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Abstract
This paper considers economic cycles that do not depend on the exogenous economic actions. More precisely, the paper develops a positive model of government behavior in order to define the intertemporal fiscal policies that are optimal for a country, determining the optimal level of the budget and the optimal level of the rate of environmental quality, as well. For this purpose, we setup an optimal control model involving the intertemporal subsidy strategies for an authoritarian (like a central European) government. It will be shown - applying the Hopf bifurcation theorem - that cyclical strategy, i.e. waves of regulation, environmental subsidies alternating with deregulation, cuts in social programmes, etc., may be optimal strategies. In this paper we propose an extremely simple optimal control model concerning budget surplus and environmental subsidies. We investigate the cyclical subsiding policies applying one bifurcation theorem. A number of propositions are stated during the solution process.

Keywords: Budget; environmental resources; subsidies; Hopf bifurcation; optimal control.

JEL Codes: E62; C61; H61; H23; Q50; C02.

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1. Introduction

In the last decades, a considerable bulk of the literature about economic business cycles is shifted on environmental resources in order to explain why and when the political decisions they are set many crucial economic variables in cyclical trajectories. A very simple argument could be the following. The evaluation of the environmental quality (made by the citizens) depends on pollution and abatement measures at a given instant of time. A farsighted regulator may exploit several such evaluations and run a program which is based on the tradeoff between subsidies devoted to abatement and budget in an optimal way. The literature on this field is fast growing and among others we refer to the studies of Feichtinger and Novak (1991), Semmler and Sieveking (2000), Fodha and Seegmueller (2012) and so on.

Moreover, the size of the budget deficits (and the resulting public debt) it is a point of concern in the most developed western countries, including European countries, USA, Japan and so on. As a result of the growing and continuously public deficits is the sharp reduction of the economic activities under the fear of inflation and depreciation. But the major concern of the above reduction would be the uncertainty of the public debt unsustainability. On the other hand, the environmental quality programs, mainly based on subsidies, are often constrained by long-term fiscal objectives which impose to control public deficits.

The search for financing mechanisms that do not increase debt burden has renewed interest in debt-for-nature swaps. Therefore, countries with debt reduce their debt burden and free up budgetary resources for environmental spending (Fodha and Seegmuller, 2014).

In this paper we setup an optimal control model for which the rate of environmental quality is maximized. The maximization program takes place under the constraint of budget surpluses (or deficits) accumulation which in turn is a crucial index for the nation primary surpluses. The budget surplus accumulation is also dependent on the opportunity cost of capital and on the cost of subsidies given in order to improve environmental quality. A second constraint that appears in the maximization process is also the most recently approved subsidies which are a measure of the instantaneous change of the overall subsidies.

The rest of paper is organized as follows. Section 2 describes the proposed model while section 3 discusses stability analysis for the same model. Section 4 gives an example with specific function forms of the model and section 5 concludes the paper.

2. The model (the rate of environmental quality is maximized)

The objective of the benevolent social planner is to maximize an intertemporal environmental quality because a higher quality offers amenities to the people, therefore it is an extra reason to improve his intentions of a good policy.

We denote by $E(t)$ the current approval rate of environmental quality at a given time instant therefore the (simplified) problem of the regulator would be:

$$\max \int_0^\infty e^{-\rho t} E(t) dt \tag{1}$$

The rate of environmental quality, variable $E(t)$, hinges on the budget surplus or deficit $B(t)$, on the total subsidies $S(t)$ and finally on the most recently approved $\sigma(t)$, therefore it is rather a function of the form $E = E(B, S, \sigma)$.

The objective (1) implies a utility w.r.t . the good environmental quality, i.e. a utility index[1] of the form $u(B,S,\sigma)=u(E(B,S,\sigma))$, with the standard concave assumptions for the rate of environmental quality

$$E_S > 0,\ E_{SS} < 0,\ E_B > 0,\ E_{BB} < 0,\ E_\sigma > 0,\ E_{\sigma\sigma} < 0 \qquad (2)$$

Subsidies, the variable S, are offered or burden (in the case of taxes – negative subsidy) in chronological order therefore the variable S is rather a result of historical adjustments, i.e. its evolution may be a sticky process. The addition of a new subsidy in the evaluation of the environmental quality acts more effective than the formerly taken, while a negative subsidy e.g. abolishing an existing pollution abatement process, will have negative results in environmental quality. The above particular types of environmental benefits and its non linearity $(E_{SS} < 0)$ it is enough to ensure interior solutions of the maximization problem (1), instead of the unwished bang – bang type solutions.

