# Quality of Schooling: Child Quantity-Quality Tradeoff, Technological Progress and Economic Growth

Saini, Swati and Keswani Mehra, Meeta

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# Quality of Schooling: Child Quantity-Quality Tradeoff, Technological Progress and Economic Growth

**Abstract**

An overlapping generations version of an R&D-based growth model ‘a la Diamond (1965) and Jones (1995) is built to examine how improvement in quality of schooling impact technical progress and long- run economic growth of an economy by influencing fertility and education decisions at household level. The results indicate that improvement in schooling quality triggers a child quantity-quality trade-off at household level when quality of schooling exceeds an endogenously determined threshold. At the household level, parents invest more in education of children and have lesser number of children in response to improvement in quality of schooling. This micro-level tradeoff has two opposing effects on aggregate human capital accumulation at macro level. Higher investment in education of a child stimulates the accumulation of human capital which fosters technical progress but the simultaneous decline in fertility rate reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. The first effect prevails over latter only when quality of schooling is higher than the threshold.

***Keywords***— fertility, quality of schooling, economic growth, demographic transition

## 1 Introduction

As per the estimates of Barro and Lee (2013), the share of population without any formal schooling in developing countries has declined from 54.6 percent in 1960 to approximately 17.4 percent in 2010. However, merely expanding access to education does not ensure that children actually learn in schools. The learning outcomes in schools closely hinge upon the quality of schooling, which has been given inadequate attention in the development policy paradigms of most developing countries until now. But recently, development policy paradigms of most countries are gradually shifting towards improving learning quality in schools than merely expanding access to education. This policy paradigm shift in education policy is also reflected in the post-2015 development agenda. Imparting quality education features as the fourth Sustainable Development Goal set by the United Nations. This shift is motivated by two factors.

First, there is growing evidence that quality of schooling matters more for economic growth. Hanushek and Kimko (2000) and Hanushek and Woessmann (2012) provide an extensive discussion of how scores from cognitive skill tests can be used to measure the quality of human capital and its effects on economic growth. They use data from six voluntary international tests of mathematics and science to build a measure of quality of education. They find that the estimate of human capital quality has a significant positive impact on growth. Several studies have since found very similar results (Bosworth & Collins, 2003; Ciccone & Papaioannou, 2009; Islam, Ang, & Madsen, 2014). Second, poor quality of schooling remains a dismal reality in developing countries. UNESCO (2014) reports that 250 million children are functionally illiterate and innumerate despite 50 percent of them having spent at least four years in school. According to the Annual Status of Education Report (ASER) (2017) survey titled "Beyond Basics", based on an assessment of 30,000 children in 28 districts of 24 states in India, only 43 percent of 14-18 year olds could do simple division after eight years of schooling. Less than half of the children surveyed could not add weights in kilograms and more than 40 percent could not tell hours and minutes from a clock. This scenario is staggering and reveals a gloomy picture about the learning outcomes of children in Indian schools. Similarly, Glewwe, Ilias, and Kremer (2010) report that teachers from rural schools in Kenya were absent 20 percent of the time; while, in Zambia and Pakistan, teachers were absent, respectively, 18 percent and 10 percent of the time (J. Das, Dercon, Habyarimana, & Krishnan, 2004; Reimers, 1993). This implies that poor quality of schooling significantly distorts the learning outcomes in schools, which in turn, has far-reaching implications on growth prospects of developing countries.

In this paper, we build an overlapping generations version of an R&D-based growth model á la Diamond (1965) and Jones (1995) to analyze how improvement in quality of schooling and the associated changes in fertility and education decisions at the micro level influence the long-run economic growth of an economy. Our work is closely related to two broad strands of literature. First, there is theoretical literature that analyzes the linkages between quality of schooling and economic growth. Many existing studies (M. Das & Guha, 2012; Gilpin & Kaganovich, 2012; Tamura, 2001) on quality of schooling and economic growth focus on explaining how determinants of quality of schooling such as teacher-student ratio and teacher quality together impact the learning process, and the consequent human capital formation and, therefore, economic growth. However, most of these studies assume exogenously determined population growth and do not consider technical progress in their models. Consequently, these studies are unable to analyze the impact of schooling quality and the resulting demographic change on R&D activities, which are a major determinant of technological development in the present world. We improve upon these papers by endogenizing both - population growth and technical change. Specifically, our work focuses on interactions between quality of schooling and demographic change, which influence total factor productivity growth and, therefore, growth prospects of an economy.

Second, our research relates to the literature linking R&D based growth with endogenous fertility and education decisions (Hashimoto & Tabata, 2016; Strulik, 2005; Strulik, Prettner, & Prskawetz, 2013). In particular, this work is closely related to Strulik et al. (2013). Strulik et al. (2013) analyze child quantity-quality trade-off by integrating R&D based innovations into a unified growth framework. They explain why high levels of total factor productivity and economic growth in modern economies are associated with low or negative population growth by considering a child quantity-quality trade-off at the household level. In their theoretical model, decisions related to fertility and education are endogenously determined by households. A substitution of child quantity, $n$, by child quality (i.e. expenditure on education), $e$, that keeps total child expenditure, $e.n$, constant sets free parental time, which can be used to earn extra income. The additional income is partly spent on education, such that the overall child expenditure rises more proportionately than child quantity falls. On the macro side of the economy, this trade-off means that the magnitude by which human capital per person, $h$ rises is larger than the magnitude by which number of persons, $L$, falls. The net impact of this micro level trade-off is that the total available human capital $h.L$ increases at the macro level. Given that human capital is the driving force for R&D, this entails a higher R&D output and higher R&D-based growth.

Although our modelling framework is similar to Strulik et al. (2013), we go beyond Strulik et al. (2013) in atleast three respects. First, the focus of our research is not on formulating a unified growth theory which explains the entire transition of an economy from Malthusian stagnation to modern growth. Instead, the purpose of this work is to build a growth model that explains the inter-linkages between quality of schooling, demographic change and technological improvements in a modern economy. Therefore, this paper focusses on characterizing two types of economies with low and high quality of schooling and examines the corresponding drivers of economic growth in these two types of economies. To the best of our knowledge, this issue is yet to be explicitly discussed in the literature. Second, our study examines the impact of a demographic transition triggered by improvement in quality of schooling. Strulik et al. (2013) focus on impact of a demographic transition induced by technological progress. Third, we extend Strulik et al. (2013) by considering two distinct channels of technological improvement - innovation and imitation. Under the innovation regime, technological improvements occur by innovating on local technology frontier whereas under imitation regime, technological progress occurs by imitating existing foreign technologies. In this respect as well, this work is an improvement over existing research in this area.

This paper is organized as follows. Section 2 discusses the basic structure of the model. Section 3 contains the key analytical results for a decentralized economy, which provide the key propositions of this study. Section 4 concludes.

# 2 The Model and Equilibrium Solutions

## 2.1 The Economic Environment

We consider a model economy populated by overlapping generations of people who live for two periods: adulthood and old age. Time is discrete and goes from 0 to $\infty$. During childhood, which is not modeled explicitly, individuals are reared and educated by their parents. All the decisions are made at the beginning of adulthood. Adults are identical in all aspects. They inelastically supply their skills in the labor market. Adults care about consumption of a homogeneous final good, number and human capital level of their children. During old age, individuals consume their savings plus interest earned on these. Abstracting from gender differences, each household has a single parent. For avoiding the indivisiblity problem, we assume that children are in continuous number. All individuals survive up to adulthood. The education of current period's children determines human capital endowment of next period's adult generation. Akin to Castelló-Climent and Hidalgo-Cabrillana (2012), human capital accumulation function depends on an exogenously given quality of education system, parental investment in education and human capital of parent. Parental investment in education is a fraction of income spent on education of each child.

The production structure of the economy closely follows Romer (1990) and Jones (1995). The economy consists of three sectors: final goods sector, intermediate goods sector and R&D sector.

## 2.2 Individuals

Individuals derive utility from $c_{1,t}$, their own consumption (of the final good) during adulthood; $c_{2,t+1}$, their own consumption during old age; $n_t$, number of children and $h_{t+1}$, human capital of children. Parents' motivation to invest in human capital of children by spending on children's education is driven by a "warm glow" of giving (Andreoni, 1989) or preference for having "higher-quality" children (Becker, 1960). The lifetime expected utility of individuals in generation $t$ is given by:

$$u_t = \log c_{1,t} + \beta_1 \log c_{2,t+1} + \beta_2 \log(h_{t+1} n_t), \tag{1}$$

where positive weights $\beta_1$ and $\beta_2$ measure the importance of future consumption and child quantity and quality relative to current consumption in the utility function. Alternatively, following De la Croix and Doepke (2004), $\beta_2$ can be interpreted as an altruism factor.

An adult's human capital is denoted by $h_t$ and the wage per unit of human capital is $w_t$. Young adults spend their income on current consumption, savings for old-age consumption and child's education expenditure. Rearing a child necessarily takes fraction $\tau \in (0,1)$ of an adult's time, which is given exogenously. Accordingly, the budget constraints for the young and old adults are given by:

$$w_t h_t (1 - \tau n_t) = c_{1,t} + s_t + e_t (w_t h_t) n_t; \tag{2}$$

$$c_{2,t+1} = (1 + r_{t+1}) s_t, \tag{3}$$

where $e_t$ is the fraction of income per child spent on education, $s_t$ is savings and $r_{t+1}$ is interest rate. Non-negativity constraints apply to all the variables.

The human capital of children, $h_{t+1}$, depends on human capital of parents, $h_t$, parental investment in education per child, $e_t$, and quality of education system, $\theta$, which is exogenously given.

$$h_{t+1} = (\mu + \theta e_t)^{\epsilon} h_t, \qquad \epsilon < 1. \tag{4}$$

The parameters satisfy $\mu \geq 1$ and $\epsilon \in (0,1)$. $\epsilon$ measures the returns to education. $\mu$ is the intergenerational human capital spillovers that are basically skills learnt by children by observing and imitating parents. The parametric restriction of $\mu \geq 1$ ensures that the growth rate of per capita human capital does not become negative when parents do not invest in education. It ensures that children will acquire knowledge and skills atleast equivalent to their parents when parents do not educate their children.The assumption that quality of schooling is an argument in human capital accumulation function is consistent with a number of studies. Hanushek, Lavy, and Hitomi (2008) find that lower-quality schools lead to higher dropout rates in case of Egyptian primary schools. Similarly, Hanushek and Woessmann (2008) find that cognitive skills, a proxy for educational quality, is positively related to individual earnings. In theoretical terms, Castelló-Climent and Hidalgo-Cabrillana (2012) have shown in their theory of human capital investment that high-quality education increases the returns to schooling and incentivizes human capital accumulation via two channels- extensive and intensive margins. Higher quality makes education accessible to more people (extensive margin), and once individuals decide to participate in higher education, higher quality increases the investment made per individual (intensive margin). Parental human capital, $h_t$, as an input in human capital accumulation technology represents intergenerational transfers of human capital, which is a common assumption in the literature (De la Croix & Doepke, 2004; Kalemli-Ozcan, 2002, 2003; Tamura, 2001).

