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Abstract

This paper clarifies a macroeconomic condition, under which households are divided between a working class and an asset-owner class. Constructing a Keynesian model, we find that if aggregate savings from profits exceed aggregate investment, workers cannot accumulate their assets, and consequently a class society is established.

JEL classification: E12, E24, E25
Keywords: Effective Demand; Keynesian; Employment Rate; Underemployment; Wage Share; Working Class

I Introduction

Piketty (2014) provides some insights on inequality in developed economies. He emphasizes that if assets grow faster than wage-income, inequality will expand between asset owners and workers. If workers, however, save a part of their wages, and thereby receive asset-income, inequality might reduce. Expanding inequality would, therefore, imply that some issue or mechanism may prevent workers from saving. This paper presents a simple explanation of such a mechanism.

The paper integrates the studies of Kalecki and Moav. Given a working class and a capitalist class, Kalecki (1971) shows how effective demand determines aggregate production. He does not, however, explain the reasons why workers cannot save a part of their wages and thereby own assets. Assuming that the saving rate increases with income, Moav (2002) and Galor and Moav (2004) explain how households are divided into two classes in the process of economic development. In their model, however, the shortage of effective demand does not play any role. The current paper explains how a shortage of effective demand results in two household groups: a working class and an asset-owner class.

II Model

Households

The utility of household i is given by:

$$u^i = (c^i - \bar{c})^\alpha (s^i + \bar{s})^{1-\alpha}, \qquad \bar{c} > 0, \quad \bar{s} > 0, \quad 0 < \alpha < 1,$$

where c^i denotes consumption, and s^i savings. The constants $\bar{c}$ and $\bar{s}$ imply that consumption has a minimum level and zero savings are allowed. As per Moav (2002), we assume that the household leaves s^i to its descendant. The budget constraint is given by:

$$c^i + s^i = y^i, \quad s^i \geq 0,$$

where s^i is non-negative because the descendant has rights for waiver from the inheritance. Disposable income y^i is given by:

$$y^i = \omega x + \pi^i, \tag{1}$$

where ω denotes a real wage rate, x an employment rate, and π^i asset income. For the sake of simplifying the analysis, we assume that the employment rate for each household coincides with the average rate in the economy: $x = N/L$ where N denotes employed labor and L denotes labor supply.[1] Each household is endowed with one unit of labor. Accordingly, L also equals the number of households. The situation with underemployment $(x < 1)$ is of concern in the

[1] Hollander (1988) uses a similar assumption on employment rates.

following discussion.

The household chooses c^i and s^i to maximize u^i. The result includes the following two cases:

Case 1: If $y^i > \frac{\delta}{1-\alpha}$, then:

$$s^i = (1-\alpha)y^i - \delta > 0, \qquad (\delta \equiv (1-\alpha)\bar{c} + \alpha\bar{s}) \tag{2}$$

$$c^i = \alpha y^i + \delta. \tag{3}$$

Case 2: If $y^i \leq \frac{\delta}{1-\alpha}$, then:

$$s^i = 0, \tag{4}$$

$$c^i = y^i. \tag{5}$$

If disposable income is relatively high, the household has positive savings (Case 1). By contrast, if disposable income is low, the entire income is spent without savings (Case 2).

The number of households, L_t, increases at the rate of n:

$$L_{t+1}/L_t = n. \tag{6}$$

Firms

We propose the following simple framework. Production Y requires both capital K and labor N, in Leontief technology:[2]

$$K = vY, \quad N = mY, \tag{7}$$

where v denotes a capital coefficient and m denotes a labor-input coefficient. Capital K_{t+1} is formed by investment I_t, and completely depreciates within one period:

$$K_{t+1} = I_t. \tag{8}$$

In period t, firms precisely forecast the amount of capital required for production in period t+1:[3]

$$K_{t+1} = vY_{t+1} = vg_{t+1}Y_t, \tag{9}$$

where g_{t+1} denotes the (gross) growth rate of production: $g_{t+1} \equiv Y_{t+1}/Y_t$.

Firms employ labor at the beginning of each period, and decide the product price P by the mark-up over the unit labor cost:

$$P = \frac{1}{\theta}(Wm), \quad 0 < \theta < 1, \tag{10}$$

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[2] In our model, an increase in labor productivity will not affect aggregate production in equilibrium because it is cancelled out by a decrease in the employment rate.

