Tax Evasion and Optimal Corporate Income Tax Rates in a Growing Economy

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Abstract

We explore how tax evasion by firms affects the growth- and welfare-maximizing rates of corporate income tax (CIT) in an endogenous growth model with productive public service. We show that the negative effect of CIT on growth is mitigated in the presence of tax evasion. This increases the benefit of raising the CIT rate for public service provision. Thus, in contrast to Barro (1990), the optimal tax rate is higher than the output elasticity of public service. Through numerical exercises, we demonstrate that the role of tax evasion by firms is quantitatively significant.

Keywords: corporate income tax, tax evasion, growth, welfare
1 Introduction

Corporate income tax (hereafter, CIT) has detrimental effects on investment by firms. Modern prevailing opinions advocate that CIT should be cut to promote economic growth. In reality, governments in developed countries have lowered the CIT rate in recent years: The average rate of CIT in OECD countries decreased from 32.5% to 24.2% between 2000 and 2017. However, because CIT is an important tax basis for public finance, cutting the CIT rate reduces public investment in infrastructure (productive public services) and may lower the rate of economic growth (e.g., Barro 1990). Therefore, setting CIT rates involves a trade-off between private investment and the provision of productive public services for economic growth, and it is an important policy issue.

In addition, tax evasion by firms is a serious problem of CIT. There is large-scale tax evasion in real economies. For example, in the US, the Internal Revenue Service (2016) reports about 44 billion dollars as the estimated tax gap of CIT on annual average from 2008 to 2010. Nonetheless, the actual audit coverage of taxed corporations is very limited, primarily because of fiscal tightness; only 1.0% of taxed returns were examined in the US in 2016 and only 3.1% of all taxed corporations were audited in Japan in 2015.

One of the most general ways of tax evasion is to underdeclare income (e.g., Allingham and Sandmo 1972). To underdeclare corporate income, firms may pretend to have lower productivity and overstate their costs. In reality, there is institutional cause for such tax evasion behavior at the microeconomic level. The CIT systems in many countries have reduction and exemption measures; deficits of corporations can be carried forward and CITs of corporations with small income are reduced or exempted. Such systems give corporations an incentive to underdeclare their profit intentionally by overstating cost. These are neither unusual nor insignificant at the macroeconomic level. For example, the National Tax Agency (2016) reports that more than two-thirds of the ordinary corporations in Japan were loss corporations between 2011 and 2015 on

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1 According to the policy stance of President Trump, the US government reduced the CIT rate to 21%. In Japan, the government plans to cut the CIT rate to about 20% gradually under some conditions.
2 In fact, CIT occupies a considerable fraction of the annual revenue of the public sector. For example, the Internal Revenue Service (2017) and the National Tax Agency (2017) report that the shares of CIT to total tax revenues were about 10% in the US in 2016 and 19% in Japan in 2015, respectively.
3 Here, the “tax gap” is the “gross tax gap” in the Internal Revenue Service’s survey. This is defined as the amount of true tax liability that is not paid voluntarily and timely. For details, see Internal Revenue Service (2016).
4 The sources of these facts are Internal Revenue Service (2017) and National Tax Agency (2016).
average. The reported losses amounted to 11.3 billion yen in 2015 and occupied a quarter of the reported total income of the corporate sector.

With these problems of CIT as motivation, this study investigates an optimal CIT policy in a growing economy with tax evasion by firms. The evaded tax payments are utilized for private investment, while a part of the tax revenue is lost. This affects the trade-off between private investment and the provision of productive public services. We are particularly interested in how CIT evasion by firms changes the optimal rate of CIT.

To tackle this problem, we construct a variety-expansion model of growth in which private firms invest to earn monopolistic profits and their investments sustain economic growth. A single final good is produced by using intermediate goods. Each intermediate good is produced by a monopolistically competitive firm. Each intermediate good firm invests to enter into business and pursues monopolistic profits. CIT discourages private investment because the monopolistic profits are subject to CIT. On the other hand, productive public services have a positive effect on the monopolistic profits of intermediate good firms. Thus, CIT has a positive effect on private investment through productive public services financed by CIT.

In our model, facing CIT, firms have an incentive to evade tax payment by underdeclaring their profits in the absence of a perfect tax enforcement system. Because each firm can avoid a part of tax payment, tax evasion weakens the discouraging effect of CIT on private investment. Simultaneously, tax evasion reduces the provision of productive public services. Thus, tax evasion by the corporate sector affects private investment, provision of productive public services, and economic growth.

In a tractable model, we obtain a qualitative result that both the growth- and welfare-maximizing CIT rates are higher than the output-elasticity of public service. This is in contrast to the familiar Barro’s (1990) rule, which indicates that the tax rate should be set at the output elasticity of public services. The mechanism behind our result is as follows. When the government raises the CIT rate, the effective CIT rate and the tax revenue rise. This increases the provision of productive public services and promotes economic growth. Simultaneously, raising the CIT rate discourages private investment and has a detrimental effect on growth. However, this negative effect on growth is weakened by tax evasion. This is because, in response to a tax hike, firms attempt to

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5 We follow Barro and Sala-i-Martin’s (2004) variety-expansion model.

6 Here, we mean that the effective tax rate is the ratio of the actually collected CIT revenue to the true profits of firms.
secure profits by increasing tax evasion. It alleviates the reduction in private investment and the tax base of CIT. CIT evasion generates the benefits of raising the CIT rate for the provision of productive public services.

Next, we extend the model to obtain quantitative results. Our quantitative analyses show that tax evasion significantly raises the optimal CIT rate. In particular, the optimal tax rate is 40% for the benchmark case, which is much higher than the standard values of the output elasticity of public service, e.g., 10%. This optimal CIT rate in our model, at 40%, is close to that in Aghion et al. (2016) mentioned below. Besides, we decompose the difference between the optimal tax rate and the output elasticity of public service into several parts including a part associated with the effect of CIT evasion. We find that the effect of CIT evasion occupies more than half of the total difference between the optimal tax rate and the output elasticity of public service for a wide range of reasonable parameter values. That is, even quantitatively, the effect of CIT evasion is the main source of the high optimal tax rate.

**Related Literature**

Largely, our study is part of the literature starting with Barro (1990), which explores optimal taxation in an endogenous growth model with productive public service (capital) (e.g., Futagami et al., 1993; Glomm and Ravikumar, 1994; Turnovsky 1997). These studies emphasize that the growth-maximizing income tax rate equals the output elasticity of public capital and it coincides with the welfare-maximizing one in the balanced growth path. This is the so-called Barro rule.

More recent studies cast some doubts on the Barro rule. Some studies (e.g., Futagami et al., 1993; Ghosh and Roy 2004; Agénor 2008) show that the welfare-maximizing tax rate is lower than the growth-maximizing one (output elasticity of public capital), while other studies (e.g., Kalaitzidakis and Kalyvitis 2004; Chang and Chang 2015) show the opposite result. While these existing studies consider neither CIT nor tax evasion, our study investigates optimal taxation,

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7Some empirical studies suggest the importance of productive public expenditure for economic growth. Abiad et al. (2016) show that increases in public investment in infrastructure raise output in both the short and long run.

8Futagami et al.’s (1993) finding works if the policy effects during the transition path caused by the accumulation of public capital is evaluated. Ghosh and Roy (2004) consider the composite output externality between the stock of public capital and the flow of public services. Kalaitzidakis and Kalyvitis (2004) consider the trade-off of public expenditure between new investment and the maintenance of existing public capital. Agénor (2008) considers public health capital in addition to general infrastructure. Chang and Chang (2015) consider an endogenous growth model with market imperfection both in goods and labor markets and show that welfare-maximizing productive government spending can be larger than growth-maximizing spending if it is financed by capital income taxes.
Among the existing studies of tax evasion and growth, we should refer to Chen (2003) and Kafkalas et al. (2014). They investigate tax evasion by household-firms in Barro’s (1990) type model, in which government incurs inspection expenditure to audit taxpayers. These authors focus on the trade-off between the government’s inspection expenditure and public investment. Chen (2003) shows that the growth-maximizing announced tax rate is higher than the output elasticity of public service, under the assumptions that inspection expenditure is proportional to output and household-firms must incur tax evasion costs. On the other hand, Kafkalas et al. (2014) assume that the government’s inspection expenditure is proportional to tax revenues. They show that, even with tax evasion, the growth-(and utility-) maximizing effective tax rate equals the output elasticity of public capital, in line with Barro’s rule.