Individuals maximize utility in eq. (1) with respect to the constraints, eqs. (2) to (4) using control variables $c_{1,t}$, $s_t$, $n_t$ and $e_t$. The solution to individuals' decision problem can either be interior, or at a corner where the individuals choose zero education. The first-order conditions yield the following solution, as in eqs. (5) to (8), for consumption and savings irrespective of whether education is in the interior or at the corner:[1]

$$c_{1,t} = \frac{w_t h_t}{1+\beta_1+\beta_2}; \tag{5}$$

$$s_t = \frac{\beta_1 w_t h_t}{1+\beta_1+\beta_2}. \tag{6}$$

For child quantity and quality, there exists a threshold level of quality of schooling. If quality of schooling falls below the threshold, adults do not spent on child quality and maximize child quantity. This constitutes the corner solution. In particular, following results are derived from the first-order conditions:

$$e_t = \begin{cases} 0, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \dfrac{\tau\theta\epsilon - \mu}{\theta(1-\epsilon)}, & \text{otherwise,} \end{cases} \tag{7}$$

$$n_t = \begin{cases} \dfrac{\beta_2\epsilon\theta}{(1+\beta_1+\beta_2)\mu}, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\theta(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)}, & \text{otherwise.} \end{cases} \tag{8}$$

Inserting eq. (7) in eq. (4), we get an equation of motion for human capital as:

$$h_{t+1} = \begin{cases} \mu^{\epsilon} h_t, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \left[\dfrac{\epsilon(\tau\theta-\mu)}{(1-\epsilon)}\right]^{\epsilon} h_t, & \text{otherwise.} \end{cases} \tag{9}$$

Below the threshold, quality of schooling is not an argument in human capital production function. Without education expenditure, human capital of next generation consists of basic skills only. From eqs. (5) to (8), irrespective of whether quality of schooling exceeds threshold or not, savings and consumption are increasing in $w_t h_t$ and there is no direct effect of income on fertility because a positive income effect of an increase in wages on fertility is balanced by a negative substitution effect. The quality of schooling has a direct bearing on child quantity and quality. The following lemma shows how quality of schooling influences fertility behavior.

**Lemma 1** *When quality of schooling is high enough to surpass the threshold, a marginal improvement in the quality of schooling triggers a child quantity-quality trade-off such that adults bear lesser number of children and invest more in education per child in response to improvement in quality of schooling. However, when quality of schooling is lower than the threshold, then it has no effect on child quality as adults do not invest in child's education and focus instead on maximizing child quantity.*

**Proof.**By investigating the corner solution in eqs. (7) and (8), it can be immediately seen that quality of schooling entails no child quantity-quality trade-off if quality of schooling falls below the threshold. Adults do not spend on education and maximize fertility. To see the effect when quality of schooling is above the threshold, we take the derivatives of the interior solution of $e_t$ and $n_t$ with respect to $\theta$ in eqs. (7) and (8). That is,

$$\frac{\partial n_t}{\partial \theta} = \frac{-\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} < 0;$$

[1]Detailed mathematical derivations are provided in Appendix A.

$$\frac{\partial e_t}{\partial \theta} = \frac{\mu}{(1-\epsilon)\theta^2} > 0.$$

When quality of schooling is less than the threshold, the derivatives of the corner solution of $e_t$ and $n_t$ with respect to $\theta$ in eqs. (7) and (8) yield:

$$\frac{\partial n_t}{\partial \theta} = \frac{\beta_2 \epsilon}{(1+\beta_1+\beta_2)\mu} > 0;$$

$$\frac{\partial e_t}{\partial \theta} = 0.$$

■

Thus, it can be seen that fertility changes are directly triggered by quality of schooling. Any improvement in quality of schooling over and above the threshold makes learning in schools more effective and, therefore, increases marginal returns to investment in human capital. Consequently, a parent reduces fertility and spends more on education per child. Thus, quality of schooling can be perceived as another plausible mechanism for triggering child quantity-quality trade-off besides other commonly proposed mechanisms such as declining child mortality (Soares, 2005), rise in life expectancy of parents (Boucekkine, Croix, & Licandro, 2003; Boucekkine, De la Croix, & Licandro, 2002; Hashimoto & Tabata, 2016; Kalemli-Ozcan, 2002, 2003), technical progress (Galor & Weil, 2000) and decline in gender wage gap (Galor & Weil, 1996). These theoretical results are in line with recent empirical findings. For example, Hanushek et al. (2008) find that lower quality of schooling leads to higher dropout rates in Egyptian primary schools. A cross-country analysis by Castelló-Climent and Hidalgo-Cabrillana (2012) reveals that quality of education has a positive effect on enrollment rates in secondary schooling only when quality of schooling is sufficiently high.

**Lemma 2** *An increase in returns to education, $\epsilon$, leads to a child quantity-quality trade-off wherein parents educate their children and bear lesser number of children when quality of schooling surpasses the threshold. However, when quality of schooling is less than the threshold, returns to education has no effect on education of children and parents maximize child fertility.*

**Proof.**Taking the derivatives of the interior solution of $e_t$ and $n_t$ with respect to $\epsilon$ in eqs. (7) and (8), one gets that:

$$\frac{\partial n_t}{\partial \epsilon} = \frac{-\beta_2 \theta}{(1+\beta_1+\beta_2)(\tau\theta-\mu)} < 0;$$

$$\frac{\partial e_t}{\partial \epsilon} = \frac{\tau\theta-\mu}{\theta(1-\epsilon)^2} > 0.$$

When quality of schooling is less than the threshold, the derivatives of the corner solution of $e_t$ and $n_t$ with respect to $\epsilon$ in eqs. (7) and (8) yield:

$$\frac{\partial n_t}{\partial \epsilon} = \frac{\beta_2 \theta}{(1+\beta_1+\beta_2)\mu} > 0;$$

$$\frac{\partial e_t}{\partial \epsilon} = 0.$$

■

This implies that returns to education is yet another factor that can trigger a child quantity-quality trade-off. High returns to education implies education makes human capital more productive. Therefore, parents invest in education of their children and decide to have lesser number of children. However, when quality of schooling is less than the threshold, then parents decide not to make any investment in the education of children and, therefore, returns to schooling has no effect on child quality and child quantity is maximized. Both Lemmas 1 and 2 will be used later in our analysis.

## 2.3 Final Goods Sector

The final homogeneous good, $Y_t$ is produced and sold in a competitive market. Firms produce the final good, $Y_t$ using human capital, $H_t^Y$, and a range of intermediate inputs. The production function for firms is defined as:

$$Y_t = (H_t^Y)^{1-\alpha} \sum_{i=1}^{A_t} x_{it}^{\alpha}, \qquad 0 < \alpha < 1. \tag{10}$$

The parameter, $\alpha$, is the capital share in final good production. The price of final good, $P_Y$, has been normalized to 1. In each period $t$, the final good producers solve the following profit maximization problem with respect to their choice of range of intermediate inputs and human capital,$H_Y$:

$$Max_{x_{it}, H_t^Y} \quad \pi_t(Y) = (H_t^Y)^{1-\alpha} \sum_{i=1}^{A_t} x_{it}^{\alpha} - w_Y H_t^Y - \sum_{i=1}^{A_t} p_{it} x_{it},$$

where $p_{it}$ is the unit monopoly price of $i$th intermediate input and $w_Y$ is the wage rate prevailing in final good sector. The first-order conditions imply that:

$$p_{it} = \alpha (H_t^Y)^{1-\alpha} x_{it}^{\alpha-1}; \tag{11}$$

$$w_Y = \frac{(1-\alpha)(H_t^Y)^{1-\alpha} \sum_{i=1}^{A_t} x_{it}^{\alpha}}{H_t^Y} = \frac{(1-\alpha) Y_t}{H_t^Y}. \tag{12}$$

Eq. (11) yields the demand for each intermediate input as:

$$x_{it} = \left[\frac{\alpha}{p_{it}}\right]^{\frac{1}{1-\alpha}} H_t^Y. \tag{13}$$

An analysis of the intermediate goods sector ensues.

## 2.4 Intermediate Goods Sector

Each intermediate good $i$ is produced by monopolist producer who holds the blueprint to produce $x_{it}$ quantity at time $t$. Each intermediate good uses only capital in a one-to-one production technology, or $x_{it} = K_{it}$. Thus, the amount of intermediate inputs produced of all types equals the aggregate capital stock of the economy.

$$\sum_{i=1}^{A_t} x_{it} = K_t. \tag{14}$$

Each $i$th intermediate good producer maximizes profits with respect to his/her choice of capital. That is,

$$Max_{x_{it}} \quad \pi_t(i) = p_{it} x_{it} - r_t K_{it} = \alpha (H_t^Y)^{1-\alpha} x_{it}^{\alpha} - r_t x_{it},$$

where the expression in r.h.s derives from substituting solution to $p_{it}$ from eq. (11) and $x_{it} = K_{it}$. $r_t$ is price per unit of capital. The first-order condition yields:

$$r_t = \alpha^2 (H_t^Y)^{1-\alpha} x_{it}^{\alpha-1}. \tag{15}$$

Using eq. (11), we get the solution to the equilibrium price as $p_{it} = p_t = \dfrac{r_t}{\alpha}$. This is the monopoly price charged as a markup over marginal cost. Note that being independent of $i$, this price is constant across all intermediate goods. From eq. (13), this implies that quantity produced of each $i$th intermediate input is the same, that is, $x_{it} = x_t = \left[\dfrac{\alpha^2}{r_t}\right]^{\frac{1}{1-\alpha}} H_t^Y$. This entails that, in equilibrium, the profit of the $i$th monopolist is given by:

$$\begin{aligned} \pi_t = p_t x_t - r_t x_t \equiv \left[\frac{r_t}{\alpha} - r_t\right] x_t \equiv \left[\frac{1-\alpha}{\alpha}\right] r_t x_t; \\ = \alpha(1-\alpha) H_Y^{1-\alpha} x_t^{\alpha}, \end{aligned} \tag{16}$$

where the last expression is derived using eq. (15) and $x_{it} = x_t$ in equilibrium. Since, in equilibrium, intermediate inputs are sold at the same price and demanded in equal quantities, aggregate physical capital is given by $K_t = A_t x_t$. Inserting this information into the production function of final good, eq. (10) simplifies to:

$$Y_t = (H_t^Y)^{1-\alpha} A_t^{1-\alpha} K_t^{\alpha}. \tag{17}$$

Accordingly, equilibrium profits of ith monopolist in eq. (16) simplifies to:

$$\pi_t = \alpha(1-\alpha)\frac{Y_t}{A_t}. \tag{18}$$

Also, since in equilibrium, $x_{it} = x_t$ and $K_t = A_t x_t$, the price per unit of capital in eq. (15) can be expressed as:

$$r_t = \alpha^2 (H_t^Y)^{1-\alpha} \left[\frac{A_t}{K_t}\right]^{1-\alpha}, \tag{19}$$

Further using eq. (17), rental rate of capital simplifies to:

$$r_t = \alpha^2 \left[\frac{Y_t}{K_t}\right]. \tag{20}$$

Next, the R&D sector is discussed.

## 2.5 R&D Sector

Under the assumption of free entry into the R&D sector, firms employ human capital, $H_t^A$, to develop new blueprints which are sold at price, $p_t^A$. This price is common to all the blueprints due to the competitive feature of the R&D sector. We consider two types of regimes that can drive R&D activities. The R&D sector produces blueprint of an intermediate variety either by imitating from the world technology frontier or by innovating upon the local technology level. Following Papageorgiou and Perez-Sebastian (2006) and Guilló, Papageorgiou, and Perez-Sebastian (2011), the production function of technology for a firm is postulated as:

$$A_{t+1} - A_t = \delta_t H_t^A, \tag{21}$$

where $A_{t+1} - A_t$ are new blueprints and $H_t^A$ is the human capital working in R&D sector. Productivity of R&D activity, $\delta_t$, is constant at the firm level but at the aggregate level, it is defined as:

$$\text{Innovation regime} : \delta_t = \bar{\delta}(H_t^A)^{\lambda-1} A_t^{\phi}; \tag{22}$$

$$\text{Imitation regime} : \delta_t = \bar{\delta}(H_t^A)^{\lambda-1} A_t^{\phi} \left[\frac{\bar{A}_t}{A_t}\right]. \tag{23}$$

