High and Low Intraday Commodity Prices: A Fractional Integration and Cointegration Approach

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Abstract

This paper examines the behaviour of high and low prices of four commodities, namely crude oil, natural gas, gold and silver, and of the corresponding ranges using both daily and intraday data at various frequencies. For this purpose, it applies fractional integration and cointegration techniques; in particular, an FCVAR model is estimated to capture both the long-run equilibrium relationships between high and low commodity prices, referred to as the range, and the long-memory properties of their linear combination. Fractional cointegration in found in all cases, with the range showing stationary and nonstationary patterns and changing substantially across the frequencies. The findings may assist investors in improving their trading strategies since high and low prices serve as entry and exit signals in the market.

Keywords: Commodity prices, intraday, fractional integration, fractional cointegration, FCVAR

JEL Classification: C22, C32, G11, G15

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1. Introduction

In financial economics, the difference between high and low intraday or daily prices is known as the range. Volatility can be expected to be higher if the range is wider. Parkinson (1980) showed that, in fact, the price range is a more efficient volatility estimator than alternatives such as the return-based estimator. It is also frequently used in technical analysis by traders in financial markets (see, e.g., Taylor and Allen, 1992). However, as pointed out by Cheung et al. (2009), focusing on the range itself might be useful if one's only purpose is to obtain an efficient proxy for the underlying volatility, but it also means discarding useful information about price behaviour that can be found in its components. Therefore, in their study, Cheung et al. (2009) analyse simultaneously both the range and daily highs and lows using daily data for various stock market indices. Because of the observation that the latter two variables generally do not diverge significantly over time, having found that they both exhibit unit roots by carrying out Augmented Dickey-Fuller (ADF) tests (Dickey and Fuller, 1979), they model their behaviour using a cointegration framework as in Johansen (1991) and Johansen and Juselius (1990) to investigate whether they are linked through a long-run equilibrium relationship, and interpreting the range as a stationary error correction term. They then show that such a model has better in-sample properties than rival ARMA specifications but does not clearly outperform them in terms of its out-of-sample properties.

Following on from Cheung et al. (2009), the present study makes a twofold contribution to the literature. First, it uses fractional integration and cointegration methods that are more general than the standard framework based on the I(0) versus I(1) dichotomy. According to the efficient market hypothesis (EMH), asset prices should be unpredictable and follow a random walk (see Fama, 1970), i.e. they should be integrated of order 1 or I(1). However, the choice between stationary I(0) and nonstationary I(1) processes is too restrictive for most financial series (Barunik and Dvorakova, 2015). Diebold and Rudebusch (1991) and Hasslers and

Wolters (1994) showed that in fact unit root tests have very low power in the context of fractional integration. Therefore our analysis below allows the differencing parameter for the individual series to take fractional values. Moreover, we adopt a fractional cointegration approach to test for the long-run relationships. Fiess and MacDonald (2002), Cheung (2007) and Cheung et al. (2009) all modelled high and low prices together with the range in a cointegration framework to analyse the foreign exchange and stock markets, respectively. However, their studies restrict the cointegrating parameter to unity (even though this is not imposed in Granger's (1986) seminal paper). By contrast, we estimate a Fractional Cointegrated Vector Autoregression (FCVAR) model (Johansen, 2008; Johansen and Nielsen, 2010, 2012) that assumes that each of the series is integrated of order d, where d can be any real value, and their order of cointegration is less than d, that is CI(d-b) with $b > 0$.

Fractional cointegration models (see also Robinson and Yajima, 2002; Nielsen and Shimotsu, 2007; etc.) are more general and have already been shown to be more suitable for many financial series (see, e.g., Caporale and Gil-Alana, 2014 and Erer et al., 2016). The FCVAR model in particular has a number of advantages over the fractional cointegration set-up of Robinson and Marrinuci (2003): it allows for multiple time series and long-run equilibrium relationships to be determined using the statistical test of MacKinnon and Nielsen (2014), and it jointly estimates the adjustment coefficients and the cointegrating relations.[1] Nielsen and Popiel (2018) provide a Matlab package for the calculation of the estimators and test statistics. Dolatabadi et al. (2016) applied the FCVAR model to analyse the relationship between spot and futures prices in future commodity markets and found more support for cointegration compared to the case when the cointegration parameters are restricted to unity.

[1] For details about the asymptotic theory for estimation and statistical inference in the FCVAR framework, see Johansen and Nielsen (2010, 2012, 2014).

Second, we provide new empirical evidence by applying these methods to analyse high and low intraday and daily prices of crude oil, natural gas, gold, and silver and their corresponding ranges; further, we carry out robustness checks by repeating the analysis at various intraday frequencies. Daily price margins obtained as differences between high and low prices can be used as a reference level to make assumptions and predictions about future developments (Barunik and Dvorakova, 2015). Also, daily high and low prices may function as a "stop-loss" indicator, containing information about liquidity provisioning and the price discovery process. Lastly, they signal "ask and bid quotes", and react to unanticipated public announcements.[2] Liu et al. (2015) examined daily returns and daily range returns dependent on close–close and the high–low prices when forecasting multifractal volatility in the Chinese stock market; they found that both daily returns and range returns have a significant impact on the future multifractal volatility, a phenomenon termed ''leverage effects'' of the positive and negative returns.

The remainder of the paper is structured as follows: Section 2 describes the econometric methodology; Section 3 presents the empirical results; Section 4 offers some concluding remarks.

2. Econometric Methodology

2.1 Testing for Fractional integration

The ADF unit root test (Dickey and Fuller, 1979) imposes the restriction of unit integration in the testing procedure; Kwiatkowski et al. (1992) suggest instead how to test the null of stationarity [I(d = 0)] against the alternative of long memory [I(d), $d > 0$] in cases where the null of a unit root is rejected by the ADF test. Fractional integration methods permit testing for

[2] Other reasons why daily high and low prices are important are summarised in Caporin et al. (2013).

fractional degrees of integration. If $\{y_t,\ t=1,2,...\}$ is the series of interest, fractional differentiation is carried out as follows:

$$(1-L)^d\ y_t = u_t, \qquad t=1,2,.... \tag{1}$$

where L is the lag operator, that is, $Ly_t = y_{t-1}$; d can be any real number, and u_t is assumed to be integrated of order 0 and denoted as $I(0)$. The time series y_t is defined to be $I(d)$, and the differencing order determines its degree of dependence: the higher the value of d, the higher is the dependence between observations far apart in time. Values of d below 1 imply that shocks have transitory effects (i.e., mean reversion occurs), whilst d equal to or above 1 implies that shocks have permanent effects.

