Model Averaging and its Use in Economics

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Summary

The method of model averaging has become an important tool to deal with model uncertainty, for example in situations where a large amount of different theories exist, as are common in economics. Model averaging is a natural and formal response to model uncertainty in a Bayesian framework, and most of the paper deals with Bayesian model averaging. The important role of the prior assumptions in these Bayesian procedures is highlighted. In addition, frequentist model averaging methods are also discussed. Numerical methods to implement these methods are explained, and I point the reader to some freely available computational resources. The main focus is on uncertainty regarding the choice of covariates in normal linear regression models, but the paper also covers other, more challenging, settings, with particular emphasis on sampling models commonly used in economics. Applications of model averaging in economics are reviewed and discussed in a wide range of areas, among which growth economics, production modelling, finance and forecasting macroeconomic quantities. (JEL: C11, C15, C20, C52, O47).

1 Introduction

This paper is about model averaging, as a solution to the problem of model uncertainty and focuses mostly on the theoretical developments over the last two decades and its uses in applications in economics. This is a topic that has now gained substantial maturity and is generating a rapidly growing literature. Thus, a survey seems timely. The discussion focuses mostly

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on uncertainty about covariate inclusion in regression models (normal linear regression and its extensions), which is arguably the most pervasive situation in economics. Advances in the context of models designed to deal with more challenging situations, such as data with dependency over time or in space or endogeneity (all quite relevant in economic applications) are also discussed. Two main strands of model averaging are distinguished: Bayesian model averaging (BMA), based on probability calculus and naturally emanating from the Bayesian paradigm by treating the model index as an unknown, just like the model parameters and specifying a prior on both; and frequentist model averaging (FMA), where the chosen weights are often determined so as to obtain desirable properties of the resulting estimators under repeated sampling and asymptotic optimality.

In particular, the aims of this paper are:

- To provide a survey of the most important methodological contributions in model averaging, especially aimed at economists. The presentation is formal, yet accessible, and uses a consistent notation. This review takes into account the latest developments, which is important in such a rapidly developing literature. Technicalities are not avoided, but some are dealt with by providing the interested reader with the relevant references. Even though the list of references is quite extensive, this is not claimed to be an exhaustive survey. Rather, it attempts to identify the most important developments that the applied economist needs to know about for an informed use of these methods. This review complements and extends other reviews and discussions; for example by Hoeting et al. (1999) on Bayesian model averaging, Clyde and George (2004) on model uncertainty, Moral-Benito (2015) on model averaging in economics and Wang et al. (2009) on frequentist model averaging. Dormann et al. (2018) present an elaborate survey of model averaging methods used in ecology. Further, a review of weighted average least squares is provided in Magnus and De Luca (2016) while Fragoso et al. (2018) develop a conceptual classification scheme to better describe the literature in Bayesian model averaging. Koop (2017) discusses the use of Bayesian model averaging or prior shrinkage as responses to the challenges posed by big data in empirical macroeconomics. This paper differs from the earlier surveys mainly through the combination of a more ambitious scope and depth and the focus on economics.

- By connecting various strands of the literature, to enhance the insight of the reader into the way these methods work and why we would use them. In particular, this paper attempts to tie together disparate literatures with roots in econometrics and statistics, such as the literature on forecasting, often in the context of time series and linked with information criteria, fundamental methodology to deal with model uncertainty and shrinkage in statistics[1], as

[1]Choosing covariates can be interpreted as a search for parsimony, which has two main approaches in Bayesian

well as more ad-hoc ways of dealing with variable selection. I also discuss some of the theoretical properties of model averaging methods.

- To discuss, in some detail, key operational aspects of the use of model averaging. In particular, the paper covers the various commonly used numerical methods to implement model averaging (both Bayesian and frequentist) in practical situations, which are often characterized by very large model spaces. For Bayesian model averaging, it is important to understand that the weights (based on posterior model probabilities) are typically quite sensitive to the prior assumptions, in contrast to the usually much more robust results for the model parameters given a specific model. In addition, this sensitivity does not vanish as the sample size grows (Kass and Raftery, 1995; Berger and Pericchi, 2001). Thus, a good understanding of the effect of (seemingly arbitrary) prior choices is critical.

- To review and discuss how model averaging has already made a difference in economics. The paper lists a number of, mostly recent, applications of model averaging methods in economics, and presents some detail on a number of areas where model averaging has furthered our understanding of economic phenomena. For example, we highlight the contributions to growth theory, where BMA has been used to shed light on the relative importance of the three main growth theories (geography, integration and institutions) for development, as well as on the existence of the so-called natural resource curse for growth; the use of BMA in combining inference on impulse responses from models with very different memory characteristics; the qualification of the importance of established early warning signals for economic crises; the combination of inference on production or cost efficiencies through different models, etc. Model averaging provides a natural common framework in which to interpret the results of different empirical analyses and as such should be an important tool for economists to resolve differences.

- To provide sensible recommendations for empirical researchers about which modelling framework to adopt and how to implement these methods in their own research. In the case of Bayesian model averaging, I recommend the use of prior structures that are easy to elicit and are naturally robust. I include a separate section on freely available computational resources that will allow applied researchers to try out these methods on their own data, without having to incur a prohibitively large investment in implementation. In making recommendations, it is inevitable that one draws upon personal experiences and preferences,

statistics: through the use of shrinkage priors, which are absolutely continuous priors that shrink coefficients to zero but where all covariates are always included in the model, and through allocating prior point mass at zero for each of the regression coefficients, which allows for formal exclusion of covariates and implies that we need to deal with many different models, which is the approach recommended here.

to some extent. Thus, I present the reader with a somewhat subjective point of view, which I believe, however, is well-supported by both theoretical and empirical results.

Given the large literature, and in order to preserve a clear focus, it is important to set some limits to the coverage of the paper. As already explained above, the paper deals mostly with covariate uncertainty in regression models, and does not address issues like the use of Bayesian model averaging in classification trees (Hernández et al., 2018) or in clustering and density estimation (Russell et al., 2015). The large literature in machine learning related to nonparametric approaches to covariate uncertainty (Hastie et al., 2009) will also largely be ignored. The present paper focuses on averaging over (mostly nontrivial) models as a principled and formal statistical response to model uncertainty and does not deal with data mining or machine learning approaches, as further briefly discussed in Subsection 2.2. In addition, this paper considers situations where the number of observations exceeds the number of potential covariates as this is most common in economics (some brief comments on the opposite case can be found in footnote 17).

As mentioned above, I discuss Bayesian and frequentist approaches to model averaging. This paper is mostly concerned with the Bayesian approach for the following reasons:

- BMA benefits from a number of appealing statistical properties, such as point estimators and predictors that minimize Mean Squared Error (MSE), and the calibration of the associated intervals (Raftery and Zheng, 2003). In addition, probabilistic prediction is optimal in the log score sense. Furthermore, BMA is typically consistent and is shown to display optimal shrinkage in high-dimensional problems. More details can be found in Subsection 3.2.

- Computationally, BMA is much easier to implement in large model spaces than FMA, since efficient MCMC algorithms are readily available.

- In contrast to FMA methods, BMA immediately leads to readily interpretable posterior model probabilities and probabilities of inclusion of possible determinants in the model.

- I personally find the finite-sample and probability-based nature of the Bayesian approach very appealing. I do realize this is, to some extent, a personal choice, but I prefer to operate within a methodological framework that immediately links to prediction and decision theory.

- There is a large amount of recent literature using the Bayesian approach to resolve model uncertainty, both in statistics and in many areas of application, among which economics

features rather prominently. Thus, this focus on Bayesian methods is in line with the majority of the literature and seems to reflect the perceived preference of many researchers in economics.

Of course, as Wright (2008) states: "One does not have to be a subjectivist Bayesian to believe in the usefulness of BMA, or of Bayesian shrinkage techniques more generally. A frequentist econometrician can interpret these methods as pragmatic devices that may be useful for out-of-sample forecasting in the face of model and parameter uncertainty."

This paper is organised as follows: in Section 2 I discuss the issue of model uncertainty and the way it can naturally be addressed through Bayesian model averaging. This section also comments on the construction of the model space and introduces the specific context of covariate uncertainty in the normal linear model. Section 3 provides a detailed account of Bayesian model averaging, focusing on the prior specification, its properties and its implementation in practice. This section also provides a discussion of various generalizations of the sampling model and of a number of more challenging models, such as dynamic models and models with endogenous covariates. Section 4 describes frequentist model averaging, its computational implementation, and its links with forecast combinations. Section 5 mentions some of the literature where model averaging methods have been applied in economics and discusses how model averaging methods have contributed to our understanding of a number of economic issues. In Section 6 some freely available computational resources are briefly discussed, and the final section concludes.

2 Model uncertainty

It is hard to overstate the importance of model uncertainty for economic modelling. Almost invariably, empirical work in economics will be subject to a large amount of uncertainty about model specifications. This may be the consequence of the existence of many different theories[2] or of many different ways in which theories can be implemented in empirical models (for example, by using various possible measures of theoretical concepts or various functional forms) or of other aspects such as assumptions about heterogeneity or independence of the observables. It is important to realize that this uncertainty is an inherent part of economic modelling, whether we acknowledge it or not. Putting on blinkers and narrowly focusing on a limited set of possible models implies that we may fail to capture important aspects of economic reality. Thus, model uncertainty affects virtually all modelling in economics and its consequences need to be taken into account. There are two main strategies that have been employed in the literature:

[2]Or perhaps more precisely, the lack of a universally accepted theory, which has been empirically verified as a (near) perfect explanation of reality, clearly a chimera in the social sciences.

- Model selection: such methods attempt to choose the best of all models considered, according to some criterion. Examples of this abound and some of the main model selection strategies used in the context of a linear regression model are briefly described in Subsection 2.2. The most important common characteristic of model selection methods is that they choose a model and then conduct inference conditionally upon the assumption that this model actually generated the data. So these methods only deal with the uncertainty in a limited sense: they try to select the "best" model, and their inference can only be relied upon if that model happens to be (a really good approximation to) the data generating process. In the much more likely case where the best model captures some aspects of reality, but there are other models that capture other aspects, model selection implies that our inference is almost always misleading, either in the sense of being systematically wrong or overly precise. Model selection methods simply condition on the chosen model and ignore all the evidence contained in the alternative models, thus typically leading to underestimation of the uncertainty.

- Model averaging: here we take into account all the models contained in the model space we consider (see Subsection 2.1) and our inference is averaged over all these models, using weights that are either derived from Bayes' theorem (BMA) or from sampling-theoretic optimality considerations (FMA). This means our inference takes into account a possible variation across models and its precision is adjusted for model uncertainty. Averaging over models is a very natural response to model uncertainty, especially in a Bayesian setting, as explained in some detail later in this section.

As it is unlikely that reality (certainly in the social sciences) can be adequately captured by any single model, it is often quite risky to rely on a single selected model for inference, forecasts and (policy) conclusions. It is much more likely that an averaging method gives a better approximation to reality and it will almost certainly improve our estimate of the uncertainty associated with our conclusions.

One could argue that the choice between model selection and model averaging methods boils down to the underlying question that one is interested in answering. If that question relates to identifying the "true" model within a model space that is known to contain the data generating process, then model selection might be the appropriate strategy. However, if the question relates to, for example, the effect of primary education on growth, then there is no reason at all to artificially condition the inference on choosing a single model. More precisely, the choice between model averaging and model selection is related to the decision problem that we aim to solve. In most typical situations, however, the implicit loss function we specify will lead to model averaging. Examples are where we are interested in maximizing accuracy of prediction or in the

estimation of covariate effects. So it makes sense to use model averaging in economics, whenever we are (as usual) interested in quantities that are not model-specific (such as predictions, effects of possible determinants, elasticities, efficiencies etc.).

As already mentioned, one important and potentially dangerous consequence of neglecting model uncertainty, either by only considering one model from the start or by choosing a single model through model selection, is that we assign more precision to our inference than is warranted by the data, and this leads to overly confident decisions and predictions. In addition, our inference can be severely biased. See Chatfield (1995) and Draper (1995) for extensive discussions of model uncertainty. In the context of the evaluation of macroeconomic policy, Brock et al. (2003) describe and analyse some approaches to dealing with the presence of uncertainty about the structure of the economic environment under study. Starting from a decision-theoretic framework, they recommend model averaging as a key tool in tackling uncertainty. Brock and Durlauf (2015) specifically focus on policy evaluation and provide an overview of different approaches, distinguishing between cases in which the analyst can and cannot provide conditional probabilities for the effects of policies.

As an example, Durlauf et al. (2012) examine the effect of different substantive assumptions about the homicide process on estimates of the deterrence effect of capital punishment[3]. Considering four different types of model uncertainty, they find a very large spread of effects, with the estimate of net lives saved per execution ranging from -63.6 (so no deterrence effect at all) to 20.9. The latter evidence was a critical part of the National Academy of Sciences report that concluded there is no evidence in favour of or against a deterrent effect of capital punishment. This clearly illustrates that the issue of model uncertainty needs to be addressed before we can answer questions such as this and many others of immediate relevance to society.

Another example, which is discussed in more detail in Subsection 5.1.1, concerns the empirical evidence for the three main types of economic growth determinants traditionally mentioned in the literature: geography, integration (trade) and institutions (often linked to property rights and rule of law). Earlier influential papers in growth theory have tended to consider only a limited number of possible models, focusing on a particular theory but without adequately covering possible alternative theories. This led Acemoglu et al. (2001) and Rodrik et al. (2004) to conclude that the quality of institutions is the only robust driver of development, while Frankel and Romer (1999) find that trade is the dominating determinant. Analyses using BMA in Lenkoski et al. (2014) and Eicher and Newiak (2013) lead to much more balanced conclusions, where all three main theories are seen to be important for growth. This highlights the importance of accounting for a large enough class of possible models and dealing with model uncertainty in a principled

[3] A systematic investigation of this issue goes back to Leamer (1983).

and statistically sound manner.

Over the last decade, there has been a rapidly growing awareness of the importance of dealing with model uncertainty for economics. As examples, the *European Economic Review* has recently published a special issue on "Model Uncertainty in Economics" which was also the subject of the 2014 Schumpeter lecture in Marinacci (2015), providing a decision-theory perspective. In addition, a book written by two Nobel laureates in economics (Hansen and Sargent, 2014), focuses specifically on the effects of model uncertainty on rational expectations equilibrium concepts.

In line with probability theory, the formal Bayesian response to dealing with uncertainty is to average. When dealing with parameter uncertainty, this involves averaging over parameter values with the posterior distribution of that parameter in order to get the predictive distribution. Analogously, model uncertainty is also resolved through averaging, but this time averaging over models with the (discrete) posterior model distribution. The latter procedure is usually called Bayesian model averaging and was already described in Leamer (1978) and later used in Min and Zellner (1993), Osiewalski and Steel (1993), Koop et al. (1997) and Raftery et al. (1997). BMA thus appears as a direct consequence of Bayes' theorem (and hence probability laws) in a model uncertainty setting and is perhaps best introduced by considering the concept of a predictive distribution, often of interest in its own right. In particular, assume we are interested in predicting the unobserved quantity y_f on the basis of the observations y. Let us denote the sampling model[4] for y_f and y jointly by $p(y_f|y, \theta_j, M_j)p(y|\theta_j, M_j)$, where M_j is the model selected from a set of K possible models, and $\theta_j \in \Theta_j$ groups the (unknown) parameters of M_j. In a Bayesian framework, any uncertainty is reflected by a probability distribution[5] so we assign a (typically continuous) prior $p(\theta_j|M_j)$ for the parameters and a discrete prior $P(M_j)$ defined on the model space. We then have all the building blocks to compute the predictive distribution as

$$p(y_f|y) = \sum_{j=1}^{K} \left[\int_{\Theta_j} p(y_f|y, \theta_j, M_j) p(\theta_j|y, M_j) \mathrm{d}\theta_j \right] P(M_j|y), \tag{1}$$

where the quantity in square brackets is the predictive distribution given M_j obtained using the posterior of θ_j given M_j, which is computed as

$$p(\theta_j|y, M_j) = \frac{p(y|\theta_j, M_j)p(\theta_j|M_j)}{\int_{\Theta_j} p(y|\theta_j, M_j)p(\theta_j|M_j)\mathrm{d}\theta_j} \equiv \frac{p(y|\theta_j, M_j)p(\theta_j|M_j)}{p(y|M_j)}, \tag{2}$$

with the second equality defining $p(y|M_j)$, which is used in computing the posterior probability

[4]For ease of notation, we will assume continuous sampling models with real-valued parameters throughout, but this can immediately be extended to other cases.

[5]Or, more generally, a measure.

assigned to M_j as follows:

$$P(M_j|y) = \frac{p(y|M_j)P(M_j)}{\sum_{i=1}^{K} p(y|M_i)P(M_i)} \equiv \frac{p(y|M_j)P(M_j)}{p(y)}. \tag{3}$$

Clearly, the predictive in (1) indeed involves averaging at two levels: over (continuous) parameter values, given each possible model, and discrete averaging over all possible models. The denominators of both averaging operations are not immediately obvious from (1), but are made explicit in (2) and (3). The denominator (or integrating constant) $p(y|M_j)$ in (2) is the so-called marginal likelihood of M_j and is a key quantity for model comparison. In particular, the Bayes factor between two models is the ratio of their marginal likelihoods and the posterior odds are directly obtained as the product of the Bayes factor and the prior odds. The denominator in (3), $p(y)$, is defined as a sum and the challenge in its calculation often lies in the sheer number of possible models, i.e. K.

Bayesian model averaging as described above is thus the formal probabilistic way of obtaining predictive inference, and is, more generally, the approach to any inference problem involving quantities of interest that are not model-specific. So it is also the Bayesian solution to conducting posterior inference on *e.g.* the effects of covariates. Formally, the posterior distribution of any quantity of interest, say Δ, which has a common interpretation across models is a mixture of the model-specific posteriors with the posterior model probabilities as weights, *i.e.*

$$P_{\Delta|y} = \sum_{j=1}^{K} P_{\Delta \,|\, y, M_j} P(M_j \,|\, y). \tag{4}$$

The rapidly growing importance of model averaging as a solution to model uncertainty is illustrated by Figure 1, which plots the citation profile over time of papers with the topic "model averaging" in the literature. The figure also indicates influential papers (with 250 citations or more) published in either economics or statistics journals[6]. A large part of the literature uses Bayesian model averaging methods, reflected in the fact that citations to papers with the topic "Bayesian" and "model averaging" account for more than 70% of the citations in Figure 1. The sheer number of recent papers in this area is evidenced by the fact that Google Scholar returns over 52,500 papers in a search for "model averaging" and over 39,000 papers when searching for "Bayesian" and "model averaging", over half of which date from the last decade (data from November 5, 2018).

[6]There are also some heavily cited papers on model averaging in a number of other application areas, in particular biology, ecology, sociology, meteorology, psychology and hydrology. The number of citations is, of course, an imperfect measure of influence and the cutoff at 250 leaves out a number of key papers, such as Brock and Durlauf (2001) and Clyde and George (2004) with both over 200 citations.

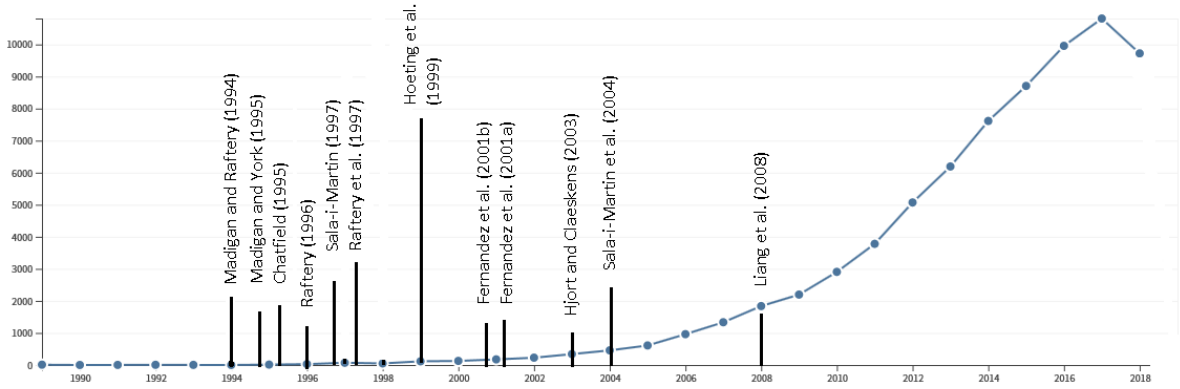

Figure 1: Total number of citations to papers with topic "model averaging" over years 1989-2018. Papers in economics or statistics journals with at least 250 citations are indicated by vertical lines proportional to the number of citations received. Source: Web of Science, November 5, 2018.

2.1 Construction of the model space

An important aspect of dealing with model uncertainty is the precise definition of the space of all models that are being considered. The idea of model averaging naturally assumes a well-defined space of possible models, over which the averaging takes place. This is normally a finite (but potentially very large) space of models, denoted by $\mathcal{M}$. There are also situations where we might consider an infinite space of models, for example when we consider data transformations of the response variable within a single family, such as the Box-Cox family[7]. In these cases where models are indexed by continuous parameters, model averaging is done by integration over these parameters and is thus perhaps less obvious. In other words, it is essentially a part of the standard Bayesian treatment of unknown parameters. Another example is given in Brock et al. (2003), who mention "hierarchical models in which the parameters of a model are themselves functions of various observables and unobservables. If these relationships are continuous, one can trace out a continuum of models." Again, Bayesian analysis of hierarchical models is quite well-established.

In economics arguably the most common case of model uncertainty is where we are unsure about which covariates should be included in a linear regression model, and the associated model space is that constructed by including all possible subsets of covariates. This case is discussed in detail in the next subsection. A minor variation is where some covariates are always included and it is inclusion or exclusion of the "doubtful" ones that defines the model space. In order to carefully construct an appropriate model space, it is useful to distinguish various common types

[7]Hoeting et al. (2002) use a number of specific values for the Box-Cox parameter, to aid interpretation, which gets us back to a finite model space

of uncertainty. Brock et al. (2003) identify three main types of uncertainty that typically need to be considered:

- Theory uncertainty. This reflects the situation where economists disagree over fundamental aspects of the economy and is, for example, illustrated by the ongoing debates over which are important drivers for economic growth (see the discussion in Subsection 5.1) or what are useful early warning signals for economic crises (see Subsection 5.4).

- Specification uncertainty. This type of uncertainty is about how the various theories that are considered will be implemented, in terms of how they are translated into specific models. Examples are the choice of available variables as a measure of theoretical constructs, the choice of lag lengths[8], parametric versus semi- or nonparametric specifications, transformations of variables, functional forms (for example, do we use linear or non-linear models) and distributional assumptions (which also include assumptions about dependence of observables).

- Heterogeneity uncertainty. This relates to model assumptions regarding different observations. Is the same model appropriate for all, or should the models include differences that are designed to accommodate observational heterogeneity? A very simple example would be to include dummies for certain classes of observations. Another example is given by Doppelhofer et al. (2016) who introduce heterogeneous measurement error variance in growth regressions.

