Navigating the oil bubble: A non-linear heterogeneous-agent dynamic model of futures oil pricing

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Abstract

We analyze short-term futures oil pricing over the 2003-2016 time-period in order to analyze the bubble-like dynamics, which characterizes the 2007-2009 years according to a large body of recent literature. Our investigation, based on a flexible three-agent model (hedgers, fundamentalist speculators and chartists), confirms the presence of a bubble price pattern, which we attribute to the strong destabilizing behaviour of fundamentalist speculators (e.g. hedge funds). The inclusion of the 2009-2016 sub-period, in spite of sharp and unexpected fluctuations in oil prices and a significant increase in the influence of geopolitical factors, fails to invalidate our financial interpretation.

Keywords: Oil pricing, Bubble, Speculation, Dynamic hedging, Logistic smooth transition, Multivariate GARCH

JEL Codes G11, G12, G18, Q40

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1. Introduction

Between 2003 and 2016, oil prices witness unprecedented fluctuations that result in two major cycles, which straddle the 2009 global financial freeze. The first cycle begins in 2003, when prices – starting from a persistent low level (about 30 dollars per barrel on average) – increase continuously. A decline, which begins in mid-2006, due mostly to the first reduction in two decades of oil demand by OECD countries, is followed, in the subsequent two years, by an extremely rapid and unexpected upswing variously attributed to shifting fundamentals, institutional changes and/or to financial bubble behaviour, upswing which is the main issue of this paper.

In more detail, Master (2008) and Sari et al. (2012), among others, attribute it to the influx of institutional investors in commodity markets that is to the financialization of the sector (on this see also Tang and Xiong 2012). Hamilton (2009) and Kesicki, on the other hand, (2010) relate it to fundamental variables (weak dollar combined with low elasticity of supply). More recently, financial herd behavior has emerged as a possible interpretation of this phenomenon, starting a new strand in the literature on the relative importance of financial determinants in commodity pricing, which Demirer et al. (2015) and Boyd et al. (2018) summarize. Regarding explanations based on bubble detection, a vast and growing strand in the literature has focused on the so-called log-period power law (LPPL) model set out by Sornette and Johansen (1997) and applied to oil pricing by Sornette et al. (2009) and Zhang and Yao (2016).

With the world economy plunged in the Great Recession and with major technological innovations (shale oil in particular) and geopolitical turmoil (Middle-East conflicts, Saudi Arabia energy policy shifts) affecting the global oil industry, a
proper identification of oil price drivers, during the second cycle (2009 – 2016), becomes more difficult. Indeed the market witnesses unprecedented - and confusing - changes in demand and supply factors, leaving it open to question, whether financial (cum bubble) drivers play a relevant role throughout the 2003 – 2016 period and, second, how alterations in standard trading patterns brought about the large price swings observed in the 2003-2009 period.

Recent empirical studies test the relevance of financial drivers, using the Commitment of Traders Futures Only report data in order to quantify, in various ways, financial speculative pressure (see, among others, Kim, 2015, and Gogolin and Kearney, 2016, and the literature quoted therein). In this paper, we address this problem building a model, which incorporates agents moved by purely financial considerations and market sentiment alongside traditional market players. This model is consistent with the LPLL approach to the detection of bubbles and complements it by disentangling the various drivers of super-exponential behaviour and of log-periodic price dynamics oscillation.

In a highly innovative article, Frankel and Froot (1986) underlined the importance of the interaction between standard financial market operators, such as chartists (or noise traders) and fundamentalist speculators, as a driver of an endogenous non-linear law of motion in foreign exchange rate dynamics. In the same vein, a large and booming literature on commodity/oil pricing, building on heterogeneous agent models (HAM) by Brock and Hommes (1997, 1998) and Westerhoff (2004), among many others, posited that agents react to differing information sets, resulting in market prices, which are weighted averages of their heterogeneous reactions.

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2 On this line, see the alternative of Deeney et al. (2015) who introduce a financial “sentiment index”, built along the lines of Baker and Wurgler (2006) in an equity market context.
Drawing inspiration from Westerhoff and Reitz (2005), Reitz and Westerhoff (2007) and Tokic (2011), we build a model in which three categories of agents interact: noise traders, fundamentalist speculators and hedgers. Noise traders react to past price changes and can either stabilize or destabilize the market, according to whether they behave as contrarians (negative feedback traders) or trend followers (positive feedback traders). Fundamentalist speculators, among whom we include financial agents, respond to deviations of market returns from equilibrium. In this case, a destabilizing behavior is due to lack of confidence in the mean-reverting nature of market prices. Finally, we account for the presence of industry investors, producers and consumers, by including them in the category of hedgers who reduce risk by using futures contracts. In this line, our model combines typical financial market behavior with dynamic hedging of commodity contracts.\(^3\)

This paper combines the different strands in the literature reviewed above, introducing some relevant innovations. Based on the Zhou-Sornette (2009) and on the Phillips et al. (2011) methodologies, we identify the presence of a bubble between January 2007 and February 2009. Our model allows us to attribute the former to shifts in the behavior of three categories of agents: feedback traders, fundamentalists and hedgers. This greatly expands the dynamics of the standard HAM pricing paradigm. We impose no a priori restrictions on the signs of the parameters of the futures returns relationship and stabilizing or destabilizing reactions of economic agents are allowed for. In the same way, no restrictions are imposed on the sign of the speed of adjustment coefficient in the logistic functions, which model the entry in (exit from) the market of these agents according to their trust in the reliability of market pricing. We also introduce two indicators to control for currency and financial market conditions, finding that changes in weighted US

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\(^3\) Along these lines, Kao et al. (2016) explicitly introduce contrarians alongside positive feedback traders and fundamentalists.
The dollar exchange rate and the VIX (VOX) index have a statistically significant impact on oil pricing patterns. More specifically, we find a negative correlation between the USD and oil prices, a finding which confirms well-known stylized facts about the oil market. We also find negative correlation between oil prices and the VOX index, which we take as indicative of a depressing effect of business uncertainty on oil price quotations.