R&D productivity depends positively on the number of already existing ideas, $A_t$, and human capital employed in R&D sector, $H_t^A$. The parameter $\bar{\delta}$ denotes general productivity in R&D. $0 < \phi < 1$ measures intertemporal knowledge spillovers (standing-on-shoulders effect) and $0 < \lambda < 1$ measures returns to R&D effort (stepping-on-toes effect). $\bar{A}_t$ is world technology frontier that grows exogenously at rate, $g_{\bar{A}}$. The standing-on-shoulders effect may arise as existing knowledge contributes to the capacity to innovate. The returns to human capital differ between the firm level and the economy-wide level. There exists constant returns to R&D effort at the firm level as revealed by eq. (21). However, the R&D technology shows diminishing returns to R&D effort as researchers generate negative externality at the aggregate level (stepping-on-toes effect). The stepping-on-toes effect may arise due to competition among multiple R&D firms to become the first to succeed at creating and patenting a new blueprint and/or process. If all other factors are held constant, an increase in R&D effort will induce increased duplication of research efforts leading to stepping-on-toes effect. Additionally, R&D productivity depends on a catch-up term, $\frac{\bar{A}_t}{A_t}$ under imitation regime. Akin to Nelson and Phelps (1966), $\frac{\bar{A}_t}{A_t}$ is the catch-up term which signifies the fact that greater the technological gap between leader and follower economy, higher the potential of the follower economy to catch up through imitation of existing technologies. Since all R&D firms end up in a symmetric equilibrium, the production function of technology under imitation regime at the aggregate level reduces to:

$$A_{t+1} - A_t = \bar{\delta}(H_t^A)^{\lambda} A_t^{\phi} \left[\frac{\bar{A}_t}{A_t}\right]. \tag{24}$$

The catch-up effect is specific to imitation regime only. Under the innovation regime, firms innovate upon the local technology level to discover new blueprints. In this case, the aggregate production function reduces to:

$$A_{t+1} - A_t = \bar{\delta}(H_t^A)^{\lambda} A_t^{\phi}. \tag{25}$$

Each firm in the R&D sector maximizes profits, given by:

$$\pi_{t,A} = p_t^A (A_{t+1} - A_t) - w_A H_t^A,$$

where $p_t^A$ is price of a blueprint, $A_{t+1} - A_t$ are number of new blueprints discovered and $w_A$ is the wage rate. Under both imitation and innovation regimes, using eq. (21), profit function of R&D firm can be expressed as:

$$\pi_{t,A} = p_t^A \delta_t (H_t^A) - w_A H_t^A. \tag{26}$$

In both the technology regimes, maximization of profits leads to following optimality condition:

$$w_A = p_t^A \delta_t. \tag{27}$$

Substituting for $\delta_t$ from eq. (22), the wage rate under innovation regime is now given by:

$$w_A^{in} = p_t^A \bar{\delta}(H_t^A)^{\lambda-1} A_t^{\phi} = \left[\frac{p_t^A \bar{\delta}(H_t^A)^{\lambda} A_t^{\phi}}{H_t^A}\right], \tag{28}$$

Similarly, wage rate under imitation regime is expressed as:

$$w_A^{im} = \frac{p_t^A \bar{\delta}(H_t^A)^{\lambda} A_t^{\phi} \frac{\bar{A}_t}{A_t}}{H_t^A}. \tag{29}$$

Using eqs. (24) and (25), the wage rate under both the technology regimes simplifies to:

$$w_A = \left[\frac{p_t^A (A_{t+1} - A_t)}{H_t^A}\right]. \tag{30}$$

where wages of scientists are increasing in price of blueprint (price of patent) and number of blueprints discovered.

The decision to produce an intermediate variety by an intermediate input producer depends on the difference between the cost of acquiring the patent for a blueprint from the R&D sector, $p_t^A$, and the monopoly profits, $\pi_t$, that can be earned by producing intermediate varieties. Given this information, the R&D sector will set the price of patent, $p_t^A$ such that it extracts the present discounted value of monopoly profits of intermediate firms. The research arbitrage condition yields the following price of blueprint:[2]

$$p_t^A = \frac{\pi_t}{r_t - g_H}. \tag{31}$$

Substituting for $\pi_t$ from eq. (18) and inserting this in eq. (30), the wage rate under both the technology regimes is derived as:

$$w_A = \frac{\alpha(1-\alpha)Y_t g_{A,t}}{(r_t - g_H)H_t^A}, \tag{32}$$

where $g_{A,t} = \dfrac{A_{t+1} - A_t}{A_t}$.

Now in equilibrium, demand for human capital in R&D sector and final good sector should add up to:

$$H_t^A + H_t^Y = H_t.$$

Assuming $\omega$ proportion of total labor force is allocated to R&D sector and $(1-\omega)$ proportion goes to final good sector, we get that:

$$H_t^A = \omega H_t \quad \& \quad H_t^Y = (1-\omega)H_t. \tag{33}$$

In equilibrium, wages in final good and R&D sectors should equalize, that is, $w_Y = w_A$. Substituting for $w_Y$ and $w_A$ from eqs. (12) and (32), we get that:

$$\frac{H_t^A}{H_t^Y} = \frac{\alpha g_{A,t}}{r_t - g_H} \tag{34}$$

[2] Detailed derivation is provided in Appendix B.

We deduce the proportion of human capital that is employed in R&D sector in equilibrium after substituting for $H_t^A$ and $H_t^Y$ in eq. (33) as:

$$\omega = \frac{\alpha g_{A,t}}{(r_t - g_H) + \alpha g_{A,t}}. \tag{35}$$

Accordingly, the equilibrium wage rate in the two sectors is given by:

$$w_A = w_Y = \frac{(1-\alpha)Y_t}{H_t^Y}. \tag{36}$$

We next characterize the BGP of our decentralized economy for both innovation and imitation regimes for two cases - a) when an economy's quality of schooling is sufficiently high, $\theta > \dfrac{\mu}{\tau\epsilon}$ and b) when quality of schooling is less than the threshold, $\theta \leq \dfrac{\mu}{\tau\epsilon}$.

# 3 Dynamics and Steady-State Properties of the Stylized Economy

## 3.1 Dynamics of the Key Variables

This section examines the dynamic properties of our stylized economy. First, we discuss the dynamics of physical factors of production. The aggregate population, $N_t$, grows at the fertility rate, $n_t$ as follows:

$$N_{t+1} = n_t N_t, \tag{37}$$

where $n_t$ is endogenously given by eq. (8). Taking child rearing time into account, the size of the workforce is given by $L_t =(1-\tau n_t)N_t$. Since child rearing costs are constant, and from eq. (8) we know that fertility rate is also constant over time, the workforce grows at the fertility rate, as:

$$L_{t+1} = n_t L_t. \tag{38}$$

Assuming that physical capital depreciates fully within a generation (that is, depreciation is 100 percent) so that next period's capital stock consists of this period's aggregate savings, the market clearing condition for capital market will be

$$K_{t+1} = s_t N_t, \tag{39}$$

where $N_t$ is the population of generation t. Inserting the solutions for savings from eq. (6) and wage rate from eq. (36) and using the fact that $N_t = L_t$ from eqs. (37) and (38), we get :

$$K_{t+1} = \frac{\beta_1(1-\alpha)Y_t H_t}{(1+\beta_1+\beta_2)H_t^Y},$$

Finally inserting for $Y_t$ from eq. (17) we get the equation governing the evolution of aggregate physical capital as:

$$K_{t+1} = B_t K_t^{\alpha} A_t^{1-\alpha} H_t (H_t^Y)^{-\alpha}, \tag{40}$$

where $B_t = \left[\frac{\beta_1}{1+\beta_1+\beta_2}\right](1-\alpha)$.

Next, we discuss the dynamics of aggregate human capital, $H_t \equiv h_t L_t$. The dynamics of per capita human capital are given by eq. (9). Using eqs. (9) and (38), the equation for aggregate human capital accumulation can be written as:

$$\frac{H_{t+1}}{H_t} = \begin{cases} \mu^{\epsilon} n_t, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \left[\frac{\epsilon(\tau\theta - \mu)}{(1-\epsilon)}\right]^{\epsilon} n_t, & \text{otherwise.} \end{cases} \tag{41}$$

Furthermore, the proportion of workforce employed in R&D sector, $\omega$, (given by eq. (35)) is constant as $g_H$ is constant as shown in eq. (41) and $r_t$ is constant along BGP as shown later in Proposition 3.2. Therefore, along BGP, we have:

$$\frac{H^Y_{t+1}}{H^Y_t} = \frac{H^A_{t+1}}{H^A_t} = \frac{H_{t+1}}{H_t}. \tag{42}$$

This follows from eq. (33).

From eqs. (24) and (25), the dynamics of total factor productivity can be expressed as:
Imitation regime:

$$A_{t+1} = A_t + \bar{\delta} H_t^{\lambda} A_t^{\phi} \left[\frac{\bar{A}_t}{A_t}\right], \tag{43}$$

Innovation regime:

$$A_{t+1} = A_t + \bar{\delta} H_t^{\lambda} A_t^{\phi}. \tag{44}$$

This system of equations fully describes the equilibrium dynamics of our model economy for all the plausible cases.The next subsection characterizes the balanced growth paths of an economy for two cases - a) when an economy's quality of education system is sufficiently high, $\theta > \dfrac{\mu}{\tau\epsilon}$, and b) when quality of schooling is less than the threshold, $\theta \leq \dfrac{\mu}{\tau\epsilon}$.

## 3.2 Characterizing the Balanced Growth Path

A balanced growth path (BGP) is a long run equilibrium of the economy, also defined as the steady state, along which growth rate of variables is either zero or constant over time. For any variable $x$, the growth rate is denoted by $g_{x,t} = (x_{t+1} - x_t)/x_t$, and its rate of change by $\tilde{g}_{x,t} = (g_{x,t+1} - g_{x,t})/g_{x,t}$. The balanced growth, thus, requires $\tilde{g}_{x,t} = 0$. We denote the growth rate of $x$ along the BGP by $g_x$, i.e., by omitting the time index for brevity. We begin by evaluating the physical capital accumulation along the BGP. From eq. (40), we deduce that:

$$1 + g_{K,t} \equiv \frac{K_{t+1}}{K_t} = \left[\frac{K_t}{K_{t-1}}\right]^{\alpha} \left[\frac{A_t}{A_{t-1}}\right]^{1-\alpha} \frac{H_t}{H_{t-1}} \left[\frac{H^Y_t}{H^Y_{t-1}}\right]^{-\alpha};$$

Using the result that at steady state, $\dfrac{K_{t+1}}{K_t} = \dfrac{K_t}{K_{t-1}}$ and $\dfrac{H^Y_{t+1}}{H^Y_t} = \dfrac{H_{t+1}}{H_t}$ from eq. (42), the above expression further simplifies to

$$\left[\frac{K_{t+1}}{K_t}\right]^{1-\alpha} = \left[\frac{A_{t+1}}{A_t}\right]^{1-\alpha} \left[\frac{H_{t+1}}{H_t}\right]^{1-\alpha}.$$

Thus, we obtain:

$$g_K = (1+g_A)(1+g_H) - 1. \tag{45}$$

The growth of physical capital and productivity are positively correlated along the steady state. Next, we consider the growth rate of total factor productivity. Under innovation regime, we observe from eq. (43) that:

$$1 + g_{A,t} \equiv \frac{A_{t+1}}{A_t} = 1 + \frac{\bar{\delta}^{\frac{1}{1-\phi}}(H_t^A)^{\frac{\lambda}{1-\phi}}}{A_t};$$

Since along BGP, l.h.s is constant, therefore, r.h.s must also be constant and this holds true when

$$(1+g_A) = [(1+g_h)n]^{\frac{\lambda}{1-\phi}}. \tag{46}$$

The r.h.s follows from the definition of aggregate human capital $H_t = h_t L_t$ and from eq. (42). Further, we observe from eq. (44) that the rate of technical progress under the imitation regime can be written as:

$$1 + g_{A,t} \equiv \frac{A_{t+1}}{A_t} = 1 + \frac{\bar{\delta}^{\frac{1}{2-\phi}}(H_t^A)^{\frac{\lambda}{2-\phi}}\bar{A}_t^{\frac{1}{2-\phi}}}{A_t}.$$

Similarly, using the definition of BGP, we derive the long-run rate of technological progress under imitation regime as:

$$(1+g_A) = (1+g_H)^{\frac{\lambda}{2-\phi}}(1+g_{\bar{A}})^{\frac{1}{2-\phi}} = [(1+g_h)n]^{\frac{\lambda}{2-\phi}}(1+g_{\bar{A}})^{\frac{1}{2-\phi}}. \tag{47}$$

Thus, under both imitation and innovation regimes, technological progress is driven by growth in aggregate human capital. Human capital accumulation improves productivity of researchers, which fosters technological progress. Besides aggregate human capital, the growth of world technology frontier is an additional driver of growth under the imitation regime. The follower economy takes advantage of existing technologies through technology adoption. Therefore, as the world technology frontier grows, it enhances the potential of follower country to catch up through imitation.

