Donna and Espin-Sanchez (2016). For a complete characterization the equilibrium when goods are strict substitutes, $K = 2$, and

[1] These descriptive statistics are obtained from *Population* and *Agricultural Census* from the *National Statistics Institute* of Spain (INE) (available online here).

[2] Obtained from *Servicios de estudios BBVA,* available online here.

[3] Obtained from the U.S. Bureau of Labor Statistics, available online here.

$-1 < \rho < 0$, see theorem 9 on p. 35 in our earlier working paper Donna and Espin-Sanchez (2012). Following the model from Section 4 in the paper, we have selected for our estimation auctions where a single person wins all units or where the last winner also won two out of the first three units (for a total of three units). This represents 54% of the total number of water units sold in our sample as displayed in Table A1. The table exhibits the frequency distribution of units sold by number of units bought by the same farmer. Overall, 42% $(5,880/13,992)$ of the units were sold in 4CU.

Selected summary statistics for the main variables are provided in Table A2. Interestingly, in the sample used in the structural estimation, the counter-intuitive positive correlation between average daily winning prices and daily rainfall recovers its "correct" sign (statistically significant at 1% level) once we condition on seasonality. The endogeneity issue arises because both demand (due to the nature of the trees) and supply (rainfall) are high during spring, generating an artificial positive correlation between the variables that is, ultimately, caused by seasonality (we further discuss seasonality below).

Table A3 displays appropriate intervals for watering citrus.

We are able to observe the identity of the winner in our data. This allows us to identify auctions where the same farmer buys all units (4CU). Complementary data from the agricultural census, where we also observe the identity of the land owners, allows us to match these characteristics to each auction's winner. Aside from the variation in these characteristics, which is important to justify our conditional-independence assumption across auctions, these data allow us to confirm specialized bidding behavior from certain outliers who own a great amount of land and, therefore, bid and win more often. This is depicted in Figure A2.

Table A1: Frequency Distribution of Units Bought by the Same Farmer

	Number of Units	Frequency	Cumulative
1CU	3,530	0.20	3,530
2CU	2,866	0.17	6,396
3CU	1,716	0.10	8,112
4CU	5,880	0.34	13,992
No bids	3,203	0.19	17,195
Total	17,195	1.00	

The table displays the frequency distribution of units in the auctions disaggregated by the units bought sequentially by the same farmer: (i) 1CU refers to the case where the same farmer buys only one unit; (ii) 2CU refers to the case where the same farmer buys two sequential units; (iii) 3CU refers to the case where the same farmer buys three sequential units; finally, (iv) 4CU refers to the case where the same farmer buys all four consecutive units. There are no observations where the same farmer buys more than four consecutive units, nor observations where the same farmer buys consecutive units across days (*e.g.* there are no observations where the same farmer buys the last units of a day-auction, and the first units of the night-auction). No bids refers to cases where auctions were run but where no one bid for the last units (see, for example, the last four units in Figure 7 in the paper).

Table A2: Summary Statistics of Selected Variables

Variable	Mean	SD	Min	Max	Obs
Rain (mm/m^2)	8.53	46.33	0	980.00	3,834
Price (pesetas)	271.61	374	.05	4,830	13,872
Land Extension (hectares)	5.54	32.24	.25	900	819
Selling Price (pesetas)	15.07	222.52	.02	5,700	964
Kg sold	5,569.70	10,003.76	0	110,000	1,000
Number of Trees	161.49	493.45	1	12,300	946

Summary Statistics of Selected Variables. *SD* stands for *Standard Deviation*, *Obs* stands for *Total Number of Observations*.

Table A3: Irrigation Requirements for Citrus Trees

Timing after planting	Month				
	Dec. - Feb.	Mar. - Apr.	May - Jun.	Jul. - Sep.	Oct. - Nov.
0 - 1 month	2 to 3 days				
2 - 3 months	3 to 5 days				
4 months to 1 year	14 days	7 to 10 days	5 to 7 days	2 to 5 days	5 to 10 days
1 to 2 years	14 to 21 days	10 to 14 days	7 to 10 days	7 to 10 days	10 to 14 days
3 years or older	21 to 30 days	14 to 21 days	14 days	10 to 14 days	14 to 21 days

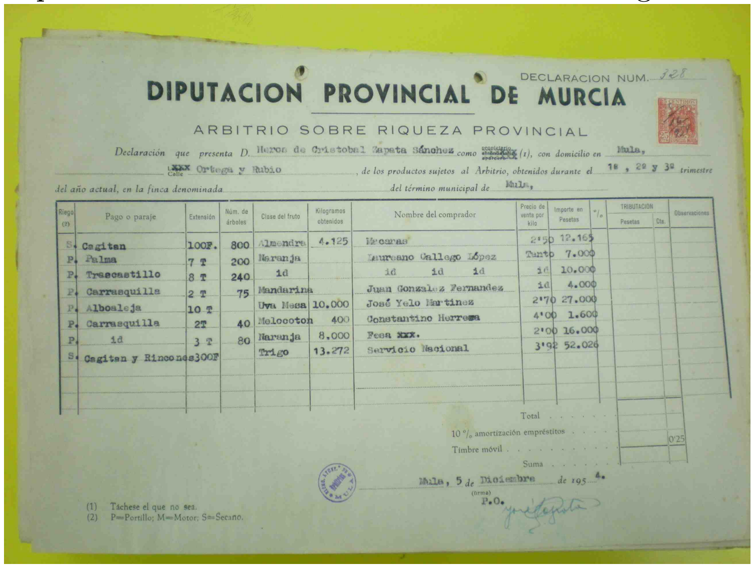
Obtained from Table 2 in Wright (2000), modified from Chott and Bradley (1997).

Table A4: Regions of the Likelihood and Covariates

			Strict Complements Region a) (equation 1)	Weak Substitutes Region b) (equation 2)	Weak Substitutes Region c) (equation 3)
Price (mean in pesetas)			322.97	260.34	253.31
Rain (mean in mm/m^2)			5.47	5.35	3.40
Percentage of observations in each	Quarter	Jan-Mar	12.2	18.2	22.0
		Apr-May	42.4	29.2	34.2
		Jul-Sep	31.4	29.9	19.5
		Oct-Dec	14.0	22.7	24.43
Percentage of observations in each	Schedule	Day	55.0	57.8	41.5
		Night	45.0	42.2	58.5
Percentage of observations in each	Weekday	Mo	23.0	25.9	41.5
		Tu	22.1	19.8	14.6
		We	20.9	20.7	17.1
		Th	19.6	17.7	9.8
		Fr	14.4	15.9	17.0

The table displays a comparison of the covariates in the three regions of the likelihood. In region a) of the likelihood, winning prices are determined by equation 1 in the paper. In region b), winning prices are determined by equation 2 in the paper. In region c), winning prices are determined by equation 3 in the paper. Note that, for each region (a, b, and c) the numbers sum up to 100% by *Quarter*. For example, for region a), $12.2 + 42.4 + 31.4 + 14.0 = 100$. The same is true for *Schedule* and *Weekday*.

Figure A1: Sample of Individual Data Obtained from the Agricultural Census



DECLARACION NUM. 328

DIPUTACION PROVINCIAL DE MURCIA

ARBITRIO SOBRE RIQUEZA PROVINCIAL

Declaración que presenta D. Heros de Cristobal Zapata Sánchez como propietario arrendatario aparcero (1), con domicilio en Mula, Calle Ortega y Rubio, de los productos sujetos al Arbitrio, obtenidos durante el 1º, 2º y 3º trimestre del año actual, en la finca denominada ______ del término municipal de Mula,

Riego (2)	Pago o paraje	Extensión	Núm. de árboles	Clase del fruto	Kilogramos obtenidos	Nombre del comprador	Precio de venta por kilo	Importe en Pesetas	%	TRIBUTACION Pesetas	Cts.	Observaciones
S.	Cagitan	100F.	800	Almendra	4.125	Mecaras	2'50	12.165				
P.	Palma	7 T	200	Naranja		Laureano Gallego López	Tanto	7.000				
P.	Trascastillo	8 T	240	id		id id id	id	10.000				
P.	Carrasquilla	2 T	75	Mandarina		Juan Gonzalez Fernandez	id	4.000				
P.	Alboaleja	10 T		Uva Mesa	10.000	José Yelo Martinez	2'70	27.000				
P.	Carrasquilla	2T	40	Melocoton	400	Constantino Herrera	4'00	1.600				
P.	id	3 T	80	Naranja	8.000	Fesa XXX.	2'00	16.000				
S.	Cagitan y Rincones	300F		Trigo	13.272	Servicio Nacional	3'92	52.026				

Total . . . . . .

10 % amortización empréstitos . . . .

Timbre móvil . . . . 0'25

Suma . . . . . .

Mula, 5 de Diciembre de 1954.

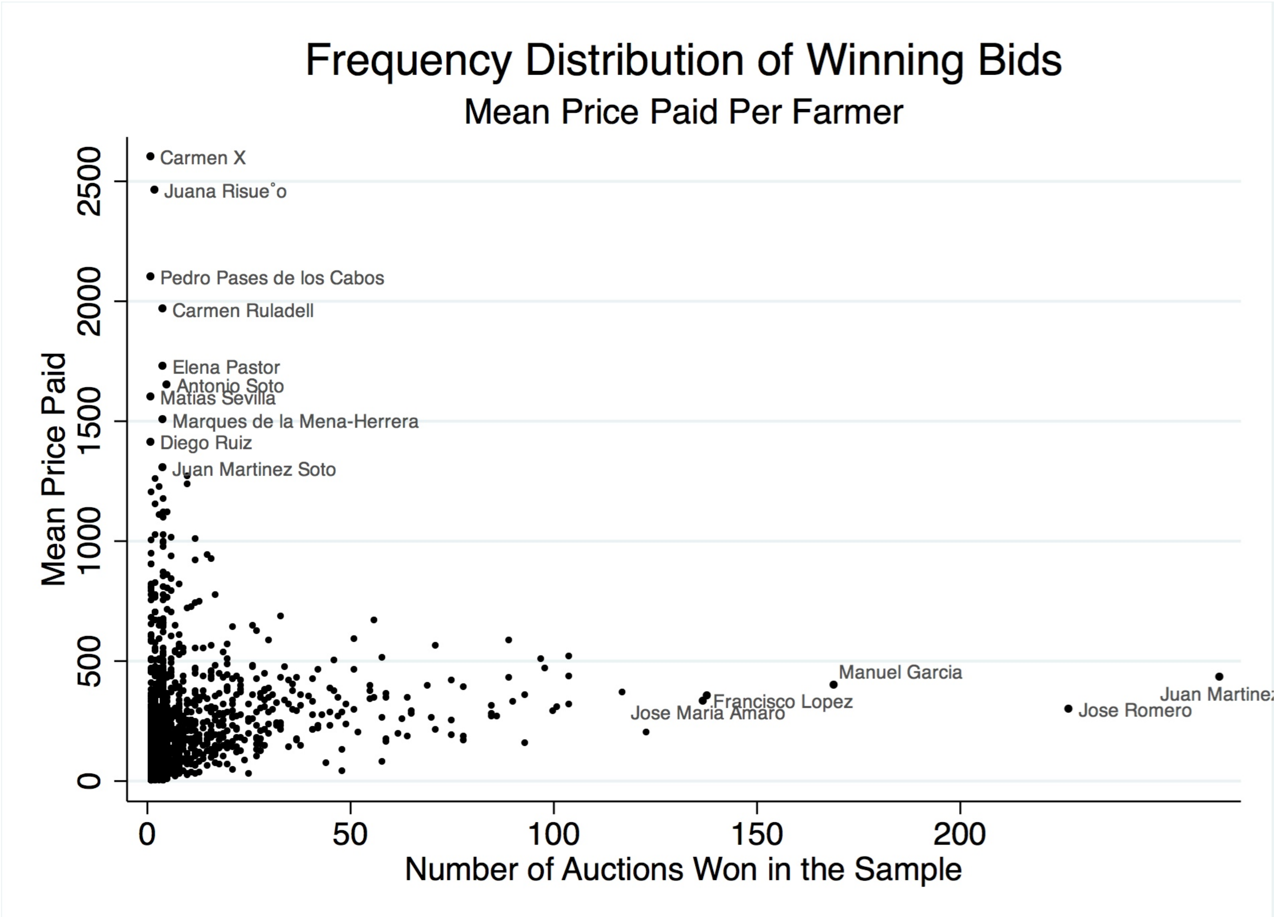
(firma)

P.O.