The main results read as follows. The bubble affects all categories of agents, in some cases reinforcing their behavior (as observed over the whole sample period) in other cases altering it. More specifically, the bubble tends to bring about a stabilizing reaction from hedgers and chartists (acting as contrarians) and to reinforce the market destabilizing behaviour of fundamentalists. These results apply to the entire sample (2003 – 2016) and are strongly corroborated over the 2003-2009 subset.

By modelling both the one-month and the three-month to expiry futures contracts, based on weekly data, we test whether contract maturity affects these patterns. As expected, in periods of turmoil and rising uncertainty, we find evidence of short-termism by rational financial agents, as the absolute values of the coefficients tend to be larger in actual value in the case of the one-month contract.

This research is structured as follows. Section 2 analyses the theoretical and empirical characteristics of our three-agent model. Section 3 sets forth the empirical estimates over the periods. Section 4, concludes the paper and provides an economic and financial interpretation of the observed oil futures price gyrations.
2. The model

2.1 Theoretical considerations

Hedging transactions are intended to reduce the risk of unwanted future cash price changes. We define the return of cash position in the oil market as \( r_{ct} = \Delta \log C_t = \Delta c_t \) where \( C_t \) is the cash (spot) oil price. In the same way, the return of futures positions is \( r_{ft} = \Delta \log F_t = \Delta f_t \), where \( F_t \) is the price of the corresponding futures contract. An investor who takes short (long) position of one unit in the oil cash market will hedge by taking a long (short) position of \( \beta \) in the futures market. This hedge ratio can be regarded as the fraction of the short (long) position that is covered by futures purchases (sales).

Prices are set in an order-driven market. Every period traders revise their long/short positions; price changes from \( t \) to \( t+1 \) are a function of their excess demands and can be parameterized by the following log-linear function

\[
f_{t+1} = f_t + \alpha(D_C^t + D_F^t + D_H^t) + \epsilon_{t+1} \tag{1}
\]

where \( \alpha \) is a positive market reaction coefficient and \( D_C^t \), \( D_F^t \) and \( D_H^t \) denote the demand of chartists (feedback traders), fundamentalists and hedgers. The residual \( \epsilon_{t+1} \) accounts for additional factors that may impact on prices. The demand of feedback traders at time \( t \) is given by

\[
D_C^t = a_1 s_{it}^t (f_t - f_{t-1}) + a_1^* r_{USD,t} + a_1^* DVOX_t \tag{2}
\]

Coefficient \( a_1 \) is positive as feedback traders expect the existing price trend to persist in the subsequent time period. They will buy the contract if \( \Delta f_t \) is positive
and sell it if $\Delta f_t$ is negative. Their overall impact is nonlinear and given by $a_t S_{i_t}^C$ where $S_{i_t}^C$ is assumed to measure the fraction of the set of feedback traders entering the market at time $t$. This fraction depends upon market conditions and is parameterized by the following logistic function

$$S_{i_t}^C = \left[1 + \exp\left\{-\gamma_C \left(\frac{|N - r_{ft-i}|}{\sigma_{r_{ft-i}}^2}\right)\right\}\right]^{-1} i = 0,1,\ldots,l \quad (3)$$

$N$ is the normal (equilibrium) return of the oil futures contract, which is defined as the following $n$-periods moving average of current and past commodity futures returns $N = \sum_{k=0}^{n-1} r_{ft-k} / n$. We assume, in this way, that oil futures returns are the algebraic sum of two stochastic components: an equilibrium level $N$ and a temporary deviation $(N - r_{ft-i})$. The value of the delay parameter $i$ is determined empirically as it depends upon the physical and institutional characteristics of WTI oil pricing. The component $|N - r_{ft-i}|/\sigma_{r_{ft-i}}^2$ is a signal to noise ratio. The larger the deviation of $r_{ft-i}$ from $N$, the stronger the perception of market disequilibrium and the larger the fraction of feedback traders that will post orders on the market. The denominator, $\sigma_{r_{ft-i}}^2$, is an index of futures price variability. It accounts for the impact of risk. A higher (lower) risk associated with higher (lower) price volatility will reduce (increase), for a given perception of market disequilibrium, the willingness of speculators to enter the market. The term $S_{i_t}^C$ can take any value in the $[0;1]$ interval depending on the sign of coefficient $\gamma_C$ as $|N - r_{ft-i}|$ ranges from 0 (when $N = r_{ft-i}$) to $+\infty$. Large deviations of $r_{ft-i}$ from normal value will bring about a decline (increase) in the number of chartists when $\gamma_C$ is negative (positive). The absolute value of $\gamma_C$ matters too. The higher the synchronization of traders’ reaction to price deviations from their normal level (a symptom of herding behaviour), the

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4 If $a_t$ is negative, negative feedback traders/contrarians stabilize the market.
larger is the value of $\gamma_C$. On the contrary, a low absolute value of this coefficient will reflect idiosyncratic reactions of traders to price disequilibria, possibly due to differing degrees of risk aversion.

The impact of the rate of change of the USD nominal trade weighted exchange rate on the behaviour of chartists is measured by coefficient $a_1^*$. Its sign is expected to be negative, as a USD appreciation will lead to a reduction in dollar denominated oil prices and vice-versa for a USD depreciation. A shift in the VOX index too will affect the demand of chartists. Its impact on oil prices is measured by coefficient $a_1^{**}$. A negative sign means that an increase (decrease) in financial uncertainty will bring about a decrease (increase) in oil prices. This coefficient is assumed to account for the impact of the assessment of financial risk outlook on chartists. As pointed out by Zhang et al. (2017), free movements of investment funds tend to swarm in and out of the oil market. An increase in stock market uncertainty will bring about an outflow of funds from oil contracts and therefore produce a decline in their price.