Next, we ascertain the growth rates of aggregate GDP and per capita consumption along BGP. From eq. (17) we observe that:

$$1 + g_{Y,t} \equiv \frac{Y_{t+1}}{Y_t} = \left[\frac{K_{t+1}}{K_t}\right]^{\alpha}\left[\frac{A_{t+1}}{A_t}\right]^{1-\alpha}\left[\frac{H_{t+1}^Y}{H_t^Y}\right]^{1-\alpha}.$$

Using eqs. (45) and (42), the long run growth rate of GDP is expressed as:

$$g_Y = (1+g_A)(1+g_H) - 1. \tag{48}$$

From eqs. (45), (46), (47) and (48), we derive the BGP of the stylized economy under the two technology regimes as:
Innovation regime:

$$g_K = g_Y = [(1+g_h)n]^{\frac{1-\phi+\lambda}{1-\phi}} - 1; \tag{49}$$

Imitation regime:

$$g_K = g_Y = [(1+g_h)n]^{\frac{2-\phi+\lambda}{2-\phi}}(1+g_{\bar{A}})^{\frac{1}{2-\phi}} - 1, \tag{50}$$

where

$$[(1+g_h)n] = (1+g_H) = \begin{cases} \dfrac{\beta_2\theta\epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}}, & \text{if } \theta \leq \dfrac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\theta\epsilon^{\epsilon}(1-\epsilon)^{1-\epsilon}}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^{1-\epsilon}}, & \text{otherwise.} \end{cases}$$

This follows after substituting the value of $n$ from eq. (8) in eq. (41).

Furthermore, we observe from the consumer's optimization exercise that:

$$\frac{c_{t+1}}{c_t} = \beta_1(1+r_{t+1}). \tag{51}$$

The r.h.s follows from substituting values of $c_t$ and $s_t$ from eqs. (5) and (6) in eq. (3). Using eqs. (20), (49) and (50), we deduce that:

$$\frac{c_{t+1}}{c_t} = \beta_1\left[1+\alpha^2\frac{Y_t}{K_t}\right]. \tag{52}$$

Along BGP, since $g_K = g_Y$, per capita consumption grows at a constant rate under both the technology regimes.

Furthermore, we derive the conditions when the economic growth rate is higher for an economy with high quality of schooling, $\theta > \frac{\mu}{\tau\epsilon}$ as compared to an economy with lower quality of schooling, $\theta \leq \frac{\mu}{\tau\epsilon}$ under the two technology regimes. We assume that when $\theta > \frac{\mu}{\tau\epsilon}$, quality of schooling is denoted by $\theta_h$ for that particular economy whereas quality of schooling is denoted by $\theta_l$ for an economy with quality of schooling less than the threshold, $\theta \leq \frac{\mu}{\tau\epsilon}$. As shown in eqs. (49) and (50), the rate of economic growth is contingent upon the rate of human capital accumulation under the two technology regimes. This means that under the two technology regimes, an economy with higher quality of schooling, $\theta_h$, grows at a higher rate as compared to an economy with a lower quality of schooling, $\theta_l$ when the following condition holds true:

$$g_{H|(\theta_h > \frac{\mu}{\tau\epsilon})} > g_{H|(\theta_l \leq \frac{\mu}{\tau\epsilon})}$$

Substituting for $g_{H|(\theta_h > \frac{\mu}{\tau\epsilon})}$ and $g_{H|(\theta_l \leq \frac{\mu}{\tau\epsilon})}$ from eqs. (49) and (50), we have:

$$\frac{\beta_2\theta_h\epsilon^{\epsilon}(1-\epsilon)^{1-\epsilon}}{(1+\beta_1+\beta_2)(\tau\theta_h-\mu)^{1-\epsilon}} > \frac{\beta_2\theta_l\epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}},$$

which on simplification yields the following condition:

$$\theta_h > \theta_l\left[\frac{(\tau\theta_h-\mu)\epsilon}{\mu(1-\epsilon)}\right]^{1-\epsilon}. \tag{53}$$

We know that,

$$\theta > \frac{\mu}{\tau\epsilon}.$$

Multiplying both sides by $\tau$ and then, subtracting $\mu$ from both sides yields

$$\frac{(\tau\theta - \mu)\epsilon}{1-\epsilon} > \mu,$$

Since $\mu \geq 1$, we have:

$$\frac{(\tau\theta - \mu)\epsilon}{1-\epsilon} > 1. \tag{54}$$

Thus, $\left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}\right]^{1-\epsilon} > 1$. Thus, we have,

**Proposition 3.1**

- *Under innovation regime, aggregate output, physical capital stock, total factor productivity and per capita consumption grow at a constant rate along the balanced growth path characterized by eqs. (49), (46) and (52).*
- *Under imitation regime, aggregate output, physical capital stock, total factor productivity and per capita consumption grow at a constant rate along the balanced growth path characterized by eqs. (50), (47) and (52).*
- *Under the two technology regimes, an economy with higher quality of schooling, $\theta_h > \frac{\mu}{\tau\epsilon}$, experiences a higher economic growth as compared to an economy with lower quality of schooling, $\theta_l \leq \frac{\mu}{\tau\epsilon}$ if the quality of schooling, $\theta_h$ is sufficiently high such that:*

$$\theta_h > \theta_l \left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}\right]^{1-\epsilon}. \tag{55}$$

Intuitively, under both the technology regimes, the self-sustaining growth path is driven by human capital accumulation when quality of schooling exceeds the threshold, $\theta > \frac{\mu}{\tau\epsilon}$. At the micro level, parents decide to have fewer number of children and invest more in their education. This follows from Lemma 1. At the macro level, this trade-off raises the rate of human capital accumulation, which encourages faster technological progress and, therefore, economic growth. Besides human capital, growth of world technology frontier is an additional driver of growth under the imitation regime via the catch-up effect.

Alternatively, when quality of schooling is less than the threshold, $\theta \leq \frac{\mu}{\tau\epsilon}$, parents do not invest in education of children and instead maximize fertility. In this case, the balanced growth path of the economy is driven only by population growth, which in turn, is determined by the fertility rate. Thus, the drivers of economic growth differ depending upon the level of quality of schooling. When quality of schooling surpasses the threshold level, economic growth is driven by human capital accumulation whereas it is driven by population growth when quality of schooling is less than the threshold.

Furthermore, a mere surpassing of the threshold level of quality schooling is not sufficient enough for an economy to experience a higher economic growth rate as compared to an economy with quality of schooling lower than the threshold level. Under the two technology regimes, quality of schooling should be high enough such that it leads to high enough investments in education of children such that the growth-stimulating effect overpowers the growth-impeding effect of quality of schooling by a larger magnitude. This, in turn, can only ensure that an economy with a higher quality of schooling ( that is, $\theta_h > \frac{\mu}{\tau\epsilon}$) experiences a higher economic growth as compared to an economy with a lower quality of schooling (that is, $\theta_l \leq \frac{\mu}{\tau\epsilon}$).

Otherwise, an economy with lower quality of schooling may experience a higher economic growth rate than an economy with higher quality of schooling for large enough values of child rearing costs, $\tau$ or for small enough value of intergenerational human capital spillovers, $\mu$ and returns to education, $\epsilon$ respectively. This follows directly from eq. (55). It can be observed that the expression, $\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}$ is increasing in $\tau$. In this particular case when the value of $\tau$ is sufficiently high, population growth rate can be a more effective driver of economic growth as the threshold value of quality of schooling for higher economic growth is so high that an economy may consider not investing an education of the future generation as a relatively more beneficial outcome . Similarly, it can be shown that:

$$\frac{\partial}{\partial\mu}\frac{(\tau\theta_h - \mu)}{\mu} = \frac{-\tau\theta_h}{\mu^2} < 0;$$
$$\frac{\partial}{\partial\epsilon}\frac{\epsilon}{1-\epsilon} = \frac{-1}{(1-\epsilon)^2} < 0.$$

This implies that the threshold value of quality of schooling for higher economic growth is decreasing in the value of $\mu$ and $\epsilon$ respectively. Thus, this threshold value of quality of schooling can be high enough for sufficiently small $\mu$ and $\epsilon$ such that population growth rate can be a more effective driver of economic growth and an economy may not invest in human capital of its future generation.

Next, we analyze the evolution of wage rate and rate of interest along the steady state. From eq. (20), under both the technology regimes, rate of interest is given by:

$$r_t = \alpha^2 \frac{Y_t}{K_t}.$$

A closer examination of the above expression for rate of interest reveals that it is constant along BGP, if $Y_t$ and $K_t$ grow at the same rate. From eqs. (49) and (50), we know that along the BGP:

$$g_K = g_Y.$$

This implies that rate of interest is constant along the BGP. Also, it is known from eq. (36) that wage rate is expressed as:

$$w_A = w_Y = \frac{(1-\alpha)Y_t}{H_t^Y}.$$

It can be observed from eq. (49) that under innovation regime:

$$(1 + g_Y) = [(1 + g_h)n]^{\frac{1-\phi+\lambda}{1-\phi}};$$

Also, from eq. (50) under imitation regime, we have:

$$(1 + g_Y) = [(1 + g_h)n]^{\frac{2-\phi+\lambda}{2-\phi}}(1 + g_{\bar{A}})^{\frac{1}{2-\phi}}.$$

Eqs. (36), (42), (49) and (50) together imply that, along BGP, wage rate under innovation regime grows at the rate:

$$g_w = [(1 + g_h)n]^{\frac{\lambda}{1-\phi}}; \tag{56}$$

And along BGP, wage rate under imitation regime grows at the rate:

$$g_w = [(1 + g_h)n]^{\frac{\lambda}{2-\phi}}(1 + g_{\bar{A}})^{\frac{1}{2-\phi}}, \tag{57}$$

where

$$[(1+g_h)n] = (1+g_H) = \begin{cases} \dfrac{\beta_2\theta\epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}}, & \text{if } \theta \le \dfrac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\theta\epsilon^{\epsilon}(1-\epsilon)^{1-\epsilon}}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^{1-\epsilon}}, & \text{otherwise} \end{cases}.$$

This yields the following proposition.

**Proposition 3.2** *When $\theta > \dfrac{\mu}{\tau\epsilon}$ or $\theta \le \dfrac{\mu}{\tau\epsilon}$:*

- *Under both the innovation and imitation regimes, rate of interest is constant along the balanced growth path.*
- *Under both the innovation and imitation regimes, the wage rate grows at the rate of technical progress given by eqs. (56) and (57) along the balanced growth path.*

Under both the technology regimes, the rate of interest is constant along the BGP. However, wage rate rises over time along BGP under both imitation and innovation regimes irrespective of quality of schooling being higher or lesser than the threshold. Growth rate of technology of the leader economy is an additional source of growth of wage rate under imitation regime.

Additionally, eqs. (49) and (50) suggest that technological progress and aggregate output are positively correlated with population growth. This implies that decline in population growth entails a decline in rate of technical progress as postulated by conventional R&D based growth models (Jones, 1995; Romer, 1990). This type of macro-level superficial examination misses the point that aggregate human capital accumulation and fertility rate are inversely related via quality-quantity trade-off at the family/household level as shown in Lemma 1 and 2. The investment in education increases and fertility rate falls simultaneously as the quality of schooling increases above the threshold. This quality-quantity trade-off implies that the effect of population growth on total factor productivity growth and GDP growth cannot be analyzed in isolation keeping human capital growth constant. This leads to the question: how does improvement in quality of schooling and returns to education affect total factor productivity growth and, therefore, economic growth by influencing fertility and education decisions?