The definition of the model space is intricately linked with the model uncertainty that is being addressed. For example, if the researcher is unsure about the functional form of the models and about covariate inclusion, both aspects should be considered in building $\mathcal{M}$. Clearly, models that are not entertained in $\mathcal{M}$ will not contribute to the model-averaged inference and the researcher will thus be blind to any insights provided by these models. Common sense should be used in choosing the model space: if one wants to shed light on the competing claims of various papers that use different functional forms and/or different covariates, it would make sense to construct a model space that combines all functional forms considered (and perhaps more variations if they are reasonable) with a wide set of possibly relevant and available covariates. The fact that such spaces can be quite large should not be an impediment.[9] In practice, not all relevant model spaces used in model averaging analyses are large. For example, to investigate the effect of capital punishment on the murder rate (see the discussion earlier in this section), Durlauf et al. (2012) build a bespoke model space by considering the following four model features: the probability model

[8] Particularly relevant in *e.g.* forecasting and VAR modelling in Subsections 5.2 and 5.3.

[9] Certainly not for a Bayesian analysis, where novel numerical methods have proven to be very efficient.

(linear or logistic regression), the specification of the covariates (relating to the probabilities of sentencing and execution), the presence of state-level heterogeneity, and the treatment of zero observations for the murder rate. In all, the model space they specify only contains 20 models, yet leads to a large range of deterrence effects. Another example of BMA with a small model space is the analysis of impulse response functions in Koop et al. (1997), who use two different popular types of univariate time series models with varying lag lengths, leading to averaging over only 32 models (see Subsection 5.2). Here the model space only reflects specification uncertainty.

It is important to distinguish between the case where the model space contains the "true" data-generating model and the case where it does not. These situations are respectively referred to as $\mathcal{M}$-closed and $\mathcal{M}$-open in the statistical literature (Bernardo and Smith, 1994). Most theoretical results (such as consistency in Subsection 3.2.1) are obtained in the simpler $\mathcal{M}$-closed case, but it is clear that in economic modelling the $\mathcal{M}$-open framework is a more realistic setting. Fortunately, model selection consistency results[10] can often be shown to extend to $\mathcal{M}$-open settings in an intuitive manner (Mukhopadhyay et al., 2015; Mukhopadhyay and Samanta, 2017) and George (1999a) states that "BMA is well suited to yield predictive improvements over single selected models when the entire model class is misspecified. In a sense, the mixture model elaboration is an expansion of the model space to include adaptive convex combinations of models. By incorporating a richer class of models, BMA can better approximate models outside the model class." A decision-theoretic approach to implementing BMA in an $\mathcal{M}$-open environment is provided in Clyde and Iversen (2013), who treat models not as an extension of the parameter space, but as part of the action space. The main objection to using BMA in the $\mathcal{M}$-open framework is the perceived logical tension between knowing the "true" model is not in $\mathcal{M}$ and assigning a prior on the models in $\mathcal{M}$. However, in keeping with most of the literature, we will assume that the user is comfortable with assigning a prior on $\mathcal{M}$, even in $\mathcal{M}$-open situations.[11]

2.2 Covariate uncertainty in the normal linear regression model

Most of the relevant literature assumes the simple case of the normal linear sampling model. This helps tractability, and it is fortunately also a model that is often used in empirical work. In addition, it is a canonical version for nonparametric regression[12], which is gaining in popularity.

[10] As explained in Subsection 3.2.1, model selection consistency is the property that the posterior probability of the data-generating model tends to unity with sample size in an $\mathcal{M}$-closed setting.

[11] Personally, I prefer to think of the prior over models as a reflection of prior beliefs about which models would be "useful proxies for" (rather than "equal to") the data-generating process, so I do not feel the $\mathcal{M}$-open setting leads to a significant additional challenge for BMA.

[12] A typical nonparametric regression approach is to approximate the unknown regression function for the mean of y given x as a linear combination of a finite number of basis functions of x.

We shall follow this tradition, and will assume for most of the paper[13] that the sampling model is normal with a mean which is a linear function of some covariates[14]. We shall further assume, again in line with the vast majority of the literature (and many real-world applications) that the model uncertainty relates to the choice of which covariates should be included in the model, *i.e.* under model j the n observations in y are generated from

$$y|\theta_j, M_j \sim N(\alpha\iota + Z_j\beta_j, \sigma^2). \tag{5}$$

Here ι represents a $n \times 1$-dimensional vector of ones, Z_j groups k_j of the possible k regressors (*i.e.* it selects k_j columns from an $n \times k$ matrix Z, corresponding to the full model) and $\beta_j \in \Re^{k_j}$ are its corresponding regression coefficients. Furthermore, all considered models contain an intercept $\alpha \in \Re$ and the scale $\sigma > 0$ has a common interpretation across all models. We standardize the regressors by subtracting their means, which makes them orthogonal to the intercept and renders the interpretation of the intercept common to all models. The model space is then formed by all possible subsets of the covariates and thus contains $K = 2^k$ models in total[15]. Therefore, the model space includes the null model (the model with only the intercept and $k_j = 0$) and the full model (the model where $Z_j = Z$ and $k_j = k$). This definition of the model space is consistent with the typical situation in economics, where theories regarding variable inclusion do not necessarily contradict each other. Brock and Durlauf (2001) refer to this as the "open-endedness" of the theory[16]. Throughout, we assume that the matrix formed by adding a column of ones to Z has full column rank[17].

This model uncertainty problem is very relevant for empirical work, especially in the social sciences where typically competing theories abound on which are the important determinants of a

[13]Section 3.9 explores some important extensions, *e.g.* to the wider class of Generalized Linear Models (GLMs) and some other modelling environments that deal with specific challenges in economics.

[14]This is not as restrictive as it may seem. It certainly does not mean that the effects of determinants on the modelled phenomenon are linear; we can simply include regressors that are nonlinear transformations of determinants, interactions etc.

[15]This can straightforwardly be changed to a (smaller) model space where some of the regressors are always included in the models.

[16]In the context of growth theory, Brock and Durlauf (2001) define this concept as "the idea that the validity of one causal theory of growth does not imply the falsity of another. So, for example, a causal relationship between inequality and growth has no implications for whether a causal relationship exists between trade policy and growth."

[17]For economic applications this is generally a reasonable assumption, as typically $n > k$, although they may be of similar orders of magnitude. In other areas such as genetics this is usually not an assumption we can make. However, it generally is enough that for each model we consider to be a serious contender the matrix formed by adding a column of ones to Z_j is of full column rank, and that is much easier to ensure. Implicitly, in such situations we would assign zero prior and posterior probability to models for which $k_j \geq n$. Formal approaches to use g-priors in situations where $k > n$ include Maruyama and George (2011) and Berger et al. (2016), based on different ways of generalizing the notion of inverse matrices.

phenomenon. Thus, the issue has received quite a lot of attention both in statistics and economics, and various approaches have been suggested. We can mention:

1. Stepwise regression: this is a sequential procedure for entering and deleting variables in a regression model based on some measure of "importance", such as the t-statistics of their estimated coefficients (typically in "backwards" selection where covariates are considered for deletion) or (adjusted) R^2 (typically in "forward" selection when candidates for inclusion are evaluated).

2. Shrinkage methods: these methods aim to find a set of sparse solutions (i.e. models with a reduced set of covariates) by shrinking coefficient estimates toward zero. Bayesian shrinkage methods rely on the use of shrinkage priors, which are such that some of the estimated regression coefficients in the full model will be close to zero. A common classical method is penalized least squares, such as LASSO (least absolute shrinkage and selection operator), introduced by Tibshirani (1996), where the regression "fit" is maximized subject to a complexity penalty. Choosing a different penalty function, Fan and Li (2001) propose the smoothly clipped absolute deviation (SCAD) penalized regression estimator.

3. Information criteria: these criteria can be viewed as the use of the classical likelihood ratio principle combined with penalized likelihood (where the penalty function depends on the model complexity). A common example is the Akaike information criterion (AIC). The Bayesian information criterion (BIC) implies a stronger complexity penalty and was originally motivated through asymptotic equivalence with a Bayes factor (Schwarz, 1978). Asymptotically, AIC selects a single model that minimizes the mean squared error of prediction. BIC, on the other hand, chooses the "correct" model with probability tending to one as the sample size grows to infinity if the model space contains a true model of finite dimension. So BIC is consistent in this setting, while AIC has better asymptotic behaviour if the true model is of infinite dimension[18]. Spiegelhalter et al. (2002) propose the Deviance information criterion (DIC) which can be interpreted as a Bayesian generalization of AIC.[19]

4. Cross-validation: the idea here is to use only part of the data for inference and to assess how well the remaining observations are predicted by the fitted model. This can be done

[18] A careful classification of the asymptotic behaviour of BIC, AIC and similar model selection criteria can be found in Shao (1997) and its discussion.

[19] DIC is quite easy to compute in practice, but has been criticized for its dependence on the parameterization and its lack of consistency.

repeatedly for random splits of the data and models can be chosen on the basis of their predictive performance.

5. Extreme Bounds Analysis (EBA): this procedure was proposed in Leamer (1983, 1985) and is based on distinguishing between "core" and "doubtful" variables. Rather than a discrete search over models that include or exclude subsets of the variables, this sensitivity analysis answers the question: how extreme can the estimates be if any linear homogenous restrictions on a selected subset of the coefficients (corresponding to doubtful covariates) are allowed? An extreme bounds analysis chooses the linear combinations of doubtful variables that, when included along with the core variables, produce the most extreme estimates for the coefficient on a selected core variable. If the extreme bounds interval is small enough to be useful, the coefficient of the core variable is reported to be "sturdy". A useful discussion of EBA and its context in economics can be found in Christensen and Miguel (2018).

6. s-values: proposed by Leamer (2016a,b) as a measure of "model ambiguity". Here σ is replaced by the ordinary least squares (OLS) estimate and no prior mass points at zero are assumed for the regression coefficients. For each coefficient, this approach finds the interval bounded by the extreme estimates (based on different prior variances, elicited through R^2); the s-value (s for sturdy) then summarizes this interval of estimates in the same way that a t-statistic summarizes a confidence interval (it simply reports the centre of the interval divided by half its width). A small s-value then indicates fragility of the effect of the associated covariate, by measuring the extent to which the sign of the estimate of a regression coefficient depends on the choice of model.

7. General-to-specific modelling: this approach starts from a general unrestricted model and uses a pre-selected set of misspecification tests as well as individual t-statistics to reduce the model to a parsimonious representation. We refer the reader to Hoover and Perez (1999) and Hendry and Krolzig (2005) for background and details. Hendry and Krolzig (2004) present an application of this technique to the cross-country growth dataset of Fernández et al. (2001b) ("the FLS data", which record average per capita GDP growth over 1960-1992 for $n = 72$ countries with $k = 41$ potential regressors).

8. The Model Confidence Set (MCS): this approach to model uncertainty consists in constructing a set of models such that it will contain the best model with a given level of confidence. This was introduced by Hansen et al. (2011) and only requires the specification of a collection of competing objects (model space) and a criterion for evaluating these objects empirically. The MCS is constructed through a sequential testing procedure, where

an equivalence test determines whether all objects in the current set are equally good. If not, then an elimination rule is used to delete an underperforming object. The same significance level is used in all tests, which allows one to control the p-value of the resulting set and each of its elements. The appropriate critical values of the tests are determined by bootstrap procedures. Hansen et al. (2011) apply their procedure to e.g. US inflation forecasting, and Wei and Cao (2017) use it for modelling Chinese house prices, using predictive elimination criteria.

9. Best subset regression of Hastie et al. (2009), called full subset regression in Hanck (2016). This method considers all possible models: for a given model size k_j it selects the best in terms of fit (the lowest sum of squared residuals). As all these models have k_j parameters, none has an unfair advantage over the others using this criterion. Of the resulting set of optimal models of a given dimension, the procedure then chooses the one with the smallest value of some criterion such as Mallows' C_p [20]. Hanck (2016) does a small simulation exercise to conclude that log runtime for complete enumeration methods is roughly linear in k, as expected. Using the FLS data and a best subset regression approach which uses a leaps and bounds algorithm (see Section 3.3) to avoid complete enumeration of all models, he finds that the best model for the FLS data has 22 (using C_p) or 23 (using BIC) variables. These are larger model sizes than indicated by typical BMA results on these data[21].

10. Bayesian variable selection methods based on decision-theory. Often such methods avoid specifying a prior on model space and employ a utility or loss function defined on an all-encompassing model, *i.e.* a model that nests all models being considered. An early contribution is Lindley (1968), who proposes to include costs in the utility function for adding covariates, while Brown et al. (1999) extend this idea to multivariate regression. Other Bayesian model selection procedures that are based on optimising some loss or utility function can be found in *e.g.* Gelfand and Ghosh (1998), Draper and Fouskakis (2000) and Dupuis and Robert (2003). Note that decision-based approaches do need the specification of a utility function, which is arguably at least as hard to formulate as a model space prior.

11. Bayesian model averaging, discussed here in detail in Section 3.

12. Frequentist model averaging, discussed in Section 4.

[20] Mallows' C_p was developed for selecting a subset of regressors in linear regression problems. For model M_j with k_j parameters $C_p = \frac{SSE_j}{\hat{\sigma}^2} - n + 2k_j$ where SSE_j is the error sum of squares from M_j and $\hat{\sigma}^2$ the estimated error variance. $E(C_p) = k_j$ (approximately) and regressions with low C_p are favoured.

[21] For example, using the prior setup later described in (6) with fixed g, Ley and Steel (2009b) find the models with highest posterior probability to have between 5 and 10 regressors for most prior choices. Using random g, the results in Ley and Steel (2012) indicate that a typical average model size is between 10 and 20.

In this list, methods 5-8 were specifically motivated by and introduced in economics. Note that all but the last two methods do not involve model averaging and essentially aim at uncovering a single "best" model (or a set of models for MCS). In other words, they are model selection methods, as opposed to the model averaging methods that we focus on here. As discussed before, model selection strategies condition the inference on the chosen model and ignore all the evidence contained in the alternative models, thus typically leading to an underestimating of the uncertainty. BMA methods can also be used for model selection, by *e.g.* simply selecting the model with the highest posterior probability[22]. Typically, the opposite is not true as most model selection methods do not specify prior probabilities on the model space and thus can not provide posterior model probabilities.

Some model averaging methods in the literature combine aspects of both frequentist and Bayesian reasoning. Such hybrid methods will be discussed along with BMA if they employ a prior over models (thus leading to posterior model probabilities and inclusion probabilities of covariates), and in the FMA section if they do not. Thus, for example BACE (Bayesian averaging of classical estimates) of Sala-i-Martin et al. (2004) will be discussed in Section 3 (Subsection 3.7) and weighted average least squares (WALS) of Magnus et al. (2010) is explained in Section 4. As a consequence, all methods discussed in Section 3 can be used for model selection, if desired, while the model averaging methods in Section 4 can not lead to model selection.

Comparisons of some methods (including the method by Benjamini and Hochberg (1995) aimed at controlling the false discovery rate) can be found in Deckers and Hanck (2014) in the context of cross-sectional growth regression. Błażejowski et al. (2018) replicate the long-term UK inflation model (annual data for 1865-1991) obtained through general-to-specific principles in Hendry (2001) and compare this with the outcomes of BACE (using $k = 20$). They find that the single model selected in Hendry (2001) contains all variables that were assigned very high posterior inclusion probabilities in BACE. However, by necessity, the model selection procedure of Hendry (2001) conditions the inference on a single model, which has a posterior probability of less than 0.1 in the BACE analysis (it is the second most probable model, with the top model obtaining 20% of the posterior mass).

Wang et al. (2009) claim that there are model selection methods that automatically incorporate model uncertainty by selecting variables and estimating parameters simultaneously. Such approaches are *e.g.* the SCAD penalized regression of Fan and Li (2001) and adaptive LASSO methods as in Zou (2006). These methods sometimes possess the so-called oracle property[23].

[22] Another possibly interesting model is the median probability model of Barbieri and Berger (2004), which is the model including those covariates which have marginal posterior inclusion probabilities of 0.5 or more. This is the best single model for prediction in orthogonal and nested correlated designs under commonly used priors.

[23] The oracle property implies that an estimating procedure identifies the "true" model asymptotically if the latter

However, the oracle property is asymptotic and assumes that the "true" model is one of the models considered (the $\mathcal{M}$-closed setting). So in the practically much more relevant context of finite samples and with true models (if they can even be formulated) outside the model space these procedures will very likely still underestimate uncertainty.

Originating in machine learning, a number of algorithms aim to construct a prediction model by combining the strengths of a collection of simpler base models, like random forests, boosting or bagging (Hastie et al., 2009). As these methods typically exchange the neat, possibly structural, interpretability of a simple linear specification for the flexibility of nonlinear and nonparametric models and cannot provide probability-based uncertainty intervals, we do not consider them in this article. Various machine learning algorithms use model averaging ideas, but they are quite different from the model averaging methods discussed in this paper in that they tend to focus on combining "poor" models, since weak base learners can be boosted to lower predictive errors than strong learners (Hastie et al., 2009), they work by always combining a large number of models and their focus is purely predictive, rather than parameter estimation or the identification of structure. In line with their main objective, they do often provide good predictive performance, especially in classification problems[24]. An intermediate method was proposed in Hernández et al. (2018), who combine elements of both Bayesian additive regression trees and random forests, to offer a model-based algorithm which can deal with high-dimensional data. For discussions on the use of machine learning methods in economics, see Varian (2014), Kapetanios and Papailias (2018) and Korobilis (2018).

3 Bayesian model averaging

The formal Bayesian response to model uncertainty is Bayesian model averaging, as already explained in Section 2. Here, BMA methods are defined as those model averaging procedures for which the weights used in the averaging are based on exact or approximate posterior model probabilities and the parameters are integrated out for prediction, so there is a prior for both models and model-specific parameters.

is part of the model space and has the optimal square root convergence rate. See Fan and Li (2001).

[24]Domingos (2000) finds that BMA can fail to beat the machine learning methods in classification problems, and conjectures that this is a consequence of BMA "overfitting", in the sense that the sensitivity of the likelihood to small changes in the data carries over to the weights in (4).

3.1 Prior Structures

As we will see, prior assumptions can be quite important for the final outcomes, especially for the posterior model probabilities used in BMA. Thus, a reasonable question is whether one can assess the quality of priors or limit the array of possible choices. Of course, the Bayesian paradigm prescribes a strict separation between the information in the data being analysed and that used for the prior[25]. In principle, any coherent[26] prior which does not use the data can be seen as "valid". Nevertheless, there are a number of legitimate questions one could (and, in my view, should) ask about the prior:

- Does it adequately capture the prior beliefs of the user? Is the prior a "sensible" reflection of prior ideas, based on aspects of the model that can be interpreted? This could, for example, be assessed through (transformations of) parameters or predictive quantities implied by the prior. At the price of making the prior data-dependent, priors can even be judged on the basis of posterior results. Leeper et al. (1996) introduce the use of priors in providing appropriate structure for Bayesian VAR modelling and propose the criterion "reasonableness of results" as a general desirable property of priors. They state that "Our procedure differs from the standard practice of empirical researchers in economics only in being less apologetic. Economists adjust their models until they both fit the data and give 'reasonable' results. There is nothing unscientific or dishonest about this. It would be unscientific or dishonest to hide results for models that fit much better than the one presented (even if the hidden model seems unreasonable), or for models that fit about as well as the one reported and support other interpretations of the data that some readers might regard as reasonable."

- Does it matter for the results? If inference and decisions regarding the question of interest are not much affected over a wide range of "sensible" prior assumptions, it indicates that you need not spend a lot of time and attention to finesse these particular prior assumptions. This desirable characteristic is called "robustness" in Brock et al. (2003). Unfortunately, when it comes to model averaging, the prior is often surprisingly important, and then it is important to find structures that enhance the robustness, such as the hierarchical structures in Subsections 3.1.2 and 3.1.3.

- What is the predictive ability (as measured by *e.g.* scoring rules)? The immediate avail-

[25]This is essentially implicit in the fact that the prior times the likelihood should define a joint distribution on the observables and the model parameters, so that *e.g.* the numerator in the last expression in (2) is really $p(y, \theta_j|M_j)$ and we can use the tools of probability calculus.

[26]This means the prior is in agreement with the usual rules of probability, and prevents "Dutch book" scenarios, which would guarantee a profit in a betting setting, irrespective of the outcome.

ability of probabilistic forecasts that formally incorporate both parameter and model uncertainty provides us with a very useful tool for checking the quality of the model. If a Bayesian model predicts unobserved data well, it reflects well upon both the likelihood and the prior components of this model. Subsection 3.2.2 provides more details in the context of model averaging.

- Are the desiderata of Bayarri et al. (2012) for "objective" priors satisfied? These key theoretical principles, such as consistency and invariance, can be used to motivate the main prior setup in this paper. Here we focus on the most commonly used prior choices, based on (6) introduced in the next subsection. These prior structures have been shown (Bayarri et al., 2012) to possess very useful properties. For example, they are measurement and group invariant and satisfy exact predictive matching.[27]

- What are the frequentist properties of the resulting Bayesian procedure? Even though frequentist arguments are, strictly speaking, not part of the underlying rationale for Bayesian inference, these procedures often perform well in repeated sampling experiments, and BMA is not an exception[28]. This is discussed in Subsection 3.2.3.

- Can it serve as a benchmark? This is mentioned in Brock et al. (2003), who argue that priors "should be flexible enough to allow for their use across similar studies and thereby facilitate comparability of results." Leeper et al. (1996) use the terminology "reference prior"[29] as a prior which "only reflects a simple summary of beliefs that are likely to be uncontroversial across a wide range of users of the analysis."

3.1.1 Priors on model parameters

When deciding on the priors for the model parameters, i.e. $p(\theta_j|M_j)$ in (2), it is important to realize that the prior needs to be proper on model-specific parameters. Indeed, any arbitrary constant in $p(\theta_j|M_j)$ will similarly affect the marginal likelihood $p(y|M_j)$ defined in (2). Thus, if this constant emanating from an improper prior multiplies $p(y|M_j)$ and not the marginal likelihoods for all other models, it clearly follows from (3) that posterior model probabilities are not determined.

[27] See Bayarri et al. (2012) for the precise definition of these criteria for "objective" model priors.

[28] However, frequentist performance necessarily depends on the assumptions made about the "true" data generating model, so there is no guarantee that BMA will do well in all situations and, for example, there is anecdotal evidence that it can perform worse in terms of, say, mean squared error than simple least squares procedures for situations with small k.

[29] In the statistical literature, this name is typically given to a prior which is somewhat similar in spirit but derived from a set of precise rules designed to minimize the information in the prior; see Bernardo and Smith (1994).

If the arbitrary constant relates to a parameter that is common to all models, it will simply cancel in the ratio (3), and for such parameters we can thus employ improper priors (Fernández et al., 2001a; Berger and Pericchi, 2001). In our normal linear model in (5), the common parameters are the intercept α and the variance σ^2, and the model-specific parameters are the β_js.