Alongside feedback traders, we posit the existence of professional (institutional) investors, labelled here fundamentalists, who exploit their oil market expertise for portfolio diversification purposes. As such, their behaviour is influenced by both futures and cash returns, as discussed below. Their demand of futures contracts at time $t$ is given by

$$D_F^t = a_2 S_F^t (N - r_{ft}) + a_2^* r_{USD,t} + a_2^{**} DVOX_t \quad (4)$$

Fundamentalists react to deviation of the futures return from its equilibrium value $N$ as defined above. The coefficient $a_2$ indicates how fundamentalists’ beliefs about market prices affect their behaviour. If the coefficient $a_2$ takes on a positive value, this indicates that the majority of fundamentalists believes that the price will revert
to its equilibrium value. This will lead them to buy if \( N > r_{ft} \) and to sell in the opposite case. If the coefficient \( a_2 \) takes on a negative value, fundamentalists, disbelieving in the mean-reverting nature of the price, will sell if \( N > r_{ft} \) and buy in the opposite case.\(^5\) In all cases, empirical findings suggest that fundamentalists enter or exit the market depending on their perception of oil price misalignment in the spot market, whereas chartists respond to futures prices as seen above. Fundamentalists base their investment strategies on more sophisticated scenarios, which necessarily include the evaluation of cash oil markets and of their underlying fundamental drivers. Consequently, we model the transition function \( S^F_{jt} \) as follows

\[
S^F_{jt} = \left[ 1 + \exp \left\{ -\gamma_F \left( \frac{|M - r_{ct-j}|}{\sigma^2_{r_{ct-j}}} \right) \right\} \right]^{-1} \quad j = 0,1,\ldots,p \quad (5)
\]

Where \( M \) is the normal (equilibrium) return of oil cash contracts, which is defined as the following \( m \)-periods moving average of current and past cash oil returns \( M = \sum_{k=0}^{m-1} r_{ct-k}/m \).\(^6\) The value of the delay parameter \( j \) is determined empirically. The component \( |M - r_{ct-j}|/\sigma^2_{r_{ct-j}} \) is a signal to noise ratio, synthesizing the dynamics of the oil spot market. Here too, the term \( S^F_{jt} \) can take any value in the \([0; 1]\) interval depending on the sign of coefficient \( \gamma_F \) as \( |M - r_{ct-j}| \) ranges from 0 (when \( M = r_{ct-j} \)) to \( +\infty \). Large deviations of \( r_{ct-j} \) from normal value will bring about a decline (an increase) in the number of fundamentalists when \( \gamma_F \) is negative (positive). Here too oil market participants are assumed to be affected by USD dollar exchange rate shifts and by changes in stock market price uncertainty, as quantified by shifts in the VOX implied volatility S&P100 index. The corresponding impacts on futures oil prices are measured by coefficients \( a^*_2 \) and \( a^{**}_2 \).

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\(^5\) See Chia et al. (2014).

\(^6\) In our empirical analysis it is assumed that \( m = n = 12 \).
Hedgers base their decisions on the return of the hedging position and on its variance. As equation (6) indicates, the return to the hedging position $r_{ht}$ is a linear combination of the returns of the cash and futures prices

$$r_{ht} = r_{ct} - \beta r_{ft} = (c_t - c_{t-1}) - \beta (f_t - f_{t-1}) \quad (6)$$

Where $\beta$ is the hedging ratio.

The variance of the portfolio revenue by unit of product is obtained:

$$\sigma_{r_{ht}}^2 = \sigma_{r_{ct}}^2 + \beta^2 \sigma_{r_{ft}}^2 - 2\beta \sigma_{r_{ct}} \sigma_{r_{ft}} \rho_{r_{ct}r_{ft}} \quad (7)$$

Where $\sigma_{r_{ct}}^2$ is the variance of the cash return, $\sigma_{r_{ft}}^2$ the variance of the futures return, $\rho_{r_{ct}r_{ft}}$ is the linear correlation coefficient between the two returns and is equal to $(\sigma_{r_{ct}r_{ft}} / \sigma_{r_{ct}} \sigma_{r_{ft}})$. The optimum hedge ratio $\beta^*$ is derived from the first order condition of the hedging position variance minimization and reads as

$$\beta^* = \frac{\sigma_{r_{ct}} \sigma_{r_{ft}} \rho_{r_{ct}r_{ft}}}{\sigma_{r_{ft}}^2} \quad (8)$$

Therefore, the optimum hedge ratio depends on the covariance between the changes in futures and cash prices and on the variance of the futures price changes.

The hedging model is extended by introducing a dynamic component. The performance of a portfolio is measured by its variance reduction with respect to the optimal percentage of hedging. Substituting $\beta^*$ in equation (7), we obtain

$$\sigma_{r_{ht}}^2 = \sigma_{r_{ct}}^2 - \left(\frac{\sigma_{r_{ct}r_{ft}}}{\sigma_{r_{ft}}^2}\right)^2 = \sigma_{r_{ct}}^2 \left(1 - \rho_{r_{ct}r_{ft}}^2\right) \quad (9)$$
Equation (10) describes the demand of futures contracts of a trader wishing to minimize the variance of her optimally hedged position

\[ D_t^H = a_3 S_H^H \sigma_r^2 + a_3^* r_{USD,t} + a_3^{**} DV OX_t \]  \hspace{1cm} (10)

An increase in the minimum portfolio variance (9) may be due to a rise in the variability of cash price changes and/or to a decrease in the correlation between the cash and futures returns. The overall impact of hedgers’ trading is nonlinear and given by \( a_3 S_H^H \) where \( S_H^H \) is assumed to measure the fraction of the set of hedgers entering the market at time \( t \), fraction which, in turn, will depend upon market conditions. The structure of the hedgers transition function is analogous to that, which governs the behavior of fundamentalists. Indeed both categories of agents respond to deviations of cash prices from their perceived equilibrium value \( M \), even if with different speeds and obviously with different goals. Based on these considerations, the transition function \( S_H^H \) is parameterized by the following logistic function, whose properties mirror those of equation (5)

\[ S_H^H = \left[ 1 + \exp\{-\gamma_H (|M - r_{ct-h}|/\sigma_r^2)\} \right]^{-1} \hspace{1cm} h = 0,1,\ldots,k \]  \hspace{1cm} (11)

The impact on hedgers’ futures demand of the USD exchange rate and of stock market uncertainty are quantified by coefficients \( a_3^* \) and \( a_3^{**} \).

