The Interdependence between Commodity-Price and GDP Cycles: A Frequency-Domain Approach

Ojeda-Joya, Jair and Jaulin-Mendez, Oscar and Bustos-Pelaez, Juan

Banco de la Republica, Banco de la Republica, Université Paris-Dauphine

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The Interdependence between Commodity-Price and GDP Cycles: A Frequency-Domain Approach[♦]

Jair N. Ojeda-Joya[♠]

Oscar Jaulin-Mendez[♣]

Juan C. Bustos-Peláez[♥]

Abstract

We study the interdependence between aggregate commodity prices and world Gross Domestic Product (GDP) by performing two empirical exercises with long-run data that starts in the 19th Century. First, we compute long-term and medium-term cycles and measure their degree of synchronization for different leads and lags. Second, we perform causality tests on the frequency domain to understand better the nature of their interdependence. Our results show first, evidence of cycle synchronization only in the case of super cycles. Second, there is causality evidence from GDP to aggregate commodity prices mostly on long-run frequencies; therefore, commodity-price trends and super-cycles are demand driven. Third, there is some causality evidence between oil-prices and GDP on both causation directions. However, oil price fluctuations cause GDP on business-cycle frequencies only. Finally, in the case of metal prices, the evidence is unclear for both causality directions implying that they are not demand driven. Overall, our results show that the interdependence between commodity prices and GDP varies significantly across types of goods and fluctuation frequencies.

[♠] Corresponding author: Senior research economist, Banco de la Republica. Cr. 7 No. 14-78, Bogotá D.C., Colombia. E-mail: jojedajo@banrep.gov.co

[♣] Economist, Banco de la Republica. Cr. 7 No. 14-78, Bogotá D.C., Colombia. E-mail: ojaulime@banrep.gov.co

[♥] Université Paris-Dauphine, Place du Maréchal de Lattre de Tassigny, 75775, Paris, France. E-mail: juancabustos.p@gmail.com

1. Introduction

Assessments on the future behavior of commodity prices usually rely on the predicted strength of economic activity in countries that are heavy importers of commodity related products (i.e. China). However, these predictions also depend on the dynamic supply-side reaction by commodity producers to price increases and on their expected economic and financial conditions[1]. This is an example of the dynamic interactions among demand, supply and prices that should be further analyzed with appropriate econometric tools. On the other hand, correlation analyses are not enough to disentangle empirically the direction of causality between commodity prices and aggregate economic activity on different fluctuation horizons.

In this study, we explore the dynamic interaction between real commodity prices (RCP) and world output fluctuations across the frequency domain. Our estimations are performed using long-term annual data for aggregate Real Commodity Prices (RCP) and aggregate world real Gross Domestic Product (GDP) starting in 1870. Instead of focusing only on their long-term trends, we analyze the interdependence across frequencies, especially, for medium and long-term cycles. First, we estimate the degree of synchronization between RCP and economic activity for different leads and lags, and for all fluctuation frequencies. Second, we perform alternative causality tests between these indicators for both directions of causality, on the frequency domain.

We perform a first causality analysis with vector error correction (VEC) models by estimating speeds of adjustment to the long-run equilibrium. We also compute instantaneous causality tests between the growth rates of RCPs and global output. In addition, we perform Granger causality tests across the frequency domain. The latter methodology is based on Breitung and Candelon (2006) and Wei (2013) who extend standard causality tests to allow for the presence of integrated and co-integrated variables and decompose the test into the frequency spectra.

Our results show first, evidence of cycle synchronization only in the case of super cycles. The highest estimated correlation is positive, between contemporaneous GDP and oil-price super cycles 2 years ahead. Second, there is causality evidence from GDP to aggregate (non-oil) commodity prices mostly on long-run frequencies; therefore, commodity-price trends and super-cycles are demand driven. Third, there is causality evidence on both directions, between oil-prices and GDP. However, oil price fluctuations cause GDP only on business-cycle frequencies. Finally, in the subgroup of metal prices, the evidence is unclear for both causality directions and therefore, we cannot conclude that they are demand driven.

This set of new results show that the story about the interdependence between commodity prices and GDP varies significantly across type of goods and fluctuation frequencies. In particular, metal prices seem not to be demand driven and there is reverse causality only in the case of oil prices and business-cycle GDP fluctuations. The rest of this paper is organized as follows. In

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[1] An example of this type of assessments is Morgan Stanley (2015). This article forecasts an increase of metal and mineral prices due to the predicted industrial recovery in China. However, its authors warn that this prediction can fail if producer companies do not use enough "supply discipline".

Section 2, we discuss the related literature. Section 3 contains our description of the data. Section 4 presents the econometric methods and Section 5 describes the empirical results. The last section concludes.

