International Capital Flows in Club of Convergence

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Abstract

We explain U-shape pattern of international capital inflows by one multi-country OLG economy and one cross-section data sample. The theory proves that capital inflows are decreasing on distance to frontier, which is measured by ratio of domestic productivity level over United States' level. The evidences not only confirm the theory but also reveal that growth is decreasing on distance to frontier for club of convergence but increasing for club of unconvergence. Therefore, Neo-Classical growth model’s implication, that capital inflows are positively correlated to growth, applies for club of convergence. However, Allocation puzzle, that capital inflows are negatively correlated to growth, works for club of unconvergence. The turning point of U-shape pattern is the productivity growth rate at world technology frontier.

Keywords: International Capital Flows, Productivity Growth, Relative Convergence.

JEL Classifications: F15, F36, F43.

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1 Introduction.

There are mixed evidences on pattern of international capital flows. Alfaro, Kalemli-Ozcan, and Volosovych (2014) confirm that net total capital inflows are increasing on growth, as implied by Neo-Classical growth model. However, Gourinchas and Jeanne (2013) establish that one economy growing faster tends to receive less inflows of capital, one phenomena labeled as Allocation puzzle. These results motivate one refreshed literature on the impact of growth on capital flows.

Figure (1.0.1) uncovers one clue on mechanism underlying the pattern of capital flows, by comparing the dependence of capital inflows on productivity growth across two groups of economies. Club of convergence includes economies with long-run growth rates being higher or at least equal to the rate of United States. And club of unconvergence gathers the rest of economies. On Graph A, for club of convergence, one economy growing faster receive more capital inflows: Neo-Classical growth model works. However, on Graph B for club of unconvergence, an increase of growth results on less capital inflows: Allocation puzzle holds. On short, the figure illustrates that the pattern of capital flows is different across club of convergence and club of unconvergence.

The divergent of international capital flows pattern across two clubs of economies suggests that the factor that accounts for convergence can be potential to explain the dependence of capital inflows on growth. Indeed, with exogenous growth, the impact of growth on capital inflows can be positive or negative. However, with endogenous growth, both growth and capital inflows can be dependent on one common factor. On that case, the correlation between growth and capital inflows can be both positive and negative, depending on the range of growth rate. In brief, one potential explanation for the dependence of capital inflows on growth can be proposed by addressing the endogeneity of growth.

Our paper employs the distance to world technology frontier to explain the mixed evidences on patterns of capital flows across countries. Indeed, one economy approaching the productivity

![Graph A: Club of Convergence](image1.png)  
![Graph B: Club of Unconvergence](image2.png)  

Figure 1.0.1: Productivity Growth and International Capital Flows
level at world frontier tends to have huger savings stock, which results on decrease of net total capital inflows. Our evidences prove that, on club of convergence, being closer to frontier also reduces the productivity growth while on club of unconvergence, it raises the growth. Therefore, the capital inflows are positively correlated to growth on club of convergence but negatively correlated to growth on club of unconvergence. In short, for one economy, the pattern of capital flows (increasing or decreasing) depends on whether it belongs to club of convergence or unconvergence.

The paper is closely related to the recent literature on the international capital flows (Lucas (1990), Gourinchas and Rey (2014), Bacchetta, Philippe, and Kenza Benhima (2015)). Song, Storesletten and Zilibotti (2011) shows that on one transition economy, the reallocation of labor from low-productivity to high-productivity firms improves overall productivity growth and reduce the inflows of capital by accelerating the domestic gross savings. The combination of high growth and capital outflows is also explained by the interaction of demographic changes and credit constraint on Coeurdacier, Guibaud and Jin (2015). Indeed, the middle-aged agents on economy with tight credit constraint save more than the same agents on economy with relaxed constraint. When the fraction of middle-aged agents over population raises, the domestic gross savings goes up, which results on the outflows of capital. However, these papers only focus on the decreasing pattern of capital flows on productivity growth, leaving aside the increasing pattern. Our analysis, based on the distance to world technology frontier, concludes that the economy’s pattern of capital flows is decreasing or increasing depend on its position on club of unconvergence or club of convergence, respectively.

On Caballero, Farhi and Gourinchas (2008), the difference on capitalization rate (i.e, the ratio of financial assets over output) across countries determines the patterns of capital flows. Indeed, since households buy foreign assets in seeking the storage of wealth, only fast-growing economy with high capitalization rate can have both high growth and inflows of capital. Otherwise, with low capitalization rate, it will have high growth but outflows of capital. However, with exogenous productivity growth, there is no role for one factor that can affect both growth and capital inflows. Therefore, their paper does not capture the U-shape on the pattern of capital flows: on data, it is decreasing then increasing. On our paper, the U-shape pattern can arise since the dependence of productivity growth on distance to world frontier is positive on club of unconvergence but negative on club of convergence.

The U-shape pattern of capital flows can exist by the impact of institution quality (IQ) of domestic credit market on growth on Matsuyama (2014). Indeed, the IQ determines the trade-off between agency cost and productivity across heterogenous investment projects. Therefore, the effect of IQ can result on the U-shaped responses of investment, which in turn regulates the capital inflows. However, the paper does not characterize the turning point on the non-linear pattern of capital flows, which is clearly stated on our paper. Indeed, we reveal that the productivity growth rate of the world technology frontier is the turning point on the U-shape pattern of capital flows.