In contrast to these studies, we do not focus on the role of inspection expenditure. Therefore, we adopt the same specification of inspection expenditure as Kafkalas et al. (2014), which does not affect Barro’s rule. Thus, we can concentrate on the role of tax evasion behavior of firms with market power.

Although the findings on optimal tax rates from Chen (2003) and Kafkalas et al. (2014) are interesting, there are some reservations. First, they consider tax evasion by household-firms, where the firm is the same as a household. Since there is no difference between CIT and a household’s income tax in their models, they do not consider the role of CIT evasion. Second, they assume a competitive goods market, and therefore, ignore tax evasion associated with profit maximization in an imperfectly competitive product market. Thus, although tax evasion affects the aggregate economy through the government budget in their models, these studies do not consider the role of tax evasion in the production side directly.

Our study is also comparable to Aghion et al. (2016). They examine the relationship among corporate taxation, growth, and welfare, focusing on corruption. Aghion et al. (2016) construct
an endogenous growth model in which public capital raises the expected returns to entrepreneurial efforts on R&D but tax revenue decreases due to the corruption of officers after tax collection. They suggest that the relationship between CIT rate and growth is an inverted-U shape and the welfare-maximizing CIT rate is 42% in the calibrated model. However, Aghion et al. (2016) do not consider tax evasion by firms but focus on the corruption between government and households. In this study, we incorporate corporate tax evasion in an endogenous growth model and suggest that the optimal CIT rate is as high as Aghion et al.’s (2016) estimate.

2 Model

Time is discrete and denoted by \( t = 0, 1, \ldots \). The economy is inhabited by the following four types of agents: producers of final output, producers of intermediate goods, a representative household, and government. An infinitely lived representative household has perfect foresight and is endowed with \( L \) unit of labor. Labor moves freely across different production sectors. The number of intermediate good in period \( t \) is \( N_t \). We assume \( N_0 = 1 \) without loss of generality.

2.1 Producers of final good

A single final output is produced by perfectly competitive producers using the following technology:

\[
Y_t = AL_{Y,t}^{1-\alpha} \int_0^{N_t} (G_t x_{i,t})^\alpha di, \quad A > 0, \quad 0 < \alpha < 1,
\]

where \( Y_t \) is output, \( L_{Y,t} \) is labor input in the final goods sector, and \( x_{i,t} \) is the input of intermediate good \( i \). Following Barro (1990), public services, \( G_t \), increases the productivity of output. We take the final output as the numeraire. Although \( \alpha \) encompasses both (i) the output elasticity of public services and (ii) the price elasticity of intermediate goods \( 1/(1 - \alpha) \), we examine the former in this simple model. In the extended model (Section 5), we separate (i) and (ii). We denote the price of intermediate good \( i \) and wage rate as \( p_{i,t} \) and \( w_t \), respectively. The profit maximization yields

\[
w_t = (1 - \alpha)AL_{Y,t}^{\alpha} \int_0^{N_t} (G_t x_{i,t})^\alpha di = (1 - \alpha) \frac{Y_t}{L_{Y,t}}, \quad (2)
\]

\[
p_{i,t} = \alpha AL_{Y,t}^{1-\alpha} G_t x_{i,t}^{\alpha-1}. \quad (3)
\]
Solving (2) and (3) with respect to $x_{i,t}$ yields the demand function for the product of firm $i$:

$$x(p_{i,t}) \equiv \frac{(1 - \alpha)Y_t}{w_t} \left( \frac{\alpha AG^*_t}{p_{i,t}} \right)^{\frac{1}{1-\alpha}}.$$

### 2.2 Producers of intermediate goods

#### 2.2.1 Entry into the intermediate goods market

Each intermediate good is produced by a monopolistically competitive firm. To operate in period $t$, each intermediate good firm must invest $\eta$ unit of the final good in period $t - 1$. Firms finance the cost of this investment by borrowing from households. Because firms must incur investment costs in each period, the planning horizon of each firm is one period, as in Young (1998). When incurring investment costs, each firm draws its productivity $b > 0$ from distribution $F(b)$. We assume that (i) $b$ is independent and identically distributed (iid) over time as well as across firms and (ii) $b$ is private information. These two assumptions are useful in describing an environment where the true tax base of each firm cannot be observable without auditing firms, as we explain in the next subsection. Besides, heterogeneity among firms’ productivity enables us to describe the realistic firm size distribution for the quantitative analysis in Section 5.

Let us denote the expected after-tax operating profit of a firm with productivity $b$ in period $t$ by $\pi_{i,t}^e$. $\pi_{i,t}^e$ depends on $b$. The next subsection discusses $\pi_{i,t}^e$ in detail. The objective of intermediate good firm $i$ that invests in period $t - 1$ is given by

$$\Pi_{i,t-1} = \frac{1}{R_{t-1}} \int \pi_{i,t}^e dF(b) - \eta, \quad (4)$$

where $R_{t-1}$ is the gross interest rate between periods $t - 1$ and $t$. Since $b$ is iid across firms, all firms face the same $\Pi_{i,t-1}$ in period $t - 1$. Free entry into the intermediate goods market implies

$$\int \pi_{i,t}^e dF(b) = R_{t-1}\eta. \quad (5)$$

#### 2.2.2 Maximization of operating profits

The price of each intermediate good $p_{i,t}$ is public information. A firm with productivity $b$ needs $1/b$ unit of labor to produce one unit of intermediate good. The true operating profit of firm $i$ is
given by
\[ \pi_{i,t} = \left( p_{i,t} - \frac{w_t}{b_t} \right) x(p_{i,t}). \] (6)

We use the word “true” to distinguish the true operating profit from the operating profit that firm \( i \) declares, \( \tilde{\pi}_{i,t} \), to the government. Since \( b \) is private information, the government cannot directly observe the true operating profit, \( \pi_{i,t} \), and thus, cannot know whether \( \tilde{\pi}_{i,t} \) equals \( \pi_{i,t} \) without auditing firms.

Let us denote the announced CIT rate by \( \tau \in (0, 1] \). The after-tax profit of each firm is given by \( \pi_{i,t} - \tau \tilde{\pi}_{i,t} \) because CIT is imposed on declared profit \( \tilde{\pi}_{i,t} \). We denote the probability of audit by \( \bar{q} \in [0, 1] \). We impose an assumption on \( \bar{q} \) later. Suppose that a firm is audited. If this firm underdeclares its operating profit (\( \tilde{\pi}_{i,t} < \pi_{i,t} \)), it has to pay a penalty of \( (1 + s)\tau(\pi_{i,t} - \tilde{\pi}_{i,t}) \), where \( s(\geq 0) \) is an additional tax rate. If this firm overdeclares its operating profit (\( \tilde{\pi}_{i,t} > \pi_{i,t} \)), the overpayment, \( \tau(\tilde{\pi}_{i,t} - \pi_{i,t}) \), is refunded. The expected after-tax operating profit of firm \( i \) is given by
\[ \pi_{i,t}^e = \pi_{i,t} - \tau \tilde{\pi}_{i,t} - \bar{q}(1 + s)\tau \cdot \max\{0, \pi_{i,t} - \tilde{\pi}_{i,t}\} + \bar{q}\tau \cdot \max\{0, \tilde{\pi}_{i,t} - \pi_{i,t}\}. \] (7)

Firm \( i \) chooses \( \tilde{\pi}_{i,t} \) and \( p_{i,t} \) to maximize \( \pi_{i,t}^e \). Given \( \pi_{i,t}, \pi_{i,t}^e \) decreases with \( \tilde{\pi}_{i,t} \) if \( \tilde{\pi}_{i,t} > \pi_{i,t} \). Thus, no firms overdeclare their operating profits. The inequality \( \tilde{\pi}_{i,t} \leq \pi_{i,t} \) must be satisfied in the following discussion.

Our specification of tax evasion behavior is essentially similar to Allingham and Sandmo (1972). They consider tax evasion by a representative household, which evades income tax by choosing declared income when its actual income is not known directly by the government. In our setting, each firm chooses its declared profit by adjusting the price level.

Using (7), we have the following lemma.