2.2 Testing for Fractional Cointegration

The definition of cointegration in the seminal paper of Engle and Granger (1987) states that, in the bivariate case, two series, x_{1t} and x_{2t} are cointegrated if both are integrated of the same order, say d, i.e., x_{1t} and $x_{2t} \approx I(d)$, and there exists a linear combination of the two, i.e, $x_{1t} - \beta x_{2t}$, which is integrated of a smaller order, say d-b, with b > 0. The parameters d and b are not constrained to be integer values, though most of the empirical applications carried out since then have assumed d = b = 1, i.e. what is usually called "standard" (or "non-fractional") cointegration. The methodology of Engle and Granger (1987) is based on two steps:

a) testing the order of integration of the individual series by using ADF (Dickey and Fuller, 1979) tests, and

b) if the two individual series are I(1), testing the order of integration of the residuals from the cointegrating regression:

$$x_{1t} = \alpha + \beta x_{2t} + u_t, \qquad t = 1,2,\ ..., \tag{2}$$

once more carrying out ADF tests with appropriately obtained critical values.

Cheung and Lai (1993) and Gil-Alana (2003) extended this approach to the fractional case by computing finite sample critical values for testing the null hypothesis of no cointegration (b = 0) against the alternative of fractional cointegration (b > 0). Other procedures were then proposed in subsequent papers (Robinson and Yajima, 2002; Robinson and Marinucci, 2003; Robinson and Hualde, 2003; Hualde and Robinson, 2007, etc.), and a survey of these methods can be found in Gil-Alana and Hualde (2009).

2.3 Testing for Cointegration in a VAR model

Consider a (*k* + 1)-dimensional vector of time series y_t , $t = 1, 2, ..., T$, each of them being $I(1)$. Johansen (1995) suggested a cointegration test based on a VAR(p) model of the form:

$$\Delta y_t \; = \; \Pi y_{t-1} \; + \; \sum_{i=1}^{k} \Gamma_i \Delta y_{t-i} \; + \; \varepsilon_t \qquad (3)$$

where (*k* + 1) is the order of autoregression including the constant term; $\Pi = \sum_{i=1}^{k+1} A_i - I$, $\Gamma_i = -\sum_{j=i+1}^{k+1} A_j$. If the rank *r* is less than *k*, then there exist $k \times r$ matrices α and β with rank *r* such that $\Pi = \alpha\beta'$, and $\beta' y_t$ is $I(0)$. The rank *r* gives the number of possible cointegrating relations, and each column of β is a cointegrating vector. The residual ε_t is (*r* + 1)-dimensional independent and identically distributed with zero mean and a variance-covariance matrix Ω . The cointegrating test involves estimating the Π matrix in an unrestricted VAR and then testing the restrictions implied by the reduced rank of Π .

2.4 The FCVAR model

The Fractionally Cointegrated Vector Autoregression (FCVAR) model is proposed in Johansen (2008), and first applied in Johansen and Nielsen (2010; 2012; 2016); its advantages are highlighted by Caporin et al. (2013). It is based on the Cointegrating VAR (CVAR) model of Johansen (1995) given in (3) above, with $\Pi = \alpha\beta'$, and using the lag operator differencing $Ly_t = y_{t-1}$ one obtains:

$$\Delta y_t = \alpha\beta' L y_t \ + \ \sum_{i=1}^{k} \Gamma_i \Delta L^i y_t \ + \ \varepsilon_t \, , \qquad (4)$$

where α and β are defined as before. By replacing the difference and lag operator Δ and $L = 1 - \Delta$ in (4) with their fractional counterparts Δ^b and $L_b = 1 - \Delta_b$, respectively, as in Johansen (2008), one obtains,

$$\Delta^b y_t = \alpha\beta' L_b y_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b^i y_t + \varepsilon_t \ , \qquad (5)$$

and with $y_t = \Delta^{d-b} x_t$, equation (5) becomes:

$$\Delta^d x_t = \alpha\beta' \Delta^{d-b} L_b x_t + \sum_{i=1}^{k} \Gamma_i \Delta^d L_b^i x_t + \varepsilon_t \ , \qquad (6)$$

where Δ^d is the fractional operator, and L_b is the fractional lag operator defined as above. The elements of $\beta' x_t$ are the cointegrating relationships in the system, where *r* represents the number of long-run equilibrium relationships, i.e. the cointegration or co-fractional rank. $\Gamma = \Gamma_i, ..., \Gamma_k$ govern the short-run dynamics. The coefficients in matrix α represent the speed of adjustment towards equilibrium for each of the variables in response to shocks. The fractional parameter d is the order of integration of the individual time series and d - b (with b < 0) is the degree of fractional cointegration, the fractional integration order of $\beta' x_t$ which is lower compared to that of x_t itself.

The specification given (6) is the so-called restricted constant version of the model by Johansen and Nielsen (2012), which is also used by Dolatabadi et al. (2016). A more general specification is the following:

$$\Delta^d x_t = \alpha \Delta^{d-b} L_b \left(\beta' x_t + \rho' \right) + \sum_{i=1}^{k} \Gamma_i \Delta^d L_b^i x_t + \xi + \varepsilon_t \qquad (7)$$

where ρ is the restricted constant term,[3] and ξ is the unrestricted constant term that generates a deterministic trend in the levels of the variables.[4]

Details of the Maximum Likelihood Estimation (MLE) of the parameter d, b, α, β and Γ and the Matlab programming code are given in Nielsen and Popiel (2018).

3. Empirical Analysis

3.1 Data Description

The series analysed are the intraday and daily high and low prices of crude oil, natural gas gold and silver. They were obtained at the following frequencies: 15 minutes (M15), 30 minutes (M30), one hour (H1), four hours (H4) and daily (D1); Table 1 gives details of the sample period. The data source is the ForexTime MT4 terminals. Considering differing frequencies is important as a robustness check (see Barunik and Dvorakova, 2015).