Our paper also belongs to several strands of theory relating growth, convergence and distance to frontier. Aghion and Howitt (2006) show that, on the club of convergence, approaching the
frontier reduces the growth rate. The reason is that, moving closer to world frontier, the economy needs to move from imitation to innovation procedure with lower probability of success. However, their paper does not explore the dependence of growth on distance to frontier on club of unconvergence. Our empirical evidences complement their theory by showing that the productivity growth is increasing on distance to frontier on the club of unconvergence. The result implies that, for club of unconvergence, the benefit from imitation of advanced technology at frontier can be large enough so that the growth rate accelerates for one economy being closer to frontier.

The impact of distance to frontier on convergence is analyzed on Acemoglu, Aghion and Zilibotti (2006). Indeed, for one economy moves closer to frontier, it changes from the investment-based strategy with large investment but little selection of high-skill managers to the innovation-based strategy with less investment but better selection of managers. And only the economy with innovation-based strategy can converge to the world technology frontier. However, within framework of closed economy, their paper does not explore the implication of distance to frontier on capital inflows. On our paper, we show that approaching the world frontier raises the domestic savings, then, reduces the inflows of capital.

The paper processes as follows. Section (2) builds up the model to analyze the role of distance to frontier on shaping the pattern of international capital flows. Next, section (3) presents the empirical evidences for both club of convergence and club of unconvergence. Finally, section (4) concludes and is followed by Appendix.

2 Theory.

2.1 Economy.

The world economy includes many large open countries. One country \((j)\) is populated with the overlapping generations of households who live for two periods: young and old. All countries use the same technology to produce one homogeneous good, which is used for consumption and investment, and is traded freely and costlessly. They also have the same structure and parameter values for the preferences and production technologies. The capital is free mobile but labor is immobile across countries. And firms are subject to changes in the domestic productivity levels and labor forces.

2.1.1 Production And Investment.

For one economy \((j)\), the output at time period \((t)\) is produced with constant-return-to-scale technology, using the capital \((K_t)\), labor force \((L_t)\) and the total factor productivity \((A_t)\).

\[
Y_t = K_t^\alpha (A_t L_t)\alpha
\]

where \((0 < \alpha < 1)\), and \(A_t\) is country-specific productivity with exogenous growth rate:

\[
A_{t+1} = (1 + g_{t+1})A_t
\]
The next-period capital stock is augmented by the current-period investment, plus the capital stock discounted by the depreciation rate \((\delta)\). The law of capital accumulation is as following:

\[
K_{t+1} = I_t + (1 - \delta)K_t
\]

(1)

One representative firm hires the labor and capital to maximize the profit.

\[
\Pi_t = Y_t - R_tK_t - w_tL_t
\]

The perfect competition on the market for factors of production implies that each factor earns its marginal product. In term of capital-effective-labor ratio \((k_t \equiv \frac{K_t}{A_tL_t})\), the interest rate \((R_t)\) and wage rate \((w_t)\) are as following:

\[
R_t = \alpha k_t^{(\alpha-1)}
\]

(2)

\[
w_t = (1 - \alpha)A_t k_t^\alpha
\]

(3)

### 2.1.2 Consumption And Savings.

For one economy \(j\), there are \(N_t\) new-born agents at the time period \((t)\). The population is assumed to growth with the exogenous rate: \(N_{t+1} = (1 + g_{t+1})N_t\). Each agent born at \((t)\) supplies one unit of labor to earn the competitive wage rate. Let \((c^y_t)\) and \((c^o_{t+1})\) denote her consumption at youth and old respectively. The lifetime utility of one agent born at \((t)\) is:

\[
U_t = u(c^y_t) + \beta u(c^o_{t+1})
\]

with the standard preferences \(u(c) = \ln(c)\). The discount factor \((\beta)\) satisfies: \(0 < \beta < 1\).

Let \((s^y_t)\) denote the saving at the end of period \((t)\) by one new-born agent. The sequence of budget constraints is as following:

\[
c^y_t + s^y_t = w_t(1 - \tau)
\]

\[
c^o_{t+1} = R_{t+1}s^y_t
\]

When young, an agent works for the wage rate \((w_t)\) and covers the income taxation \((\tau)\). She allocates the income between consumption and saving. At old, she receives the interest rate on savings and consume all income.

With the log utility function, the saving is a constant fraction of disposable income, the income after taxation duty fulfillment.

\[
s^y_t = \frac{\beta}{(1 + \beta)}(1 - \tau)w_t
\]

(4)

### 2.1.3 R&D Expenditure And Productivity Growth.

Let \((Z_t)\) denote the public expenditure on the Research and Development (R&D) by the government in country \((j)\). We assume that the balanced public budget holds for all time period. Therefore, the income tax revenue equals to the R&D expenditure:

\[
\tau w_t = Z_t
\]

(5)
Let \((\bar{A}_t)\) denotes the productivity level at the world technology frontier: \(\bar{A}_t \equiv \max_j (A^j_t)\). As on Aghion and Howitt (2008), the productivity growth rate is assumed to depend on the target level of productivity, which is, in our model, given by the technology level at world frontier.

\[ g^A_t = \phi\left(\frac{Z_t}{\bar{A}_t}\right) \]

whereby the function \(\phi(x)\) is continuous, increasing, concave and satisfies: \(\lim_{x \to \infty} < \infty\).