Substituting equations (2), (4) and (10) in equation (1) we have the following futures prices relationship

\[ r_{ft+1} = \theta_1 S_t^C r_{ft} + \theta_2 S_t^F (N - r_{ft}) + \theta_3 S_H^H \left( \sigma_r^2 - \left( \sigma_r^2 r_{ft} / \sigma_{r_{ft}}^2 \right) \right) + \theta_4 r_{USD,t} + \theta_5 DV OX_t + e_{ft+1} \]  \hspace{1cm} (12)
Where $\theta_1 = a_a_1$, $\theta_2 = a_a_2$, $\theta_3 = a_a_3$, $\theta_4 = a(a_1^* + a_2^* + a_3^*)$ and $\theta_5 = a(a_1^{**} + a_2^{**} + a_3^{**})$

Equation (12) relates futures returns to their previous period values, to the deviation of these values from their long run equilibrium $N$, and to the past variability of the optimally hedged positions of oil traders and oil producers. Economic theory posits that spot and futures prices are jointly determined for any given commodity (Stein 1961). Our investigation thus includes two equations accounting, respectively, for the behavior of spot and futures price returns together with their covariance. The conditional mean equation for $r_{ct}$ is modelled as an error correction relationship (Equation 13), where spot prices adjust to futures prices, which play the price discovery role. In the long run, indeed, a cointegration relationship between cash and futures prices holds and plays the role of attractor for the short-run cash price adjustments.

$$r_{ct+1} = b_0 + \sum_{z=0}^{n} b_{1z}r_{ct-z} + \sum_{w=0}^{m} b_{2w}r_{ft-w} + \theta(c_t - \lambda f_t) + e_{ct+1} \quad (13)$$

2.2. The empirical model

Futures and cash price rates of return are conditionally heteroskedastic when data are sampled with a weekly frequency – as we do in this paper – and a GARCH approach is used to model the second moments that enter equation (12). Equation (14), the empirical counterpart of equation (13) above, parameterizes the conditional mean of the cash returns whereas equation (15), the counterpart of equation (12), illustrates futures pricing by hedgers and speculators.
We adapt the model set out above to the bubble-like environment, by including a slope dummy $DB_t$ in equation (15), which is equal to 1 over the bubble period and 0 otherwise, enabling us to assess the different reaction of the three categories of agents. We identify the bubble period using the approaches of Zhou and Sornette (2009) and of Phillips et al. (2011) and find that it spans the 2007-2009 years as we discuss in more detail in Section 3.

\[ r_{ct} = d_0 + \sum_{iz=1}^{m} d_{cz}r_{ct-iz} + \sum_{iw=1}^{m} d_{fw}r_{ft-w} + \zeta (f_{t-1} - \lambda_0 - \lambda_1 c_{t-1}) + \nu_{ct} \quad (14) \]

\[ r_{ft} = \]

\[ g_0 + (g_1 + b_1 DB_{t-1})S_{it-1}^C r_{ft-1} + (g_2 + b_2 DB_{t-1})S_{jt-1}^F (N - r_{ft-1}) + (g_3 + b_3 DB_{t-1})S_{ht-1}^H h_{ft-1}^2 + g_4 r_{USD,t-1} + g_5 DVOX_{t-1} + \nu_{ft} \quad (15) \]

\[ S_{it-1}^C = \left[ 1 + \exp \left\{ -\gamma C \left( \frac{N - r_{ft-1-i}}{h_{ft-i}^2} \right) \right\} \right]^{-1} \quad (16) \]

\[ S_{jt-1}^F = \left[ 1 + \exp \left\{ -\gamma F \left( \frac{M - r_{ct-1-j}}{h_{ct-j}^2} \right) \right\} \right]^{-1} \quad (17) \]

\[ S_{ht-1}^H = \left[ 1 + \exp \left\{ -\gamma H \left( \frac{M - r_{ct-1-h}}{h_{ct-h}^2} \right) \right\} \right]^{-1} \quad (18) \]

\[ \nu_t = [\nu_{ct} \, \nu_{ft}] \quad (19) \]

\[ \nu_t | \Omega_{t-1} \sim N(0, H_t) \quad (20) \]

\[ H_t = \Delta_t R \Delta_t \quad (21) \]

\[ R = \begin{bmatrix} 1 & \rho_{r_c r_f} \\ \rho_{r_c r_f} & 1 \end{bmatrix} \quad (22) \]

\[ \Delta_t = \begin{bmatrix} h_{r_{ct}} & 0 \\ 0 & h_{r_{ft}} \end{bmatrix} \quad (22') \]

\[ h_{r_{ct}}^2 = \omega_c + \alpha_c v_{r_{ct}-1} + \beta_c h_{r_{ct}}^2 \quad (23) \]

\[ h_{r_{ft}}^2 = \omega_f + \alpha_f v_{r_{ft}-1} + \beta_f h_{r_{ft}}^2 \quad (23') \]

\[ h_{r_{ht-1}}^2 = \left( h_{r_{ct}}^2 - \frac{(h_{r_{ct-1}r_{ft-1}}^2)^2}{h_{r_{ft}}^2} \right) \quad (24) \]
Our empirical model allows for a complex characterization of the interaction among different categories of economic agents, who react to deviations of market prices from their equilibrium values, in ways, which can be stabilizing or destabilizing.

If $g_1$ is positive (negative), chartists (contrarians) destabilize (stabilize) the market, acting as positive (negative) feedback traders.\(^7\) If $\gamma$ ($\gamma_C$ in equation (16)) is positive (negative), the relative number of feedback traders, present in the market, grows (declines) with the deviation of $r_{ft-1-i}$ from its moving average value $N$.

Turning to fundamentalist speculators, the negative value of $g_2$ deserves specific comment. Fundamentalists may indeed believe that the persistence in the misalignment between the equilibrium and the current rate of return on futures contracts will last for some time and persist to buy (sell) if $r_{ft} > N$ ($r_{ft} < N$). This is a symptom of the failure of the price signaling process during periods of turbulence and is consistent with fundamentalists destabilizing the market, their traditional stabilizing behaviour being associated with a positive value of $g_2$. As for the negative sign of $\gamma$ ($\gamma_F$ in Equation (17)), Shleifer and Vishny (1997) explain it by the wariness of fundamentalists to enter the market if trades based on their own forecasts turn out to be persistently incorrect. In this case, a growing disequilibrium between the cash return and its equilibrium value will bring about a decline in the number of fundamentalists active in the market.