(1) Táchese el que no sea.
(2) P=Portillo; M=Motor; S=Secano.

Notes: Sample Card from a Farmer Obtained from the Agricultural Census. Individual characteristics include: farmers' name (that we match to the names in the auctions), type of land and location, area, number of trees, production and the price at which this production was sold in the census year.

Figure A2: Winning Farmers and Winning Prices



The figure displays the distribution of Bids per Person (number of auction that the farmer won in the whole sample) and the Mean Price (in the auctions that the farmer won). Each point correspond to a different bidder. In total, there are 537 different points (farmers) who won auctions. Outliers are in red with labels.

B Proofs and Extensions.

In all the proofs below we consider the cases where the number of players is sufficiently high to ensure potential competition in all stages, which correspond to the empirical application. In particular we assume that $N > K + 2$. We also restrict attention to the cases where c is very small compared to $\rho_k \cdot v_i$.

B.1 Strict Complements.

Assumption 1 [A1]: $\rho_1 \leq \rho_4$.

Assumption 2 [A2]: $\rho_1 + \rho_2 \leq \rho_3 + \rho_4$.

Proposition 1. *In a* strict complements *regime (i.e., when A1 and A2 both hold) the pure strategy symmetric Perfect Bayesian Equilibrium is:*

- *First auction:*

- *Participation: bidder i will always participate in the first auction, i.e.* $y_i^1 = 1$.
- *Bidding Strategy:* $b_i^1(v_i) = \sum_{k=1}^{4} \rho_k \cdot v_i - 4c$.

- *Second, third, and fourth auctions:*
  - *Participation: bidder i participates in all the remaining auctions if, and only if, she won the first auction, i.e.* $y_i^2 = y_i^3 = y_i^4 = 1$ *if, and only if,* $x_i^1 = 1$.
  - *Bidding Strategy: If bidder i participates in the second, third or fourth auction ($y_i^k = 1$ for $k = 2, 3, 4$), she will continue bidding until the price reaches her own valuation for the remaining goods,* $b_i^l(v_i) = \sum_{k=l}^{4} \rho_k \cdot v_i - (4 - l)\, c$.

Proof. The first step of this proof consists of proving that in any revealing equilibrium, i.e., an equilibrium where it becomes common knowledge after the first round who is the bidder with the highest valuation, only the winner (the bidder with the highest type) will enter the remaining auctions, and pay the cost c in each of them. Since both a direct mechanism and the sequential auction will give the same utility to the winner, and both will give the four objects to the bidder with the highest valuation, the total utility for the winner should be $W_i \equiv \sum_{k=1}^{4} \rho_k \cdot v_i - 4c - \left[\sum_{k=1}^{4} \rho_k \cdot v_j - 4c\right] = \sum_{k=1}^{4} \rho_k \cdot (v_i - v_j)$ in both cases, where j is the bidder with the second highest valuation. For the case of a direct mechanism, we assume that there is a cost of communication for each of the auctions, which should be paid by every bidder who wants to win the object.

The second step is to show that the winner will pay $\sum_{k=1}^{4} \rho_k \cdot v_j - 4c$ in the first auction. This payment, together with the utility the winner gets from the four goods, $\sum_{k=1}^{4} \rho_k \cdot v_i$, and the cost of entering all the auctions, $4c$, will give her the same utility as in the direct mechanism: $\sum_{k=1}^{4} \rho_k \cdot v_i - \left(\sum_{k=1}^{4} \rho_k \cdot v_j - 4c\right) - 4c = \sum_{k=1}^{4} \rho_k \cdot v_i - \sum_{k=1}^{4} \rho_k \cdot v_j \equiv W_i$. The utility for the winner in the second auction is $\rho_2 \cdot v_i - c$, because the equilibrium price in the second auction is zero, and the utility for the loser is zero. Similarly, the utility for the winner is $\rho_3 \cdot v_i - c$ in the third auction and $\rho_4 \cdot v_i - c$ in the fourth auction. Thus the total value of winning the auction is $\sum_{k=1}^{4} \rho_k \cdot v_i - 4c$. This four-unit auction is equivalent to the following single-unit auction. Let us define $z_i = \sum_{k=1}^{4} \rho_k \cdot v_i - 4c$ and consider a single-object auction in which the valuation for the good for bidder i is z_i. Thus, we return to the standard single-unit auction, and $b_i = z_i = \sum_{k=1}^{4} \rho_k \cdot v_i - 4c$ is a weakly dominant strategy.

The third step is to show that given this set of strategies, there are no profitable deviations for the other players. In particular, since the sequence of ρ_k need not be strictly increasing, some players might find it profitable to bid for the remaining units in either the second, third, or fourth auctions. If a loser in the first auction, player j, chooses to deviate and bid in the second auction for the remaining units, her utility, after entering the second auction, of winning the the three remaining units is $\sum_{k=1}^{3} \rho_k \cdot v_j - 2c$, since she will win three units. The utility for the winner in the first auction, player i, of winning the remaining three units is $\sum_{k=2}^{4}\rho_k \cdot v_i - 2c$, since she already has one unit. There is no profitable deviation whenever $\sum_{k=1}^{3}\rho_k \cdot v_j - 2c < \sum_{k=2}^{4}\rho_k \cdot v_i - 2c$. Rearranging we get $\rho_1 \cdot v_j < \rho_4 \cdot v_i$, which always hold due to A1 and the fact that player i was the winner in the first auction, i.e. $v_i > v_j$. We also need to check that there is no profitable deviation in the third auction. In this case, the utility of winning the remaining two units for player j is $\sum_{k=1}^{2}\rho_k \cdot v_j - c$ and for player i is $\sum_{k=3}^{4}\rho_k \cdot v_i - c$. Rearranging we get $(\rho_1 + \rho_2)\, v_j < (\rho_3 + \rho_4)\, v_i$, which always hold due to A2 and the fact that player i was the winner in the first auction, i.e. $v_i > v_j$. Finally we need to check that no loser wants to deviate in the last auction, that is $\rho_1 \cdot v_j < \rho_4 \cdot v_i$, which again is always true by A1.

When A1 and A2 hold, the winner in the first auction always has a greater valuation for the remaining units than the other players and hence she will deter them from entering.

We have proven that in any revealing equilibrium only the winner will enter the remaining auctions. Her utilities in the second, third, and fourth auctions are $\rho_2 \cdot v_i - c$, $\rho_3 \cdot v_i - c$ and $\rho_4 \cdot v_i - c$, respectively. Since her utility in the first auction is $\rho_1 \cdot v_i - c$, the revenue equivalence theorem shows that she will bid $b_i^1 = \sum_{k=1}^{4}\rho_k \cdot v_i - 4c$ and pay $p^1 = \sum_{k=1}^{4}\rho_k \cdot v_j - 4c$. Hence, this is a revealing equilibrium. Therefore, we have shown that, given the payoffs in the second, third, and fourth auctions, there is only one possible payoff and one possible bid for every player in the first auction. Hence, this is also the unique symmetric equilibrium in pure strategies. $\square$

B.2 Weak Substitutes.

Lemma 1. *In a* weak substitutes regime *(i.e., when neither A1 nor A2 holds) the probability that a bidder different from the winner enters the last auction is decreasing in the participation cost, c. Moreover, this probability goes to* 1 *when c goes to zero, that is*

$$\lim_{c \to 0} \left\{ \Pr \left(\sum_{j \neq i} y_j^4 \geq 1 \mid x_i^1 = 1 \right) \right\} = 1.$$

Proof. The full solution for equilibrium in this case is not provided here for simplicity. The results are available upon request. In the equilibrium with weak substitutes, the bidding function in the first auction is strictly increasing for low values of v_i, then it is flat (pooling) without a jump for intermediate values of v_i, and then it resembles a step function with jumps and locally flat regions. If the valuation of the highest bidder lies in the first region, the equilibrium is similar to the case of complements. In the other regions, the valuations are not fully revealed and more than one player will enter the remaining auctions with positive probability. It can be shown that the support of the first region, where the bidding function is strictly increasing, shrinks as $c \to 0$. Hence, in the limit, the relevant case corresponds to a bidding function that is not strictly increasing.

If the equilibrium bidding in the first auction is constant or is a step function, then the analysis is less restrictive.[4] In a pooling bidding region, bidders with valuations within the same interval bid the same amount, and ties are broken randomly. In that case all the losers of the tiebreaker will enter the following auction. The winner of the tiebreaker may or may not enter the following auction.

[4] At the limit when $c = 0$, the equilibrium is not pooling, as shown by Black and De Meza (1992).