**Assumption 2.1.1.** For country \(j\) on club of convergence, the R&D expenditure and productivity level grow with the same rate by the world technology frontier.

\[
\begin{align*}
\bar{A}_t &= (1 + g^A_t)\bar{A}_{t-1} \\
A^j_t &= (1 + g^A_t)A^j_{t-1} \\
Z^j_t &= (1 + g^A_t)Z_{t-1}
\end{align*}
\]

The ratio of R&D expenditure over world technology frontier \((Z_t/\bar{A}_t)\) is constant over time across countries on the club of convergence, since they are assumed to have the same growth rate. Moreover, the distance to world technology frontier is different across countries.

\[
\frac{Z^j_t}{\bar{A}_t} = \frac{Z_0}{\bar{A}_0}, \forall j \\
\frac{A^j_t}{\bar{A}_t} \neq \frac{A^h_t}{\bar{A}_t}, \forall j \neq h
\]

By using (3) to rewrite the public budget balance (5), we have:

\[
\tau k^g_t = \frac{1}{(1 - \alpha)} \frac{Z_t \bar{A}_t}{\bar{A}_t \bar{A}_t}
\]

The tax rate is a function of the distance to frontier \((\frac{A^j_t}{\bar{A}_t})\). Therefore, given the same ratio of R&D expenditure over \((\bar{A})\), the difference on distance to frontier would determine the difference on taxation and on disposable income. As one result, the savings are different across countries, which motivates the cross-border capital flows.

**2.2 Equilibrium.**

**2.2.1 Autarky.**

At autarky, the aggregate savings by young agents must be equal to the investment.

\[
S^y_t \equiv N_t s^y_t = I_t
\]

By aggregating the saving rate (4) and taking into account the wage rate (3), the share of labor income over output \((w_t N_t = (1 - \alpha)Y_t)\), the gross savings per output ratio is as following:

\[
S^y_t = \frac{\beta}{(1 + \beta)(1 - \tau)(1 - \alpha)}Y_t
\]
Replacing the last equation on the law of capital accumulation (1), the long-run equilibrium capital-effective-labor ratio is determined by the equation:

$$(1 + g_A^{t+1})(1 + g_N^{t+1})k_{t+1} = \frac{\beta}{(1 + \beta)}(1 - \alpha)(1 - \tau)k_t^\alpha + (1 - \delta)k_t$$

The autarky steady state with perfect foresight is characterized on the following Theorem.

**Theorem 2.2.1.** There exists an unique, stable steady state for an autarky economy. At steady state, $\frac{\partial R}{\partial (1 + g_A)} > 0; \frac{\partial R}{\partial (1 + g_N)} > 0$.

**Proof.** Appendix

At steady state ($k_{t+1} = k_t = k$), the interest rate given by (2) as:

$$R = \frac{\alpha}{(1 - \alpha)} \frac{(1 + \beta)(1 + g_A)(1 + g_N) - 1 + \delta}{(1 - \tau)}$$

(8)

At the long-run equilibrium with constant growth rate, the ratio of R&D expenditure over productivity level is constant: $Z_t/A_t = Z/A$. Then, by employing the implicit function theorem for public budget (6) at steady state, the tax rate is decreasing on the distance to frontier:

$$\tau k(\tau)^\alpha = \frac{1}{(1 - \alpha)} \frac{Z A}{\bar{A} A} \Rightarrow \frac{\partial \tau}{\partial (A/\bar{A})} < 0$$

(9)

The dependence of income taxation on the distance to frontier plays one crucial role on the theoretical model. Indeed, through the tax rate, the movement toward the world frontier reduces the tax rate, and raises the disposable income. Therefore, with the constant marginal saving rate, the aggregate savings stock goes up, which pushes down the autarky interest rate. In brief, one economy being closer to world frontier would have lower interest rate. We summarize the result on the following Lemma.

**Lemma 2.2.1.** At autarky steady state, one economy $j$ being closer to the world frontier would have huger domestic gross savings stock and lower interest rate.

$$\frac{\partial (S^n/Y)}{\partial (A/A)} > 0; \frac{\partial R}{\partial (A/A)} < 0$$

**Proof.** By (7) and (9), we have:

$$\frac{\partial (S^n/Y)}{\partial (A/A)} = \frac{\partial (S^n/Y)}{\partial \tau} \frac{\partial \tau}{\partial (A/A)} > 0$$

By (8), the autarky interest rate is decreasing on the tax rate: $\frac{\partial R}{\partial \tau} > 0$. Then,

$$\frac{\partial R}{\partial (A/A)} = \frac{\partial R}{\partial \tau} \frac{\partial \tau}{\partial (A/A)} < 0$$
2.2.2 Integration.

At integration, the world capital market clearing condition requires that the aggregate savings by young agents must be equal to aggregate investment.

\[ \Sigma_j S^y_{t} = \Sigma_j I^j_t \]

The law of capital accumulation at integration is as following:

\[ \Sigma_j K^j_{t+1} = \Sigma_j S^j_t + (1 - \delta) \Sigma_j K^j_t \]

The free mobility of capital implies that the capital-effective-labor ratio is equalized across countries:

\[ k^j_t = k^w_t, \forall j. \]

And the integration capital accumulation level at long-run satisfies the equation:

\[ (1 + g^{A}_{t+1})(1 + g^{N}_{t+1}) k^w_{t+1} = \frac{\beta}{(1 + \beta)}(1 - \alpha)(1 - \bar{\tau})(k^w_{t})^\alpha + (1 - \delta)k^w_t \]

whereby, the world average tax rate is \( \bar{\tau} \equiv \Sigma_j \lambda^j \tau^j \), with the constant share \( \lambda^j \equiv \frac{A^j_t N^j_t}{\Sigma A^j_t N^j_t} \), given the symmetric growth rates across countries implied by Assumption (2.1.1).

