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Abstract

This paper proposes simple tax policies that can alleviate the distortive effects of royalties. We consider a Cournot duopoly under decreasing returns where one of the firms has a patented technology that it can license to its rival using combinations of royalties and fixed fees. Under optimal licensing policies for the patentee, stronger diseconomies of scale result in lower market prices. It is possible to construct tax-transfer schemes for the firms that are Pareto improving as well as deficit neutral, i.e., these taxes lower market prices and collect sufficient revenue to compensate firms for their losses from taxation without incurring any deficit (JEL: D21, D43, D45, L13)

Keywords: Decreasing returns; royalty; quantity tax; deficit neutrality

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1 Introduction

A patent grants an innovator monopoly rights over its innovation for a given period of time. It seeks to provide incentives to innovate as well as to diffuse innovations. Since Arrow (1962), it has been recognized that patents often create a conflict between private and social incentives of innovation. An innovation that results in an efficient production technology may not yield its full benefits to consumers due to distortions created by specific patent licensing policies.[1] In particular, if a patent holder is one of the incumbent firms in an oligopoly, it has incentives to use royalties to license its innovation to its rivals (see, e.g., Shapiro, 1985; Wang, 1998; Sen and Tauman, 2007). Royalties give competitive edge to the innovator, but distort the innovation and lead to higher prices.[2]

In this paper we identify a situation where, with a patent system in place, the government can adopt simple deficit-neutral policies that alleviate the distortions created by licensing practices. These policies are based on quantity taxation to firms that lower market prices. Furthermore, the revenue collected by the government is sufficiently high to compensate firms for their losses due to taxation. As a result these policies strictly improve the welfare of consumers as well as the firms in the market.

We carry out our analysis in a Cournot duopoly where the competing firms have a production technology that exhibits decreasing returns to scale. One of the firms has a patent on a cost-reducing technological innovation. The patent holder can either use the new technology exclusively or license it to its rival by using combinations of upfront fees and royalties. The starting point of our analysis is the characterization of optimal licensing policies. We show that it is optimal for the patent holder to sell the new technology irrespective of the magnitude of innovation. Moreover, optimal policies always involve positive royalties.[3] Royalties in our model interact with decreasing returns to generate certain counter-intuitive effects which are absent under constant returns. In particular, the rate of optimal royalty falls with the strength of decreasing returns. As a result decreasing returns affect market price through two channels. Higher decreasing returns have the direct effect of raising the price, but also have the indirect effect of lowering royalties which reduces the price. We identify robust regions where the indirect effect dominates the direct effect and consequently stronger decreasing returns result in lower market price.

Consider the regions where stronger decreasing returns lead to lower price. Suppose the

[1]Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985, 1986) first studied the problem of patent licensing in oligopolies. Since then, the literature has been extended to address issues such as informational asymmetries (Gallini and Wright, 1990; Choi, 2001), product differentiation (Muto, 1993; Faulí-Oller and Sandonís, 2002; Colombo, 2012), incumbent innovators (Marjit, 1990; Wang, 1998; Kamien and Tauman, 2002; Sen and Tauman, 2007), sequential competition (Filippini, 2005), intertemporal aspects (Jensen, 1992; Avagyan et al., 2014) and vertical integration (Filippini and Vergari, 2017). See Kamien (1992) and Bhattacharya et al. (2014) for surveys.

[2]Royalties often raise anticompetitive concerns in actual licensing practices. For instance, five wireless technology companies filed complaints at European Commission against their competitor Qualcomm alleging that royalties charged by Qualcomm were "excessive and disproportionate" (Dombey et al., 2005). The Federal Trade Commission (FTC) in the US recently investigated allegations that use of high royalties by Google on some patents was slowing down its competitors (Morran, 2013). One standard policy intervention to remedy the resulting inefficiency could be to provide either direct or indirect subsidies to firms; however, fiscal constraints may prevent governments from following such policies.

[3]In a duopoly with constant returns, it is not optimal for the patent holder to license innovations that are *drastic* (i.e., innovations that are significant enough to create a monopoly), but non-drastic innovations are licensed by using royalties (see Sen and Tauman, 2018).

government introduces a *quantity tax* on each firm which is increasing and convex in the quantity it sells. As the marginal tax is increasing in quantity, this tax is "progressive" in nature. The introduction of this tax effectively strengthens the magnitude of decreasing returns for firms. Within the parametric configuration of the model, we show that it is possible for the government to design this tax in a way that reduces the market price, but doing so necessarily makes one or both firms worse off. However, the tax generates sufficient revenue to compensate firms for their losses. This makes it possible to design tax and transfer schemes (that is, taxation together with lump-sum transfers to firms) that make consumers as well as the firms better off. The scheme finances itself and thus achieves deficit neutrality.

Although progressive income taxes are quite common in most countries, progressive corporate taxes are sometimes criticized to be anticompetitive. Royalties are generally considered to be anticompetitive too. This paper shows that selective and prudent use of one anticompetitive instrument (progressive tax)[4] may weaken the effect of the other one (royalty) and lead to better market outcomes.

Our work is not the first that traces the unexpected impact of taxation in an oligopoly. Studying commodity taxation in a vertical differentiated duopoly with endogenous quality, Cremer and Thisse (1994) show that an ad valorem tax lowers the qualities of the products in the market, which in turn lowers the cost of production and market prices. They also show that there always exists a small ad valorem tax that can raise social welfare via redistribution of the tax receipts. Along similar lines, de Meza et al. (1995) show that imposing an ad-valorem tax in Cournot markets can reduce market prices. Their result is driven by the presence of increasing returns to scale. The introduction of the tax induces some firms to exit, allowing the remaining firms to exploit returns to scale so efficiently that the post-tax price falls below the pre-tax level. By contrast, the exactly opposite factor—decreasing returns to scale—drives our result.

This paper is also related to another strand of literature that looks at Pareto-improving tax policies in economies with imperfectly competitive markets. Konishi et al. (1990) provide welfare improving tax-transfer schemes in a general equilibrium model with one competitive and one non-competitive production sector. Dillén (1995) considers efficiency-restoring policies in a general equilibrium framework with imperfect competition. Ushio (2000) studies the impact of small commodity taxes in a Cournot market. Wang (2018) shows that in economies where all commodities are taxable, the optimal tax rates should equalize the Lerner indexes of commodities.