2. Literature Review

Understanding the evolution of real commodity prices (RCP) is very important for policy makers and research economists not only in producer economies but also in those economies where importing commodities is crucial for their industries. For this reason, this topic has received a great deal of attention in the economics literature. We focus on recent works that have tried to identify the empirical properties and determinants of RCPs.

Schumpeter (1939) initially studied long-term RCP cycles and explained them through his theory of creative destruction. This hypothesis relies on the prosperity and stagnation phases resulting from evolving technological innovations. Thus, RCPs increase during prosperity phases due to the investments needed to implement the new technology. RCPs then fall during the stagnation phase once the new technology is standardized. A few recent papers follow a similar line, for example, Harvey et al (2017) show recent evidence of the technological determinants of RCPs using data starting in the 17th century.

Cashin and McDermott (2002) is one the first studies that use time-series econometrics to analyze aggregate RCPs and find that, in the period 1862-1999, there is a significant long-run downward trend. However, this trend is small compared to the typically increasing volatility of these RCP. In particular, rapid and unexpected fluctuations are more important than trends during recent decades, with important implications for the design of macroeconomic policies.

Cuddington and Jerrett (2008) and Jerrett and Cuddington (2008) apply the Band-Pass (BP) filter to real metal prices to estimate their super cycles which are defined to last between 20 and 70 years[2]. They identify three super cycles during the period 1850-2006, which are highly correlated for a set of metal RCPs. They point out that the times of occurrence of these supercycles coincide with the industrialization and urbanization of different regions of the world. For instance, the cycle starting in the 1990s coincides with the strong growth and industrialization observed in China.

Erten and Ocampo (2013) use similar methods to identify supercycles for aggregate RCP indexes of metals, agriculture and the real oil price. They identify 4 supercycles for each time series in the period 1865-2010 and find that World GDP mostly drives all of them. On the other hand, real oil-price cycles are drivers of long run GDP fluctuations. Finally, they find evidence of the

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[2] Diverse studies estimate the cyclical components of business activity using filtering methodologies based on BP filters. Recent examples of these works are Comin and Gertler (2006), Borio (2014) and Drehmann et al. (2012). The Band-Pass (BP) filter was developed by Baxter and King (1999) and Christiano and Fitzgerald (2003).

Prebisch-Singer hypothesis (a long-run downward trend) for tropical agricultural commodities[3]. Jacks (2013) also computes supercyles for long-run RCP series and characterizes the most recent phases of these cycles with similar qualitative results. Gil-Alana and Gupta (2014) apply an alternative statistical methodology to compute long-term cycles of real oil prices.

Another relevant contribution is Alquist and Coibion (2014) who study the interdependence between non-energy RCPs and economic activity. These authors use a factor-based decomposition of RCP movements with theory-based restrictions. Their findings imply a significant role for supply-based shocks, including those originating from energy RCPs. They also identify an economic activity factor that is able to explain a good portion of RCP fluctuations on the business-cycle frequency.

Our document studies the interdependence between RCP cycles and global economic activity with special focus on the case of long and medium-term cycles. We accomplish this goal by computing long and medium-term cycles of these variables and by computing correlation coefficients between GDP and RCP cycles for alternative leads and lags. However, our main contribution with respect to Erten and Ocampo (2013), and others, is that we study the specific interdependence between World GDP and RCPs using Granger causality tests on the frequency domain. This methodology allows identifying the not only the direction of causality but also the frequency ranges for which this causality is more evident.

3. Data Description

We use the non-oil Real Commodity Price (RCP) index originally developed by Grilli and Yang (1988) and then extended by Ocampo and Parra (2010). This annual index is composed of 24 commodities for the period 1865-1961 and 32 commodities during 1962-2010. Using IMF data, we extend this index until 2013. Next, Erten and Ocampo (2013) using data from the World Economic Outlook, Global Financial Data and West Texas International, constructed a real oil price series for the period 1875-2010[4]. We also extend this real price until 2013.

We use the Manufacturing Unit Value (MUV) as the deflator of RCP series. The advantage of working with this deflator is that it includes only prices of tradable goods that are directly comparable to RCPs. The United Nations and the World Bank develop and update the MUV index.

Global real GDP is in 1990 International Geary-Khamis dollars. This index is originally from Maddison (2004) data and spans 1820-2003. The Groningen Growth and Development Centre's

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[3] Other recent papers have documented the relation between recent high-growth periods in developing economies and the dynamics of RCP. Collier and Goderis (2012), Garnaut (2012) and Byrne et al (2013) discuss this relation. In addition, Baffes and Etienne (2016) as well as Winkelried (2016) provide recent evidence of the Prebisch- Singer hypothesis.