This can be answered by carrying out comparative dynamics with respect to these parameters that has been done in the next subsection.

## 3.3 Comparative Dynamics Analyses of the Balanced Growth Path

### 3.3.1 Comparative Dynamics w.r.t Quality of Schooling, $\theta$

Since total factor productivity growth and economic growth depend on the rate of human capital accumulation under both the regimes of technological improvement, we first carry out comparative dynamics of the growth rate of aggregate human capital with respect to schooling quality. We take the derivative of the growth rate of aggregate human capital with respect to schooling quality, $\theta$, when it exceeds the threshold, that is, $\theta > \dfrac{\mu}{\tau\epsilon}$. Detailed derivations of eqs. (58) and (59) are provided in Appendix C.

$$\frac{\partial g_H}{\partial \theta} = \left[\frac{(1+g_h)\beta_2(\epsilon\tau\theta-\mu)(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}\right] > 0, \tag{58}$$

in view of $\epsilon < 1$. Thus, in the aggregate, the growth rate of technology increases in response to an increase in schooling quality which sustains economic growth in the long-run.

We next analyze the derivative of the growth rate of aggregate human capital with respect to schooling quality when it is less than the threshold, that is, $\theta \leq \frac{\mu}{\tau\epsilon}$. We get that,

$$\frac{\partial g_H}{\partial \theta} = \left[\frac{\beta_2 \epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}}\right] > 0. \tag{59}$$

Thus, it can be deduced that,

**Proposition 3.3** *The long-run rate of technical progress, $g_A$, and economic growth, $g_Y$, increase in response to an improvement in the quality of schooling on account of different channels, depending upon the quality of schooling, $\theta$:*

- *When $\theta > \frac{\mu}{\tau\epsilon}$, $g_A$ and $g_Y$ are increasing in $\theta$ due to higher rate of human capital accumulation under both the regimes of technological improvement.*
- *When $\theta \leq \frac{\mu}{\tau\epsilon}$, $g_A$ and $g_Y$ are increasing in $\theta$ due to higher population growth rate under both the regimes of technological improvement .*

The intuitive explanation for the impact of a change in quality of schooling on the long-run rate of technical progress and economic growth is as follows. When quality of schooling surpasses the threshold, it has two opposing effects on human capital accumulation. We know from Lemma 1 that an improvement in quality of schooling increases investment in the education of a child. This stimulates the accumulation of human capital which fosters technical progress leading to higher economic growth in the economy. This effect can be regarded as the growth-stimulating effect. The increase in education is also accompanied by a decline in fertility rate as the quality of education improves. This constitutes the growth-impeding effect that reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. Total factor productivity growth and economic growth will accelerate or decelerate depending upon the relative magnitude of the two effects. As shown by eq. (58), the growth-stimulating effect overpowers the growth-impeding effect of a change in quality of schooling when quality of schooling exceeds the threshold, that is, $\theta > \frac{\mu}{\tau\epsilon}$.

When quality of schooling is less than the threshold, parents do not educate their children and instead focus on having more children. In this particular case, there exist no growth-stimulating and growth-impeding effects of quality of schooling. In this case, the rate of technical progress and economic growth increase in response to an increase in the quality of schooling solely due to higher population growth, as parents focus on maximizing fertility when quality of schooling is less than the threshold.

Thus contingent upon the quality of schooling, there are two different channels at work which foster technical progress and economic growth. This result is similar to Hashimoto and Tabata (2016) finding about old-age survival probability and economic growth. They find that in economies in which old-age survival probability is sufficiently low, an increase in old-age survival probability motivates individuals to invest more in their own education, accelerating the accumulation of per capita human capital and, thereby, enhancing the long-run growth rate of the economy. However, in economies where old-age survival probability is sufficiently high, an increase in old age survival probability will lead to decline in population growth rates, thereby lowering the long-run growth rate of the economy.

We next consider the comparative dynamics of growth rate of per capita output, $g_y = Y_t/L_t$ with respect to $\theta$. At steady state, growth rate of per capita output under innovation regime is given by:

$$g_y = (1 + g_h)^{\frac{1-\phi+\lambda}{1-\phi}} n^{\frac{\lambda}{1-\phi}} - 1. \tag{60}$$

And under imitation regime, it is given by:

$$g_y = (1 + g_{\bar{A}})^{\frac{1}{2-\phi}} (1 + g_h)^{\frac{2-\phi+\lambda}{2-\phi}} n^{\frac{\lambda}{2-\phi}} - 1. \tag{61}$$

We first consider the case when, $\theta > \dfrac{\mu}{\tau\epsilon}$. Differentiating $g_y$ with respect to $\theta$ under innovation regime yields:[3]

$$\frac{\partial g_y}{\partial \theta} = \frac{1 + g_y}{1 - \phi} \left[ \frac{(1 - \phi + \lambda)\tau\epsilon}{\tau\theta - \mu} - \frac{\mu\lambda}{\theta(\tau\theta - \mu)} \right] \tag{62}$$

It can be observed that $\dfrac{\partial g_y}{\partial \theta} > 0$, if

$$\frac{(1 - \phi + \lambda)\tau\epsilon}{(\tau\theta - \mu)} > \frac{\mu\lambda}{\theta(\tau\theta - \mu)},$$

which simplifies to

$$\theta > \frac{\mu\lambda}{\tau\epsilon(1 - \phi + \lambda)}.$$

This holds true as $\theta > \dfrac{\mu}{\tau\epsilon}$, $\lambda < 1$ and $\phi < 1$. Thus,

$$\frac{\partial g_y}{\partial \theta} > 0. \tag{63}$$

Similarly, under imitation regime, differentiating per capita income growth rate with respect to $\theta$ yields:

$$\frac{\partial g_y}{\partial \theta} = \frac{1 + g_y}{2 - \phi} \left[ \frac{(2 - \phi + \lambda)\tau\epsilon}{\tau\theta - \mu} - \frac{\mu\lambda}{\theta(\tau\theta - \mu)} \right]. \tag{64}$$

Now, $\dfrac{\partial g_y}{\partial \theta} > 0$ if

$$\frac{(2 - \phi + \lambda)\tau\epsilon}{\tau\theta - \mu} > \frac{\mu\lambda}{\theta(\tau\theta - \mu)},$$

which, upon simplification, yields:

$$\theta > \frac{\mu\lambda}{\tau\epsilon(2 - \phi + \lambda)}.$$

[3] Detailed derivations of eqs. (62) and (64) are provided in Appendix D.

This holds true as $\theta > \dfrac{\mu}{\tau\epsilon}$, $\lambda < 1$ and $\phi < 1$. Therefore,

$$\frac{\partial g_y}{\partial \theta} > 0. \tag{65}$$

When $\theta \leq \dfrac{\mu}{\tau\epsilon}$, differentiating per capita income growth rate with respect to $\theta$ yields:[4]
Innovation regime:

$$\frac{\partial g_y}{\partial \theta} = \frac{(1+g_y)\lambda}{\theta(1-\phi)} > 0; \tag{66}$$

Imitation regime:

$$\frac{\partial g_y}{\partial \theta} = \frac{(1+g_y)\lambda}{\theta(2-\phi)} > 0, \tag{67}$$

as $\phi < 1$.

Next, we determine when the growth rate of per capita income will be higher for an economy with higher quality of schooling, $\theta > \dfrac{\mu}{\tau\epsilon}$, as compared to an economy with lower quality of schooling, $\theta \leq \dfrac{\mu}{\tau\epsilon}$, under the two technology regimes. We assume that when $\theta > \dfrac{\mu}{\tau\epsilon}$, quality of schooling is denoted by $\theta_h$ for that particular economy, whereas quality of schooling is denoted by $\theta_l$ for an economy with quality of schooling less than the threshold, $\theta \leq \dfrac{\mu}{\tau\epsilon}$. Derivations provided in Appendix E show that an economy with higher quality of schooling, $\theta_h$, will experience a higher per capita income growth rate as compared to an economy with a lower quality of schooling, $\theta_l$, if the following conditions hold under the individual technology regimes. Innovation regime:

$$\theta_h > \theta_l \left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}\right]^{\frac{\lambda - \epsilon(1-\phi+\lambda)}{\lambda}}; \tag{68}$$

Imitation regime:

$$\theta_h > \theta_l \left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}\right]^{\frac{\lambda - \epsilon(2-\phi+\lambda)}{\lambda}}. \tag{69}$$

Suppose, we postulate that,

$$\frac{\lambda - \epsilon(1-\phi+\lambda)}{\lambda} > 1, \quad \text{and} \quad \frac{\lambda - \epsilon(2-\phi+\lambda)}{\lambda} > 1.$$

which can be simplified to yield the following expressions:

$$\frac{-\epsilon(1-\phi+\lambda)}{\lambda} > 0, \quad \text{and} \quad \frac{-\epsilon(2-\phi+\lambda)}{\lambda} > 0.$$

[4] Detailed derivations of eqs. (66) and (67) are again provided in Appendix D.

This is a contradiction as $\phi < 1$, $\lambda < 1$ and $\epsilon < 1$. Therefore,

$$\frac{\lambda - \epsilon(1-\phi+\lambda)}{\lambda} < 1, \quad \text{and}$$
$$\frac{\lambda - \epsilon(2-\phi+\lambda)}{\lambda} < 1. \tag{70}$$

Further, it is known from eq. (54) that $\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)} > 1$. Therefore, eqs. (70) and (54) together imply that

$$\left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}\right]^{\frac{\lambda-\epsilon(1-\phi+\lambda)}{\lambda}} > 1, \quad \text{and}$$
$$\left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}\right]^{\frac{\lambda-\epsilon(2-\phi+\lambda)}{\lambda}} > 1. \tag{71}$$

Thus, from eqs. (63), (65) (66) and (67), it can be deduced that,

**Proposition 3.4**

- *For $\theta > \frac{\mu}{\tau\epsilon}$ and $\theta \leq \frac{\mu}{\tau\epsilon}$, $g_y$ is unambiguously increasing in $\theta$ along the balanced growth path under both the innovation and imitation regimes of technological improvement.*
- *An economy with higher quality of schooling, $\theta_h > \frac{\mu}{\tau\epsilon}$, will experience a higher per capita income growth rate as compared to an economy with lower quality of schooling, $\theta_l \leq \frac{\mu}{\tau\epsilon}$ if the quality of schooling, $\theta_h$, is sufficiently high captured by the following:*

$$\textit{Innovation regime:} \theta_h > \theta_l \left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}\right]^{\frac{\lambda-\epsilon(1-\phi+\lambda)}{\lambda}} > \theta_l;$$
$$\textit{Imitation regime:} \theta_h > \theta_l \left[\frac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}\right]^{\frac{\lambda-\epsilon(2-\phi+\lambda)}{\lambda}} > \theta_l.$$

The growth rate of per capita income along the BGP can be expressed as $(1 + g_y) = \frac{(1 + g_Y)}{n}$. It is known from Proposition 3.3 that the economic growth rate, $g_Y$, is increasing in $\theta$ as growth-stimulating effect dominates the growth-impeding effect of quality of schooling when it exceeds the threshold. Also, it is known from Lemma 1 that parents bear a lower number of children in response to an improvement in quality of schooling. Thus, the fertility rate or the population growth rate is decreasing in $\theta$. Consequently, the growth rate of per capita income rises as quality of schooling improves under both the technology regimes.

Further, it has been shown in Lemma 1 that parents maximize fertility and do not spend on education of their children when $\theta \leq \frac{\mu}{\tau\epsilon}$. It follows from Proposition 3.3 that the rate of technical progress and economic growth increase in response to an increase in the quality of schooling solely due to higher population growth. This higher population growth rate raises economic growth rate via two channels. First, higher aggregate human capital accumulation fosters technical progress and, therefore, economic growth. Secondly, higher aggregate human capital accumulation leads to higher economic growth directly as human capital enters as an input in the production function of the final good, $Y_t$. It can be observed from eqs. (60) and (61) that quality of schooling raises population growth rate by a lesser proportion as compared to the proportionate rise in growth rate of aggregate output under both the regimes of technical improvement. As a result, growth rate of per capita income is increasing in quality of schooling under both the technology regimes.