Finally our paper is indirectly related to the literature on Pigouvian taxation (e.g., Pigou, 1932; Baumol, 1972; Greenwald and Stiglitz, 1986) which studies the role of taxes in correcting externalities generated by market activities. In a market where production results in negative externalities, Carlton and Loury (1980, 1986) argue that imposition of a Pigouvian tax (that is, a per unit tax) on firms leads to socially efficient outcome in the short run but a lump-sum transfer is also required in the long run. Geanakoplos and Polemarchakis (2008) show that in pure exchange economies with separable externalities, a tax-transfer scheme can be used to Pareto improve upon competitive equilibria. In contrast to both of these papers, we look at imperfectly competitive markets, so the choices of firms affect prices. Progressive quantity taxes in our model "correct" the licensing contracts in a way which is beneficial for consumers. Although we study the role of taxes in the context of technology transfer rather than externalities, our paper shares the broad theme that taxes can be used to improve market outcomes.

[4]There is a large literature on non-linear commodity taxes (see, for example, Atkinson and Stiglitz, 1976; Naito, 1999; Saez, 2002).

The paper is organized as follows. Section 2 presents the model and derives optimal licensing policies. Section 3 discusses the inverse relation between market price and decreasing returns. Finally, Section 4 analyzes tax-transfer schemes and their welfare impact. All proofs are placed in the Appendix.

2 The model

Demand: Consider a Cournot duopoly with firms 1 and 2. For $i = 1, 2$, let q_i be the quantity produced by firm i and let Q be the industry output, i.e., $Q = q_1 + q_2$. Denoting by p the market price, the inverse demand function of the industry is $p = \max\{a - Q, 0\}$ where $a > 0$.

Cost: There is an existing technology under which both firms operate under the linear marginal cost function

$$\mu(q) = bq + c \text{ where } 0 < c < a \text{ and } b > 0 \tag{1}$$

We assume that the fixed cost of production is 0. As $\mu(q)$ is increasing in q, the technology exhibits decreasing returns to scale.

New technology: One of the firms, say firm 1, is granted a patent for a new technology that leads to a reduction in the production cost. Specifically the new technology results in the marginal cost function

$$\mu^{\varepsilon}(q) = bq + c - \varepsilon \text{ where } 0 < \varepsilon < c \tag{2}$$

So ε is the magnitude of innovation.[5]

Licensing policies: The patent holder (firm 1) carries out its production with the new technology. It may also license the new technology to its rival firm 2. The set of licensing policies available to firm 1 is the set of all combinations of a unit royalty[6] $r \geq 0$ and an upfront fee $\alpha \geq 0$, so a typical policy is given by (r, α). If firm 2 does not have a license, its marginal cost is $\mu(q)$. If it has a license under a policy (r, α), its *effective* marginal cost becomes $\mu^{\varepsilon}(q) + r$. Assuming firm 2 will not accept a policy with $r > \varepsilon$, it is sufficient to consider royalties $r \in [0, \varepsilon]$.

The licensing game G_b: The strategic interaction between the two firms is modeled as an extensive-form game G_b that has three stages. In the first stage, firm 1 decides whether to license the new technology to firm 2 or not. If firm 1 licenses, it offers a policy (r, α) where $r \in [0, \varepsilon]$ and $\alpha \geq 0$. In the second stage, firm 2 decides whether to accept or reject a licensing offer. In the third stage, the two firms compete in quantities. If firm 2 operates under a policy (r, α) and produces q units, it pays $rq + \alpha$ to firm 1. We look at Subgame Perfect Nash Equilibrium (SPNE) outcomes of this game.

[5] We consider the innovation to be a downward shift of the marginal cost line which lowers c without affecting the slope b. This makes our analysis comparable to the existing literature as by taking $b = 0$, we have the standard case of constant returns.

[6] We look at unit royalties, that is, a licensee pays a royalty for each unit it sells. Studying patent licensing with decreasing returns, Colombo and Filippini (2016) look at a larger class of royalty contracts that include ad valorem and revenue-based royalties. They show that optimal kind of royalty contracts depends on the extent of convexity of cost functions. Using the standard set of licensing contracts (unit royalties plus fixed fees), our primary objective is to look at the possibility of Pareto-improving taxes.

2.1 Cournot equilibrium

We begin with the last stage of the game where firms compete as Cournot duopolists. If firm 2 does not have a license, the payoff of firm i is simply its profit in the Cournot duopoly. Denote this profit by $\widehat{\pi}_i(q_1, q_2)$. Then

$$\widehat{\pi}_i(q_1, q_2) = (a - q_1 - q_2)q_i - [(c - \varepsilon + (i-1)\varepsilon]q_i - bq_i^2/2, \;\; i = 1, 2 \tag{3}$$

Denoting by $\pi_i(r, q_1, q_2)$ the duopoly profit of firm i when firm 2 has a license with rate of royalty r,

$$\pi_i(r, q_1, q_2) = (a - q_1 - q_2)q_i - [c - \varepsilon + (i-1)r]q_i - bq_i^2/2, \;\; i = 1, 2 \tag{4}$$

When there is licensing the total payoff Π_1 of firm 1 is the sum of its duopoly profit and licensing revenue. For firm 2, note from (4) that royalty r is part of its marginal cost and royalty payments are already included in its duopoly profit. So firm 2's total payoff Π_2 is its duopoly profit net of fixed fees. So under a licensing policy (r, α), we have

$$\Pi_1(r, \alpha) = \pi_1(r, q_1, q_2) + rq_2 + \alpha, \;\; \Pi_2(r, \alpha) = \pi_2(r, q_1, q_2) - \alpha \tag{5}$$

To determine the SPNE of G, we need to consider Nash equilibrium (NE) outcomes of the Cournot duopoly where firms choose q_1, q_2 simultaneously and obtain profits given by (3) or (4). Lemma A2 in the Appendix shows that this duopoly has a unique NE. Let $\widehat{q}_i, \widehat{\pi}_i$ denote the Cournot (NE) output, profit of firm i in the no-licensing case; let $q_i(r), \pi_i(r)$ be the corresponding expressions when licensing occurs under royalty r.

2.2 Technology transfer

Under a licensing policy (r, α), by (5) the payoffs are given by

$$\Pi_1(r, \alpha) = \pi_1(r) + rq_2(r) + \alpha, \; \Pi_2(r, \alpha) = \pi_2(r) - \alpha$$

If firm 2 rejects the policy, it obtains $\widehat{\pi}_2$. Hence for any $r \in [0, \varepsilon]$, it is optimal for firm 1 to set the fixed fee $\alpha = \pi_2(r) - \widehat{\pi}_2$, making firm 2 just indifferent between accepting and rejecting the licensing offer. Therefore, if firm 1 decides to offer a license, its problem reduces to choosing $r \in [0, \varepsilon]$ to maximize

$$\Pi_1(r) = \pi_1(r) + rq_2(r) + \pi_2(r) - \widehat{\pi}_2 \tag{6}$$

The notion of *drastic* innovation will be useful to characterize optimal licensing policies. A cost-reducing innovation is drastic (Arrow 1962) if it is significant enough to create a monopoly whenever one firm only uses the new technology; otherwise it is non-drastic. An innovation of magnitude ε is drastic if $\varepsilon \geq (b+1)(a-c)$ and it is non-drastic if $\varepsilon < (b+1)(a-c)$ (see Lemma A2 in the Appendix).