[3] We do not consider economic fluctuations caused by incorrect forecasts.

where W denotes a nominal wage rate,[4] Wm the unit labor cost, and $1/\theta$ the mark-up ratio.[5] From (10), the real wage rate ω becomes:

$$\omega = \frac{W}{P} = \frac{\theta}{m}. \quad (11)$$

From (7) and (11), we have the total wage:

$$\omega N = \omega m Y = \theta Y, \quad (12)$$

where θ implies the wage share in income. Firms distribute the whole profit to households. Accordingly, the total asset-income is given by:

$$\textstyle\sum_{i=1}^{L} \pi^i = Y - \omega N = (1-\theta)Y. \quad (13)$$

III Non-Class Society

Aggregate consumption

We term the case in which all households make positive savings as the non-class society. These households have the same consumption function (3) (Case 1 in section II). The aggregate income of households, $\sum_{i=1}^{L} y^i$, will coincide with aggregate production Y, if the whole profit is distributed to households (as per (13)). Taking (3) into account, aggregate consumption C is given by:

$$C = \textstyle\sum_{i=1}^{L} c^i = \alpha\left[\sum_{i=1}^{L} y^i\right] + \delta L = \alpha Y + \delta L. \quad (14)$$

Equilibrium

From (8) and (9), investment is:

$$I_t = v g_{t+1} Y_t. \quad (15)$$

From (14), (15), and the equilibrium condition ($Y_t = C_t + I_t$), we obtain the equilibrium production:

$$Y_t = \frac{\delta L_t}{1 - \alpha - v g_{t+1}}. \quad (16)$$

Let us examine an economy in the steady state: $g_{t+1} = g_{t+2} = g$[6]. Then, from (16), the growth rate g is:

$$g = \frac{Y_{t+1}}{Y_t} = \frac{L_{t+1}}{L_t} = n, \quad (17)$$

and therefore equation (16) turns out to be:

$$Y = \frac{\delta L}{1 - \alpha - vn}, \quad (18)$$

where we assume:

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[4] Nominal wages are assumed not to be zero in the case of underemployment.
[5] We use similar price settings to Kalecki (1971, Ch. 5).
[6] Our focus is on comparative statics in the steady state.

$$1-\alpha-vn>0. \tag{19}$$

Taking (7) and (18) into account, the employment rate x is given by:

$$x=\frac{N}{L}=\frac{mY}{L}=\frac{m\delta}{1-\alpha-vn}. \tag{20}$$

Let us assume that workers have no asset initially. As their income is ωx, equilibrium in a non-class society requires that Case 1 holds for the workers, i.e.,

$$\omega x>\delta/(1-\alpha). \tag{21}$$

Taking (20) into account, inequality (21) becomes:

$$(1-\alpha)(1-\theta)<vn, \tag{22}$$

which implies $(1-\alpha)(1-\theta)Y<I$. (Notice that $vnY=vgY=I$.) In other words, aggregate savings from profits are smaller than aggregate investment. Thus, we have the following proposition:

Proposition 1: If aggregate savings from profits are smaller than aggregate investment, workers who initially have no assets make positive savings and leave assets to their descendants. So, after the next generation, all households become asset holders.

There is an intuitive explanation for Proposition 1. Suppose that all the households (including workers) save. The product market is equilibrated on the condition that aggregate savings are equal to aggregate investment. Taking (18) into consideration, this condition is expressed by:

$$(1-\alpha)Y-\delta L=vnY,$$

which can be rewritten as:

$$[(1-\alpha)(1-\theta)Y-vnY]+[(1-\alpha)\theta Y-\delta L]=0,$$

and therefore,

$$[(1-\alpha)(1-\theta)Y-I]+[(1-\alpha)\omega x-\delta]L=0.$$

The first part with square brackets in the above equation represents aggregate savings from profits minus aggregate investment, while the second part represents savings from wage-income minus a constant part of consumption. As (21) shows, when workers can save, the second part is positive, and so the first part must be negative. Thus, condition (22) is necessary for positive savings from wage-income.

IV Class Society

Consumption of workers and asset owners

In the case of a class society, workers who own no initial assets do not save. Suppose that workers' income ωx satisfies the following condition:

$$\omega x \leq \frac{\delta}{1-\alpha} \ , \tag{23}$$

which means that Case 2 in section II applies to workers. Workers' saving s^w is zero accordingly, and workers' consumption c^w is given by:

$$c^w = \omega x. \tag{24}$$

For simplicity's sake, let us assume that each asset holder owns the same amount of initial assets and receives the same asset income π. We further assume that Case 1 holds for the asset owners.[7] Then, from (1) and (3), their consumption c^k is given by:

$$c^k = \alpha[\omega x + \pi] + \delta. \tag{25}$$

Aggregate consumption

Let η stand for the ratio of workers to all households (and $1-\eta$ for the ratio of asset owners). Accordingly, (13) becomes $\pi(1-\eta)L = (1-\theta)Y$. Taking this equation together with (24), (25) and (12), gives an aggregate consumption:

$$C = c^w \eta L + c^k(1-\eta)L = [\eta + \alpha(1-\eta)]\theta Y + \alpha(1-\theta)Y + (1-\eta)\delta L.$$