Also, a mere surpassing of the threshold level of quality schooling is not sufficient enough for an economy to experience a higher growth rate of per capita output as compared to an economy with quality of schooling lower than the threshold level. The intuitive explanation for this result is similar to the explanation given for the condition stated in Proposition 3.1. Under the two technology regimes, quality of schooling should be sufficiently high such that it leads to high enough investments in education of children, entailing that the growth-stimulating effect overpowers the growth-impeding effect of quality of schooling by a larger magnitude. Conversely, an economy with low quality of schooling may experience a higher per capita output growth rate than an economy with high quality of schooling for large enough values of child rearing costs, $\tau$, or for small enough value of inter-generational human capital spillovers, $\mu$ and returns to education, $\epsilon$ respectively. This follows directly from eq. (55). Next, the comparative dynamics with respect to returns to education, $\epsilon$, are discussed.

### 3.3.2 Comparative Dynamics w.r.t Returns to Education, $\epsilon$

We first analyze the derivative of the growth rate of aggregate human capital with respect to returns to education. We get that,[5]

$$\frac{\partial g_H}{\partial \epsilon} = (1 + g_H) log \left[ \frac{(\tau\theta - \mu)\epsilon}{1 - \epsilon} \right] > 0 \tag{72}$$

as $\dfrac{(\tau\theta - \mu)\epsilon}{1 - \epsilon} > 1$ from eq. (54). Similarly, when $\theta \leq \dfrac{\mu}{\tau\epsilon}$, the derivative of the growth rate of aggregate human capital with respect to returns to education yields:

$$\frac{\partial g_H}{\partial \epsilon} = (1 + g_H) \left[ \frac{1}{\epsilon} + log\mu \right] > 0. \tag{73}$$

Thus, we have,

**Proposition 3.5** *The long-run rate of technical progress, $g_A$, and aggregate output, $g_Y$, increase in response to an increase in returns to education, $\epsilon$, on account of different channels, depending upon the quality of schooling, $\theta$:*

- *When $\theta > \dfrac{\mu}{\tau\epsilon}$, $g_A$ and $g_Y$ are increasing in $\epsilon$ due to higher rate of human capital accumulation under both the regimes of technological improvement.*
- *When $\theta \leq \dfrac{\mu}{\tau\epsilon}$, $g_A$ and $g_Y$ are increasing in $\epsilon$ due to higher inter-generational human capital spillovers and higher population growth rate.*

The intuitive explanation for the impact of a change in returns to education on the long-run rate of technical progress and economic growth is as follows. We know from Lemma 2 that an increase in returns to education triggers a child quantity-quality trade-off at the micro level. The threshold value of quality of schooling, $\dfrac{\mu}{\tau\epsilon}$, is decreasing in the value of $\epsilon$. This implies that, ceteris paribus, this critical threshold value decreases as returns to schooling increase when quality of schooling

[5] Detailed derivations of eqs. (72) and (73) are provided in Appendix F.

exceeds the threshold. Therefore, parents educate their children and bear lower number of children in response to an increase in returns to education. Similar to the impact of quality of schooling, this micro level trade-off generates a growth-stimulating effect and a growth-impeding effect at the macro level. The growth-stimulating effect overpowers the growth-impeding effect of a change in returns to education when quality of schooling exceeds the threshold. Resultantly, an increase in returns to education yields higher rate of technical progress and, therefore, higher economic growth under both innovation and imitation regimes of technological improvement.

Alternatively, when quality of schooling is less than the threshold, parents do not educate their children and, instead, focus on having more children. Therefore, similar to the effect of quality of schooling, the rate of technical progress increases in response to an increase in the returns to education due to higher population growth. Additionally, it can be observed from eq. (9) that inter-generational human capital spillovers become more productive and spur growth rate of per capita human capital as returns to education increase. On the whole, when $\theta \leq \dfrac{\mu}{\tau\epsilon}$, an increase in returns to education yield higher rate of technical progress and, therefore, economic growth under both innovation and imitation regimes due to higher growth rate of population and higher inter-generational human capital spillovers.

Lastly, we examine the impact of a change in returns to education, $\epsilon$, on the growth rate of per capita income along the BGP under the two technology regimes. When $\theta > \dfrac{\mu}{\tau\epsilon}$, differentiating per capita income growth rate with respect to $\epsilon$ under the innovation and imitation regimes, yields:[6]
Innovation Regime:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{(1-\phi+\lambda)(1+g_y)}{1-\phi}\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] - \frac{(1+g_y)\lambda}{(1-\epsilon)(1-\phi)}. \tag{74}$$

Now, it can be observed that $\dfrac{\partial g_y}{\partial \epsilon} > 0$, if

$$(1-\phi+\lambda)\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] > \frac{\lambda}{1-\epsilon},$$

which simplifies to

$$1 + \frac{(1-\phi)}{\lambda} + \frac{(1-\epsilon)(1-\phi+\lambda)}{\lambda} log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 1. \tag{75}$$

The above expression can be re-expressed as:

$$\frac{(1-\phi)}{\lambda} + \frac{(1-\epsilon)(1-\phi+\lambda)}{\lambda} log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 0. \tag{76}$$

Similarly, under imitation regime, we have,

$$\frac{\partial g_y}{\partial \epsilon} = \frac{(2-\phi+\lambda)(1+g_y)}{2-\phi}\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] - \frac{(1+g_y)\lambda}{(1-\epsilon)(2-\phi)}. \tag{77}$$

Now, $\dfrac{\partial g_y}{\partial \epsilon} > 0$, if

$$(2-\phi+\lambda)\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] > \frac{\lambda}{1-\epsilon}, \tag{78}$$

[6] Detailed derivations of eqs. (74) and (77) are provided in Appendix G.

which simplifies to

$$\frac{(2-\phi)}{\lambda} + \frac{(1-\epsilon)(2-\phi+\lambda)}{\lambda} log \frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 0. \tag{79}$$

We know that $\theta > \dfrac{\mu}{\tau\epsilon}$, $\epsilon < 1$, $\lambda < 1$ and $\phi < 1$. Also, $\dfrac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 1$ from eq. (54). Therefore, the expressions in eqs. (76) and (79) hold true. We next, consider the case when $\theta \leq \dfrac{\mu}{\tau\epsilon}$. Differentiating per capita income growth rate with respect to $\epsilon$ yields the following.[7]
Innovation regime:

$$\frac{\partial g_y}{\partial \epsilon} = (1+g_y)\left[1+\frac{\lambda}{1-\phi}\right] log\mu + \frac{\lambda(1+g_y)}{(1-\phi)\epsilon} > 0, \quad \text{and} \tag{80}$$

Imitation regime:

$$\frac{\partial g_y}{\partial \epsilon} = (1+g_y)\left[1+\frac{\lambda}{2-\phi}\right] log\mu + \frac{\lambda(1+g_y)}{(2-\phi)\epsilon} > 0 \tag{81}$$

since $\phi < 1$ and $\mu \geq 1$. An examination of these derivatives yields the following results.

**Proposition 3.6** *When $\theta > \dfrac{\mu}{\tau\epsilon}$ and $\theta \leq \dfrac{\mu}{\tau\epsilon}$ , $g_y$ is unambiguously increasing in $\epsilon$ along the balanced growth path under both the regimes of technological improvement.*

Intuitively, we know that per capita income is given by $y_t = Y_t/L_t$. The growth rate of per capita income along the BGP can be expressed as $(1+g_y) = \dfrac{(1+g_Y)}{n}$. It is known from Proposition 3.5 that the economic growth rate, $g_Y$, is increasing in $\epsilon$. Also, it is known from Lemma 2 that parents bear lower number of children in response to a rise in returns to education. Thus, the population growth rate is decreasing in returns to education, $\epsilon$. Consequently, the growth rate of per capita income rises as returns to education increase under both the technology regimes.

Furthermore, it follows from Proposition 3.5 that an increase in returns to education yields higher rate of technical progress and, therefore, economic growth under both innovation and imitation regimes due to higher growth rate of population and higher inter-generational human capital spillovers when $\theta \leq \dfrac{\mu}{\tau\epsilon}$. Similar to the effect of quality of schooling, it can be observed from eqs. (60) and (61) that returns to education raise population growth rate by a lesser proportion as compared to the proportionate rise in growth rate of aggregate output under both the technology regimes. Therefore, growth rate of per capita income rises as returns to education rises.

This completes the characterization of the balanced growth path of our decentralized economy.

# 4 Discussion

This paper formulates an analytical framework to analyze the impact of quality of schooling on economic growth of an economy under imitation and innovation regimes.We characterize two types of economies. The first type is an innovation economy where technological improvements occur by innovating upon the local technology frontier. The second type is an imitation economy where technological progress occurs by imitating existing foreign technologies. We find that the quality

[7]Detailed derivations of eqs. (80) and (81) are again provided in Appendix G.

of schooling triggers a child quantity-quality trade-off at the micro level when quality of schooling surpasses an endogenously determined threshold under both the technology regimes. When quality of schooling surpasses the threshold, parents invest in the education of their children and bear lesser number of children. However, parents focus on maximizing fertility and do not educate their children when quality of schooling is less than the threshold. This micro-level trade-off generates two types of effects on economic growth at the macro level - a growth-stimulating effect and a growth-impeding effect. Our results show that the former effect dominates over latter only when the quality of schooling is higher than the threshold, and the economy is on a self-sustaining growth path. Alternatively, when the quality of schooling is less than the threshold, parents do not educate their children and focus, instead on maximizing fertility. Higher fertility rate leads to higher population growth, which propels economic growth rate under both innovation and imitation regimes.

Furthermore, a mere surpassing of the threshold level of quality schooling is not sufficient enough for an economy to experience a higher growth rate of per capita output as compared to an economy with quality of schooling lower than the threshold level. Under the two technology regimes, quality of schooling should be high enough such that it leads to high enough investments in education of children, entailing that the growth-stimulating effect overpowers the growth-impeding effect of quality of schooling by a larger magnitude. This, in turn, can only ensure that an economy with quality of schooling above the threshold exhibits a higher per capita income growth as compared to an economy with quality of schooling less than the threshold.