**Lemma 1:** Suppose that a firm setting the price at \( \tilde{p}_{i,t} \) declares its operating profits truthfully, \( \tilde{\pi}_{i,t} = \pi_{i,t} \). Then, the declared profit of this firm satisfies
\[ \tilde{\pi}_{i,t} = (1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t}). \] (8)

**Proof:** See Appendix A.
Since the contraposition of a true claim is also true, we can conclude that if a firm does not declare (8), it does not declare its true profit. Thus, we assume that any firms whose declared profits do not satisfy (8) are audited with the probability of one. In addition, we assume that if a firm declares (8), the firm is audited with a probability that is lower than one. Note that (8) is a necessary condition for true declaration of profits. Thus, a firm declaring (8) does not necessarily declare its profit truthfully.

**Assumption:** Consider a firm that sets the price at $\tilde{p}_{i,t}$.

(i) If the firm’s declared profit does not satisfy (8), the firm is audited with a probability of one, $\bar{q} = 1$.

(ii) If the firm’s declared profit satisfies (8), the firm is audited with a probability that is lower than one, $\bar{q} = q \in [0, 1)$.

Since $\tilde{\pi}_{i,t} \leq \pi_{i,t}$, we now rewrite (7) as $\pi_{i,t}^e = (1 - \hat{\tau})\pi_{i,t}$, where $\hat{\tau}$ is the effective CIT rate and is defined as

$$\hat{\tau} \equiv \left\{ \left[ 1 - \bar{q}(1 + s) \right] \frac{\tilde{\pi}_{i,t}}{\pi_{i,t}} + \bar{q}(1 + s) \right\} \tau.$$

The following proposition shows each firm’s decisions on profit declaration and pricing.

**Proposition 1.** (i) Suppose that $1 - q(1 + s) \leq 0$. Then, all firms declare their true profit $\tilde{\pi}_{i,t} = \pi_{i,t}$. Firms with productivity $b_i$ set the price at $\tilde{p}_{i,t} = w_i / (\alpha b_i)$. The declared profit satisfies (8). The true and the expected after-tax operating profit are, respectively, given by

$$\pi_{i,t} = \tilde{\pi}_{i,t} = (1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t}) \quad \text{and} \quad \pi_{i,t}^e = (1 - \hat{\tau})(1 - \alpha)\tilde{p}_{i,t}x(\tilde{p}_{i,t}).$$

The effective CIT rate coincides with the announced CIT rate, $\hat{\tau} = \tau$.

(ii) Suppose that $1 - q(1 + s) > 0$. Firms with productivity $b_i$ set the price at

$$\tilde{p}_{i,t} = \frac{w_i}{\alpha b_i} \Gamma(\tau), \quad \text{where} \quad \Gamma(\tau) \equiv \frac{1 - q(1 + s)\tau}{1 - q(1 + s)\tau - (1 - \alpha)[1 - q(1 + s)]\tau} > 1.$$

The declared profit satisfies (8). The true and the expected after-tax operating profit are, respec-
tively, given by

\[ \pi_{i,t} = (1 - \alpha \Gamma(\tau)^{-1}) \tilde{p}_{i,t} x(\tilde{p}_{i,t}) > \tilde{\pi}_{i,t}, \quad (12) \]

\[ \pi_{e,i,t} = (1 - \tilde{\tau}) (1 - \alpha \Gamma(\tau)^{-1}) \tilde{p}_{i,t} x(\tilde{p}_{i,t}). \quad (13) \]

The inequality \( \pi_{i,t} > \tilde{\pi}_{i,t} \) indicates that all firms underdeclare their operating profits. The effective CIT rate, \( \tilde{\tau} \in \left(0, \frac{1 + \alpha q (1 + s)}{1 + \alpha}\right] \), satisfies that \( \tilde{\tau} < \tau \).

**Proof:** See Appendix B.

Proposition 1 states that the firms control their declared profits according to whether tax evasion is beneficial. If the expected additional tax rate is higher than the rate under honest declaration \( (1 \leq q(1 + s)) \), firms have no incentive to underdeclare profits. Thus, they declare true profits \( (\tilde{\pi}_{i,t} = \pi_{i,t}) \).

In contrast, if the expected additional tax rate is lower than the rate under honest declaration \( (1 > q(1 + s)) \), firms choose to evade CIT by underdeclaring profits \( (\tilde{\pi}_{i,t} < \pi_{i,t}) \). This is captured by the term \( \Gamma(\tau) \) in (12), because the only difference between the true profit \( \pi \) in (10) and declared profit \( \tilde{\pi} \) in (12) is the presence of \( \Gamma(\tau) \). \(^{11}\)

Note the following. When \( q(1 + s) > 1 \), because firms declare true profit and tax evasion does not occur, the CIT has nothing to do with firms’ control of their profits. However, when \( q(1 + s) < 1 \), the CIT rate affects tax evasion substantially through the term, \( \Gamma(\tau) \). By \( \Gamma'(\tau) > 0 \), an increase in the CIT rate causes a larger difference between the true profit \( \pi_{i,t} \) and declared profit \( \tilde{\pi}_{i,t} \). Thus, we find that it encourages tax evasion. Appendix C shows that \( d(\tau/\tilde{\tau})/d\tau > 0 \) if and only if \( \Gamma'(\tau) > 0 \). \(^{12}\)

**Remarks**

The properties \( d\tilde{\tau}/d\tau > 0 \) and \( d(\tau/\tilde{\tau})/d\tau > 0 \) are in line with Roubini and Sala-i-Martin (1995) and Kafkalas et al. (2014).\(^{13}\) However, the tax evasion considered in our model departs from

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\(^{11}\)They underdeclare profits by pretending to be lower productivity firms, which leads to a higher price setting \( (\Gamma(\tau) > 1) \) as represented by (11).

\(^{12}\)Equation (9) indicates that the degree of tax evasion in response to a tax hike, \( d(\pi_{i,t}/\tilde{\pi}_{i,t})/d\tau \), is reduced to \( d(\tau/\tilde{\tau})/d\tau \).

\(^{13}\)We obtain \( d(\tau/\tilde{\tau})/dq < 0 \) (see Appendix C) as in Roubini and Sala-i-Martin (1995) and Kafkalas et al. (2014). Both studies assume that the difference between the announced and the effective tax rate increases with the announced tax rate \( \tau \) and decreases with the detection probability \( q \).
these previous studies in the following respects.

First, tax evasion in our model is derived endogenously from the microfoundation of firms’
tax evasion behavior, in contrast to the *ad-hoc* expression by Roubini and Sala-i-Martin (1995).

Second, Chen (2003) and Kafkalas et al. (2014) do not consider tax evasion associated with
profit maximization in an imperfectly competitive product market. Thus, there is no relationship
between the tax rates and the firms’ choice of profits and declarations of profits in their models.
In contrast, in our model, the CIT rate affects them substantially, as represented by $\Gamma(\tau) > 1$ and
$\Gamma'(\tau) > 0$.

### 2.3 Household

The population size is constant at one. The utility function of a representative household is

$$U_0 = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t u(C_t), \quad u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}, \quad \sigma > 0. \quad (14)$$

$u(C_t) = \ln C_t$, when $\sigma = 1$. Here, $C_t$, $\rho(>0)$ and $1/\sigma$ denote consumption in period $t$, the
subjective discount rate, and the intertemporal elasticity of substitution, respectively. The rep-
resentative household supplies $L$ unit of labor inelastically. The household’s budget constraint
is given by $W_t = R_{t-1}W_{t-1} + w_tL - C_t$, where $W_{t-1}$ is assets at the end of period $t - 1$. The
household’s utility maximization yields

$$\frac{C_{t+1}}{C_t} = \left( \frac{R_t}{1 + \rho} \right)^{1/\sigma}, \quad \text{(15)}$$

and the transversality condition (TVC) is

$$\lim_{t \to \infty} \frac{C_t^{-\sigma}W_{t-1}}{(1 + \rho)^t} = 0. \quad \text{(16)}$$

### 2.4 Government

We assume that the government keeps a balanced budget in each period. From (7) and (9), the
aggregate tax revenue of the government is

$$\tau \int_0^{\bar{N}_t} \int \{ \tau \tilde{\pi}_{i,t} + q(1 + s) \tau [\pi_{i,t} - \tilde{\pi}_{i,t}] \} dF(b) di = \tilde{\tau} \int_0^{\bar{N}_t} \pi_{i,t} dF(b) di = \tilde{\tau} N_t \int \pi_{i,t} dF(b).$$