We are in a position to characterize SPNE of the game G_b. Towards this end, for $b > 0$ denote

$$\ell(b) := (b+1)^2/(b^3 + 5b^2 + 7b + 1) \tag{7}$$

Note that $0 < \ell(b) < b + 1$ and $\ell(b)$ is decreasing, with $\lim_{b \to \infty} \ell(b) = 1$.

Proposition 1 *For any $b > 0$, the game G_b has a unique SPNE outcome. It has the following properties.*

(I) *Regardless of whether the innovation is drastic or non-drastic, licensing always occurs.*

(II) *Firm 2 always obtains the net payoff* $\widehat{\pi}_2$ *and it is just indifferent between accepting and rejecting the licensing offer. This payoff is zero for drastic innovations and positive for non-drastic innovations.*

(III) *The licensing policy at the SPNE is given as follows, where* $\theta(b, a-c) := \ell(b)(a-c)$.

(a) *If* $\varepsilon \leq \theta(b, a-c)$, *the licensing policy has royalty* ε *and no fixed fee.*

(b) *If* $\varepsilon > \theta(b, a-c)$, $\exists\ r_b(\varepsilon) \in (0, \varepsilon)$ *such that the licensing policy has royalty* $r_b(\varepsilon)$ *and fixed fee* $\alpha = \pi_2(r_b(\varepsilon)) - \widehat{\pi}_2 > 0$. *The royalty rate* $r_b(\varepsilon)$ *is decreasing in* b.

To see the intuition of this result, first consider a non-drastic innovation. Under the pure royalty policy $r = \varepsilon$, the effective marginal cost of firm 2 becomes $\mu^{\varepsilon}(q) + \varepsilon = \mu(q)$, as in the case of no licensing. This implies that quantities and prices under licensing and no licensing are identical. Hence the profit of firm 1 is the same under these two regimes, i.e., $\pi_1(\varepsilon) = \widehat{\pi}_1$. Positive royalty revenue tilts the scale in favor of licensing for firm 1. This shows technology transfer is generally optimal for non-drastic innovations.

For drastic innovations, firm 1 can become a monopolist by using the innovation exclusively, but still it prefers to license to its rival. This is because under decreasing returns to scale, production of a large output by a single firm creates cost inefficiencies. Hence firm 1 has incentive to keep firm 2 in the market: the presence of two active firms increases efficiency and results in higher surplus than the monopoly profit, which firm 1 then extracts via a fixed fee.

Optimal combinations of fees and royalties are determined by two factors. First, firm 1 intends to create a relatively inefficient rival and second, it has to consider how the rival's efficiency affects its own marginal cost. Under decreasing returns these two factors work in opposite direction as an inefficient rival implies higher output and higher marginal cost for firm 1. The relation between the magnitude of innovation and the strength of decreasing returns plays an important role to resolve this conflict, as we explain below.

Note that the threshold $\theta(b, a-c)$ is decreasing in b. So the result of Proposition 1(III) can be restated in terms of b. If b is low then the impact of decreasing returns on firm 1's marginal cost is relatively low, so the incentive of creating an inefficient rival is more important. Accordingly, a pure royalty policy with maximum royalty equal to ε is used. If b is high, the impact of decreasing returns is high and firm 1 prefers to provide a certain degree of cost efficiency to firm 2 to ensure that firm 1 avoids operating in relatively inefficient production zones. For this reason the rate of royalty is set below ε. This also explains why the royalty rate $r_b(\varepsilon)$ is decreasing in b. Higher values of b means firm 1 faces higher marginal cost. By designing a royalty that decreases in b, firm 1 "commits" to a relatively smaller quantity, thus staying in a relatively efficient production zone.

3 Post-licensing price

Now we study how the post-licensing price behaves in relation to the strength of diseconomies of scale (represented by b). Proposition 2 identifies a paradox. Using optimal licensing policies, we find ranges of parameter values under which an increase in b lowers market price.

Proposition 2 *Let* $p^0(b)$ *be the post-licensing Cournot price in the SPNE of* G_b. *There exist* $\widehat{\varepsilon} \in (0, a-c)$ *and* $\widehat{b} > 0$ *such that*

(I) *If* $0 < \varepsilon \leq \widehat{\varepsilon}$, $p^0(b)$ *is increasing for all* $b > 0$.

(II) *If* $\widehat{\varepsilon} < \varepsilon < a-c$, $\exists\ \widetilde{b}(\varepsilon) \in (0, \widehat{b})$ *such that* $p^0(b)$ *is increasing for* $b \in (0, \widetilde{b}(\varepsilon))$, *decreasing for* $b \in (\widetilde{b}(\varepsilon), \widehat{b})$ *and increasing for* $b > \widehat{b}$.

(III) *If* $\varepsilon \geq a - c$, $p^0(b)$ *is decreasing for* $b \in (0, \widehat{b})$ *and increasing for* $b > \widehat{b}$.

Let $r^*(b)$ be the optimal royalty for firm 1. By Proposition 1, $r^*(b) \in \{\varepsilon, r_b(\varepsilon)\}$. Let the industry output be $Q^* = Q(b, r^*(b))$. An increase in b creates a direct and an indirect effect on Q^*: the first effect captures the standard impact of an increase in marginal cost and has a negative sign. The second effect operates via the royalty rate. The overall marginal effect of b on Q^* (given by dQ^*/db) can be decomposed as

$$dQ^*/db = \underbrace{\partial Q^*/\partial b}_{<0} + \underbrace{\partial Q^*/\partial r}_{<0} \cdot \underbrace{\partial r^*/\partial b}_{\leqq 0} \tag{8}$$

If $r^*(b) = \varepsilon$, the indirect effect of b is zero and the industry output decreases (so price increases) in b. If $r^*(b) = r_b(\varepsilon)$, (8) becomes

$$dQ^*/db = \underbrace{\partial Q^*/\partial b}_{<0} + \underbrace{\partial Q^*/\partial r}_{<0} \cdot \underbrace{\partial r_b(\varepsilon)/\partial b}_{<0} \tag{9}$$

In this case, the indirect effect is positive as an increase in b reduces the level of royalty. As a result of the two opposite effects, the industry output is an inverse u-shaped function in b, so the price is a u-shaped function of b.