If the equilibrium bidding strategies in the first auction are strictly increasing, then, after the first auction, it will become common knowledge that $v_1 \sim \tilde{F}_1(v_1) \equiv F_1(v_1 \mid v_1 > v_2)$, where bidder 1 is the winner of the first auction and bidder 2 is the second highest bidder in the first auction, i.e. the bidder with the highest valuation among the losers. The expected utility of entering the last auction for bidder 2 is:[5]

$$\rho_1 v_2 \Pr[\rho_1 v_2 > \rho_4 v_1] - c - p^4 = \rho_1 v_2 \tilde{F}_1\left(\frac{\rho_1}{\rho_4} v_2\right) - c - p^4. \tag{B.1}$$

If no losers from the first auction enter the last auction, then $p^4 = 0$ and in particular we know bidder 2's expected utility from entering is nonpositive, so $\rho_1 v_2 \tilde{F}_1\left(\frac{\rho_1}{\rho_4} v_2\right) \leq c$. Thus, if $\rho_1 v_2 \tilde{F}_1\left(\frac{\rho_1}{\rho_4} v_2\right) > c$, then at least one loser must enter the last auction. Notice that the left hand side is strictly positive since $\frac{\rho_1}{\rho_4} v_2 > v_2$ by the weak substitutes assumption. Since c only appears on the right hand side, the probability of this event is decreasing in c. Formally, we have:

$$\lim_{c \to 0} \left\{ \Pr\left(\sum_{j \neq i} y_j^4 \geq 1 \mid x_i^1 = 1 \right) \right\} \geq \lim_{c \to 0} \left\{ \Pr\left[\rho_1 v_2 \tilde{F}_1\left(\frac{\rho_1}{\rho_4} v_2\right) > c \right] \right\} = 1.$$

It is worth noticing that the result only applies when $\rho_1 > \rho_4$. Otherwise the term inside the probability becomes $([0 > c])$, which is impossible and thus, the limit is equal to zero, which is the result in the strict complements regime. □

Lemma 2. *In a* weak substitutes *regime (i.e., when neither A1 nor A2 holds) it is a weakly dominant strategy for all bidders to bid their marginal valuations in the last auction, conditional on entering the auction.*

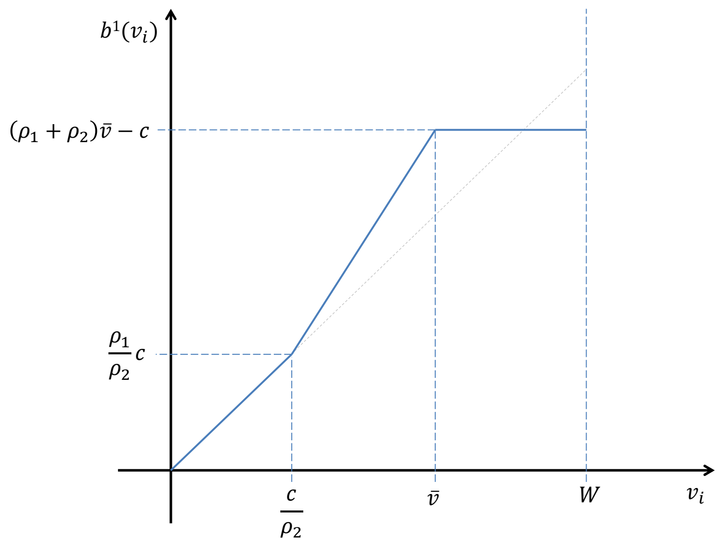
Proof. In this case, there is no equilibrium in weakly dominant strategies. However, in any PBE, the bidding strategy in the last auction will always be to bid one's own valuation, conditional on entering. Moreover, any strategy involving entering the last auction and bidding something either than one's valuation is dominated either by not entering the last auction or by entering and bidding one's valuation. □

B.3 Equilibrium with two substitutes goods

In this section we fully characterize the equilibrium when there are two bidders, two units and goods are substitutes ($\rho_1 > \rho_2$). This "simple" case is already fairly complicated and our intention here is to show how the equilibrium looks like. Solving for the equilibrium in the general case would be more cumbersome. For simplicity of exposition, we restrict the parameter space for the analysis here. The assumptions below are needed in order to have only one "flat" bidding area and no "jumps" in the bidding function. The equilibrium could still be characterize without the assumptions below, but the full characterization would involve several cases and would be too cumbersome. Here we assume that the distribution of types F has a bounded support and the maximum value that v_i can take is W. We also need to define the value of $\overline{v}$ as the value that solves this equation

[5] Note that the winner in the first auction has complete information in the remaining auctions, because all equilibrium bidding strategies are strictly increasing and everybody observed the prices at which each loser dropped. Hence, a necessary condition for equilibrium is that if bidder 2 wins the last auction, bidder 1 does not enter that auction.

Figure A3: Equilibrium bidding strategies in the first auction.



Notes: The figure depicts a typical equilibrium bidding strategies for the first auction. In this case, $\rho_1 = 1$, thus for low values the bidding function coincides with the 45 degree line. Also, we take the case with $\rho_1 + \rho_2 > 1$, thus for moderate values the slope is greater than the 45 degree line. Finally, the support of the distribution of types is bounded above by W thus the bidding function stops there.

$$Prob\left[\rho_2 v_j < \rho_1 \overline{v} + c \middle| v_j \geq \overline{v}\right] \rho_1 \overline{v} - c = 0$$

Here $\overline{v}$ is defined as the value of v_i such that a bidder with that valuation would be indifferent to participating in the second auction given she lost the first auction. Notice that at this point the true valuation of bidder i is common knowledge since her bidding strategy was strictly increasing (see below). At this point bidder i knows that bidder j has a valuation greater than $\overline{v}$, thus bidder j will only enter when her utility from doing so is positive (see below). Also, upon entering, since it is the last auction, each bidder will bid their valuation for that good. Thus, the expected value of participating in the second auction is the probability that bidder i (wth type $\overline{v}$ and valuation for the good $\rho_1\overline{v}$) wins the auction times his valuation minus the entry cost. Notice that, since bidder j knows the valuation of bidder i she will only enter the auction if she expects to win. That, if bidder i wins it is because bidder j did not entered and she gets the good for free.

With this definition, we can know describe the basic structure of a symmetric equilibrium as shown in Figure A3. The equilibrium bidding strategy for the first auction has the following structure, up to some threshold (c) bidders bid their valuations for the first unit. Then, from there to another threshold ($\overline{v}$) bidders bid higher than their valuation according to a strictly increasing function. Finally, after $\overline{v}$, bidders pool. Depending on the values of the parameters, there can be multiple pooling steps, i.e. bidders pool for a while, then at a certain valuation, the bidding function jumps discretely to a higher pooling region. To avoid the complication of multiple pooling regions in the main analysis we make the following assumptions:

Assumption A3:

$$\overline{v} \geq \frac{2\rho_2}{\rho_1+\rho_2}W$$

Along with this assumption, in order to simplify the analysis further, we want to assume that no bidder who wins the first auction from the "pooling" region, will want to enter the second auction. This simplifies the analysis greatly.

Assumption A4:

$$\{\rho_2 W - E\left[v_l | v_l > \overline{v}\right]\} \, Prob\left[\rho_2 W < \rho_1 v_l | v_l > \overline{v}\right] < c$$

Assumption A4 ensures that the winner of the first auction never enters the second auction.[6]

Theorem 1. *Under assumptions A3 & A4, the unique symmetric equilibrium satisfies the following properties:*

(a) First Auction:

- *All bidders participate in the first auction $y_i^1 = 1$ for all i.*
- *The bidding strategy for bidder i is*

$$b_i^1(v_i) = b^1(v_i) = \begin{cases} \rho_1 v_i & \text{if } v_i \leq \frac{c}{\rho_2} \\ (\rho_1+\rho_2)v_i - c & \text{if } \frac{c}{\rho_2} < v_i \leq \overline{v} \\ (\rho_1+\rho_2)\overline{v} - c & \text{if } v_i \geq \overline{v} \end{cases}.$$

(b) Second Auction:

- *Case 1: No pooling in the first auction (at most one bidder has valuation above $\overline{v}$). Bidder i's participation function $(y_{i-np}^2(v_i, x_i^1))$ is*

$$y_{i-np}^2(v_i, 0) = 0$$

$$y_i^2(v_{i-np}, 1) = \begin{cases} 0 & \text{if } v_i \leq \frac{c}{\rho_2} \\ 1 & v_i > \frac{c}{\rho_2} \end{cases}$$

- *Case 2: Pooling in the first auction (both bidders above $\overline{v}$). Bidder i's participation function $(y_{i-p}^2(v_i, x_i^1))$ is:*

$$y_{i-p}^2(v_i, 0) = 1$$

$$y_{i-p}^2(v_i, 1) = 0$$

- *If bidder i participates in the second auction, his bidding strategy is:*

$$b^2(v_i, x_i^1) = \begin{cases} \rho_1 v_i & \text{if } x_i^1 = 0 \\ \rho_2 v_i & \text{if } x_i^1 = 1 \end{cases}$$

[6]When assumption $A1$ holds but assumption $A2$ does not, the winner in the first auction may enter the second auction, even when both bidders have valuations above $\bar{v}$. The proof in this case is analogous to the one below.

Proof. The proof will proceed by backwards induction. Without loss of generality, assume bidder 1 is the bidder with the highest valuation, $v_1 > v_2$. Notice that when $c \to 0$ the probability that we are in the pooling region goes to one. This is because $c \to 0$ implies $\overline{v} \to 0$. When the number of bidders increases this probability also increases.

2-(c): From the bidders' perspective, once the participation decision has been made, the final auction is a standard English auction. As a result, it is a dominant strategy of this stage game for each bidder to bid their valuations.

2-(b)-Case-2: This is the case after pooling in the first auction, that is $v_1, v_2 \geq \overline{v}$. We will write down the relevant expected utilities from entering and argue that any deviation (in pure strategies) results in negative expected utility. In this case, since the winner of the auction is determined by a random probability lottery, we will refer to the winner and loser valuations as v_w and v_l, respectively. This is simply to emphasize that in this pooling region, the entry decision only depends on the outcome of the first auction, not the relative valuations. First, consider the loser of the first auction. The strategy states that she enters the second auction. The only possible deviation in pure strategies is not to enter the second auction, but given the winner's strategy (not to enter), we get:

$$\begin{aligned} EU[y^2_{i-p}(v_l, 0) = 1 | y^2_{i-p}(v_i, 1) = 0] =& \rho_1 v_l - c \geq \rho_1 \overline{v} - c \\ \geq Prob\left(\rho_2 v_w < \rho_1 \overline{v} \middle| v_w > \overline{v}\right) \cdot \rho_1 \overline{v} - c =& 0 = EU[y^2_{i-p}(v_l, 0) = 0 | y^2_{i-p}(v_i, 1) = 0] \end{aligned}$$

Where the first inequality comes from the fact that both bidders have valuations above $\overline{v}$, the second inequality comes from the fact a probability is equal or smaller than 1 and the second equality comes from the definition of $\overline{v}$. The first and third equalities are just the computation of the expected utility of the loser under the equilibrium strategy and the potential deviation respectively. Hence, there is no profitable deviation for the loser.

Now consider the winner of the first auction. By following the strategy $y^2_{i-p}(\cdot, \cdot)$, this bidder will not enter the auction and get an expected utility of zero. Now, the only possible pure strategy deviation is to enter the auction, which gives him an expected utility of:[7]

$$\begin{aligned} & Prob\left(\rho_1 v_l < \rho_2 v_w | v_l > \overline{v}\right) \cdot \{\rho_2 v_w - E\left[\rho_1 v_l | v_l > \overline{v}\right]\} - c \\ \leq \quad & Prob\left(\rho_1 v_l < \rho_2 v_w | v_l > \overline{v}\right) \cdot \left[\rho_2 W - E\left[\rho_1 v_l | v_l > \overline{v}\right] - c\right] \leq 0 \end{aligned}$$

where the first inequality comes from W being the upper bound of the distribution, i.e., $v_w \leq W$ and the second inequality comes from $A2$. Thus, by entering, the winner will now get negative utility and, therefore, she has no profitable deviation.[8]

2-(a)-Case-1: Now consider the participation decision when no pooling occurred in the first auction, that is: $v_2 \leq \overline{v} < v_1$. Consider bidder 1. Her strategy is simply a threshold strategy. As long as his valuation for the second good is larger than the cost of entry, she will enter the second auction. Since bidder 2 will not enter, bidder 1 will win the second object and pay a price of 0. Therefore, assuming bidder 2 follows $y^2_{i-np}(\cdot, 0)$, bidder 1's strategy is optimal. We must show that given this strategy by bidder 1, the strategy of bidder 2 is

[7]Notice that when both bidders are pooling, they only know that the valuation of the other bidder is above $\overline{v}$.