Here, we use the fact that $b$ is iid across firms. This revenue is allocated to productive government spending, $G_t$, and inspection expenditure to detect
tax evasion, $M_t$. Thus, the budget constraint of the government is given by

$$\tilde{\tau} N_t \int \pi_{i,t} dF(b) = G_t + M_t.$$  \hfill (17)

We assume that spending a constant fraction, $Q(q)$, of government revenue on $M_t$ leads to a successful detection of tax evasion by a firm with probability $q$, where $Q'(q) > 0$, $Q(q) \in [0,1]$ for $q \in [0,1]$, and $Q(0) = 0$. Therefore, the inspection expenditure is given by $M_t = Q(q)\tilde{\tau} N_t \int \pi_{i,t} dF(b)$. This specification of $M_t$ is in line with Kafkalas et al. (2014). Thus, (17) reduces to

$$G_t = [1 - Q(q)]\tilde{\tau} N_t \int \pi_{i,t} dF(b).$$  \hfill (18)

As mentioned in Introduction, such a specification allows us to isolate the role of tax evasion by firms with market power for pursuing Barro’s rule. This is because Kafkalas et al.’s (2014) result indicates that Barro’s rule holds even with the tax evasion of perfectly competitive firms under inspection expenditure of the form in (18).

3 Equilibrium

The labor market clears as

$$L = L_{Y,t} + N_t \int \frac{x_{i,t}}{b} dF(b).$$  \hfill (19)

The entry cost of intermediate good market $\eta$ is financed by borrowing from households. Since $N_{t+1}$ firms invest in period $t$, the asset market equilibrium condition is given by $W_t = \eta N_{t+1}$.

The final good market clears as $Y_t = C_t + \eta N_{t+1} + G_t + M_t$.

We next characterize the dynamic system and the steady state of the economy. Appendix D derives the following dynamic system with respect to $z_t \equiv C_t / N_t$.

$$z_{t+1} = \frac{\eta^{1-\frac{1}{\sigma}}[(1 - \tilde{\tau})(1 - \alpha \Theta^{-1})\alpha \Omega(\tilde{\tau})/(1 + \rho)]^{1/\sigma} z_t}{[1 - \tilde{\tau}(1 - \alpha \Theta^{-1})\alpha \Omega(\tilde{\tau}) - z_t]}.$$  \hfill (20)

where

$$\Theta = \begin{cases} 
1 & \text{if } 1 - q(1 + s) \leq 0 \\
\Gamma(\tau) & \text{if } 1 - q(1 + s) > 0,
\end{cases}$$
and

\[
\frac{Y_t}{N_t} = \Omega(\bar{\tau}) \equiv A^{\frac{1}{1-\alpha}} \left[ (1 - Q(q)) \tilde{\tau} (1 - \alpha \Theta^{-1}) \alpha \right]^{\frac{\alpha}{1-\alpha}} (1 - \alpha) \Theta \alpha^{\frac{2\alpha}{1-\alpha}} \\
\times \left[ \frac{L}{(1 - \alpha) \Theta + \alpha^2} \right]^{\frac{1}{1-\alpha}} \left[ \int b^{\frac{\alpha}{1-\alpha}} dF(b) \right].
\] (21)

From (20), we arrive at the following proposition.

**Proposition 2.** A unique steady state exists. In the steady state, \(z_t\) takes the following constant value:

\[
\hat{z} = \left[ 1 - \tilde{\tau} (1 - \alpha \Theta^{-1}) \alpha \right] \Omega(\bar{\tau}) - \eta \left[ (1 - \tilde{\tau}) (1 - \alpha \Theta^{-1}) \alpha (1 + \rho)^{-1} \eta^{-1} \Omega(\bar{\tau}) \right]^{1/\sigma} \in (0, \bar{z}),
\] (22)

where \(\bar{z} \equiv [1 - \tilde{\tau} (1 - \alpha \Theta^{-1}) \alpha] \Omega(\bar{\tau})\). In the steady state, \(C_t, N_t, \) and \(Y_t\) grow at the same constant rate:

\[
\hat{g} = \left[ (1 - \tilde{\tau}) (1 - \alpha \Theta^{-1}) \alpha (1 + \rho)^{-1} \eta^{-1} \Omega(\bar{\tau}) \right]^{1/\sigma}.
\] (23)

The economy jumps to the steady state initially.

**Proof:** See Appendix E.

### 4 Optimal CIT Rates

As mentioned in Section 2, the CIT rate substantially affects the tax evasion behavior of firms in the imperfectly competitive market. Therefore, we analyze the growth- and welfare-maximizing CIT rates under such CIT evasion.

#### 4.1 Growth-maximizing CIT Rate

We obtain the following proposition.

\[^{14}\text{Note that the parameters } q \text{ and } s \text{ are really endogenously selected by the governments in that the government controls tax enforcement and punishment against tax evasion. However, to concentrate on the choice of optimal CIT and keep the analyses simple, we ignore these policy dimensions and assume that they are exogenous parameters. This assumption is in line with Johansen (2010) who assume that parameters related to tax enforcement and punishment against profit shifting by multinational firms are exogenous.}\]
Proposition 3. Let us denote the growth-maximizing announced CIT rate and growth-maximizing effective CIT rate as $\tau^{GM}$ and $\tilde{\tau}^{GM}$, respectively.

1. When $1 - q(1 + s) \leq 0$ and each firm declares its true operating profit, $\tilde{\tau}^{GM} = \tau^{GM} = \alpha$ holds.

2. When $1 - q(1 + s) > 0$ and each firm underdeclares its operating profit, $\tau^{GM} > \tilde{\tau}^{GM} > \alpha$ holds.

Proof: See Appendix F.

The first part of Proposition 3 is in line with Barro’s (1990) rule, that is, the tax rate that maximizes long-run growth equals the output elasticity of public services, $\alpha$. This result is also consistent with that of Kafkalas et al. (2014), indicating that even with government spending for detection, the growth-maximizing tax rate coincides with the output elasticity of public services.

Importantly, the mechanism behind this result is the same as that of Barro (1990). In Barro’s model, the growth-maximizing rule is attributed to the following trade-off between income tax and growth. On the one hand, an increase in income tax decreases the net interest rate ($(1 - \text{tax rate}) \times \text{interest rate}$) and has a negative effect on growth. On the other hand, an increase in income tax boosts productive government spending and raises the interest rate. This has a positive effect on growth.

The interest rate in our model is determined through the free entry condition of intermediate goods firms as

$$R_{t-1} = \eta^{-1} \int \pi_{i,t} e dF(b) = \eta^{-1} (1 - \tilde{\tau})(1 - \alpha \Theta^{-1}) \alpha Y_t / N_t. \quad (24)$$

Here, $Y_t / N_t$, given in (21), is positively affected by

$$G_t = [1 - Q(q)] \tilde{\tau} N_t \int \pi_{i,t} dF(b) = [1 - Q(q)] \tilde{\tau} (1 - \alpha \Theta^{-1}) \alpha Y_t. \quad (25)$$

This induces essentially the same trade-off as Barro (1990). Thus, in the absence of tax evasion ($\Theta = 1$), we obtain the same result as Barro (1990).

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15See Appendix D for the derivation of (24) and (25).
The second part of Proposition 3 states that with underdeclaration of profit, the growth-maximizing announced CIT rate is higher than the growth-maximizing effective CIT rate, $\tau_{GM} > \tilde{\tau}_{GM}$. Moreover, both are higher than the output elasticity of public services, $\alpha$. That is, tax evasion by firms increases the growth-maximizing tax rates, $\tau_{GM}$ and $\tilde{\tau}_{GM}$. $\tau_{GM} > \tilde{\tau}_{GM}$ stems simply from the evasion of CIT by firms. Then, we consider the intuition behind the result of $\tilde{\tau}_{GM} > \alpha$ here.