The ranges identified in Proposition 2 are driven by optimal licensing policies. Recall by Proposition 1 that if the magnitude of innovation is low, the optimal royalty is ε for any b. So for small innovations, price increases in b. If the magnitude of innovation is large, the royalty is $r_b(\varepsilon)$ for any b; then the price is u-shaped in b. For intermediate magnitudes of the innovation, the royalty is ε for small values of b and it is $r_b(\varepsilon)$ for large values of b. As a result, price is increasing when b is relatively small and it is u-shaped when b is relatively large.

Remark Our comparative statics exercise has focused on changes of b. Alternatively, we could examine the constant term c of the marginal cost line. The impact on industry output would again be captured by an expression similar to (8). However, in this case the direct effect would always dominate the indirect effect.

4 Pareto-improving taxes

In this section we restrict to cases where the post-licensing price $p^0(b)$ is decreasing in b. Let $\varepsilon > \widehat{\varepsilon}$ and define

$$\underline{b}(\varepsilon) := \begin{cases} \widetilde{b}(\varepsilon) & \text{if } \widehat{\varepsilon} < \varepsilon < a-c \\ 0 & \text{if } \varepsilon \geq a-c \end{cases} \tag{10}$$

By Proposition 2, if $\varepsilon > \widehat{\varepsilon}$, then $p^0(b)$ is decreasing for $b \in (\underline{b}(\varepsilon), \widehat{b})$. In what follows, we show that in this case it is possible to design a *quantity tax* that lowers the post-licensing market price. Moreover this tax generates sufficient revenue to offset the potential losses of firms from this taxation.

Let $t(q)$ be a tax policy that applies to any firm, that is, any firm that sells q units has to pay tax $t(q)$. Let us take

$$t(q) = \tau q^2/2 \text{ where } \tau > 0 \tag{11}$$

As $t'(q) = \tau q > 0$ for $q > 0$, the marginal tax under tax policy (11) is increasing in q, i.e., this tax policy is *progressive*. It has the effect of raising each firm's marginal cost by τq, so the strategic interaction between the firms under tax policy (11) is the same as the interaction when there is no tax but the marginal costs under old and new technologies are $(b+\tau)q + c$ and $(b+\tau)q + c - \varepsilon$. Accordingly, under tax policy (11) firms play the extensive-form game[7] $G_{b+\tau}$. Denoting by $p^\tau(b)$ the post-licensing price under tax policy (11), we conclude that

$$p^\tau(b) = p^0(b+\tau) \tag{12}$$

For $i = 1, 2$, let q_i^τ be the quantity and Π_i^τ the payoff of firm i at the SPNE of $G_{b+\tau}$. Let S^τ be the sum of payoffs of firms $1, 2$ and T^τ the tax revenue at the SPNE, i.e.,

$$S^\tau := \Pi_1^\tau + \Pi_2^\tau \text{ and } T^\tau := \tau[(q_1^\tau)^2 + (q_2^\tau)^2]/2 \tag{13}$$

Note that S^0 and T^0 correspond to the values when there is no tax (i.e., $\tau = 0$); in particular, $T^0 = 0$.

Definition A *tax and transfer scheme*, denoted by $(t(.), f_1, f_2)$, is a scheme under which any firm $i = 1, 2$: (i) pays quantity tax according to function $t(q)$ and (ii) receives a lump-sum transfer f_i.

Proposition 3 *Let $\varepsilon > \widehat{\varepsilon}$, $\tau \in (0, \widehat{b} - \underline{b}(\varepsilon))$ and $b \in I^\tau \equiv (\underline{b}(\varepsilon), \widehat{b} - \tau)$. The introduction of tax* (11) *has the following effects.*

(I) *$p^\tau(b) < p^0(b)$, i.e., the tax lowers the post-licensing price.*

(II) *There exists $\underline{\tau} \in (0, \widehat{b} - \underline{b}(\varepsilon))$ such that for all $\tau \in (0, \underline{\tau})$:*

(i) *$S^\tau < S^0$ i.e., the tax reduces the sum of payoffs of two firms. Consequently it makes one or both firms worse off.*

(ii) *$T^\tau > S^0 - S^\tau$ i.e., the tax generates sufficient revenue to offset the sum of losses of firms from the tax.*

(iii) *There exists a tax and transfer scheme $(t(.), f_1, f_2)$ where firm $i = 1, 2$ pays tax according to* (11) *and receives a lump-sum transfer f_i such that* (a) *the scheme makes consumers better off compared to no taxes,* (b) *$\Pi_i^\tau + f_i > \Pi_i^0$ for $i = 1, 2$ i.e., the scheme makes both firms better off compared to no taxes and* (c) *$f_1 + f_2 = T^\tau$, i.e., the scheme is deficit-neutral.*

Remark Observe by Proposition 3(II)(i) that tax (11) makes at least one firm worse off. If it makes both firms worse off, then any tax and transfer scheme satisfying Proposition 3(II)(iii) must have positive lump-sum transfer for both firms, i.e., $f_i > 0$ for $i = 1, 2$. However, if tax (11) makes firm i better off and firm j worse off, then f_i could be negative (i.e. i might be required to make a lump-sum transfer, which together with the tax revenue will compensate for the losses of j), but i would still be better off compared to no taxes.

[7] Recall that when there are no taxes, the game played between two firms with existing technology (1) and new technology (2) is given by G_b, whose SPNE has been characterized in Proposition 1.

The quantity tax essentially strengthens the diseconomies of scale and consequently lowers the rate of optimal royalty. The tax has two effects on the market price: (i) the direct effect by strengthening diseconomies of scale and (ii) the indirect effect by lowering the rate of royalty. Proposition 3 shows that the indirect effect dominates and the market price falls. The increase in the industry output allows the government to collect sufficient revenue in order to compensate the firms. As a result this tax-transfer scheme is strictly Pareto improving: it improves welfare of consumers as well as firms.

One question that arises is whether we can have Pareto-improving tax transfer schemes with a linear quantity tax, that is, a tax policy where the tax rate is constant in the quantity of each firm. Take for example the tax policy $t(q) = \tau q$. Then the market price will again be affected by τ via two channels. However, the net effect will always be such that the price is increasing in τ, so the result cannot be reproduced with a linear tax.

To conclude, considering a Cournot duopoly model with technology transfer, we have shown that under optimal licensing policies, higher diseconomies of scale can benefit consumers in terms of lower prices. Using this result we have proposed simple tax-transfer schemes that enhance welfare without incurring any deficit. Technology transfer across firms via licensing is quite frequent in practice. Our analysis suggests that in designing policies, it will be useful to adequately consider the role of licensing.

Appendix

Let $i, j = 1, 2$ and $i \neq j$. By (3) and (4), the profit function of firm i is

$$\pi_i = (a - q_i - q_j)q_i - (c - \delta)q_i - bq_i^2/2 \tag{14}$$

for some $\delta \in [0, \varepsilon]$ where (a) for $i = 1$, $\delta = \varepsilon$ and (b) for $i = 2$, $\delta = 0$ if firm 2 does not have a license and $\delta = \varepsilon - r \in [0, \varepsilon]$ if it has a license with royalty r.