**Theorem 2.2.2.** There exists an unique, stable steady state for an integration economy. At steady state, \( \max_{(A^j/\bar{A})} R^j(A^j/\bar{A}) < R^w < \min_{(A^j/\bar{A})} R^j(A^j/\bar{A}) \).

**Proof.** Appendix

At world integration steady state, the world interest rate depends on the economic growth and average tax rate:

\[ R^w = \frac{\alpha}{(1 - \alpha)} \left(1 + \beta\right) \left(1 + g^A\right) \left(1 + g^N\right) - 1 + \delta \]

\[ \left(1 - \bar{\tau}\right) \]

(10)

The convergence of autarky interest rate to one common world one is the motivation for the cross-border capital flows. In particular, the capital flows from the economy with low autarky interest rate to the one with high autarky interest rate. The flow of capital would end only when the capital-effective-labor ratio and the corresponding interest rate are equalized across countries. On the next section, we would show that the difference on the distance to frontier shapes the pattern of international capital flows through its impact on the autarky interest rate.

2.3 International Capital Flows.

The aggregate savings stock of the young (i.e, \( S^y_t \equiv N_t s^y_t \)) has two components\(^1\), the net foreign assets \( B_{t+1} \) and the capital stock that will be used in the production in the next period \( K_{t+1} \).

\[ S^y_t = B_{t+1} + K_{t+1} \]

\(^1\)Obstfeld and Rogoff (1996), Chapter 3.
By dividing both side of previous equation by the next-period output, we have:

\[ \frac{B_{t+1}}{Y_{t+1}} = \frac{S_t^w}{Y_t} \frac{Y_t}{Y_{t+1}} \frac{K_{t+1}}{Y_{t+1}} \]

Replacing the gross savings stock (7), the growth rate of output (i.e, \( Y_{t+1}/Y_t = (1+g^A_{t+1})(1+g^N_{t+1}) \)), and the capital income share (\( R^w_{t+1}K_{t+1} = \alpha Y_{t+1} \)), we get the ratio of net stock of foreign assets over output as:

\[ \frac{B_{t+1}}{Y_{t+1}} = \beta \left( 1 - \alpha \right) \left( 1 - \tau \right) \frac{1}{(1+\beta)(1+g^A_{t+1})(1+g^N_{t+1})} - \frac{\alpha}{R^w_{t+1}} \]

At steady state with constant growth rate of productivity and labor force (\( g^A_t = g^A, g^N_t = g^N, \forall t \)), the net foreign assets at the beginning of period is also constant.

\[ \frac{B}{Y} = \frac{\beta}{(1+\beta)} \left( 1 - \alpha \right) \left( 1 - \tau \right) \frac{1}{(1+g^A)(1+g^N)} \]

Replacing the world interest rate (10) into (11), the steady-state net foreign assets with full depreciation (\( \delta = 1 \)) are as following:

\[ \frac{B}{Y} = \frac{\beta}{(1+\beta)} \left( 1 - \alpha \right) \left( \tau - \tau(A/\bar{A}) \right) \frac{1}{(1+g^A)(1+g^N)} \]  

(12)

The current account is the changes in the net stock of foreign assets, i.e, \( CA_t = B_{t+1} - B_t \).

\[ \frac{CA_t}{Y_t} = \frac{B_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t} - \frac{B_t}{Y_t} \]  

(13)

Evaluating the current account (13) at steady state, then using the net foreign assets (12), we have the net total capital inflows, which is measured by negative value of current account, as following:

\[ \frac{-CA}{Y} = (g^A + g^N + g^A g^N) \left( \frac{-B}{Y} \right) = \frac{\beta}{(1+\beta)} \frac{(1 - \alpha)(g^A + g^N + g^A g^N)}{(1+g^A)(1+g^N)} (\tau(A/\bar{A}) - \bar{\tau}) \]  

(14)

Since a smaller distance to world frontier (i.e, \( A/\bar{A} \) raises) lowers the income tax rate, it would reduce the net total capital inflows. On other words, approaching the world frontier reduces the inflows of capital. The result is summarized on the following Proposition.

**Proposition 2.3.1.** At integration steady state, the net total capital inflows are decreasing on the distance to world frontier:

\[ \frac{\partial \left( -CA/Y \right)}{\partial(A/\bar{A})} < 0. \]

**Proof.** By (9) and (14): \( \frac{\partial \tau}{\partial(A/\bar{A})} < 0; \frac{\partial(-CA/Y)}{\partial \tau} > 0. \)

Since the income tax rate is a function of the distance to frontier, we have:

\[ \frac{\partial(-CA/Y)}{\partial(A/\bar{A})} = \frac{\partial(-CA/Y)}{\partial \tau} \frac{\partial \tau}{\partial(A/\bar{A})} < 0. \]
The dependence of net total capital inflows on distance to world frontier relies on the domestic savings. Indeed, for one economy moves closer to world technology frontier (i.e, higher $A'/\bar{A}$), the disposable income increases, which, in turn, raises the savings stock. With the given investment demand, the autarky interest rate goes down, and the economy makes the foreign investment to the rest of world in seeking a higher rate of return. In short, for one economy approaches the world frontier, the increase of savings reduces the inflows of capital.

One direct implication is that the distance to frontier can shape the dependence pattern of capital flows on productivity growth. For club of convergence, Aghion and Howitt (2008) shows that closer to world frontier one economy is, lower productivity growth rate it experiences. Indeed, moving to the world frontier requires the economy to change the type of innovation from imitation with high probability of success to invention with low probability of success. Therefore, the growth rate of economy goes down when it approach the frontier. In combination with our result on the negative impact of distance to frontier on the capital inflows, a lower distance to frontier reduces both the productivity growth and capital inflows. Therefore, the net total capital inflows are positively correlated to the growth rate on the club of convergence.