As we have seen after Proposition 1, in response to a tax hike, each intermediate good firm increases tax evasion. This secures the expected operating after-tax profit, $\pi_{e(i,t)}$, and private investment. Because the true operating profit, $\pi_{i,t}$, is also secured, the tax base for public service provision ($N_t \int \pi_{i,t} dF(b)$) is maintained. These are caused by the effect of CIT evasion associated with profit maximization in an imperfectly competitive product market, which are captured by the term $1 - \alpha \Theta - 1$ in (24) and (25), where $\Theta = \Gamma(\tau)$ and $\Gamma'(\tau) > 0$. Hereafter, we call this simply the effect of CIT evasion. The effect of CIT evasion mitigates the negative effect of CIT on growth and increases the benefit of raising the CIT rate for the provision of productive public services.

Our result ($\tilde{\tau}_{GM} > \alpha$) is different from Kafkalas et al. (2014), who advocate that Barro’s rule holds ($\tilde{\tau}_{GM} = \alpha$) even in the economy with tax evasion. While tax evasion does not affect firms’ decision-making in Kafkalas et al (2014), our model includes the effect of CIT evasion as mentioned above, which causes $\tilde{\tau}_{GM} > \alpha$. In Chen’s (2003) model, although the growth-maximizing announced tax rate is higher than the output elasticity of public service, it is ambiguous whether so is the growth-maximizing effective tax rate. This is because the household-firms need to pay some tangible cost to evade tax and it is included in the definition of the effective tax rate of that model. Therefore, Chen’s (2003) result is not necessarily suitable for a rigorous comparison with those of Kafkalas et al. (2014) and this paper.

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16 Indeed, the disparity between the announced and effective CIT rates is enlarged by the strong tax evasion due to the tax hike. At a glance, this is likely to damage public service provision. However, at the same time, such an increase in tax evasion holds the tax base, $N_t \int \pi_{i,t} dF(b) = (1 - \alpha \Theta - 1) \alpha Y_t$.

17 Except for the effect of CIT evasion, there exist general equilibrium effects in our model. An increase in $\tilde{\pi}_{i,t}$ reduces the demands of intermediate good in the final good sector, $x(\tilde{\pi}_{i,t})$. This exerts the following opposite effects on $R_{t-1}$. On the one hand, a fall in $x(\tilde{\pi}_{i,t})$ reduces $\pi_{e(i,t)}$ and lowers $R_{t-1}$. On the other hand, a fall in $x(\tilde{\pi}_{i,t})$ causes a labor shift from the intermediate to final good sector, which increases final output and raises $R_{t-1}$. However, we find that these general equilibrium effects are not the most important ones, as follows. By taking the logarithm of the growth rate (23) and differentiating it with respect to the effective tax rate $\tilde{\tau}$, we find that the sum of the opposing general equilibrium effects is negative; the negative effect of a fall in $x(\tilde{\pi}_{i,t})$ dominates the positive effect of a rise in the final output by shifting labor to the final good sector. Therefore, the primary force of raising the growth-maximizing effective tax rate is the direct effect, which we mentioned in the text.

18 In Chen’s (2003) model, although the growth-maximizing announced tax rate is higher than the output elasticity of public service, it is ambiguous whether so is the growth-maximizing effective tax rate. This is because the household-firms need to pay some tangible cost to evade tax and it is included in the definition of the effective tax rate of that model. Therefore, Chen’s (2003) result is not necessarily suitable for a rigorous comparison with those of Kafkalas et al. (2014) and this paper.
4.2 Welfare-maximizing CIT Rate

Next, we analyze the welfare-maximizing CIT rate. Using the balanced growth rate, $\hat{g}$, the equilibrium path of consumption is given by $C_t = \hat{g}tC_0 = \hat{g}t\hat{z}$. Substituting it into the lifetime utility function of the representative household, (14), we obtain

$$U_0 = \frac{\hat{z}^{1-\sigma}}{(1 - \sigma)[1 - (1 + \rho)^{-1}\hat{g}^{1-\sigma}]},$$

where $1 > (1 + \rho)^{-1}\hat{g}^{1-\sigma}$ holds by the TVC. The social welfare is determined by the initial level of consumption and the long-run growth rate, both of which depend on the announced CIT rate, $\tau$. Let us denote the welfare-maximizing announced CIT rate by $\tau_{WM}$.

Proposition 4.

1. When each firm declares its true operating profit, $1 - q(1 + s) \leq 0$, $\tau_{WM} > \tau_{GM} = \alpha$ holds. Therefore, the welfare-maximizing announced CIT rate, $\tau_{WM}$, is higher than the growth-maximizing announced CIT rate, $\tau_{GM}$.

2. Suppose that $1 - q(1 + s) > 0$ and $q = 0$. Then, each firm understates its operating profit and a marginal increase in the announced CIT rate at the growth-maximizing CIT rate improves social welfare.

Proof: See Appendix G.

Proposition 4 shows that the welfare-maximizing CIT is higher than the growth-maximizing one. On the one hand, a marginal increase in CIT from $\tau_{GM}$ does not affect the growth rate, because the first order effect vanishes at $\tau = \tau_{GM}$. On the other hand, it increases current consumption, because labor income, which is exempt from taxation, is raised by increases in productive public services (see Appendix G). Therefore, the welfare-maximizing CIT rate is higher than the growth-maximizing one.

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19 The economy jumps onto the balanced growth path in the initial period, as explained in section 3. Remember that we assumed $N_0 = 1$.

20 In the case of underdeclaration of profits by firms ($1 - q(1 + s) > 0$), we cannot reach such a statement when $q \neq 0$. However, in Section 5, we conjecture that the statement may be true for the other values of $q$ because we could not find any counterexample.
5 Quantitative Analysis

Propositions 3 and 4 summarize that the following three effects make $\tau^{WM}$ larger than the output elasticity of public services with tax evasion by firms $(1 - q(1 + s) > 0)$.

The first effect is represented by $\tau^{WM} > \tau^{GM}$; the welfare-maximizing announced tax rate is higher than the growth-maximizing one. This is because the tax base is CIT, and wage income is exempt from taxation as we have seen in Proposition 4. We call this the tax base effect. The second effect is represented by $\tau^{GM} > \tilde{\tau}^{GM}$. This stems from the degree of tax evasion by firms, $(\tau > \tilde{\tau})$. We call this the difference in tax rates. The third effect is represented by $\tilde{\tau}^{GM} > \alpha$. This is attributable to the effect of CIT evasion, as mentioned in subsection 4.1.

The objective of this section is to investigate how high the welfare maximizing CIT rate $\tau^{WM}$ is and which of the three effects contributes most to it. To solve these quantitative problems, we extend the above base model in this section.

5.1 Extended Model

We start with a small revision of the model, because the problematic restriction on the parameter lies in the previous form of production technology. The output-elasticity of public services $\alpha$ must be set at the price elasticity of intermediate good, $1/(1 - \alpha)$. To resolve this, we change production technology (1) into

$$Y_t = AL^{1-\alpha} \int_0^{N_t} (a(G_t, N_t) x_{i,t})^\alpha di,$$

where

$$a(G_t, N_t) = G_t^{\epsilon} N_t^{1-\epsilon}, \quad 0 < \epsilon < 1.$$

The composite externality (28) represents a combination of the role of knowledge spillover, as in Benassy (1998), together with productive public services, as in Barro (1990).

We adopt this form of the composite externality for the following two reasons. First, as will become evident below and as stated by Chatterjee and Turnovsky (2012), it helps provide a plausible calibration of the aggregate economy, something that is generically problematic in the conventional one-sector endogenous growth model.\textsuperscript{21} Under (27) and (28), the output-elasticity of

\textsuperscript{21}Chatterjee and Turnovsky (2012) consider the composite externality from physical capital, as in Romer (1986)
public services is \( \beta \equiv \alpha \epsilon (< \alpha) \), which differentiates \( \alpha \) from the output-elasticity of public services. Second, as the following Remark shows, although we take an additional externality (the spillover of knowledge) into account, the basic property of the benchmark model is maintained.

**Remark.** *The qualitative results do not change in the extended model. By replacing \( \alpha \) with \( \beta \), the same results as Proposition 3 and Proposition 4 hold.*

Proof: See Appendix H.

### 5.2 Calibration

To conduct numerical exercises, we set the baseline parameter value as in Table 1. Appendix I provides details of our calibration.