Lemma A1 *Suppose firm i has profit function (14) for some $\delta \in [0, \varepsilon]$. For any $b > 0$, firm i has a unique best response to any quantity q_j chosen by its rival firm j, given as*

$$B^\delta(q_j) = \begin{cases} (a - c + \delta - q_j)/(b + 2), \text{ if } q_j \in [0, a - c + \delta) \\ 0, \text{ if } q_j \geq a - c + \delta \end{cases} \tag{15}$$

Proof Follows by standard computations. ■

Lemma A2

(i) *The monopoly problem where the monopolist has the new technology has a unique solution. The monopoly price is $p_M = c + [(b+1)(a-c) - \varepsilon]/(b+2)$, the monopoly quantity is $q_M = (a-c+\varepsilon)/(b+2)$ and the monopoly profit is $\pi_M = (b+2)[q_M]^2/2$. Furthermore, $p_M \gtreqless c \Leftrightarrow \varepsilon \lesseqgtr (b+1)(a-c)$.*

(ii) *Suppose firm 2 has a license under rate of royalty r. The resulting Cournot duopoly game has a unique NE. Let $p(r)$ be the Cournot (NE) price and $q_i(r), \pi_i(r)$ be the Cournot quantity, profit of firm $i = 1, 2$.*

(a) *If $\varepsilon < (b + 1)(a - c)$, then $p(r) = c + [(b + 1)(a - c) - 2\varepsilon + r]/(b + 3)$; $q_1(r) = [(b+1)(a-c+\varepsilon)+r]/(b+1)(b+3)$, $q_2(r) = [(b+1)(a-c+\varepsilon)-(b+2)r]/(b+1)(b+3)$ and $\pi_i(r) = (b + 2)[q_i(r)]^2/2$ for $i = 1, 2$. Furthermore, $p(r), q_1(r), \pi_1(r)$ are increasing and $q_2(r), \pi_2(r)$ are decreasing in r.*

(b) *If $\varepsilon \geq (b+1)(a-c)$, then there exists $\overline{r}_b(\varepsilon) \equiv (b+1)(a-c+\varepsilon)/(b+2) \in (0, \varepsilon]$ such that if $r \in [0, \overline{r}_b(\varepsilon))$ NE price, outputs and profits are same as in* (a). *If $r \in [\overline{r}_b(\varepsilon), \varepsilon]$, then firm* 2 *does not produce and $p(r) = p_M$, $q_1(r) = q_M$, i.e, a monopoly is created with firm* 1.

(c) *For $\varepsilon < (b+1)(a-c)$, $q_1(r)q_2(r)$ is decreasing for $r \in [0, \varepsilon]$; for $\varepsilon \geq (b+1)(a-c)$, $q_1(r)q_2(r)$ is decreasing for $r \in [0, \overline{r}_b(\varepsilon)]$ and $q_1(r)q_2(r) = 0$ for $r \in [\overline{r}_b(\varepsilon), \varepsilon]$.*

(iii) *Suppose firm* 2 *does not have a license. The resulting Cournot duopoly game has a unique NE. Let $\widehat{p}$ be the Cournot price and $\widehat{q}_i$ be the Cournot quantity for $i = 1, 2$.*

(a) *If $\varepsilon < (b+1)(a-c)$, then $\widehat{p} = c + [(b+1)(a-c) - \varepsilon]/(b+3) > c$; $\widehat{q}_1 = [(b+1)(a-c) + (b+2)\varepsilon]/(b+1)(b+3)$, $\widehat{q}_2 = [(b+1)(a-c) - \varepsilon]/(b+1)(b+3)$ and $\widehat{\pi}_i = (b+2)[\widehat{q}_i]^2/2$ for $i = 1, 2$.*

(b) *If $\varepsilon \geq (b+1)(a-c)$, then $\widehat{p} = p_M \leq c$. Firm* 2 *does not produce and a monopoly is created with firm* 1.

(c) *$\pi_i(\varepsilon) = \widehat{\pi}_i$ for $i = 1, 2$ and $p(\varepsilon) = \widehat{p}$.*

Proof (i) Follows by standard computations.

(ii) By Lemma A1, the best response function of firm 1 to q_2 is $B^{\varepsilon}(q_2)$. If firm 2 has a license with rate of royalty r, its best response function to q_1 is $B^{\varepsilon - r}(q_1)$. The results of (a)-(b) follow by solving the system of two best response equations. To prove (c), let $\rho(r) := q_1(r)q_2(r)$. Using the expressions of $q_1(r), q_2(r)$ from (a)-(b), observe that whenever $\rho(r) > 0$, it is an inverse u-shaped quadratic function. Noting that $\rho'(0) = -(a - c + \varepsilon)/(b+3)^2 < 0$, it follows that $\rho'(r) < 0$ for $r \geq 0$, which proves the result.

(iii) As before, the best response function of firm 1 is $B^{\varepsilon}(q_2)$. If firm 2 does not have a license, its best response function is $B^0(q_1)$. The results of (a)-(b) follow by solving the system of two best response equations. The result of (c) follows by noting that $B^{\varepsilon - r}(q_1)$ equals $B^0(q_1)$ when $r = \varepsilon$. ■

Let $F_b(p)$ be the profit of the monopolist under the new technology (2) evaluated at price p, i.e.,

$$F_b(p) := (p - c + \varepsilon)Q - bQ^2/2 \text{ where } Q = \max\{a - p, 0\} \tag{16}$$

As $\pi_1(r) = p(r)q_1(r) - (c - \varepsilon)q_1(r) - b[q_1(r)]^2/2$ and $\pi_2(r) = p(r)q_2(r) - (c - \varepsilon + r)q_2(r) - b[q_2(r)]^2/2$, denoting $Q(r) := q_1(r) + q_2(r)$ to be the industry output, from (6) we have

$$\Pi_1(r) = (p(r) - c + \varepsilon)Q(r) - b[(q_1(r))^2 + (q_2(r))^2]/2 - \widehat{\pi}_2$$

Using (16) in the expression above, it follows that

$$\Pi_1(r) = F_b(p(r)) + bq_1(r)q_2(r) - \widehat{\pi}_2 \tag{17}$$

Since firm 2 obtains $\widehat{\pi}_2$ in any SPNE of G_b, it follows from (17) that if the royalty rate is r in an SPNE of G_b, then the sum of payoffs of two firms is

$$\Pi_1(r) + \Pi_2(r) = F_b(p(r)) + bq_1(r)q_2(r) \tag{18}$$

Proof of Proposition 1 (I) To prove part (I), we consider two cases.