INSERT TABLE 1 ABOUT HERE

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[3] The constant term is restricted to be of the form $\alpha\rho'$, which is the mean level of each long-run equilibrium relationship, that is $E\left(\beta' y_t + \rho'\right) = 0$.

[4] Both (6) and (7) include the standard cointegrated VAR model of Engle-Granger as the special case when $d = b = 1$.

Cheung (2007) and Barunik and Dvorakova (2015) argue that, although daily high and low asset prices might be non-stationary, they could share a common trend, i.e. be cointegrated; in that case, "Range", defined as the difference between highs and lows, should be a stationary series. Figure 1 shows all three series for crude oil, natural gas, gold and silver. In the case of crude oil, high and low prices move relatively closely. By contrast, they appear to diverge at times in the case of natural gas, especially in 2013 and 2014. Divergence occurs also in the case of gold and silver prices.

INSERT FIGURE 1 ABOUT HERE

As a first step, we carry out ADF tests for the highs and lows, as well as the corresponding range (see Table 2). The test statistics imply that the unit root null cannot be rejected for the high and low price series in any case, whilst the first-differenced series are stationary. As for the range series, all series (daily and intraday) appear to be stationary except for natural gas at the 1-hour and 4-hour frequencies, which suggest they are cointegrated as argued by Cheung (2007).

INSERT TABLE 2 ABOUT HERE

3.2 Univariate Analysis

Granger (1980) and Granger and Joyeux (1980) advocated the use of an Autoregressive Fractionally Integrated Moving Average (ARFIMA) model that they showed to have better forecasting properties than ARIMA specifications. The fractional differencing parameter can be estimated using a variety of parametric, semi-parametric or non-parametric methods. Here we use the Exact Local Whittle (ELW) estimator proposed by Robinson (1995a) and Shimotsu and Phillips (2005) as well as the Geweke and Porter-Hudak (GPH, 1983) estimator in the improved version due to Robinson (1995b).[5]

The ELW and GPH results are reported in Tables 3 and 4, respectively. The estimated value of d for the highs and lows is close to 1 in all cases, regardless of the estimation method used and the periodogram ordinates considered. By contrast, the integration order of the range series is lower, but above 0.5 in most cases, which suggests that high and low prices might be cointegrated.

INSERT TABLE 3 ABOUT HERE

INSERT TABLE 4 ABOUT HERE

3.3 Multivariate Analysis

To establish whether there exists a long-run equilibrium relationship between the variables of interest, tests of the homogeneity of the fractional integration order must be carried out in the first instance. The results reported in Table 5 are based on the test statistic proposed by Robinson and Yajima (2002), though identical conclusions were obtained with Hualde (2013). They show that the null hypothesis of equal orders of integration cannot be rejected for any of the high and low price series except in one single case, and therefore it is legitimate to test for cointegration.

INSERT TABLE 5 ABOUT HERE

First a grid-search is conducted for the optimal k value for the cointegration rank test using minimum information criteria, where $k = 5$ is the maximum value considered in each case. Then, for each chosen k value, the corresponding fractional cointegration rank test of Johansen and Nielsen (2012) is carried out. The results are reported in Table 6: the null hypothesis of rank 0 against the alternative of rank 1 can be rejected in all cases, which leads

[5] These methods are appropriate for stationary series. If they are nonstationary, first differences are taken, and 1 is added to the estimated value of d. There exist alternative methods which are valid in the nonstationary case (Abadir et al., 2007, Shao, 2010, etc.) but they require additional user-chosen parameters.

to the conclusion that there is cointegrating relationship between high and low commodity prices.

INSERT TABLE 6 ABOUT HERE

Therefore the estimated FCVAR model includes only one cointegrating vector; the lag length selected on the basis of standard information criteria is 5 (note that MacKinnon and Nielsen, 2014, argued instead that one lag is generally sufficient to whiten the residuals in the FCVAR case). The same initial values set for *d* and *b* for the rank test are used again for the FCVAR estimation. Two specifications are considered, with unrestricted and restricted constant terms, as previously mentioned. Table 7 reports the results for the unrestricted constant case; all cointegrating coefficients as well as the *d* and *b* parameters are statistically significant. The model has been estimated for $d \neq b$. If the restriction $d = b$ is satisfied, and d = b = 1, then one is back to the CVAR model of Johansen (1995), and the order of integration of the range is 0 (that is I(d-b) = 0); although this has been rejected on the basis of the ELW and GPH results (see Table 3 and 4). The estimates of the cointegration vector in the unrestricted constant case reported in the 5th column of Table 7 are very close to the vector (1, -1, c) where "c" is the constant; therefore we then impose the restriction (1, -1) for the cointegrating vector without a constant, which now represents the range itself. The corresponding results are reported in Table 8 and are rather similar to those obtained in the unrestricted constant case.

INSERT TABLE 7 ABOUT HERE

INSERT TABLE 8 ABOUT HERE

The residual diagnostic tests based on the Q statistic suggest that there is no autocorrelation in the residuals for either FCVAR specification.[6]

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[6] The estimation results for the other parameters of the FCVAR such as the speed of adjustment towards equilibrium (α) and the short-run dynamics for each variables (Γ) are available on request.

Table 9 provides an overview of the estimation results for the integration order of the range, namely the ELW and GPH estimates of d for each of the periodogram values, as well as the FCVAR results for both the unrestricted and restricted constant cases. It is noteworthy that the estimated values for gold are generally low compared to estimates for other commodities, thus, the range for gold can be characterised as a stationary series with long memory. For example, in the case of the daily price for gold, the estimated *d* values based on GPH and ELW are constrained between 0.34 and 0.41, while the FCVAR estimates are 0.29 and 0.299, for the restricted and unrestricted cases, respectively. The next commodity to gold's performance in terms of fractional d for the range is silver. In this case, the FCVAR estimates of d for the range are quite similar to those of gold, and the daily frequency price is found in the stationary long memory range. As we know silver is a substitute for gold, and the market attracts many individual traders and buyers around the world, thus price changes and volatility in the two commodities are expected to be similar. For the cases of crude oil and natural gas, the estimated values of d in the range are generally above 0.5, that is in the nonstationary long memory range. The estimates for daily prices across the two commodities are fairly similar. It is assumed that variations in the values of *d* across different price frequencies are the result of price jumps that occur during that period.