[4] We thank Bilge Erten and Jose A. Ocampo for sharing with us their database on commodity prices.

Total Economy database updated it until 2008. We update this series until 2013 using the World Economic Outlook and International Financial Statistics.

4. Econometric Methods and Results

4.1. Estimating Cyclical Components

We use the asymmetric Band-Pass (BP) filter developed by Christiano and Fitzgerald (2003), to estimate the long-term and medium-term cyclical components of real commodity prices (RCP) and global activity. All series are in natural logarithms. We decompose each time series into four components: long-term trend (LT), super cycle (SC), medium-term cycle (MTC), and other components (OC).

- Long-Term Trend and Super cycles: Following Cuddington and Jerrett (2008), and Erten and Ocampo (2013), we define the long-term trend as the group of frequencies with periodicities longer than 70 years. Additionally, super cycles correspond to periodicities spanning between 20 and 70 years.

- Medium-Term Cycles: Following Comin and Gertler (2006) and Drehmann et al. (2012), we define medium-term cycles to have periodicities between 8 and 20 years.

- Other Components: cyclical components with periodicities below eight years.

Hence, the log level of every series is the sum of their four components, as expressed in Equation (1):

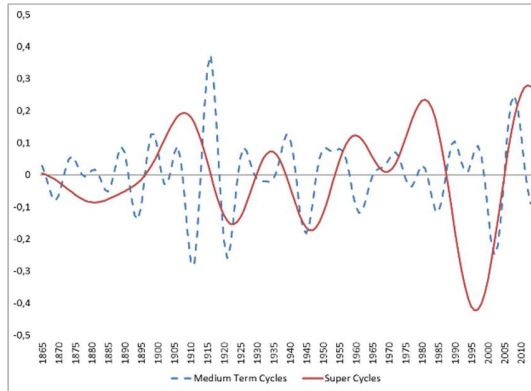$$LX_t \equiv LT_t + SC_t + MTC_t + OC_t \qquad (1)$$

4.2 Results of the Estimation of Cycles

Figure 1 shows the results of the frequency-based decomposition for RCPs and global GDP. The y-axis measures the percentage distance with respect to the long-run trend. The estimated super cycles for real metal prices (Graph A) show an important trough in 1997 (-42%) and the most recent peak in 2012 (27.9%). In the same graph, medium-term cycles show a recent peak in 2008 (24.8%) that follows a trough in 2002 (-21%).

Graph B shows the estimated cycles for the aggregate RCP non-oil index described in Section 3. Super cycles in this case also shows an important trough in 1997 (-26.5%) and the end of the sample (2013) seems to be near a peak of around 20%. However, the most recent identified peak corresponds to 1978 (14.1%). Recent medium-term cycles are smaller with the most recent trough in 2003 (-13.7%) followed by a peak in 2009 (11%). Notice that since this index also includes metal prices, super cycles in Graphs A and B have some similarities.

Figure 1. Results of the Decomposition into Medium-Term and Super Cycles of Commodity Prices and GDP
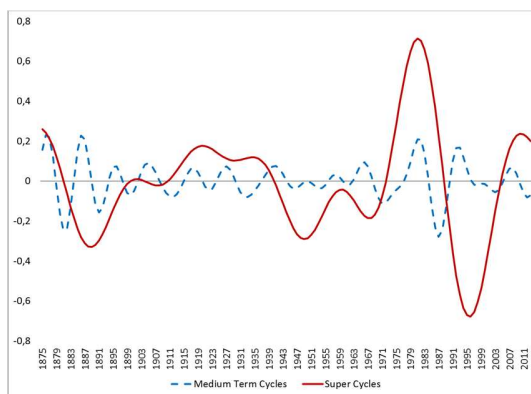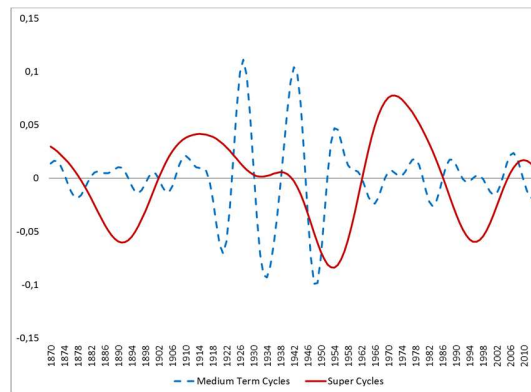
A. Real Metal Price

B. Real Non-Oil Price

C. Real Oil Price

D. Real World GDP

Source: Authors' calculations

The estimated cycles for the real oil price are in Graph C. These super cycles have a very important trough in 1996 (-67.8%) which is followed by a peak in 2010 (23.7%). The previous estimated peak in 1981 is also very important (71.3%). Medium-term cycles are much smaller with the most recently estimated peak in 2008 (6.6%) followed by a trough in 2012 (-8.2%).