Case 1 $\varepsilon \geq (b+1)(a-c)$ (drastic innovations): For this case without a license firm 2 obtains zero profit and firm 1 becomes a monopolist, i.e., $\widehat{\pi}_2 = 0$ and $\widehat{\pi}_1 = \pi_M$. By

Lemma A2(ii)(b), $\exists\, \overline{r}_b(\varepsilon) \in [0, \varepsilon]$ such that if 1 chooses $r \in [\overline{r}_b(\varepsilon), \varepsilon]$, then $\Pi_1(r) = \widehat{\pi}_1 = \pi_M$. Therefore it is sufficient to consider policies with $r \in [0, \overline{r}_b(\varepsilon)]$.

Using the results of Lemma A2 in (17), it follows that

$$\Pi_1'(r) = [(b+1)^2(a-c+\varepsilon) - h(b)r]/(b+1)^2(b+3)^2 \text{ where } h(b) := (b+2)(b^2+4b+1) \quad (19)$$

Since $h(b) > 0$, we have

$$\Pi_1'(r) \gtreqless 0 \Leftrightarrow r \lesseqgtr r_b(\varepsilon) \equiv (b+1)^2(a-c+\varepsilon)/h(b) > 0 \quad (20)$$

As $\overline{r}_b(\varepsilon) - r_b(\varepsilon) = b(b+1)(b+3)(a-c+\varepsilon)/h(b) > 0$, we conclude that over $r \in [0, \overline{r}_b(\varepsilon)]$, the unique maximum of $\Pi_1(r)$ is attained at $r = r_b(\varepsilon)$. Thus, in particular, $\Pi_1(r_b(\varepsilon)) > \Pi_1(\overline{r}_b(\varepsilon)) = \pi_M$. This proves that it is optimal for firm 1 to license to firm 2 with rate of royalty $r = r_b(\varepsilon)$ and fee $\pi_2(r_b(\varepsilon)) - \widehat{\pi}_2 = \pi_2(r_b(\varepsilon)) > 0$. This proves the result for drastic innovations.

Case 2 $\varepsilon < (b+1)(a-c)$ (non-drastic innovations): By Lemma A2(iii)(c), $\widehat{\pi}_i = \pi_i(\varepsilon)$ for $i = 1, 2$. In particular, if firm 1 does not license, it obtains $\widehat{\pi}_1 = \pi_1(\varepsilon)$. If it offers a license with royalty $r = \varepsilon$, then by (6), firm 1 obtains $\Pi_1(\varepsilon) = \pi_1(\varepsilon) + \varepsilon q_2(\varepsilon) + \pi_2(\varepsilon) - \widehat{\pi}_2 = \pi_1(\varepsilon) + \varepsilon q_2(\varepsilon) > \pi_1(\varepsilon)$ (since $q_2(\varepsilon) > 0$ for a non-drastic innovation). This proves that it is optimal for firm 1 to offer a license to firm 2 for non-drastic innovations. This completes the proof of (I).

(II) Note by the discussion preceding equation (6) that $\alpha = \pi_2(r) - \widehat{\pi}_2$, so firm 2 obtains $\widehat{\pi}_2$. The last statement of (II) follows by noting that $\widehat{\pi}_2 = 0$ if the innovation is drastic and $\widehat{\pi}_2 > 0$ if the innovation is non-drastic.

(III) For the case $\varepsilon \geq (b+1)(a-c)$, the (unique SPNE) licensing policy has been already characterized in Case 1 of the proof of part (I). So consider $\varepsilon < (b+1)(a-c)$. Applying Lemma A2(ii)(a) in (17), in this case $\Pi_1'(r)$ satisfies (19) and (20) for any $r \in [0, \varepsilon]$. Observe that $\varepsilon - r_b(\varepsilon) = w(b)[\varepsilon - \ell(b)(a-c)]/h(b)$ where $\ell(b)$ is given in (7) and $h(b)$ is given in (19). Hence $\varepsilon \gtreqless r_b(\varepsilon) \Leftrightarrow \varepsilon \gtreqless \ell(b)(a-c)$.

If $\varepsilon > \ell(b)(a-c)$, then $\varepsilon > r_b(\varepsilon)$ and by (20), the unique maximum of $\Pi_1(r)$ is attained at $r = r_b(\varepsilon)$; the resulting fixed fee is $\pi_2(r_b(\varepsilon)) - \widehat{\pi}_2 = \pi_2(r_b(\varepsilon)) - \pi_2(\varepsilon)$ (since $\pi_2(r)$ is decreasing and $r_b(\varepsilon) < \varepsilon$, the fee is positive). This proves the result for $\ell(b)(a-c) < \varepsilon < (b+1)(a-c)$. Together with Case 1 of the proof of (I), this characterizes SPNE licensing policy for $\varepsilon > \ell(b)(a-c)$ (part (III)(b)). Using the expression of $r_b(\varepsilon)$ from (20), its properties follow by standard computations.

If $\varepsilon \leq \ell(b)(a-c)$, then $r_b(\varepsilon) \geq \varepsilon$ and by (20), the unique maximum of $\Pi_1(r)$ is attained at $r = \varepsilon$ where the fixed fee is $\pi_2(\varepsilon) - \widehat{\pi}_2 = \pi_2(\varepsilon) - \pi_2(\varepsilon) = 0$. This proves the result for $\varepsilon \leq \ell(b)(a-c)$ (part (III)(a)). ■

Proof of Proposition 2 Recall that $p(r) = c + [(b+1)(a-c) - 2\varepsilon + r]/(b+3)$ is the Cournot price with royalty r when both firms are active in the market. Then,

$$p^0 = \begin{cases} p(r_b(\varepsilon)) \text{ if } \varepsilon > \ell(b)(a-c) \\ p(\varepsilon) \text{ if } \varepsilon \leq \ell(b)(a-c) \end{cases} \quad (21)$$

Noting that $r_b(\varepsilon) = (b+1)^2(a-c+\varepsilon)/(b+2)(b^2+4b+1)$, it follows that

$$\partial p(r_b(\varepsilon))/\partial b = y(b)(a-c+\varepsilon)/[h(b)]^2 \text{ where } y(b) := 2b^4 + 10b^3 + 15b^2 + 4b - 1$$

and there exists $\widehat{b} \in (0.15, 0.16)$ such that $\partial p(r_b(\varepsilon))/\partial b \gtreqless 0 \Leftrightarrow b \gtreqless \widehat{b}$. We consider the following cases.