In this paper, we will primarily focus on the prior structure proposed by Fernández et al. (2001a), which is in line with the majority of the current literature[30]. Fernández et al. (2001a) start from a proper conjugate prior specification, but then adopt Jeffreys-style non-informative priors for the common parameters α and σ^2. For the model-specific regression coefficients β_j, they propose a g-prior specification (Zellner, 1986) for the covariance structure[31]. The prior density[32] is then as follows:

$$p(\alpha, \beta_j, \sigma \mid M_j) \propto \sigma^{-1} f_N^{k_j}(\beta_j | 0, \sigma^2 g (Z_j' Z_j)^{-1}), \tag{6}$$

where $f_N^q(\cdot | m, V)$ denotes the density function of a q-dimensional Normal distribution with mean m and covariance matrix V. It is worth pointing out that the dependence of the g-prior on the design matrix is not in conflict with the usual Bayesian precept that the prior should not involve the data, since the model in (5) is a model for y given Z_j, so we simply condition on the regressors throughout the analysis. The regression coefficients not appearing in M_j are exactly zero, represented by a prior point mass at zero. The amount of prior information requested from the user is limited to a single scalar $g > 0$, which can either be fixed or assigned a hyper-prior distribution. In addition, the marginal likelihood for each model (and thus the Bayes factor between each pair of models) can be calculated in closed form (Fernández et al., 2001a). In particular, the posterior distribution for the model parameters has an analytically known form as follows:

$$\begin{aligned} p(\beta_j \mid \alpha, \sigma, M_j) &= f_N^{k_j}(\beta_j \mid \delta (Z_j' Z_j)^{-1} Z_j' y, \sigma^2 \delta (Z_j' Z_j)^{-1}) & (7) \\ p(\alpha \mid \sigma, M_j) &= f_N^1(\alpha \mid \bar{y}, \sigma^2/n) & (8) \\ p(\sigma^{-2} \mid M_j) &= f_{Ga}\left(\sigma^{-2} \mid \frac{n-1}{2}, \frac{s_\delta}{2}\right), & (9) \end{aligned}$$

where $\delta = g/(1+g)$, $\bar{y} = \frac{1}{n}\sum_{i=1}^n y_i$, $s_\delta = \left[\delta y' Q_{X_j} y + (1-\delta)(y - \bar{y}\iota)'(y - \bar{y}\iota)\right]$ with $Q_W = I_n - W(W'W)^{-1}W'$ for a full column rank matrix W and $X_j = (\iota : Z_j)$ (assumed of full column rank, see footnote 17). Furthermore, $f_{Ga}(\cdot \mid a, b)$ is the density function of a Gamma distribution

[30] A textbook treatment of this approach can be found in Chapter 11 of Koop (2003).

[31] In line with most of the literature, in this paper g denotes a variance factor rather than a precision factor as in Fernández et al. (2001a). Interestingly, the g-prior appears earlier in the context of combining forecasts by Diebold and Pauly (1990), who use a regression-based forecast combination framework as a means to introduce shrinkage in the weights and adopt an empirical Bayes (see Subsection 3.1.3) approach to selecting g.

[32] For the null model, the prior is simply $p(\alpha, \sigma) \propto \sigma^{-1}$.

with mean a/b. The conditional independence between β_j and α (given σ) is a consequence of demeaning the regressors. After integrating out the model parameters as above, we can write the marginal likelihood as

$$p(y|M_j) \propto (1+g)^{\frac{n-1-k_j}{2}}[1+g(1-R_j^2)]^{-\frac{n-1}{2}}, \tag{10}$$

where R_j^2 is the usual coefficient of determination for model M_j, defined through $1-R_j^2 = y'Q_{X_j}y/[(y-\bar{y}\iota)'(y-\bar{y}\iota)]$, and the proportionality constant is the same for all models, including the null model. In addition, for each model M_j, the marginal posterior distribution of the regression coefficients β_j is a k_j-variate Student-t distribution with $n-1$ degrees of freedom, location $\delta(Z_j'Z_j)^{-1}Z_j'y$ (which is the mean if $n > 2$) and scale matrix $\delta s_\delta(Z_j'Z_j)^{-1}$ (and variance $\frac{\delta s_\delta}{n-3}(Z_j'Z_j)^{-1}$ if $n > 3$). The out-of-sample predictive distribution for each given model (which in a regression model will of course also depend on the covariate values associated with the observations we want to predict) is also a Student-t distribution with $n-1$ degrees of freedom. Details can be found in equation (3.6) of Fernández et al. (2001a). Following (4), we can then conduct posterior or predictive inference by simply averaging these model-specific distributions using the posterior model weights computed (as in (3)) from (10) and the prior model distributions described in the next subsection.

There are a number of suggestions in the literature for the choice of fixed values for g, among which the most popular ones are:

- The unit information prior of Kass and Wasserman (1995) corresponds to the amount of information contained in one observation. For regular parametric families, the "amount of information" is defined through Fisher information. This gives us $g = n$, and leads to log Bayes factors that behave asymptotically like the BIC (Fernández et al., 2001a).
- The risk inflation criterion prior, proposed by Foster and George (1994), is based on the Risk inflation criterion (RIC) which leads to $g = k^2$ using a minimax perspective.
- The benchmark prior of Fernández et al. (2001a). They examine various choices of g depending on the sample size n or the model dimension k and recommend $g = \max(n, k^2)$.

When faced with a variety of possible prior choices for g, a natural Bayesian response is to formulate a hyperprior on g. This was already implicit in Zellner and Siow (1980) who use a Cauchy prior on the regression coefficients, corresponding to an inverse gamma prior on g. This idea was investigated further in Liang et al. (2008), where hyperpriors on g are shown to alleviate certain paradoxes that appear with fixed choices for g. Sections 3.1.3 and 3.2.1 will provide more detail.

The g-prior is a relatively well-understood and convenient prior with nice properties, such as invariance under rescaling and translation of the covariates (and more generally, invariant to reparameterization under affine transformations), and automatic adaptation to situations with near-collinearity between different covariates (Robert, 2007, p. 193). It can also be interpreted as the conditional posterior of the regression coefficients given a locally uniform prior and an imaginary sample of zeros with design matrix Z_j and a scaled error variance.

This idea of imaginary data is also related to the power prior approach (Ibrahim and Chen, 2000), initially developed on the basis of the availability of historical data (*i.e.* data arising from previous similar studies). In addition, the device of imaginary training samples forms the basis of the expected-posterior prior (Pérez and Berger, 2002). In Fouskakis and Ntzoufras (2016b) the power-conditional-expected-posterior prior is developed by combining the power prior and the expected-posterior prior approaches for the regression parameters conditional on the error variance.

Som et al. (2015) introduce the block hyper-g/n prior for so-called "poly-shrinkage", which is a collection of ordinary mixtures of g-priors applied separately to groups of predictors. Their motivation is to avoid certain paradoxes, related to different asymptotic behaviour for different subsets of predictors. Min and Sun (2016) consider the situation of grouped covariates (occurring, for example, in ANOVA models where each factor has various levels) and propose separate g-priors for the associated groups of regression coefficients. This also circumvents the fact that in ANOVA models the full design matrix is often not of full rank.

A similar idea is used in Zhang et al. (2016) where a two-component extension of the g-prior is proposed, with each regressor being assigned one of two possible values for g. Their prior is proper by treating the intercept as part of the regression vector in the g-prior and by using a "vague" proper prior[33] on σ^2. They focus mostly on variable selection.

A somewhat different approach was advocated by George and McCulloch (1993, 1997), who use a prior on the regression coefficient which does not include point masses at zero. In particular, they propose a normal prior with mean zero on the entire k-dimensional vector of regression coefficients β given the model M_j which assigns a small prior variance to the coefficients of the variables that are "inactive"[34] in M_j and a larger variance to the remaining coefficients. In addition, their overall prior is proper and does not assume a common intercept.

[33]Note that this implies the necessity to choose the associated hyperparameters in a sensible manner, which is nontrivial as what is sensible depends on the scaling of the data.

[34]Formally, all variables appear in all models, but the coefficients of some variables will be shrunk to zero by the prior, indicating that their role in the model is negligible.

Raftery et al. (1997) propose yet another approach and use a proper conjugate[35] prior with a diagonal covariance structure for the regression coefficients (except for categorical predictors where a g-prior structure is used).

3.1.2 Priors over models

The prior $P(M_j)$ on model space is typically constructed by considering the probability of inclusion of each covariate. If the latter is the same for each variable, say w, and we assume inclusions are prior independent, then

$$P(M_j) = w^{k_j}(1-w)^{k-k_j}. \tag{11}$$

This implies that prior odds will favour larger models if $w > 0.5$ and the opposite if $w < 0.5$. For $w = 0.5$ all model have equal prior probability $1/k$. Defining model size as the number of included regressors in a model, a simple way to elicit w is through the prior mean model size, which is wk.[36] As the choice of w can have a substantial effect on the results, various authors (Brown et al., 1998; Clyde and George, 2004; Ley and Steel, 2009b; Scott and Berger, 2010) have suggested to put a Beta(a, b) hyperprior on w. This results in

$$P(M_j) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \frac{\Gamma(a+k_j)\Gamma(b+k-k_j)}{\Gamma(a+b+k)}, \tag{12}$$

which leads to much less informative priors in terms of model size. Ley and Steel (2009b) compare both approaches and suggest choosing $a = 1$ and $b = (k-m)/m$, where m is the chosen prior mean model size. This means that the user only needs to specify a value for m. The large differences between the priors in (12) and (11) can be illustrated by the prior odds they imply. Figure 2 compares the log prior odds induced by the fixed and random w priors, in the situation where $k = 67$ (corresponding to the growth dataset first used in Sala-i-Martin et al. (2004)) and using $m = 7, 33.5$ and 50. For fixed w, this corresponds to $w = 7/67, w = 1/2$ and $w = 50/67$ while for random w, we have used the specification of Ley and Steel (2009b). The figure displays the prior odds in favour of a model with $k_i = 10$ versus models with varying k_j.

Note that the random w case always leads to down-weighting of models with k_j around $k/2$, irrespectively of m. This counteracts the fact that there are many more models with k_j around $k/2$ in the model space than of size nearer to 0 or k.[37] In contrast, the prior with fixed w does not take the number of models at each k_j into account and simply always favours larger models

[35] Conjugate prior distributions combine analytically with the likelihood to give a posterior in the same class of distributions as the prior.

[36] So, if our prior belief about mean model size is m, then we simply choose $w = m/k$.

[37] This reflects the multiplicity issue analysed more generally in Scott and Berger (2010) who propose to use (12) with $a = b = 1$ implying a prior mean model size of $k/2$. The number of models with k_j regressors in $\mathcal{M}$ is given

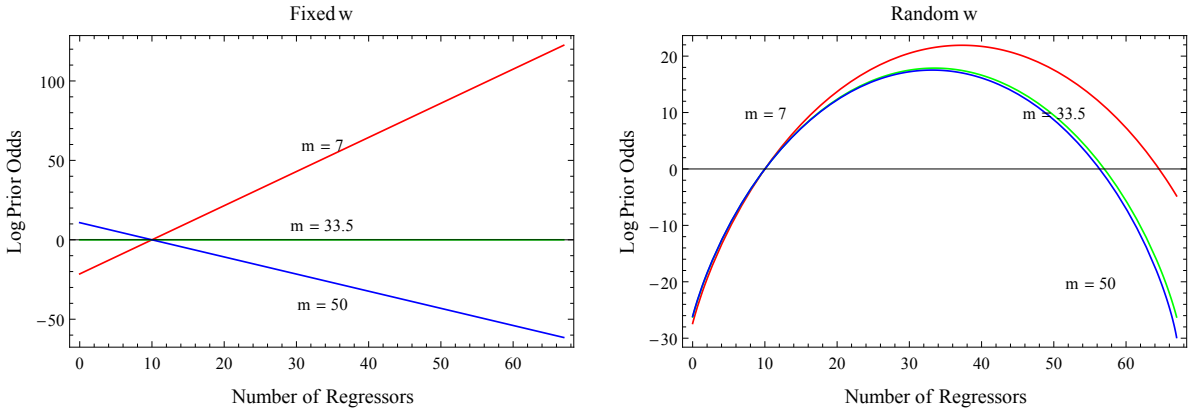

Figure 2: Log of Prior Odds: $k_i = 10$ vs varying k_j. From Ley and Steel (2009b).

when $m > k/2$ and smaller ones when $m < k/2$. Note also the much wider range of values that the log prior odds take in the case of fixed w. Thus, the choice of m is critical for the prior with fixed w, but much less so for the hierarchical prior structure, which is naturally adaptive to the data observed.

It is often useful to elicit prior ideas by focusing on model size, as it is an easily understood concept. In addition, there will often be a preference for somewhat smaller models due to their interpretability and simplicity. The particular choice of $m = 7$ (mentioned above) was used in Sala-i-Martin et al. (2004) in the context of growth regression and has become a rather popular choice in a variety of applied contexts. Sala-i-Martin et al. (2004) sensibly argue that the prior mean model size should not be linearly increasing with k, but provide little motivation for specifically choosing $m = 7$. The origins of this choice may be related to computational restrictions faced by earlier empirical work (*e.g.* the EBA analysis of Levine and Renelt (1992) was conducted on a restricted set of models that never had more than 8 regressors). I think that any particular prior choice should be considered within the appropriate context and I would encourage the use of sensitivity analyses and robust prior structures (such as the hierarchical prior leading to (12)). Giannone et al. (2018) investigate whether sparse modelling is a good approach to predictive problems in economics on the basis of a number of datasets from macro, micro and finance. They find that artificially tight model priors[38] focused on small models induce sparsity at the expense of predictive performance and model fit. They conclude that "predictive model

by $\binom{k}{k_j}$. For example, with $k = 67$, we have 1 model with $k_j = 0$ and $k_j = k$, 8.7×10^8 models with $k_j = 7$ and $k_j = 60$ and a massive 1.4×10^{19} models with $k_j = 33$ and 34.

[38]They use a very tight prior indeed, which corresponds to a prior mean model size $m = k/(k+1)$ which is less than one!

uncertainty seems too pervasive to be treated as statistically negligible. The right approach to scientific reporting is thus to assess and fully convey this uncertainty, rather than understating it through the use of dogmatic (prior) assumptions favoring low dimensional models."

George (1999b) raises the issue of "dilution", which occurs when posterior probabilities are spread among many similar models, and suggest that prior model probabilities could have a built-in adjustment to compensate for dilution by down-weighting prior probabilities on sets of similar models. George (2010) suggests three distinct approaches for the construction of these so-called "dilution priors", based on tessellation determined neighbourhoods, collinearity adjustments, and pairwise distances between models. Dilution priors were implemented in economics by Durlauf et al. (2008) to represent priors that are uniform on theories (*i.e.* neighbourhoods of similar models) rather than on individual models, using a collinearity adjustment factor. A form of dilution prior in the context of models with interactions of covariates is the heredity prior of Chipman et al. (1997) where interaction are only allowed to be included if both main effects are included (strong heredity) or at least one of the main effects (weak heredity). In the context of examining the sources of growth in Africa, Crespo Cuaresma (2011) comments that the use of a strong heredity prior leads to different conclusions than the use of a uniform prior in the original paper by Masanjala and Papageorgiou (2008).[39] Either prior is, of course, perfectly acceptable, but it is clear that the user needs to reflect which one best captures the user's own prior ideas and intended interpretation of the results. Using the same data, Moser and Hofmarcher (2014) compare a uniform prior with a strong heredity prior and a tesselation dilution prior and find quite similar predictive performance (as measured by LPS and CRPS, explained in Section 3.2.2) but large differences in posterior inclusion probabilities (probably related to the fact that both types of dilution priors are likely to have quite different responses to multicollinearity).

Womack et al. (2015) propose viewing the model space as a partially ordered set. When the number of covariates increases, an isometry argument leads to the Poisson distribution as the unique, natural limiting prior over model dimension. This limiting prior is derived using two constructions that view an individual model as though it is a "local" null hypothesis and compares its prior probability to the probability of the alternatives that nest it. They show that this prior induces a posterior that concentrates on a finite true model asymptotically.

Another interesting recent development is the use of a loss function to assign a model prior. Equating information loss as measured by the expected minimum Kullback-Leibler divergence between any model and its nearest model and by the "self-information loss"[40] while adding an

[39] See also Papageorgiou (2011), which is a reply to the comment by Crespo Cuaresma.

[40] This is a loss function (also known as the log-loss function) for probability statements, which is given by the negative logarithm of the probability.

adjustment for complexity, Villa and Lee (2016) propose the prior $P(M_j) = \exp(-ck_j)$ for some $c > 0$. This builds on an idea of Villa and Walker (2015).

3.1.3 Empirical Bayes versus Hierarchical Priors

The prior in (6) and (11) only depends on two scalar quantities, g and w. Nevertheless, these quantities can have quite a large influence on the posterior model probabilities and it is very challenging to find a single default choice for g and w that performs well in all cases, as explained in *e.g.* Fernández et al. (2001a), Berger and Pericchi (2001) and Ley and Steel (2009b). One way of reducing the impact of such prior choices on the outcome is to use hyperpriors on w and g, which fits seamlessly with the Bayesian paradigm. Hierarchical priors on w are relatively easy to deal with and were already discussed in the previous section.

Zellner and Siow (1980) used a multivariate Cauchy prior on the regression coefficients rather than the normal prior in (6). This was inspired by the argument in Jeffreys (1961) in favour of heavy-tailed priors[41]. Since a Cauchy is a scale mixture of normals, this means that implicitly the Zellner-Siow prior uses an Inverse-Gamma$(1/2, n/2)$ prior on g.

Liang et al. (2008) introduce the hyper-g priors, which correspond to the following family of priors:

$$p(g) = \frac{a-2}{2}(1+g)^{-a/2} \tag{13}$$

where $a > 2$ in order to have a proper distribution for $g > 0$. This includes the priors proposed in Strawderman (1971) in the context of the normal means problem. A value of $a = 4$ was suggested by Cui and George (2008) for model selection with known σ, while Liang et al. (2008) recommend values $2 < a \leq 4$. Feldkircher and Zeugner (2009) propose to use a hyper-g prior with a value of a that leads to the same mean of the, so-called, shrinkage factor[42] $\delta = g/(1+g)$ as for the unit information or the RIC prior. Ley and Steel (2012) consider the more general class of beta priors on the shrinkage factor where a Beta(b, c) prior on δ induces the following prior on g:

$$p(g) = \frac{\Gamma(b+c)}{\Gamma(b)\Gamma(c)} g^{b-1}(1+g)^{-(b+c)}. \tag{14}$$

[41] The reason for this was the limiting behaviour of the resulting Bayes factors as we consider models with better and better fit. In this case, you would want these Bayes factors, with respect to the null model, to tend to infinity. This criterion is called "information consistency" in Bayarri et al. (2012) and its absence is termed "information paradox" in Liang et al. (2008).

[42] The name "shrinkage factor" derives from the fact that the posterior mean of the regression coefficients for a given model is the OLS estimator times this shrinkage factor, as clearly shown in (7) and the ensuing discussion.

This is an inverted beta distribution[43] (Zellner, 1971, p. 375) which clearly reduces to the hyper-g prior in (13) for $b = 1$ and $c = (a/2) - 1$. Generally, the hierarchical prior on g implies that the marginal likelihood of a given model is not analytically known, but is the integral of (10) with respect to the prior of g. Liang et al. (2008) propose the use of a Laplace approximation for this integral, while Ley and Steel (2012) use a Gibbs sampler approach to include g in the Markov chain Monte Carlo procedure (see footnote 54). Some authors have proposed Beta shrinkage priors as in (14) that lead to analytical marginal likelihoods by making the prior depend on the model: the robust prior of Bayarri et al. (2012) truncate the prior domain to $g > [(n+1)/(k_j + 1)] - 1$ and Maruyama and George (2011) adopt the choice $b + c = (n - k_j - 1)/2$ with $c < 1/2$. However, the truncation of the robust prior is potentially problematic for cases where n is much larger than a typical model size (as is often the case in economic applications). Ley and Steel (2012) propose to use the beta shrinkage prior in (14) with mean shrinkage equal to the one corresponding to the benchmark prior of Fernández et al. (2001a) and the second parameter chosen to ensure a reasonable prior variance. They term this the benchmark beta prior and recommend using $b = c \max\{n, k^2\}$ and $c = 0.01$.

An alternative way of dealing with the problem of selecting g and w is to adopt so-called "empirical Bayes" (EB) procedures, which use the data to suggest appropriate values to choose for w and g. Of course, this amounts to using data information in selecting the prior, so is not formally in line with the Bayesian paradigm. Often, such EB methods are adopted for reasons of convenience and because they are sometimes shown to have good properties. In particular, they provide "automatic" calibration of the prior and avoid the (relatively small) computational complications that typically arise when we adopt a hyperprior on g. Motivated by information theory, Hansen and Yu (2001) proposed a local EB method which uses a different g for each model estimated by maximizing the marginal likelihood. George and Foster (2000) develop a global EB approach, which assumes one common g and w for all models and borrows strength from all models by estimating g and w through maximizing the marginal likelihood, averaged over all models. Liang et al. (2008) propose specific ways of estimating g in this context.

There is some evidence in the literature regarding comparisons between fully Bayes and EB procedures: Cui and George (2008) largely favour (global) EB in the context of known σ and $k = n$, whereas Liang et al. (2008) find that there is little difference between EB and fully Bayes procedures (with unknown σ and $k < n$). Scott and Berger (2010) focus on EB and fully Bayesian ways of dealing with w, which, respectively, use maximum likelihood[44] and a Beta(1,1) or uniform hyperprior on w. They remark that both fully Bayesian and EB procedures exhibit

[43] Also known as a gamma-gamma distribution (Bernardo and Smith, 1994, p. 120).

[44] This is the value of w that maximizes the marginal likelihood of w summed over model space, or $\arg\max_w p(y)$ in (3), which can be referred to as type-II maximum likelihood.

clear multiplicity adjustment: as the number of noise variables increases, the posterior inclusion probabilities of variables decrease (the analysis with fixed w shows no such adjustment; see also footnote 37). However, they highlight some theoretical differences, for example the fact that EB will assign probability one to either the full model or the null model whenever one of these models has the largest marginal likelihood. They also show rather important differences in various applications, one of which uses data on GDP growth. Overall, they recommend the use of fully Bayesian procedures. Li and Clyde (2017) compare EB and fully Bayes procedures in the more general GLM context (see Section 3.9.1), and find that local EB does badly in simulations from the null model in that it almost always selects the full model.

3.2 Properties of BMA

3.2.1 Consistency and paradoxes

One of the desiderata in Bayarri et al. (2012) for objective model selection priors is model selection consistency (introduced by Fernández et al. (2001a)), which implies that if data have been generated by $M_j \in \mathcal{M}$, then the posterior probability of M_j should converge to unity with sample size[45]. Fernández et al. (2001a) present general conditions for the case with non-random g and show that consistency holds for *e.g.* the unit information and benchmark priors (but not for the RIC prior). When we consider hierarchical priors on g, model selection consistency is achieved by the Zellner-Siow prior in Zellner and Siow (1980) but not by local and global EB priors nor by the hyper-g prior in Liang et al. (2008), who therefore introduce a consistent modification, the hyper-g/n prior, which corresponds to a beta distribution on $g/(n+g)$. Consistency is shown to hold for the priors of Maruyama and George (2011), Feldkircher and Zeugner (2009) (based on the unit information prior) and the benchmark beta prior of Ley and Steel (2012).

Moreno et al. (2015) consider model selection consistency when the number of potential regressors k grows with sample size. Consistency is found to depend not only on the priors for the model parameters, but also on the priors in model space. They conclude that if $k = O(n^b)$, the unit information prior, the Zellner-Siow prior and the intrinsic prior[46] lead to consistency for $0 \leq b < 1/2$ under the uniform prior over model space, while consistency holds for $0 \leq b \leq 1$ if

[45] So this is a concept defined in the $\mathcal{M}$-closed framework, mentioned in Subsection 2.1.

