Shipping the Good Apples Out: Alchian-Allen Theorem of Various Qualities

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Abstract

This paper explores conditions for which Alchian-Allen theorem–the finer quality product is relatively more purchased than the lower one in regions where additional costs are imposed–holds in models of various but finite qualities. In this paper, "finer qualities raise ratios to lower ones in response to transportation costs" defines the various qualities version of Alchian-Allen theorem. Then we find Alchian-Allen theorem holds if and only if consumptions of neighbor qualities are not so different and weighted averages of compensated elasticities by inverse of prices are decreasing in qualities. In order to provide plausible intuitions, some sufficiency conditions are also suggested.

JEL Classifications: *D00; F19*

Keywords: *Alchian-Allen theorem; various qualities; substitution effect; fixed transportation cost*

1 Introduction

Our world is made up with many commodities with similar but whose qualities vary widely. A good model to investigate those qualities and consumption patterns in terms of transportation costs is given by Alchian and Allen's model. The theorem provided by Alchian and Allen suggests consumers will purchase relatively more ratio of higher quality products–or, higher priced–in response to the rise in prices by transportation costs (Alchian-Allen theorem; see, for example, their textbook: Alchian and Allen 1967, pp. 63-64).

Empirical examples are easily found. For example, Pritchett and Chamberlain (1993) involve Alchian-Allen theorem to explain the relatively low price of local slaves sold in New Orleans compared with slaves from Old South, which is distant from Louisiana. Another example is also found in American history of trade; the tariff of cotton textiles. From 1816 to 1857, the minimum valuation tariff and ad valorem tariff were imposed on British textiles. As a result, the average per yard FOB price of British textiles became much higher than the average per yard price of local textiles, which indicates finer products were exported by British manufacturer to American market. Manufacturers in the United States also specialized in production of coarse textiles and gained their competitiveness in those periods, which resulted in some failures of British merchants to sell inferior quality textiles after the drastic tariff cut in 1857 (see, for example, Temin 1988, and Irwin and Temin 2001).[1]

[1]Empirical verifications of Alchian-Allen theorem are also done, for example, by Hummels and Skiba (2004); Staten and Umbeck (1989); and Bertonazzi, Maloney and McCormick (1993). Their results reinforce the validity of the theorem.

The aim of this paper is to consider the case in which qualities vary widely–as such, high ends, middle classes, and low ends–and provide conditions of relations among those qualities for Alchian-Allen theorem to hold. Models of more than three goods with two qualities are considered by Gould and Segall (1969); Borcherding and Silberberg (1978); and Bauman (2004). Yet, its conditions in various qualities are not well known in general. Evaluating those multiple quality levels, we can find intra-quality relations and some plausible conditions in our economy. It reinforces arguments of Alchian-Allen theorem in broad cases.

2 Alchian-Allen Theorem

The model is constructed based on Borcherding and Silberberg (1978) but see also Umbeck (1980). Suppose there are $n+1$ normal goods; n different qualities of good x denoted by x_i for $i = 1, 2, \ldots, n$ and y a Hicksian composite good or "all other goods." I assume all goods are normal and x_i has higher quality than x_{i+1}, so that, x_1 has the finest quality and x_n the lowest. Suppose p_i denote the FOB price of x_i interms of the price of y. Then, for $\boldsymbol{p} = (p_1, \ldots, p_n)$ and a given utility level U, Hicksian demand functions are given by $x_i = x_i(\boldsymbol{p}, U)$ for $i = 1, \ldots, n$ and $y = y(\boldsymbol{p}, U)$. Using Hicksian demand functions enables us to eliminate the influence of the income effect.

I assume all qualities are consumed and y substitutes x; h's are differentiable with respect to prices; $\partial y/\partial p_i > 0$, $y > 0$, and $x_i > 0$ for all i. In cases of multiple those "all other goods" are studied by Bauman (2004). Yet, I leave open possibilities that some qualities complement other qualities as well as substituting.

Let t be the common fixed transportation cost on trading x. Thus respective CIF prices of each quality are given by $p_i + t$. In order to evaluate influences of shipping costs on consumption patterns, consider $a_{i,i+1} = \partial(x_i/x_{i+1})/\partial t$ for $i = 1, 2, \ldots, n-1$, which is given by

$$\begin{aligned} a_{i,i+1} &= \frac{x_{i+1} \sum_{s=1}^{n} \partial x_i/\partial p_s - x_i \sum_{j=1}^{n} \partial x_{i+1}/\partial p_s}{{x_{i+1}}^2} \\ &= \left(\frac{x_i}{x_{i+1}} \right) \times \left[\sum_{s=1}^{n} \frac{\varepsilon_{i,s} - \varepsilon_{i+1,s}}{p_s} \right], \end{aligned} \tag{1}$$

where $\varepsilon_{i,s} = (p_s/x_i) \cdot (\partial x_i/\partial p_s)$ is its compensated elasticity. In the case of $i = 1, 2$, this corresponds to Alchian-Allen theorem proved by Borcherding and Silberberg. However, in various qualities more than three, Hick's third law (Hicks 1946, pp. 309-311) is less useful. In this paper, $3 \leqslant n < +\infty$ is assumed.