INSERT TABLE 9 ABOUT HERE

4. Conclusions

This paper examines the behaviour of high and low prices of four commodities, namely crude oil, natural gas, gold and silver, and of the corresponding ranges using both daily and intraday data at various frequencies. In contrast to previous studies such as Cheung et al. (2009) restricting the integration parameter *d* to be an integer and the cointegration coefficient to be unity, this paper adopts a general fractional integration and cointegration framework that allows *d* to be any real value, and thus it allows for richer dynamics. In particular, the cointegration analysis was carried out using the fractional cointegrating VAR framework (FCVAR), which can capture both the long-run equilibrium relationships between high and low commodity prices and the long-memory properties of their linear combination. Both stationary and non-stationary ranges were found to exhibit long memory, which implies that highs and lows will deviate in the long run, in contrast to the findings of other studies (e.g., Cheung et al., 2009) providing evidence that highs and lows tend not to drift too far apart.

This work provides an alternative method for dealing with asset prices, different from the log-price-difference transformation usually employed in the literature (see Caporin et al, 2013). By assuming that prices are I(1) processes, unit cointegration implies lack of predictability of price differences; while in this work we have applied a slightly more sophisticated approach to dealing with such predictions. Our results are similar to those obtained in Barunik and Dvorakova (2015).

The evidence of long memory found in the range of the series suggests the potential predictability of the variance in a model for the mean dynamics of high and low commodity prices. Thus, better predictions of volatility might be obtained with this approach, remediating the flaws of the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models in predicting accurately the conditional volatility series (Andersen and Bollerslev, 1998; Hansen and Lunde, 2005). The findings in this work will assist investors in improving their trading strategies since high and low prices serve as entry and exit signals in the market. More curious readers can extend the strategy employed here beyond risk analysis and management.

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Table 1: Data description and Sample

Commodity	Intraday frequency	Sample range	Sample size, T
Crude oil	15mins (M15)	01/10/2018 - 09/11/2018	2728
	30mins (M30)	29/08/2018 - 09/11/2018	2391
	1 hour (H1)	27/06/2018 - 09/11/2018	2242
	4 hour (H4)	03/07/2017 - 09/11/2018	2098
	Daily (D1)	13/09/2012 - 09/11/2018	1593
Natural gas	15mins (M15)	10/10/2018 - 09/11/2018	2048
	30mins (M30)	28/08/2018 - 09/11/2018	2436
	1 hour (H1)	27/06/2018 - 09/11/2018	2242
	4 hour (H4)	03/07/2017 - 09/11/2018	2098
	Daily (D1)	24/06/2010 - 09/11/2018	2146
Gold	15mins (M15)	08/08/2018 - 09/11/2018	6130
	30mins (M30)	09/07/2018 - 09/11/2018	4099
	1 hour (H1)	07/05/2018 - 09/11/2018	3073
	4 hour (H4)	13/06/2018 - 09/11/2018	641
	Daily (D1)	30/06/2016 - 09/11/2018	611
Silver	15mins (M15)	08/08/2018 - 09/11/2018	6054
	30mins (M30)	09/07/2018 - 09/11/2018	4099
	1 hour (H1)	07/05/2018 - 09/11/2018	3073
	4 hour (H4)	12/05/2017 - 09/11/2018	2315
	Daily (D1)	17/07/2014 - 09/11/2018	1131