*Insert Table 1 here.*

The distribution of firms’ productivity is determined to make the curvature of the distribution function of firm sizes in the model equal that of the Pareto distribution estimated with US data by Axtell (2001). This requires \( \psi = 1.059 \). We choose \( \alpha = 0.8620 \) so that the markup rate of firms \( \mu = \Gamma(\tau)/\alpha - 1 \) takes 20\%, which is a standard value of markup rate of firms (e.g., Rotemberg and Woodford (1999)).

We set the parameter to measure the knowledge spillover, \( \epsilon \), to 0.1160, so that the output elasticity of public services, \( \beta \), equals 0.1. Although the estimates of the elasticity vary among some empirical studies, 0.1 is one of the reasonable values of \( \beta \). 22

In this subsection, we provide the baseline value of announced CIT rate, \( \tau = 0.2706 \), to determine the baseline balanced growth rate because the balanced growth rate depends on \( \tau \) in this model economy. This value of \( \tau (=0.2706) \) is the average CIT rate in OECD countries from

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22From an empirical point of view, the output elasticity of infrastructure (or productive public services) has been estimated and examined using data from many countries. Recent empirical studies (e.g., Röller and Wavorman, 2001; Shioji, 2001; Esfahani and Ramírez, 2003; Kamps, 2006; Bom and Ligthart, 2014) indicate that the output elasticity of infrastructure (or productive public services) lies in the range of 0.1–0.2, on average. More recent studies by Bom and Ligthart, 2014 and Caldeón et al. (2015) indicate that the output elasticity of infrastructure is around 0.1.
2000 to 2017. 23 We set penalty tax rate, \( s \), to 0.5, according to Fullerton and Karayannis (1994), who take this value as a normal rate in the US.

We set the benchmark value of audit rate, \( q \), to 0.096. To obtain this value, we utilize the statistics provided by the Internal Revenue Service. Each IRS Data Book between 2000 and 2017 provides the actual ratio of the examined corporations to all corporations under a classification by firm size. 24 We choose the class of the smallest size of the large corporations. 25 This is because such a class occupies a significantly large part (about 60%) of the large corporations, which are corporations above a certain business size. Besides, the ratios of the audited corporations vary greatly across the classes and so does the number of corporations in the classes. This means that taking the average audit rate among the classes is unreasonable. Therefore, we set the audit rate in such a manner. Later, we confirm the robustness of our results for a range of \( q \), including \( q = 0.089 \), the value adopted by Fullerton and Karayannis (1994).

We specify the functional form of \( Q(\cdot) \) by \( Q(q) = kq \), where \( k \) is a constant. We set \( k = 0.167 \) to make \( Q(q) \) equal the ratio of inspection cost to the CIT revenue in the US on average from 2000 to 2017. 26 However, not only the value but also the functional form does not change our results because the growth rate of our model is independent of them: see (23).

We set \( \sigma = 1.5 \) and \( \rho = 0.0204 \) according to Jones et al. (1993). These are the standard values used in quantifying growth models. We set \( L = 1 \) for normalization. Finally, we choose the scale parameter, \( A \), and the cost of developing one intermediate good, \( \eta \) such that the balanced growth rate equals 2%. 27

The above benchmark parameter set realizes the case of tax evasion, \( q(1 + s) − 1 < 0 \).

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23 The value of \( \tau \) does not strongly affect the levels of growth- and welfare-maximizing CIT rates. Therefore, the choice of the baseline value of \( \tau \) makes little difference in the quantitative results of our numerical exercises.

24 We can collect the data from the archive of the prior year IRS Data Books in the Internal Revenue Service’s website. These are downloadable at https://www.irs.gov/statistics/soi-tax-stats-prior-year-irs-data-books.

25 According to the definition by the Internal Revenue Service, large corporations are those with assets greater than 10 million dollars. The upper bound of the asset size of the smallest class is 50 million dollars.

26 The source is IRS Data Book between 2000 and 2017.

27 We do not eliminate the scale effect explicitly in this calibration because we obtain the same results in Section 5.3 if we do. Following the method in pp. 302 of Barro and Sala-i-Martin (2004), we can eliminate the scale effect by imposing the relation \( \eta = B\alpha^{\frac{1}{\alpha-1}}L^{\frac{1}{\alpha-1}} \), where \( B > 0 \). Given an arbitrary positive number \( B \), we set \( \eta \) satisfying the above equation. Given such \( \eta \), we set the scale parameter \( A \) to fix the growth rate to 2%. This modification leads to the same quantities in Table 3. We can also adjust the value of \( \eta \) by varying \( B \).
5.3 Results

Main Results

Insert Table 2 here.

Table 2 provides the welfare-maximizing announced CIT rate, $\tau^{WM}$, the growth-maximizing announced rate, $\tau^{GM}$, and the growth-maximizing effective CIT rate, $\tilde{\tau}^{GM}$ for the benchmark case. We find that $\tau^{WM} = 0.4025$, $\tau^{GM} = 0.3921$, and $\tilde{\tau}^{GM} = 0.3133$. The value of the optimal CIT rate, 0.4025, is close to the estimated value of 42% by Aghion et al. (2016). As Propositions 3 and 4 indicate, $\tau^{WM}$, $\tau^{GM}$, and $\tilde{\tau}^{GM}$ are all larger than $\beta (= 0.1)$. Importantly, the difference between $\tau^{WM}$ and $\beta$ is 0.3025, which is quite large.

The fourth, fifth, and sixth rows of Table 2 provide a decomposition of the total effect, $\tau^{WM} - \beta$. The effect of CIT evasion, $\tilde{\tau}^{GM} - \beta$, amounts to 0.2133 and is largest of the three effects. Thus, the effect of CIT evasion is the primary source of the high optimal CIT rate quantitatively.

Robustness

The effect of CIT evasion depends on the markup rate $\mu (= \Gamma(\tau)/\alpha - 1)$ because it is related to their market power. Therefore, we calculate the optimal CIT rates for the various markup rates: see Table 3. 28

Insert Table 3 here.

Around the benchmark value (e.g., the case of markup rate = 0.1, ..., 0.5), we find that both $\tau^{GM}$ and $\tau^{WM}$ are much higher than $\beta$. In particular, the effect of CIT evasion, $\tilde{\tau}^{GM} - \beta$, is significantly large for the alternative markup rates. This indicates that the effect of CIT evasion is the main source of the high optimal CIT rates.

Insert Table 4 here.

Unsurprisingly, the level of the optimal CIT rate substantially changes according to the output elasticity of public services. However, we find that the impact of tax evasion remains relatively

28 We choose the values of $\alpha$ so that the markup rates take the values listed in Table 3. See also Appendix I for details of the calibration.
strong for the alternative values of \( \beta \), the output elasticity of public services. In Table 4, we provide the ratio of the welfare-maximizing CIT rate to the output elasticity of public services, \( \tau_{WM} / \beta \). Remember that the criterion in the comparison with the Barro rule is \( \beta \). Thus, for example, although \( \tau_{WM} \approx 0.1 \) in the case of \( \beta = 0.025 \), we can interpret this rate as a relatively large value. Because the ratios in Table 4 show that the optimal CIT rates are much higher than \( \beta \), we can confirm the effect of tax evasion on the optimal CIT rate.

Next, we calculate the share of the effect of CIT evasion in the total effect, \( \frac{\beta_{GM} - \beta}{\tau_{WM} - \beta} \), for the alternative values of \( \beta \). In the benchmark case, the share is 70%. For any case, the effect of CIT evasion occupies more than half of the total effect. This ensures the robustness of the relative importance of the effect of CIT evasion.

Finally, we conduct a robustness check of the main result with respect to audit rate \( q \).

Insert Figure 1 here.

Figure 1 illustrates the result. Indeed, the level of the optimal CIT rate changes according to \( q \). However, for the various alternative values of \( q \), we can confirm that the effect of CIT evasion is the dominant factor of the high optimal CIT rate relative to the output elasticity of public services.

6 Conclusion

This study investigates the optimal CIT in an endogenous growth model with productive public services, incorporating tax evasion by monopolistically competitive firms of intermediate goods. We show that the growth- and welfare-maximizing CIT rates are higher than the output elasticity of productive public services. Thus, in view of tax evasion by firms, CIT should be higher than the output elasticity of public services. This is mainly because the effect of CIT evasion mitigates the negative effect of CIT on growth and increases the benefit of raising the CIT rate for the provision of productive public services.