[8]Note that this result depends only on $A2$. If $A2$ does not hold, but $A1$ does, then there could be an equilibrium where the winner also enters the second auction if his valuation is high enough.

optimal. The threshold of bidder 2, $\overline{v}$, is the solution to:

$$Prob\left[\rho_2 v_1 < \rho_1 \overline{v} + c \middle| v_1 \geq \overline{v}\right] \rho_1 \overline{v} - c = 0$$

That is, since bidder 2's valuation will become public after the first auction when there is no pooling, bidder 1 will enter whenever his valuation is such that he will have a positive utility, that is when he will win the auction and pay the entry cost $\rho_2 v_1 - \rho_1 \overline{v} - c \geq 0$. We have to show that this threshold is still binding when bidder 2 knows that bidder 1 will enter with any valuation above c/ρ_1, i.e. for any undominated strategy. In this case, the only deviation to consider is that bidder 2 enters after losing for some valuation less than $\overline{v}$. First, note that, if $\rho_1 v_2 < c$, by entering, bidder 2 will get a negative utility, regardless of bidder 1's entry choice. There is one case left to analyze: $\rho_1 v_2 \geq c$. According to the equilibrium strategies, bidder 1 will always enter. Then, by deviating and entering, bidder 2 gets:

$$\begin{aligned} & Prob\left(\rho_2 v_1 < \rho_1 v_2 | v_1 > v_2\right) \cdot \rho_1 v_2 - c \\ \leq \quad & Prob\left(\rho_2 v_1 < \rho_1 v_2 + c | v_1 > v_2\right) \cdot \rho_1 v_2 - c \quad \leq 0, \quad \forall\, v_2 < \overline{v} \end{aligned}$$

where the first inequality always holds since the probability below is smaller than the probability above and the second inequality comes from the definition of $\overline{v}$ and the fact that $v_2 \leq \overline{v}$. Since not entering gives 0 expected utility, this deviation is not profitable.
Thus, there is no profitable deviation from the second auction participation strategy.

1-(b): We need to show that if bidders are following the bidding strategy for the first auction as described in Theorem 1 above then they have no profitable deviations. We will consider each part of the function separately and show that any deviation results in lower utility for the bidders. For simplicity we assume that $v_1 > v_2$.

$\boldsymbol{\rho_2 v_i \leq c}$: Consider an upwards deviation, $\tilde{b}^1(v_i) = \rho_1 v_i + \varepsilon$, for $\varepsilon > 0$. This will not affect bidder 1. She will win with probability 1 and still pay v_2.[9] Consider bidder 2. If she follows $\tilde{b}^1(\cdot)$, she will either still lose and get 0 or she will now win with positive probability and get utility:

$$U_2 = \rho_1 (v_2 - v_1) < 0$$

Thus, no bidder will gain from deviating from the prescribed strategy upwards. Consider a downwards deviation. If bidder 2 deviates, she will lose with probability one, and will get utility of 0. The same is true for the case where $v_1 = v_2$: either deviation still results in utility of 0. However, if bidder 1 plays $\tilde{b}(v_1) = \rho_1 v_1 - \varepsilon$, with positive probability, he will lose when he is the highest type, thus getting 0 utility when previously she was getting $\rho_1 (v_1 - v_2) > 0$. Thus, downward deviations are also not profitable. Therefore, when $\rho_2 v_i < c$, the strategy in Theorem 1 has no profitable deviations. Notice that in this case the entry cost for the second auction is so high that no bidder will enter the second auction, thus the game becomes a second price auction with a single good, and bidding your own valuation for that good is a weakly dominant strategy.

$\boldsymbol{\frac{c}{\rho_2} < v_i < \overline{v}}$: Agents bid above their valuation for the first object, because if they win, they will also get the second object for free, after paying the entry cost to the second auction. Thus, the "effective" valuation of the first object is $(\rho_1 + \rho_2)v_i - c$. Consider an upwards deviation, $\tilde{b}^1(v_i) = (\rho_1 + \rho_2)v_i - c + \varepsilon$. Bidder 1 will still win with probability one

[9] In the case where $v_1 = v_2$, any upward deviation will still result in a utility of 0 or negative.

and pay the same price. By bidding $\tilde{b}^1(v_2)$, bidder 2 would win the first auction with positive probability. His utility would be:

$$\rho_1 v_2 - [(\rho_1 + \rho_2) v_1 - c] + \rho_2 v_2 - c = (\rho_1 + \rho_2)(v_2 - v_1) < 0$$

Thus, an upwards deviation makes no bidder better off, and some strictly worse off. Now, consider a downwards deviation, $\tilde{b}^1(v_i) = (\rho_1 + \rho_2) v_i - c - \varepsilon$. Bidder 2 will still lose with probability 1. Now, bidder 1 may lose, and thus obtains utility of 0 from deviating. Playing the prescribed strategy would give him positive utility:

$$v_1 - b(v_2) + \rho_2 v_1 - c \geq v_1 - (\rho_1 + \rho_2) v_2 - c + \rho_2 v_1 - c = (\rho_1 + \rho_2) \cdot (v_1 - v_2) > 0$$

Thus, no downwards deviation is profitable.

$\mathbf{v_i > \overline{v}}$: The first step is to show that no jump in bidding occurs at $\overline{v}$. Then, we will show that bidders will pool for all values between $\overline{v}$ and W. The difference between this and the previous case is that the loser of the first auction now gets positive utility from entering the second auction. Consider the case where $v_i = \overline{v} + \varepsilon$. We now show that such agents will bid $(\rho_1 + \rho_2)\overline{v} - c$. Assuming ties are broken with an equal probability, the utility to agent i bidding $b^1(v_i)$ is:

$$\begin{aligned} U_i &= \frac{1}{2}\{\rho_1 v_i - [(\rho_1 + \rho_2)\overline{v} - c]\} + \frac{1}{2}[\rho_1 v_i - c] \\ &= \rho_1 v_i - \left(\frac{\rho_1 + \rho_2}{2}\right)\overline{v} \\ &= \left(\frac{\rho_1}{2}\right)(v_i - \overline{v}) + \frac{1}{2}(\rho_1 v_i - \rho_2 \overline{v}) \\ &> 0 \text{ if } \rho_1 > \rho_2 \end{aligned} \tag{B.2}$$

This is true for all $\varepsilon > 0$. Now we show that deviating from this results in lower utility. First, consider an upward deviation, $\tilde{b}^1(v_i) = (\rho_1 + \rho_2)\overline{v} - c + \varepsilon$. $A1$ implies that bidders prefer to stay in the "pool" than deviating upwards and getting both goods and paying $(\rho_1 + \rho_2)\overline{v} - c$ for the first unit and the entry cost c for the second. If upon deviating upward, the loser instead enters the second auction. This will give the winner a utility of:

$$\begin{aligned} U_i &= \rho_1 v_i - [(\rho_1 + \rho_2)\overline{v} - c] + 0 \\ &= (v_i - \overline{v})\rho_1 - (\rho_1 + \rho_2)\overline{v} + c \\ &\xrightarrow{\varepsilon \to 0} c - (\rho_1 + \rho_2)\overline{v} \\ &< 0 \end{aligned}$$

by assumption. Thus, bidders will not bid strictly higher at $\overline{v}$. Moreover, $A2$ implies that no bidder with type $\overline{v} < v_i < W$ will enter the second auction upon winning the first one

Now consider a downward deviation for bidder i with $v_i > \overline{v}$, $\tilde{b}^1(v_i) = (\rho_1 + \rho_2)\overline{v} - c - \varepsilon$. Since both players have valuations above $\overline{v}$ a downward deviation means that the equilibrium price will be lower than $(\rho_1 + \rho_2)\overline{v} - c$ and thus the winner (with type v_w) will enter the second auction, believing than the other player has a valuation below $\overline{v}$. Then, the player who deviates will not get the first object and will have to compete for the second object with the other player. Playing the equilibrium strategy gives

$$\frac{1}{2}\left\{\rho_1 v_i - \left[(\rho_1+\rho_2)\,\overline{v} - c\right] + \rho_1 v_i - c\right\} = \rho_1 v_i - \left(\frac{\rho_1+\rho_2}{2}\right)\overline{v}$$

The alternative strategy gives:

$$Prob\,(\rho_1 v_i > \rho_2 v_w | v_w > \overline{v}) \cdot \{\rho_2 v_w - E\left[\rho_2 v_w | v_w > \overline{v}\right]\} - c$$

We need to show then that

$$\begin{array}{rcl}
\rho_1 v_i - \left(\frac{\rho_1+\rho_2}{2}\right)\overline{v} & \geq & Prob\,(\rho_1 v_i > \rho_2 v_w | v_w > \overline{v}) \cdot \{\rho_2 v_i - E\left[\rho_2 v_w | v_w > \overline{v}\right]\} - c \\
\rho_1 v_i - \left(\frac{\rho_1+\rho_2}{2}\right)\overline{v} & \geq & \{\rho_2 v_i - E\left[\rho_2 v_w | v_w > \overline{v}\right]\} \\
\rho_1 v_i - \left(\frac{\rho_1+\rho_2}{2}\right)\overline{v} & \geq & \rho_2 v_i - \rho_2 \overline{v} \\
2\rho_1 v_i - \rho_1 \overline{v} & \geq & 2\rho_2 v_i - \rho_2 \overline{v} \\
2 v_i & \geq & \overline{v}
\end{array}$$

which is always true. Notice that this result holds for any value of c. Under the equilibrium strategy the entry costs does not play any role, since they are discounted in the bidding in the first auction.

□

C Regime Determination

To determine the regime being played, we use the model's prediction of the prices in each regime. According to the model, when goods are strict complements a price of zero is paid by the winner of the first unit, for the second, third, and fourth units. In practice (according to the auctioneer who ran the auctions, see Botia, interview), bidders did not pay a price of zero, but symbolic prices that are "close" to zero as can be seen in Panel B in Figure 1 in the paper. Thus, when goods are complements the difference between the price paid for the first and the remaining units is large (or, analogously, the ratio is large). When goods are substitutes, the units might be bought by different bidders and the prices of all four units are similar as can be seen in Panel A in Figure 1 in the paper.

