Financial shocks and endogenous labor market participation

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Abstract

This article studies the effects of financial shocks on the labor market when participation in the labor force is endogenous. Previous research concerning endogenous participation produced models that generated a counterfactually procyclical unemployment rate and a positively sloped Beveridge curve. This paper shows that collateral constraints alone are not able to produce correlations in line with the data. However, financial shocks, that change the collateral requirements, are responsible for most of the movements on the labor market and generate a countercyclical unemployment and a negatively slopped Beveridge curve.

JEL Codes: E32,E44,J63,J64

Keywords: Endogenous participation, Financial shocks

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1 Introduction

The Great recession of 2008 showed the importance of understanding the effects of financial frictions on the economy. A significant drop in the employment rate as well as in GDP was accompanied by a tightening of credit conditions and a delayed decrease in the labor market participation rate. However, the 2008 financial crisis was not an isolated episode. As it can be seen in Figure 1, which shows the detrended\(^1\) GDP, employment, labor force and tightening standards for the US economy, a similar pattern occurred in previous recessions even if on a smaller scale.

The economy and the labor market are not immune to fluctuations in the financial market. From Figure 1 we can see that there is a negative correlation between tightening standards in the financial market, measured as the net percentage of domestic banks tightening standards for commercial and industrial Loans, and the cyclical component of GDP and the employment rate.

According to Figure 2, which shows the cross correlation of GDP with selected labor market variables, the labor force participation rate is procyclical. In addition, the evidence, provided by Elsby et al (2015) [10], shows that up to a third of labor market volatility can be explained by the participation margin. Furthermore, policy makers are increasingly interested in the movements in the labor participation margin (Bernanke(2012) [2], Draghi (2014)[8] and Yellen (2014) [28]).

This article investigates the interaction of labor market participation, financial frictions and the business cycle and aims to understand what the aggregate implications of endogenous labor participation are, amid financial shocks.

The model developed in this article is able to generate moments that are on par with the data. That is, a negatively slopped Baveridge curve, a counter cyclical unemployment rate, a pro cyclical labor force participation rate and less volatile wages. Furthermore, the introduction of the participation margin gives the household and extra instrument to smooth its consumption. As a consequence, the volatility of unemployment and the labor market tightness is reduced. This result gives an extra strength to the Shimer puzzle since models would overestimate the volatility of unemployment when the participation margin is absent.

The following model generates a frictional financial market by adding a collateral constraint, as in Kiyotaki and Moore (1997) [17] and Jermann and Quadrini (2012) [15], to the investment decision of firms. The labor market friction with endogenous participation is similar to the ones in Shimer (2013) [25] and Campolmi and Gnocchi (2016) [6].

\(^1\)Variables detrended following Hamilton (2017)[14]
The financial frictions of this model are closely related to the seminal paper of Jermann and Quadrini (2012) [15]. The authors set up a framework where firms are also bound to a collateral constraint. Furthermore, bonds and the payment of dividends are also subject to frictions. They show that when an economy is hit by a negative financial shock, firms need to decrease investments and the number of workers. The work of Garín (2015) studies the effect of financial shocks on unemployment dynamics when the labor market is subject to search and matching frictions. In addition, Zanetti (2017) studies the interaction of financial and labor market frictions by incorporating a job destruction shock. Epstein et al (2017) [11] also have model with collateral constrains and a labor market with on the job search. This paper aims to improve the literature by adding a participation margin to the collateral constrain and frictional labor market framework.

Table 1 presents the correlation matrix of the detrended variables seen in the US data. The key points of this table are that there is a negative correlation between unemployment and vacancies, the unemployment rate is strongly counter-cyclical, and the labor force participation is pro-cyclical. The Filter suggested by Hamilton
Table 1: Correlation matrix (Data)

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>U</th>
<th>v</th>
<th>θ</th>
<th>n</th>
<th>w</th>
<th>lf</th>
<th>I</th>
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</thead>
<tbody>
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<td>-0.82</td>
<td>0.80</td>
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<td>0.86</td>
<td>0.53</td>
<td>0.46</td>
<td>0.82</td>
</tr>
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<td>-0.83</td>
<td>-0.67</td>
<td>-0.32</td>
<td>-0.76</td>
<td></td>
</tr>
<tr>
<td>v</td>
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<td>0.79</td>
<td>-0.64</td>
<td>0.38</td>
<td>0.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ</td>
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<td>0.83</td>
<td>-0.13</td>
<td>0.36</td>
<td>0.67</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>n</td>
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<td>-0.15</td>
<td>0.63</td>
<td>0.74</td>
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<td></td>
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<tr>
<td>w</td>
<td>1</td>
<td>-0.58</td>
<td>-0.14</td>
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<td></td>
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</tr>
</tbody>
</table>

Figure 2: Cross-correlation

Ebbel (2011) [9] proposes a

(2017) [14] generates a correlation between GDP and the labor force that is four times higher than the one produced with the HP-filter.

One problem of search models with endogenous labor supply is that they produce a positive correlation between unemployment and vacancies (Ravn (2008) [21], Veracierto (2008) [26]). That is, the Beveridge curve is positively sloped. The intuition behind this result is the following, if a firm increases the level of vacancies, because of a positive shock, the job finding probability and the resulting wages will also go up. This in turn increases the benefit of participating in the labor market. As a result, the unemployment goes up.

Several authors tried to overcome this problem. Ebbel (2011) [9] proposes a
new calibration strategy. Another approach is to make wages less volatile to curb the incentives of nonparticipants or job seekers to join or leave the labor market. Shimer (2013) [25] showed that models with endogenous participation are able to produce the expected labor market statistics when wages are rigid. Burda et al (2016) [5] introduced countercyclical payroll taxes, resulting in a decrease in net wage volatility, to generate a negatively sloped Beveridge curve.

The following model is able to tackle all three problems with models with endogenous participation, namely: the negative correlation between unemployment and vacancies; the countercyclicality of unemployment, and the procyclicality of the labor force. The introduction of financial frictions enables the generation of these results. In addition, an unemployment insurance rule that takes into account the unemployed previous history helps bring the moments closer to the data.

Another benefit of the introduction of financial frictions to this kind of modeling is that it is normally capable of generating more volatility in labor market variables (Petrosky-Nadeau (2014) [19]). The following model produces high volatility in unemployment and vacancies. However, the introduction of the participation margin decreases the labor tightness volatility.