3 Evidences.

After description of dataset, we investigate the dependence of net total capital inflows on productivity growth to test the main theory implied by Proposition (2.3.1). Then, we extend the empirical analysis to account for the divergence of capital flows pattern across club of convergence and unconvergence, illustrated by Figure (1.0.1).

3.1 Descriptive Statistics.

The dataset is one cross-section sample of about 170 observations, covering both developing and advanced economies. Each variable is the value averaged over 1980-2013 so that the analysis can focus on the long-run equilibrium.

The net total capital inflows ($anegCA2y$) are measured by the averaged negative value of current account per Gross Domestic Product (GDP). Scaling by GDP rules out the country-size effect. The data source is the updated and extended version of dataset of net private and public capital flows constructed by Alfaro, Kalemli-Ozcan, and Volosovych (2014). The database is one panel sample for many economies, spanning from 1980 to 2013. Moreover, it also incorporates the data from various sources, from International Financial Statistics, and the external wealth of nations dataset by Lane and Milesi-Ferretti (2007).

The productivity growth rate ($aGDPpcgrowth$) is the averaged growth rate of GDP per capita, which is the GDP at constant national 2005 prices, divided by population. As implied by the Neoclassical growth model (Solow (1956)), at the long-run equilibrium, the growth rate of income per capita is equal to the growth rate of total factor productivity. Moreover, using the GDP
per capita growth rate can overcome the controversy on the measurement of productivity level, which arising from the exact estimation of capital stock at the initial time, or from the dependence of productivity on capital and labor employed by firms. Recently, Alfaró, Kalemli-Ozcan, and Volosovych (2014) also use the growth rate of GDP per capita to investigate the dependence pattern of net total capital inflows on growth. For computation, we explore the GDP from Pen World Table 8.1 (2015) and the population from World Development Indicators.

The savings are the averaged gross savings (including both private and public savings) scaled by the Gross National Income (GNI) to preclude the country-size effect. Indeed, the measurement is just in line with the theoretical model on which the balanced public budget implies the coincidence of private and aggregate savings. Furthermore, GNI is employed to illustrate more exactly the aggregate income since it accounts for the net foreign income gained on the international investment position.

The distance to world frontier (adistance) for one economy is the averaged ratio of domestic productivity level over the value of United States, which is considered as the world technology frontier. An increase of distance to frontier means the domestic productivity level is closer to the world frontier. The data is from Pen World Table 8.1.

For the classification of club of convergence and club of unconvergence, following Aghion and Howitt (2008), we compute the ratio of productivity growth rate of one economy over the rate of United States. If the ratio is greater or equal to 1, the country belongs to club of convergence. Otherwise, it is on club of unconvergence. The computation is consistent to the definition of relative convergence, by which one economy converges successfully to world technology frontier if it attains the same growth rate by the economy at frontier.

Table (3.1.1) shows the descriptive statistics for the data sample. The net total capital inflows have the mean at 4.78% and standard deviation at 7.5%. In comparison with the capital inflows, the productivity growth has both lower mean (1.8%) and deviation (1.85%) while the gross savings have much higher mean (20%) and deviation (10.5%). For the distance to world frontier, it also exhibits quite large standard deviation at 0.27 from the lowest value at 0.15 to the highest one at 1.5. In brief, the data set offers rich variation for exploring the mechanism underlying the pattern of international capital flows on the club of convergence.

Table 3.1.1: Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Total Capital Inflows (%) (anegCA2y)</td>
<td>175</td>
<td>4.781286</td>
<td>7.523386</td>
<td>-13.82</td>
<td>38.93</td>
</tr>
<tr>
<td>Productivity Growth Rate (%) (aGDPpcgrowth)</td>
<td>160</td>
<td>1.881766</td>
<td>1.852483</td>
<td>-1.409</td>
<td>10.206</td>
</tr>
<tr>
<td>Gross Savings (%) (aS2GNI)</td>
<td>172</td>
<td>20.03227</td>
<td>10.52318</td>
<td>-32.236</td>
<td>49.752</td>
</tr>
<tr>
<td>Distance to World Frontier (USA=1) (adistance)</td>
<td>107</td>
<td>.6580185</td>
<td>.2755747</td>
<td>.15389</td>
<td>1.542014</td>
</tr>
</tbody>
</table>
3.2 Distance to World Frontier, Gross Savings and Capital Flows.

Figure (3.2.1) illustrates the dependence of gross savings on the distance to world technology frontier. On club of convergence, Graph A shows that for one economy moves toward the world frontier, its gross savings accelerates. The same pattern applies for the club of unconvergence on Graph B. Since the net total capital inflows are the gap between domestic investment and savings, the figure suggests the capital inflows can decline for one economy is closer to world frontier.