Thus, the difference between the price paid for the first and the remaining units is negligible (or, analogously, the ratio is low). This allows to separate the data into the four categories described in the paper by looking at the identities of the winner (*i.e.* whether the same bidder bought all the four units), and the difference (or ratio) between the price paid for the first and the remaining units. Thus, to classify each regime, we use: (1) the identity of the winner (*i.e.* whether the same bidder bought all the four units), and (2) the difference (or ratio) between the price paid for the first and the remaining units. Specifically, we classify a regime as complements when the ratio between the prices in the first auction relative to the mean price in the second, third, and fourth auctions is greater than a threshold of 5. This is just another way of identifying "symbolic" or "token" prices paid in the second, third, and fourth auctions. We have experimented with a number of different thresholds (*e.g.* 2, 3, and 4.) and specifications (*e.g.* using the averages of prices paid in the second, third, and fourth auctions, the median, the maximum, the minimum, *etc.*), and obtained almost identical results.

In principle one could estimate a simultaneous equation switching regression model along the lines of Porter (1983) and Ellison (1994). In such model, the parameters that characterize demand would be estimated as in this section conditional on the regime classification, the regime classification would be unknown, and the parameters governing the distribution of the regime classification would be estimated by, e.g., an adaptation of the E-M algorithm (Kiefer 1980). However, in our case the regime is observed, not unobserved as it is in Porter's case. We can determine the regime classification directly by examining the data, from the observed prices. The first-order distinguishing feature of the regime are the predicted price patterns by the theoretical model. Panels A and B in Figure 1 show one instance of how these patterns look throughout the data. This is the most straightforward and, thus, our preferred approach.

D Predicted Prices and High Definition Figures.

In this Subsection we describe how we compute the predicted prices of Figure 8 in the paper. We also present four high definition versions of Figure 8 in the paper.

D.1 Predicted Prices.

In all cases, Figure 8 (in the paper) and Figures A4-A7 compares the average monthly prices from the data (using the sample from the three regions of the likelihood as defined in section 4 in the paper) and average monthly prices predicted by the following three models.

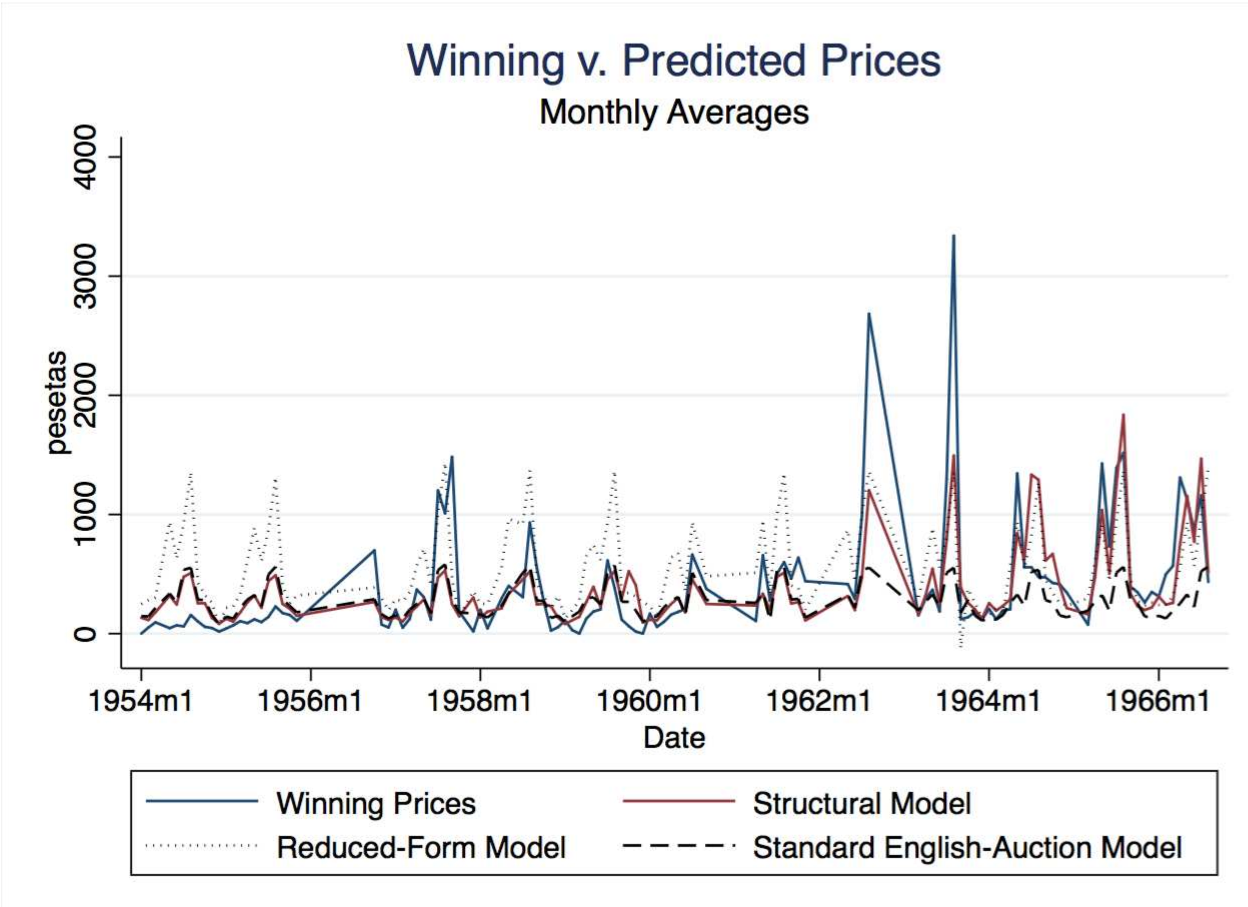
Structural Model. We use the estimates from specification 3 in Tables 5 and 6 in the paper. Then we use the model equations (equations 1, 2, and 3 along with equations 5 and 7 in the paper) and the observed covariates from the data to simulate predicted prices for each region of the likelihood. We then compute the monthly averages of these predicted prices.

Standard English Auction Model. We use the estimates from specification 5 in Tables 5 and 6 in the paper. We then repeat the procedure done for the structural model setting the SC and DMR equal to zero. Note that the estimate of the mean valuation of this model is different from the estimate of the mean valuation from the structural model (see section 5 in the paper for further details). So the v_t^i differ when simulating these models.

Reduced-Form Model. Using the same sample as for the structural and standard english auction models, we run en OLS regression of winning prices on Past Rain, unit (3 dummy variables), weekday (4 dummy variables), schedule (1 dummy variable), month (11 dummy variables), year (12 dummy variables), and individual fixed effects, in addition to a constant (for details about the reduced-form specification see Table 2 discussed in Subsection 2.4 in the paper). We then use the estimated OLS coefficients to compute the predicted prices. We then compute the monthly averages of these predicted prices.

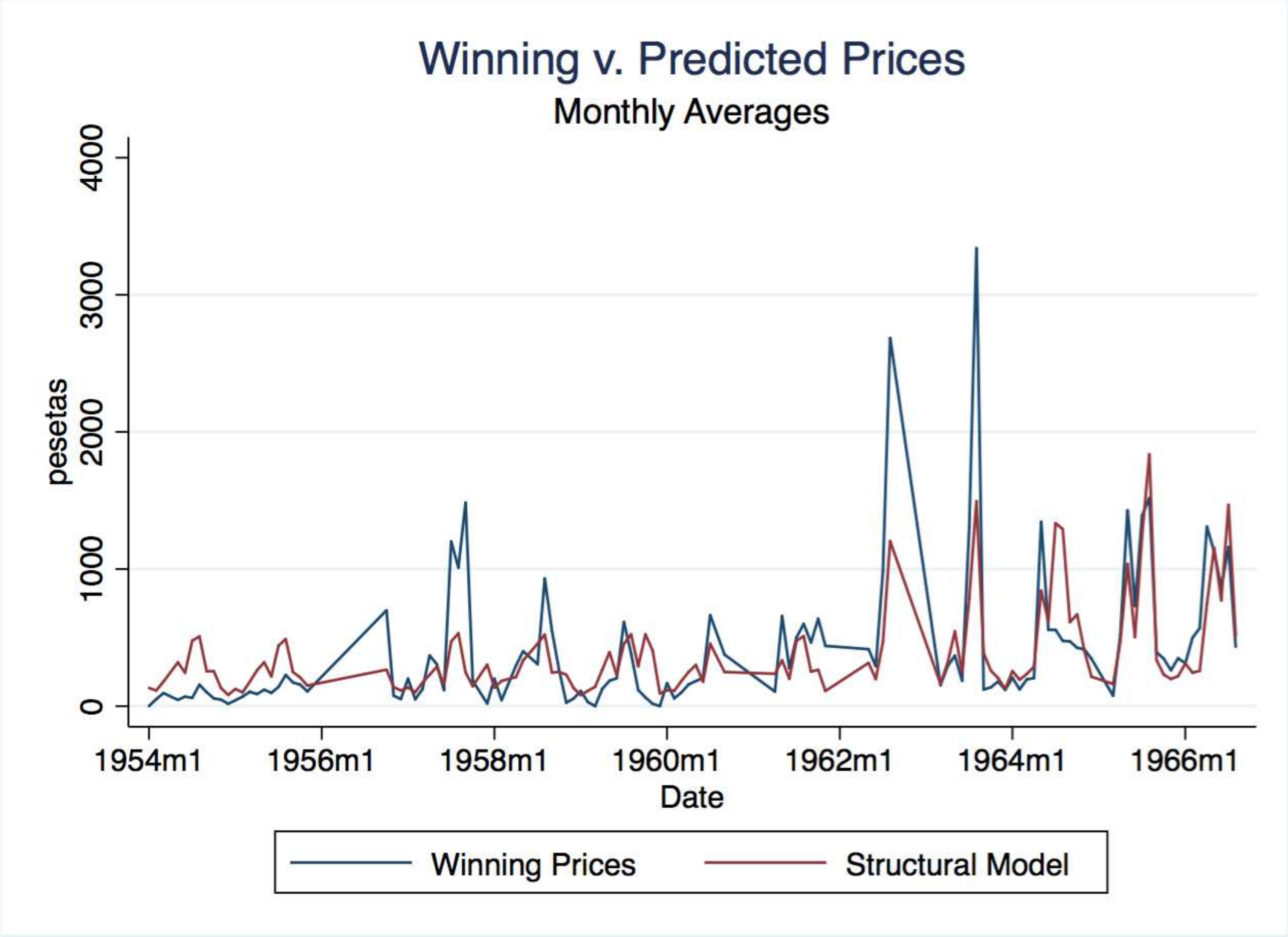
D.2 High Definition Figures.