Even though the model is able to decrease the volatility of wages, it is still too volatile. For that reason, an extension was made where only a fraction of workers are able to renegotiate their salary each period. This staggered wage bargaining model based on Gertler and Trigari (2009) [13] generates a recovery labor pattern similar to the one seen in the great recession. That is, the GDP recovers faster than the unemployment rate. Afterwards the employment rate goes back to its pre-crisis level and lastly the labor market participation returns to its steady state level. Therefore, the participation rate remains contracted longer than the other labor market variables.

There are also other papers that link financial frictions and employment. Wesselbaum (2016) [27] established a link between financial frictions, by Tobin’s Q, and labor market frictions, by the matching efficiency. Boeri et al (2017) [3] integrate the labor search model of DMP with the Holmstrom and Tirole model of demand for liquidity. They show that high leveraged firms tend to experience higher employment losses during a recession. This paper is also related to Buera et al (2015)[4] that created a model with heterogeneous agents and labor and credit market frictions.

The paper is organized as follows: Section 2 presents the model. Section 3 describes the calibration. Section 4 presents the results. Section 5 presents an extension. Section 6 concludes the paper.
2 Model

The following model describes an economy with two agents: households and firms. Households work in the market for the firms and produce a home good. Non-participants (those not in the labor market) are fully dedicated to home production. On the other hand, participants in the labor market are either employed or looking for employment. The household consumes market and home goods and saves by acquiring firm issued bonds.

Firms use capital and labor in production. Given a timing mismatch, firms need to post vacancies and pay salaries and interests before the income from production is received. For that reason, an intra-period loan is made where capital is used as collateral. Therefore, capital has two functions. It is an input in the production function and it is also used as collateral. A financial shock affects the amount of collateral needed for the intra-period loan. Since capital is used also for production, the financial shock has a direct impact on the firms capacity to post vacancies and pay wages.

Households see the impact of financial shocks on their incentives to participate in the labor market through wages and the job finding probability. If, for example, a financial shock generates a decrease in employment and wages, then households would have an incentive to leave the labor market. This movement would mechanically decrease the unemployment rate, not because jobs were created, but because there is less people looking for work. Therefore, financial shocks have extra effects on the labor market when participation is endogenous. The following section describes the model in more detail from the perspective of each agent.

2.1 Households

The economy is populated by a representative household family of measure one which is divided into workers, unemployed and those that are outside of the labor market, the non-participants. It is assumed that there is perfect income insurance within the household members. That is, agents within the household share their resources like in Merz (1995) [18] and Andolfatto (1996) [1]. Consumption and a home good, produced by those who are not working, enter the utility function that reads:

\[ U(c, h) = \frac{c^{1-\sigma}}{1-\sigma} + \chi \frac{h^{1-\nu}}{1-\nu} \]  

(1)

Following Campolmi and Gnocchi (2016) [6], the home production function has
the following form.

\[ h_t = (1 - n_t - U_t)^{1-\alpha_h} \]  

Non-participants dedicate all of their time to home production, while those unemployed spend all their time looking for employment.

The Household save in bonds \((b_t)\) issued by firms, pay a lump-sum tax \((T_t)\) and receive income from wages \((w_t)\), unemployment insurance \((UI_t)\) and dividends from the firms \((d_t)\).

Therefore, the pooled budget constraint of the household is:

\[ C_t + \frac{b_{t+1}}{1+r_t} \leq b_t + w_t n_t + d_t + UI_t - T_t \]  

### 2.2 Labor Market

At the beginning of each period, the firm chooses the amount of job vacancies posted \((v_t)\). Then, the household chooses the measure, \(S_t\), of agents that are going to search for a job. Given, \(v_t\) and \(S_t\), a measure \(m(v_t, S_t)\) of labor relations are formed. The matching technology, originated from Den Haan et al (2000) [7], has the desired properties that matching probabilities lie in the unit interval and has the following form

\[ m(v_t, S_t) = \frac{v_t S_t}{(v_t^\gamma + S_t^\gamma)^{1/\gamma}} \]  

The labor market tightness is \(\theta_t = \frac{v_t}{S_t}\). Hence, the job finding rate is \(p(\theta_t) = \frac{m(v_t, S_t)}{S_t}\) and the job filling rate is \(q(\theta_t) = \frac{m(v_t, S_t)}{v_t}\). At the end of each period the mass of, end of period, unemployed agents is \(u_t = (1 - p(\theta_t))S_t\) and the unemployment rate is:

\[ U_t = \frac{u_t}{n_t + u_t} \]

The timing of the events goes as follows. At the end of each quarter, an exogenous proportion, \(\delta_n\), of workers get separated from their jobs. In the following period, after \(s_t\) and \(v_t\) are chosen, a measure \(m(v_t, s_t)\) of labor relations are created. Once a match is made, it becomes active in the same period. This is not a strong hypothesis since a period corresponds to a quarter. Therefore, the employment rate evolves according to the following law of motion

\[ n_t = (1 - \delta_n)n_{t-1} + m(v_t, s_t) \]
2.3 Unemployment insurance

There is unemployment insurance in the economy. The unemployed are entitled to the benefit $\bar{w}$ for as long as they are unemployed. However, only those looking for a job that lost their employment contract can receive the unemployment benefit. That is, job seekers originated from outside the labor force are not entitled to receive it. This distinction is necessary because, if all job seekers were entitled to the benefit, those outside of the labor market would have an extra and unrealistic incentive to search for a job. There is no informational problem in the economy. Policy makers are able to verify perfectly who is unemployed. Hence, the government’s total expenditure in unemployment insurance is

$$UI_t = \sum_{i=0}^{\infty} \prod_{j=1}^{i} (1 - p(\theta_{t-j})) \delta_n n_{t-i} \bar{w}$$

By this formula, it is required to keep track of the level of employment and the job finding rate from $t = 0$ to $t = -\infty$. In order to make the model more tractable and remembering that the importance of past employment levels on the total amount of unemployment insurance decreases exponentially, a simplifying assumption is made. All employment rates and job finding probabilities from before $t - 1$ are assumed to have its steady state value. That is, $p(\theta_{t-j}) = p(\theta)$ and $n_{t-j} = n$ for $j \geq 2$. Where $p(\theta)$ and $n$ are the respective steady state values. The formula becomes

$$UI_t = \bar{w} \delta_n (1 - p(\theta_t)) \left( n_{t-1} + \frac{(1 - p(\theta))n}{p(\theta)} \right)$$

(7)