[46] Intrinsic priors were introduced to justify the intrinsic Bayes factors (Berger and Pericchi, 1996). In principle, these are often based on improper reference (see footnote 29) or Jeffreys priors and the use of a so-called minimal training sample to convert the improper prior to a proper posterior. The latter is then used as a prior for the remaining data, so that Bayes factors can be computed. As the outcome depends on the arbitrary choice of the minimal training sample, such Bayes factors are typically "averaged" over all possible training samples. Intrinsic priors are priors that (at least asymptotically) mimic these intrinsic Bayes factors.

we use a Beta(1,1) hyperprior on w in (12). Wang and Maruyama (2016) investigate Bayes factor consistency associated with the prior structure in (6) for the problem of comparing nonnested models under a variety of scenarios where model dimension grows with sample size. They show that in some cases, the Bayes factor is consistent whichever the true model is, and that in others, the consistency depends on the pseudo-distance between the models. In addition, they find that the asymptotic behaviour of Bayes factors and intrinsic Bayes factors are quite similar.

Mukhopadhyay et al. (2015) show that in situations where the true model is not one of the candidate models (the $\mathcal{M}$-open setting), the use of g-priors leads to selecting a model that is in an intuitive sense closest to the true model. In addition, the loss incurred in estimating the unknown regression function under the selected model tends to that under the true model. These results have been shown under appropriate conditions on the rate of growth of g as n grows and for both the cases when the number of potential predictors remains fixed and when $k = O(n^b)$ for some $0 < b < 1$.[47] Mukhopadhyay and Samanta (2017) extend this to the situation of mixtures of g-priors and derive consistency properties for growing k under a modification of the Zellner-Siow prior, that continue to hold for more general error distributions.

Using Laplace approximations, Xiang et al. (2016) prove that in the case of hyper-g priors with growing model sizes, the Bayes factor is consistent when $k = O(n^b)$ for some $0 < b \leq 1$, even when the true model is the null model. For the case when the true model is not the null model, they show that Bayes factors are always consistent when the true model is nested within the model under consideration, and they give conditions for the non-nested case. In the specific context of analysis-of-variance (ANOVA) models, Wang (2017) shows that the Zellner-Siow prior and the beta shrinkage prior of Maruyama and George (2011) yield inconsistent Bayes factors when k is proportional to n due to the presence of an inconsistency region around the null model. To solve the latter inconsistency, Wang (2017) propose a variation on the hyper-g/n prior, which generalizes the prior arising from a Beta distribution on $g/(\frac{n}{k} + g)$. Consistency for the power-expected-posterior approach using independent Jeffreys baseline priors is shown by Fouskakis and Ntzoufras (2016a).

Sparks et al. (2015) consider posterior consistency for parameter estimation, rather than model selection. They consider posterior consistency under the sup vector norm (weaker than the usual l_2 norm) in situations where k grows with sample size and derive necessary and sufficient conditions for consistency under the standard g-prior, the empirical Bayes specification of

[47] Unlike Moreno et al. (2015), they do not explicitly find different results for different priors on the model space, which looks like an apparent contradiction. However, their results are derived under an assumption (their (A3)) bounding the ratio of prior model probabilities. Note from our Figure 2 that ratio tends to be much smaller when we use a hyperprior on w.

George and Foster (2000) and the hyper-g and Zellner-Siow mixture priors.

In addition, BMA also has important optimality properties in terms of shrinkage in high-dimensional problems. In particular, Castillo et al. (2015) prove that BMA in linear regression leads to an optimal rate of contraction of the posterior on the regression coefficients to a sparse "true" data-generating model (a model where many of the coefficients are zero), provided the prior sufficiently penalizes model complexity. Rossell and Telesca (2017) show that BMA leads to fast shrinkage for spurious coefficients (and explore so-called nonlocal priors that provide even faster shrinkage in the BMA context).

A related issue is that Bayes factors can asymptotically behave in the same way as information criteria. Kass and Wasserman (1995) investigate the relationship between BIC (see Section 2.2) and Bayes factors using unit information priors for testing non-nested hypotheses and Fernández et al. (2001a) show that log Bayes factors with $g_j = n/f(k_j)$ (with $f(\cdot)$ some function which is finite for finite arguments) tend to BIC. When k is fixed, this asymptotic equivalence to BIC extends to the Zellner-Siow and Maruyama and George (2011) priors (Wang, 2017) and also the intrinsic prior (Moreno et al., 2015).

Liang et al. (2008) remark that analyses with fixed g tend to lead to a number of paradoxical results. They mention the Bartlett (or Lindley) paradox, which is induced by the fact that very large values of g will induce support for the null model, irrespective of the data[48]. Another paradox they explore is the information paradox, where as R_i^2 tends to one, the Bayes factor in favour of M_i versus, say, the null model does not tend to ∞ but to a constant depending on g (see also footnote 41). From (16) in Subsection 3.4 this latter limit is $(\frac{w}{1-w})^{k_i}(1+g)^{(n-k_i-1)/2}$. Liang et al. (2008) show that this information paradox is resolved by local or global EB methods, but also by using hyperpriors $p(g)$ that satisfy $\int (1+g)^{(n-k_i-1)/2} p(g) \mathrm{d}g = \infty$ for all $k_i \leq k$, which is the case for the Zellner-Siow prior, the hyper-g prior and the benchmark beta priors (the latter two subject to a condition, which is satisfied in most practical cases).

3.2.2 Predictive performance

Since any statistical model will typically not eliminate uncertainty and it is important to capture this uncertainty in forecasting, it is sensible to consider probabilistic forecasts, which have become quite popular in many fields. In economics, important forecasts such as the quarterly Bank of England inflation report are presented in terms of predictive distributions, and in the field of finance the area of risk management focuses on probabilistic forecasts of portfolio values. Rather

[48] This can be seen immediately by considering (16) in Subsection 3.4, which behaves like a constant times $(1+g)^{(k_j-k_i)/2}$ as $g \to \infty$.

than having to condition on estimated parameters, the Bayesian framework has the advantage that predictive inference can be conducted on the basis on the predictive distribution, as in (1) where all uncertainty regarding the parameters and the model is properly incorporated. This can be used to address a genuine interest in predictive questions, but also as a model-evaluation exercise. In particular, if a model estimated on a subset of the data manages to more or less accurately predict data that were not used in the estimation of the model, that intuitively suggests satisfactory performance.

In order to make this intuition a bit more precise, scoring rules provide useful summary measures for the evaluation of probabilistic forecasts. Suppose the forecaster wishes to optimize the scoring rule. If the scoring rule is proper, the forecaster has no incentive to predict any other distribution than his or her true belief for the forecast distribution. Details can be found in Gneiting and Raftery (2007).

Two important aspects of probabilistic forecasts are calibration and sharpness. Calibration refers to the compatibility between the forecasts and the observations and is a joint property of the predictions and the events that materialize. Sharpness refers to the concentration of the predictive distributions and is a property of the forecasts only. Proper scoring rules address both of these issues simultaneously. Popular scoring rules, used in assessing predictive performance in the context of BMA are

- The logarithmic predictive score (LPS), which is the negative of the logarithm of the predictive density evaluated at the observation. This was introduced in Good (1952) and used in the BMA context in Madigan et al. (1995), Fernández et al. (2001a,b) and Ley and Steel (2009b).

- The continuous ranked probability score (CRPS). The CRPS measures the difference between the predicted and the observed cumulative distributions as follows[49]:

$$CRPS(Q,x) = \int_{-\infty}^{\infty} \left[Q(y) - \mathbb{1}(y \geq x)\right]^2 \mathrm{d}y, \tag{15}$$

  where Q is the predictive distribution, x is the observed outcome and $\mathbb{1}(\cdot)$ is the indicator function. CRPS was found in Gneiting and Raftery (2007) to be less sensitive to outliers than LPS and was introduced in the context of growth regressions by Eicher et al. (2011).

Simple point forecasts do not allow us to take into account the uncertainty associated with the prediction, but are popular in view of their simplicity, especially in more complicated models

[49] An alternative expression is given by Gneiting and Raftery (2007) as $CRPS(Q,x) = E_Q|X-x| - \frac{1}{2}E_Q|X-Z|$, where X and Z are independent copies of a random variable with distribution function Q and finite first moment. This shows that CRPS generalizes the absolute error, to which it reduces if Q is a point forecast.

incorporating *e.g.* dynamic aspects or endogenous regressors. Such models are often evaluated in terms of the mean squared forecast error or the mean absolute forecast error calculated with respect to a point forecast.

There is a well-established literature indicating the predictive advantages of BMA. For example, Madigan and Raftery (1994) state that BMA predicts at least as well[50] as any single model in terms of LPS and Min and Zellner (1993) show that expected squared error loss of point (predictive mean) forecasts is always minimized by BMA provided the model space includes the model that generated the data[51]. On the basis of empirical results, Raftery et al. (1997) report that predictive coverage is improved by BMA with respect to prediction based on a single model. Similar results were obtained by Fernández et al. (2001a), who also use LPS as a model evaluation criterion in order to compare various choices of g in the prior (6). Fernández et al. (2001b) find, on the basis of LPS that BMA predicts substantially better than single models (such as the model with highest posterior probability) in growth data. Ley and Steel (2009b) corroborate these findings, especially with a hyperprior on w, as used in (12). Piironen and Vehtari (2017) focus on model selection methods, but state that "From the predictive point of view, best results are generally obtained by accounting for the model uncertainty and forming the full BMA solution over the candidate models, and one should not expect to do better by selection." In the context of volatility forecasting of non-ferrous metal futures, Lyócsa et al. (2017) show that averaging of forecasts substantially improves the results, especially where the averaging is conducted through BMA.

3.2.3 Frequentist performance of point estimates

Point estimation is not a particularly natural Bayesian concept, and the Bayesian paradigm does not rely on frequentist considerations (unlike FMA, where often the model weights are chosen to optimize some frequentist criterion). Nevertheless, it is interesting to know the properties of point estimates resulting from BMA under repeated sampling. Whenever we consider sampling-theoretic properties, we require an assumption about the sampling model under which the properties will be studied. Using the assumption that the data is generated by the predictive in (1),

[50]This optimality holds under the assumption that the data is generated by the predictive in (1) rather than a single "true" model. George (1999a) comments that "It is tempting to criticize BMA because it does not offer better average predictive performance than a correctly specified single model. However, this fact is irrelevant when model uncertainty is present because specification of the correct model with certainty is then an unavailable procedure. In most practical applications, the probability of selecting the correct model is less than 1, and a mixture model elaboration seems appropriate."

[51]The importance of this latter assumption is discussed in Diebold (1991), who points out that violation of this assumption could negatively affect large sample performance of BMA in point forecasting.

Raftery and Zheng (2003) state a number of important results about Bayesian model selection and BMA. Firstly, they state a result by Jeffreys (1961), which is that for two nested models, model choice based on Bayes factors minimizes the sum of type I and type II errors. Secondly, they mention properties of BMA point estimates (of parameters and observables). In particular, BMA leads to point estimates that minimize MSE and BMA estimation intervals are calibrated in the sense that the average coverage probability of a BMA interval with posterior probability α is at least equal to α. Of course, this does not provide any guarantee for the frequentist properties when the actual observations come from a different distribution, but there exists no compelling argument for any particular choice of such a "true" distribution. I believe it is highly unlikely in economics (as in most other areas of application) that the data are truly generated by a single model within the model space (assumed in *e.g.* Castillo et al. (2015)) or a local neighbourhood around such a model that shrinks with sample size[52] (as in *e.g* Claeskens and Hjort (2003) and Hjort and Claeskens (2003)) (see also footnote 50).

3.3 BMA in practice: Numerical methods

One advantage of the prior structure in (6) is that integration of the model parameters can be conducted analytically, and the Bayes factor between any two given models can be computed quite easily, given g, through (10). The main computational challenge is then constituted by the often very large model space, which makes complete enumeration impossible. In other words, we simply can not try all possible models, as there are far too many of them[53].

A first possible approach is to (drastically) reduce the number of models under consideration. One way to do this is the Occam's window algorithm, which was proposed by Madigan and Raftery (1994) for graphical models and extended to linear regression in Raftery et al. (1997). It uses a search strategy to weed out the models that are clearly dominated by others in terms of posterior model probabilities and models that have more likely submodels nested within them. An algorithm for finding the best models is the so-called leaps and bounds method used by Raftery (1995) for BMA, based on the all-subsets regression algorithm of Furnival and Wilson (1974). The resulting set of best models can then still be reduced further through Occam's window if required. The Occam's window and the leaps and bounds algorithms are among the methods implemented in the BMA `R` package of Raftery et al. (2010) and the leaps and bounds algorithm

[52]Insightful comments concerning the practical relevance of this particular assumption can be found in Raftery and Zheng (2003) and other discussions of Claeskens and Hjort (2003) and Hjort and Claeskens (2003).

[53]In areas such as growth economics, we may have up to $k = 100$ potential covariates. This implies a model space consisting of $K = 2^{100} = 1.26 \times 10^{30}$ models. Even with fast processors, this dramatically exceeds the number of models that can be dealt with exhaustively. In other fields, the model space can even be much larger: for example, in genetics k is the number of genes and can well be of the order of tens of thousands.

was used in *e.g.* Masanjala and Papageorgiou (2008) and Eicher et al. (2011).

However, this tricky issue of exploring very large model spaces is now mostly dealt with through so-called Markov chain Monte Carlo (MCMC) methods[54]. In particular, a popular strategy is to run an MCMC algorithm in model space, sampling the models that are most promising: the one that is most commonly used is a random-walk Metropolis sampler usually referred to as MC^3, introduced in Madigan and York (1995) and used in *e.g.* Raftery et al. (1997) and Fernández et al. (2001a). On the basis of the application in Masanjala and Papageorgiou (2008), Crespo Cuaresma (2011) finds that MC^3 leads to rather different results from the leaps and bounds method, which does not seem to explore the model space sufficiently well.

The original prior in George and McCulloch (1993) is not conjugate in that the prior variance of β does not involve σ^2 (unlike (6)); this means that marginal likelihoods are not available analytically, but an MCMC algorithm can easily be implemented by a Gibbs sampler on the joint space of the parameters and the models. This procedure is usually denoted as Stochastic Search Variable Selection (SSVS). George and McCulloch (1997) also introduce an alternative prior which is conjugate, leading to an analytical expression for the marginal likelihoods and inference can then be conducted using an MCMC sampler over only the model space (like MC^3).

Clyde et al. (2011) remark that while the standard algorithms MC^3 and SSVS are easy to implement, they may mix poorly when covariates are highly correlated. More advanced algorithms that utilize other proposals can then be considered, such as adaptive MCMC[55] (Nott and Kohn, 2005) or evolutionary Monte Carlo (Liang and Wong, 2000). Clyde et al. (2011) propose a Bayesian adaptive sampling algorithm (BAS), that samples models without replacement from the space of models. In particular, the probability of a model being sampled is proportional to some

[54] Suppose we have a distribution, say π, of which we do not know the properties analytically and which is difficult to simulate from directly. MCMC methods construct a Markov chain that has π as its invariant distribution and conduct inference from the generated chain. The draws in the chain are of course correlated, but ergodic theory still forms a valid basis for inference. Various algorithms can be used to generate such a Markov chain. An important one is the Metropolis-Hastings algorithm, which takes an arbitrary Markov chain and adjusts it using a simple accept-reject mechanism to ensure the stationarity of π for the resulting Markov chain. Fairly mild conditions then ensure that the values in the realized chain actually converge to draws from π. Another well-known algorithm is the Gibbs sampler, which partitions the vector of random variables which have distribution π into components and replaces each component by a draw from its conditional distribution given the current values of all other components. Various combinations of these algorithms are also popular (*e.g.* a Gibbs sampler where one or more conditionals are not easy to draw from directly and are treated through a Metropolis-Hastings algorithm). More details can be found in *e.g.* Robert and Casella (2004) and Chib (2011).

[55] MCMC methods often require certain parameters (of the proposal distribution) to be appropriately tuned for the algorithm to perform well. Adaptive MCMC algorithms achieve such tuning automatically. See Atchadé and Rosenthal (2005) for an introduction.

probability mass function with known normalizing constant. Every time a new model is sampled, one needs to account for its mass by subtracting off its probability from the probability mass function to ensure that there is no duplication and then draw a new model from the renormalized distribution. The model space is represented by a binary tree structure indicating inclusion or exclusion of each variable, and marginal posterior inclusion probabilities are set at an initial estimate and then adaptively updated using the marginal likelihoods from the sampled models.

Generic numerical methods were compared in García-Donato and Martínez-Beneito (2013), who identify two different strategies:

i) MCMC methods to sample from the posterior (3) in combination with estimation based on model visit frequencies and

ii) searching methods looking for "good" models with estimation based on renormalization (*i.e.* with weights defined by the analytic expression of posterior probabilities, such as in (16)).

Despite the fact that it may, at first sight, appear that ii) should be a more efficient strategy, they show that i) is potentially more precise than ii) which could be biased by the searching procedure. Nevertheless, implementations of ii) have lead to fruitful contributions, and a lot of the most frequently used software (see Section 6) uses this method. Of course, if the algorithm simply generates a chain through model space in line with the posterior model probabilities (such as MC^3 using the prior in (6)) then both strategies can be used to conduct inference on quantities of interest, *i.e.* to compute the model probabilities to be used in (4). Indeed, Fernández et al. (2001a) suggest the use of the correlation between posterior model probabilities based on i) and ii) as an indicator of convergence of the chain. However, some other methods only lend themselves to one of the strategies above. For example, the prior of George and McCulloch (1993) does not lead to closed form expressions for the marginal likelihood, so SVSS based on this prior necessarily follows the empirical strategy i). Examples of methods that can only use strategy ii) are BAS in Clyde et al. (2011), which only samples each model once, and the implementation in Raftery (1995) based on a leaps and bound algorithm which is used only to identify the top models. These methods need to use the renormalization strategy, as model visit frequencies are not an approximation to posterior model probabilities in their case.

MC^3 uses a Metropolis sampler which proposes models from a small neighbourhood of the current model, say M_j, namely all models with one covariate less or more. Whereas this works well for moderate values of k, it is not efficient in situations with large k where we expect parsimonious models to fit the data well. This is because the standard MC^3 algorithm (using a uniform distribution on the model neighbourhood) will propose to add a covariate with probability

$(k - k_j)/k$, which is close to 1 if $k >> k_j$. Therefore, the algorithm will much more frequently propose to add a variable than to delete one. However, the acceptance rate of adding a new variable is equal to the acceptance rate of deleting a variable if the chain is in equilibrium. Thus, a large number of adding moves are rejected and this leads to a low between-model acceptance rate. Brown et al. (1998) extend the MC^3 proposal by adding a "swap" move where one included and one excluded covariate are selected at random and the proposed model is the one where they are swapped. They suggest to generate a candidate model by either using the MC^3 move or the swap move. Lamnisos et al. (2009) extend this further by decoupling the MC^3 move into an "add" and a "delete" move (to avoid proposing many more additions than deletions) and uniformly at random choosing whether the candidate model is generated from an "add", a "delete" or a "swap" move. In addition, they allow for less local moves by adding, deleting or swapping more than one covariate at a time. The size of the blocks of variables used for these moves is drawn from a binomial distribution. This allows for faster exploration of the model space. In Lamnisos et al. (2013) an adaptive MCMC sampler is introduced where the success probability of the binomial distribution (used for block size) is tuned adaptively to generate a target acceptance probability of the proposed models. They successfully manage to deal with problems like finding genetic links to colon tumours with $n = 62$ and $k = 1224$ genes in a (more challenging) probit model context (see Section 3.9.2), where their algorithm is almost 30 times more efficient[56] than MC^3 and the adaptive Gibbs sampler of Nott and Kohn (2005). Problems with even larger k can be dealt with through more sophisticated adaptive MCMC algorithms. Griffin et al. (2017) propose such algorithms which exploit the observation that in these settings the vast majority of the inclusion indicators of the variables will be virtually uncorrelated a posteriori. They are shown to lead to orders of magnitude improvements in efficiency compared to the standard Metropolis-Hastings algorithm, and are successfully applied to extremely challenging problems (*e.g.* with $k = 22,576$ possible covariates and $n = 60$ observations).

In situations with more complicated models, it may not be practical to run an MCMC chain over model space. One option would be to use an easily computed approximation to the marginal likelihoods and to base the MCMC model moves on that approximation (discussed in Subsection 3.7), but this may not always work well. If the number of models is not too large, another option is to treat all K models separately and to compute their marginal likelihoods directly so that BMA inference can be implemented. This is not a straightforward calculation on the basis of MCMC output for each model, but there are a number of standard methods in the statistics literature. Chib (2011) discusses the method of Chib (1995) which is essentially based on Bayes'

[56]The efficiency is here standardized by CPU time. Generally, the efficiency of a Monte Carlo method is proportional to the reciprocal of the variance of the sample mean estimator normalized by the size of the generated sample.

formula evaluated at a particular value of the parameters. Another popular method is bridge sampling (Meng and Wong, 1996), which is a generalization of importance sampling with a high degree of robustness with respect to the relative tail behaviour of the importance function. For nested models, we can often compute Bayes factors through the Savage-Dickey density ratio (Verdinelli and Wasserman, 1995), which is typically both easy to calculate and accurate. An in-depth discussion of all of these methods can be found in DiCiccio et al. (1997).

3.4 Role of the prior

It has long been understood that the effect of the prior distribution on posterior model probabilities can be much more pronounced than its effect on posterior inference given a model (Kass and Raftery, 1995; Fernández et al., 2001a). Thus, it is important to better understand the role of the prior assumptions in BMA. While Fernández et al. (2001a) examined the effects of choosing fixed values for g, a more systematic investigation of the interplay between g and w was conducted in Ley and Steel (2009b) and Eicher et al. (2011).

From combining the marginal likelihood in (10) and the model space prior in (11), we obtain the posterior odds between models, given g and w:

$$\frac{P(M_i \mid y, w, g)}{P(M_j \mid y, w, g)} = \left(\frac{w}{1-w}\right)^{k_i - k_j} (1+g)^{\frac{k_j - k_i}{2}} \left(\frac{1+g(1-R_i^2)}{1+g(1-R_j^2)}\right)^{-\frac{n-1}{2}}. \tag{16}$$

The three factors on the right-hand side of (16) correspond to, respectively, a model size (or complexity) penalty induced by the prior odds on the model space, a model size penalty resulting from the marginal likelihood (Bayes factor) and a lack-of-fit penalty from the marginal likelihood. It is clear that for fixed g and w, the complexity penalty increases with g and decreases with w (see also the discussion in Section 3.8 and in Eicher et al. (2011)). Ley and Steel (2012) consider each of the three factors separately, and define penalties as minus the logarithm of the corresponding odds factor, which ties in well with classical information criteria, which, in some cases, correspond to the limits of log posterior odds (Fernández et al., 2001a). The complexity penalty induced by the prior odds can be in favour of the smaller or the larger model, whereas the penalties induced by the Bayes factor are always in favour of the smaller and the better fitting models.