Finally, Graph D shows the estimated world GDP cycles. The most recent trough in its super cycle took place in 1996 (-6%) which is followed by a peak in 2010 (1.7%). On the other hand, its medium-term cycles show a trough in 2001 (-1.6%) followed by a peak in 2007 (2.4%), at the end of the sample a new trough seems likely to show up one or two years ahead.

Table 1: Amplitude and Duration of Super Cycles

	Super Cycles				
	Amplitude*		Duration**		
	Upward Phase	Downward Phase	Upward Phase	Downward Phase	Cycles
Metal price	34.63%	-34.15%	16.00	12.75	26.00
Non-Oil price	17.94%	-23.00%	14.00	16.50	31.00
Oil price	43.56%	-40.94%	11.50	10.40	21.60
World GDP	8.60%	-8.90%	15.25	19.33	32.00

* Average percentage variation from trough to peak and from peak to trough. ** Average number of years.
Source: Authors' calculations

Table 1 describes the amplitude and duration of the estimated super cycles. While oil prices have the widest fluctuations (amplitudes greater than 40%), GDP has the least volatile ones. Furthermore, oil price cycles are the shortest (21.6 years), while non-oil price and GDP super cycles are the longest on average (31 and 32 years, respectively).

Table 2: Amplitude and Duration of Medium-Term Cycles

| | Medium-Term Cycles | | | | |
| | Amplitude* | | Duration** | | |
	Upward Phase	Downward Phase	Upward Phase	Downward Phase	Cycles
Metal price	21.08%	-20.24%	4.80	4.79	9.57
Non-Oil price	14.24%	-14.39%	5.23	5.42	10.58
Oil price	18.45%	-19.35%	5.38	4.71	10.15
World GDP	5.74%	-5.68%	4.85	5.62	10.46

* Average percentage variation from trough to peak and from peak to trough. ** Average number of years.
Source: Authors' calculations

Table 2 is analogous to Table 1 and describes features of medium-term cycles. In this case, metal prices have the widest fluctuations while GDP, again, has the least volatile cycles. Interestingly, the duration is similar (approximately 10 years) across all four variables in Table 2. This duration is slightly longer for non-oil RCPs (10.58 years). In sum, RCP cycles are clearly more volatile than GDP cycles. In addition, oil-price cycles tend to be shorter and more volatile than in the case of non-oil RCP.

4.3. Analyzing the Synchronization of Cycles

We study the degree of synchronization by estimating correlation coefficients. This measure is a linear relationship that indicates both the strength and direction of the interdependence between two stationary time series.

Equation 2 defines the correlation coefficient for each pair of cycles (X_i, X_j)

$$C_C(p) = \frac{Cov\,(X_{it}, X_{jt-p})}{(\sigma X_{it})(\sigma X_{jt-p})} \tag{2}$$

This coefficient $C_C(p)$ takes values between -1 (negative synchronization) and 1 (positive synchronization). To make a statistical inference, we perform tests on whether $C_C(p)$ is significantly different from 0. Following Hevia (2008), we perform these significance tests using a GMM approach along with the delta method for the estimation of variance. We test the null hypothesis: $Cc = 0$, against the alternative: $Cc \neq 0$ for all pairs of cycles under study[5].

In equation (2), p represents the number of lags. By estimating the synchronization between the cycle of one variable and the lagged cycle of another variable, we try to assess their dynamic relationship. Although this measure does not formally establish causality, it is helpful to

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[5] We also compute the synchronization measure proposed by Harding and Pagan (2006) with qualitatively similar results.

understand the interrelation between peaks or troughs and future phases of cycles. These results are in Tables 3 and 4.

Table 3 shows the correlation coefficient between super cycles with leads and lags ranging from 0 to 10 years. A first result is that in most columns of Table 3, the highest correlations are for contemporaneous cycles (0 lags). The highest contemporaneous coefficient in the table is 0.635, which is between oil prices and GDP. The highest non-contemporaneous correlation is 0.674, between GDP and oil-price super cycles three years ahead. This result shows a potential positive causal effect between these variables. Another possible causal effect is from metal prices to GDP super cycles three years ahead with a correlation of 0.539.

Table 3: Correlation Coefficient between Super Cycles.