2.4 Problem of the Household

Given the above, the household solves the following problem:

$$H(n_{t-1}, b_t) = \max_{C_t, S_t, b_{t+1}, n_t} U(C_t, h_t) + \beta E_t [H(n_t, b_{t+1})]$$

$$C_t + \frac{b_{t+1}}{1 + r} \leq b_t + \bar{w} n_t + d_t + UI_t - T_t$$

$$n_t = (1 - \delta_n) n_{t-1} + p(\theta) S_t$$

(8)

Which delivers the following optimality conditions

$$UC_t = \beta (1 + r_t) E_t [UC_{t+1}]$$

(9)

$^2$Detailed derivations and FOCs are in the appendix.
\[
U_{ht} = \frac{w_t U_{ht}}{P(\theta_t)} = w_t U_{C_t} + \beta E_t \left( (1 - P(\theta_t+1))U_{C_{t+1}} + \left( \frac{1 - \delta}{P(\theta_{t+1})}U_{h_{t+1}} + \bar{w} \delta_n \right) \right) \tag{10}
\]

Equation 9 is the standard Euler equation. Equation 10 is the optimality condition for labor decisions. The cost of searching for a job, in the LHS, is equal to the marginal benefit of the job plus its continuation value. From this equation it is possible to verify the distortion caused by the labor market friction. If we assume that the job finding probability is one, making the labor market perfect, we would have that \(w_t = \frac{U_{ht}}{U_{C_t}}\). Since there is no disutility from working in the model, the equilibrium wage would be the marginal rate of substitution between the market and home good. Deviations from this price indicate the magnitude of the market imperfection.

2.5 Firms

There is a measure one of firms that maximize the stream of dividends \(d_t\) subject to the following budget constraint

\[
F(a_t, k_t, n_t) + \frac{b_{t+1}}{R_t} + (1 - \delta)k_t = \varphi(d_t) + w_t n_t + k_{t+1} + g_t v_t + b_t
\]

where \(F(a_t, k_t, n_t)\) is the production function, \(b_t\) is the firms’ debt issued to households, \(R_t = 1 + (1 - \tau) r_t\) is the effective gross interest rate, \(i_t = k_{t+1} - (1 - \delta)k_t\) is investment, \(v_t\) is the number of job vacancy postings and \(g_t = \Gamma\) is the cost of posting a job vacancy. Since there are no idiosyncratic shocks in the model, there is no reason for a firm to act differently from another one. Therefore, they are treated as one representative firm.

Following Jermann and Quadrini (2012) [15] I assume that the firm’s payout costs are quadratic.

\[
\varphi(d_t) = d_t + \kappa (d_t - \bar{d})^2 \tag{11}
\]

Where \(\bar{d}\) is the steady state value for dividend payout. This quadratic cost can be justified by the firm shareholders’ preferences for a smooth distribution of dividends throughout time. Garín (2015) [12] instead of distributing dividends to households, opted to make risk averse capitalists own the firms. In this way there is no need for quadratic costs. Both modeling choices deliver the similar mechanics.

Since firms receive revenues after production takes place (but within the same period) they need to make an intra-period loan, \(l_t\), to finance working capital. That
is used to pay wages, vacancy costs, new capital investments and to refinance their debt.

\begin{equation}
l_t = \varphi(d_t) + w_t n_t + k_{t+1} - (1 - \delta_k)k_t + g_t v_t + b_t - \frac{b_{t+1}}{R_t} = F(a_t, k_t, n_t)
\end{equation}

Firms can choose to default the intra-period loan. If this happens, the firms collateral, capital \((k_t)\), gets confiscated. However, there is a possibility, \(\xi_t\), that the collateral has no worth, in case of defaulting \(l_t\), the firms are subject to an enforcement constraint.

\begin{equation}
\xi_t \left( k_{t+1} - \frac{b_{t+1}}{1 + r} \right) \geq l_t
\end{equation}

Where \(\xi_t\) is the probability that the collateral has value and the financier can use the collateral. Jermann and Quadrini [15] showed that, if firms are bounded to this collateral constraint, it is optimal to always make the intra-period loan repayment. Using the definition of \(l_t\) and replacing in the budget constraint, assuming \(\tau = 0\) for expositional clarity, we have

\begin{equation}
\xi_t \left( (1 - \delta_k)k_t - w_t n_t - b_t - \varphi(d_t) - g_t v_t \right) \geq (1 - \xi_t)F(a_t, k_t, n_t)
\end{equation}

If the economy is hit by a negative TFP shock (reduction in \(a_t\)), then the output, \textit{ceteris paribus}, would shrink relaxing the collateral constraint. This would allow the firm to increase the number of workers or the amount of dividends paid. On the other hand, if there is a decrease in \(\xi_t\), the collateral constraint becomes more stringent and the firms would need to decrease the amount of vacancies posted or dividends paid.

\subsection{2.6 Problem of the firm}

Given the above description of the production sector, firms have the following problem.

\begin{align}
V(n_{t-1}, b_t, k_t) &= \max_{d_t, n_t, k_{t+1}, b_{t+1}, n_t} d_t + \mathbb{E} [\beta_{t,t+1}V(n_t, b_{t+1}, k_{t+1})] \\
\varphi(d_t) + w_t n_t + k_{t+1} - (1 - \delta_k)k_t + g_t v_t + b_t \leq \frac{b_{t+1}}{R_t} + F(a_t, k_t, n_t) & (\lambda_1) \\
n_t = (1 - \delta_n)n_{t-1} + q(\theta_t)v_t & (\lambda_2) \\
\xi_t \left( k_{t+1} - \frac{b_{t+1}}{1 + r} \right) \geq F(a_t, k_t, n_t) & (\mu)
\end{align}

(14)
where $\lambda_1$, $\lambda_2$ and $\mu$ are the Lagrange multipliers. The optimality conditions of this problem are

$$
E_t \left[ \beta_{t,t+1} \left( \frac{(1 - \delta_k) + F_{k_{t+1}}}{\varphi_d(d_{t+1})} - \mu_{t+1} F_{k_{t+1}} \right) \right] + \xi_t \mu_t = \frac{1}{\varphi_d(d_t)} \tag{15}
$$

$$
E_t \left[ \beta_{t,t+1} \frac{(1 - \delta_n) g_{t+1}}{\varphi_d(d_{t+1}) q(\theta_{t+1})} \right] + \left[ F_{n_{t+1}} - w_t \right] \frac{g_t}{\varphi_d(d_t) q(\theta_t)} = \mu_t F_{n_t} \tag{16}
$$

$$
\frac{\xi_t \mu_t R_t \varphi_d(d_t)}{1 + r_t} + E_t \left[ \beta_{t,t+1} \frac{\varphi_d(d_t) R_t}{\varphi_d(d_{t+1})} \right] = 1 \tag{17}
$$

Equation 15 presents the firm’s optimality condition with respect to capital. It is possible to see two sources of frictions. The marginal dividend payout cost $\varphi_d(d_t)$ and collateral constraint measured by the Lagrange multiplier $\mu$. If none of these frictions were present, that is, $\varphi_d(d_t) = 1$ and $\mu = 0$, in the steady state we would have that $F_k = \frac{1}{\beta} + \delta_k = r + \delta_k$. The model would deliver the optimal amount of capital. Since $\varphi_d(d_t) \neq 1$ outside the steady state, if we assume that $\varphi_d(d_t) \geq \varphi_d(d_{t+1})$ the resulting marginal product of capital would be higher than the optimal one. That is, there would be less capital in the economy. However, $\mu$ has the opposite effect on the economy. There would be more capital in equilibrium.