Coming to hedgers, the following considerations apply. As Cifarelli (2013, p.161) explains, an increase in $\sigma^2_{rct}$ can be produced either by an increase or a decrease in crude oil prices. As Equation (9) indicates the hedged portfolio variance $\sigma^2_{rht}$ depends on the variance of cash prices $\sigma^2_{rct}$ and on the squared correlation coefficient between cash and futures prices $\rho^2_{rct \cdot rft}$. Whenever – as is the case in our estimates – correlation between the two prices is stable over time, hedgers will react to changes

\(^7\) The standard justification for the presence of contrarians is that some feedback traders may believe that prices have overshot a reasonable equilibrium value (Wan and Kao, 2009).
in cash prices only. Coefficient $g_3$ is expected to be negative if in the previous period(s) the cash price rate of change is positive and positive if in the previous period(s) the cash price rate of change is negative. Long positions in commodities (by producers) are associated with short positions in futures contracts, whereas short positions in commodities (by e.g. traders or consumers) are associated with long positions in futures contracts. If the commodity cash price rises (falls), the producer is likely, in the subsequent time period, to increase (reduce) his planned future sales. In order to hedge against future spot price declines he is going to raise (decrease) his hedging position by selling more (less) futures contracts. The futures price will fall (rise) and the coefficient of the hedged position variability $g_3$ will be negative (positive). The behavior of either traders or consumers causes the same sign shifts. If the commodity price declines (rises) traders will face, in the following period, an increase (decrease) in demand and increase (reduce) their short positions commitments in the cash market, and in order to hedge against futures price rises, will raise (cut) their long positions in the futures market bringing about a futures price increase (decrease).

3. Empirical results

The paper uses weekly data in order to measure the impact of the financial crisis on the dynamics of futures oil pricing. Our sample spans the time interval from 2 January 2003 to 12 January 2016 and includes two major cycles, terminated by abrupt downswings in 2009 and in 2015. We analyze in this paper futures oil price dynamics over the full sample and in order to investigate its peculiar properties, over the highly controversial 2003-2009 time period. The oil spot price $C_t$ is the WTI spot price FOB (US dollars per barrel), the futures oil price $F_t$ is provided by the EIA
database.\footnote{Futures contract 1 expires on the third business day prior to the 25th calendar day of the month preceding the delivery month. If the 25th calendar day of the month is a non-business day, trading ceases on the 3rd business day prior to the business day preceding the 25th calendar day. Contract 3 corresponds to the second successive delivery month following contract 1.} Figure 1 exhibits the series themselves and summary statistics of the rates of returns over the full sample and the second sub-sample are set out in Table 1. Figure 1 depicts oil cash and futures prices in levels (left-hand panels) and in first log-differences (right-hand panels). Price levels provide visual insights into the bubble-like price behaviour, which our analysis tries to explain. Leaving out the “Great Moderation”, our sample period is characterized – from 2005 to mid 2008 - by a positive trend, interrupted by a sharp spike, followed by an equally outsized downswing.

As pointed out in the recent literature, a defining characteristic of bubble dynamics is the super-exponential behavior of prices. In order to detect it, we perform both the D-test of Zhou and Sornette (2009) and the SADF (supremum right-tail ADF) bubble test of Phillips et al. (2011).\footnote{The test results are set out in the appendix.} We find that our series conform with this behaviour from January 2007 to July 2008. The tests we are using here indentify the shorter super-exponential price upswing, which ends in June 2008, i.e. the first part of the bubble only. In the empirical estimates we use the entire bubble, which includes also the 2008-2009 downswing, in our dummy time interval. The bubble dummy $DB_t$, therefore, takes a value of 1 over the longer 12 January 2007 - 13 February 2009 time interval.
As expected, the rates of return in Table 1 are strongly serially correlated and conditionally heteroskedastic, volatility clustering being extremely large between 2008 and 2009 and again at the end of the sample period. The data in first
differences are always stationary, as shown by ADF test statistics, non-normally distributed and affected by nonlinearities.

### Table 1. Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Spot price return $r_{ct}$</th>
<th>Futures contract 1 return $r_{f1t}$</th>
<th>Futures contract 3 return $r_{f3t}$</th>
<th>Spot price return $r_{ct}$</th>
<th>Futures contract 1 return $r_{f1t}$</th>
<th>Futures contract 3 return $r_{f3t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.0001</td>
<td>0.0015</td>
<td>0.0003</td>
<td>0.0024</td>
<td>0.0024</td>
<td>0.0026</td>
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<tr>
<td><strong>Std. dev.</strong></td>
<td>0.0424</td>
<td>0.0402</td>
<td>0.0365</td>
<td>0.0479</td>
<td>0.0446</td>
<td>0.0398</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>-0.4680</td>
<td>-0.4635</td>
<td>-0.3939</td>
<td>-0.6728</td>
<td>-0.6638</td>
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<td><strong>Kurtosis</strong></td>
<td>8.6338</td>
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<td>4.3949</td>
<td>8.2117</td>
<td>4.2909</td>
<td>3.9708</td>
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<td><strong>JB</strong></td>
<td>751.3448</td>
<td>99.5258</td>
<td>72.7132</td>
<td>435.7910</td>
<td>51.5757</td>
<td>32.8132</td>
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<tr>
<td><strong>AR1</strong></td>
<td>16.753</td>
<td>14.645</td>
<td>29.954</td>
<td>7.2194</td>
<td>14.472</td>
<td>15.861</td>
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<td><strong>AR5</strong></td>
<td>28.214</td>
<td>22.022</td>
<td>39.545</td>
<td>17.941</td>
<td>21.522</td>
<td>23.133</td>
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<tr>
<td><strong>ARCH1</strong></td>
<td>115.270</td>
<td>63.395</td>
<td>37.663</td>
<td>69.367</td>
<td>45.072</td>
<td>29.491</td>
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<tr>
<td><strong>ARCH5</strong></td>
<td>267.560</td>
<td>213.470</td>
<td>114.70</td>
<td>159.750</td>
<td>162.00</td>
<td>94.582</td>
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<tr>
<td><strong>BDS2</strong></td>
<td>8.1306</td>
<td>8.3545</td>
<td>6.8262</td>
<td>5.9710</td>
<td>6.5189</td>
<td>4.24045</td>
</tr>
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</table>