# A Solution to Household's Optimization Exercise

The utility function is described as follows:
Maximize

$$
\begin{aligned}
u_t = \log c_{1,t} + \beta_1 \log c_{2,t+1} + \beta_2 \log(h_{t+1} n_t) \\
\text{subject to} \\
w_t h_t(1 - \tau n_t) = c_{1,t} + s_t + e_t(w_t h_t) n_t \\
c_{2,t+1} = (1 + r_{t+1}) s_t \\
h_{t+1} = (\mu + \theta e_t)^\epsilon h_t, \quad \epsilon < 1
\end{aligned}
$$

After substituting for $c_{2,t+1}$ and $h_{t+1}$, the langragean for this problem is formulated as :

$$
\begin{aligned}
L = \log c_{1,t} + \beta_1 \log[(1 + r_{t+1}) s_t] + \beta_2 \log n_t + \beta_2 \epsilon \log(\mu + \theta e_t) + \beta_2 \epsilon \log h_t \\
+ \psi[w_t h_t(1 - \tau n_t) - c_{1,t} - s_t - e_t n_t (w_t h_t)]
\end{aligned}
$$

The choice variables are $c_{1,t}$, $s_t$,$e_t$ and $n_t$.The first-order conditions are:

$$\frac{\partial L}{\partial c_{1,t}} = 0 \Leftrightarrow \frac{1}{c_{1,t}} - \psi = 0 \Leftrightarrow c_{1,t} = \frac{1}{\psi}. \tag{A.1}$$

$$\frac{\partial L}{\partial s_t} = 0 \Leftrightarrow \frac{\beta_1}{s_t} - \psi = 0 \Leftrightarrow s_t = \frac{\beta_1}{\psi}. \tag{A.2}$$

$$\frac{\partial L}{\partial n_t} = 0 \Leftrightarrow \frac{\beta_2}{n_t} - \psi \tau w_t h_t - \psi e_t w_t h_t = 0 \Leftrightarrow \frac{\beta_2}{n_t} = \psi[\tau + e_t] w_t h_t \Leftrightarrow n_t = \frac{\beta_2}{\psi[\tau + e_t] w_t h_t}. \tag{A.3}$$

$$\frac{\partial L}{\partial e_t} = 0 \Leftrightarrow \frac{\beta_2 \epsilon \theta}{\mu + \theta e_t} - \psi n_t w_t h_t = 0 \Leftrightarrow n_t = \frac{\beta_2 \epsilon \theta}{\psi[\mu + \theta e_t] w_t h_t}. \tag{A.4}$$

From eqs. (A.3) and (A.4), the l.h.s can be equated to yield:

$$\mu + \theta e_t = \epsilon \theta [\tau + e_t] \Leftrightarrow \mu - \epsilon \theta \tau = e_t \theta [\epsilon - 1]$$

$$e_t = \frac{\mu - \epsilon \theta \tau}{\theta(\epsilon - 1)} = \frac{\epsilon \theta \tau - \mu}{\theta(1 - \epsilon)}$$

Hence, we have:

$$e_t = \begin{cases} 0, & \text{if } \theta \leq \frac{\mu}{\tau \epsilon}; \\ \frac{\tau \theta \epsilon - \mu}{\theta(1 - \epsilon)}, & \text{otherwise.} \end{cases} \tag{A.5}$$

Next, we know that the budget constraint is given by:

$$w_t h_t(1 - \tau n_t) = c_{1,t} + s_t + e_t(w_t h_t) n_t.$$

From eq. (A.3), $e_t n_t(w_t h_t)$ can be expressed as:

$$e_t n_t(w_t h_t) = \frac{\beta_2}{\psi} - \tau n_t w_t h_t. \tag{A.6}$$

Substituting from eqs. (A.1), (A.2) and (A.6), the budget constraint can be expressed as:

$$w_t h_t - \tau n_t w_t h_t = \frac{1}{\psi} + \frac{\beta_1}{\psi} + \frac{\beta_2}{\psi} - \tau n_t w_t h_t$$

which on simplifying leads to:

$$\psi = \frac{1 + \beta_1 + \beta_2}{w_t h_t} \tag{A.7}$$

whose substitution into eqs. (A.1) and (A.2) yields:

$$c_{1,t} = \frac{w_t h_t}{1 + \beta_1 + \beta_2};$$
$$s_t = \frac{\beta_1 w_t h_t}{1 + \beta_1 + \beta_2},$$

Substituting for $e_t$ from eq. (A.5) and for $\psi$ from eq. (A.7) in eq. (A.3), yields:

$$n_t = \begin{cases} \dfrac{\beta_2 \epsilon \theta}{(1 + \beta_1 + \beta_2)\mu}, & \text{if } \theta \leq \dfrac{\mu}{\tau \epsilon}; \\ \dfrac{\beta_2 \theta (1 - \epsilon)}{(1 + \beta_1 + \beta_2)(\tau \theta - \mu)}, & \text{otherwise.} \end{cases}$$

This completes the solution to the utility maximization exercise of households.

# B Derivation of Eq. (31)

The research arbitrage argument goes as follows. Suppose an intermediate firm faces two options. First, it can make an investment of $p_t^A$ in physical capital and earn the market rate of interest, $r_t$. Alternatively, it can purchase a patent, earn profits in one period and, then, sell the patent. In equilibrium, the rate of return from both these investments should be the same. That is,

$$r_t p_t^A = \pi_t + (p_{t+1}^A - p_t^A).$$

The l.h.s of this equation is the interest earned from investing in physical capital. The r.h.s is the sum of the profits earned and the capital gain/loss resulting from the change in price of patents over time. Rearranging the above equation yields the following research arbitrage condition:

$$r_t = \frac{\pi_t}{p_t^A} + \left[\frac{p_{t+1}^A - p_t^A}{p_t^A}\right]. \tag{B.1}$$

This equation states that R&D sector charges a price of blueprint, $p_t^A$, such that intermediate input producers are indifferent between purchasing a blueprint to produce an intermediate variety and not producing the intermediate variety at all. The dividend rate, given by, $\dfrac{\pi_t}{p_t^A}$. and the capital gain/loss, $\dfrac{p_{t+1}^A - p_t^A}{p_t^A}$, equal the market rate of return on investment, $r_t$. We know from Proposition

3.2 that $r_t$ is constant along BGP. This implies l.h.s of eq. (B.1) is constant. For r.h.s to be constant, following condition should hold true.

$$\frac{\pi_{t+1}}{\pi_t} = \frac{p^A_{t+1}}{p^A_t}.$$

From eq. (18), we have:

$$\pi_t = \alpha(1-\alpha)\frac{Y_t}{A_t}.$$

This implies:

$$\frac{\pi_{t+1}}{\pi_t} = \frac{\frac{Y_{t+1}}{Y_t}}{\frac{A_{t+1}}{A_t}} \tag{B.2}$$

Now, it is known from eq. (48) that along BGP,

$$g_Y = (1+g_A)(1+g_H) - 1.$$

Inserting in eq. (B.2), we deduce:

$$\frac{\pi_{t+1}}{\pi_t} = \frac{H_{t+1}}{H_t} = (1+g_H).$$

Thus, eq. (B.1) holds true if

$$\frac{\pi_{t+1}}{\pi_t} = \frac{p^A_{t+1}}{p^A_t} = (1+g_H).$$

Inserting in eq. (B.1) and solving for $p^A_t$, we get

$$p^A_t = \frac{\pi_t}{r_t - g_H}.$$

This completes derivation of eq. (31).

# C Derivations of Eqs. (58) and (59)

We know that $(1+g_H) = (1+g_h).n$. Differentiating both the sides w.r.t $\theta$ yields:

$$\frac{\partial g_H}{\partial \theta} = (1+g_h)\frac{\partial n}{\partial \theta} + n\frac{\partial g_h}{\partial \theta}. \tag{C.1}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, we know from Lemma 1 that:

$$\frac{\partial n_t}{\partial \theta} = -\frac{\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2},$$

and it is given $(1+g_h) = \left[\frac{\epsilon(\tau\theta-\mu)}{(1-\epsilon)}\right]^{\epsilon}$ from eq (9). Differentiating $g_h$ w.r.t $\theta$, we get that:

$$\frac{\partial g_h}{\partial \theta} = \left[\frac{\epsilon(\tau\theta-\epsilon)}{1-\epsilon}\right]^{\epsilon} . \frac{\epsilon\tau}{\tau\theta-\mu} = \frac{(1+g_h)\epsilon\tau}{\tau\theta-\mu}. \tag{C.2}$$

Substituting this into eq. (C.1), we get that:

$$\frac{\partial g_H}{\partial \theta} = (1+g_H)\frac{-\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} + (1+g_h)\frac{\epsilon\tau}{\tau\theta-\mu} * n,$$

Substituting for $n$ from eq. (8),

$$\begin{aligned} &= (1+g_h)\left[\frac{\epsilon\tau\beta_2\theta(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} - \frac{\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}\right] \\ &= (1+g_h)[\epsilon\tau\theta-\mu]\frac{\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}. \end{aligned}$$

Alternatively, when $\theta \leq \frac{\mu}{\tau\epsilon}$, we know from Lemma 1 that:

$$\frac{\partial n_t}{\partial \theta} = \frac{\beta_2\epsilon}{(1+\beta_1+\beta_2)\mu},$$

and it is given $(1+g_h) = \mu^{\epsilon}$ from eq. (9). Differentiating $g_h$ w.r.t $\theta$ yields:

$$\frac{\partial g_h}{\partial \theta} = 0. \tag{C.3}$$

Substituting this into eq. (C.1), we get that:

$$\frac{\partial g_H}{\partial \theta} = (1+g_H)\frac{\beta_2\epsilon}{(1+\beta_1+\beta_2)\mu} = \frac{\beta_2\epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}}$$

This completes derivation of eqs. (58) and (59).

# D Derivations of Eqs. (62), (64), (66) and (67)

The growth rate of per capita income can be expressed as:

$$(1+g_y) = \frac{(1+g_Y)}{n}. \tag{D.1}$$

Under innovation regime, substituting for $(1+g_Y)$ from eq. (49) and simplifying, we get:

$$(1+g_y) = (1+g_h)^{\frac{1-\phi+\lambda}{1-\phi}} n^{\frac{\lambda}{1-\phi}}. \tag{D.2}$$

Taking log on both sides,

$$log(1+g_y) = \frac{1-\phi+\lambda}{1-\phi}log(1+g_h) + \frac{\lambda}{1-\phi}logn.$$

Differentiating w.r.t $\theta$, yields

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial \theta} = \frac{1-\phi+\lambda}{(1+g_h)1-\phi}\frac{\partial g_h}{\partial \theta} + \frac{\lambda}{1-\phi(n)}\frac{\partial n}{\partial \theta}, \tag{D.3}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial n}{\partial \theta}$ from Lemma 1 and $\frac{\partial g_h}{\partial \theta}$ from eq. (C.2) yields:

$$\frac{\partial g_y}{\partial \theta} = \frac{1+g_y}{1-\phi}\left[\frac{(1-\phi+\lambda)\tau\epsilon}{\tau\theta-\mu} - \frac{\mu\lambda}{\theta(\tau\theta-\mu)}\right].$$

Next, we consider the case when $\theta \leq \frac{\mu}{\tau\epsilon}$.
Substituting for n from eq. (8), $\frac{\partial g_h}{\partial \theta}$ from eq.(C.3) and $\frac{\partial n}{\partial \theta}$ from Lemma 1 in eq. (D4), we deduce:

$$\frac{\partial g_y}{\partial \theta} = \left[\frac{\lambda(1+g_y)}{\theta(1-\phi)}\right].$$

We now, consider the imitation regime.
Substituting for $(1+g_Y)$ from eq. (50) and simplifying, we get:

$$(1+g_y) = (1+g_{\bar{A}})^{\frac{1}{2-\phi}}(1+g_h)^{\frac{2-\phi+\lambda}{2-\phi}} n^{\frac{\lambda}{2-\phi}}. \tag{D.4}$$

Taking log on both sides,

$$log(1+g_y) = \frac{1}{(2-\phi)}log(1+g_{\bar{A}}) + \frac{2-\phi+\lambda}{(2-\phi)}log(1+g_h) + \frac{\lambda}{2-\phi}logn; \tag{D.5}$$

Differentiating w.r.t $\theta$, we get:

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial \theta} = \frac{2-\phi+\lambda}{2-\phi(1+g_h)}\frac{\partial g_h}{\partial \theta} + \frac{\lambda}{2-\phi(n)}\frac{\partial n}{\partial \theta}. \tag{D.6}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial n}{\partial \theta}$ from Lemma 1 and $\frac{\partial g_h}{\partial \theta}$ from eq. (C.2), we have:

$$\frac{\partial g_y}{\partial \theta} = \frac{1+g_y}{2-\phi}\left[\frac{(2-\phi+\lambda)\tau\epsilon}{\tau\theta-\mu} - \frac{\mu\lambda}{\theta(\tau\theta-\mu)}\right].$$

We next, consider the case when $\theta \leq \frac{\mu}{\tau\epsilon}$.
When $\theta \leq \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial g_h}{\partial \theta}$ from eq.(C.3) and $\frac{\partial n}{\partial \theta}$ from Lemma 1 in eq. (D.6) yields:

$$\frac{\partial g_y}{\partial \theta} = \left[\frac{\lambda(1+g_y)}{\theta(2-\phi)}\right].$$

This completes derivations of eqs. (62), (64), (66) and (67).