	GDP-Metal prices		GDP-Non-Oil prices		GDP-Oil prices	
	Lags of GDP	Lags of price	Lags of GDP	Lags of price	Lags of GDP	Lags of price
0	0.486***	0.486***	0.408**	0.408**	0.635***	0.635***
1	0.453***	0.514***	0.408**	0.405**	0.661***	0.597***
2	0.409**	0.532***	0.399*	0.396**	0.674***	0.545***
3	0.357**	0.539***	0.379*	0.383**	0.674***	0.482***
4	0.298	0.534***	0.350*	0.365**	0.664***	0.408***
5	0.234	0.517***	0.309*	0.346**	0.643***	0.324**
6	0.165	0.488***	0.256	0.327**	0.613***	0.233*
7	0.094	0.449***	0.193	0.309**	0.576***	0.136
8	0.022	0.401***	0.120	0.293**	0.531***	0.038
9	-0.050	0.346**	0.039	0.280**	0.481***	-0.059
10	-0.121	0.288*	-0.049	0.270**	0.426**	-0.152

*, ** and *** are significant at the 90%, 95% and 99% confidence levels, respectively.
Source: Authors' calculations

Table 4 shows the correlation coefficients between the medium-term cycles of GDP and prices, for up to 10 leads and lags. Notice that these correlation coefficients are low and only a few of them are statistically significant. The highest correlations (in absolute value) are those between lagged (four and five lags) GDP cycles and non-oil prices. These correlations show a possible negative causal effect from world GDP to aggregate (non-oil) price cycles.

In summary, there is abundant evidence of cycle synchronization in the case of very long-term frequencies (super cycles). This co-movement is mostly important between contemporaneous GDP and RCP fluctuations. There is also some evidence of dynamic relationships that we further study with causality tests. In the case of medium-term cycles, there is no contemporaneous synchronization and only a few inter-temporal correlations are significant[6].

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[6] We also performed this analysis with Kendall's Tau correlations with very similar conclusions.

Table 4: Correlation Coefficient between Medium-Term Cycles

| | GDP-Metal prices | | GDP-Non-Oil prices | | GDP-Oil prices | |
	Lags of GDP	Lags of price	Lags of GDP	Lags of price	Lags of GDP	Lags of price
0	0.179	0.179	0.265	0.265	0.083	0.083
1	0.093	0.205***	0.096	0.346**	0.111	0.049
2	-0.022	0.159	-0.114	0.323**	0.124***	0.019
3	-0.131	0.050	-0.299*	0.214	0.110	-0.008
4	-0.204	-0.089	-0.400***	0.061	0.061	-0.034
5	-0.227**	-0.210	-0.384**	-0.089	-0.013	-0.061
6	-0.198*	-0.268	-0.255	-0.197	-0.093	-0.088
7	-0.128	-0.230**	-0.055	-0.242	-0.157	-0.106
8	-0.032	-0.100	0.156	-0.227	-0.181***	-0.106
9	0.071	0.081	0.314**	-0.172	-0.149***	-0.079
10	0.164	0.252	0.376**	-0.098	-0.075	-0.025

*, ** and *** are significant at the 90%, 95% and 99% confidence levels, respectively.
Source: Authors' calculations

4.4 Standard Causality Tests

In this section, we perform standard causality tests between real commodity prices (RCP) and GDP for both directions of causality. These results are to be contrasted with those obtained in the previous correlation analysis and then, with causality tests on the frequency domain (Section 4.5). The first step is performing unit-root and co-integration tests to all three price-GDP pairs. These results show that all RCPs and GDP series are I(1). Furthermore, all three RCP series have co-integration relations with GDP as shown in Tables A1 and A2 in the Appendix.

We perform VEC estimations in these three cases using bivariate models with intercept in the cointegration vector. We use the Akaike information criterion for lag length selection. These lengths as well as the co-integration elasticity are in Table 5. Table A3 in the Appendix shows the results of tests for residual normality and autocorrelation.

Table 5 – Long-Term Relation between Commodity Prices and GDP

Co-integrated variables	Lag Length	Long-term Elasticity
Non-Oil prices and GDP	3	-0.2***
Metal prices and GDP	1	0.017
Oil prices and GDP	2	0.631***

*, ** and *** are significant at the 90%, 95% and 99% confidence levels, respectively. The elasticity corresponds to the coefficient of the cointegration relation.
Source: Authors' calculations

Table 5 shows the estimated co-integration relations between RCPs and GDP. Their interpretation implies that a 10% GDP increase is typically associated with a 2% reduction of the total (non-oil) RCP index, a 0.17% increase in real metal prices and a 6.3% increase in real

oil prices. Please notice that these coefficients show a long-term relation between endogenous variables and are therefore compatible with both directions of causality. In addition, the elasticity between metal prices and GDP is small and therefore not significantly different from zero. We allow such a small coefficient because the co-integration test in this case is conclusive and highly significant (see Table A2).