The terms on the right side implies consumption from wages, consumption from profits, and the constant consumption of asset owners. Removing duplications, we obtain:

$$C = (1-\alpha)\eta\theta Y + \alpha Y + (1-\eta)\delta L. \tag{26}$$

Equilibrium

From (26), $I = vnY$ and $Y = C + I$, we have the aggregate production in a class society:[8]

$$Y = \frac{(1-\eta)\delta}{(1-\alpha)(1-\eta\theta) - vn} L. \tag{27}$$

The employment rate x is given by:

$$x = \frac{N}{L} = \frac{mY}{L} = m\frac{(1-\eta)\delta}{(1-\alpha)(1-\eta\theta) - vn} \ , \tag{28}$$

and inequality (23) becomes:

$$\theta\frac{(1-\eta)\delta}{(1-\alpha)(1-\eta\theta) - vn} \leq \frac{\delta}{1-\alpha}. \tag{29}$$

Rearranging (29), we obtain:

$$0 \leq \frac{(1-\alpha)(1-\theta) - vn}{(1-\alpha)(1-\alpha - vn)}\delta. \tag{30}$$

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[7] It will be easily confirmed that the disposable income of an asset owner, y^k, satisfies $y^k > \delta/(1-\alpha)$ in equilibrium.

[8] The denominator in (27) is positive under inequality (31), which is derived later.

As the denominator in (30) is positive ((19) applies), inequality (30) implies:

$$(1-\alpha)(1-\theta)-vn \geq 0. \quad (31)$$

Thus, we have the following proposition that is dual to Proposition 1:

Proposition 2: If aggregate savings from profits are equal to or larger than aggregate investment, workers who initially own no assets create no savings. As a result, their descendants will not own initial assets. Thus, households will be divided into a working class and an asset-owner class for generations.[9]

V Keynesian 45° Line Diagram

The different cases of a class and non-class society can be depicted by a Keynesian 45° line diagram. The two lines in Figure 1 show the aggregate demand curves of these two cases. The line corresponding to a class society is steeper because a worker's marginal propensity to consume is unity (see (24)). If $Y \leq \delta L/(1-\alpha)\theta$, then $s^w = 0$ and the society becomes a class society. If $Y > \delta L/(1-\alpha)\theta$, then $s^w > 0$ and a non-class society forms. Therefore, the effective aggregate demand is segment *ABD*. Figure 2 depicts how the class society appears in equilibrium. In this case, the effective demand curve cuts the 45° line on the lower left side of point *B*. Figure 3 depicts the case of a non-class society, in which the effective demand curve cuts the 45° line on the upper right side of point *B*. The former case occurs under condition (31), while the latter occurs under condition (22).

If the wage-share in income, θ, is small, and the growth rate n is low, the condition of a class society (31) is likely to hold. In such circumstances, an increase in Y will yield a small increase in aggregate demand. The aggregate demand curve will be relatively flat and cuts the 45° line at a low level of Y. The low level of equilibrium production lowers the employment rate, and so discourages workers from saving. Thus, workers' descendants also do not own initial assets.

VI Conclusion

This paper investigates what divides households between working class and asset-owner class. Our theoretical finding is that in a class society, aggregate savings from profits exceed aggregate investment. Under this condition, the shortage of effective demand causes a low level of aggregate production, leading to a low employment rate, which prevents workers from leaving assets to their

[9] The condition (31) can be rewritten as $(1-\alpha)(1-\theta)/v \geq n$, which implies that the earning rate of assets multiplied by the marginal propensity to save is greater than the rate of economic growth. This is similar to Piketty's "$r > g$."

descendants. Consequently, the working class is likely to become immobile. Thus, a class society would be established. The low wage-share in income and the low rate of growth make such a circumstance more likely.

References

Galor O. and Moav O. (2004). From physical to human capital accumulation: inequality and the process of development. *Review of Economic Studies*, 71, 1001-1026.

Hollander, H. (1988). Increasing Returns and the Foundations of Unemployment Theory: A Note. *Economic Journal*, 98, 165-71.

Kalecki M. (1971). *Selected Essays on the Dynamics of the Capitalist Economy 1933-1970*. Cambridge (Eng.): Cambridge University Press.

Moav O. (2002). Income distribution and macroeconomics: the persistence of inequality in a convex technology framework. *Economics Letters*, 75, 187-192.

Piketty T. (2014). *Capital in the Twenty-first Century*. Cambridge (Mass.): Belknap Press of Harvard University.