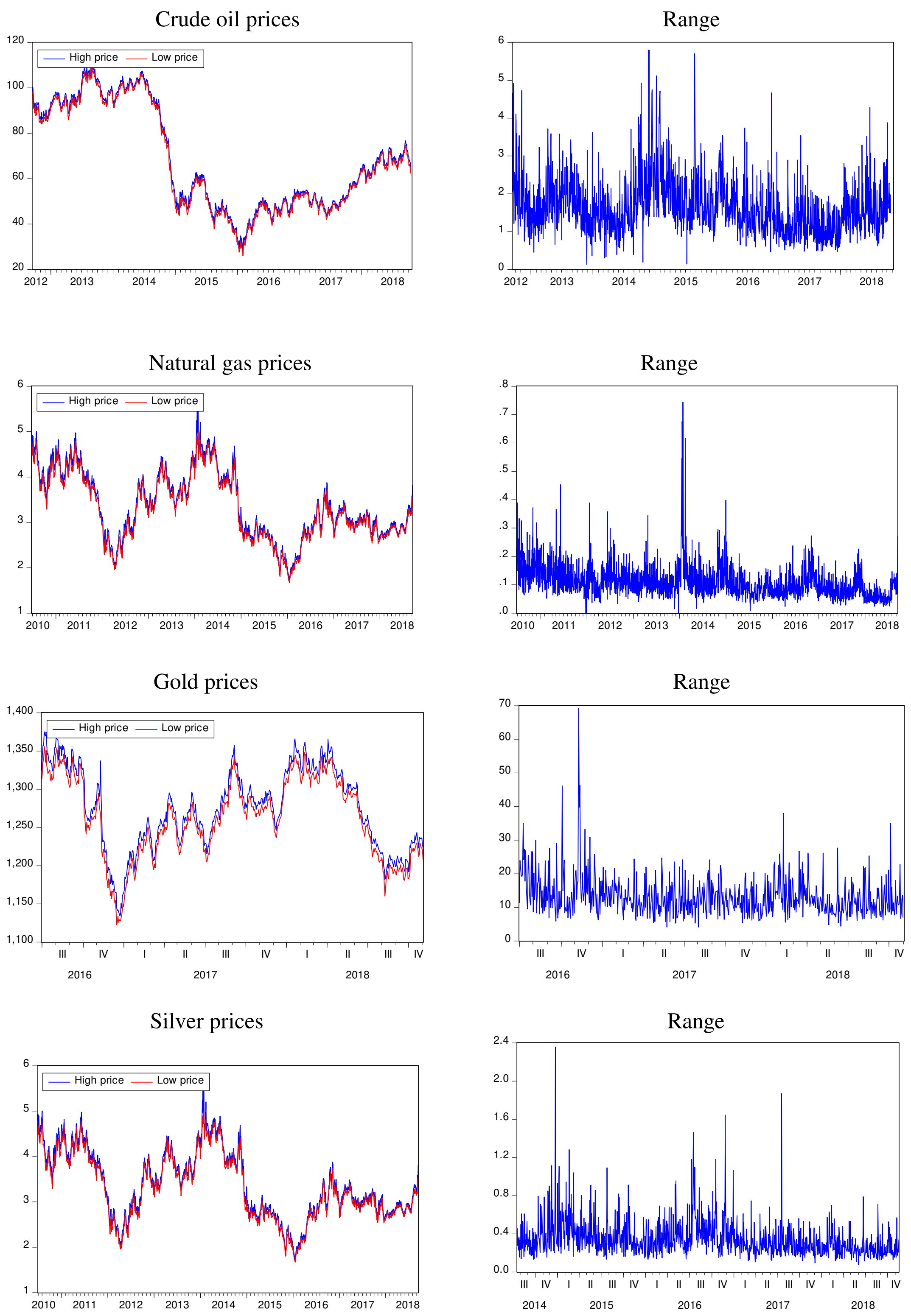

Figure 1: High and Low prices of Crude oil, Natural gas, Gold and Silver (left) and corresponding price ranges (right)

Table 2: P-values of ADF test for level series, first differences and range of high and low prices (Note, "c" denotes constant, "t" denotes time trend in the ADF test regression model).

		ADF_H						ADF_L						ADF_R		
		Level			**First difference**			**Level**			**First difference**			**Level**		
Commod.	**Series freq.**	**none**	**c**	**c, t**	**none**	**c**	**c, t**	**none**	**c**	**c, t**	**none**	**c**	**c, t**	**none**	**c**	**c, t**
Crude	15mins (M15)	0.0315	0.9852	0.0084	0.0001	0.0001	0.0000	0.0415	0.9791	0.0056	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Oil	30mins (M30)	0.2137	0.9912	0.9872	0.0001	0.0000	0.0000	0.2246	0.9842	0.9797	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
	1 hour (H1)	0.2967	0.8915	0.9697	0.0000	0.0000	0.0000	0.3120	0.8478	0.9521	0.0001	0.0000	0.0000	0.3433	0.0000	0.0001
	4 hour (H4)	0.8292	0.3625	0.9510	0.0000	0.0000	0.0000	0.8193	0.2274	0.8072	0.0000	0.0000	0.0000	0.3905	0.0043	0.0001
	Daily (D1)	0.1747	0.5656	0.9280	0.0000	0.0000	0.0000	0.1860	0.5704	0.9241	0.0000	0.0000	0.0000	0.1856	0.0001	0.0004
Natural	15mins (M15)	0.8984	0.9415	0.8509	0.0000	0.0000	0.0000	0.9132	0.9698	0.9080	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
gas	30mins (M30)	0.9807	0.9712	0.5314	0.0000	0.0000	0.0000	0.9896	0.9878	0.7377	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
	1 hour (H1)	0.9671	0.9953	0.8220	0.0000	0.0000	0.0000	0.9706	0.9970	0.8563	0.0000	0.0000	0.0000	0.7105	0.4495	0.0125
	4 hour (H4)	0.8549	0.7148	0.9233	0.0000	0.0000	0.0000	0.8512	0.6178	0.8687	0.0000	0.0000	0.0000	0.6028	0.3040	0.6878
	Daily (D1)	0.3345	0.0533	0.2095	0.0000	0.0000	0.0000	0.3024	0.0478	0.1869	0.0000	0.0000	0.0000	0.0292	0.0000	0.0000
Gold	15mins (M15)	0.6523	0.2893	0.1239	0.0001	0.0001	0.0000	0.6516	0.2490	0.0874	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
	30mins (M30)	0.3774	0.0380	0.1666	0.0001	0.0001	0.0000	0.3793	0.0378	0.1880	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
	1 hour (H1)	0.1919	0.3681	0.7439	0.0001	0.0001	0.0000	0.1978	0.3478	0.7322	0.0001	0.0001	0.0000	0.1801	0.0000	0.0000
	4 hour (H4)	0.2340	0.0406	0.2572	0.0000	0.0000	0.0000	0.2332	0.0314	0.2392	0.0000	0.0000	0.0000	0.2107	0.0000	0.0000
	Daily (D1)	0.4707	0.3141	0.6267	0.0001	0.0001	0.0000	0.4389	0.2853	0.5966	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
Silver	15mins (M15)	0.2782	0.0307	0.1339	0.0001	0.0001	0.0000	0.2683	0.0330	0.1439	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
	30mins (M30)	0.1479	0.1807	0.3151	0.0000	0.0000	0.0000	0.1591	0.1855	0.3189	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
	1 hour (H1)	0.1851	0.7573	0.3000	0.0000	0.0000	0.0000	0.1875	0.7468	0.2942	0.0001	0.0001	0.0000	0.2158	0.0000	0.0000
	4 hour (H4)	0.4130	0.5873	0.6873	0.0000	0.0000	0.0000	0.3897	0.7792	0.2823	0.0001	0.0001	0.0000	0.0843	0.0000	0.0000
	Daily (D1)	0.2323	0.0365	0.1429	0.0000	0.0000	0.0000	0.2309	0.0376	0.1476	0.0001	0.0001	0.0000	0.1186	0.0000	0.0000

Table 3: Estimates of Exact Local Whittle (ELW) of fractional integration parameter *d* for high (d_H), low prices (d_L) and their differences, the range (d_R) for two bandwidths m for the number of periodogram ordinates (m = $T^{0.5}$ and m = $T^{0.6}$). s.e.(d_R) is the standard error of d_R. In bold, evidence of significant long range dependency, that is 0<d<1.