The error-correction representation allows estimating the effects of deviations from the co-integration relation on each endogenous variable. In particular, it is possible to estimate a coefficient for the speed of adjustment. If this coefficient is significant, then there is causality from the errors of the co-integration equation to the left-hand side variable. Erten and Ocampo (2013) use this approach to test for long-run causality. We also perform speed of convergence tests using our updated database, see Table 6.

Table 6 - Speed of Adjustment Coefficients

VEC System	Commodity price variation	GDP variation
Non-Oil prices and GDP	-0.149***	-0.010
Metal prices and GDP	-0.09***	-0.029***
Oil prices and GDP	-0.003	-0.004***

*, ** and *** stand for rejection of the non-significance null hypothesis at 90%, 95% and 99% confidence levels, respectively
Source: Authors' calculations

The results in Table 6 show coefficients with the correct sign and statistically significant in most cases. This result is in line with the obtained by Erten and Ocampo (2013) about RCPs adjustment to their co-integration vectors. These speed-of-adjustment coefficients show evidence of a gradual and significant RCP adjustment from the cointegration vector. Only in the case of real oil prices is this coefficient not significantly different from zero, when considering RCP variations.

According to Lütkepohl (2007), x_t is Granger-causal for z_{t+1} when z_{t+1} can be predicted more efficiently if the information of x_t is taken into account in addition to all information available up to and including period t. Therefore, Granger causality tests analyze more directly the dynamic effects of variations of one variable (x_t) on future values of another (z_{t+1}). We apply these tests within our RCP-GDP VEC systems for both directions of causality.

Table 7 shows a matrix of p-values in which the directions of causality go from the row variables to the column variables. These results imply that there is Granger (dynamic) causality from GDP to non-oil RCP, but there is not such evidence from GDP to oil or metal prices. On the other hand, there is evidence of Granger causality from RCPs (oil and non-oil) to GDP.

Table 7: P-values of Granger Causality Tests within VEC systems

		To	
		GDP	Metals
From	GDP		0.469
	Metals	0.5588	
		GDP	Non-oil
	GDP		0.0358**
	Non-oil	0.0051***	
		GDP	Oil
	GDP		0.9304
	Oil	0.0026***	

*, ** and *** stand for rejection of *H0* (no causality) at the 90%, 95% and 99% confidence levels, respectively.

Source: Authors' calculations

Instantaneous causality occurs, following Lütkepohl (2007), when in period t, adding x_{t+1} to the information set helps to improve the forecast of z_{t+1}. This definition is symmetric because instantaneous causality between x_t and z_t implies a similar result in the reverse direction. We apply tests for this definition to our three VEC systems. Table 8 shows strong evidence of instantaneous causality between GDP and all three RCPs.

Table 8 – Instantaneous Causality Test

VEC System	Symmetric test p-value
Non-Oil prices and GDP	0.0001***
Metal prices and GDP	0.0000***
Oil prices and GDP	0.0158**

*, ** and *** stand for rejection of H0 (no causality) at 90%, 95% and 99% confidence levels, respectively.

Source: Authors' calculations

We summarize the previous results individually. There is clear evidence that world GDP drives the non-oil commodity price index using all causality tests. This result is consistent with the proposition that RCPs cycles are demand driven (Erten and Ocampo, 2013). In the case of real metal prices, there is no evidence of Granger causality from GDP to prices although its speed of adjustment coefficient is significant. Furthermore, we do not find any significant evidence of causality from GDP to real oil prices, except for instantaneous causality (Table 8). On the other hand, for all three commodity-price indices there is some evidence of causality from prices to GDP. Real oil prices have the clearest evidence for this direction of causality.

These standard causality tests are able to detect causality within all fluctuation frequencies. One of our goals is studying the interdependence between GDP and commodity prices in the case of

medium and long-term cycles only. Therefore, we need to use an appropriate methodology as explained in the following sub-section.

4.5 Testing for Granger Causality on the Frequency Domain

As mentioned previously, we are interested in testing for causality between real commodity prices (RCP) and GDP across the frequency domain, but with a special focus on medium- and long-term fluctuations. Following Wei (2013), we use a procedure to estimate Vector Auto Regressions (VAR), which is robust to the integration and co-integration properties of the involved series. In this framework, we compute Granger causality tests across the frequency domain by applying the methodology devised by Breitung and Candelon (2006).

Let us consider a VAR ($p+d$) on the level of the series, such that p is the optimal lag order and d is the maximum order of integration. Toda and Yamamoto (1995) show that including these additional d lags helps to correct asymptotically the estimation distortions, which are associated to the presence of integrated and co-integrated variables. Dolado and Lütkepohl (1996) developed the appropriate significance tests for the estimated coefficients. Therefore, we estimate the following equation:

$$y_t = \mu + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \cdots + \Phi_p y_{t-p} + \cdots + \Phi_{p+d} y_{t-(p+d)} + \varepsilon_t \tag{3}$$

In Equation (3), $y_t = [x_t, z_t]'$ is the vector of variables, μ is the constant term, Φ_j are the coefficient matrices for each lag j, and ε_t is the error term.