Figure A4: Winning and Estimated Prices



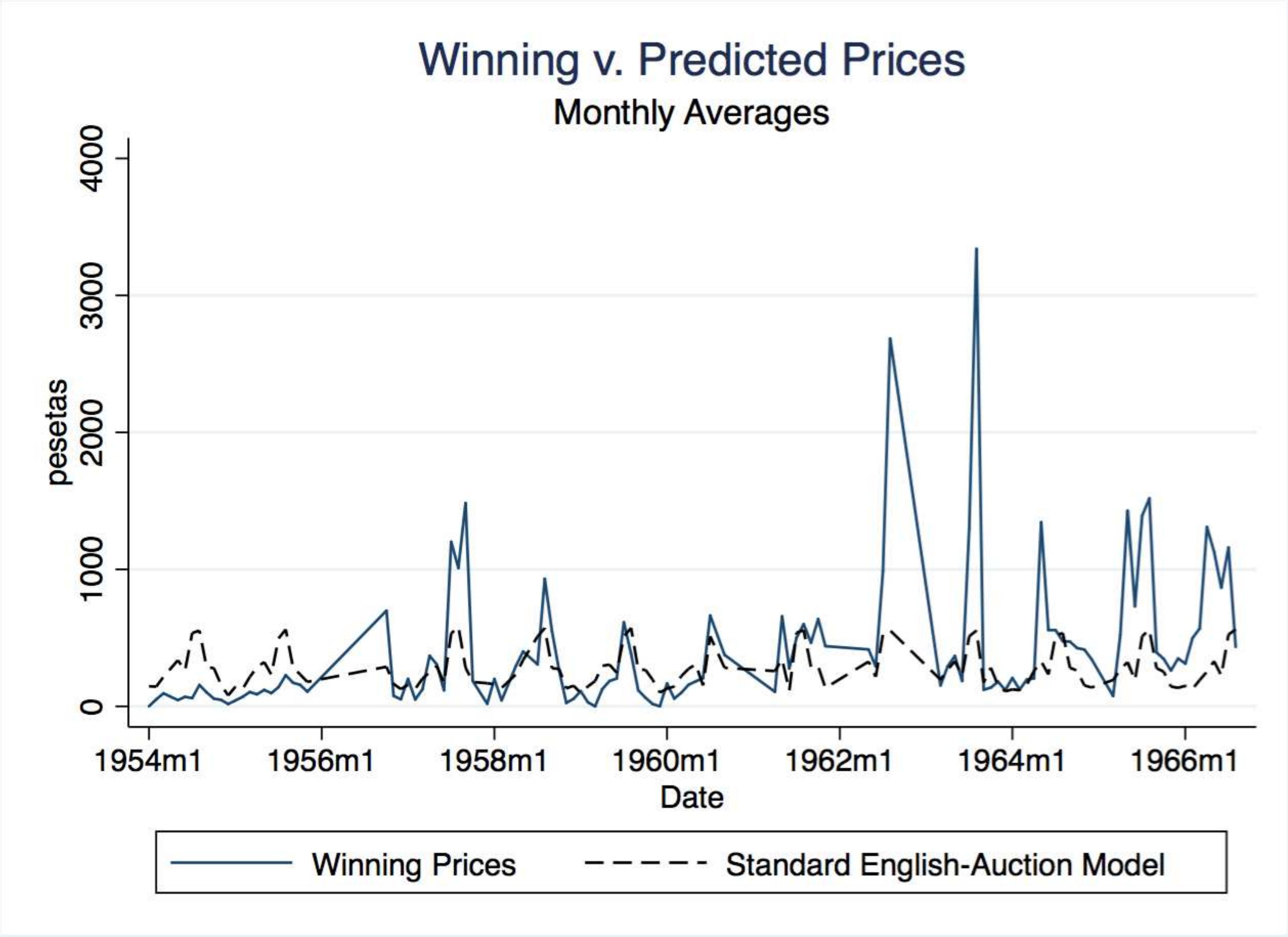
This is a high definition version of Figure 8 in the paper. The figure displays real prices against predicted prices using three different models: (i) our structural model (specification 3 in Tables 5 and 6 in the paper), (ii) a standard (button) English auction model (specification 5 in Tables 5 and 6 in the paper), and (iii) a reduced-form model for the sample using as regressors: *Past Rain*, unit (3 dummy variables), weekday (4 dummy variables), schedule (1 dummy variable), month (11 dummy variables), year (12 dummy variables), and individual fixed effects, in addition to a constant. The graph shows the mean monthly averages of the prices.

Figure A5: Winning and Estimated Prices



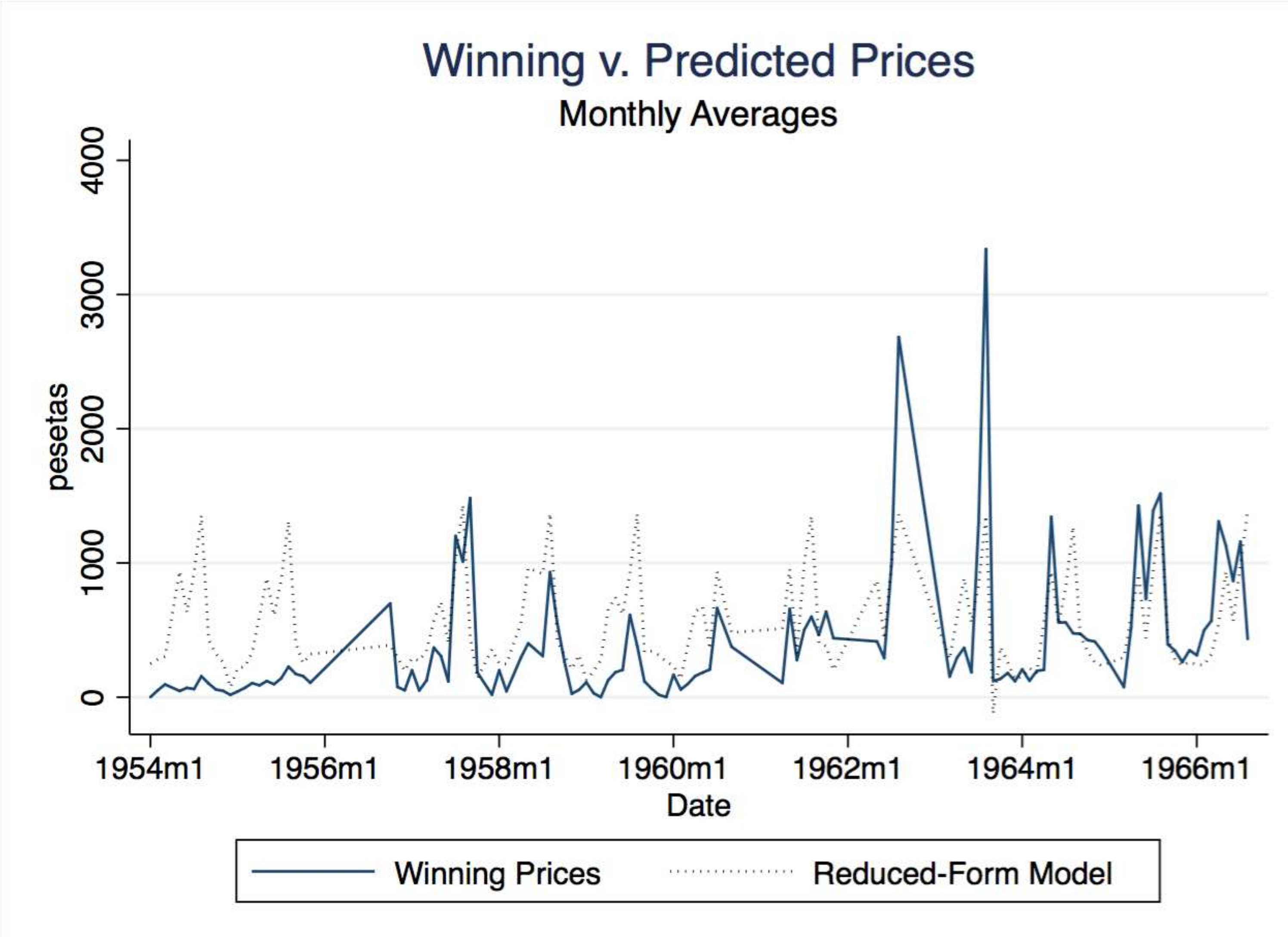
This is a high definition version of Figure 8 in the paper. The figure displays real prices against predicted prices using our structural model (specification 3 in Tables 5 and 6 in the paper). The graph shows the mean monthly averages of the prices.

Figure A6: Winning and Estimated Prices



This is a high definition version of Figure 8 in the paper. The figure displays real prices against predicted prices using a standard (button) English auction model (specification 5 in Tables 5 and 6 in the paper). The graph shows the mean monthly averages of the prices.

Figure A7: Winning and Estimated Prices



This is a high definition version of Figure 8 in the paper. The figure displays real prices against predicted prices using a reduced-form model for the sample using as regressors: *Past Rain*, unit (3 dummy variables), weekday (4 dummy variables), schedule (1 dummy variable), month (11 dummy variables), year (12 dummy variables), and individual fixed effects, in addition to a constant. The graph shows the mean monthly averages of the prices.

E Estimation: Implementation Details.

Note from equation 6 in the paper that, conditional on θ, the log-likelihood function is the sum of three components: a part associated with the winning bids where the same bidder wins all four units and goods are strict complements, a part where the same bidder wins all four units and goods are substitutes, and a last part where the last winner also bought two out of the first three units, three units in total. To recover the parameters we follow an iterative two stage procedure. In the first stage, conditional on θ, we recover the parameter vector $\gamma \in \mathbb{R}^{19}$ by MLE using the distribution of the $(N-1)^{th}$ order statistic.[10] In the second stage, conditional on γ, we recover $\theta \in \mathbb{R}^5$ by generalized method of moments (GMM, Hansen 1982) using the model moment equations. We then iterate these two stages until convergence.[11] Implementing the estimation in two stages results in substantial computational savings (to compute the bootstrapped standard errors) as we express analytically the solution of the gradient of the GMM estimator.[12] [13]

[10]For the exponential distribution we obtain this distribution numerically. For the Exponentiated Gamma distribution we are able to obtain a closed-form solution of this expression (see footnote 31 in the paper). This motivated its use originally (see Donna and Espin-Sanchez (2012)).

[11]For the initial condition we use a consistent estimate of γ and θ obtained by full maximum likelihood estimation using equation 6 in the paper (*e.g.* Aguirregabiria and Mira (2002) and Aguirregabiria and Mira (2007)).

[12]For details about the moment equations, the empirical analogues, and the gradient that we use in our program see Section E in this online appendix.

[13]We performed a Monte Carlo study to evaluate how well the proposed estimation procedure performs in our setting. Using a smaller subset of parameters (7 instead of 22) we did not register substantial improvement in efficiency of MLE relative to the two-stage estimator. We obtained similar point estimates. The Monte Carlo study is available online in the earlier working paper (Donna and Espin-Sanchez 2012).