Notes. Probability values in square brackets; JB: Jarque-Bera normality test; ARk: Ljung-Box test statistic for k-th order serial correlation of the time series; ARCHk: Ljung-Box test statistic for k-th order serial correlation of the squared time series; ADF(c, n): Augmented Dickey Fuller unit root test statistic, with a constant term and n-th order autoregressive component; BDSk: test statistic, with embedding dimension k, of the null that the time series, filtered for a first order autoregressive structure, is independently and identically distributed.

Indeed, the BDS test statistics of Brock et al. (1987) strongly reject, with embedding dimension 2, the null hypothesis that the rates of returns, filtered for first order serial dependence are iid. (Analogous results are obtained for the unfiltered returns, with embedding dimensions varying from 2 to 6). From a qualitative point of view, the properties of the data, which span the sub-sample (2003-2009), are analogous to the full sample ones.
3.1 Empirical analysis over the full sample

The full sample estimates (02/01/2003 – 12/01/2016) of the model can be found in Table 2. The parameterization of equation (15) is justified by the strategy set out in Teräsvirta (1994). At first, the lag of the autoregressive futures log difference is selected using the Akaike Information Criterion: a one-week lag provides the best fit. A test of linearity against the non-linear parameterization of equation (15) is performed following the procedure of Luukkonen at al. (1988), as modified by Wan and Kao (2009). The transition functions (16), (17) and (18) are replaced in equation (15) by a third order Taylor series approximation. The following auxiliary equation is estimated

\[ r_{ft} = \pi_0 + \pi_1 r_{ft-1} + \pi_2 r_{ft-1} y_{t-1-i} + \pi_3 r_{ft-1} y_{t-1-i}^2 + \pi_4 r_{ft-1} y_{t-1-i}^3 + \]
\[ + \mu_1 (N - r_{ft-1}) + \mu_2 (N - r_{ft-1}) x_{t-1-j} + \mu_3 (N - r_{ft-1}) x_{t-1-j}^2 + \mu_4 (N - r_{ft-1}) x_{t-1-j}^3 + \]
\[ + \delta_1 h_{r_{ht-1}}^2 + \delta_2 h_{r_{ht-1}}^2 x_{t-1-h} + \delta_3 h_{r_{ht-1}}^2 x_{t-1-h}^2 + \delta_4 h_{r_{ht-1}}^2 x_{t-1-h}^3 + \]
\[ + \tau_1 r_{USD,t-1} + \tau_2 DVOX_t \]
\[ + \epsilon_t \]

where, \( x_{it-k} = |M - r_{ct-1-k}|, \) \( k = j, h \) and \( y_{t-i} = |N - r_{ft-1-i}| \)

We test linearity against STAR modeling - for various values of \( i, j \) and \( h \) - performing LM tests of the null hypothesis \( H_0: \pi_2 = \pi_3 = \pi_4 = \mu_2 = \mu_3 = \mu_4 = \delta_2 = \delta_3 = \delta_4 = 0 \). We have also tested linearity against STAR modeling for chartists, fundamentalists and hedgers in isolation. That is, we have performed the following LM tests of the null hypotheses \( H_{0c}: \pi_2 = \pi_3 = \pi_4 = 0; H_{0f}: \mu_2 = \mu_3 = \mu_4 = 0; \) and \( H_{0h}: \delta_2 = \delta_3 = \delta_4 = 0 \). For the values of the delay parameters of the first row of Tables 2
and 3, the Teräsvirta Non-linearity Test (TNT) statistics uniformly reject $H_0$, $H_{0C}$ and $H_{0H}$ in the case of the full-sample estimates and fail to reject $H_{0F}$ only, in the case of fundamentalists operating with the three month futures contract. Our non-linear parameterization is thus convincingly justified by the data and the time-varying fractions of chartists, fundamentalists and hedgers in equation (15) are parameterized using equations (16), (17) and (18).\textsuperscript{10} The overall quality of fit of the model is satisfactory. The estimated parameters are significantly different from zero and our GARCH model captures the conditional heteroskedasticity of the residuals.\textsuperscript{11} The usual misspecification tests indicate that the standardized residuals $\eta_t$ are always well behaved; for each system $E[\eta_t] = 0$, $E[\eta_t^2] = 1$ and $\eta_t^2$ is serially uncorrelated. The BDS2 tests, moreover, fail to reject the null that the standardized residuals are iid. The nonlinearities detected in the return time series of Table 1 are filtered away by the model.

The main results emerging from the estimates reported on Table 2 may be summarized as follows. First, we find evidence that both feedback traders and fundamentalists exert a price destabilizing effect over the whole sample, captured by the coefficients $g_1$ and $g_2$, in the case of Fut-1 and also in the case of Fut-3 as far as feedback traders are concerned. The bubble exacerbates the destabilizing behaviour of fundamentalists (i.e. hedge funds), as the highly significant negative value of coefficient $b_2$ indicates, while dampening the effect of feedback trading, as the negative sign of the coefficient $b_1$ suggests. In the same way, during the bubble and particularly for the Fut-1 contract, hedgers stabilize the market as we explain in detail at the end of section 2. Overall, our estimates indicate that market

\textsuperscript{10} The Taylor procedure allows us to reject the alternative ESTAR parameterization of the transition function. For the sake of parsimony these tests are not reported here. It should be noticed that rejection of the $H_{0C}$, $H_{0F}$ and $H_{0H}$ hypotheses implies also the rejection of the hypotheses that chartists, fundamentalists and hedgers fail to affect the behavior of the futures contracts rates of change, justifying, in this way, the three-agent model parameterization of our paper.