Ley and Steel (2012) find that the choice of the hyperpriors on g and w can have a large effect on the induced penalties for model complexity but does not materially affect the impact of the relative fit of the models. They also investigate how the overall complexity penalty behaves if we integrate over g and w. Figure 3 plots the logarithm of the approximate posterior odds for M_i versus M_j as a function of k_j when fixing $k_i = 10$, for different values of the prior mean model

size, m, using a beta hyperprior on w as in Ley and Steel (2009b) and the benchmark beta prior on g in (14) with $c = 0.01$. We use $n = 72$ and $k = 41$ (as in the FLS growth data) and assume $R_i^2 = R_j^2$. We contrast these graphs with those for fixed values of θ and g (corresponding to the values over which the priors are centered) as derived from (16). Whereas the log posterior odds are linear in $(k_i - k_j)$ for fixed values of θ and g, they are much less extreme for the random θ and g case, and consistently penalize models of size around $k/2$. This reflects the multiplicity penalty (see Section 3.1.2) which is implicit in the prior and analyzed in Scott and Berger (2010) in a more general context, and in Ley and Steel (2009b) in this same setting. The behaviour is qualitatively similar to that of the prior odds in Figure 2. The difference with Figure 2 is that we now consider the complexity penalty in the posterior, which also includes an (always positive) size penalty resulting from the Bayes factor. No fixed w can induce a multiplicity correction. As in Figure 2, the (relatively arbitrary) choice of m matters very little for the case with random w (and g), whereas it makes a substantial difference if we keep w (and g) fixed.

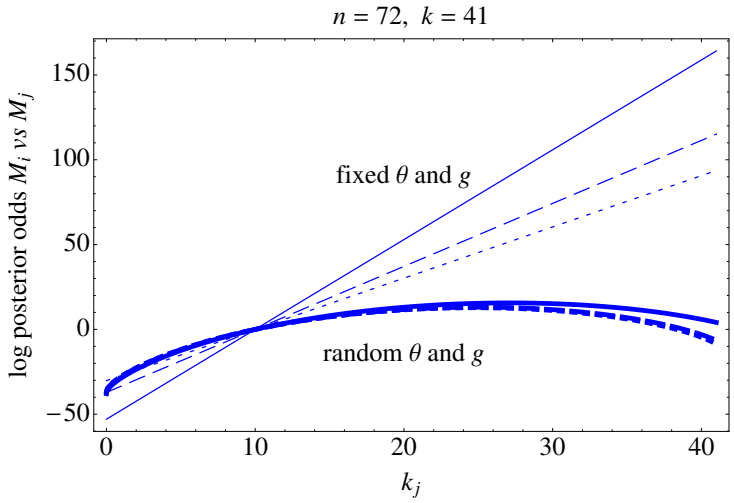

Figure 3: Posterior odds as a function of k_j when $k_i = 10$ with equal fit, using $m = 7$ (solid), $m = k/2$ (dashed), and $m = 2k/3$ (dotted). Bold lines correspond to random w and g. From Ley and Steel (2012).

Thus, marginalising out the posterior model probabilities with the hyperpriors on w and g induces a much flatter model size penalty over the entire range of model sizes. This then makes the analysis less dependent on (usually arbitrary) prior assumptions and increases the relative importance of the data contribution (the model fit) to the posterior odds.

3.5 Data Robustness

Generally, in economics, the quality of the data may be problematic. An important issue is whether policy conclusions and key insights change when data are revised to eliminate errors, incorporate improved data or account for new price benchmarks. For example, the Penn World Table (PWT) income data, a dataset frequently used in cross-country empirical work in economics, have undergone periodic revisions. Ciccone and Jarociński (2010) applied the methodologies of Fernández et al. (2001b) and Sala-i-Martin et al. (2004) for investigating the determinants of cross-country growth to data generated as in Sala-i-Martin et al. (2004) from three different versions of the PWT, versions 6.0-6.2. Both methods led to substantial variations in posterior inclusion probabilities of certain covariates between the different datasets.

It is, of course, not surprising that solving a really complicated problem (assessing the posterior distribution on a model space that contains huge quantities of models) on the basis of a relatively small number of observations is challenging, and if we modify the data in the absence of strong prior information, we can expect some (perhaps even dramatic) changes in our inference. Clearly, if we add prior information such changes would normally be mitigated. Perhaps the most relevant question in practice is whether we can conduct meaningful inference using BMA with the kinds of prior structures that we have discussed in this paper, such as (6), on the basis of available data.

Using BMA, Feldkircher and Zeugner (2012) examine more in detail what causes the lack of robustness found in Ciccone and Jarociński (2010). One first conclusion is that the changes are roughly halved if the analyses with the different PWT data use the same set of countries. They also stress that the use of the fixed value of g as in the benchmark prior leads to a very large g and it is clear from (16) that this amplifies the effect of differences in the fit on posterior odds. Thus, small differences in the data can have substantial impact on the results. They propose to use a hyper-g prior which allows the model to adjust g to the data, and this dramatically reduces the instability. Interestingly, this is not a stronger prior, but a less informative one. The important thing is that fixing g at a value which is not warranted by the data quality leads to an exaggerated impact of small difference in model fit. They find that the analysis with stochastic g leads to much smaller values of g. The same behaviour was also found in Ley and Steel (2012) where three datasets were analysed: two cross-country growth datasets as in Fernández et al. (2001b) (with $n = 72$ and $k = 41$) and Sala-i-Martin et al. (2004) (with $n = 88$ and $k = 67$) and the returns-to-schooling data of Tobias and Li (2004) ($n = 1190$ and $k = 26$). In all these examples, the data favour[57] values of g in the range 15-50, which contrasts rather sharply with the fixed

[57] This can be inferred from the likelihood which is marginalised with respect to all parameters but g and averaged over models; see expression (9) in Ley and Steel (2012) which is plotted in their Figure 9.

values of g that the benchmark prior would suggest, namely 1681, 4489 and 1190, respectively. As a consequence of the smaller g, differences between models will be less pronounced and this can be seen as a quite reasonable reaction to relatively low-quality data.

Rockey and Temple (2016) consider restricting the model space by imposing the presence of initial GDP per capita and regional dummies, *i.e.* effectively using a more informative prior on the model space. They conclude that this enhances robustness even when the analysis is extended to more recent vintages of the Penn World Table (they also consider PWT 6.3-8.0).

3.6 Collinearity and Jointness

One of the primary outputs of a BMA analysis is the posterior distribution of the regression coefficients, which is a mixed distribution (for each coefficient a continuous distribution with mass point at zero, reflecting exclusion of the associated regressor) of dimension k, which is often large. Thus, this is a particularly hard object to describe adequately. Summarizing this posterior distribution merely by its k marginals is obviously a gross simplification and fails to capture the truly multivariate nature of this distribution. Thus, efforts have been made to define measures that more adequately reflect the posterior distribution. Such measures should be well suited for extracting relevant pieces of information. It is important that they provide additional insight into properties of the posterior that are of particular interest, and that they are easy to interpret. Ley and Steel (2007) and Doppelhofer and Weeks (2009) propose various measures of "jointness", or the tendency of variables to appear together in a regression model. Ley and Steel (2007) formulate four desirable criteria for such measures to possess:

- Interpretability: any jointness measure should have either a formal statistical or a clear intuitive meaning in terms of jointness.
- Calibration: values of the jointness measure should be calibrated against some clearly defined scale, derived from either formal statistical or intuitive arguments.
- Extreme jointness: the situation where two variables always appear together should lead to the jointness measure reaching its value reflecting maximum jointness.
- Definition: the jointness measure should always be defined whenever at least one of the variables considered is included with positive probability.

The jointness measure proposed in Ley and Steel (2007) satisfies all of these criteria and is defined as the posterior odds ratio between those models that include a set of variables and the models that only include proper subsets. If we consider the simple case of bivariate jointness between

variables i and j, and we define the events $\tilde{\imath}$ and $\tilde{\jmath}$ as the exclusion of i and j, respectively, this measure is the probability of joint inclusion relative to the probability of including either regressor, but not both:

$$\mathcal{J}_{ij}^{\circ} = \frac{P(i \cap j \,|\, y)}{P(i \cap \tilde{\jmath} \,|\, y) + P(\tilde{\imath} \cap j \,|\, y)}.$$

An alternative measure, suggested by Doppelhofer and Weeks (2009), takes the form

$$\mathcal{J}_{ij} = \ln \frac{P(i \cap j \,|\, y) P(\tilde{\imath} \cap \tilde{\jmath} \,|\, y)}{P(i \cap \tilde{\jmath} \,|\, y) P(\tilde{\imath} \cap j \,|\, y)},$$

which has the interpretation of the log of the posterior odds of including i given that j is included divided by the posterior odds of including i given that j is not included. Ley and Steel (2009a) discuss how these and another jointness measure proposed by Strachan (2009) compare on the basis of the criteria above. They highlight that $\mathcal{J}_{ij}$ is undefined when a variable is always included or excluded, and for such cases Doppelhofer and Weeks (2009) propose to use the log posterior odds of inclusion of the variable with inclusion probability in $(0, 1)$. This, however, implies that pairwise jointness involving covariates that are almost always in the model will depend entirely on the low-probability models not involving this variable. As $\mathcal{J}^{\circ}$ is a posterior odds ratio, its values can be immediately interpreted as evidence in favour of jointness (values above one) or disjointness (values below one, suggesting that variables are more likely to appear on their own than jointly). Disjointness can occur, *e.g.*, when variables are highly collinear and are proxies or substitutes for each-other. In the context of two growth data sets, Ley and Steel (2007) find evidence of jointness only between important variables, which are complements in that each of them has a separate role to play in explaining growth. They find many more occurrences of disjointness, where regressors are substitutes and really should not appear together. However, these latter regressors tend to be fairly unimportant drivers of growth.

Man (2018) compares different jointness measures using data from a variety of disciplines and finds that results differ substantially between the measures of Doppelhofer and Weeks (2009) on the one hand and Ley and Steel (2007) on the other hand. In contrast, results appear quite robust across different prior choices. Man (2018) suggests the use of composite indicators, which combine the information contained in the different concepts, often by simply averaging over different indicators. Given the large differences in the definitions and the properties of the jointness measures considered, I would actually expect to find considerable differences. I would recommend selecting a measure that makes sense to the user while making sure the interpretation of the results is warranted by the properties of the specific measure chosen. The use of composite indicators, however interesting it may be from the perspective of combining information, seems to me to make interpretation much harder.

Ghosh and Ghattas (2015) investigate the consequences of strong collinearity for Bayesian variable selection. They find that strong collinearity may lead to a multimodal posterior distribution over models, in which joint summaries are more appropriate than marginal summaries. They recommend a routine calculation of the joint inclusion probabilities for correlated covariates, in addition to marginal inclusion probabilities, for assessing the importance of regressors in Bayesian variable selection.

Crespo Cuaresma et al. (2016) propose a different approach to deal with patterns of inclusion such as jointness among covariates. They use a two-step approach starting from the posterior model distribution obtained from BMA methods, and then use clustering methods based on latent class analysis to unveil clusters of model profiles. Inference in the second step is based on Dirichlet process clustering methods. They also indicate that the jointness measures proposed in the literature (and mentioned earlier in this subsection) relate closely to measures used in data mining (see their footnote 1). These links are further explored in Hofmarcher et al. (2018), who propose a new measure of bivariate jointness which is a regularised version of the so-called Yule's Q association coefficient, used in the machine learning literature on association rules. They use insights from the latter to extend the set of desirable criteria outlined above, and show they are satisfied by the measure they propose.

3.7 Approximations and hybrids

The use of the prior structure in (6) for the linear normal model in (5) immediately leads to a closed-form marginal likelihood, but for other Bayesian models this may not be the case. In particular, more complex models (such as described in Section 3.9) often do not allow for an analytic expression. One approach to addressing this problem is to use an approximation to the marginal likelihood, which is based on the ideas underlying the development of BIC (or the Schwarz criterion). In normal (or, more generally, regular[58]) models, BIC can be shown (see Schwarz, 1978 and Raftery, 1995) to provide an asymptotic approximation to the log Bayes factor. In the specific context of the normal linear model with prior (6), Fernández et al. (2001a) provide a direct link between the BIC approximation and the choice of $g = n$ (the unit information prior, which essentially leaves the asymptotics unaffected). Thus, in situations where closed-form expressions for the Bayes factors are not available (or very costly to compute), BIC has been used to approximate the actual Bayes factor. For example, some available procedures for models with endogenous regressors and models assuming Student-t sampling are based on BIC approximations to the marginal likelihood.

[58]Regular models are such that the sampling distribution of the maximum likelihood estimator is asymptotically normal around the true value with covariance matrix equal to the inverse expected Fisher information matrix.

An alternative approximation of posterior model probabilities is through the (smoothed) AIC. Burnham and Anderson (2002) provide a Bayesian justification for AIC (with a different prior over the models than the BIC approximation) and suggest the use of AIC-based weights as posterior model probabilities. The smoothed AIC approximation is used in the context of assessing the pricing determinants of credit default swaps in Pelster and Vilsmeier (2018)[59].

Sala-i-Martin et al. (2004) use asymptotic reasoning in the specific setting of the linear model with a g-prior to avoid specifying a prior on the model parameters and arrive at a BIC approximation in this manner. They call the resulting procedure BACE (Bayesian averaging of classical estimates). This approach was generalized to panel data by Moral-Benito (2012), who proposes Bayesian averaging of maximum likelihood estimates (BAMLE). In a similar vein, hybrids of frequentist and Bayesian methods were used in Durlauf et al. (2008) and Durlauf et al. (2012) to deal with the difficult issue of endogenous regressors (later discussed in Subsection 3.9.7). In particular, they propose to use BIC approximations to posterior model probabilities for averaging over classical two-stage least squares (2SLS) estimates. Durlauf et al. (2008) comment that: "Hybrids of this type are controversial from the perspective of the philosophical foundations of statistics and we do not pursue such issues here. Our concern is exclusively with communicating the evidentiary support across regressions; our use of averaging is simply a way of combining cross-model information and our posterior probabilities are simply relative weights."

3.8 Prior robustness: illusion or not?

Previous sections have already stressed the importance of the choices of the hyperparameters and have made the point that settings for g and w are both very important for the results. However, there are examples in the literature where rather different choices of these hyperparameters led to relatively similar conclusions, which might create the impression that these choices are not that critical. For example, if we do not put hyperpriors on g and w, we note that the settings used in Fernández et al. (2001b) and in Sala-i-Martin et al. (2004) lead to rather similar results in the analysis of the growth data of Sala-i-Martin et al. (2004), which have $n = 88$ and $k = 67$. The choices made in Fernández et al. (2001b) are $w = 0.5$ and $g = k^2$, whereas the BACE analysis in Sala-i-Martin et al. (2004) is based on $w = 7/k$ (giving a prior mean model size

[59]Pelster and Vilsmeier (2018) comment that "Burnham and Anderson (2002) strongly suggest replacing BIC by the (smoothed) Akaike Information Criterion (S-AIC) since BIC aims to identify the models with the highest probability of being the true model for the data assuming a true model exists. Since a true model in reality does not exist, BIC tends to assign too much weight to the best model. AIC, by the contrary, tries to select the model that most adequately fits the unknown model, and can be interpreted as the probability that a model is the expected best model in repeated samples. Hansen (2007) reports rather poor performance of BIC weights compared to S-AIC weights, particularly if the sample size is large."

$m = 7$) and $g = n$. As BACE attempts to avoid specifying a prior on the model parameters, the latter is not explicit, but follows from the expression used in Sala-i-Martin et al. (2004) for the Bayes factors, which is a close approximation to Bayes factors for the model with prior (6) using $g = n$. There is, however, an important tradeoff between values for g and w, which was visually clarified in Ley and Steel (2009b) and was also mentioned in Eicher et al. (2011). In particular, Ley and Steel (2009b) present a version of Figure 4 which shows the contours in (g, m) space of the values of fit (R_i^2) of Model M_i that would give it equal posterior probability to M_j, when $n = 88, k = 67, k_i = 8, k_j = 7$, and $R_j^2 = 0.75$.

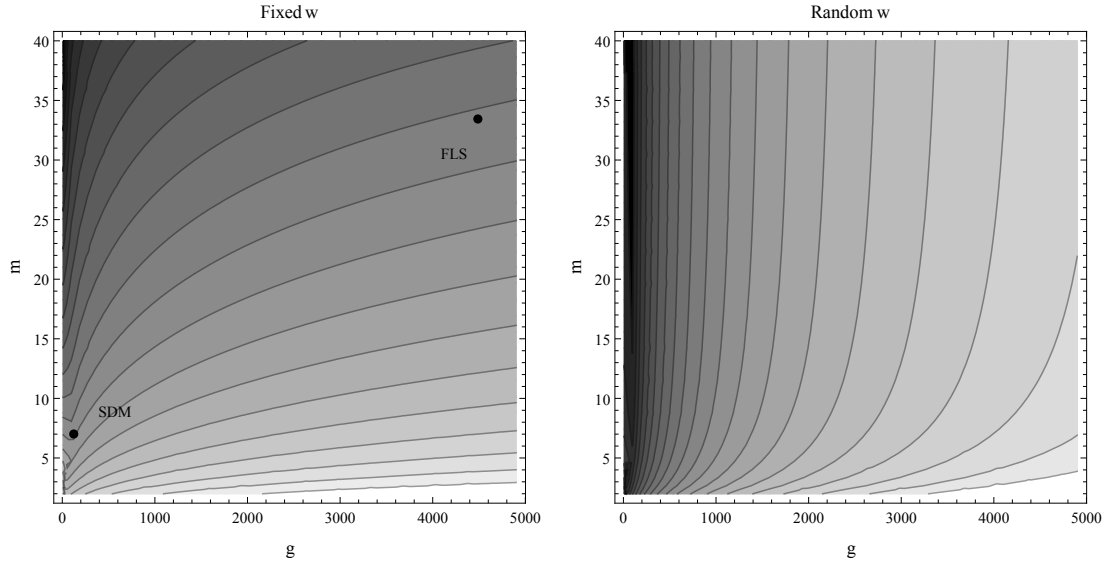

Figure 4: Equal Posterior Probability Contours for different values of R_i^2, using $n = 88, k = 67, k_i = 8, k_j = 7$, and $R_j^2 = 0.75$. The left panel is for fixed w and also indicates the choices of (g, m) in Fernández et al. (2001b) (FLS) and Sala-i-Martin et al. (2004) (SDM). The right panel corresponds to random w. Adapted from Ley and Steel (2009b).

From the left panel in Figure 4 the particular combinations of (g, w) values underlying the analyses in Fernández et al. (2001b) and in Sala-i-Martin et al. (2004) are on contours that are quite close, and thus require a very similar increase in R^2 to compensate for an extra regressor (in fact, the exact values are $R_i^2 = 0.7731$ for FLS and $R_i^2 = 0.7751$ for SDM). Remember from Section 3.4 that the model complexity penalty increases with g and decreases with w (or $m = wk$), so the effects of increasing both g and w (as in Fernández et al. (2001b) with respect to Sala-i-Martin et al. (2004)) can cancel each-other out, as they do here.

In conclusion, it turns out that certain (often used) combinations happen to give quite similar results. However, this does not mean that results are generally robust with respect to these

choices, and there is ample evidence in the literature (Ley and Steel, 2009b; Eicher et al., 2011) that these choices matter quite crucially. Also, it is important to point out that making certain prior assumptions implicit (as is done in BACE) does not mean they no longer matter. Rather, it seems to me more useful to be transparent about prior choices and to attempt to robustify the analysis by using prior structures that are less susceptible to subjectively chosen quantities. This is illustrated in the right panel of Figure 4, where the equal probability contours are drawn for the case with a Beta$(1,(k-m)/m)$ hyperprior on w. As discussed in Section 3.1.2, this prior is much less informative, which means that the actual choice of m matters much less and the trade-off between g and w has almost disappeared. A hyperprior can also be adopted for g, as in Section 3.1.3, which would further robustify the analysis (see also Section 3.4).

3.9 Other sampling models

This section describes the use of BMA in the context of other sampling models, which are sometimes fairly straightforward extensions of the normal linear regression model (for example, the Hoeting et al. (1996) model for outliers in Section 3.9.4 or the Student-t model mentioned in Section 3.9.5) and sometimes imply substantial challenges in terms of prior elicitation and numerical implementation. Many of the models below are inspired by issues arising in economics, such as dynamic models, spatial models, models for panel data and models with endogenous covariates.

3.9.1 Generalized linear models

Generalized Linear Models (GLMs) describe a more general class of models (McCullagh and Nelder, 1989) that covers the normal linear regression model but also regression models where the response variable is non-normal, such as binomial (e.g. logistic or logit regression models, probit models), Poisson, multinomial (e.g. ordered response models, proportional odds models) or gamma distributed.

An early contribution to BMA with GLMs is Raftery (1996), who proposes to use approximations for the Bayes factors, based on the Laplace method for integrals. He also suggests a way to elicit reasonable (but data-dependent) proper priors.

Sabanés Bové and Held (2011b) consider the interpretation of the g-prior in linear models as the conditional posterior of the regression coefficients given a locally uniform prior and an imaginary sample of zeros with design matrix Z_j and a scaled error variance, and extend this to the GLM context. Asymptotically, this leads to a prior which is very similar to the standard g-prior, except that it has an extra scale factor c and a weighting matrix W in the covariance structure. In many cases, $c = 1$ and $W = I$, which leads to exactly the same structure as (6).

This idea was already used in the conjugate prior proposed by Chen and Ibrahim (2003), although they only considered the case with $W = I$ and do not treat the intercept separately. For priors on g, Sabanés Bové and Held (2011b) consider a Zellner-Siow prior and a hyper-g/n prior. Both choices are shown to lead to consistent model selection in Wu et al. (2016).

The priors on the model parameters designed for GLMs in Li and Clyde (2017) employ a different type of "centering" of the covariates (induced by the observed information matrix at the maximum likelihood estimator (MLE) of the coefficients), leading to a g-prior that displays local orthogonality properties at the MLE. In addition, they use a wider class of (potentially truncated) hyper-priors for g[60]. Their results rely on approximations, and, more importantly, their prior structures are data-dependent (depending on y, not just the design matrix). Interestingly, on the basis of theoretical and empirical findings in the GLM context, they recommend similar hyper-priors[61] as recommended by Ley and Steel (2012) in a linear regression setting.

The power-conditional-expected-posterior prior of Fouskakis and Ntzoufras (2016b) has also been extended to the GLM setting in Perrakis et al. (2015).

3.9.2 Probit models

A popular approach for modelling dichotomous responses uses the probit model, which is an example of a GLM. If we observe $y_1, \ldots, y_n$ taking the values either zero or one, this model assumes that the probability that $y_i = 1$ is modeled by $y_i|\eta_i \sim \text{Bernoulli}(\Phi(\eta_i))$ where Φ is the cumulative distribution function of a standard normal random variable and $\eta = (\eta_1, \eta_2, \ldots, \eta_n)'$ is a vector of linear predictors modelled as $\eta = \alpha\iota + Z_j\beta_j$, where α, β_j and Z_j are as in (5).

Typical priors have a product structure with a normal prior on β_j (for example a g-prior) and an improper uniform on α. Generally, posterior inference for the probit model can be facilitated by using the data augmentation approach of Albert and Chib (1993).

When dealing with model uncertainty, this model is often analysed through a Markov chain Monte Carlo method on the joint space of models and model parameters, since the marginal likelihood is no longer analytically available. This complicates matters with respect to the linear regression model as this space is larger than model space and the dimension of the model parameters varies with the model. Thus, reversible jump Metropolis-Hastings methods (Green, 1995) are typically used here. Details and comparison of popular algorithms can be found in Lamnisos et al. (2009).

[60] In particular, they use the class of compound confluent hypergeometric distributions, which contains most hyperpriors used in the literature as special cases.

[61] Namely, the hyper-g/n prior and the benchmark beta prior.