Let $\theta_{12,j}$ be the (1,2) element of the coefficient matrix Φ_j and $\beta = [\theta_{12,1}, \theta_{12,2}, \ldots, \theta_{12,p}]'$. To test for Granger causality, we should contrast the following null hypothesis:

$$H_0: R\beta = 0 \tag{4}$$

Notice that if R were an identity matrix of order p, the null hypothesis in Equation (4) would correspond to the conventional Granger causality test. The approach by Breitung and Candelon (2006) changes this null hypothesis by modifying the linear restrictions on their parameters:

$$R = \begin{bmatrix} \cos(\omega) & \cos(2\omega) \ldots \cos(p\omega) \\ \sin(\omega) & \sin(2\omega) \ldots \sin(p\omega) \end{bmatrix} \qquad \omega \in (0, \pi). \tag{5}$$

The Wald test statistic computed from Equations 4 and 5 is asymptotically distributed $\chi^2(2)$ for each $\omega \epsilon (0, \pi)$. We compare this tests statistic with their respective critical values for alternative confidence levels.

4.6. Results of Causality Tests on the Frequency Domain

Figure 2 is composed of three figures that show the results of the Granger causality tests across frequencies. Each graph shows results only for the frequency range that goes from 0 to $\pi/2 \approx$ 1.57 radians, implying that we focus on fluctuations lasting longer than 4 years, approximately (Table 9). The goal of this exercise is learning more about the drivers of medium and long-term cyclical fluctuations of commodity prices.

Table 9: Ranges for Frequencies

	Radians	
	From	To
Trend	0.0	0.09
Long-Term Cycles	0.091	0.31
Medium-Term Cycles	0.311	0.79
Between 4 and 8 years	0.791	$\pi/2$

Source: Authors' calculations

On panel A, in Figure 2, we graph causality tests between aggregate (non-oil) RCP and real GDP versus a 90% critical value. The only Granger causality evidence goes from GDP to RCP on frequencies similar to long-term cycles in Table 9. These results confirm that demand drives (non-oil) commodity price super cycles in line with the results by Erten and Ocampo (2013).
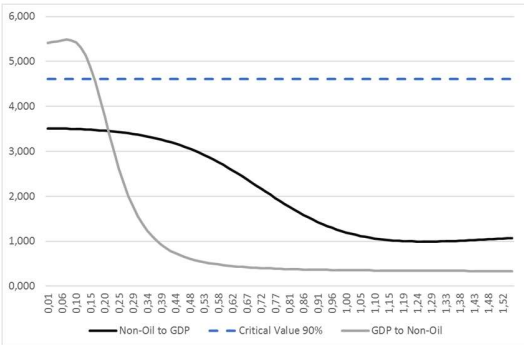
Panel B shows analogous results between real metal prices and GDP. In this case, the test statistics do not vary with the frequency range due to the low number of lags selected by information criteria. There is no evidence of Granger causality between these two variables and the result is the same across frequencies and for both directions of causality. Therefore, we are detecting a subset of the Non-Oil RCP index, metal prices, which is not driven by aggregate demand. This result is consistent with the standard causality tests described in Section 4.4.

Panel C studies the causality relations between real oil prices and GDP. There is causality evidence from oil prices to economic activity on two frequency ranges. First, in the case of very low frequencies including a common long-run trend, consistent with the cointegration results described in Table 5. Second, we find causality evidence on higher frequencies related to the business cycle, as also reported above in Section 4.4. Panel C also shows that there is Granger causality evidence from GDP to oil prices on the lowest fluctuation frequencies: Long-run trends and super cycles. This result demonstrates that world aggregate demand also drives oil-price long-term cycles. The key difference in the case of oil prices is that there is evidence for both directions of causality. This result is consistent with Alquist et al (2013) who find that it is very difficult to obtain good short-term oil-price predictions using macroeconomic aggregates.
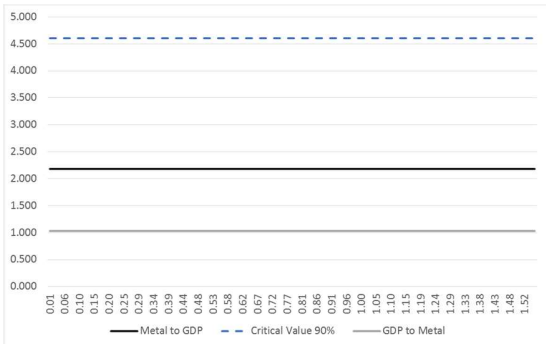
Notice also that in Figure 2, causality evidence from GDP to RCP (Panel A and C) is restricted to the lower range of frequencies including the trend and a small portion of long-term cycles as defined in Table 9. If the causality tests incorporate the complete long-term range in the null hypothesis, then causality would be rejected since the appropriate test statistic in this case corresponds to the minimum in the same range, which is much lower than the critical value. This result follows Breitung and Schreiber (2016) and implies that the causality result does not apply to all super-cycles but only to those with the lowest frequencies.