\textsuperscript{11} The t-ratios reported in the tables are based on the robust quasi-maximum likelihood estimation procedure of Bollerslev and Wooldridge (1992).
participate in the contract.

Table 2. Full sample estimates: 02/01/2003 – 12/01/2016

<table>
<thead>
<tr>
<th>m = 1, n = 3</th>
<th>i=0, j=2, h=8</th>
<th>m = 1, n = 3</th>
<th>i=0, j=1, h=11</th>
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<tbody>
<tr>
<td>r_{ct}</td>
<td>r_{ct}</td>
<td>r_{ct}</td>
<td>r_{ct}</td>
</tr>
<tr>
<td>d_0</td>
<td>1.079 (51.146)</td>
<td>g_0</td>
<td>0.232 (10.512)</td>
</tr>
<tr>
<td>d_{c1}</td>
<td>-0.059 (-9.840)</td>
<td>g_1</td>
<td>0.295 (21.814)</td>
</tr>
<tr>
<td>d_{c2}</td>
<td>-0.000 (-0.056)</td>
<td>g_2</td>
<td>-0.117 (-8.310)</td>
</tr>
<tr>
<td>d_{c3}</td>
<td>-0.027 (-4.301)</td>
<td>g_3</td>
<td>1.171 (3.196)</td>
</tr>
<tr>
<td>d_{c4}</td>
<td>0.243 (41.093)</td>
<td>γ_c</td>
<td>0.460 (-4.751)</td>
</tr>
<tr>
<td>θ</td>
<td>0.644 (53.360)</td>
<td>γ_f</td>
<td>0.370 (-2.399)</td>
</tr>
<tr>
<td>λ_0</td>
<td>0.013 (39.932)</td>
<td>γ_H</td>
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</tr>
<tr>
<td>λ_1</td>
<td>1.000 (12778.9)</td>
<td>λ_1</td>
<td>1.006 (148.414)</td>
</tr>
<tr>
<td>ρ_{c_f}</td>
<td>0.986 (47.488)</td>
<td>LLF</td>
<td>-2427.50</td>
</tr>
<tr>
<td>α_c</td>
<td>0.681 (57.352)</td>
<td>α_f</td>
<td>0.711 (57.352)</td>
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<tr>
<td>β_c</td>
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<td>α_f</td>
<td>0.091 (98.611)</td>
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<tr>
<td>β_r</td>
<td>0.633 (39.037)</td>
<td>β_r</td>
<td>0.685 (774.673)</td>
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<tr>
<td>T.N.T.</td>
<td>7.124 (0.000)</td>
<td>C F H</td>
<td>4.629 (0.003)</td>
</tr>
<tr>
<td>BDS2</td>
<td>0.3250</td>
<td>BDS2</td>
<td>0.6636</td>
</tr>
</tbody>
</table>

E[η_{ct}] = E[η_{ct}]^2 / E[η_{ct}]^2
E[η_{ct}] = E[η_{ct}]^2 / E[η_{ct}]^2
E[η_{ct}] = E[η_{ct}]^2 / E[η_{ct}]^2

Notes. Probability values in square brackets; Sk.: Skewness; Kurt.: Excess Kurtosis; JB: Jarque-Bera normality test; ARK: Ljung-Box test statistic for k-th order serial correlation of the time series; ARCHK: Ljung-Box test statistic for k-th order serial correlation of the squared time series; T.N.T.: Terasvirta (1994) test of nonlinearity applied to the chartists’ (C), fundamentalists’ (F) and hedgers’ (H) transition functions and to the three transition functions simultaneously (C F H). BDSK: test statistic, with embedding dimension k, of the null that the standardized residuals are independently and identically distributed.

Indeed the three LSTAR dynamics γ-coefficient are always positive and significant in the case of Fut-3 and negative in the case of Fut-1. Coefficient
$g_4$ estimates are negative for both contracts and capture the well-known negative impact of a USD appreciation on oil prices. The negative sign of the $g_5$ coefficient is more relevant and disproves the hypothesis that a greater stock market perception of risk resulted in an outflow of financial funds into commodity investment. Indeed, the opposite seems to have been the case, the latter occurring in spite of rise in financial market uncertainty.

Figures 2 and 3 provide a graphical assessment of the effects on futures returns of the different degree of consensus of chartists (left panel), fundamentalists (middle panel) and hedgers (right panel) with respect to deviations of price from perceived equilibrium and of trust in this perception. As discussed above, these deviations bring about an increase (decline) in the number of market participants, and thus in the absolute value of the impact on the futures rate of return, depending upon the positive (negative) sign of the estimated $\gamma$ coefficient, the absolute value of which (i.e. degree of consensus) being reflected in the slope of the curves.\textsuperscript{12} The impact of the bubble is relevant and brings about a shift in the pattern of the graphs (see the dots in the circles) and corresponds - in response to a growing deviation of the rates of return from their normal values - to a decrease in the destabilizing behaviour of chartists and to an increase in the destabilizing behaviour of fundamentalists. The case of the hedgers is more complex, since they become stabilizers in the case of the Fut-1 contract only.

\textsuperscript{12} Each graph contains a scatter plot of the impact of each group of agents on futures returns (regression coefficient multiplied by the value of the LSTAR transition function) and the deviations of the transition variable from its equilibrium value. We report the former on the vertical axis and the latter on the horizontal one. For the sake of clarity, we have interpolated the scatter plots using local first order polynomial regressions with bandwidth based on the nearest neighbor approach. The local regressions are performed on a sub sample selected according to the Cleveland (1993) procedure and involves about 100 evaluation points. Tricube weights are used in the weighted regressions used to minimize the weighted sum of squared residuals. The bandwidth span of each local regression is set to 0.3.
As shown in Figure 2, which applies to Fut-1 returns, the price destabilizing impact of chartists weakens during the bubble for any given deviation of futures returns from perceived normal values (left-panel). On the contrary, fundamentalists strongly destabilize the market during the bubble (mid-panel), overshadowing the price-moderating effect of hedging (right-panel).

Coming to Fut-3 returns, we obtain qualitatively analogous results (see Figure 3) in the case of chartists and fundamentalists, who both contribute to price
destabilization during the bubble. This effect is augmented but marginally by hedgers.