Case 1 $\varepsilon \geq a - c$: Since $\ell(0) = 1$ and $\ell(b)$ is decreasing, for this case $\varepsilon > \ell(b)(a - c)$ for all $b > 0$ and by (21), $p^0 = p(r_b(\varepsilon))$. So for this case, p^0 is decreasing for $b \in (0, \widehat{b})$ and increasing for $b > \widehat{b}$ which proves (III).

Case 2 $0 < \varepsilon < a-c$: Since $\ell(0) = 1$, $\ell(b)$ is decreasing and $\lim_{b\to\infty} \ell(b) = 0$, there exists $\widetilde{b}(\varepsilon) > 0$ such that $\varepsilon \gtreqless \ell(b)(a - c) \Leftrightarrow b \gtreqless \widetilde{b}(\varepsilon)$. From the properties of $\ell(b)$, it follows that $\widetilde{b}(\varepsilon)$ is decreasing with $\lim_{\varepsilon\uparrow a-c} \widetilde{b}(\varepsilon) = 0$ and $\lim_{\varepsilon\downarrow 0} \widetilde{b}(\varepsilon) = \infty$. Therefore, there exists $\widehat{\varepsilon} \in (0, a-c)$ such that $\widehat{b} \lesseqgtr \widetilde{b}(\varepsilon) \Leftrightarrow \varepsilon \lesseqgtr \widehat{\varepsilon}$.

Case 2(a) $0 < \varepsilon \leq \widehat{\varepsilon}$: For this case, $\widehat{b} \leq \widetilde{b}(\varepsilon)$. If $b \in (0, \widetilde{b}(\varepsilon)]$, then $\varepsilon \leq \ell(b)(a - c)$ and by (21), $p^0 = p(\varepsilon)$, which is increasing in b. If $b > \widetilde{b}(\varepsilon)$, then $\varepsilon > \ell(b)(a - c)$ and $p^0 = p(r_b(\varepsilon))$ which is increasing in b (since $b > \widetilde{b}(\varepsilon) \geq \widehat{b}$). By continuity of p^0 it follows that p^0 is increasing for all $b > 0$. This proves (I).

Case 2(b) $\widehat{\varepsilon} < \varepsilon < a - c$: For this case, $\widehat{b} > \widetilde{b}(\varepsilon)$. If $b \in (0, \widetilde{b}(\varepsilon)]$, then $\varepsilon \leq \ell(b)(a-c)$ and $p^0 = p(\varepsilon)$, which is increasing in b. If $b > \widetilde{b}(\varepsilon)$, then $\varepsilon > \ell(b)(a-c)$ and $p^0 = p(r_b(\varepsilon))$, which is decreasing for $b \in (\widetilde{b}(\varepsilon), \widehat{b})$ and increasing otherwise. Therefore for this case p^0 is increasing for $b \in (0, \widetilde{b}(\varepsilon))$, decreasing for $b \in (\widetilde{b}(\varepsilon), \widehat{b})$ and increasing for $b > \widehat{b}$. This proves (II). ■

Lemma A3 *Let $\tau \in (0, \widehat{b} - \underline{b}(\varepsilon))$ and $b \in I^\tau \equiv (\underline{b}(\varepsilon), \widehat{b} - \tau)$. Consider the functions given in (13) and define U^τ to be the sum of tax revenue and payoffs of the firms at the SPNE. As functions of τ, S^τ is decreasing and T^τ, U^τ are both increasing at $\tau = 0$.*

Proof As the SPNE price of $G_{b+\tau}$ is $p^0(b + \tau)$, from (18) it follows that the sum of payoffs of firms at the SPNE is

$$S^\tau = F_{b+\tau}(p^0(b + \tau)) + bq_1^\tau q_2^\tau \tag{22}$$

where $F_b(p)$ is given in (16). Since $b + \tau \in (\underline{b}(\varepsilon), \widehat{b})$, from the proof of Proposition 2 we know that at the SPNE of $G_{b+\tau}$, firm 1 sets royalty $r_{b+\tau}(\varepsilon)$ where $r_b(\varepsilon)$ is given in (20). The quantities q_1^τ, q_2^τ and the price $p^0(b + \tau)$ is obtained from Lemma A2(ii) by replacing b by $b + \tau$ and taking $r = r_{b+\tau}(\varepsilon)$. Denoting $\kappa(b) := (a - c + \varepsilon)^2/2[h(b)]^2 > 0$, from (13) and (22) we have

$$\frac{\mathrm{d}S^\tau}{\mathrm{d}\tau}[\tau = 0] = -\kappa(b)(b + 1)(2b^3 + 6b^2 + 3b + 1) < 0,$$

$$\frac{\mathrm{d}T^\tau}{\mathrm{d}\tau}[\tau = 0] = \kappa(b)(2b^4 + 10b^3 + 15b^2 + 6b + 1) > 0, \quad \frac{\mathrm{d}U^\tau}{\mathrm{d}\tau}[\tau = 0] = \kappa(b)2b(b^2 + 3b + 1) > 0$$

This proves the result. ■

Proof of Proposition 3 (I) For any $b \in I^\tau$, both $b, b + \tau \in (\underline{b}(\varepsilon), \widehat{b})$ and by Proposition 2, $p^0(b + \tau) < p^0(b)$. Using (12), it follows that $p^\tau(b) = p^0(b + \tau) < p^0(b)$.

(II) Since as function of τ, S^τ is decreasing and U^τ is increasing at $\tau = 0$ (by Lemma A3), there exists a sufficiently small $\underline{\tau} \in (0, \widehat{b} - \underline{b}(\varepsilon))$ such that for all $\tau \in (0, \underline{\tau})$:

$$S^\tau < S^0 \text{ and } U^\tau > U^0 \tag{23}$$

The first inequality of (23) proves (i). As $U^\tau = S^\tau + T^\tau$ and $T^0 = 0$, the last inequality of (23) implies

$$T^\tau > S^0 - S^\tau = (\Pi_1^0 - \Pi_1^\tau) + (\Pi_2^0 - \Pi_2^\tau) \tag{24}$$

which proves (ii).

Since lump-sum transfer do not affect market prices, (iii)(a) is immediate from (I). Denote $T^\tau - (S^0 - S^\tau) \equiv d > 0$ and take $f_i = \Pi_i^0 - \Pi_i^\tau + d/2$ for $i = 1, 2$. Then $\Pi_i^\tau + f_i = \Pi_i^0 + d/2 > \Pi_i^0$ for $i = 1, 2$ and by (24), $f_1 + f_2 = S^0 - S^\tau + d = T^\tau$. This proves (b) and (c). ■

References

Arrow, K. J. (1962), "Economic Welfare and the Allocation of Resources for Invention," in: R. R. Nelson (ed.), *The Rate and Direction of Inventive Activity: Economic and Social Factors*, Princeton University Press, pp. 609-625.