With multiple qualities, how can we state the Alchian-Allen theorem? The original statement is; consumers raise ratio of consumption on better qualities relatively more than lower qualities in response to rising prices by transportation costs. According to the original statement, we can define the various qualities version of Alchian-Allen effect as the case in which ratios of relatively finer quality products change more than that of lower quality ones for all qualities. For example, the percentage change in x_i shall be larger than that of x_{i+1} or inferior ones. Keeping in mind that $a_{i,i} = 0$ and $a_{i,j} = -a_{j,i}$, we can define Alchian-Allen theorem as follows.

Definition 1 (Alchian-Allen Theorem) *Suppose there are multiple qualities* $i, j, k, \ell = 1, 2, \ldots, n$ *such that* $i < j$ *and* $k < \ell$. *The various qualities version of Alchian-Allen theorem is defined by;* $0 < a_{i,j} < a_{k,\ell}$ *holds* $\forall i, j, k, \ell$ *such that* $j - i < \ell - k$.

Definition 1 implies the set of $a_{i,j}$ has a partial order (Table 1). Practically, for all $i = 1, 2, \ldots, n-2$, we need to show the relations among $a_{i,i+1}$, $a_{i,i+2}$, and $a_{i+1,i+2}$, which is completed by showing (i) $a_{i,i+1} < a_{i,i+2}$ and (ii) $a_{i+1,i} > a_{i,i+1}$ (Table 2). Then (i) and (ii) imply $a_{i-1,i+1} > a_{i,i+2}$ for $2 \leqslant i \leqslant n-2$.

Remark 1 *Let* $3 \leqslant n < +\infty$. *Alchian-Allen theorem holds if and only if* $a_{i,i+1} < a_{i,i+2}$ *and* $a_{i,i+2} > a_{i+1,i+2}$ *for* $i = 1, 2, \ldots, n-2$.

Define $\phi(i) = \sum_{s=1}^{n}(-\varepsilon_{i,s}/p_s)$ and $\Phi(j, i) = \phi(j) - \phi(i)$. Then $a_{i,i+1} - a_{i,i+2}$ and $a_{i,i+2} - a_{i+1,i+2}$ are respectively given by

$$a_{i,i+1} - a_{i,i+2} = \left(\frac{x_i}{x_{i+1}}\right) \times \Phi(i+1, i) - \left(\frac{x_i}{x_{i+2}}\right) \times \Phi(i+2, i), \tag{2}$$

$$a_{i,i+2} - a_{i+1,i+2} = \left(\frac{x_i}{x_{i+2}}\right) \times \Phi(i+2, i) - \left(\frac{x_{i+1}}{x_{i+2}}\right) \times \Phi(i+2, i+1). \tag{3}$$

Theorem 1 *Let* $3 \leqslant n < +\infty$. *Alchian-Allen theorem holds if and only if* $\phi(i+1) > \phi(i)$ *for* $i = 1, 2, \ldots, n-1$ *and the following condition is satisfied for* $i = 1, 2, \ldots, n-2$;

$$\frac{\Phi(i+1, i)}{\Phi(i+2, i)} \times x_{i+2} < x_{i+1} < \frac{\Phi(i+2, i)}{\Phi(i+2, i+1)} \times x_i \tag{A}$$

Proof: Let $3 \leqslant n < +\infty$ and $1 \leqslant i < j \leqslant n$. $\phi(j) > \phi(i)$ implies $\Phi(j, i) > 0$ and then $a_{i,j} > 0$. If $\exists i, j$, $a_{i,j} \leqslant 0$, Alchian-Allen theorem vanishes (Definition 1). Thus, $\forall i, j$, $a_{i,j} > 0$ is required, which is the same as requiring $\phi(i+1) > \phi(i)$, for $i = 1, 2, \ldots, n-1$.

Let $a_{i,j} > 0$ and consider conditions for which $a_{i,i+1} - a_{i,i+2}$ to be negative and $a_{i,i+2} - a_{i+1,i+2}$ to be positive to get condition (A). Therefore, only if $a_{i,j} > 0$ and condition (A) hold, Alchian-Allen theorem holds. If Alchian-Allen theorem holds, $a_{i,j} > 0$ and condition (A) hold by its definition.