# E Derivations of Eqs. (68) and (69)

We derive the conditions when an economy with high quality of schooling $\left(\theta > \dfrac{\mu}{\tau\epsilon}\right)$ exhibits a higher per capita output growth rate as compared to an economy with lower quality of schooling $\left(\theta \leq \dfrac{\mu}{\tau\epsilon}\right)$. We assume that when $\theta > \dfrac{\mu}{\tau\epsilon}$, quality of schooling is denoted by $\theta_h$ for that particular economy whereas quality of schooling is denoted by $\theta_l$ for an economy with quality of schooling less than the threshold $\left(\theta \leq \dfrac{\mu}{\tau\epsilon}\right)$. An economy with higher schooling quality ($\theta_h$) will grow at a higher rate as compared to an economy with lower quality of schooling ($\theta_l$) when the following condition holds true under both the technology regimes:

$$g_{y,\theta_h} > g_{y,\theta_l} \tag{E.1}$$

We first, derive the condition for innovation economy. Substituting from eq. (60), we have

$$(1+g_{h,\theta_h})^{\frac{1-\phi+\lambda}{1-\phi}}(n_{\theta_h})^{\frac{\lambda}{1-\phi}} > (1+g_{h,\theta_l})^{\frac{1-\phi+\lambda}{1-\phi}}(n_{\theta_l})^{\frac{\lambda}{1-\phi}}, \tag{E.2}$$

where $g_{h,\theta_h}$ and $g_{h,\theta_l}$ are per capita human capital accumulation rates when $\theta > \dfrac{\mu}{\tau\epsilon}$ and $\theta \leq \dfrac{\mu}{\tau\epsilon}$ respectively and $n_{\theta_h}$ and $n_{\theta_l}$ are the fertility rates when $\theta > \dfrac{\mu}{\tau\epsilon}$ and $\theta \leq \dfrac{\mu}{\tau\epsilon}$ respectively. Substituting from eqs. (9) and (8) for $(1+g_{h,\theta_h})$, $(1+g_{h,\theta_l})$, $n_{\theta_h}$ and $n_{\theta_l}$ to get:

$$\left[\frac{(\tau\theta_h-\mu)\epsilon}{1-\epsilon}\right]^{\epsilon(1-\phi+\lambda} \left[\frac{\theta_h(1-\epsilon)}{(\tau\theta_h-\mu)}\right]^{\lambda} > \mu^{\epsilon(1-\phi+\lambda}\left[\frac{\epsilon\theta_l}{\mu}\right]^{\lambda}$$

which on simplification, yields:

$$\theta_h^{\lambda} > \left[\frac{\mu(1-\epsilon)}{\epsilon(\tau\theta_h-\mu)}\right]^{\epsilon(1-\phi+\lambda} \left[\frac{\epsilon\theta_l(\tau\theta_h-\mu)}{\mu(1-\epsilon)}\right]^{\lambda},$$

This can be further simplified to:

$$\theta_h > \theta_l \left[\frac{\epsilon(\tau\theta_h-\mu)}{\mu(1-\epsilon)}\right]^{\dfrac{\lambda-\epsilon(1-\phi+\lambda)}{\lambda}}.$$

Similarly, we derive the condition for imitation economy. Substituting from eq. (61), we get

$$(1+g_{\bar{A}})^{\frac{1}{2-\phi}}(1+g_{h,\theta_h})^{\frac{2-\phi+\lambda}{2-\phi}}(n_{\theta_h})^{\frac{\lambda}{2-\phi}} > (1+g_{\bar{A}})^{\frac{1}{2-\phi}}(1+g_{h,\theta_l})^{\frac{2-\phi+\lambda}{1-\phi}}(n_{\theta_l})^{\frac{\lambda}{2-\phi}}.$$

Substituting from eqs. (9) and (8) for $(1+g_{h,\theta_h})$, $(1+g_{h,\theta_l})$, $n_{\theta_h}$ and $n_{\theta_l}$, we derive:

$$\left[\frac{(\tau\theta_h-\mu)\epsilon}{1-\epsilon}\right]^{\epsilon(2-\phi+\lambda} \left[\frac{\theta_h(1-\epsilon)}{(\tau\theta_h-\mu)}\right]^{\lambda} > \mu^{\epsilon(2-\phi+\lambda}\left[\frac{\epsilon\theta_l}{\mu}\right]^{\lambda},$$

which simplifies to:

$$\theta_h^{\lambda} > \left[\frac{\mu(1-\epsilon)}{\epsilon(\tau\theta_h-\mu)}\right]^{\epsilon(2-\phi+\lambda} \left[\frac{\epsilon\theta_l(\tau\theta_h-\mu)}{\mu(1-\epsilon)}\right]^{\lambda},$$

This can be re-expressed as:

$$\theta_h > \theta_l \left[\frac{\epsilon(\tau\theta_h-\mu)}{\mu(1-\epsilon)}\right]^{\dfrac{\lambda-\epsilon(2-\phi+\lambda)}{\lambda}}.$$

This completes derivations of eqs. (68) and (69).

# F Derivations of Eqs. (72) and (73)

We know that:

$$(1+g_H) = (1+g_h).n$$

Differentiating both the sides w.r.t $\epsilon$, we get that:

$$\frac{\partial g_H}{\partial \epsilon} = (1+g_h)\frac{\partial n}{\partial \epsilon} + n\frac{\partial g_h}{.}\partial\epsilon \tag{F.1}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, we know from the interior solution of eq. (9) that:

$$1+g_h = \left[\frac{\epsilon(\tau\theta-\mu)}{(1-\epsilon)}\right]^{\epsilon}.$$

Taking log on both sides,

$$log(1+g_h) = \epsilon log\epsilon + \epsilon log(\tau\theta-\mu) - \epsilon log(1-\epsilon),$$

Differentiating w.r.t $\epsilon$, we get the following expression:

$$\frac{1}{1+g_h}\frac{\partial g_h}{\partial \epsilon} = 1 + log\epsilon + log(\tau\theta-\mu) + \frac{\epsilon}{1-\epsilon} - log(1-\epsilon),$$

which on simplification reduces to:

$$\frac{\partial g_h}{\partial \epsilon} = (1+g_h)\left[log\frac{1}{(1-\epsilon)} + \frac{\epsilon(\tau\theta-\mu)}{1-\epsilon}\right]. \tag{F.2}$$

Also, from Lemma 2, we have $\frac{\partial n_t}{\partial \epsilon} = \frac{-\beta_2\theta}{(1+\beta_1+\beta_2)(\tau\theta-\mu)}$. Substituting for $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 and $\frac{\partial g_h}{\partial \epsilon}$ from eq. (F.2) into eq. (F.1), we derive that:

$$\frac{\partial g_H}{\partial \epsilon} = \frac{-\beta_2\theta(1+g_h)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)} + (1+g_h)n\left[\frac{1}{(1-\epsilon)} + log\frac{\epsilon(\tau\theta-\mu)}{1-\epsilon}\right]$$

Substituting for $\frac{\beta_2\theta}{(1+\beta_1+\beta_2)(\tau\theta-\mu)}$ from eq. (8), yields:

$$\begin{aligned} &= (1+g_h)\left[n\left[\frac{1}{(1-\epsilon)} + log\frac{\epsilon(\tau\theta-\mu)}{1-\epsilon}\right] - \frac{n}{1-\epsilon}\right] \\ &= (1+g_H)log\left[\frac{\epsilon(\tau\theta-\mu)}{1-\epsilon}\right]. \end{aligned} \tag{F.3}$$

We next derive the expression for $\frac{\partial g_H}{\partial \epsilon}$ when $\theta \leq \frac{\mu}{\tau\epsilon}$.
When $\theta \leq \frac{\mu}{\tau\epsilon}$, it is known from eq. (9) that:

$$(1+g_h) = \mu^{\epsilon}.$$

Taking log on both sides,

$$log(1+g_h) = \epsilon log\mu,$$

Differentiating $g_h$ w.r.t $\epsilon$ yields:

$$\frac{\partial g_h}{\partial \epsilon} = (1+g_h)log\mu. \tag{F.4}$$

Further, we derive know from Lemma 2 that $\dfrac{\partial n_t}{\partial \epsilon} = \dfrac{\beta_2\theta}{(1+\beta_1+\beta_2)\mu}$. Substituting for $\dfrac{\partial n}{\partial \epsilon}$ and $\frac{\partial g_h}{\partial \epsilon}$ from eq. (F.4) into eq. (F.1), we deduce that:

$$\frac{\partial g_H}{\partial \epsilon} = \frac{\beta_2\theta(1+g_h)}{(1+\beta_1+\beta_2)\mu} + (1+g_h)nlog\mu.$$

Substituting for $\dfrac{\beta_2\theta}{(1+\beta_1+\beta_2)\mu)}$ from eq. (8) when $\theta \leq \frac{\mu}{\tau\epsilon}$, we derive that:

$$= (1+g_H)\left[\frac{1}{\epsilon} + log\mu\right]. \tag{F.5}$$

This completes the derivations of eqs. (72) and (73).

# G Derivations of Eqs. (74), (77), (80) and (81)

It is known from eq. (D.2) that the growth rate of per capita income under innovation regime is given by:

$$(1+g_y) = (1+g_h)^{\frac{1-\phi+\lambda}{1-\phi}} n^{\frac{\lambda}{1-\phi}}. \tag{G.1}$$

Taking log on both sides,

$$log(1+g_y) = \frac{1-\phi+\lambda}{1-\phi} log(1+g_h) + \frac{\lambda}{1-\phi} logn.$$

Differentiating w.r.t $\epsilon$ yields:

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial \epsilon} = \frac{1-\phi+\lambda}{(1+g_h)1-\phi}\frac{\partial g_h}{\partial \epsilon} + \frac{\lambda}{1-\phi(n)}\frac{\partial n}{\partial \epsilon}. \tag{G.2}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\dfrac{\partial n}{\partial \epsilon}$ from Lemma 2 and $\dfrac{1}{1+g_h}\dfrac{\partial g_h}{\partial \epsilon}$ from eq. (F.2), we get:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{(1-\phi+\lambda)(1+g_y)}{1-\phi}\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] - \frac{(1+g_y)\lambda}{(1-\epsilon)(1-\phi)}. \tag{G.3}$$

We next, consider the case where $\theta \leq \frac{\mu}{\tau\epsilon}$.
Substituting for n from eq. (8), $\dfrac{\partial g_h}{\partial \epsilon}$ from eq. (F.4) and $\dfrac{\partial n}{\partial \epsilon}$ from Lemma 2 in eq. (G.3) yields:

$$\frac{\partial g_y}{\partial \epsilon} = (1+g_y)log\mu\left[1+\frac{\lambda}{1-\phi}\right] + \frac{\lambda(1+g_y)}{(1-\phi)\epsilon}.$$

We next, derive the expression for $\frac{\partial g_y}{\partial \epsilon}$ under imitation regime.
We know from eq. (D.5) that:

$$log(1+g_y) = \frac{1}{(2-\phi)}log(1+g_{\bar{A}}) + \frac{2-\phi+\lambda}{(2-\phi)}log(1+g_h) + \frac{\lambda}{2-\phi}logn,$$

Differentiating w.r.t $\epsilon$, we get:

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial \epsilon} = \frac{2-\phi+\lambda}{2-\phi(1+g_h)}\frac{\partial g_h}{\partial \epsilon} + \frac{\lambda}{2-\phi(n)}\frac{\partial n}{\partial \epsilon}. \tag{G.4}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 and $\frac{\partial g_h}{\partial \epsilon}$ from eq. (F.2) yields:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{(2-\phi+\lambda)(1+g_y)}{2-\phi}\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] - \frac{(1+g_y)\lambda}{(1-\epsilon)(2-\phi)}.$$

Alternatively, when $\theta \leq \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial g_h}{\partial \epsilon}$ from eq. (F.4) and $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 in eq. (G.4) yields:

$$\frac{\partial g_y}{\partial \epsilon} = (1+g_y)log\mu\left[1+\frac{\lambda}{2-\phi}\right] + \frac{\lambda(1+g_y)}{(2-\phi)\epsilon}.$$

This completes derivations of eqs. (74), (77), (80) and (81).

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