		Bandwidths			$ELW_{m=T^{0.5}}$				$ELW_{m=T^{0.6}}$			
Commodity	**Series Frequency**	**T**	**$T^{0.5}$**	**$T^{0.6}$**	**d_H**	**d_L**	**d_R**	**s.e.(d_R)**	**d_H**	**d_L**	**d_R**	**s.e.(d_R)**
Crude oil	15mins (M15)	2728	52	115	1.0290	1.0291	**0.2362**	0.0693	1.0095	1.0088	**0.4454**	0.0466
	30mins (M30)	2391	48	106	1.0397	1.0397	**0.7846**	0.0722	1.0186	1.0191	**0.2477**	0.0490
	1 hour (H1)	2242	47	102	1.0351	1.0351	**0.8886**	0.0729	1.0257	1.0261	**0.6807**	0.0495
	4 hour (H4)	2098	45	98	1.0679	1.0689	**0.7457**	0.0745	1.0350	1.0341	**0.7974**	0.0505
	Daily (D1)	1593	39	83	1.0109	1.0140	**0.7790**	0.0801	1.0358	1.0395	**0.7353**	0.0549
Natural gas	15mins (M15)	2048	45	97	0.9049	0.9117	-0.1599	0.0745	1.0335	1.0410	**0.4762**	0.0508
	30mins (M30)	2436	49	107	1.0056	1.0064	**0.6982**	0.0714	0.9910	0.9937	**0.2337**	0.0483
	1 hour (H1)	2242	47	102	1.0055	1.0061	**0.8305**	0.0729	0.9909	0.9923	**0.0697**	0.0495
	4 hour (H4)	2098	45	98	1.0019	1.0037	**0.8662**	0.0745	0.9798	0.9793	**0.8681**	0.0505
	Daily (D1)	2146	46	99	0.9145	0.9210	**0.6489**	0.0737	0.9375	0.9423	**0.6870**	0.0737
Gold	15mins (M15)	6130	78	187	1.0082	1.0084	**0.6157**	0.0566	1.0094	1.0092	**0.6157**	0.0366
	30mins (M30)	4099	64	147	1.0186	1.0185	**0.7736**	0.0625	1.0066	1.0066	**0.6476**	0.0412
	1 hour (H1)	3073	55	123	1.0110	1.0109	**0.6834**	0.0674	1.0066	1.0064	**0.5630**	0.0451
	4 hour (H4)	641	25	48	0.8108	0.8131	0.0844	0.1000	0.8394	0.8239	-0.0181	0.0722
	Daily (D1)	611	24	46	0.9042	0.9148	**0.3461**	0.1021	0.9105	0.9261	**0.3281**	0.0737
Silver	15mins (M15)	6054	77	185	1.0199	1.0201	**0.6375**	0.0570	1.0091	1.0088	**0.3758**	0.0368
	30mins (M30)	4099	64	147	1.0182	1.0181	**0.7899**	0.0625	1.0068	1.0067	**0.6580**	0.0412
	1 hour (H1)	3073	55	123	1.0157	1.0157	**0.7514**	0.0674	1.0086	1.0083	**0.5694**	0.0451
	4 hour (H4)	2315	48	104	1.0299	1.0305	**0.7459**	0.0722	1.0226	1.0212	**0.6226**	0.0490
	Daily (D1)	1131	33	67	0.9765	0.9764	**0.8845**	0.0870	0.9849	0.9841	**0.7129**	0.0611

Table 4: Estimates of Geweke and Porter-Hudak (GPH) of fractional integration parameter *d* for high (d_H), low prices (d_L) and their differences, the range (d_R) for two bandwidths m for the number of periodogram ordinates ($m = T^{0.5}$ and $m = T^{0.6}$). s.e.(d_R) is the standard error of d_R. In bold, evidence of significant long range dependency, that is 0<d<1.

		Bandwidths			**$GPH_{m = T^{0.5}}$**				**$GPH_{m = T^{0.6}}$**			
Commodity	**Series Frequency**	**T**	**$T^{0.5}$**	**$T^{0.6}$**	**d_H**	**d_L**	**d_R**	**s.e.(d_R)**	**d_H**	**d_L**	**d_R**	**s.e.(d_R)**
Crude oil	15mins (M15)	2728	52	115	1.0377	1.0375	**0.4104**	0.1007	0.9314	0.9319	**0.4828**	0.0643
	30mins (M30)	2391	48	106	1.0872	1.0868	**0.8243**	0.1055	1.0307	1.0310	**0.4070**	0.0673
	1 hour (H1)	2242	47	102	1.0840	1.0831	**0.9548**	0.1068	1.0422	1.0431	**0.7052**	0.0687
	4 hour (H4)	2098	45	98	1.0943	1.0905	**0.7563**	0.1096	1.0737	1.0717	**0.8273**	0.0702
	Daily (D1)	1593	39	83	1.0444	1.0500	**0.8612**	0.1194	1.0620	1.0660	**0.7737**	0.0771
Natural gas	15mins (M15)	2048	45	97	0.9667	1.0301	0.1049	0.1096	1.0457	1.0422	**0.3125**	0.0707
	30mins (M30)	2436	49	107	1.0337	1.0364	**0.7021**	0.1043	0.9896	0.9986	**0.3671**	0.0669
	1 hour (H1)	2242	47	102	1.0782	1.0808	**0.8548**	0.1068	1.0096	1.0116	**0.6797**	0.0687
	4 hour (H4)	2098	45	98	0.9904	0.9908	**0.9210**	0.1096	0.9661	0.9635	**0.9266**	0.0702
	Daily (D1)	2146	46	99	0.9416	0.9406	**0.6246**	0.1082	0.9993	1.0029	**0.6670**	0.0699
Gold	15mins (M15)	6130	78	187	0.8852	0.8856	**0.5397**	0.0798	1.0148	1.0131	**0.4089**	0.0494
	30mins (M30)	4099	64	147	0.8571	0.8583	**0.6029**	0.0893	0.7223	0.7227	**0.4764**	0.0562
	1 hour (H1)	3073	55	123	1.0133	1.0134	**0.6856**	0.0975	0.9529	0.9522	**0.5610**	0.0620
	4 hour (H4)	641	25	48	0.8372	0.8272	0.0581	0.1570	0.7753	0.7431	0.0378	0.1058
	Daily (D1)	611	24	46	0.8452	0.8782	**0.3485**	0.1611	0.8889	0.8983	**0.4140**	0.1085
Silver	15mins (M15)	6054	77	185	1.0505	1.0505	**0.5965**	0.0804	1.0304	1.0296	**0.4164**	0.0497
	30mins (M30)	4099	64	147	0.9333	0.9333	**0.5532**	0.0893	0.8357	0.8355	**0.5002**	0.0562
	1 hour (H1)	3073	55	123	1.0302	1.0315	**0.7252**	0.0975	0.9831	0.9819	**0.5600**	0.0620
	4 hour (H4)	2315	48	104	1.0361	1.0347	**0.6697**	0.1055	1.0173	1.0194	**0.5936**	0.0680
	Daily (D1)	1131	33	67	1.0100	1.0166	**0.8861**	.1321	1.0026	1.0003	**0.6815**	0.0870