Equation 16 presents the firm’s optimality condition with respect to labor. There are three sources of frictions, two financial and one from the labor market. If no frictions were present, the conditions would become $F_{n_t} = w_t$. If there were no search frictions, $g_t = 0$, the condition would be $F_{n_t} = \frac{\varphi_d(d_t) w_t}{1 - \mu_t}$. Both of these frictions increase the cost of the firm and, therefore, the labor demand is smaller than optimal.

### 2.7 Wage determination

The equilibrium wage is determined by Nash bargaining. When a match occurs, the newly hired employee and the employer split the surplus of the match. For the worker, the outside option is to become unemployed and for the firm the outside option is to lose the match and have to wait another period to fill the vacancy.

The marginal surplus of working for the household is

$$
\frac{H^E_t}{U_c} = w_t - \frac{U_{b_{t+1}}}{U_c} + E_t \left[ \frac{\beta}{U_c} \left( (1 - \delta_n (1 - p(\theta_{t+1}))) H^E_{t+1} + \delta_n (1 - p(\theta_{t+1})) H^U_{t+1} \right) \right] \tag{18}
$$

The marginal surplus of losing a job and becoming unemployed for the household

---

3Detailed derivations are in the appendix.
is

$$\frac{H_t^U}{U_c} = \bar{w} - \frac{U_{ht}}{U_c} + \mathbb{E}_t \left[ \frac{\beta}{U_c} \left( p(\theta_{t+1})H_{t+1}^E + (1 - p(\theta_{t+1}))H_{t+1}^U \right) \right] \quad (19)$$

The marginal surplus of an additional worker for the firm is

$$\frac{\partial V_t}{\partial n_t} \bigg|_{v=v_t} = V_t^j = \frac{F_{nt} - w_t}{\varphi_d(d_t)} - \mu_tF_{nt} + \mathbb{E}_t \left[ \beta_{t,t+1}(1 - \delta_n)V_{t+1}^j + \delta_nV_{t+1}^v \right] \quad (20)$$

Given the free entry condition we have that the value of an additional vacancy for the firm is zero. Therefore,

$$V_t^j = \frac{g_t}{\varphi_d(d_t)q(\theta_t)} \quad (21)$$

Since wages are determined by Nash bargaining, the agreed wage rate is the one where a fraction \( \eta \) of the overall match surplus is allocated to the worker and a fraction \((1 - \eta)\) to the firms. Therefore, it is possible to derive the following solution\(^4\)

$$\eta \left[ \frac{H_{t}^E - H_{t}^U}{U_c} \right] = \eta \varphi_d(d_t)V_t^j \quad (22)$$

Using 18, 19, 21 and 22 it is possible determine the wage equation that has the following form.

$$w_t = \eta \left[ 1 - \mu_t\varphi_d(d_t) \right] F_{nt} + (1 - \eta)\bar{w} + \mathbb{E}_t \left[ \beta_{t,t+1}(1 - \delta_n) \left( \frac{\varphi_d(d_t)}{\varphi_d(d_{t+1})} - (1 - p(\theta_{t+1})) \right) (1 - \varphi_d(d_{t+1})q(\theta_{t+1})) \right] \quad (23)$$

The resulting bargaining outcome in Equation 23 shows that shocks have non trivial effects on the equilibrium wage. A negative TFP shock will decrease the marginal product of labor \( (F_{nt}) \). However, as shown in Equation 13 the same shock relaxes the collateral constraint and diminishes \( \mu_t \). Therefore, it is unclear how a negative TFP shock affects wages.

The friction deriving from the dividend costs also play a role on wages. The higher the friction, smaller the equilibrium wage will be. However, there is also a dynamic effect. If firms expect the friction to increase, that is \( \varphi_d(d_{t+1}) > \varphi_d(d_t) \), then wages will go down.

\(^4\)Detailed derivations are in the appendix
2.8 Government

The government collects a lump-sum tax $T_t$ in order to finance the subsidies to the firms borrowing, the unemployment insurance and its own expenses ($G$). Therefore, the government budget constraint is

$$ T_t = \frac{b_{t+1}}{R_t} - \frac{b_{t+1}}{(1 + r_t)} + G + UI_t $$

(24)

2.9 Equilibrium

The equilibrium can be defined by the equations 9, 10 12, 15, 16, 17, 23, and the households, firms and government budget constraint.

It is good to notice that given the financial frictions, the collateral constraint is always binding in the steady state.

**Proposition 1.** The collateral constraint is binding in the non-stochastic steady state.

**Proof.** Suppose not, that is $\mu = 0$. Then, Equation 17 reads

$$ \beta R = 1 $$

and from Equation 9 we have

$$ \beta (1 + r) = 1 $$

Therefore, $R = (1 + r) \iff \tau = 0$

\[ \square \]

3 Calibration

Table 2 shows the values used in the calibration of the model. Most values like the households’ risk aversion and the discount factor are standard in the literature. The parameters concerning home production $\nu$ and $\alpha_h$ come from Campolmi and Gnocchi (2016) [6]. The utility from home production $\chi$ was set so that 30% of households would be non-participants. The steady state level of the lender recovery probability, $\xi$, was set so that the ratio between bonds and GDP would be 2.3. The dividend deviation cost $\kappa$ and the matrix $A$ comes from the recalibration done by Pfeifer (2016) [20]. The government parameters $G$ and $\tau$ are from Jermann and Quadrini (2012) [15]. The labor market parameters were set to match the steady state level of unemployment of 6% and the labor market tightness of 0.8. The labor separation rate and depreciation of capital are also standard. The first column of Table 3 shows the steady state values of the model.
Table 2: Calibration values