Atkinson, A. B., and J. E. Stiglitz (1976), "The Design of Tax Structure: Direct versus Indirect Taxation," *Journal of Public Economics*, 6(1-2), 55-75.

Avagyan, V., M. Esteban-Bravo, and J. M. Vidal-Sanz (2014), "Licensing Radical Product Innovations to Speed up the Diffusion," *European Journal of Operational Research*, 239(2), 542-555.

Baumol, W. (1972), "On Taxation and the Control of Externalities," *American Economic Review*, 62(3), 307-322.

Bhattacharya, S., C. d'Aspremont, S. Guriev, D. Sen, and Y. Tauman (2014), Cooperation in R&D: Patenting, Licensing, and Contracting. in: K. Chatterjee and W. Samuelson (eds.), *Game Theory and Business Applications*, Springer, pp. 265-286.

Carlton, D. W., and G. C. Loury (1980), "The Limitations of Pigouvian Taxes as a Long-Run Remedy for Externalities," *Quarterly Journal of Economics*, 95(3), 559-566.

Carlton, D. W., and G. C. Loury (1986), "The Limitation of Pigouvian Taxes as a Long-Run Remedy for Externalities: An Extension of Results," *Quarterly Journal of Economics*, 101(3), 631-634.

Choi, J. P. (2001), "Technology Transfer with Moral Hazard," *International Journal of Industrial Organization*, 19(1-2), 249-266.

Colombo, S. (2012), "A Comment on "Welfare Reducing Licensing"," *Games and Economic Behavior*, 76(2), 515-518.

Colombo, S., and L. Filippini (2016), "Revenue Royalties," *Journal of Economics*, 118(1), 47-76.

Cremer, H., and J-F. Thisse (1994), "Commodity Taxation in a Differentiated Oligopoly," *International Economic Review*, 35(3), 613-633.

de Meza, D., J. Maloney, and G. D. Myles (1995), "Price-reducing Taxation," *Economics Letters*, 47(1), 77-81.

Dombey, D., M. Palmer, and C. Nuttall (2005, October 28), "Qualcomm Rivals take Case to EU," *Financial Times*, Retrieved from https://www.ft.com/content/abf20f38-47ba-11da-a949-00000e2511c8 accessed November 26, 2018

Dillén, M. (1995), "Corrective Tax and Subsidy Policies in Economies with Bertrand Competition," *Journal of Public Economics*, 58(2), 267-282.

Faulí-Oller, R., and J. Sandonís (2002), "Welfare Reducing Licensing," *Games and Economic Behavior*, 41(2), 192-205.

Filippini, L. (2005), "Licensing Contract in a Stackelberg Model," *Manchester School*, 73(5), 582-598.

Filippini, L., and C. Vergari (2017), "Vertical Integration Smooths Innovation Diffusion," *B.E. Journal of Economic Analysis & Policy*, 17(3), 1-22.

Gallini, N. T., and B. D. Wright (1990), "Technology Transfer under Asymmetric Information," *RAND Journal of Economics*, 21(1), 147-160.

Geanakoplos, J., and H. M. Polemarchakis (2008), "Pareto Improving Taxes," *Journal of Mathematical Economics*, 44(7-8), 682-696.

Greenwald, B., and J. E. Stiglitz (1986), "Externalities in Economies with Imperfect Information and Incomplete Markets," *Quarterly Journal of Economics*, 101(2), 229-264.

Jensen, R. (1992), "Dynamic Patent Licensing," *International Journal of Industrial Organization*, 10(3), 349-368.

Kamien, M. I. (1992), "Patent Licensing," in: R. Aumann and S. Hart (eds.), *Handbook of Game Theory with Economic Applications*, Elsevier, pp. 331-354.

Kamien, M. I., and Y. Tauman (1984), "The Private Value of a Patent: A Game Theoretic Analysis," *Journal of Economics*, 4 (Supplement), 93-118.

Kamien, M. I., and Y. Tauman (1986), "Fees versus Royalties and the Private Value of a Patent," *Quarterly Journal of Economics*, 101(3), 471-491.

Kamien, M. I., and Y. Tauman (2002), "Patent Licensing: The Inside Story," *Manchester School*, 70(1), 7-15.

Katz, M. L., and C. Shapiro (1985), "On the Licensing of Innovations," *Rand Journal of Economics*, 16(4), 504-520.

Katz, M. L., and C. Shapiro (1986), "How to License Intangible Property," *Quarterly Journal of Economics*, 101(3), 567-589.

Konishi H., M. Okuno-Fujiwara, and K. Suzumura (1990), "Oligopolistic Competition and Economic Welfare: A General Equilibrium Analysis of Entry Regulation and Tax-subsidy Schemes," *Journal of Public Economics*, 42(1), 67-88.

Marjit, S. (1990), "On a Non-cooperative Theory of Technology Transfer," *Economics Letters*, 33(3), 293-298.

Morran, C. (2013, January 3), "Google settles with FTC, agrees to change Anticompetitive Business Practices," *Consumerist*, Retrieved from https://consumerist.com/2013/01/03/google-settles-with-ftc-agrees-to-change-anticompetitive-business-practices/ accessed November 26, 2018

Muto, S. (1993), "On Licensing Policies in Bertrand Competition," *Games and Economic Behavior*, 5(2), 257-267.

Naito, H. (1999), "Re-examination of Uniform Commodity Taxes under a Non-linear Income Tax System and its Implication for Production Efficiency," *Journal of Public Economics*, 71(2), 165-188.

Pigou, A. C. (1932), *The Economics of Welfare*, Palgrave Macmillan (2013 edition)

Saez, E. (2002), "The Desirability of Commodity Taxation under Non-linear Income Taxation and Heterogeneous Tastes," *Journal of Public Economics*, 83(2), 217-230.

Sen, D., and Y. Tauman (2007), "General Licensing Schemes for a Cost-reducing Innovation," *Games and Economic Behavior*, 59(1), 163-186.

Sen, D., and Y. Tauman (2018), "Patent Licensing in a Cournot Oligopoly: General Results," *Mathematical Social Sciences*, 96, 37-48.

Shapiro, C. (1985), "Patent Licensing and R&D Rivalry," *American Economic Review, Papers and Proceedings*, 75(2), 25-30.

Ushio Y. (2000), "Welfare Effects of Commodity Taxation in Cournot Oligopoly," *Japanese Economic Review*, 51(2), 268-273.

Wang, X. H. (1998), "Fee versus Royalty Licensing in a Cournot Duopoly Model," *Economics Letters*, 60(1), 55-62.

Wang, H. (2018), "Optimal Indirect Taxes and Subsidies under Imperfect Competition," *Journal of Institutional and Theoretical Economics*, 174(2), 334-350.

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