3.9.3 Generalized additive models

Generalized additive models are generalized linear models in which the linear predictor depends linearly on unknown smooth functions of the covariates, so these models can account for non-linear effects; see e.g. Hastie et al. (2009). In the context of the additive model[62], Sabanés Bové and Held (2011a) consider using fractional polynomials for these smooth functions in combination with a hyper-g prior. They combine covariate uncertainty with flexible modelling of additive effects by expanding the model space to include different powers of each potential regressor. To explore this large model space, they propose an MCMC algorithm which adapts the Occam's window strategy of Raftery et al. (1997). Using splines for the smooth functions, Sabanés Bové et al. (2015) propose hyper-g priors based on an iterative weighted least squares approximation to the nonnormal likelihood. They conduct inference using an algorithm which is quite similar to that in Sabanés Bové and Held (2011b).

3.9.4 Outliers

The occurrence of outliers (atypical observations) is a general problem that may affect both parameter estimation and model selection, and the issue is especially relevant if the modelling assumptions are restrictive, for example by imposing normality (*i.e.* thin tails). In the context of normal linear regression, Hoeting et al. (1996) propose a Bayesian method for simultaneous variable selection and outlier identification, using variable inflation to model outliers. They use a proper prior and recommend the use of a pre-screening procedure to generate a list of potential outliers, which are then used to define the model space to consider. Ho (2015) applies this methodology to explore the cross-country variation in the output impact of the global financial crisis in 2008-9.

Outliers are also accommodated in Doppelhofer et al. (2016). In the context of growth data, they also introduce heteroscedastic measurement error, with variance potentially differing with country and data vintage. The model also accounts for vintage fixed effects and outliers. They use data from eight vintages of the PWT (extending the data used in Sala-i-Martin et al. (2004)) to estimate the model, and conclude that 18 variables are relatively robustly associated with GDP growth over the period 1960 to 1996, even when outliers are allowed for. The quality of the data seems to improve in later vintages and varies quite a bit among the different countries. They estimate the model using JAGS, a generic MCMC software package which determines the choice of sampling strategy, but this approach is very computer-intensive[63].

[62] This is where the link function is the identity link, so we have a normally distributed response variable.

[63] They comment that a single MCMC run takes about a week to complete, even with the use of multiple computers

Of course, the use of more flexible error distributions such as scale mixtures of normals (like, for example, the Student-t regression model mentioned in the next section) can be viewed as a way to make the results more robust against outliers.

3.9.5 Non-normal errors

Doppelhofer and Weeks (2011) use a Student-t model as the sampling model, instead of the normal model in (5) in order to make inference more robust with respect to outliers and unmodelled heterogeneity. They consider either fixing the degrees of freedom parameter of the Student-t distribution or estimating it and they use the representation of a Student-t as a continuous scale mixture of normals. Throughout, they approximate posterior model probabilities by the normality-based BIC, so the posterior model probabilities remain unaffected and only the estimates of the model parameters are affected[64]. Oberdabernig et al. (2018) use a Student-t sampling model with fixed degrees of freedom in a spatial BMA framework to investigate the drivers of differences in democracy levels across countries.

Non-normality can, of course, also be accommodated by transformations of the data. Hoeting et al. (2002) combine selection of covariates with the simultaneous choice of a transformation of the dependent variable within the Box-Cox family of transformations. Charitidou et al. (2018) consider four different families of transformations along with covariate uncertainty and use model averaging based on intrinsic and fractional Bayes factors.

3.9.6 Dynamic models

In the context of simple AR(F)IMA time-series models, BMA was used in *e.g.* Koop et al. (1997), which will be discussed in more detail in Subsection 5.2.

Raftery et al. (2010) propose the idea of using state-space models in order to allow for the forecasting model to change over time while also allowing for coefficients in each model to evolve over time. Due to the use of approximations, the computations essentially boil down to the Kalman filter. In particular, they use the following dynamic linear model, where the subscript indicates time $t = 1, \ldots, T$:

$$y_t \sim N(z_t^{(j)\prime}\theta_t^{(j)}, H^{(j)}) \tag{17}$$

$$\theta_t^{(j)} \sim N(\theta_{t-1}^{(j)}, Q_t^{(j)}), \tag{18}$$

and parallel chains.

[64] For each model they propose a simple Gibbs sampler setup after augmenting with the mixing variables.

and the superscript is the model index with models differing in the choice of covariates in the first equation. Choosing $Q_t^{(j)}$ sequences is not required as they propose to use a forgetting factor (discount factor) on the variance of the state equation (18). Using another forgetting factor, Raftery et al. (2010) approximate the model probabilities at each point in time, which greatly simplifies the calculations. Dynamic model averaging (DMA) is where these model weights are used to average in order to conduct inference, such as predictions, and dynamic model selection (DMS) uses a single model for such inference (typically the one with the highest posterior probability) at each point in time. Koop and Korobilis (2012) apply DMA and DMS to inflation forecasting, and find that the best predictors change considerably over time and that DMA and DMS lead to improved forecasts with respect to the usual autoregressive and time-varying-parameter models. Drachal (2016) investigates the determinants of monthly spot oil prices between 1986 and 2015, using Dynamic Model Averaging (DMA) and Dynamic Model Selection (DMS). Although some interesting patterns over time were revealed, no significant evidence was found that DMA is superior in terms of forecast accuracy over, for example, a simple ARIMA model (although this seems to be based only on point forecasts, and not on predictive scores). Finally, Onorante and Raftery (2016) introduce a dynamic Occam's window to deal with larger model spaces.

van der Maas (2014) proposes a dynamic BMA framework that allows for time variation in the set of variables that is included in the model, as well as structural breaks in the intercept and conditional variance. This framework is then applied to real-time forecasting of inflation.

Other time-varying Bayesian model weight schemes are considered in Hoogerheide et al. (2010), who find that they outperform other combination forecasting schemes in terms of predictive and economic gains. They suggest forecast combinations based on a regression approach with the predictions of different models as regressors and with time-varying regression coefficients.

Another increasingly important area of dynamic model averaging is the use of BMA methods for ensembles of deterministic models, which was introduced for probabilistic weather forecasting in Raftery et al. (2005). BMA is used to post-process the forecasts provided by ensembles[65], where each forecast is associated with a predictive distribution and weights are computed by maximum likelihood (implemented through an expectation-maximization algorithm).

[65] An ensemble is a collection of runs of deterministic models which differ in initial conditions or physical model assumptions.

3.9.7 Endogeneity

If one or more of the covariates is correlated with the error term in the equation corresponding to (5), we talk of endogeneity. In particular, we consider the following extension of the model in (5):

$$y = \alpha\iota + x\gamma + Z_j\beta_j + \varepsilon \tag{19}$$

$$x = W\delta + \nu, \tag{20}$$

where x is an endogenous regressor[66] and W is a set of instruments, which are independent of ε. Finally, the error terms corresponding to observation i are identically and independently distributed as follows:

$$(\varepsilon_i, \nu_i)' \sim N(0, \Sigma), \tag{21}$$

with $\Sigma = (\sigma_{ij})$ a 2×2 covariance matrix. It is well-known that whenever $\sigma_{12} \neq 0$ this introduces a bias in the OLS estimator of γ and a standard classical approach is the use of 2SLS estimators instead. For BMA it also leads to misleading inference on coefficients and model probabilities, even as sample size grows, as shown in Miloschewski (2016).

Tsangarides (2004) and Durlauf et al. (2008) consider the issue of endogenous regressors in a BMA context. Durlauf et al. (2008) focus on uncertainty surrounding the selection of the endogenous and exogenous variables and propose to average over 2SLS model-specific estimates for each single model. Durlauf et al. (2012) consider model averaging across just-identified models (with as many instruments as endogenous regressors). In this case, model-specific 2SLS estimates coincide with LIML estimates, which means that likelihood-based BIC weights have some formal justification.

Lenkoski et al. (2014) extend BMA to formally account for model uncertainty not only in the selection of endogenous and exogenous variables, but also in the selection of instruments. They propose a two-step procedure that first averages across the first-stage models (*i.e.* linear regressions of the endogenous variables on the instruments) and then, given the fitted endogenous regressors from the first stage, it again takes averages in the second stage. Both steps use BIC weights. Their approach, named two-stage BMA (2SBMA), was used in Eicher and Kuenzel (2016) in the context of establishing the effect of trade on growth, where feedback (and thus endogeneity) can be expected.

Koop et al. (2012) use simulated tempering to design an MCMC method that can deal with BMA in the endogeneity context in one step. It is, however, quite a complicated and computa-

[66] For simplicity, we focus the presentation on the case with one endogenous regressor, but this can immediately be extended.

tionally costly algorithm and it is nontrivial to implement.

Karl and Lenkoski (2012) propose IVBMA, which is based on the Gibbs sampler of Rossi et al. (2006) for instrumental variables models and use conditional Bayes factors to include model uncertainty in this Gibbs algorithm. It hinges on certain restrictions (*e.g.* joint Normality of the errors is important and the prior needs to be conditionally conjugate), but the algorithm is very efficient and is implemented in an `R`-package (see Section 6).

3.9.8 Panel data and individual effects

Panel (or longitudinal) data contain information on individuals ($i = 1, \ldots, N$) over different time periods ($t = 1, \ldots, T$). Correlation between covariates and error term might arise through a time-invariant individual effect, denoted by η_i in the model

$$y_{it} = z_{it}'\beta + \eta_i + \epsilon_{it}. \tag{22}$$

Moral-Benito (2012) uses BMA in such a panel setting with strictly exogenous regressors (uncorrelated with the ϵ_{it}s but correlated with the individual effects). In this framework, the vector of regressors can also include a lagged dependent variable (y_{it-1}) which is then correlated with ϵ_{it-1}. Moral-Benito (2012) considers such a dynamic panel model within the BMA approach by combining the likelihood function discussed in Alvarez and Arellano (2003) with the unit information g-prior. Distinguishing between "within" and "between" estimators in a BMA context is advocated by Desbordes et al. (2018), who stress that cross-sectional and time-series relationships can be quite different. They use a BIC approximation for the log Bayes factors and average over classical estimators (both discussed in Subsection 3.7).

Tsangarides (2004) addresses the issues of endogenous and omitted variables by incorporating a panel data system Generalized Method of Moments (GMM) estimator. This was extended to the limited information BMA (LIBMA) approach of Mirestean and Tsangarides (2016) by Chen et al. (2018), in the context of short-T panel models with endogenous covariates using a GMM approximation to the likelihood. They then employ a BIC approximation of the limited information marginal likelihood. Moral-Benito (2016) remarks on the controversial nature of combining frequentist GMM procedures with BMA, as it is not firmly rooted in formal statistical foundations and GMM methods may require mean stationarity. Thus, Moral-Benito (2016) proposes the use of a suitable likelihood function (derived in Moral-Benito (2013)) for dynamic panel data with fixed effects and weakly exogenous[67] regressors, which is argued to be the most

[67]This implies that past shocks to the dependent variable can be correlated with current covariates, so that there is feedback from the dependent variable to the covariates

relevant form of endogeneity in the growth regression context. Posterior model probabilities are based on the BIC approximation of the log Bayes factors with a unit-information g-prior (see Section 3.7) and on a uniform prior over model space.

León-González and Montolio (2015) develop BMA methods for models for panel data with individual effects and endogenous regressors, taking into account the uncertainty regarding the choice of instruments and exogeneity restrictions. They use reversible jump MCMC methods (developed by Koop et al. (2012)) to deal with a model space that includes models that differ in the set of regressors, instruments, and exogeneity restrictions in a panel data context.

3.9.9 Spatial data

If we wish to capture spatial interactions in the data, the model for panel data in (22) can be extended to a Spatial Autoregressive (SAR) panel model as follows:

$$y_{it} = \rho \sum_{j=1}^{N} w_{ij} y_{jt} + z_{it}' \beta + \eta_i + \xi_t + \epsilon_{it}, \tag{23}$$

where $i = 1, \ldots, N$ denotes spatial location and w_{ij} is the $(i,j)^{th}$ element of the spatial weight matrix reflecting spatial proximity of the N regions, with $w_{ii} = 0$ and the matrix is normalized to have row-sums of unity. Finally, there are regional effects η_i and time effects $\xi_t, t = 1 \ldots, T$. BMA in this model was used in LeSage (2014), building on earlier work, such as LeSage and Parent (2007). Crespo Cuaresma et al. (2018) use SAR models to jointly model income growth and human capital accumulation and mitigate the computational requirements by using an approximation based on spatial eigenvector filtering as in Crespo Cuaresma and Feldkircher (2013). Hortas-Rico and Rios (2016) investigate the drivers of urban income inequality using Spanish municipal data. They follow the framework of LeSage and Parent (2007) to incorporate spatial effects in the BMA analysis. Piribauer and Crespo Cuaresma (2016) compare the relative performance of the BMA methods used in LeSage and Parent (2007) with two different versions of the SVSS method (see Section 3.3) for spatial autoregressive models. On simulation data the SVSS approaches tended to perform better in terms of both in-sample predictive performance and computational efficiency.

An alternative approach was proposed by Dearmon and Smith (2016), who use the nonparametric technique of Gaussian process regression to accommodate spatial patterns and develop a BMA version of this approach. They apply it to the FLS growth data augmented with spatial information.

3.9.10 Duration models

BMA methods for duration models were first examined by Volinsky et al. (1997) in the context of proportional hazard models and based on a BIC approximation. Kourtellos and Tsangarides (2015) set out to uncover the correlates of the duration of growth spells. In particular, they investigate the relationship between inequality, redistribution, and the duration of growth spells in the presence of other possible determinants. They employ BMA for Cox hazards models and extend the BMA method developed by Volinsky et al. (1997) to allow for time-dependent covariates in order to properly account for the time-varying feedback effect of the variables on the duration of growth spells. Traczynski (2017) uses a Bayesian model-averaging approach for predicting firm bankruptcies and defaults at a 12-month horizon using hazard models. The analysis is based on a Laplace approximations for the marginal likelihood, arising from the logistic likelihood and a g-prior. On model space, a collinearity-adjusted dilution prior is chosen. Exact BMA methodology was used to identify risk factors associated with dropout and delayed graduation in higher education in Vallejos and Steel (2017), who employ a discrete time competing risks survival model, dealing simultaneously with university outcomes and its associated temporal component. For each choice of regressors, this amounts to a multinomial logistic regression model, which is a special case of a GLM. They use the prior as in Sabanés Bové and Held (2011b) in combination with the hyper-g/n prior (see Subsection 3.2.1).

4 Frequentist model averaging

Frequentist methods[68] are inherently different to Bayesian methods, as they tend to focus on estimators and their properties (often, but not always, in an asymptotic setting) and do not require the specification of a prior on the parameters. Instead, parameters are treated as fixed, yet unknown, and are not assigned any probabilistic interpretation associated with prior knowledge or learning from data. Whereas Bayesian inference on parameters typically centers around the uncertainty (captured by a full posterior distribution) that remains after observing the sample in question, frequentist methods usually focus on estimators that have desirable properties in the context of repeated sampling from a given experiment.

Early examples of FMA can be found in the forecasting literature, such as the forecast combinations of Bates and Granger (1969). This literature on forecast combinations (discussed here more in detail in Subsection 4.3) has become quite voluminous, see *e.g.* Granger (1989) and

[68]These are based on the "classical" statistical methodology which still underlies most introductory textbooks in statistics and econometrics.

Stock and Watson (2006) for reviews, while useful surveys of FMA can be found in Burnham and Anderson (2002), Wang et al. (2009), Ullah and Wang (2013) and Dormann et al. (2018).

In the context of the linear regression model in (5), FMA estimators can be described as

$$\hat{\beta}_{FMA} = \sum_{j=1}^{K} \omega_j \hat{\beta}_j, \tag{24}$$

where $\hat{\beta}_j$ is an estimator based on model j and $\omega_j, j = 1 \ldots, K$ are weights in the unit simplex within $\Re^K$. The critical choice is then how to choose the weights.

Buckland et al. (1997) construct weights based on different information criteria. They propose using

$$\omega_j = \frac{\exp(-I_j/2)}{\sum_{i=1}^{K} \exp(-I_i/2)}, \tag{25}$$

where I_j is an information criterion for model j, which can be the AIC or the BIC. Burnham and Anderson (2002) recommend the use of a modified AIC criterion, which has an additional small-sample second order bias correction term. They argue that this modified AIC should be used whenever $n/k < 40$.

Hjort and Claeskens (2003) build a general large-sample likelihood framework to describe limiting distributions and risk properties of estimators post-selection as well as of model averaged estimators. A crucial tool for this analysis is their assumption of local misspecification, which allows them to derive asymptotic results in a framework that mimicks the effects of model uncertainty (however, see footnote 52). Their approach also explicitly takes modeling bias into account. Besides suggesting various FMA procedures (based on e.g. AIC, the focused information criterion, FIC[69], of Claeskens and Hjort (2003) and empirical Bayes ideas), they provide a frequentist view of the performance of BMA schemes (in the sense of limiting distributions and large sample approximations to risks).

Hansen (2007) proposed a least squares model averaging estimator with model weights selected by minimizing the Mallows' criterion (C_p as defined in footnote 20). This estimator, known as Mallows model averaging (MMA), is easily implementable for linear regression models and has certain asymptotic optimality properties, since the Mallows' criterion is asymptotically equivalent to the squared predictive error. Therefore, the MMA estimator minimizes the squared error in large samples. Hansen (2007) shows that the weight vector chosen by MMA achieves optimality in the sense conveyed by Li (1987). The assumed data generating process is an infinite linear regression model, but the model space is kept quite small by asking the user to order the

[69]The underlying idea of FIC is that it focuses the analysis on parameters of interest, by ignoring the risk or loss associated with parameters that are not of interest.

regressors and by considering only the sequence of approximating models $M_1, \ldots, M_K$ where the jth model uses the first k_j covariates, with $0 \leq k_1 < k_2 < \cdots < k_K$ for some $K \leq n$ such that the matrix of covariates for M_K is of full column rank.

Hansen and Racine (2012) introduced another estimator within the FMA framework called jackknife model averaging (JMA) that selects appropriate weights for averaging models by minimizing a cross-validation (leave-one-out) criterion. JMA is asymptotically optimal in the sense of reaching the lowest possible squared errors over the class of linear estimators. Unlike MMA, JMA has optimality properties under heteroscedastic errors and when the candidate models are non-nested.

Liu (2015) derives the limiting distributions of least squares averaging estimators for linear regression models in a local asymptotic framework. The averaging estimators with fixed weights are shown to be asymptotically normal and a plug-in averaging estimator is proposed that minimizes the sample analog of the asymptotic mean squared error. This estimator is compared with the FIC, MMA and JMA estimators. The asymptotic distributions of averaging estimators with data-dependent weights are shown to be nonstandard and a simple procedure to construct valid confidence intervals is proposed.

Liu et al. (2016) extend MMA to linear regression models with heteroscedastic errors, and propose a model averaging method that combines generalized least squares (GLS) estimators. They derive C_p-like criteria to determine the model weights and show they are optimal in the sense of asymptotically achieving the smallest possible MSE. They also consider feasible versions using both parametric and nonparametric estimates of the error variances. Their objective is to obtain an estimator that generates a smaller MSE, which they achieve by choosing weights to minimize an estimate of the MSE. They compare their methods with those of Magnus et al. (2011), who also average feasible GLS estimators.

Most asymptotically optimal FMA methods have been developed for linear models, but Zhang et al. (2016) specifically consider GLMs (see Section 3.9.1) and generalized linear mixed-effects models[70] and propose weights based on a plug-in estimator of the Kullback-Leibler loss plus a penalty term. They prove asymptotic optimality (in terms of Kullback-Leibler loss) for fixed or growing numbers of covariates. Another extension of the linear model is the partial linear model, which includes both linear and nonparametric components in the regression function. FMA for such models is investigated in Zhang and Wang (2018) who do not consider estimation of the parameters, but focus on finding weights that minimize the squared prediction error. They use kernel smoothing to estimate each individual model, which implies a serious computa-

[70]These models are GLMs with so-called random effects, *e.g.* effects that are subject-specific in a panel data context.

tional burden when the number of candidate models is large. For larger model spaces, Zhang and Wang (2018) recommend a model screening step (see Subsection 4.2) prior to model averaging. This semiparametric setting is taken one step further in Zhu et al. (2018), who consider partial linear models with varying coefficients. They also allow for heteroscedastic errors and assume the "true" model is infinite-dimensional, as *e.g.* in Hansen (2007). They propose Mallows-type weights based on minimizing expected predictive squared error, and prove asymptotic optimality of the resulting FMA estimator (in terms of squared error loss). Li et al. (2018) also propose an FMA approach based on semiparametric varying coefficient models, using MMA weights. In a fully nonparametric regression context, Henderson and Parmeter (2016) propose a nonparametric regression estimator averaged over the choices of kernel, bandwidth selection mechanism and local-polynomial order, while Tu (2018) uses local linear kernel estimation of individual models and combines them with weights derived through maximizing entropy.

FMA was used for forecasting with factor-augmented regression models in Cheng and Hansen (2015). In the context of growth theory, Sala-i-Martin (1997) uses (24), and focuses on the "level of confidence"[71], using weights that are either uniform or based on the maximized likelihood.

Another model-averaging procedure that has been proposed in Magnus et al. (2010) and reviewed in Magnus and De Luca (2016) is weighted average least squares (WALS), which can be viewed as being in between BMA and FMA. The weights it implies in (24) can be given a Bayesian justification. However, it assumes no prior on the model space and thus can not produce inference on posterior model probabilities. WALS is easier to compute than BMA or FMA, but quite a bit harder to explain and inherently linked to a nested linear regression setting. Magnus and De Luca (2016) provide an in-depth description of WALS and its relation to BMA and FMA. They state: "The WALS procedure surveyed in this paper is a Bayesian combination of frequentist estimators. The parameters of each model are estimated by constrained least squares, hence frequentist. However, after implementing a semiorthogonal transformation to the auxiliary regressors, the weighting scheme is developed on the basis of a Bayesian approach in order to obtain desirable theoretical properties such as admissibility and a proper treatment of ignorance. The final result is a model-average estimator that assumes an intermediate position between strict BMA and strict FMA estimators [....] Finally we emphasize (again) that WALS is a model-average procedure, not a model-selection procedure. At the end we cannot and do not want to answer the question: which model is best? This brings with it certain restrictions. For example, WALS cannot handle jointness (Ley and Steel, 2007; Doppelhofer and Weeks, 2009). The concept of jointness refers to the dependence between explanatory variables in the poste-

[71]This is defined as the maximum probability mass one side of zero for a Normal distribution centred at the estimated regression coefficient with the corresponding estimated variance.

rior distribution, and available measures of jointness depend on posterior inclusion probabilities of the explanatory variables, which WALS does not provide." The choice of the weight function in WALS is based on risk (or MSE) considerations, and can be given a frequentist and a Bayesian flavour: in the latter framework admissibility is ensured, so it is favoured by Magnus and De Luca (2016). In particular, for transformations of the regressions coefficients they consider robust[72] priors in the class of reflected generalized gamma distributions (with a Laplace prior as a (non-robust) special case). An extension called Hierarchical WALS was proposed in Magnus and Wang (2014) to jointly deal with uncertainty in concepts and in measurements within each concept, in the spirit of dilution priors (see Section 3.1.2). Generally, however, WALS is rather closely linked to the context of covariate uncertainty in a normal linear regression framework and the possibility of extensions to other settings seems limited. One important extension is to the general GLM framework, developed in De Luca et al. (2018). They base WALS on a combination of one-step MLEs and consider its asymptotic frequentist properties in the $\mathcal{M}$-closed local misspecification framework of Hjort and Claeskens (2003).