Table 5: Test statistics for the equality of integration orders based on ELW estimates

		Bandwidths			Test statistic	
Commodity	**Series Frequency**	**T**	**$T^{0.5}$**	**$T^{0.6}$**	**m = $T^{0.5}$**	**m = $T^{0.6}$**
Crude oil	15mins (M15)	2728	52	115	**0.0104**	**-0.0344**
	30mins (M30)	2391	48	106	**0.0192**	**-0.0318**
	1 hour (H1)	2242	47	102	**0.0470**	**-0.0918**
	4 hour (H4)	2098	45	98	**0.1710**	**0.1959**
	Daily (D1)	1593	39	83	**-0.2184**	**-0.3319**
Natural gas	15mins (M15)	2048	45	97	-2.8529	**0.3394**
	30mins (M30)	2436	49	107	**-0.1322**	**-0.9630**
	1 hour (H1)	2242	47	102	**-0,1222**	**-0.2040**
	4 hour (H4)	2098	45	98	**-0.0180**	**0.2547**
	Daily (D1)	2146	46	99	**0.0460**	**-0.3564**
Gold	15mins (M15)	6130	78	187	**-0.0311**	**0.3178**
	30mins (M30)	4099	64	147	**-0.0767**	**-0.0588**
	1 hour (H1)	3073	55	123	**-0.0055**	**0.0860**
	4 hour (H4)	641	25	48	**0.2499**	**1.5456**
	Daily (D1)	611	24	46	**-0.7919**	**-0.4324**
Silver	15mins (M15)	6054	77	185	**0.0000**	**0.1480**
	30mins (M30)	4099	64	147	**0.0000**	**0.0293**
	1 hour (H1)	3073	55	123	**-0.0715**	**0.1476**
	4 hour (H4)	2315	48	104	**0.0672**	**-0.2183**
	Daily (D1)	1131	33	67	**-0.2178**	**0.1540**

In bold, evidence of equal orders of integration at the 5% level.

Table 6: Fractional Cointegration Rank test by Johansen and Nielsen (2012)

			r = 0			r = 1			r = 2	
Commodity Prices	**Series Frequency**	**k_{max} (3)**	**d**	**b**	**LR**	**d**	**b**	**LR**	**d**	**b**
Crude oil	15mins (M15)	0	0.839	0.500	882.80	**1.002**	**0.358**	**0.190**	1.006	0.362
	30mins (M30)	3	0.800	0.100	94.578	**0.800**	**0.100**	**0.160**	0.800	0.100
	1 hour (H1)	3	0.800	0.100	176.42	**0.817**	**0.122**	**0.315**	0.821	0.119
	4 hour (H4)	3	1.084	0.100	68.057	**1.200**	**0.688**	**8.819**	1.200	0.673
	Daily (D1)	2	0.800	0.100	64.155	**1.003**	**0.264**	**3.686**	1.020	0.255
Natural gas	15mins (M15)	0	0.843	0.500	544.46	**1.004**	**0.359**	**0.114**	1.008	0.365
	30mins (M30)	3	0.800	0.100	125.77	**0.800**	**0.100**	**0.222**	0.800	0.100
	1 hour (H1)	3	0.800	0.100	119.33	**0.800**	**0.246**	**0.266**	0.800	0.245
	4 hour (H4)	3	1.104	0.100	38.508	**1.200**	**0.664**	**7.225**	1.200	0.658
	Daily (D1)	1	0.800	0.100	108.69	**0.937**	**0.336**	**15.86**	1.043	0.100
Gold	15mins (M15)	0	0.839	0.499	1808.6	**1.001**	**0.510**	**0.061**	1.002	0.510
	30mins (M30)	1	0.800	0.100	179.23	**0.970**	**0.894**	**0.314**	0.969	0.888
	1 hour (H1)	0	0.849	0.500	929.66	**0.990**	**0.990**	**0.044**	0.989	0.900
	4 hour (H4)	2	0.800	0.100	62.669	**0.997**	**0.878**	**0.145**	1.002	0.893
	Daily (D1)	0	0.820	0.500	345.83	**1.015**	**0.716**	**0.086**	1.020	0.722
Silver	15mins (M15)	0	0.841	0.500	1709.4	**1.001**	**0.501**	**0.120**	1.001	0.500
	30mins (M30)	1	0.800	0.100	147.93	**0.971**	**0.898**	**0.041**	0.970	0.896
	1 hour (H1)	3	0.800	0.100	69.139	**0.800**	**0.100**	**0.161**	0.800	0.100
	4 hour (H4)	2	0.800	0.100	123.91	**1.056**	**0.653**	**0.630**	1.065	0.656
	Daily (D1)	0	0.819	0.500	531.62	**1.011**	**0.738**	**0.565**	1.022	0.752

Note, maximum k is set at 3 and this gives the order of the error correction mechanism in the FCVAR system. The LR is the Likelihood Ratio statistics, computed for rank r = 0 and 1. This is not available for rank 2 since we are not rejecting any more rank.