29 As we mentioned in the calibration section, the estimations of \( \beta \) lie in the neighborhood of 0.10 in existing empirical studies. Although the quantitative performance of the model heavily depends on \( \beta \), the range considered here is sufficiently wide to keep the model quantitatively plausible.

30 In particular, consider \( q = 0.089 \). This is the value chosen in Fullerton and Karayannis (1994). Table 5 shows that the results do not change for the benchmark parameter (other than \( q \)).

20
Under the plausible parameter values, our numerical exercises show that the effect of CIT evasion is significantly large and the optimal level of the CIT rate is much higher than the output elasticity of public services.
References


Appendix

A Proof of Lemma 1

Suppose that a firm declares its profit truthfully ($\tilde{\pi}_{i,t} = \pi_{i,t}$). Then, (7) can be rewritten as $\pi_{i,t}^e = (1 - \tau)\pi_{i,t}$. This is maximized at $p_{i,t} = w_t/(\alpha b_i)$. From (6), we know that the firm’s true profit is given by $\pi_{i,t} = (1 - \alpha)p_{i,t}x(p_{i,t})$. Thus, if a firm setting the price at $\tilde{p}_{i,t}$ declares its operating profit truthfully, (8) holds for the firm.

B Proof of Proposition 1

Proof of (i): Since $\tilde{\pi}_{i,t} \leq \pi_{i,t}$, (7) shows that $\pi_{i,t}^e = \{1 - q(1 + s)\tau\}\pi_{i,t} - \{1 - q(1 + s)\}\tau\tilde{\pi}_{i,t}$. Thus, if $1 - q(1 + s) < 0$, $\pi_{i,t}^e$ increases with $\tilde{\pi}_{i,t}$. Therefore, firms declare the largest $\tilde{\pi}_{i,t}$, which is equal to $\pi_{i,t}$ because of $\tilde{\pi}_{i,t} \leq \pi_{i,t}$.

Since all firms declare their true profit, we have $\pi_{i,t}^e = (1 - \tau)\pi_{i,t}$. The maximization of $\pi_{i,t}^e = (1 - \tau)\pi_{i,t}$ yields $\tilde{p}_{i,t} = w_t/(\alpha b_i)$. We can obtain the true profit by substituting $\tilde{p}_{i,t}$ into (6). The expected after-tax profit follows from $\pi_{i,t}^e = (1 - \tau)\pi_{i,t}$. Since $\tilde{\pi}_{i,t} = \pi_{i,t}$ holds in (9), we have $\tilde{\tau} = \tau$.

Next, if $1 - q(1 + s) = 0$, we obtain $\pi_{i,t}^e = (1 - \tau)\pi_{i,t}$. In this case, the expected after-tax profit is indifferent between whether a firm declares its profit truthfully. The maximization of $\pi_{i,t}^e = (1 - \tau)\pi_{i,t}$ yields the same results as in the case of truth-telling firms, $\tilde{\pi}_{i,t} = \pi_{i,t}$, $\tilde{p}_{i,t} = w_t/(\alpha b_i)$, and $\tilde{\tau} = \tau$.

Proof of (ii): We first prove the following lemma.

Lemma 2: Suppose $1 - q(1 + s) > 0$. If a firm sets the price at $\tilde{p}_{i,t}$, then the declared profit of the firm satisfies (8).

Proof of Lemma 2: To prove this lemma, we consider (a) firms that declare their operating profits truthfully and (b) firms that declare their operating profits dishonestly.

(a) Consider firms that declare their operating profits truthfully. Lemma 1 shows that the declared profit of these firms satisfies (8).

(b) We next consider the following two types of dishonest firms: those that do not declare (8) and those that declare (8).

We begin with a firm of the first type that declares $\tilde{\pi}_{i,t}$, which is not equal to (8). From
Assumption (i), this firm is audited with probability one, $\bar{q} = 1$. Thus, the expected after-tax profit of this firm is given by

$$\tilde{\pi}_{i,t} = \pi_{i,t} - \tau \tilde{\pi}_{i,t} - 1 \times (1 + s)\tau \left( \pi_{i,t} - \tilde{\pi}_{i,t} \right).$$  \hspace{1cm} \text{(B.1)}$$

We next consider a dishonest firm that declares (8). From Assumption (ii), this firm is audited with a probability lower than one, $\bar{q} = q \in [0, 1)$. The expected after-tax profit of this type of dishonest firms is given by

$$\tilde{\pi}_{i,t} = \pi_{i,t} - \tau \tilde{\pi}_{i,t} - q(1 + s)\tau (\pi_{i,t} - \tilde{\pi}_{i,t}).$$  \hspace{1cm} \text{(B.2)}$$

Suppose $\tilde{\pi}_{i,t} < \tilde{\pi}_{i,t}$ holds. This implies that a dishonest firm declares $\tilde{\pi}_{i,t}$, which is not equal to (8). Using (B.1) and (B.2), we rearrange $\tilde{\pi}_{i,t} < \tilde{\pi}_{i,t}$ as follows:

$$\pi_{i,t} - \tau \tilde{\pi}_{i,t} - q(1 + s)\tau (\pi_{i,t} - \tilde{\pi}_{i,t}) < \pi_{i,t} - \tau \tilde{\pi}_{i,t} - (1 + s)\tau (\pi_{i,t} - \tilde{\pi}_{i,t}) \Leftrightarrow (1 + s)(1 - q)\pi_{i,t} - [1 - q(1 + s)]\tilde{\pi}_{i,t} < s \tilde{\pi}_{i,t},$$

$$\Rightarrow (1 + s)(1 - q)\pi_{i,t} - [1 - q(1 + s)]\tilde{\pi}_{i,t} < s \pi_{i,t},$$

$$\Leftrightarrow [1 - q(1 + s)]\pi_{i,t} < [1 - q(1 + s)]\tilde{\pi}_{i,t}.$$  \hspace{1cm} \text{The third line uses the fact $\tilde{\pi}_{i,t} \leq \pi_{i,t}$.} If $1 - q(1 + s) > 0$, the inequality in the last line indicates $\pi < \tilde{\pi}_{i,t}$, which contradicts $\tilde{\pi}_{i,t} \leq \pi_{i,t}$. Thus, dishonest firms declare (8). Lemma 2 is proved.

From Lemma 2, we have that $\pi_{i,t} = (1 - \tilde{\tau})\pi_{i,t} = [1 - q(1 + s)\tau]\pi_{i,t} - [1 - q(1 + s)]\tau \tilde{\pi}_{i,t}$, where $\pi_{i,t}$ is given by (8). Firms maximize this $\pi_{i,t}$ by choosing $p_{i,t}$. The first-order condition is given by

$$\frac{\partial \pi_{i,t}}{\partial p_{i,t}} = L_{Y,t}(\alpha A)\frac{1}{1 - \alpha} G_{t}^{\alpha} \frac{\tau}{p_{i,t} - 1} \left[ \frac{1 - q(1 + s)\tau}{(1 - \alpha)b_i} \left( -\alpha b_i + \frac{w_t}{p_{i,t}} \right) + [1 - q(1 + s)]\tau \alpha \right] = 0$$

$$\text{(B.3)}$$

Since $1 - q(1 + s) > 0$ holds, we have that $0 < (1 - \alpha)[1 - q(1 + s)]\tau < [1 - q(1 + s)]\tau < 1 - q(1 + s)\tau$, which ensures the second-order condition. Solving this condition yields (11).

Since $1 - q(1 + s) > 0$ and $(1 - \alpha)[1 - q(1 + s)]\tau < 1 - q(1 + s)\tau$, we have $\Gamma(\tau) > 1$, and
hence, \( \tilde{p}_{i,t} > w_t / (\alpha b_t) \). If we substitute (11) into (6), we obtain (12). From \( \pi^{c}_{i,t} = (1 - \tilde{\tau}) \pi_{i,t} \) and (12), we obtain (13).