3.2 Robustness analysis

The inclusion of observations from the second 2009-2016 cycle, with its own specific characteristics, may introduce new factors that alter the behavioural reaction of the agents and affect our bubble investigation. In order to focus on the interpretation of the latter, we restrict the sample to the first cycle, which spans the 2 January 2003 to 30 December 2009 time period, and we obtain surprisingly similar results. This finding suggests that our analysis identifies a specific thread of the oil price dynamics, over the 2007-2009 time interval, which is independent of the sample length.

The quality of fit of the estimates set forth in Table 3 is highly satisfactory, all the coefficients are significant at the standard levels of significance and a perusal of the usual tests finds no evidence of model misspecification. Feedback traders and fundamentalists tend to destabilize prices since, for both contracts, coefficient \( g_1 \) and \( g_2 \) estimates take, respectively, positive and negative values. Here too the bubble brings about differing results. Feedback traders become contrarians (\( b_1 \) is large and negative), a fact that may reflect their fear of impeding price collapse. On the other hand, the destabilizing behaviour of fundamentalists becomes more incisive (\( b_2 \) is large and negative). Hedgers, as in the full sample estimates, tend to stabilize prices during the bubble upswing, since coefficient \( b_3 \) estimates are large and negative. As for the gamma estimates, the only difference between the full sample and first period estimates, is that feedback traders in the Fut-1 sub-market tend to trust price dynamics, and – the \( \gamma_c \) estimates being positive - enter the market for large deviations of current from equilibrium futures prices.
The strong similarity of the full sample and first period results is also conducive to the conclusion that the financialisation of the oil market is a permanent, irreversible...
phenomenon. The extension of the investigation to the second 2009 - 2016 cycle fails to weaken the quality of the system’s fit and does not alter its economic interpretation, the signs of the coefficients being almost always unchanged.

4. Conclusions

A recent and growing literature explains the bubble that characterized oil prices between 2007 and 2009 as the result of utility maximization by rational heterogeneous agents, interacting and possibly influencing each other. In this paper, we combine this approach with the HAM models of Westerhoff and Reitz and the LPPL model of Johansen et al. (2000), which combines a super-exponential pattern with bouts of negative feedback loops of price collapse. More precisely, our analysis, based on a flexible three-agent model, which controls also for exchange rate and equity market risk perception, attributes the bubble mostly to the destabilizing behaviour of fundamentalist speculators. Among these we include institutional investors, ETFs and hedge funds, as conventionally done in the literature. This reaction reinforces the standard price destabilizing effect caused by chartists. The extension of the sample to the post-bubble period (2009 – 2016) does not seem to invalidate our financial interpretation in spite of sharp and unexpected fluctuations in oil prices and a significant increase in the influence of geopolitical factors. Indeed, speculation plays a clear-cut destabilizing role over the entire sample period, due to the joint reaction of chartists and fundamentalists. Our results are thus in line with Zhang et al. (2017) among others.
References


Appendix on statistical bubble detection

D-test of “super exponential” growth price behaviour (Zhou and Sornette, 2009)

Let \( D = \frac{(RMS_1 - RMS_2)}{RMS_1} \) be a relative goodness of fit statistic

where \( RMS_1 \) and \( RMS_2 \) denote the root-mean-square of the residuals of the following (log) price equation estimates

\[
\log(p_t) = a + bt + \epsilon_t \quad (A.1)
\]

\[
\log(p_t) = a + bt + ct^2 + \epsilon_t \quad (A.2)
\]

(A.1) is the standard geometric random walk and coincides with (A.2), if the null \( c=0 \) holds. For \( c>0 \), (A.2) parameterizes a price process growing super exponentially. The D statistic measures the relative difference of the improvement of the fits resulting from the additional quadratic term in (A.2). The larger is D the more probable is the rejection of the null \( c=0 \) and the relevance of the quadratic term, i.e. of the bubble model. Zhou and Sornette (2009, p. 872) suggest that the time series is not in a bubble regime if (1) \( D \leq 0.25 \), or (2) \( c \) is not positive. Over the
time periods set out in the table below the values of the D-test statistics are supportive of a super-exponential (Fut-1 and Fut-3 price) behaviour.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>D-statistic Fut-1</td>
<td>0.1781</td>
<td>0.7559</td>
<td>0.7146</td>
<td>0.3916</td>
</tr>
<tr>
<td>D-statistic Fut-3</td>
<td>0.2739</td>
<td>0.6649</td>
<td>0.1396</td>
<td>0.3711</td>
</tr>
</tbody>
</table>

**SADF test (Phillips et al., 2011) of explosive behaviour (bubble) identification**

![Graphs showing SADF tests](image)

Notes. Critical values obtained using a Monte Carlo simulation with 1000 replications, initial window of 54 observations and null model: $x_t = dT^x + x_{t-1} + e_t$, where it is assumed that d=1 and e=1.

Phillips et al. (2011) propose comparing each element of the estimated right-tail rolling ADF sequence to the corresponding right-tailed critical values of the standard ADF statistic to identify a bubble initiating at time $T$. The estimated origination point of a bubble is the first chronological observation, denoted by $T_{re}$, in which $ADF_r$ crosses the corresponding critical value from below, while the estimated termination point is the first chronological observation after $T_{re}$, denoted by $T_{rf}$, in which $ADF_r$ crosses the critical value from above. Formally, the estimates of the bubble period (as fractions of the sample) are defined by

$$r_e = \inf (r : ADF_r > c\nu_r^{\beta r})$$

$r_e \in [0, 1]$

$$r_r = \inf (r : ADF_r < c\nu_r^{\beta r})$$

$r_r \in [0, 1]$

where $c\nu_r^{\beta r}$ is the $100(1 - \beta_r)$% critical value of the standard ADF statistic based on $[T_{re}]$ observations (see Caspi, 2017, p. 7, for more details). Both graphs suggest that the SADF statistics exceed the right-tail ADF critical values, rejecting thus the
null of unit root in favour of the explosive alternative, from the beginning of 2007 to the end of 2008, and corroborate in this way our bubble timing hypothesis.