F Moment Conditions, Gradient and Empirical Analogues

1 Moment Conditions

1.1 Moments

To recover the structural parameters, α and β , we use the following moment conditions. To simplify the notation, let $\theta = \left(\alpha, \beta_0^C, \beta_1^C, \beta_0^S, \beta_1^C\right)$, $Y_t \equiv \left(p_t, R_t, v_{N-1:N,t}, \{D_t^j\}_{j\in\{a,b,c\}}, c\right)$ and $\bar{x}_j = \frac{1}{\sum_{t=1}^T 1[D_{j,t}=1]} \sum_{t=1}^T D_{j,t} x_t$, $j \epsilon \{a, b, c\}$.

$$f_1(Y_t, \theta) \equiv \mathbb{E}\left(D_{a,t} \sum_{k=1}^4 p_t^k - \left[4 - \alpha - 6\left(\beta_0^C + \beta_1^C D_{a,t} R_t\right)\right] D_{a,t} v_{N-1:N,t} + 3c\right) = 0$$

$$f_2(Y_t, \theta) \equiv \mathbb{E}\left(D_{b,t} p_t^4 - (1-\alpha) D_{b,t} v_{N-1:N,t}^t + c\right) = 0$$

$$f_3(Y_t, \theta) \equiv \mathbb{E}\left(D_{c,t} p_t^4 - \left[1 - \beta_0^S + \beta_1^S D_{c,t} R_t\right] D_{c,t} v_{N-1:N,t} + c\right) = 0$$

$$f_4(Y_t,\theta) \equiv \mathbb{E}\left(\left[D_{a,t}\sum_{k=1}^{4}p_t^k - \bar{p}_a\right]^2 - \left[\left(4-\alpha-6\beta_0^C\right)\left(D_{a,t}v_{N-1:N,t} - \bar{v}_{N-1:N}\right)\right.\right.$$
$$\left.\left. - 6\beta_1^C\left(D_{a,t}R_t v_{N-1:N,t} - \bar{R}_a\bar{v}_{N-1:N}\right)^2\right)\right. = 0$$

Or, equivalently:

$$f_1\left(Y_t,\theta\right) = \mathbb{E}\left(D_{a,t}\sum_{k=1}^{4}p_t^k\right) - \left[4-\alpha-6\left(\beta_0^C + \beta_1^C\mathbb{E}\left(\sum_{t=1}^{T}D_{a,t}R_t\right)\right)\right]\mathbb{E}\left(D_{a,t}v_{N-1:N,t}\right) + 3c = 0$$

$$f_2\left(Y_t,\theta\right) = \mathbb{E}\left(D_{b,t}p_t^4\right) - (1-\alpha)\mathbb{E}\left(D_{b,t}v_{N-1:N,t}\right) + c = 0$$

$$f_3\left(Y_t,\theta\right) = \mathbb{E}\left(D_{c,t}p_t^4\right) - \left[1-\beta_0^C + \beta_1^C\mathbb{E}\left(D_{c,t}R_t\right)\right]\mathbb{E}\left(D_{c,t}v_{N-1:N,t}\right) + c = 0$$

$$f_4\left(Y_t,\theta\right) \equiv \mathbb{V}\left(D_{a,t}\sum_{k=1}^{4}p_t^k\right) - \left(4-\alpha-6\beta_0^C\right)^2\mathbb{V}\left(D_{a,t}v_{N-1:N,t}\right)$$
$$+ 12\left(4-\alpha-6\beta_0^C\right)\beta_1^C\mathbb{E}\left(D_{a,t}R_{a,t}\right)\mathbb{V}\left(D_{a,t}v_{N-1:N,t}\right) - 36\left(\beta_1^C\right)^2 C_{a,t} = 0$$

$$f_5\left(Y_t,\theta\right) \equiv \mathbb{V}\left(D_{b,t}p_{b,t}^4\right) - (1-\alpha)^2\,\mathbb{V}\left(D_{b,t}v_{N-1:N,t}\right) = 0$$

$$
\begin{aligned}
f_6\left(Y_t,\theta\right) \equiv & \,\mathbb{V}\left(D_{c,t}p_{c,t}^4\right) - \left(1-\beta_0^S\right)^2 \mathbb{V}\left(D_{c,t}v_{N-1:N,t}\right) \\
& - 2\left(1-\beta_0^S\right)\beta_1^S \mathbb{E}\left(D_{c,t}R_{c,t}\right)\mathbb{V}\left(D_{c,t}v_{N-1:N,t}\right) + \left(\beta_1^C\right)^2 C_{c,t} = 0
\end{aligned}
$$

where $\mathbb{V}\left(\cdot\right)$ denotes variance and:

$$
C_{j,t} \equiv \left[\mathbb{E}\left(D_{j,t}R_t\right)\right]^2 \mathbb{V}\left(D_{j,t}v_{N-1:N,t}\right) + \left[\mathbb{E}\left(D_{j,t}v_{N-1:N,t}\right)\right]^2 \mathbb{V}\left(D_{j,t}R_t\right) + \mathbb{V}\left(D_{j,t}v_{N-1:N,t}\right)\mathbb{V}\left(D_{j,t}R_t\right)
$$

, $j \epsilon \{a, c\}$.

1.2 Moments Empirical Analogues

For the estimation we use the empirical analogues, $\hat{f}_i(Y_t,\theta) = \frac{1}{T}\sum_{t=1}^T f_i(Y_t,\theta)$, $i = 1, \ldots, 6$. Specifically:

$$
\begin{aligned}
\hat{f}_1\left(Y_t,\theta\right) = & \frac{1}{\sum_{t=1}^T 1\left[D_{a,t}=1\right]}\sum_{t=1}^T D_{a,t}\sum_{k=1}^4 p_t^k - \left[4-\alpha-6\left(\beta_0^C + \beta_1^C \frac{1}{\sum_{t=1}^T 1\left[D_{a,t}=1\right]}\sum_{t=1}^T D_{a,t}R_t\right)\right] \\
& \frac{1}{\sum_{t=1}^T 1\left[D_{a,t}=1\right]}\sum_{t=1}^T D_{a,t}v_{N-1:N,t} + 3c = 0
\end{aligned}
$$

$$
\hat{f}_2\left(Y_t,\theta\right) = \frac{1}{\sum_{t=1}^T 1\left[D_{b,t}=1\right]}\sum_{t=1}^T D_{b,t}p_t^4 - (1-\alpha)\frac{1}{\sum_{t=1}^T 1\left[D_{b,t}=1\right]}\sum_{t=1}^T D_{b,t}v_{N-1:N,t} + c = 0
$$

$$
\begin{aligned}
\hat{f}_3\left(Y_t,\theta\right) = & \frac{1}{\sum_{t=1}^T 1\left[D_{c,t}=1\right]}\sum_{t=1}^T D_{c,t}p_t^4 - \left[1-\beta_0^C + \beta_1^C \frac{1}{\sum_{t=1}^T 1\left[D_{c,t}=1\right]}\sum_{t=1}^T D_{c,t}R_t\right] \\
& \frac{1}{\sum_{t=1}^T 1\left[D_{c,t}=1\right]}\sum_{t=1}^T D_{c,t}v_{N-1:N,t} + c = 0
\end{aligned}
$$

$$
\begin{aligned}
\hat{f}_4\left(Y_t,\theta\right)= & \frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]}\sum_{t=1}^{T}\left[D_{a,t}\sum_{k=1}^{4}p_t^k-\frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]}\sum_{t=1}^{T}D_{a,t}\sum_{k=1}^{4}p_t^k\right]^2 \\
& -\left(4-\alpha-6\beta_0^C\right)^2\frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]}\sum_{t=1}^{T}\left[D_{a,t}v_{N-1:N,t}-\frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]}\sum_{t=1}^{T}D_{a,t}v_{N-1:N,t}\right]^2 \\
& +12\left(4-\alpha-6\beta_0^C\right)\beta_1^C\frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]}\sum_{t=1}^{T}D_{a,t}R_t\frac{1}{\sum_{t=1}^{T} 1} \\
& \left[D_{a,t}=1\right]\sum_{t=1}^{T}\left[D_{a,t}v_{N-1:N,t}-\frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]}\sum_{t=1}^{T}D_{a,t}v_{N-1:N,t}\right]^2-36\left(\beta_1^C\right)^2C_{a,t}=0
\end{aligned}
$$

$$
\begin{aligned}
\hat{f}_5\left(Y_t,\theta\right)= & \frac{1}{\sum_{t=1}^{T} 1\left[D_{b,t}=1\right]}\sum_{t=1}^{T}\left[D_{b,t}p_t^4-\frac{1}{\sum_{t=1}^{T} 1\left[D_{b,t}=1\right]}\sum_{t=1}^{T}D_{b,t}p_t^4\right]^2 \\
& -\left(1-\alpha\right)^2\frac{1}{\sum_{t=1}^{T} 1\left[D_{b,t}=1\right]}\sum_{t=1}^{T}\left(D_{b,t}v_{N-1:N,t}-\frac{1}{\sum_{t=1}^{T} 1\left[D_{b,t}=1\right]}\sum_{t=1}^{T}D_{b,t}v_{N-1:N,t}\right)^2=0
\end{aligned}
$$

$$
\begin{aligned}
\hat{f}_6\left(Y_t,\theta\right)= & \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]}\sum_{t=1}^{T}\left[D_{c,t}p_t^4-\frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]}\sum_{t=1}^{T}D_{c,t}p_t^4\right]^2-\left(1-\beta_0^S\right)^2\frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \\
& \sum_{t=1}^{T}\left[D_{c,t}v_{N-1:N,t}-\frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]}\sum_{t=1}^{T}D_{c,t}v_{N-1:N,t}\right]^2-2\left(1-\beta_0^S\right)\beta_1^S\frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \\
& \sum_{t=1}^{T}D_{c,t}R_t\frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]}\sum_{t=1}^{T}\left[D_{c,t}v_{N-1:N,t}-\frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]}\sum_{t=1}^{T}D_{c,t}v_{N-1:N,t}\right]^2 \\
& +\left(\beta_1^C\right)^2C_{c,t}=0
\end{aligned}
$$

where for $j \epsilon \{a, c\}$:

$$\hat{C}_{j,t} \equiv \left[\frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} R_t\right]^2 \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T}\left[D_{j,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} v_{N-1:N,t}\right]^2$$
$$+\left[\frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} v_{N-1:N,t}\right]^2 \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T}\left[D_{j,t} R_t - \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} R_t\right]^2$$
$$+\left[\frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} v_{N-1:N,t}\right]^2 \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T}\left[D_{j,t} R_t - \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} R_t\right]^2$$

2 Gradient

2.1 Gradient

Let $\hat{F}(\theta) = \begin{bmatrix} \hat{f}_1(\theta) & \cdots & \hat{f}_6(\theta) \end{bmatrix}'$, $\hat{W}$ be a positive-definite weighting matrix (computed based on our data), and m' denotes transposition. The GMM estimator is given by:

$$\hat{\theta} = \underset{\theta \epsilon \Theta}{arg\,min} \hat{F}(\theta)' \hat{W} \hat{F}(\theta)$$

where Θ is the (compact) parameter set obtained from our model in Section 3.