Wagner and Hlouskova (2015) consider frequentist model averaging for principal components augmented regressions illustrated with the FLS data set on economic growth determinants. In addition, they compare and contrast their method and findings with BMA and with the WALS approach, finding some differences but also some variables that are important in all analyses. Another comparison of different methods on growth data can be found in Amini and Parmeter (2011). They consider BMA, MMA and WALS and find that results (in as far as they can be compared: for example, MMA and WALS do not provide posterior inclusion probabilities) for three growth data sets are roughly similar. In the context of GLMs, a comparison of BMA (as in Chen and Ibrahim (2003), various FMA methods (*e.g.* based on smoothed versions of AIC, BIC and FIC), a number of model selection procedures (*e.g.* LASSO, SCAD and stepwise regression) and WALS is conducted by De Luca et al. (2018). They find that WALS performs similarly to BMA and FMA based on FIC, and that these model averaging procedures outperform the model selection methods.

4.1 Estimating the sampling variance of FMA estimators

The construction of confidence intervals associated with FMA estimators has to take into account the model uncertainty, in order to achieve the expected coverage probability. The traditional classical confidence interval is derived conditional upon the chosen model, and thus underestimates uncertainty and leads to confidence intervals that are too narrow. Buckland et al. (1997) propose

[72]This is a specific implementation of the robustness idea in Subsection 3.1: a robust prior has fatter tails than a normal and is often defined as a prior for which $p'(x)/p(x) \to 0$ as $x \to \infty$.

an adjustment to the standard error to be used in this traditional formula, but Hjort and Claeskens (2003) show that the asymptotic distribution of the FMA estimator in (24) is not normal in their local misspecification framework. They derive the limiting distribution and suggest an expression for a confidence interval with correct coverage probability. Wang and Zhou (2013) prove that this confidence interval is asymptotically equivalent to that constructed based only on the full model estimator (which is very easy to compute). In fact, they also show that the same equivalence result holds for the varying-coefficient partially linear model. This means that if the interest of the user is only in interval estimation (for linear combinations of regression coefficients), there appears to be no need to consider FMA and the full model should be able to provide the appropriate intervals. However, in simulations FMA does beat model selection procedures and the full model in terms of finite sample MSE performance, which suggests that for point estimation there is a clear advantage to model averaging.

4.2 Computational aspects of FMA

An important difference between BMA and FMA is that the latter does not lead to estimated model probabilities[73]. A consequence of this for computation in large model spaces is that simple MCMC algorithms where models are visited in line with their posterior probabilities are not readily available. This restricts the ability of FMA methods to deal with large model spaces, as it would need to enumerate the large number of models in (24). In addition, some approaches, like JMA or other cross-validation schemes require non-trivial computational effort for each model weight computation.

Thus, researches with an interest in applying FMA often rely on ways of reducing the model space. In the literature, this has been implemented in a number of ways:

- Conduct a preliminary model-screening step to remove the least interesting models before applying FMA. This was done in *e,g,* Claeskens et al. (2006) (for logistic models), Zhang et al. (2013) (for forecasting discrete response time series) and Zhang et al. (2016) (for GLMs). Inference is then conducted conditionally upon the outcome of the screening, so that the uncertainty involved in this first step is not addressed by the model averaging. This approach is not just used for FMA, as some BMA methods also include a preliminary screening step: in particular, the use of Occam's window (see Subsection 3.3) proposed for graphical models in Madigan and Raftery (1994) and applied to linear regression in Raftery et al. (1997).

[73]Nevertheless, Burnham and Anderson (2002) have suggested interpreting the estimated weights with AIC as model probabilities.

- In the context of covariate uncertainty in the linear model, apply an orthogonal transformation of the regressors. WALS critically relies on a semiorthogonal transformation which effectively reduces the computational burden of WALS to the order k, in a model space of dimension 2^k. Of course, this does not allow us to conduct inference on inclusion probabilities of the covariates and the inference is about estimating the effects of covariates. For growth regressions with large model spaces, Amini and Parmeter (2012) introduce an operational version of MMA by using the same semiorthogonal transformations as adopted in WALS.

- Always include a subset of the covariates in each model. This was already used as a device to simplify computations in early work in growth regressions, such as the EBA in Levine and Renelt (1992), and is also found in WALS and *e.g.* the analysis of Wagner and Hlouskova (2015).

4.3 Combining forecasts

4.3.1 The forecast combination puzzle

As mentioned earlier, there is a large literature in forecasting which combines point forecasts from different models in an equation such as (24) to provide more stable and better-performing forecasts. Of course, the choice of weights in combination forecasting is important. For example, we could consider weighting better forecasts more heavily. In addition, time-varying weights have been suggested. Stock and Watson (2004) examine a number of weighting schemes in terms of the accuracy of point forecasts and find that forecast combinations can perform well in comparison with single models, but that the best weighting schemes are often the ones that incorporate little or no data adaptivity. This empirical fact that "optimally" estimated weights often perform worse than equal weights in terms of mean squared forecast error is known as the "forecast combination puzzle".

Smith and Wallis (2009) provide an explanation for this phenomenon in terms of the additional uncertainty associated with estimating the weights. Their analysis shows that if the optimal weights are close to equal, then a simple average of competing forecasts (*i.e.* assuming the weights are equal) can dominate a forecast combination with weights that are estimated from the data. Along similar lines, Claeskens et al. (2015) also provide an explanation for the forecast combination puzzle, but they explicitly treat the estimated weights as random and consider the derivation of the optimal weights and their estimation jointly.

4.3.2 Density forecast combinations

There is an increasing awareness of the importance of probabilistic or density forecasts, as described in Section 3.2.2. Thus, a recent literature has emerged on density forecast combinations or weighted linear combinations (pools) of prediction models. Density forecasts combinations were discussed in Wallis (2005) and further developed by Hall and Mitchell (2007), where the combination weights are chosen to minimize the Kullback-Leibler "distance" between the predicted and true but unknown density. The latter is equivalent to optimizing LPS as defined in Section 3.2.2. The properties of such prediction pools are examined in some detail in Geweke and Amisano (2011), who show that including models that are clearly inferior to others in the pool can substantially improve prediction. Also, they illustrate that weights are not an indication of a predictive model's contribution to log score. This approach is extended by Kapetanios et al. (2015), who allow for more general specifications of the combination weights, by letting them depend on the variable to be forecast. They specifically investigate piecewise linear weight functions and show that estimation by optimizing LPS leads to consistency and asymptotic normality[74]. They also illustrate the advantages over density forecast combinations with constant weights using simulated and real data.

5 Applications in Economics

There is a large and rapidly growing literature where model averaging techniques are used to tackle empirical problems in economics. Before the introduction of model averaging methods, model uncertainty was typically dealt with in a less formalized manner and perhaps even simply ignored in many cases. Without attempting to be exhaustive, this chapter briefly mentions some examples of model averaging in economic problems and highlights some instances where model averaging has provided new empirical insights.

5.1 Growth regressions

Traditionally, growth theory has been an area where many potential determinants have been suggested and empirical evidence has struggled to resolve the open-endedness of the theory (see footnote 16). Extensive discussions of model uncertainty in the context of economic growth can be found in Brock et al. (2003) and Temple (2000). Starting from neoclassical growth theory, Barro (1991) investigates extensions by considering partial correlations and adding variables one

[74]Formally, this is shown for known thresholds of the piecewise linear weights, and is conjectured to hold for unknown threshold parameters.

by one to OLS regressions. By including human capital as a determinant (captured by school enrollment) he finds empirical support for the neoclassical convergence theory (*i.e.* that poorer countries tend to grow faster than richer ones). Rodrik et al. (2004) distinguish three main strands of theories in the literature about drivers for growth: geography (which determines natural resources and also influences agricultural productivity and the quality of human resources), international trade (linked with market integration) and institutions (more in particular, property rights and the rule of law)[75]. Of course, there is a myriad of possible ways in which these theoretical determinants could be measured in practice, leading to a large collection of possible models in line with one or more of these theories. To make empirical implementation even more complex, only the first theoretical source of growth (geography) can be safely assumed to be exogenous, and for the other theories we need to be aware of the possibility that variables capturing these effects may well be endogenous (see Subsection 3.9.7). The typical treatment of the latter issue requires us to make choices regarding instrumental variables, yet again increasing the number of potential models. Early attempts at finding a solution include the use of EBA (see Section 2.2) in Levine and Renelt (1992) who investigate the robustness of the results from linear regressions and find that very few regressors pass the extreme bounds test, while Sala-i-Martin (1997) employs a less severe test based on the "level of confidence" of individual regressors averaged over models (uniformly or with weights proportional to the likelihoods). These more or less intuitive but ad-hoc approaches were precursors to a more formal treatment through BMA discussed and implemented in Brock and Durlauf (2001) and Fernández et al. (2001b). Hendry and Krolzig (2004) present an application of general-to-specific modelling (see Section 2.2) in growth theory, as an alternative to BMA. However, there is a long list of applications in this area where model averaging is used, and some examples are given below, organised in terms of possible drivers of growth.

5.1.1 Institutions

As mentioned above, the theory that institutions are a crucial driver for growth is often specifically linked to property rights and the rule of law. Acemoglu et al. (2001) argue that private property rights (as measured by government risk of expropriation) are a key determinant of growth, and that the security of such property rights is crucially dependent on the colonial history. This leads them to use mortality rates of the first European settlers as an instrument for the current institutions in those countries in a 2SLS regression analysis. They find that institutions have a large effect on GDP per capita, and judge this effect to be robust with respect to adding other potential

[75] Durlauf (2018) provides a very complete discussion of the empirical evidence for links between institutions and growth, concluding that institutions matter.

determinants to their regression model. However, these extra variables are essentially added one by one, so that the number of different models actually presented is quite limited. Also, they do not consider any trade variables. In a similar fashion, Frankel and Romer (1999) focus on trade as a growth driver and present empirical evidence to support that theory, but without controlling for the effects of institutions. A somewhat broader empirical framework is used in Rodrik et al. (2004), who aim to provide a comparison of the three main growth theories mentioned in the previous subsection. They use three different samples and take into account the potential endogeneity of institution and integration variables. For the former, they choose rule of law and as an integration variable, they select the ratio of trade to GDP. They use the instrument in Acemoglu et al. (2001), which they replace for the largest sample by two other instruments: the fraction of the population speaking English and the fraction speaking another European language as their first language. As an additional instrument they adopt the constructed trade share as estimated from a gravity model in Frankel and Romer (1999). Geography is measured by the distance from the equator. Rodrik et al. (2004) conclude that institutions are the main driver of growth between the three theories investigated. But again, this result hinges upon specific choices of variables to represent the theories, specific choices of instruments and consideration of a very limited number of models with other possible determinants. A systematic model averaging approach is implemented in Lenkoski et al. (2014), using their 2SBMA methodology as explained in Subsection 3.9.7. They reexamine the data used by Rodrik et al. (2004) through 2SBMA to carefully address the model and instrument uncertainty. Once they allow for additional theories in the model space, they start finding important differences with the Rodrik et al. (2004) conclusions. In particular, they conclude that all three main growth theories (geography, integration and institutions) play an important part in development. A similar 2SBMA analysis in Eicher and Newiak (2013) also includes intellectual property rights. Measuring intellectual property rights by the enforcement of patents, Eicher and Newiak (2013) find evidence of strong positive effects of both intellectual and physical property rights, as well as some effects to do with geography (malaria and tropics both have a negative impact on development). The influence of trade on growth is analysed in Eicher and Kuenzel (2016), again using the two-stage BMA approach of Lenkoski et al. (2014). They find that sectoral export diversity serves as a crucial growth determinant for low-income countries, and that this effect decreases with the level of development. They also find strong evidence for institutional effects on growth.

All these model averaging results take into account many models with different instruments and different growth determinants, and thus allow for a much more complete and nuanced inference from the data than the earlier studies, more adequately reflecting the uncertainty of any conclusions drawn. In addition, the instrument constructed by Acemoglu et al. (2001) has been criticised by Albouy (2012), due to the problems in obtaining reliable historical mortality data.

However, a model averaging approach that takes account of the uncertainty in the instruments will simply downweigh inappropriate instruments. For example, settler mortality does not receive a lot of weight in the instrumental variables equation of Lenkoski et al. (2014) for rule of law[76] and those of Eicher and Newiak (2013) for patent protection and rule of law.[77] Model averaging is particularly natural and useful in the context of instrumental variables, where theory is at best indicative and data quality may be a serious issue.

5.1.2 Energy consumption

The question of whether energy consumption is a critical driver of economic growth is investigated in Camarero et al. (2015). This relates to an important debate in economics between competing economic theories: ecological economic theory (which considers the scarcity of resources as a limiting factor for growth) and neoclassical growth theory (where it is assumed that technological progress and substitution possibilities may serve to circumvent energy scarcity problems). There are various earlier studies that concentrate on the bivariate relationship between energy consumption and economic growth, but of course the introduction of other relevant covariates is key. In order to resolve this in a formal manner, they use the BMA framework on annual US data (both aggregate and sectoral) from 1949 to 2010, with up to 32 possible covariates. Camarero et al. (2015) find that energy consumption is an important determinant of aggregate GDP growth (but their model does not investigate whether energy consumption really appears as an endogenous regressor, so that they can not assess whether there is also feedback) and also identify energy intensity, energy efficiency, the share of nuclear power and public spending as important covariates. Sectoral results support the conclusion about the importance of energy consumption, but show some variation regarding the other important determinants.

5.1.3 Government spending

The effect of government investment versus government consumption on growth in a period of fiscal consolidation in developed economies is analysed in Jovanovic (2017). Using BMA and a dilution prior (based on the determinant of the correlation matrix), it is found that public investment is likely to have a bigger impact on GDP than public consumption in the countries with high public debt. Also, and more controversially, the (investment) multiplier is likely to be higher in countries with high public debt than in countries with lower public debt. The results suggest that

[76] Although settler mortality does seem to have some use an an instrument for integration.

[77] But when replacing patent protection by patent enforcement, settler mortality actually becomes an important instrument.

fiscal consolidation should be accompanied by increased public investment. Using the dataset from Sala-i-Martin et al. (2004) and a prior which replaces the g-prior in (6) by a prior structure with a covariance for β_j proportional to an identity matrix and independence between β_j and σ, Lee and Chen (2018) come to largely similar conclusions as previous studies using g-priors. However, they find some differences: one is that both government consumption share and government share of GDP are important drivers for growth, with a positive effect. They conjecture that the difference with the results in Sala-i-Martin et al. (2004) (finding negative effects and less importance for these variables) is a consequence of the different prior in combination with the high pairwise correlation between these covariates.

5.1.4 Natural resources

As discussed in Frankel (2012), a surprising empirical fact is that the raw data (based on average growth of countries for 1970-2008) certainly do not suggest a positive correlation between natural resource wealth and economic growth. This is often termed the "natural resource curse" and Frankel (2012) lists six possible mechanisms: the long-run trend of world prices for commodities; volatility in commodity prices; permanent crowding out of manufacturing, where spillover effects are thought to be concentrated; autocratic or oligarchic institutions; anarchic institutions, such as unenforceable property rights, unsustainably rapid depletion, or civil war; and cyclical expansion of the nontraded sector via the Dutch disease.

On the basis of a relatively small number of cross-country growth regressions, Sachs and Warner (2001) conclude that the curse is real and that the most likely explanation could be through crowding out of growth-stimulating activity. They find that resource-abundant countries tend to be high-price economies and that contributes to less export-led growth in these countries. Using BMA, Arin and Braunfels (2018) examine the existence of the natural resource curse focusing on the empirical links between oil rents and long-term growth. They find that oil revenues have a robust positive effect on growth. When they include interactions and treat them simply as additional covariates, they find that the positive effect can mostly be attributed to the interaction of institutional quality and oil revenues, which would suggest that institutional quality is a necessary condition for oil revenues to have a growth-enhancing effect. However, if they use a prior that adheres to the strong heredity principle (see Section 3.1.2), they find instead that the main effect of oil rents dominates. In any case, their BMA results lead them to the conclusion "that oil rents have a robust positive effect on economic growth and that the resource curse is a red herring."

5.1.5 Other determinants

A specific focus on the effect of measures of fiscal federalism on growth was adopted in Asatryan and Feld (2015). They conclude that, after controlling for unobserved country heterogeneity, no robust effects of federalism on growth can be found.

Man (2015) investigates whether competition in the economic and political arenas is a robust determinant of aggregate growth, and whether there exists jointness among competition variables versus other growth determinants. This study also provides a comparison with EBA and with "reasonable extreme bounds analysis", which also takes the fit of the models into account. Evidence is found for the importance and positive impact on growth of financial market competition, which appears complementary to other important growth determinants. Competition in other areas does not emerge as a driver of economic growth.

Piribauer (2016) estimates growth patterns across European regions in a spatial econometric framework, building on threshold estimation approaches (Hansen, 2000) to account for structural heterogeneity in the observations. The paper uses the prior structure by George and McCulloch (1993, 1997) with SSVS (see Section 3.3), and concludes that initial income, human capital endowments, infrastructure accessibility, and the age structure of the population all appear to be robust driving forces of income growth.

Lanzafame (2016) derives the natural or potential growth rates of Asian economies (using a Kalman filter on a state-space model) and investigates the determinants of potential growth rates through BMA methods (while always including some of the regressors). He finds evidence of robust links with various aspects of institutional quality, the technology gap with the US, trade, tertiary education, and the growth rate of the working-age population.

5.2 Inflation and Output Forecasting

In the context of time series modelling with ARIMA and ARFIMA models, BMA was used for posterior inference on impulse responses[78] for real GNP in Koop et al. (1997). This is an example where the behaviour of the quantity of interest (impulse response) is very different between the groups of models considered. ARFIMA (Autoregressive Fractionally Integrated Moving Average) models allow for long memory behaviour which is characterized by a fractional differencing parameter $\delta \in (-1, 0.5)$[79]. Processes with $\delta > 0$ are called long memory processes, while for $\delta < 0$ we have intermediate memory. The special case of $\delta = 0$ corresponds to the more fa-

[78]The impulse response function measures the effect of a unitary shock on an economic quantity as a function of time. For a stationary process, impulse responses are the coefficients of its (infinite) moving average representation.

[79]For this range of values of δ the process (expressed in first differences) is stationary and invertible.

miliar ARIMA class, which leads to an impulse response function that tends to values on the positive real line when the time horizon goes to infinity. In contrast, ARFIMA models, lead only to limiting impulse responses of either zero (when $\delta < 0$) or ∞ (for $\delta > 0$). Even though the real interest in practice may be in finite time horizons, these limiting results of course impact on the entire impulse response function. This fact actually led Hauser et al. (1999) (who use a frequentist model selection approach) to conclude that ARFIMA models are not appropriate for inference on persistence, in conflict with the earlier recommendation in Diebold and Rudebusch (1989). However, by using model averaging rather than model selection, we are not forced to choose either one of these models, but we can formally combine the (often necessarily disparate) inference on persistence and take proper account of the uncertainty. In Koop et al. (1997), the model space chosen is characterized by different combinations of autoregressive and moving average lag lengths (from 0 to 3) and includes both ARIMA and ARFIMA models for real US GNP. Thus, the posterior distribution of impulse responses is bimodal for (medium and long run) finite horizons and for the impulse response at infinity it contains point masses at zero and ∞.

Cogley and Sargent (2005) consider Bayesian averaging of three models for inflation using dynamic model weights. Another paper that uses time-varying BMA methods for inflation forecasting is van der Maas (2014). The related strategy of dynamic model averaging, due to Raftery et al. (2010) and described in Section 3.9.6, was used in Koop and Korobilis (2012). Forecasting inflation using BMA has also been examined in Eklund and Karlsson (2007), who propose the use of so-called predictive weights in the model averaging, rather than the usual BMA based on posterior model probabilities. Shi (2016) models and forecasts quarterly US inflation and finds that Bayesian model averaging with regime switching leads to substantial improvements in forecast performance over simple benchmark approaches (e.g. random-walk or recursive OLS forecasts) and pure BMA or Markov switching models. Ouysse (2016) considers point and density forecasts of monthly US inflation and output growth that are generated using principal components regression (PCR) and BMA. A comparison between 24 BMA specifications and 2 PCR ones in an out-of-sample, 10-year rolling event evaluation leads to the conclusion that PCR methods perform best for predicting deviations of output and inflation from their expected paths, whereas BMA methods perform best for predicting tail events. Thus, risk-neutral policy-makers may prefer the PCR approach, while the BMA approach would be the best option for a prudential, risk-averse forecaster.

Bencivelli et al. (2017) investigate the use of BMA for forecasting GDP relative to simple bridge models[80] and factor models. They conclude that for the Euro area, BMA bridge models

[80]Bridge models relate information published at monthly frequency to quarterly national account data, and are used for producing timely "now-casts" of economic activity.

produce smaller forecast errors than a small-scale dynamic factor model and an indirect bridge model obtained by aggregating country-specific models.

Ductor and Leiva-Leon (2016) investigate the time-varying interdependence among the economic cycles of the major world economies since the 1980's. They use a BMA panel data approach (with the model in (22) including a time trend) to find the determinants of pairwise desynchronization between the business cycles of countries. They also apply WALS and find that it indicates the same main determinants as BMA.

A probit model is used for forecasting US recession periods in Aijun et al. (2018). They use a Gibbs sampler based on SSVS (but with point masses for the coefficients of the excluded regressors), and adopt a generalized double Pareto prior (which is a scale mixture of normals) for the included regression parameters along with a dilution prior over models based on the correlation between the covariates. Their empirical results on monthly U.S. data (from 1959:02 until 2009:02) with 108 potential covariates suggest the method performs well relative to the main competitors.

5.3 VAR and DSGE modelling

A popular econometric framework for jointly modelling several variables is the vector autoregressive (VAR) model. Koop (2017) provides an intuitive and accessible overview of Bayesian methods for inference with these types of models. BMA methodology has been applied by Garratt et al. (2003) for probability forecasting of inflation and output growth in the context of a small long-run structural vector error-correcting model of the UK economy. George et al. (2008) apply BMA ideas in VARs using SSVS methods with priors which do not induce exact zero restrictions on the coefficients, as in George and McCulloch (1993). Koop and Korobilis (2016) extend this to Panel VARs where the restrictions of interest involve interdependencies between and heterogeneities across cross-sectional units.

Feldkircher and Huber (2016) use a Bayesian VAR model to explore the international spillovers of expansionary US aggregate demand and supply shocks, and of a contractionary US monetary policy shock. They use SVSS and find evidence for significant spillovers, mostly transmitted through financial channels and with some notable cross-regional variety.

BMA methods for the more restricted dynamic stochastic general equilibrium (DSGE) models were used in Strachan and van Dijk (2013), with a particular interest in the effects of investment-specific and neutral technology shocks. Evidence from US quarterly data from 1948-2009 suggests a break in the entire model structure around 1984, after which technology shocks appear to account for all stochastic trends. Investment-specific technology shocks seem more important

for business cycle volatility than neutral technology shocks.