Regarding budget deficits there at least two reasons that they affect negatively the environmental quality. First, environmental quality may suffer from budget deficits because with a nonbalanced budget it is impossible the government to pay for environmental subsidies. The general rule in the classical economic literature is "running a deficit was considered an immoral by the public so the regulators at that time transgressed this norm with great peril" (Mueller, 1989). Second, the supposition of the variable E rather as a utility instead of environmental quality gives the possible interpretation: the surpluses in budget gives to the social regulator the ability to

[1] One model for which the utility index is maximized discussed in another paper

engage in prestigious environmental projects or to augment pollution abatement without the necessity to increase taxes[2].

Finally the regulator has to solve the following optimal control problem

$$\max_{\sigma} \int_{0}^{\infty} e^{-\rho t} E(B,S,\sigma) dt \qquad (3)$$

$$\text{subject to} \quad dB/dt = \dot{B} = rB - C(S) \qquad (4)$$

$$\text{and} \qquad dS/dt = \dot{S} = \sigma \qquad (5)$$

with control (strategic) variable the variable σ which is equivalent to the decision to offer or abolish subsidy, a decision which is highly depended on the budget constraint.

The variable S represents the amount of total subsidies received. The function $C(S)$ is the cost function associated with the financial burden of the subsidies and it is supposed in the convex fashion, i.e. $C' > 0,\ C'' \geq 0,\ C(0) = 0,\ C'(0) = 1$. Moreover, it is worth to mention that may be a deadweight loss in the case that the social cost $C(S)$ exceed the amount S of the subsidy payment, i.e. in the case $C(S) > S$. The additional expenses that may create the above divergence $C(S) > S$ are called by Becker "deadweight costs" (Becker, 1983) and could be e.g. a disincentive effect or the costs associated with the expansion of institutions in order to manage subsidy payments.

It is worth noting that for small amount of subsidies there is no deadweight costs, i.e. for $S \approx 0$ implies $C(S) \approx S$, due to the assumption $C'(0) = 1$. Moreover, due to the convexity assumption of the cost function, for $S > 0$ implied $C(S) > S$

[2] For more information on cost differential and optimal abatement of air pollution see Halkos (1993, 1994, 1996) while for fiscal spending and environmental quality see Halkos and Paizanos (2016a,b,c; 2017) and for controlling polluting firms see Halkos and Papageorgiou 2014, 2016, 2017, 2018).

meaning that with increased subsidies the deadweight costs increases. The latter convexity effect could be justified first by the presence of many pressure groups and therefore, due to the competition amongst the groups, the proposals with lower costs win, and second by the disincentive effect caused the transfer payments.

Another parameter of the model, the interest rate r, is earned on budget surpluses while is paid in the case of deficits, and moreover it is assumed lower than the discount rate of social planner, i.e. $\rho > r$.

In the model under consideration, it was made the simplified assumption that all the taxes are ear marked for the provision of public goods. This fact is due to the equation (4) which, in turn, implies that all the taxes and expenses provide public goods and moreover they are balanced, therefore simplifying our arguments.

We proceed with the solution analysis of the optimal control problem (3) – (5) in the usual way. The necessary and sufficient conditions are summarized below

$$H = E + \lambda(rB - C) + \mu\sigma \tag{6}$$

$$H_\sigma = E_\sigma + \mu \tag{7}$$

$$\dot{\lambda} = (\rho - \delta)\lambda - E_B \tag{8}$$

$$\dot{\mu} = \rho\mu + \lambda C - E_S \tag{9}$$

$$\lim_{\rho\to\infty} e^{-\rho t}\lambda(t)B(t) = 0 \tag{10}$$

$$\lim_{\rho\to\infty} e^{-\rho t}\mu(t)S(t) = 0 \tag{11}$$

While the stationary solution of the states and costates follows form the solution of the system below

$$B_\infty = C(S_\infty)/r \tag{12}$$

$$\lambda_\infty = E_B/(\rho - r) \tag{13}$$

$$\mu_\infty = \left[(\rho - r)E_S - E_B C'\right] / \left[\rho(\rho - r)\right] \quad (14)$$

$$\sigma_\infty = 0 \quad (15)$$

$$E_\sigma\left(B_\infty, S_\infty, 0\right) = -\mu_\infty \quad (16)$$

In the next section we consider the stability analysis of the solution strategy.

3. Stability analysis

For the solution purposes we assume that there exists an interior solution for the optimal policy $E(B,S,\sigma)$, which is ensured by the boundary conditions. According to Pontryagin's maximum principle we solve equation $H_\sigma = 0$ with respect to the control variable σ i.e., $\sigma = \phi(B,S,\mu)$. The so called canonical system of equations in (B,S,λ,μ) is produced by substitutions into the differential equations of the state and costate variables.