Figure 3.2.1: Distance to World Frontier and Gross Savings

Graph A: Club of Convergence

Graph B: Club of Unconvergence

Table 3.2.1: Regression Results of Gross Savings ($aS_2GNI$) on Distance to Frontier: Full Sample (col.1), Club of Convergence (col.2) and Club of Unconvergence (col.3)

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to World Frontier</td>
<td>12.70***</td>
<td>7.212*</td>
<td>13.71***</td>
</tr>
<tr>
<td>($adistance$)</td>
<td>(2.354)</td>
<td>(3.925)</td>
<td>(3.092)</td>
</tr>
<tr>
<td>Constant</td>
<td>13.19***</td>
<td>18.92***</td>
<td>10.51***</td>
</tr>
<tr>
<td></td>
<td>(1.684)</td>
<td>(2.920)</td>
<td>(1.981)</td>
</tr>
<tr>
<td>Observations</td>
<td>106</td>
<td>54</td>
<td>50</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.219</td>
<td>0.061</td>
<td>0.291</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1
Table (3.2.1) reports the regression results of gross savings on the distance to world frontier. The evidences confirm the mechanism on Lemma (2.2.1) that the domestic savings goes up for one economy approaches the frontier. Indeed, on Column 1 for the full sample, 10% increase of distance to frontier raises the savings by 12.7%. On Column 2 for club of convergence, the effect is lower with the coefficient of 7.2%. The same pattern holds for club of unconvergence on Column 3 with the magnitude at 13.71%. In brief, the table lays the ground for the positive impact of distance to world frontier on the aggregate savings on both the club of convergence and club of unconvergence.

Figure 3.2.2: Distance to World Frontier and International Capital Flows

Graph A: Club of Convergence  
Graph B: Club of Unconvergence

Table 3.2.2: Regression Results of Net Total Capital Inflows ($anegCA2y$) on Distance to Frontier: Full Sample (col.1), Club of Convergence (col.2) and Club of Unconvergence (col.3)

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to World Frontier ($adistance$)</td>
<td>-11.13***</td>
<td>-12.16***</td>
<td>-10.44***</td>
</tr>
<tr>
<td></td>
<td>(1.427)</td>
<td>(2.437)</td>
<td>(2.050)</td>
</tr>
<tr>
<td>Constant</td>
<td>9.755***</td>
<td>10.05***</td>
<td>9.732***</td>
</tr>
<tr>
<td></td>
<td>(1.021)</td>
<td>(1.813)</td>
<td>(1.313)</td>
</tr>
<tr>
<td>Observations</td>
<td>106</td>
<td>54</td>
<td>50</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.369</td>
<td>0.324</td>
<td>0.351</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1
Figure (3.2.2) shows the dependence pattern of net total capital inflows on the distance to world frontier. On Graph A for club of convergence, one country being closer to frontier tends to make the foreign investment, or experience the outflows of capital. The same pattern applies for the club of unconvergence on Graph B. In short, the figure reveals that the tendency to make foreign investment for one economy steps toward the world technology frontier.

Table (3.2.2) presents the regression results of capital flows on the distance to frontier. On Column 1, for full sample, the net total capital inflows reduce by about 11.13% for each increase of 10% on the distance to frontier. On Column 2, for club of convergence, the net total capital inflows reacts more strongly to the increase of distance to frontier, with the coefficient of 12.16%. The same pattern applies for the club of unconvergence on Column 3 with the coefficient of 10.44%. In brief, the evidences confirms the theoretical result on Proposition (2.3.1) that the net total capital inflows are decreasing on the distance to frontier.

In sum, the empirical analysis uncovers the crucial role of distance to frontier on shaping the pattern of international capital flows. When one economy is closer to the world technology frontier, its savings goes up, which leads to the surge on the foreign investment or the reduction of net total capital inflows. Moreover, by data investigation, we find out that the pattern emerges not only on the club of convergence but also on the club of unconvergence. On next section, we will employ this new finding to explain the dependence pattern of capital inflows on productivity growth.

### 3.3 Distance to World Frontier, Productivity Growth and Capital Flows.

Figure (3.3.1) illustrates the dependence of productivity growth on the distance to world frontier. On Graph A, for one economy moves toward the world frontier, its growth rate slows down. On Graph B, for the club of unconvergence, the growth rate tends to increase for one economy approaches the world frontier. The increasing path of growth on club of unconvergence complements to the implication of endogenous growth theory on club of convergence by Aghion and Howitt (2008). Indeed, for convergence to world frontier, one economy needs to change from imitation to invention of new technology. Since the probability of success for invention is lower, the frequency of innovation goes down and the growth rate decelerates. Moreover, the positive dependence of growth rate on the distance to frontier is one new finding, which is only revealed by the data but not yet explained by the theory. In brief, the dependence pattern of productivity growth on the distance to world frontier is different across the club of convergence and club of unconvergence.

Table (3.3.1) reports the regression results of productivity growth on the distance to world frontier. On Column 1, for the full sample, the growth rate is one quadratic function of distance to frontier: it is increasing then decreasing. On Column 2, for club of convergence, the growth rate decreases by 1.48% for an increase of 10% on the distance to frontier. On Column 3, for club of unconvergence, the growth rate raises by 1.6% for 10% increase on the distance to frontier. In short, the evidences shows that the productivity growth is decreasing on the distance to frontier for the club of convergence but increasing for the club of unconvergence.
The distance to frontier can affect the dependence pattern of capital inflows on the productivity growth. For club of convergence, one economy being closer to the world technology frontier experiences both a lower productivity growth rate and less inflows of capital. Therefore, the net total capital inflows are positively correlated to the productivity growth. However, for club of unconvergence, approaching the world frontier raises the productivity growth while reduces the inflows of capital. Therefore, the net total capital inflows are negatively correlated to the productivity growth. On short, the impact of distance to frontier on both capital inflows and productivity
growth results on the non-linearity on the pattern of international capital flows.