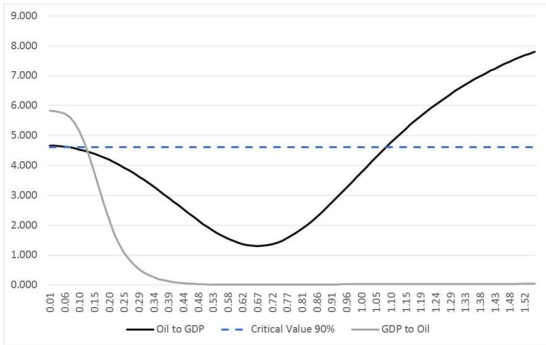Figure 2. Granger Causality Tests on the Frequency Domain between GDP and Prices

A. Causality Tests between Real Non-Oil Prices and Real GDP

B. Causality Tests between Real Metal Prices and Real GDP

C. Causality Tests between Real Oil Prices and Real GDP

Source: Authors' calculations

5. Conclusions

In this paper, we study the relation between real commodity prices (RCP) and world GDP with a special interest in medium and long-term fluctuations. First, we find that there is significant synchronization between the long-term cycles of these variables. Standard causality tests are able to confirm this relation from GDP to RCPs, in the case of the aggregate non-oil price index. Second, there is causality evidence from oil prices to GDP. However, since we compute these tests within multivariate systems of differentiated variables, it is difficult to capture well the slow dynamics of super cycles.

Therefore, we estimate Granger causality tests on the frequency-domain following Breitung and Candelon (2006) and Wei (2013). These causality tests a interpreted for specific ranges of frequencies and are appropriate to study low frequencies since they are performed using the levels of GDP and prices. Our results show that world GDP does cause commodity super cycles in the case of the aggregate index and oil prices. This causality evidence does not hold in the case of metal prices. On the other hand, we find that real oil prices are drivers of world GDP fluctuations across both business cycle and long-term frequencies.

Finally, it is important to point out a few policy implications from our findings. If supply-side variables drive medium and long-term metal-price cycles, policymakers in metal-rich countries should not worry about the effect of large GDP swings on future metal prices. For the rest of commodities, policymakers in producer economies should closely follow world aggregate demand since it is a crucial driver of their real prices in the long-term. Finally, policymakers should consider in their analyses that real oil-price fluctuations have casual effects on medium and long-term world GDP fluctuations.

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Appendix

Table A1 – ADF Unit Root Test

Variable	ADF	
	Level	First Difference
Real World GDP (LY)	1.33645	-7.454642***
Real Metal Price (LM)	-2.49719	-10.87977***
Real Oil Price (LO)	-1.64504	-10.61684***
Real Non-Oil Price (LT)	-2.15142	-10.19042***

*, ** and *** stand for rejection of H0: unit root with confidence levels of
99%, 95% and 90%, respectively.
Source: Authors' calculations

Table A2 – Johansen Co-Integration Test for price-GDP

	Johansen Co-integration Trace Test			
	Null Hypothesis	Alternative	λ trace stat.	Prob.
LT and LY	$r \leq 0$	$r = 1$	17.71	0.0228**
	$r \leq 1$	$r = 2$	2.58	0.1081
LM and LY	$r \leq 0$	$r = 1$	31.94	0.0001 ***
	$r \leq 1$	$r = 2$	0.00	0.951
LO and LY	$r \leq 0$	$r = 1$	33.15	0.0005***
	$r \leq 1$	$r = 2$	4.92	0.2922

*, ** and *** stand for rejection of the null hypothesis with confidence
levels of 99%, 95% and 90%, respectively.
Source: Authors' calculations

Table A3 – Normality and Autocorrelation Tests

Co-integrated variables [1]	Normality test (Doornik and Hansen, 2008)	LM type test for auto-correlation
LT and LY	3.8454	17.59***
LM and LY	8.53*	4.8
LO and LY	23.53	4.07

*, ** and *** are significant at 10%, 5% and 1%, respectively
[1] A few time-dummy variables were added to each co-integration system to correct for outliers and getting closer
to the requirement on residual normality.
Source: Authors' calculations