The gradient is given by:

$$\frac{\partial F(\theta)}{\partial \theta'} = \begin{bmatrix} \frac{\partial f_1(\theta)}{\partial \alpha} & \frac{\partial f_1(\theta)}{\partial \beta_0^C} & \frac{\partial f_1(\theta)}{\partial \beta_1^C} & \frac{\partial f_1(\theta)}{\partial \beta_0^S} & \frac{\partial f_1(\theta)}{\partial \beta_1^S} \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \frac{\partial f_6(\theta)}{\partial \alpha} & \cdots & \cdots & \cdots & \frac{\partial f_6(\theta)}{\partial \beta_1^S} \end{bmatrix}$$

Where:

$$\frac{\partial F(\theta)}{\partial \alpha} = \begin{array}{c} \frac{\partial f_1(\theta)}{\partial \alpha} \\ \frac{\partial f_2(\theta)}{\partial \alpha} \\ \frac{\partial f_3(\theta)}{\partial \alpha} \\ \frac{\partial f_4(\theta)}{\partial \alpha} \\ \frac{\partial f_5(\theta)}{\partial \alpha} \\ \frac{\partial f_6(\theta)}{\partial \alpha} \end{array} = 2 \begin{bmatrix} f_1(\theta)\,\mathbb{E}(D_{a,t}v_{N-1:N,t}) \\ f_2(\theta)\,\mathbb{E}(D_{b,t}v_{N-1:N,t}) \\ 0 \\ 2f_4(\theta)\left[\left(4-\alpha-6\beta_0^C\right)-6\beta_1^C\mathbb{E}(D_{a,t}R_t)\right]\mathbb{V}(D_{a,t}v_{N-1:N,t}) \\ 2f_5(\theta)(1-\alpha)\,\mathbb{V}(D_{b,t}v_{N-1:N,t}) \\ 0 \end{bmatrix}$$

$$\frac{\partial F(\theta)}{\partial \beta_0^C} = \begin{array}{c} \frac{\partial f_1(\theta)}{\partial \beta_0^C} \\ \frac{\partial f_2(\theta)}{\partial \beta_0^C} \\ \frac{\partial f_3(\theta)}{\partial \beta_0^C} \\ \frac{\partial f_4(\theta)}{\partial \beta_0^C} \\ \frac{\partial f_5(\theta)}{\partial \beta_0^C} \\ \frac{\partial f_6(\theta)}{\partial \beta_0^C} \end{array} = 12 \begin{bmatrix} f_1(\theta)\,\mathbb{E}(D_{a,t}v_{N-1:N,t}) \\ 0 \\ 0 \\ 2f_4(\theta)\left[\left(4-\alpha-6\beta_0^C\right)-6\beta_1^C\mathbb{E}(D_{a,t}R_t)\right]\mathbb{V}(D_{a,t}v_{N-1:N,t}) \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{\partial F(\theta)}{\partial \beta_1^C} = \begin{array}{c} \frac{\partial f_1(\theta)}{\partial \beta_1^C} \\ \frac{\partial f_2(\theta)}{\partial \beta_1^C} \\ \frac{\partial f_3(\theta)}{\partial \beta_1^C} \\ \frac{\partial f_4(\theta)}{\partial \beta_1^C} \\ \frac{\partial f_5(\theta)}{\partial \beta_1^C} \\ \frac{\partial f_6(\theta)}{\partial \beta_1^C} \end{array} = 12 \begin{bmatrix} f_1(\theta)\,\mathbb{E}(D_{a,t}R_t)\,\mathbb{E}(D_{a,t}v_{N-1:N,t}) \\ 0 \\ 0 \\ 2f_4(\theta)\left[\left(4-\alpha-6\beta_0^C\right)\mathbb{E}(D_{a,t}R_t)\,\mathbb{V}(D_{a,t}v_{N-1:N,t})-6\beta_1^C C_{a,t}\right] \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{\partial F(\theta)}{\partial \beta_0^S} = \begin{matrix} \frac{\partial f_1(\theta)}{\partial \beta_0^S} \\ \frac{\partial f_2(\theta)}{\partial \beta_0^S} \\ \frac{\partial f_3(\theta)}{\partial \beta_0^S} \\ \frac{\partial f_4(\theta)}{\partial \beta_0^S} \\ \frac{\partial f_5(\theta)}{\partial \beta_0^S} \\ \frac{\partial f_6(\theta)}{\partial \beta_0^S} \end{matrix} = 2 \begin{bmatrix} 0 \\ 0 \\ f_3(\theta)\,\mathbb{E}\left(D_{c,t} v_{N-1:N,t}\right) \\ 0 \\ 0 \\ 2f(\theta)\left[\left(1-\beta_0^S\right) + \beta_1^S \mathbb{E}\left(D_{c,t} R_t\right)\right] \mathbb{V}\left(D_{c,t} v_{N-1:N,t}\right) \end{bmatrix}$$

$$\frac{\partial F(\theta)}{\partial \beta_1^S} = \begin{matrix} \frac{\partial f_1(\theta)}{\partial \beta_1^S} \\ \frac{\partial f_2(\theta)}{\partial \beta_1^S} \\ \frac{\partial f_3(\theta)}{\partial \beta_1^S} \\ \frac{\partial f_4(\theta)}{\partial \beta_1^S} \\ \frac{\partial f_5(\theta)}{\partial \beta_1^S} \\ \frac{\partial f_6(\theta)}{\partial \beta_1^S} \end{matrix} = 2 \begin{bmatrix} 0 \\ 0 \\ f_3(\theta)\,\mathbb{E}\left(D_{c,t} R_t\right) \mathbb{E}\left(D_{c,t} v_{N-1:N,t}\right) \\ 0 \\ 0 \\ 2f_6(\theta)\left[-\left(1-\beta_0^S\right) \mathbb{E}\left(D_{c,t} R_t\right) \mathbb{V}\left(D_{c,t} v_{N-1:N,t}\right) + \beta_1^S C_{c,t}\right] \end{bmatrix}$$

2.2 Gradient Empirical Analogue

Finally, the empirical analogue we use in the estimation is given by:

$$\frac{\partial \hat{F}(\theta)}{\partial \alpha} = 2 \begin{bmatrix} \hat{f}_1(\theta) \frac{1}{\sum_{t=1}^T 1\left[D_{a,t}=1\right]} \sum_{t=1}^T D_{a,t} v_{N-1:N,t} \\ \hat{f}_2(\theta) \frac{1}{\sum_{t=1}^T 1\left[D_{b,t}=1\right]} \sum_{t=1}^T D_{b,t} v_{N-1:N,t} \\ 0 \\ \frac{\partial \hat{f}_4(\theta)}{\partial \alpha} \\ 2\hat{f}_5(\theta)(1-\alpha) \frac{1}{\sum_{t=1}^T 1\left[D_{b,t}=1\right]} \sum_{t=1}^T \left[D_{b,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^T 1\left[D_{b,t}=1\right]} \sum_{t=1}^T D_{b,t} v_{N-1:N,t}\right]^2 \\ 0 \end{bmatrix}$$

where:

$$\frac{\partial \hat{f}_4(\theta)}{\partial \alpha} = 2\hat{f}_4(\theta)\left[\left(4-\alpha-6\beta_0^C\right)-6\beta_1^C\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\sum_{t=1}^T D_{a,t}R_t\right]\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}$$
$$\sum_{t=1}^T\left[D_{a,t}v_{N-1:N,t}-\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\sum_{t=1}^T D_{a,t}v_{N-1:N,t}\right]^2$$

$$\frac{\partial \hat{F}(\theta)}{\partial \beta_0^C} = 12\begin{bmatrix}\hat{f}_1(\theta)\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\sum_{t=1}^T D_{a,t}v_{N-1:N,t}\\ 0\\ 0\\ \frac{\partial \hat{f}_4(\theta)}{\partial \beta_0^C}\\ 0\\ 0\end{bmatrix}$$

where:

$$\frac{\partial \hat{f}_4(\theta)}{\partial \beta_0^C} = 2\hat{f}_4(\theta)\left[\left(4-\alpha-6\beta_0^C\right)-6\beta_1^C\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\sum_{t=1}^T D_{a,t}R_t\right]\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}$$
$$\sum_{t=1}^T\left[D_{a,t}v_{N-1:N,t}-\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\sum_{t=1}^T D_{a,t}v_{N-1:N,t}\right]^2$$

$$\frac{\partial \hat{F}(\theta)}{\partial \beta_1^C} = 12\begin{bmatrix}\hat{f}_1(\theta)\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\sum_{t=1}^T D_{a,t}R_t\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\sum_{t=1}^T D_{a,t}v_{N-1:N,t}\\ 0\\ 0\\ \frac{\partial \hat{f}_4(\theta)}{\partial \beta_1^C}\\ 0\\ 0\end{bmatrix}$$

where:

$$\frac{\partial \hat{f}_4(\theta)}{\partial \beta_1^C} = 2\hat{f}_4(\theta)\left[\left(4-\alpha-6\beta_0^C\right)\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\sum_{t=1}^T D_{a,t}R_t\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\right.$$
$$\left.\sum_{t=1}^T\left[D_{a,t}v_{N-1:N,t}-\frac{1}{\sum_{t=1}^T 1[D_{a,t}=1]}\sum_{t=1}^T D_{a,t}v_{N-1:N,t}\right]^2-6\beta_1^C\hat{C}_{a,t}\right]$$

$$\frac{\partial \hat{F}(\theta)}{\partial \beta_0^S} = 2 \begin{bmatrix} 0 \\ 0 \\ \hat{f}_3(\theta) \frac{1}{\sum_{t=1}^T 1[D_{c,t}=1]} \sum_{t=1}^T D_{c,t} v_{N-1:N,t} \\ 0 \\ 0 \\ \frac{\partial \hat{f}_5(\theta)}{\partial \beta_0^S} \end{bmatrix}$$

where:

$$\begin{aligned} \frac{\partial \hat{f}_5(\theta)}{\partial \beta_0^S} =& 2\hat{f}_6(\theta) \left[\left(1 - \beta_0^S\right) + \beta_1^S \frac{1}{\sum_{t=1}^T 1[D_{c,t}=1]} \sum_{t=1}^T D_{c,t} R_t \right] \frac{1}{\sum_{t=1}^T 1[D_{c,t}=1]} \\ & \sum_{t=1}^T \left[D_{c,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^T 1[D_{c,t}=1]} \sum_{t=1}^T D_{c,t} v_{N-1:N,t} \right]^2 \end{aligned}$$

$$\frac{\partial \hat{F}(\theta)}{\partial \beta_1^S} = 2 \begin{bmatrix} 0 \\ 0 \\ \hat{f}_3(\theta) \frac{1}{\sum_{t=1}^T 1[D_{c,t}=1]} \sum_{t=1}^T D_{c,t} R_t \frac{1}{\sum_{t=1}^T 1[D_{c,t}=1]} \sum_{t=1}^T D_{c,t} v_{N-1:N,t} \\ 0 \\ 0 \\ \frac{\partial \hat{f}_6(\theta)}{\partial \beta_1^S} \end{bmatrix}$$

where:

$$\begin{aligned} \frac{\partial \hat{f}_6(\theta)}{\partial \beta_1^S} =& 2\hat{f}_6(\theta) \left[-\left(1 - \beta_0^S\right) \frac{1}{\sum_{t=1}^T 1[D_{c,t}=1]} \sum_{t=1}^T D_{c,t} R_t \frac{1}{\sum_{t=1}^T 1[D_{c,t}=1]} \right. \\ & \left. \sum_{t=1}^T \left[D_{c,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^T 1[D_{c,t}=1]} \sum_{t=1}^T D_{c,t} v_{N-1:N,t} \right]^2 + \beta_1^S \hat{C}_{c,t} \right. \end{aligned}$$

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