Suppose $n \longmapsto +\infty$. Then the consumption of the lowest quality product cannot be determined in general and the condition (A) may not be evaluated at its limit. ■

The requirements of the theorem implies weighted sum of compensated elasticities by prices of the lower quality products must be larger than that of the finer quality one and neighbor qualities must be consumed not to be different from each other except for the lowest and the finest qualities. As a matter of the form of condition (A), the lowest quality products can be consumed significantly smaller than the second lowest quality and, in contrast, the finest quality products can be consumed significantly larger than the second finest one. Actually, however, those quantities are constrained by their budgets and supplies in the market. In the two-quality model, we cannot observe those characteristics because only the finest and the lowest appear in the model.

3 Sufficiency

I study some sufficiency conditions that are intuitively more familiar with empirical applications. To begin with, consider the conditions on consumption patterns. From the definition of Φ, the following operation is true;

$$\Phi(i+2,i) = \Phi(i+2,i+1) + \Phi(i+1,i). \tag{4}$$

Then $\Phi(i+1,i) < \Phi(i+2,i)$ and $\Phi(i+1,i+2) < \Phi(i+2,i)$. Therefore, if consumers have preferences such that $\phi(i+1) > \phi(i)$ and they purchase more finer quality products than lower quality ones, which is $x_1 \geqslant x_2 \geqslant \cdots \geqslant x_n$ (> 0), those two conditions provide a sufficiency condition of Alchian-Allen theorem.

In turn, suppose $x_1 \geqslant x_2 \geqslant \cdots \geqslant x_n$ does not hold. Then condition (A) of Theorem 1 indicates the finer and the lower quality products bind consumption of middle quality products and consumptions of those middle quality products are not so large comparing to the finer one but not so small comparing to the worse one. Otherwise, Alchian-Allen theorem cannot hold. Figure 1 depicts an example of consumption patterns of each quality–consumptions are not necessarily either increasing or decreasing in qualities but conform to the lower and upper bounds determined by consumptions of the lower and finer quality products. Those arguments are also pointed out by Gould and Segall (1969).

Next, consider the condition $\phi(i+1) > \phi(i)$ to provide other sufficient conditions. Although, in proofs of the theorem, orders of prices were not mentioned, for more intuitions, hereafter, prices are supposed to be higher as qualities are better;

Assumption 1 *Finer products are better priced;* $p_1 > p_2 > \cdots > p_n > 0$.

Let $\varepsilon_i = (p_i/y) \cdot (\partial y/\partial p_i)$ be the compensated elasticity of y. Applying Assumption 1, $\Phi(i+1) - \Phi(i)$ satisfies

$$\sum_{s=1}^{n} \frac{\varepsilon_{i,s} - \varepsilon_{i+1,s}}{p_s} > \frac{\sum_{s=1}^{n}(\varepsilon_{i,s} - \varepsilon_{i+1,s})}{p_1} \equiv \frac{\varepsilon_{i+1}/p_{i+1} - \varepsilon_i/p_i}{p_1/y}, \tag{5}$$

where the last equivalence follows from Hick's third law and symmetry of substitution terms in Hicksian demand functions. Then we can say Alchian-Allen theorem holds if y substitutes x's and $\varepsilon_{i+1}/\varepsilon_i > p_{i+1}/p_i$. In this sense, since there are various qualities, the highest and the lowest qualities are not necessarily close substitutes as it is required by Borcherding and Silberberg (1978). With multiple other goods, Bauman (2004) uses another mechanism–sums of compensated elasticities tend to be closer to each other as goods increase–to get Alchian-Allen theorem likely to be feasible in the sense of Borcherding and Silberberg.

As a slight extension of the various qualities model, consider to apply the result of Bauman to get $\varepsilon_{i+1} \simeq \varepsilon_i$ for $i = 1, 2, \ldots, n-1$. Then, if Assumption 1 is true, (5) implies Alchian-Allen theorem holds because

$$\sum_{s=1}^{n} \frac{\varepsilon_{i,s} - \varepsilon_{i+1,s}}{p_s} > \frac{\varepsilon_{i+1}/p_{i+1} - \varepsilon_i/p_i}{p_1/y} \simeq \frac{\varepsilon \times (1/p_{i+1} - 1/p_i)}{p_1/y} > 0, \tag{6}$$

where $\varepsilon \simeq \varepsilon_i \simeq \varepsilon_{i+1}$.

4 Conclusions

Under the given definition of the various qualities version of Alchian-Allen theorem, its necessary and sufficient conditions have been provided. Within its definition, qualities just order ratios of change but prices. An contribution of this study was to unveil relations of consumption patters among qualities. Alchian-Allen theorem requires consumptions of each quality must not significantly smaller than the next lower quality one but it must not significantly larger than the next finer quality one. In other words, Alchian-Allen theorem holds if and only if consumers has no significantly different taste over closest qualities except for the lowest quality and the finest quality.