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Labor utility curvature parameter</td>
<td>5</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Preference for home over market goods</td>
<td>0.06</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>replacement rate of the unemployment insurance</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Marginal return to labor</td>
<td>1/3</td>
</tr>
<tr>
<td>$\alpha_h$</td>
<td>Marginal return to labor in home good</td>
<td>1/3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9926</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>dividend deviation cost</td>
<td>0.08</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Worker’s bargaining power</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Matching function efficiency</td>
<td>2</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Lender recovery probability</td>
<td>0.115</td>
</tr>
<tr>
<td>$G$</td>
<td>Government purchases</td>
<td>0.66</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax</td>
<td>0.35</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Vacancy posting cost</td>
<td>1.15</td>
</tr>
<tr>
<td>$\delta_n$</td>
<td>Labor separation rate</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>Capital depreciation</td>
<td>0.025</td>
</tr>
<tr>
<td>$A$</td>
<td>Matrix for the shocks process</td>
<td>$\begin{bmatrix} 0.9736 &amp; -0.0287 \ 0.1509 &amp; 0.9363 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

The Financial efficiency ($\xi_t$) and productivity ($a_t$) variables follow the same structure as in Jermann and Quadrini(2012) [15]. That is

$$\begin{pmatrix} \hat{a}_t \\ \hat{\xi}_t \end{pmatrix} = A \begin{pmatrix} \hat{a}_{t-1} \\ \hat{\xi}_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{a_t} \\ \varepsilon_{\xi_t} \end{pmatrix}$$

(25)

Table 3 shows the steady state values achieved with the described calibration. It is worth noting that the steady state in the two other versions of the model, with exogenous labor participation and staggered wages, are identical to the one in the full model.

4 Results

Table 4 shows the relative business cycles standard deviations based on the US data (described in the appendix), and the data produced by several specifications of the model. All variables were detrended according to the method suggested by Hamilton (2017) [14], using a forecasting horizon ($h$) of 8 periods and a linear projection using the variables’ four most recent values ($p=4$).

The model was calibrated so that the relative volatility of the labor force would be equal to the one seen in the data. Row (ii) shows the variables’ volatility of the
Table 3: Steady state

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product (Y)</td>
<td>2.15</td>
<td>Employment (n)</td>
<td>0.65</td>
</tr>
<tr>
<td>Capital (K)</td>
<td>23.44</td>
<td>Unemployment rate (U)</td>
<td>0.06</td>
</tr>
<tr>
<td>Investment (I)</td>
<td>0.59</td>
<td>Job seekers (S)</td>
<td>0.11</td>
</tr>
<tr>
<td>Consumption (C)</td>
<td>0.80</td>
<td>Vacancies (v)</td>
<td>0.08</td>
</tr>
<tr>
<td>Home production (h)</td>
<td>0.46</td>
<td>Labor force (LF)</td>
<td>0.69</td>
</tr>
<tr>
<td>Dividends (d)</td>
<td>0.15</td>
<td>Labor market tightness (θ)</td>
<td>0.78</td>
</tr>
<tr>
<td>Bonds (b)</td>
<td>4.81</td>
<td>p(θ)</td>
<td>0.61</td>
</tr>
<tr>
<td>Wage (w)</td>
<td>1.99</td>
<td>q(θ)</td>
<td>0.79</td>
</tr>
</tbody>
</table>

full model. That is, the model with endogenous participation and flexible wages. Row (iii) shows the volatility for the model without the participation margin. The table shows that the introduction of the participation margin increases the volatility of employment, bringing it closer to the data, and decreases the volatility of wages when compared with the model with the exogenous labor force.

This improvement comes with a decrease in the volatility of unemployment, and labor market tightness (θ). Since, now, households can optimally choose the amount of agents in the labor market. They do so as a consumption smoothing mechanism. Therefore, this result adds strength to the Shimer puzzle. If search and matching model of the labor market were able to generate the correct volatility of labor market tightness, the model with exogenous participation would lead to an overestimation of the volatility.

As seen in the literature, Vercierto (2008) [26] and Ravn (2008) [21], search and matching models with endogenous participation produce three counterfactual results. A positive correlation between GDP and unemployment, and between unemployment and vacancies, and a negative correlation between the labor force and GDP.

Rows (vi) and (vii) of Table 4 show the standard deviation of the full model when only the financial or TFP shocks are present. In both specifications, the volatility of all labor market variables increases. The puzzling part of this result is why the labor market volatility decreases once we combine both shocks. Table 5 shows the these three correlation for both models. The full model, with financial friction but only with TFP shocks, still produces a pro-cyclical unemployment rate. However, the model with only financial shocks produces correlations in line with the data. Therefore, the volatility of the labor market decreases when the model has both shocks because of the TFP shock, which produces movements in a counterfactual direction and curbs the total volatility of the model.
Table 6 and Table 7 show the correlations of the model with and without the endogenous participation margin. It is possible to see that the full model is able to generate correlations in the right direction.

### Table 4: Relative standard deviation

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>U</th>
<th>v</th>
<th>θ</th>
<th>n</th>
<th>w</th>
<th>LF</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Data</td>
<td>1</td>
<td>6.42</td>
<td>6.59</td>
<td>12.71</td>
<td>0.63</td>
<td>0.67</td>
<td>0.24</td>
</tr>
<tr>
<td>(ii)</td>
<td>Full model</td>
<td>1</td>
<td>4.55</td>
<td>4.10</td>
<td>3.62</td>
<td>0.47</td>
<td>1.17</td>
<td>0.24</td>
</tr>
<tr>
<td>(iii)</td>
<td>Exo model</td>
<td>1</td>
<td>6.34</td>
<td>4.31</td>
<td>5.09</td>
<td>0.41</td>
<td>1.41</td>
<td>-</td>
</tr>
<tr>
<td>(iv)</td>
<td>Staggered wage</td>
<td>1</td>
<td>5.03</td>
<td>4.39</td>
<td>4.06</td>
<td>0.50</td>
<td>0.82</td>
<td>0.23</td>
</tr>
<tr>
<td>(v)</td>
<td>Staggered wage (exo)</td>
<td>1</td>
<td>6.82</td>
<td>4.66</td>
<td>5.45</td>
<td>0.44</td>
<td>0.95</td>
<td>-</td>
</tr>
<tr>
<td>(vi)</td>
<td>Only financial shocks</td>
<td>1</td>
<td>9.07</td>
<td>10.03</td>
<td>7.65</td>
<td>1.07</td>
<td>1.93</td>
<td>0.59</td>
</tr>
<tr>
<td>(vii)</td>
<td>Only TFP shocks</td>
<td>1</td>
<td>5.77</td>
<td>5.70</td>
<td>4.74</td>
<td>0.58</td>
<td>1.33</td>
<td>0.35</td>
</tr>
</tbody>
</table>