5.4 Crises and finance

There is a substantial literature on early warning signals for economic crises going back many decades, illustrated by the review in Frankel and Saravelos (2012) and forcefully brought back to attention by the global financial crisis of 2008-9. Unfortunately, the established early warning signals did not provide clear alerts for this recent crisis (see, for example, Rose and Spiegel (2011)). Feldkircher et al. (2014) focus on finding leading indicators for exchange market pressures during the crisis and their BMA results indicate that inflation plays an important aggravating role, whereas international reserves act as a mitigating factor. Early warning signals are also investigated in Christofides et al. (2016) who consider four different crisis dimensions: banking, balance of payments, exchange rate pressure, and recession and use balanced samples in a BMA analysis. They do not identify any single early warning signal for all dimensions of the 2008 crisis, but find that the importance of such signals is specific to the particular dimension of the crisis being examined. They argue that the consensus in the previous literature about early warning signals (such as "foreign currency reserves" and "exchange rate overvaluations" mentioned in, for example, Frankel and Saravelos (2012)) hinges critically on the fact that many earlier analyses only considered models that were in line with a particular theory. Taking theory uncertainty into account in a formal statistical framework, the conclusions change.

Following the work of Rose and Spiegel (2011) and the earlier BMA approach of Giannone et al. (2011), Feldkircher (2014) uses BMA to identify the main macroeconomic and financial market conditions that help explain the real economic effects of the crisis of 2008-9. He finds that countries with strong pre-crisis growth in credit and/or in real activity tended to be less resilient. Ho (2015) investigates the causes of the crisis, using BMA, BACE and the approach of Hoeting et al. (1996) (see Section 3.9.4) to deal with outliers, and finds that the three methods lead to broadly similar results. The same question about the determinants of the 2008 crisis was addressed in Chen et al. (2017), who use a hierarchical prior structure with groups of variables (grouped according to a common theory about the origins of the crisis) and individual variables within each group. They use BMA to deal with uncertainty at both levels and find that "financial policies and trade linkages are the most relevant groups with regard to the relative macroeconomic performance of different countries during the crisis. Within the selected groups, a number of pre-existing financial proxies, along with measures of trade linkages, were significantly correlated with real downturns during the crisis. Controlling for both variable uncertainty and group uncertainty, our group variable selection approach is able to identify more variables that are significantly correlated with crisis intensity than those found in past studies that select variables

individually."

The drivers of financial contagion after currency crises were investigated through BMA methods in Dasgupta et al. (2011). They use a probit model for the occurrence of a currency crisis in 54 to 71 countries for four years in the 1990s and find that institutional similarity is an important predictor of financial contagion during emerging market crises. Puy (2016) investigates the global and regional dynamics in equity and bond flows, using data on portfolio investments from international mutual funds. In addition, he finds strong evidence of global contagion. To assess the determinants of contagion, he regresses the fraction of variance of equity and bond funding attributable to the world factor on a set of 14 structural variables, using both WALS and BMA. Both point towards distance and political risk as robust drives of contagion, Puy (2016) concludes that "sudden surges/stops tend to strike fragile countries, *i.e.* emerging markets with unstable political systems and poor connection to the main financial centers."

Moral-Benito and Roehn (2016) explore the relationship between financial market regulation and current account balances. They use a dynamic panel model and combine the BMA methodology with a likelihood-based estimator that accommodates both persistence and unobserved heterogeneity. In their investigation of the determinants of current account balances, Desbordes et al. (2018) highlight "three features which are likely to be shared by many panel data applications: high model uncertainty, presence of slope heterogeneity, and potential divergence in short-run and long-run effects".

The use of BMA in forecasting exchange rates by Wright (2008) leads to the conclusion that BMA provides slightly better out-of-sample forecasts (measured by mean squared prediction errors) than the traditional random walk benchmark. This is confirmed by Ribeiro (2017), who also argues that a bootstrap-based method, called bumping, performs even better. Iyke (2015) analyses the real exchange rate in Mauritius using BMA. Different priors are adopted, including empirical Bayes. There are attempts to control for multicollinearity in the macro determinants using three competing model priors incorporating dilution, among which the tessellation prior and the weak heredity prior (see Section 3.1.2). Adler and Grisse (2017) examine behavioral equilibrium exchange rates models, which relate a long-run cointegration relationship between real exchange rates to fundamental macroeconomic variables, in a panel regression across currencies. They use BACE to deal with model uncertainty and find that some variables (central bank reserves, government consumption, private credit, real interest rates and the terms of trade) are robustly linked with real exchange rates. The introduction of fixed country effects in the models greatly improves the fit to real exchange rates over time.

BMA applied to a meta-analysis is used by Zigraiova and Havranek (2016) to investigate the relationship between bank competition and financial stability. They find some evidence of

publication bias[81] but encounter no clear link between bank competition and stability, even when correcting for publication bias and potential misspecifications.

Devereux and Dwyer (2016) examine the output costs associated with 150 banking crises using cross country data for the years after 1970. They use BMA to identify important determinants of output changes after crises and conclude that for high-income countries the behavior of real GDP after a banking crisis is most closely associated with prior economic conditions, where above-average changes in credit tend to be associated with larger expected decreases in real GDP. For low-income economies, the existence of a stock market and deposit insurance are linked with quicker recovery of real GDP.

Pelster and Vilsmeier (2018) use Bayesian Model Averaging to assess the pricing-determinants of credit default swaps. They use an autoregressive distributed lag model with time-invariant fixed effects and approximate posterior model probabilities on the basis of smoothed AIC. They conclude that credit default swaps price dynamics can be mainly explained by factors describing firms' sensitivity to extreme market movements, in particular variables measuring tail dependence (based on so-called dynamic copula models).

Horvath et al. (2017) explore the determinants of financial development as measured by financial depth (both for banks and stock markets), the efficiency of financial intermediaries (both for banks and stock markets), financial stability and access to finance. They use BMA to analyse financial development in 80 countries using nearly 40 different explanatory variables and find that the rule of law is a major factor in influencing financial development regardless of the measure used. In addition, they conclude that the level of economic development matters and that greater wealth inequality is associated with greater stock market depth, although it does not matter for the development of the banking sector or for the efficiency of stock markets and banks.

The determinants of US monetary policy are investigated in Wölfel and Weber (2017), who conclude from a BMA analysis that over the long-run (1960-2014) the important variables in explaining the Federal Funds Rate are inflation, unemployment rates and long-term interest rates. Using samples starting in 1973 (post Bretton-Woods) and 1982 (real-time data), the fiscal deficit and monetary aggregates were also found to be relevant. Wölfel and Weber (2017) also account for parameter instability through the introduction of an unknown number of structural breaks and find strong support for models with such breaks, although they conclude that there is less

[81] Generally, this is the situation that the probability of a result being reported in the literature (*i.e.* of the paper being published) depends on the sign or statistical significance of the estimated effect. In this case, the authors found some evidence that some authors of primary studies tend to discard estimates inconsistent with the competition-fragility hypothesis, one of the two main hypotheses in this area. For an in-depth discussion of publication bias and meta-analyses in economics, see Christensen and Miguel (2018).

evidence for structural break since the 1990s.

Watson and Deller (2017) consider the relationship between economic diversity and unemployment in the light of the economic shocks provided by the recent "Great Recession". They use a spatial BMA model allowing for spatial spillover effects on data from US counties with a Herfindahl diversity index computed across 87 different sectors. They conclude that increased economic diversity within the county itself is associated with significantly reduced unemployment rates across all years of the sample (2007-2014). The economic diversity of neighbours is only strongly associated with reduced unemployment rates at the height of the Great Recession.

Ng et al. (2016) investigate the relevance of social capital in stock market development using BMA methods and conclude that trust is a robust and positive determinant of stock market depth and liquidity.

BMA was used to identify the leading indicators of financial stress in 25 OECD countries by Vašíček et al. (2017). They find that financial stress is difficult to predict out of sample, either modelling all countries at the same time (as a panel) or individually.

5.5 Production modelling

For production or cost modelling through stochastic frontier models, there exists a large amount of uncertainty regarding the distributional assumptions, particularly for the one-sided errors that capture inefficiencies. There is no theoretical guidance and a number of different distributions are used in empirical work. Given the typical lack of observations per firm (just one in cross-sectional analyses), the distribution used has a potentially large effect on the inference on firm efficiencies[82], a key aspect of interest in this type of modelling.

Bayesian methods were introduced in this context by van den Broeck et al. (1994). They deal with the uncertainty regarding the specification of the inefficiency distribution through BMA, so that inference on all aspects of interest (such as elasticities, returns to scale, firm-specific posterior and predictive efficiencies, etc.) is appropriately averaged over the models considered. Other, more ad-hoc, approaches to providing model averaged estimates for some features of productivity or inefficiency appear, for example, in Sickles (2005), who takes simple averages of technical efficiency estimates for US banks across a range of alternative stochastic frontier panel data models, with the aim of providing "a clear and informative summary". This is formalized

[82]For example, the exponential inefficiency distribution is a popular choice, as it is relatively easy to work with, but because of its shape (with a lot of mass near zero) it will tend to lead to a cluster of highly efficient firms; this contrasts with other, more flexible, distributions, such as the gamma or generalized gamma distributions or mixtures of distributions; see Griffin and Steel (2008).

in Parmeter et al. (2016), who develop FMA methods for (cross-sectional) stochastic frontier models. In particular, they consider models that allow for the variance of the symmetric model error and the parameters of the inefficiency distribution to depend on possible covariates, and the uncertainty resides in which are the appropriate sets of covariates in these distributions and which covariates should be included in the frontier itself (typically functions of inputs for a production frontier, but there could be other covariates, such as time in a panel setup, etc.). One method averages inefficiencies and works for nested structures, and another is a J-fold cross-validation method similar to JMA, which can be used when inefficiency distributions are not nested.

McKenzie (2016) considers three different stochastic frontier models with varying degrees of flexibility in the dynamics of productivity change and technological growth, and uses Bayesian model averaging to conduct inference on productivity growth of railroads.

Cost efficiency in the Polish electricity distribution sector is analysed through Bayesian methods in Makieła and Osiewalski (2018). Given the complexity of the models involved, they do not run a chain over model space but treat models separately, using the so-called corrected arithmetic mean estimator for the marginal likelihoods (this involves importance sampling after trimming the parameter space). They use BMA in carefully chosen model spaces that address uncertainty regarding the included covariates and also regarding the existence of firm inefficiencies.

5.6 Other applications

Havranek et al. (2015) use BMA in a meta-analysis of intertemporal substitution in consumption. Havranek and Sokolova (2016) investigate the mean excess sensitivity reported in studies estimating consumption Euler equations. Using BMA methods, they control for 48 variables related to the context in which estimates are obtained in a sample of 2,788 estimates reported in 133 published studies. Reported mean excess sensitivity seems materially affected by demographic variables, publication bias and liquidity constraints and they conclude that the permanent income hypothesis seems a pretty good approximation of the actual behavior of the average consumer. Havranek et al. (2017) consider estimates of habit formation in consumption in 81 published studies and try and relate differences in the estimates to various characteristics of the studies. They use BMA (with MC^3) and FMA[83] and find broadly similar results using both methods. Another example of the use of BMA in meta-analysis is Philips (2016) who investigates political budget cycles and finds support for some of the context-conditional theories in that literature.

[83]Here they follow the approach suggested by Amini and Parmeter (2012), who build on Magnus et al. (2010) and use orthogonalization of the covariate space, thus reducing the number of models that need to be estimated from 2^k to k. In individual regressions they use inverse-variance weights to account for the estimated dependent variable issue.

The determinants of export diversification are examined in Jetter and Ramírex Hassan (2015) who conclude that Primary school enrollment has a robust positive effect on export diversification, whereas the share of natural resources in gross domestic product lowers diversification levels. Using the IVBMA approach of Karl and Lenkoski (2012) (see Subsection 3.9.7) they find that these findings are robust to accounting for endogeneity.

Kourtellos et al. (2016) use BMA methods to investigate the variation in intergenerational spatial mobility across commuter zones in the US using model priors based on the dilution idea. Their results show substantial evidence of heterogeneity, which suggests exploring nonlinearities in the spatial mobility process.

Returns to education have been examined through BMA in Tobias and Li (2004). Koop et al. (2012) use their instrumental variables BMA method in this context. Cordero et al. (2016) use BMA methods to assess the determinants of cognitive and non-cognitive educational outcomes in Spain. The link between tuition fees and demand for higher education has been investigated in Havranek et al. (2018) in a meta-analysis framework. After accounting for publication bias, they conclude that the mean effect of tuition fees on enrolment is close to zero. BMA and FMA approaches (the latter using the orthogonal transformation of Amini and Parmeter (2012)) lead to very similar results and also indicate that enrolment of male students and students at private universities does tend to decrease with tuition fees.

Daude et al. (2016) investigate the drivers of productive capabilities (which are important for growth) using BACE based on bias-corrected least squares dummy variable estimates (Kiviet, 1995) in a dynamic panel context with country-specific effects.

Through spatial BMA, Oberdabernig et al. (2018) examine democracy determinants and find that spatial spillovers are important even after controlling for a large number of geographical covariates, using a student-t version of the SAR model (see subsection 3.9.9) with fixed degrees of freedom. Also employing a model with spatial effects, Hortas-Rico and Rios (2016) examine the main drivers of urban income inequality using Spanish municipal data.

Cohen et al. (2016) investigate the social acceptance of power transmission lines using an EU survey. An ordered probit model was used to model the level of acceptance and the fixed country effects of that regression were then used as dependent variables in a BMA analysis, to further explain the heterogeneity between the 27 countries covered in the survey.

In order to identify the main determinants of corruption, Jetter and Parmeter (2018) apply IVBMA to corruption data with a large number of endogenous regressors, using lagged values as instruments. They conclude that institutional characteristics (*e.g.* rule of law, government effectiveness and urbanization rate) and the extent of primary schooling emerge as important

predictors, while they find little support for historical, time-invariant cultural, and geographic determinants.

Pham (2017) investigates the impact of different globalization dimensions (both economic and non-economic) on the informal sector and shadow economy in developing countries. The methodology of León-González and Montolio (2015) is used to deal with endogenous regressors as well as country-specific fixed effects.

The effect of the abundance of resources on the efficiency of resource usage is explored in Hartwell (2016). This paper considers 130 countries over various time frames from 1970 to 2011, both resource-abundant and resource-scarce, to ascertain a link between abundance of resources and less efficient usage of those resources. Efficiency is measured by *e.g.* gas or oil consumption per unit of GDP, and three-stage least squares estimates are obtained for a system of equations. Model averaging is then conducted according to WALS. The paper concludes that for resource-abundant countries, the improvement of property rights will lead to a more environmentally sustainable resource usage.

Wei and Cao (2017) use dynamic model averaging (DMA) to forecast the growth rate of house prices in 30 major Chinese cities. They use the MCS test (see Section 2.2) to conclude that DMA achieves significantly higher forecasting accuracy than other models in both the recursive and rolling forecasting modes. They find that the importance of predictors for Chinese house prices varies substantially over time and that the Google search index for house prices has recently surpassed the forecasting ability of traditional macroeconomic variables. Housing prices in Hong Kong were analysed in Magnus et al. (2011) using a GLS version of WALS.

Robust determinants of bilateral trade are investigated in Chen et al. (2018), using their LIBMA methodology (see Section 3.9.8). They find evidence of trade persistence and of important roles for the exchange rate regime, several of the traditional "core" variables of the trade gravity model as well as trade creation and diversion through trade agreements. They stress that neglecting to address model uncertainty, dynamics, and endogeneity simultaneously would lead to quite different conclusions.

6 Software and resources

The free availability of software is generally very important for the adoption of methodology by applied users. There are a number of publicly available computational resources for conducting BMA. Early contributions are the code by Raftery et al. (1997) (now published as an `R` package in Raftery et al. (2010)) and the `Fortran` code used by Fernández et al. (2001a).

Recently, a number of R-packages have been created, in particular the frequently used BMS package (Feldkircher and Zeugner, 2014). Details about BMS are given in Zeugner and Feldkircher (2015). Two other well-known R-packages are BAS (Clyde, 2017), explained in Clyde et al. (2011), and BayesVarSel (García-Donato and Forte, 2015), described in García-Donato and Forte (2018). When endogenous regressors are suspected, the R-package ivbma (Lenkoski, Karl, and Neudecker, 2014) implements the method of Karl and Lenkoski (2012). For situations where we wish to allow for flexible nonlinear effects of the regressors, inference for (generalized) additive models as in Sabanés Bové and Held (2011a) and Sabanés Bové et al. (2015) can be conducted by the packages glmBfp (Gravestock and Sabanés Bové, 2017) on CRAN and hypergsplines (Sabanés Bové, 2011) on R-Forge, respectively. For dynamic models, an efficient implementation of the DMA methodology of Raftery et al. (2010) is provided in the R package eDMA (Catania and Nonejad, 2017), as described in Catania and Nonejad (2018). This software uses parallel computing if shared memory multiple processors hardware is available. The model confidence set approach (as described in Section 2.2) can be implemented through the R package MCS (Catania and Bernardi, 2017) as described in Bernardi and Catania (2018). Finally, the R packages MuMIn (Bartoń, 2016) and AICcmodavg (Mazerolle, 2017) contain a wide range of different information-theoretic model selection and FMA methods.

In addition, code exists in other computing environments; for example LeSage (2015) describes Matlab code for BMA with spatial models. Błażejowski and Kwiatkowski (2015, 2018) present packages that implements Bayesian model averaging (including jointness measures) and BACE in gretl.[84] Perrakis and Ntzoufras (2018) describe an implementation in WinBUGS (using Gibbs sampling over all parameters and the models) for BMA under hyperpriors on g.

Using the BMS package Amini and Parmeter (2012) successfully replicate the BMA results of Fernández et al. (2001b), Masanjala and Papageorgiou (2008) and Doppelhofer and Weeks (2009). Forte et al. (2018) provide a systematic review of R-packages publicly available in CRAN for Bayesian model selection and model averaging in normal linear regression models. In particular, they examine in detail the packages BAS, BayesFactor (Morey et al., 2015), BayesVarSel, BMS and mombf (Rossell et al., 2014) and highlight differences in priors that can be accommodated (within the class described in (6)), numerical implementation and posterior summaries provided. All packages lead to very similar results on a number of real data sets, and generally provide reliable inference within 10 minutes of running time on a simple PC for problems up to $p = 100$ or so covariates. They find that BAS is overall faster than the other packages considered but with a very high cost in terms of memory requirements and, overall, they recommend BAS with estimation based on model visit frequencies[85]. If memory restrictions are an issue (for mod-

[84] Gretl is a free, open-source software (written in C) for econometric analysis with a graphical user interface.

[85] The BAS package also has the option to use the sampling method (without replacement) called Bayesian Adap-

erately large p or long runs) then BayesVarSel is a good choice for small or moderate values of n, while BMS is preferable when n is large.

A number of researchers have made useful BMA and FMA resources freely available:

- Clyde: `http://stat.duke.edu/~clyde/software` for BAS and her papers can be found at `http://www2.stat.duke.edu/~clyde/research/`.
- Feldkircher and Zeugner: `http://bms.zeugner.eu/resources/` a dedicated BMA resource page with lots of free software and introductory material.
- Hansen: `https://www.ssc.wisc.edu/~bhansen/progs/progs_ma.html` contains code (in R, Matlab, Gauss and STATA) implementing MMA and JMA.
- Magnus: `http://www.janmagnus.nl/items/WALS.pdf` for MATLAB and Stata implementations of WALS, described in Magnus and De Luca (2011).
- Raftery: `https://www.stat.washington.edu/raftery/Research/bma.html` for his BMA papers and `http://www.stat.washington.edu/raftery/software.html` for software and data.
- Steel: `http://www.warwick.ac.uk/go/msteel/steel_homepage/bma` has BMA papers that I contributed to as well as code (Fortran) and data.

7 Conclusions

Model uncertainty is a pervasive (and often not fully recognized) problem in economic applications. It is important to realize that simply ignoring the problem (and sticking to a single model, as is the traditional approach) is clearly not a solution, and has the dangerous consequence of presenting conclusions with an excess of confidence, since the results do not take into account the host of other possible models that could have been tried. Even if we do acknowledge the existence of other models, the use of model selection techniques and presenting our inference conditional upon the single chosen model typically leads to an underestimation of our uncertainty and can induce important biases[86]. Of course, for applying model averaging it is key that we define an ap-

tive Sampling (BAS) described in Clyde et al. (2011), which is based on renormalization and leads to less accurate estimates in line with the comments in Section 3.3.

[86]Model selection can only be a valid strategy in certain circumstances, for example if we are interested in the identity of the "true" model and if the latter is part of the model space or if all other models in the appropriate model space are massively dominated by a single model. Both situations are unlikely to occur in economic applications.

propriate model space, which adequately reflects all the uncertainty in the problem, for example in terms of theory, implementation or modelling assumptions.

In my view, the proliferation of "empirically supported" theories in economics, for example, on what are key drivers for growth, or which indicators can serve as early warning signals for financial crises, is to an important extent due to the tendency of investigators to focus their empirical analyses on models that are largely in line with their own preferred theories. This means that debates often remain unresolved, and I firmly believe that model averaging with appropriately chosen model spaces, encompassing (at least) all available theories, can provide very compelling evidence in such situations. I interpret the rapidly growing use of model averaging methods in economics as a recognition of how much can be gained by adopting principled approaches to the resolution of model uncertainty.

Model averaging methods broadly come in two flavours: Bayesian (BMA) and frequentist (FMA). The choice between BMA versus FMA is to some extent a matter of taste and may depend on the particular focus and aims of the investigation. For this author, the theoretically optimal, finite sample nature of BMA makes it particularly attractive for use in situations of model uncertainty. Also, the availability of posterior inclusion probabilities for the regressors and model probabilities (which also allows for model selection if required) seem to be clear advantages of BMA. In addition, BMA is easily implemented in very large model spaces and possesses some attractive properties (as detailed in Subsection 3.2).

Clearly, priors matter for BMA and it is crucial to be aware of this. Looking for solutions that do not depend on prior assumptions at all can realistically only be achieved on the surface by hiding the implicit prior assumptions underneath. I believe it is much preferable to be explicit about the prior assumptions and the recent research in prior sensitivity can serve to highlight which aspects of the prior are particularly critical for the results and how we can "robustify" our prior choices. A recommended way to do this is through the use of hyperpriors on hyperparameters such as w and g, given a prior structure such as the one in (6). We can then, typically, make reasonable choices for our robustified priors by eliciting simple quantities, such as prior mean model size. The resulting prior avoids being unintentionally informative and has the extra advantage of making the analysis more adaptive to the data. For example, in cases of weak or unreliable data it will tend to favour smaller values of g, avoiding unwarranted precise distinctions between models. This may well lead to larger model sizes, but that can easily be counteracted by choosing a prior on the model space that is centered over smaller models.

Sensitivity analysis (over a range of different priors and different sampling models) is indispensable if we want to convince our colleagues, clients and policy makers. Providing an explicit mapping from these many assumptions to the main results is a key aspect of careful applied re-

search, and should not be neglected. There are many things that theory and prior desiderata can tell us, but there will always remain a lot that is up to the user, and then it is important to try and capture a wide array of possible reasonable assumptions underlying the analysis. In essence, this is the key message of model averaging and we should take it to heart whenever we do empirical research, certainly in non-experimental sciences such as economics.

I believe that model averaging is well on its way to take its rightful place in the toolbox of the economist, as a powerful methodology for the resolution of uncertainty. The free availability of well-documented and efficient software should stimulate its wider adoption in the profession. Besides formally accounting for uncertainty (as expressed by our choice of model space) in our inference and policy conclusions, it can, in my view, also contribute to constructive and focused communication within economics. In particular, model averaging creates an excellent unifying platform to highlight and explain the reasons for differences in empirical findings, through *e.g.* differences in model space, priors in BMA or weights in FMA.

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