In empirical studies, some conditions are useful to verify the existance of Alchian-Allen effects. For example, if every finer quality product is more purchased, one of conditions for Alchian-Allen theorem is satisfied. If finer quality products are not necessarily more purchased than lower quality products, it will be a criteria whether consumption of each quality are not so different from their neighbor qualities.

Sufficiency in compensated elasticities also have provided good outlooks in cases of which finer qualities are priced higher. In those cases, orders in compensated elasticities–finer products are less likely substituted–are consistent with pricing strategies of suppliers–pricing higher for less elastic products. Within such contexts, we also need to be careful about the condition on consumption patterns as well. If lower quality products are significantly relatively more consumed, Alchian-Allen theorem cannot hold. In those cases, bad apples will be shipped more than good apples to distant areas It still depends on each case of each market.

In our actual economy, conditions such that "consumptions of closer qualities are not so different from each other" and "finer qualities are priced higher" are not peculiar assumptions. If qualities are similar, usually there is no significant difference in preferences of consumers. If qualities are finer, it is natural to be priced higher. It is also intuitively plausible to regard finer qualities are less substitutable to other goods than lower qualities because those consumers who prefer finer ones admire those products much more than lower qualities, and then their elasticities of finer qualities are likely to be less elastic. In this vein, Alchian-Allen theorem seems to hold in broad cases in our economy. Empirically, if the surveyed good is regarded as either having sufficient other goods or no other good influence on the good, applying the results of Bauman (2004) and this paper, Alchian-Allen theorem can be verified by looking at consumption patterns as well as their responses to transaction costs.

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Table 1: Relations among $a_{i,j}$

$$
\begin{array}{ccccccccc}
a_{1,2} & \rightarrow & a_{1,3} & \rightarrow & a_{1,4} & \rightarrow & \cdots & \rightarrow & a_{1,n} \\
 & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 & & a_{2,3} & \rightarrow & a_{2,4} & \rightarrow & \cdots & \rightarrow & a_{2,n} \\
 & & & & \uparrow & & \uparrow & & \uparrow \\
 & & & & a_{3,4} & \rightarrow & \cdots & \rightarrow & a_{3,n} \\
 & & & & & & \uparrow & & \uparrow \\
 & & & & & & \ddots & \rightarrow & \vdots \\
 & & & & & & & & \uparrow \\
 & & & & & & & & a_{n-1,n}
\end{array}
$$

Table 2: Relations among $a_{i,j}$ which shall be shown

$$
\begin{array}{ccc}
a_{i,i+1} & \rightarrow & a_{i,i+2} \\
 & & \uparrow \\
 & & a_{i+1,i+2}
\end{array}
$$

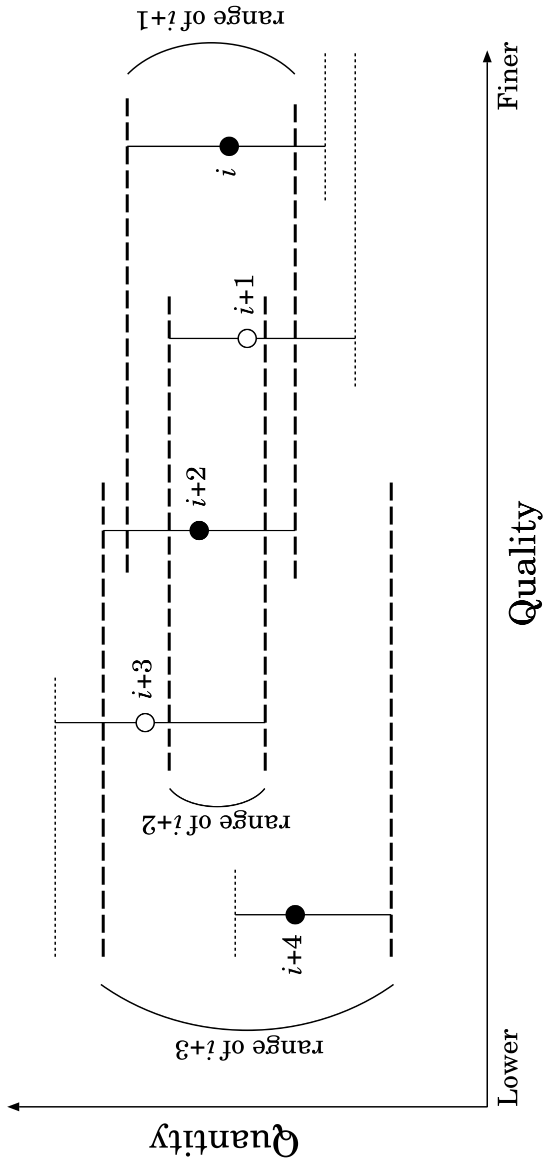

Figure 1: An Example of Consumption Pattern