### Table 5: Selected correlations of models with only one shock

<table>
<thead>
<tr>
<th></th>
<th>TFP shocks</th>
<th>Financial shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(corr(U, Y))</td>
<td>0.05</td>
<td>-0.75</td>
</tr>
<tr>
<td>(corr(U, v))</td>
<td>-0.40</td>
<td>-0.37</td>
</tr>
<tr>
<td>(corr(LF, Y))</td>
<td>0.31</td>
<td>0.63</td>
</tr>
</tbody>
</table>

### Table 6: Correlation Matrix (full model)

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>U</th>
<th>v</th>
<th>θ</th>
<th>n</th>
<th>w</th>
<th>LF</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1</td>
<td>-0.74</td>
<td>0.44</td>
<td>0.71</td>
<td>0.63</td>
<td>0.96</td>
<td>0.39</td>
<td>0.97</td>
</tr>
<tr>
<td>U</td>
<td>1</td>
<td>-0.44</td>
<td>-0.87</td>
<td>-0.92</td>
<td>-0.89</td>
<td>-0.59</td>
<td>-0.72</td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>1</td>
<td>0.82</td>
<td>0.68</td>
<td>0.67</td>
<td>0.79</td>
<td>0.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>1</td>
<td>0.95</td>
<td>0.94</td>
<td>0.82</td>
<td>0.61</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>1</td>
<td>0.87</td>
<td>0.83</td>
<td>0.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w</td>
<td>1</td>
<td>0.65</td>
<td>0.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LF</td>
<td>1</td>
<td></td>
<td></td>
<td>0.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3 Shows the IRF of a negative financial shock to the full model and the one with exogenous participation. This shock represents a decrease in the collateral recovery probability \(\xi_t\). Following the shock firms are obliged to reduce their intertemporal loan given the increase in collateral requirements. Hence, there is a reduction in vacancies and investment. Since the job destruction rate is constant, by assumption, there is a reduction in employment that leads to a reduction in production. In the exogenous participation case, all the job losses are transformed into unemployment. However, when the participation is endogenous, agents can leave the labor market and dedicate themselves to home production. Therefore, the
Table 7: Correlation Matrix (Exogenous LF)

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>U</th>
<th>v</th>
<th>θ</th>
<th>n</th>
<th>w</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1</td>
<td>-0.62</td>
<td>0.42</td>
<td>0.58</td>
<td>0.63</td>
<td>0.82</td>
<td>0.97</td>
</tr>
<tr>
<td>U</td>
<td>1</td>
<td>-0.74</td>
<td>-0.96</td>
<td>-1.00</td>
<td>-0.94</td>
<td>-0.54</td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>1</td>
<td>0.93</td>
<td>0.74</td>
<td>0.77</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>1</td>
<td>0.96</td>
<td>0.93</td>
<td>0.47</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>1</td>
<td>0.94</td>
<td>0.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w</td>
<td>1</td>
<td>0.73</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

impact of financial shocks on unemployment is halved when agents can leave the labor market.

Figure 3: IRF of a negative financial shock($a_t$)

As noted by Garín (2015) [12] even if the framework with collateral constraints improves the performance of the model, it also generates a counterfactual result, the increase in consumption following a negative financial shock. After the shock, firms decrease their level of borrowing. Since households are now saving less, they balance their budget constraint by consuming more. As is shown in Figure 4, the introduction of endogenous labor supply reduces the increase in consumption, but does not make this counterfactual result disappear. Even if the importance of financial shocks on consumption is negligible when compared with TFP shocks. Adding
another saving mechanism option to households could dissipate this result.

Figure 4: IRF of a negative productivity shock\(a_t\)

Figure 4 shows the IRF of a negative TFP shock to the full model and the one with exogenous participation. Following a negative TFP shock, there is a decrease in production and investment. However, the drop in employment is greater in the endogenous case even if the increase in unemployment is slightly smaller. This happens because households leave the labor market given the adverse job finding probabilities. Since now there are less people looking for employment, the amount of new matches decrease.

Since the households are better off with the participation margin, which gives them an extra mechanism to smooth the impact of negative shocks - if they were not, the optimal behavior would be to keep the labor force constant - the outside option of working is improved (when compared with the exogenous participation case). Their bargaining position is improved and the drop in wages is mitigated. Therefore, the drop in consumption is smaller in the endogenous participation case.

4.1 Variance decomposition

In order to understand the magnitude and importance of financial shocks on the economy and on the labor market, the variance decomposition of the TFP and Fi-
financial shocks were computed. The results can be seen in Table 8. GDP, investment and wages are predominantly influenced by the TFP shock and the labor market variables (unemployment, employment, labor market tightness and participation rate) are mostly influenced by the financial shock. This asserts the importance of financial shock on the economy and, in particular, the labor market.

The variance decomposition was also calculated for the model with exogenous participation. The results, available in Table 10 in the appendix, are quite similar to the ones of the full model. The main difference is that in the exogenous participation case, the financial shocks gain extra importance in the volatility of labor market variables.

## 5 Staggered wage bargaining

In the following section, an extension of the full model is elaborated. It aims to address the small variance of the labor market tightness and the high volatility of wages. This is done by not allowing every labor relation to renegotiate their wages every period.

As pointed out by Shimer (2005) [23] search and matching models deliver too little movement in the labor market tightness when compared to output. The data shows that the standard deviation of the former is 12 times higher than the latter, as seen in Table 4. The reason for that is the high volatility of wages. Shocks are absorbed by price adjustments and, as a result, the labor market quantities do not vary much.

To be able to dampen the high variance of wages, I use the fact that wages are not renegotiated every period. As in Gertler and Trigari (2009) [13], I assume that only a fraction (quarter) of firms are able to renegotiate wages in each period. There is no nominal of real wage rigidity, the introduction of staggered wages only makes
wages less responsive to shocks.

As it can be seen in Table 4, the introduction of staggered wage bargaining is able to decrease the standard deviation of wages and increase the standard deviation of the labor market tightness.

Figure 5 and Figure 6 compares the impulse response originated from a negative financial and TFP shock to the model with staggered wages and the baseline model (when all the wages are renegotiated every period). There are minor differences between the responses of both models to a financial shock. However, the model produces significantly different responses to a TFP shock. Since, now, firms cannot renegotiate all wages, there is a greater reduction in investment and vacancies following the shock.