The main question of the entire study hinges upon whether the optimal equilibrium strategy is cyclical, stable or unstable, while the cyclicality is characterized in the sense of stable limit cycles according to Wirl (1992). The two major tools of limit cycles analysis are first the Hopf bifurcation theorem which requires the analytical expressions of the eigenvalues of the linear approximation of the above canonical system of equations and second a theorem founded by the economist Dockner (1985) which allows the explicit calculation of the latter eigenvalues.

Therefore, we calculate the Jacobian matrix for equations $(12)-(16)$ and in the next step we compute the eigenvalues, according to Dockner's formula (Dockner, 1985). The Jacobian matrix will be

$$J=\begin{bmatrix} r & -C' & 0 & 0 \\ -\dfrac{E_{\sigma B}}{E_{\sigma\sigma}} & -\dfrac{E_{\sigma S}}{E_{\sigma\sigma}} & 0 & -\dfrac{1}{E_{\sigma\sigma}} \\ -\dfrac{E_{BB}E_{\sigma\sigma}-E_{\sigma B}^2}{E_{\sigma\sigma}} & -\dfrac{E_{BS}E_{\sigma\sigma}-E_{\sigma B}E_{\sigma S}}{E_{\sigma\sigma}} & \rho-r & \dfrac{E_{\sigma B}}{E_{\sigma\sigma}} \\ -\dfrac{E_{BS}E_{\sigma\sigma}-E_{\sigma S}E_{\sigma B}}{E_{\sigma\sigma}} & -\dfrac{E_{SS}E_{\sigma\sigma}-E_{\sigma S}^2}{E_{\sigma\sigma}}+\dfrac{E_B C''}{\rho-r} & C' & \rho+\dfrac{E_{\sigma S}}{E_{\sigma\sigma}} \end{bmatrix} \quad (17)$$

According to Dockner's theorem (Dockner, 1985) the four eigenvalues of the canonical equations $(12)-(16)$ would be $e_{1,2,3,4}=\frac{1}{2}\rho\pm\sqrt{\left(\frac{1}{2}\rho\right)^2-\frac{1}{2}K\pm\frac{1}{2}\sqrt{K^2-4\|J\|}}$ where:

$$K=\begin{Vmatrix} \dfrac{\partial\dot{B}}{\partial B} & \dfrac{\partial\dot{B}}{\partial\lambda} \\ \dfrac{\partial\dot{\lambda}}{\partial B} & \dfrac{\partial\dot{\lambda}}{\partial\lambda} \end{Vmatrix}+\begin{Vmatrix} \dfrac{\partial\dot{S}}{\partial S} & \dfrac{\partial\dot{S}}{\partial\mu} \\ \dfrac{\partial\dot{\mu}}{\partial S} & \dfrac{\partial\dot{\mu}}{\partial\mu} \end{Vmatrix}+2\begin{Vmatrix} \dfrac{\partial\dot{B}}{\partial S} & \dfrac{\partial\dot{B}}{\partial\mu} \\ \dfrac{\partial\dot{\lambda}}{\partial S} & \dfrac{\partial\dot{\lambda}}{\partial\mu} \end{Vmatrix}$$

therefore, the determinant of the Jacobian (17) is the following

$$\|J\|=\frac{\left\{\left[E_{BB}C'^2+2E_{BS}C'r+E_{SS}r^2\right]-E_{BS}C'\rho-E_{SS}r\rho+E_B C''r-\rho(\rho-r)\left[E_{\sigma S}r+E_{\sigma B}C'\right]\right\}}{E_{\sigma\sigma}} \quad (18)$$

while coefficient K is

$$K=r(\rho-r)+\frac{\left[E_B C''/(\rho-r)\right]-E_{SS}-\rho E_{\sigma S}}{E_{\sigma\sigma}}+2\frac{-E_{\sigma B}C'}{E_{\sigma\sigma}} \quad (19)$$

Applying Dockner's theorem (Dockner, 1985) the eigenvalues are given by

$$e_{1,2,3,4}=\tfrac{1}{2}\rho\pm\sqrt{\left(\tfrac{1}{2}\rho\right)^2-\tfrac{1}{2}K\pm\tfrac{1}{2}\sqrt{K^2-4\|J\|}} \quad (20)$$