Table (3.3.2) shows the regression results of net total capital inflows on the productivity growth. On Column 1, for full sample, the dependence pattern of net total capital inflows on productivity growth is non-linear: it follows quadratic function. The result is similar to the finding by Hung (2017) on the non-linear pattern of international capital flows. However, while the author divides the sample into two equalized subsamples, we compare the pattern of capital flows across club of convergence and club of unconvergence. In details, on Column 2, for club of convergence, the capital inflows are increasing on growth: 10% increase on growth raises the inflows of capital by 1.15%. However, on Column 3, for club of unconvergence, they are decreasing on growth; 10% increase on growth reduces the inflows of capital by 1.61%. In brief, the divergence on coefficients of productivity growth across two clubs of economies justifies the role of distance to frontier on shaping the non-linear pattern of international capital flows.

Table 3.3.2: Regression Results of Net Total Capital Inflows (anegCA2y) on Productivity Growth: Full Sample (col.1), Club of Convergence (col.2) and Club of Unconvergence (col.3)

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity Growth (aGDPpcgrowth)</td>
<td>-1.598***</td>
<td>1.151**</td>
<td>-1.612**</td>
</tr>
<tr>
<td>(aGDPpcgrowth2)</td>
<td>(0.584)</td>
<td>(0.442)</td>
<td>(0.714)</td>
</tr>
<tr>
<td>Squared Value of (aGDPpcgrowth)</td>
<td>0.269***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(aGDPpcgrowth2)</td>
<td>(0.0806)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>5.734***</td>
<td>0.495</td>
<td>4.461***</td>
</tr>
<tr>
<td></td>
<td>(0.890)</td>
<td>(1.529)</td>
<td>(0.729)</td>
</tr>
<tr>
<td>Observations</td>
<td>158</td>
<td>87</td>
<td>50</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.067</td>
<td>0.074</td>
<td>0.096</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Figure (3.3.2) summarizes the role of distance to world frontier on determining the pattern of international capital flows for the full sample of about 160 economies. On Panel C, the productivity growth is increasing on the distance to frontier for the club of unconvergence, then decreasing for the club of convergence. On Panel B, the capital inflows are decreasing on the distance to frontier for both clubs of economies. Indeed, by both theory and evidences, we prove that approaching the world frontier raises the domestic savings, which, in turn, reduces the inflows of capital. The combination of two panels explain the non-linearity of capital flows illustrated by Panel A. In particular, on club of unconvergence, the capital inflows are negatively correlated to growth since the distance to frontier raises the growth (left half of Panel C) but reduces the capital inflows (left
half of Panel B). However, on club of convergence, they are positively correlated to growth since the distance to frontier reduces both growth (right half of Panel C) and capital inflows (right half of Panel B). In sum, the decreasing pattern of net total capital inflows on the distance to frontier is the key to explain the non-linear pattern of capital flows.

Panel A: Capital Flows and Productivity Growth

Panel B: Capital Flows and Distance to World Frontier

Panel C: Productivity Growth and Distance to World Frontier

Figure 3.3.2: Pattern of International Capital Flows
4 Conclusion.

We construct one multi-country OLG economy to establish that the net total capital inflows are decreasing on the distance to frontier. The empirical evidences on one cross-section sample of about 170 economies support the theory. Moreover, by dividing the full sample on club of convergence and club of unconvergence, our analysis can account for non-linear pattern of international capital flows. In details, on club of convergence: approaching the world frontier reduces both capital inflows and productivity growth. Therefore, the capital inflows are positively correlated to growth: Neo-Classical growth model works. On club of unconvergence: approaching the world frontier reduces the capital inflows but raises the growth. Therefore, the capital inflows are negatively correlate to growth: Allocation puzzle applies.

For the policy implication, one economy needs to be on club of convergence so that it will receives more foreign investment for an improvement of productivity growth. In particular, the required growth rate is greater or at least equal to 1.68%, the value at the world technology frontier (United States) over 1980-2013. Otherwise, on the club of unconvergence, one economy tends to receive less capital inflows for a higher growth rate.

On the future research avenue, we can extend the paper to characterize the investment demand by firms. One potential way can be to take into account the selection of high-skill entrepreneurs as on Acemoglu, Aghion and Zilibotti (2006).

References


A Appendix.

A.1 Definition of Equilibrium.

Definition A.1.1. The autarky temporal equilibrium.

Given the value determined by the last period \( s_{t-1}^y, I_{t-1} = N_{t-1} s_{t-1}^y \) and the expected interest rate \( R_{t+1}^e \), the temporary equilibrium of time \( t \) is a list of prices (interest rate \( R_t \), wage rate \( w_t \)) and individual allocations \( (c_t^y, c_t^o, s_t^y) \), aggregate variables \( (K_t, L_t, Y_t, k_t, I_t, Z_t) \) such that:

1. Profit is maximized and Utility is maximized subjected to budget constraints.

2. Market clearing conditions are satisfied:

   a. Labor market: \( L_t = N_t \)

   b. Capital market: \( I_t = S_t^y = N_t s_t^y \)
Proof. The law of capital accumulation at the world integration economy:

\[ c \text{ Good market: } N_t c_t^c + N_t s_t^c + N_{t-1} c_t^c + Z_t = Y_t \]

Definition A.1.2. The autarky inter-temporal equilibrium.
Given an initial capital stock \( k_0 = K_0/(A_{t-1}N_{t-1}) \), an autarky inter-temporal equilibrium with perfect foresight is a sequence of temporary equilibria that satisfies for all \( t > 0 \) the conditions that:

\[
R_{t+1}^c = R_{t+1} = \alpha k_{t+1}^{(α-1)}
\]
\[
(1 + g^A)(1 + g^N)k_{t+1} = N_t s_t^g(w(k_t), τ, R_{t+1}) + (1 - δ)k_t
\]