Notice, that now the shock generates a positive response on the labor market. The costs to participate in the labor market increased since the job finding probability decreased together with the labor market tightness. However, the benefit of participating in the labor market increased since the wages did not decrease as much as in the baseline and the reduction of consumption increased the marginal utility of consumption.

Figure 5: IRFs Staggered wage bargaining (financial shock)

Figure 7 shows the IRF of GDP, employment, unemployment and the labor force for the model with staggered wage bargaining. We can see that the unemployment
Figure 6: IRFs Staggered wage bargaining (TFP shock)

Figure 7: Labor market recovery to a financial shock
rate recovers to its pre-shock level before the employment rate and the labor force participation has an even slower recovery. This picture fits with the aftermath of the financial crisis when the unemployment rate recovered to its pre-crisis level and the employment rate was still depressed. This late response of the participation margin is also on par with lagged correlation between GDP and labor force participation seen in Figure 2 and the recovery rate of the US labor market after the great recession. It is worth noticing that this jobless recovery effect - Schmitt-Grohé and Uribe (2017) [22] - completely disappears without the endogenous participation margin.  

6 Final remarks

In this article I show the importance of the labor participation margin amid financial frictions. This was done by incorporating the labor participation margin in a model with search and matching frictions in the labor market, and financial market frictions in the form of collateral constraints.

One of the main problems of models with endogenous participation is that they produce counterfactual moments. This paper shows that the introduction of financial shocks is key for generating realistic moments. Also, the same model with collateral constraints but without financial shocks still produces counterfactual results.

The labor market participation generates an extra mechanism for which the household can smooth consumption and utility. For that reason, the volatility of the labor market tightness is reduced. This result gives strength to the Shimer puzzle since it demonstrates that models with exogenous participation would naturally overestimate the labor market volatility.

Moreover, the model, when extended with staggered wage bargaining, produces an asymmetric recovery of the labor market variables to a financial shock. The unemployment rate recovers faster than the employment and labor force. This behavior mimics the recovery of the US economy after the great recession.

---

5Figure 8 in the appendix.
References


A Appendix: Detailed derivations

A.1 Problem of the Household

Given the above, the household solves the following problem:

\[
H(n_{t-1}, b_t) = \max_{C_t, S_t, b_{t+1}, n_t} U(C_t, h_t) + \beta E_t [H(n_t, b_{t+1})] \\
C_t + \frac{b_{t+1}}{1 + r} \leq b_t + w_t n_t + d_t + U I_t - T_t (\lambda_1) \\
n_t = (1 - \delta_n) n_{t-1} + p(\theta) S_t (\lambda_2)
\]

(26)

FOCs

\[
C_t : U_{C_t} - \lambda_{1t} = 0 \\
S_t : -\chi_u (1 - p(\theta_t)) U_{hE} + p(\theta_t) \lambda_{2t} = 0
\]

where \( U_{hE} = \chi h_t^{\alpha_h} (1 - \alpha_h) (1 - n_t) - \chi_u (1 - p(\theta_t)) S_t \) \( \alpha_h \)

\[
b_{t+1} : \beta H_{b_{t+1}} - \frac{\lambda_{1t}}{1 + r_t} = 0
\]

where \( H_{b_{t+1}} = E_t [\lambda_{1t+1}] \), therefore,

\[
U_{C_t} = \beta (1 + r_t) E_t [U_{C_{t+1}}]
\]

(27)

\[
n_t : -U_{hE} + \beta H_{n_t} + w_t \lambda_{1t} - \lambda_{2t} = 0
\]

where \( H_{n_t} = (1 - \delta_n) \lambda_{2t+1} + \delta_n (1 - p(\theta_t)) \lambda_{1t} \)

\[
\frac{U_{h_{nt}}}{P(\theta_t)} = w_t U_{C_t} + \beta E_t \left[ (1 - P(\theta_{t+1})) U_{C_{t+1}} + \left( \frac{1 - \delta_n}{P(\theta_{t+1})} U_{h_{nt+1}} + \tilde{w} \delta_n \right) \right]
\]

(28)
A.2 Problem of the firm

Given the above description of the production sector, firms have the following problem.

\[ V(n_{t-1}, b_t, k_t) = \max_{d_t, n_t, k_{t+1}, b_{t+1}, n_{t+1}} \left\{ d_t + E \left[ \beta_{t,t+1} V(n_t, b_{t+1}, k_{t+1}) \right] \right\} \]

\[ \varphi(d_t) + w_t n_t + k_{t+1} - (1 - \delta_k) k_t + g_t v_t + b_t \leq \frac{b_{t+1}}{R_t} + F(a_t, k_t, n_t) \left( \lambda_1 \right) \]

\[ n_t = (1 - \delta_n)n_{t-1} + q(\theta_t)v_t \left( \lambda_2 \right) \]

\[ \xi_t \left( k_{t+1} - \frac{b_{t+1}}{1 + r} \right) \geq F(a_t, k_t, n_t) \left( \mu \right) \]

FOCs

\[ d_t : \quad 1 - \varphi_d(d_t) \lambda_{1t} = 0 \]

\[ v_t : \quad -g_t \lambda_{1t} + q(\theta_t) \lambda_{2t} = 0 \]

\[ k_{t+1} : \quad \mathbb{E}_t \left[ \beta_{t,t+1} V_{k_{t+1}} \right] - \lambda_{1t} + \xi_t \mu_t = 0 \]

\[ b_{t+1} : \quad \mathbb{E}_t \left[ \beta_{t,t+1} V_{b_{t+1}} \right] + \frac{\lambda_{1t}}{R_t} - \frac{\xi_t \mu_t}{1 + r_t} = 0 \]

\[ E_t : \quad \mathbb{E}_t \left[ \beta_{t,t+1} V_{E_t} \right] + (F_{n_t} - w_t) \lambda_{1t} - \lambda_{2t} - F_{n_t} \mu_t = 0 \]