In order to simplify the analysis that follows we assume that the function that represents the rate of environmental quality $E(B,S,\sigma)$ is additive, which implies that its crossed derivatives are vanish. Moreover we assume linear dependence with respect to the variable which represents subsidies S implying that the second derivative of the same rate function vanishes, i.e. $E_{SS} = 0$. Finally, after the above simplifications, the expressions (18) and (19) reduce into

$$\|J\| = \frac{\left[E_{BB}C'^2 + E_B C'' r\right]}{E_{\sigma\sigma}} \tag{21}$$

$$K = r(\rho - r) + \frac{E_B C''}{(\rho - r)E_{\sigma\sigma}} \tag{22}$$

Since we interested to facilitate cyclical strategies, we choose from the equilibrium properties, as these proposed by Dockner and Feichtinger (1991), the appropriate case[3]. This case entails two purely imaginary eigenvalues of the Jacobian $\|J\|$, which corresponds to the following conditions:

$$\|J\| \geq \left(\tfrac{1}{2}K\right)^2 \tag{C.1}$$

$$\|J\| - \left(\tfrac{1}{2}K\right)^2 - \tfrac{1}{2}\rho^2 K = 0 \tag{C.2}$$

Therefore, in order to apply the Hopf bifurcation theorem the necessary conditions are $\|J\| > 0$ and $K > 0$. Further inspection of the crucial variables $\|J\|$, K , for the interesting case which favors cyclical strategies, reveals that the second order partial derivative of environmental quality w.r.t. budget E_{BB} must be sufficient negative, while the same first derivative E_B and the first derivative of the cost function C' must be small enough, the inequality $\rho > r$ as well must hold and the absolute value $|E_{\sigma\sigma}|$ must be large. These algebraic conditions have the following economic sense.

[3] Another one case is, according to Dockner and Feichtinger (1991), the case at which the saddle point stability with two real roots is encountered, but here is out of interest, and the inequalities that satisfied in that case are respectively: $0 < \|J\| \leq \left(\tfrac{1}{2}K\right)^2$, $K < 0$

Proposition 1.

The economic cyclical strategies for the proposed model $(3)-(5)$ are feasible if the following conditions are met:

(a) The subsidy programs produces low deadweight costs (i.e. $C' \approx 0$)

(b) There exists sufficient concavity of the evaluation of budget surpluses ($E_{BB} < 0$)

(c) A strong concavity of the rate of environmental quality with respect to most recent subsidy concessions, which means that the marginal gain of environmental quality from granting an additional euro declines rapidly with respect to the concessions; or conversely any substantial deregulation bears large costs in environmental quality, are favorable for those conditions that are necessary for cyclical policies.

Therefore, an efficient rent seeking process favors cyclical strategies. To see that we choose the bifurcation point $\widehat{E}_{\sigma\sigma}$ by solving equation $(C.2)$ also taking into account the simplified versions (21) and (22)

$$\widehat{E}_{\sigma\sigma} = \frac{4E_{BB}}{r(\rho - r)\left[r(\rho - r) + 2\rho^2\right]} \qquad (23)$$

The resulting value of the bifurcation parameter(23) suggests that for a small discount rate ρ $(\rho > r)$ satisfies equality $(C.2)$ for large $E_{\sigma\sigma}$. Therefore, the assumptions based on highly discounted rates are not necessary to do, but these assumptions are helpful in explaining the cyclical policies.

Finally regarding stability, it is worth noting that further motions from the bifurcation point leads, according to Dockner and Feichtinger (1991), to complete instability of the canonical system of equations, which is not welcomed.

Furthermore, it is worth noting that the above analysis is restricted in the obvious case at which the Hamiltonian function (6) is maximized. This in turn implies that the restriction $\rho > r$ is crucial for the concavity of the Hamiltonian (and therefore for its maximization). Technically speaking in the case at $\rho < r$, according to (13) the costate variable λ changes its sign, at the steady states, below to zero, i.e. $\lambda_{\infty} < 0$, which in turn violates the concavity of the Hamiltonian function and therefore the maximization condition becomes invalid. Moreover no stationary environmental policy $E(B,S,\sigma)$ exists which satisfies solution (16) and concavity conditions (2) simultaneously. The latter, because $\lambda_{\infty} < 0$, implies $\mu_{\infty} > 0$ which contradicts condition $E_{\sigma} > 0$. The next proposition summarizes in economic terms the above discussion.

Proposition 2

In the case the discount rate ρ of the regulator is below the opportunity cost of capital *r* the optimal policy boils down to tax today in order to accumulate budget surpluses which allow larger subsidies in the future.