Table 7: FCVAR estimation results (no restriction)

Commodity	Series Frequency	$\hat{d}$	$\hat{b}$	$\hat{\beta}$
Crude oil	15mins (M15)	1.002 (0.015)	0.358 (0.048)	[1.000, -0.993, -0.639]
	30mins (M30)	0.800 (0.048)	0.100 (0.000)	[1.000, -1.003, -0.019]
	1 hour (H1)	0.817 (0.027)	0.122 (0.002)	[1.000, -1.003, -0.050]
	4 hour (H4)	1.200 (0.028)	0.688 (0.050)	[1.000, -1.000, -0.503]
	Daily (D1)	1.003 (0.050)	0.264 (0.040)	[1.000, -0.989, -2.019]
Natural gas	15mins (M15)	1.004 (0.017)	0.359 (0.053)	[1.000, -1.022, 0.061]
	30mins (M30)	0.800 (0.035)	0.100 (0.003)	[1.000, -1.005, 0.001]
	1 hour (H1)	0.800 (0.026)	0.246 (0.015)	[1.000, -1.009, 0.014]
	4 hour (H4)	1.200 (0.036)	0.664 (0.059)	[1.000, -1.015, 0.018]
	Daily (D1)	0.937 (0.043)	0.336 (0.095)	[1.000, -1.043, 0.050]
Gold	15mins (M15)	1.001 (0.010)	0.510 (0.029)	[1.000, -1.000, -1.277]
	30mins (M30)	0.970 (0.020)	0.894 (0.038)	[1.000, -0.999, -3.135]
	1 hour (H1)	0.990 (0.014)	0.900 (0.034)	[1.000, -0.998, -4.062]
	4 hour (H4)	0.997 (0.054)	0.878 (0.878)	[1.000, -0.996, -9.856]
	Daily (D1)	1.015 (0.032)	0.716 (0.032)	[1.000, -1.026, 18.423]
Silver	15mins (M15)	1.001 (0.010)	0.501 (0.029)	[1.000, -1.005, 0.056]
	30mins (M30)	0.971 (0.020)	0.898 (0.037)	[1.000, -0.998, -0.064]
	1 hour (H1)	0.800 (0.044)	0.100 (0.005)	[1.000, -1.003, -0.003]
	4 hour (H4)	1.056 (0.028)	0.653 (0.084)	[1.000, -1.006, -0.004]
	Daily (D1)	1.011 (0.024)	0.738 (0.046)	[1.000, -1.019, -0.027]

Table 8: FCVAR estimation results (with restriction)

Commodity	**Series Frequency**	$\hat{d}$	$\hat{b}$	$\hat{\beta}$
Crude oil	15mins (M15)	1.002 (0.015)	0.357 (0.047)	[1.000, -1.000]
	30mins (M30)	0.800 (0.048)	0.100 (0.002)	[1.000, -1.000]
	1 hour (H1)	0.887 (0.027)	0.100 (0.002)	[1.000, -1.000]
	4 hour (H4)	1.200 (0.027)	0.744 (0.047)	[1.000, -1.000]
	Daily (D1)	1.133 (0.043)	0.159 (0.000)	[1.000, -1.000]
Natural gas	15mins (M15)	1.004 (0.017)	0.358 (0.054)	[1.000, -1.000]
	30mins (M30)	0.800 (0.033)	0.100 (0.003)	[1.000, -1.000]
	1 hour (H1)	0.971 (0.023)	0.100 (0.002)	[1.000, -1.000]
	4 hour (H4)	1.200 (0.040)	0.644 (0.068)	[1.000, -1.000]
	Daily (D1)	0.910 (0.049)	0.378 (0.080)	[1.000, -1.000]
Gold	15mins (M15)	1.001 (0.010)	0.510 (0.029)	[1.000, -1.000]
	30mins (M30)	0.977 (0.020)	0.897 (0.037)	[1.000, -1.000]
	1 hour (H1)	0.993 (0.014)	0.896 (0.031)	[1.000, -1.000]
	4 hour (H4)	1.014 (0.052)	0.837 (0.133)	[1.000, -1.000]
	Daily (D1)	1.016 (0.032)	0.726 (0.066)	[1.000, -1.000]
Silver	15mins (M15)	1.001 (0.010)	0.502 (0.029)	[1.000, -1.000]
	30mins (M30)	0.977 (0.020)	0.891 (0.035)	[1.000, -1.000]
	1 hour (H1)	0.800 (0.041)	0.100 (0.005)	[1.000, -1.000]
	4 hour (H4)	1.057 (0.028)	0.651 (0.084)	[1.000, -1.000]
	Daily (D1)	1.011 (0.024)	0.738 (0.046)	[1.000, -1.000]

Table 9: Comparison of integration orders of range. Note "R" denotes a model with restrictions on the cointegrating vector, and "NR" denotes a model without restrictions.

		GPH		ELW		FCVAR	
Commodity	**Series Frequency**	**m=$T^{0.5}$**	**m=$T^{0.6}$**	**m=$T^{0.5}$**	**m=$T^{0.6}$**	**R**	**NR**
Crude oil	15mins (M15)	0.410	0.483	0.236	0.445	0.645	0.644
	30mins (M30)	0.824	0.407	0.785	0.248	0.700	0.700
	1 hour (H1)	0.955	0.705	0.889	0.681	0.787	0.695
	4 hour (H4)	0.756	0.827	0.746	0.797	0.456	0.512
	Daily (D1)	0.861	0.774	0.779	0.735	0.974	0.739
Natural gas	15mins (M15)	0.105	0.313	-0.160	0.476	0.646	0.645
	30mins (M30)	0.702	0.367	0.698	0.234	0.700	0.700
	1 hour (H1)	0.855	0.680	0.831	0.070	0.871	0.554
	4 hour (H4)	0.921	0.927	0.866	0.868	0.556	0.536
	Daily (D1)	0.625	0.667	0.649	0.687	0.532	0.601
Gold	15mins (M15)	0.540	0.409	0.616	0.616	0.491	0.491
	30mins (M30)	0.603	0.476	0.774	0.648	0.080	0.076
	1 hour (H1)	0.686	0.561	0.683	0.563	0.097	0.090
	4 hour (H4)	0.058	0.038	0.084	-0.018	0.177	0.119
	Daily (D1)	0.349	0.414	0.346	0.328	0.290	0.299
Silver	15mins (M15)	0.597	0.416	0.638	0.376	0.499	0.500
	30mins (M30)	0.553	0.500	0.790	0.658	0.086	0.073
	1 hour (H1)	0.725	0.560	0.751	0.569	0.700	0.700
	4 hour (H4)	0.670	0.594	0.746	0.623	0.406	0.403
	Daily (D1)	0.886	0.682	0.885	0.713	0.273	0.273