Envelope conditions

\[ V_{k_t} = [(1 - \delta_k) + F_{k_t}] \lambda_{1t} - F_{k_t} \mu_t \]

\[ V_{n_{t-1}} = (1 - \delta_n) \lambda_{2t} \]

\[ V_{b_t} = -\lambda_{1t} \]

\[ \mathbb{E}_t \left[ \beta_{t,t+1} \left( \frac{((1 - \delta_k) + F_{k_{t+1}})}{\varphi_d(d_{t+1})} - \mu_{t+1} F_{k_{t+1}} \right) \right] + \xi_t \mu_t = \frac{1}{\varphi_d(d_t)} \]
\[
\mathbb{E}_t \left[ \beta_{t,t+1} (1 - \delta_n) g_{t+1} \varphi_d(d_{t+1}) q(\theta_{t+1}) \right] + \frac{[F_{nt} - w_t]}{\varphi_d(d_t)} - \frac{g_t}{\varphi_d(d_t) q(\theta_t)} = \mu_t F_{nt} \quad (31)
\]

\[
\frac{\xi_t \mu_t R_t \varphi_d(d_t)}{1 + r_t} + \mathbb{E}_t \left[ \frac{\beta_{t,t+1} \varphi_d(d_t) R_t}{\varphi_d(d_{t+1})} \right] = 1 \quad (32)
\]

### A.3 Wage determination

The marginal surplus of an additional worker for the household is

\[
\frac{H^E_t}{U_c} = w_t - \frac{U_{hE}}{U_c} + \mathbb{E}_t \left[ \frac{\beta}{U_c} \left( (1 - \delta_n(1 - p(\theta_{t+1})))H^E_{t+1} + \delta_n(1 - p(\theta_{t+1})))H^U_{t+1} \right) \right] \quad (33)
\]

\[
\frac{H^U_t}{U_c} = \bar{w} - \chi \frac{U_{hE}}{U_c} + \mathbb{E}_t \left[ \frac{\beta}{U_c} \left( p(\theta_{t+1})H^E_{t+1} + (1 - p(\theta_{t+1})))H^U_{t+1} \right) \right] \quad (34)
\]

\[
\frac{H^E_t - H^U_t}{U_c} = w_t - \bar{w} + \mathbb{E}_t \left[ \frac{\beta}{U_c} \left( (1 - \delta_n)((1 - p(\theta_{t+1})))H^E_{t+1} - H^U_{t+1} \right) \right] \quad (35)
\]

The marginal surplus of an additional worker for the firm is

\[
\frac{\partial V_i}{\partial n_i} \mid_{v=v_i} = V^j_i = \frac{F_{nt} - w_t}{\varphi_d(d_t)} - \mu_t F_{nt} + \mathbb{E}_t \left[ \beta_{t,t+1}((1 - \delta_n)V^j_{t+1} + \delta_n V^e_{t+1}) \right] \quad (36)
\]

Given the free entry condition we have that the value of an additional vacancy for the firm is zero. Therefore,

\[
V^e_i = -\frac{g_t}{\varphi_d(d_t)} + q(\theta_t) V^j_i + (1 - q(\theta_t)) \mathbb{E}_t \left[ \beta_{t,t+1} V^e_{t+1} \right]
\]

\[
V^j_i = \frac{g_t}{\varphi_d(d_t) q(\theta_t)} \quad (37)
\]

Wages are determined by Nash bargaining.

\[
w_t = \text{argmax} \left[ \frac{H^E_t - H^U_t}{U_c} \right] = \text{argmax} \eta \ln \left( \frac{H^E_t - H^U_t}{U_c} \right) + (1 - \eta) \ln V^j_i \quad (38)
\]

Therefore, its possible to derive the following solution

\[
(1 - \eta) \left[ \frac{H^E_t - H^U_t}{U_c} \right] = \eta \varphi_d(d_t) V^j_i \quad (39)
\]
Using 35, 37 and 39 it is possible determine the wage equation that has the following form.

\[ w_t = \eta (1 - \mu t \varphi_d(d_t)) F_n t + (1 - \eta) \bar{w} + \left( \beta_{t,t+1} \eta \left( \frac{\varphi_d(d_t)}{\varphi_d(d_{t+1})} - (1 - p(\theta_{t+1})) \right) (1 - \delta_n) \frac{g_{t+1}}{q(\theta_{t+1})} \right) \]

(40)

B Appendix: Data description

The data used in the paper were taken from FRED and have the following codes:

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
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<td>Unemployment Rate: Aged 15-64: All Persons for the United States, Percent,</td>
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<tr>
<td></td>
<td>Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>JTSJOL</td>
<td>Job Openings: Total Nonfarm, Level in Thousands, Quarterly, Seasonally</td>
</tr>
<tr>
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<td>Adjusted</td>
</tr>
<tr>
<td>LREM64TTUSQ156S</td>
<td>Employment Rate: Aged 15-64: All Persons for the United States, Percent,</td>
</tr>
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<td></td>
<td>Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>UNEMPLOY</td>
<td>Unemployment Level, Thousands of Persons, Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>LRAC64TTUSQ156S</td>
<td>Activity Rate: Aged 15-64: All Persons for the United States, Percent,</td>
</tr>
<tr>
<td></td>
<td>Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>CES05000000003</td>
<td>Average Hourly Earnings of All Employees: Total Private, Dollars per Hour,</td>
</tr>
<tr>
<td></td>
<td>Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>A939RX0Q048SBEA</td>
<td>Real gross domestic product per capita, Chained 2009 Dollars, Quarterly,</td>
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<tr>
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<td>Seasonally Adjusted Annual Rate</td>
</tr>
<tr>
<td>LES1252881600Q</td>
<td>Employed full time: Median usual weekly real earnings: Wage and salary</td>
</tr>
<tr>
<td></td>
<td>workers: 16 years and over, 1982-84 CPI Adjusted Dollars, Quarterly,</td>
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<tr>
<td></td>
<td>Seasonally Adjusted</td>
</tr>
<tr>
<td>DRTSCILM</td>
<td>Net Percentage of Domestic Banks Tightening Standards for Commercial and</td>
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<tr>
<td></td>
<td>Industrial Loans to Large and Middle-Market Firms, Percent, Quarterly,</td>
</tr>
<tr>
<td></td>
<td>Not Seasonally Adjusted</td>
</tr>
<tr>
<td>GPDI</td>
<td>Gross Private Domestic Investment, Billions of Dollars, Quarterly,</td>
</tr>
<tr>
<td></td>
<td>Seasonally Adjusted Annual Rate</td>
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C Appendix: Exogenous labor force

Figure 8: Labor market recovery to a financial shock (Exogenous participation)

![Graph showing labor market recovery](image)

Table 10: Variance decomposition

<table>
<thead>
<tr>
<th>Variable</th>
<th>TFP shock</th>
<th>Credit shock</th>
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<tr>
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<td>U</td>
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<td>v</td>
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### Appendix: Staggered wage model

#### Table 11: Correlations - staggered wage model

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<th>U</th>
<th>v</th>
<th>θ</th>
<th>n</th>
<th>w</th>
<th>lf</th>
<th>I</th>
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<tr>
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<td>0.67</td>
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