Definition A.1.3. The integration temporal equilibrium.
At each country \( j \), given the value determined by the last period \( (s_t^{y,j}, Σ_jI_t^j = Σ_jN_t^j s_t^{y,j}) \) and the expected interest rate \( (R_{t+1}^c) \), the temporary equilibrium of time \( t \) is a list of world interest rate \( (R_t^w) \), domestic wage rate \( (w_t^j) \), and individual allocations \( (c_t^{y,j}, c_t^{o,j}, s_t^{y,j}) \), aggregate variables \( (K_t^I, L_t^I, Y_t^I, k_t^w, I_t^w, Z_t^I) \) such that:

1. Profit is maximized and Utility is maximized subjected to budget constraints at country \( j \).
2. Market clearing conditions are satisfied:
   a) Labor market at each country \( j \): \( L_t^I = N_t^j \)
   b) World Capital market: \( Σ_jI_t^I = Σ_jN_t^j s_t^{y,j} \)
   c) World Good market: \( Σ_j(N_t^j c_t^{y,j} + N_t^j s_t^{y,j} + N_{t-1}^j c_t^{o,j} + Z_t^I) = Σ_jY_t^j \)

Definition A.1.4. The integration inter-temporal equilibrium.
Given an initial world capital stock \( k_0^w = Σ_jK_0^j/Σ_j(A_{t-1}^jN_{t-1}^j) \), an integration inter-temporal equilibrium with perfect foresight is a sequence of temporary equilibria that satisfies for all \( t > 0 \) the conditions that:

\[
R_{t+1}^{c,w} = R_{t+1}^w = \alpha(k_{t+1}^w)^{(α-1)}
\]
\[
(1 + g^A)(1 + g^N)k_{t+1}^w = Σ_jN_t^j s_t^g(w_t^j(k_t^w), τ^j, R_{t+1}^w) + (1 - δ)k_t^w
\]

A.2 Proofs.
We prove the Theorem (2.2.2). The proof for Theorem (2.2.1) is the similar, for one closed economy.

**Theorem 2.2.2.** There exists an unique, stable steady state for an integration economy. At steady state, \( \max(A_{t-1}^j/Ā)R^j(A_{t-1}^j/Ā) < R^w < \min(A_{t-1}^j/Ā)R^j(A_{t-1}^j/Ā) \).

**Proof.** The law of capital accumulation at the world integration economy:

\[
k_{t+1}^w = \frac{1}{(1 + g_{t+1}^c)(1 + g_{t+1}^N)}(\frac{β}{1 + β}(1 - α)(1 - τ)(k_{t}^w)^α + (1 - δ)k_t^w)
\]
1. Existence.
Define $\Delta \equiv k_{t+1}^w - \frac{1}{(1 + g_{t+1}^A)(1 + g_{t+1}^N)} \left( \frac{\beta}{1 + \beta} (1 - \alpha)(1 - \bar{\tau})(k_t^w)^\alpha + (1 - \delta)k_t^w \right)$.

\[
\lim_{k_{t+1}^w \to 0} \Delta < 0; \quad \lim_{k_{t+1}^w \to \infty} \Delta > 0
\]

$\Rightarrow$ There exist the solution for the equation ($\Delta = 0$).

2. Uniqueness.
Let define $G(k_{t+1}^w) \equiv \frac{1}{(1 + g_{t+1}^A)(1 + g_{t+1}^N)} \left( \frac{\beta}{1 + \beta} (1 - \alpha)(1 - \bar{\tau})(k_t^w)^\alpha + (1 - \delta)k_t^w \right)$, as the function of ($k_{t+1}^w$) with respect to ($k_t^w$).

\[
\frac{\partial G(k_t^w)}{\partial k_t^w} = \frac{1}{(1 + g_{t+1}^A)(1 + g_{t+1}^N)} \left( \frac{\beta}{1 + \beta} \alpha(1 - \alpha)(1 - \bar{\tau})k_t^w(\alpha - 1) + 1 - \delta \right) > 0
\]

$\Rightarrow G(k_t^w)$ is a monotonic increasing function, which converges to one positive value if any.

For $0 < \alpha < 1, 0 < \delta < 1$,

\[
\lim_{k_t^w \to \infty} k_{t+1}^w = \lim_{k_t^w \to \infty} \frac{1}{(1 + g_{t+1}^A)(1 + g_{t+1}^N)} \left( \frac{\beta}{1 + \beta} (1 - \alpha)(1 - \bar{\tau})k_t^w(\alpha - 1) + 1 - \delta \right)
\]

\[
= \frac{(1 - \delta)}{(1 + g^A)(1 + g^N)} < 1
\]

$\Rightarrow$ The curve of $G(k_t^w)$ is below the 45 degree line for a large enough value of ($k_t^w$). Therefore, the solution for ($\Delta = 0$) is stable.

For the world interest rate, we have:

\[
\frac{1}{R^w} = \frac{(1 - \alpha)}{\alpha} \frac{\beta}{(1 + \beta)} \frac{(1 - \bar{\tau})}{(1 + g^A)(1 + g^N) - 1 + \delta}
\]

\[
= \sum_j \lambda_j \frac{(1 - \alpha)}{\alpha} \frac{\beta}{(1 + \beta)} \frac{(1 - \tau^j (A^j / \bar{A}))}{(1 + g^A)(1 + g^N) - 1 + \delta}
\]

$\Rightarrow \max_{(A^j / \bar{A})} R^j (A^j / \bar{A}) < R^w < \min_{(A^j / \bar{A})} R^j (A^j / \bar{A})$