4. Cyclical policies (an example)

The following paradigm is an application of the above proposed model with specific functional forms of functions involved. For this purpose we consider the form of the function of the rate of environmental quality

$$E(B,S,\sigma) = a_0 S + b_0 (B - B_{\min})^{\beta} + \sigma + \tfrac{1}{2}\gamma\sigma^2 \qquad (24)$$

with the following parameters $a_0 > 0,\ b_0 > 0,\ 0 < \beta < 1,\ \gamma < 0,\ B_{\min} < 0$

and the cost function associated with the financial burden of the subsidies in the following convex form

$$C(S) = S + \tfrac{1}{2}kS^2 \ , \ k > 0 \qquad (25)$$

The specification (24) only represents a weighted average of the individual contributions to E, moreover we suppose that the weight from the rate of environmental quality bonus with respect to most recently approved subsidies σ is set to one (1) at equilibrium. Therefore, at the equilibrium, condition (16) implies $E_{\sigma} = 1 = -\mu_{\infty}$, since $\sigma_{\infty} = 0$. In the same specification (24) the weights a_0, b_0 determine whether a surplus or a deficit describe the stationary solution. The parameter $B_{\min}$ shows how budget surpluses and deficits are appreciated, and the superscript β , at which the amount $(B - B_{\min})$ has been raised, constrains budget deficits to $B > B_{\min}$.

The application of equation (22) in the parameters of our example reveals that the parameter $E_{\sigma\sigma} = \gamma$ is the first candidate choice as the bifurcation parameter since this choice satisfies the condition $K > 0$ and moreover the same choice doesn't affect the equilibrium position. According to Dockner and Feichtinger (Dockner and Feichtinger, 1991), the choice of γ, as the bifurcation parameter, implies that we vary γ until the bifurcation curve $\|J\| = \left(\tfrac{1}{2}K\right)^2 - \tfrac{1}{2}\rho^2 K$ is crossed, which is possible with the above specifications.

Let us consider the parameters $a_0 = 1,\ b_0 = 50,\ B_{\min} = -1,\ \ \rho = 1,\ \delta = 0{,}2$ and $\beta = 0{,}1$. With these parameter values the model admits an equilibrium with budget surplus $B_{\infty} = 2{,}57$ which in turn facilitates equilibrium subsidies $S_{\infty} = 0{,}51$.

Lowering the value of the parameter b_0 leads in a deficit of the national budget therefore in turn could lead in taxation (negative subsidy) as the equilibrium strategy. For $|\gamma|$ small the system of canonical equations exhibit saddle point stability, but the bifurcation curve is crossed at point $\hat{\gamma} = -4,793$ as numerical calculations confirm (Hassard et al, 1981). In the above point a stable limit cycle is born, therefore a family of stable cyclical strategies exists for smaller values of γ but sufficiently close to $-4,793$, i.e. for $\gamma \leq -4,793$.

5. Conclusions

In this paper we setup a very simple optimal control model for which the main concern of the central regulator was to steer the undertaken subsidy's decision to its target, in an optimal way, but under the constraint of the national budget. For this purpose we treat the overall subsidies offered as an accumulated variable. In this accumulated variable, the instant changes about the given subsidy they are added onto the historically already offered, constituting therefore the overall subsidy.

The above treatment of the subsidies is similar to the case of consumption behavior. According to Becker and Murphy (1988) a wide variety of consumer behavior is consistent with utility maximization. In the same framework of Becker and Murphy enters the addictive behavior of the consumers as a characteristic at which an increase in the past consumption causes present consumption to rise.

Since consumption includes all the goods without constraints we enlarge the above behavior in the case of the subsidies, assuming the subsidies as consumption goods. Moreover, the past subsidies offered is summarized by a stock of subsidies that, together with current subsidy, affects current utility. This definition implicitly assumes that subsidies accumulate a single stock (subsidies capital). The latter is the simplest case in literature, the one capital accumulation, but there are more complex cases of two and more capital accumulations.

In our note we explain the causes for the occurrence of cyclical subsidies trajectories. We show that, for the case of subsidy consumption capital, subsidies trajectories will always be monotonic. Hence, cyclical subsidies paths expressed as stable limit cycles or damped (explosive) oscillations require a subsidy that accumulates at least two stocks. It is the interaction of these two stocks that causes irregular behavior. In the main result of the paper (Proposition 1) we show that if present subsidy offered is positively correlated with past subsidies but is negatively correlated with the other stock (i.e. the national budget stock) then cyclical subsidies patterns are possible.

This implies that the cyclical policies of subsidies require two counter–balancing effects: the consuming and the satiating one. The first force acts like an addictive force which causes the current subsidies consumption to increase as past subsidies accumulate, while the second force causing current subsidies consumption to decline because of its costs.

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