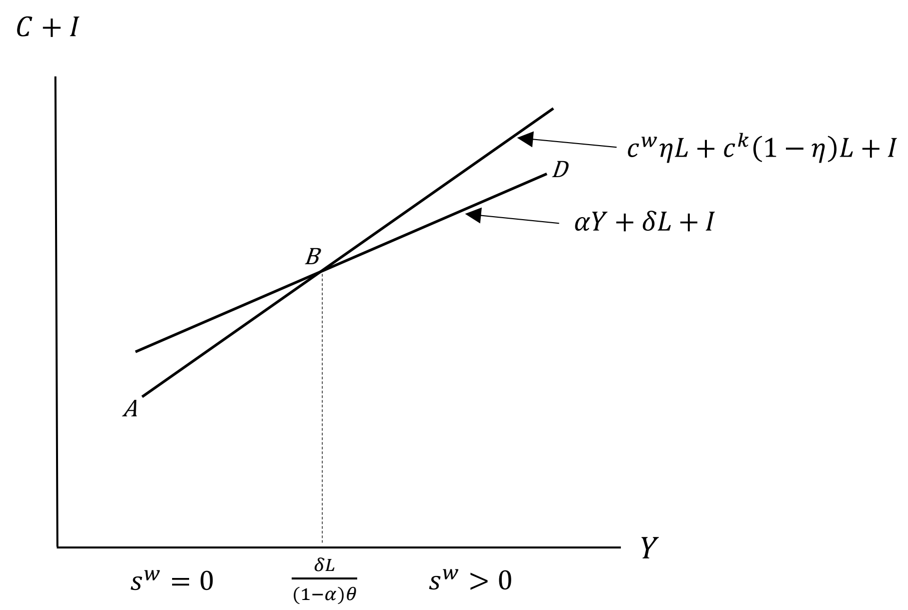

Figure 1. Aggregate demand curves. The two lines show the aggregate demand curves of a class society and a non-class society. The line corresponding to a class society is steeper because a worker's marginal propensity to consume is unity. If $Y \leq \delta L/(1-\alpha)\theta$, then $s^w = 0$ and actually the society becomes a class society. If $Y > \delta L/(1-\alpha)\theta$, then $s^w > 0$ and a non-class society forms. Therefore, the effective aggregate demand is segment *ABD*.

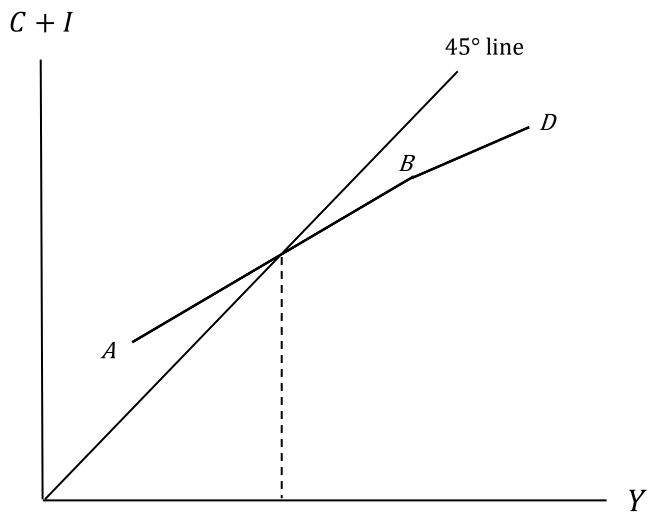

Figure 2. The case of a class society. The effective demand curve *ABD* cuts the 45° line on the lower left side of point *B*.

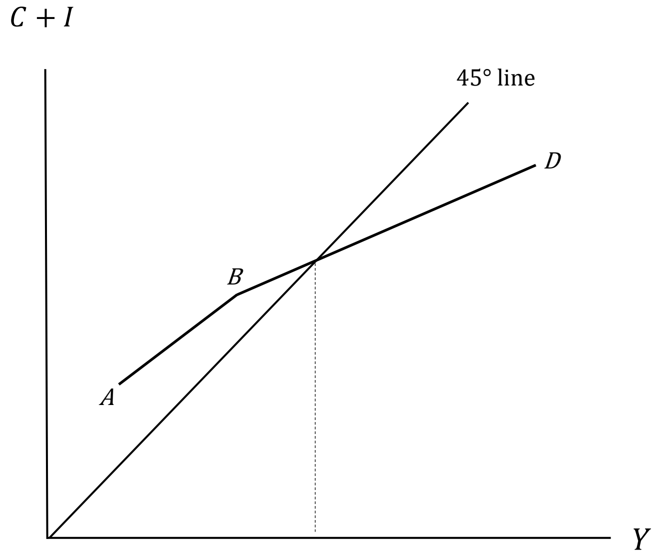

Figure 3. The case of a non-class society. The effective demand curve *ABD* cuts the 45° line on the upper right side of point *B*.