Here, let us define

\[
\gamma(\tau) \equiv \Gamma(\tau)^{-1} = 1 - \frac{(1 - \alpha)[1 - q(1 + s)]\tau}{1 - q(1 + s)\tau} (< 1). \tag{B.4}
\]

Substituting (8) and (12) into (9), we have

\[
\frac{\dot{\tilde{\tau}}}{\tau} = [1 - q(1 + s)] \frac{1 - \alpha}{1 - \alpha \gamma(\tau)} + q(1 + s) \tag{B.5}
\]

First, we can easily show that \( \tilde{\tau} < \tau \) because of \( [1 - q(1 + s)] \frac{1 - \alpha}{1 - \alpha \gamma(\tau)} + q(1 + s) - 1 = [1 - q(1 + s)] \left[ \frac{1 - \alpha}{1 - \alpha \gamma(\tau)} - 1 \right] < 0 \), where \( \gamma(\tau) < 1 \) and \( 1 - q(1 + s) > 0 \).

Second, from the definition of \( \gamma(\tau) \) and (B.5), \( \lim_{\tau \to 0} \dot{\tilde{\tau}} = 0 \) and \( \dot{\tilde{\tau}}|_{\tau = 1} = \frac{1 + \alpha q(1 + s)}{1 + \alpha} \), and therefore, we have \( \tilde{\tau} \in \left(0, \frac{1 + \alpha q(1 + s)}{1 + \alpha}\right] \).

C Relationship between \( \tilde{\tau} \) and \( \tau \)

From (B.4), we obtain

\[
\gamma'(\tau) = -\frac{(1 - \alpha)[1 - q(1 + s)]}{[1 - q(1 + s)\tau]^2} < 0, \tag{C.1}
\]

\[
\frac{d\gamma(\tau)}{dq} = \frac{(1 - \alpha)(1 - \tau)\tau(1 + s)}{[1 - q(1 + s)\tau]^2} > 0, \tag{C.2}
\]

or both \( \Gamma'(\tau) > 0 \) and \( d\Gamma(\tau)/dq < 0 \). From (B.5) and (C.1), we obtain \( d(\tau/\tilde{\tau})/d\tau > 0 \). From (B.5), (C.2), and \( \gamma(\tau) < 1 \), we obtain \( d(\tau/\tilde{\tau})/d(\tau/q(1 + s)) < 0 \).

Finally, we prove \( d\tilde{\tau}/d\tau > 0 \). From (B.5),

\[
\frac{d\tilde{\tau}}{d\tau} = [1 - q(1 + s)](1 - \alpha) \frac{1 - \alpha \gamma(\tau) + \alpha \tau \gamma'(\tau)}{(1 - \alpha \gamma(\tau))^2} + q(1 + s) \tag{C.3}
\]

Thus, we have \( \frac{d\tilde{\tau}}{d\tau} > 0 \), if and only if

\[
1 - \alpha \gamma(\tau) + \alpha \tau \gamma'(\tau) > \frac{(1 - \alpha \gamma(\tau))^2 q(1 + s)}{[1 - q(1 + s)](1 - \alpha)} \tag{C.4}
\]

From \( \gamma'(\tau) < 0 \) and \( \gamma''(\tau) < 0 \), \( \frac{d}{d\tau} [1 - \alpha \gamma(\tau) + \alpha \tau \gamma'(\tau)] = -\alpha \gamma'(\tau) + \alpha \tau \gamma''(\tau) < 0 \) holds. This
indicates that the LHS of (C.4) is decreasing in $\tau$. Furthermore, the RHS of (C.4) is decreasing in $\tau$ because of $\gamma(\tau) < 1$ and $\gamma'(\tau) < 0$. Thus, $\frac{d\tilde{t}}{d\tau} > 0$ for any $\tau \in (0, 1)$ if the minimum value of the LHS of (C.4), $1 - \alpha \gamma(1) + \alpha \tau \gamma'(1)$, is larger than the maximum value of the RHS, $\frac{\left[1 - \alpha \gamma(0)\right]^2 q(1 + s)}{[1 - q(1 + s)][1 - \alpha]}$. Using $\gamma(0) = 1$, $\gamma(1) = \alpha$ and $\gamma'(1) = -\frac{1 - \alpha}{1 - q(1 + s)}$, we obtain

$$1 - \alpha \gamma(1) + \alpha \tau \gamma'(1) - \left\{ \frac{\left[1 - \alpha \gamma(0)\right]^2 q(1 + s)}{[1 - q(1 + s)][1 - \alpha]} \right\} = (1 - \alpha) \left( 1 - \alpha q(1 + s) \right) > 0, \quad (C.5)$$

and therefore, $\frac{dt}{d\tau} > 0$ for any $\tau \in (0, 1)$.

**D Derivation of equilibrium conditions**

The price level of firm $i$ and $j$ whose productivity is $b_i$ and $b_j$ is $\tilde{p}_{i,t} = \Theta \alpha b_i w_t$ and $\tilde{p}_{j,t} = \Theta \alpha b_j w_t$, where $\Theta = 1(\Gamma(\tilde{\tau}))$ for $1 - q(1 + s) \leq (>) 0$. Combining these with (3) yields $\frac{p_{i,t}}{p_{j,t}} = \left( \frac{x_{i,t}}{x_{j,t}} \right)^{1/1 - \alpha} = \frac{b_j}{b_i}$. Thus, we have $x_{i,t} = \left( \frac{b_j}{b_i} \right)^{1/1 - \alpha} x_{j,t}$. This together with (1), (2), and (3) rewrites $\tilde{p}_{i,t} = \Theta \alpha b_i w_t$ into

$$x_{i,t} = \frac{\alpha^2 L_{Y,t}}{(1 - \alpha) \Theta b_i^{1 - \alpha} \int b^{1 - \alpha} dF(b) N_t}. \quad (D.1)$$

From (19) and (D.1), we obtain labor employed in the final good sector as follows:

$$L_{Y,t} = \frac{(1 - \alpha) \Theta}{(1 - \alpha) \Theta + \alpha^2 L}. \quad (D.2)$$

Substituting (D.2) into (D.1) leads to

$$x(\tilde{p}_{i,t}) = \left[ \frac{\alpha^2}{(1 - \alpha) \Theta + \alpha^2} \right] \left( \frac{b_j^{1 - \alpha}}{b_i^{1 - \alpha}} \right) \frac{L}{\int b^{1 - \alpha} dF(b) N_t}. \quad (D.3)$$

From (1) and (3), we obtain $\int_0^{N_t} p_{i,t} x_{i,t} di = \alpha Y_t$. Combining $\int_0^{N_t} p_{i,t} x_{i,t} di = \alpha Y_t$ with $\pi_{i,t} = (1 - \tilde{\tau})(1 - \alpha \Theta^{-1}) \hat{p}_{i,t} x(\hat{p}_{i,t})$, we obtain the total expected after-tax operating profit, $N_t \int \pi_{i,t} dF(b) = (1 - \tilde{\tau})(1 - \alpha \Theta^{-1}) \alpha Y_t$. By combining the total expected after-tax operating profit with (5), we obtain the gross interest rate

$$R_{t-1} = \frac{(1 - \tilde{\tau})(1 - \alpha \Theta^{-1}) \alpha Y_t}{\eta N_t}. \quad (D.4)$$
Combining \( \pi_{i,t} = (1 - \alpha \Theta^{-1}) \tilde{p}_{i,t} \epsilon x(\tilde{p}_{i,t}) \) with \( \int_0^{N_t} p_i x_i dt = \alpha Y_t \), the budget constraint of the government (17) is rewritten into \( G_t + M_t = \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha Y_t \). Dividing both sides of the final good market clearing condition, \( Y_t = C_t + \eta N_{t+1} + G_t + M_t \), by \( N_t \) and using \( G_t + M_t = \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha Y_t \) yield

\[
\frac{N_{t+1}}{N_t} = \frac{1}{\eta} \left\{ \left[ 1 - \tilde{\rho} (1 - \alpha \Theta^{-1}) \alpha \right] \frac{Y_t}{N_t} - \frac{C_t}{N_t} \right\}, \tag{D.5}
\]

and substituting (D.4) into (15), we obtain

\[
\frac{C_{t+1}}{C_t} = \left[ \frac{(1 - \tilde{\rho}) (1 - \alpha \Theta^{-1}) \alpha Y_{t+1}}{(1 + \rho) \eta \frac{N_{t+1}}{N_t}} \right]^{1/\sigma}. \tag{D.6}
\]

Substituting \( \pi_{i,t} = (1 - \alpha \Theta^{-1}) \tilde{p}_{i,t} x(\tilde{p}_{i,t}) \) into (18) reduces to \( G_t = [1 - \hat{Q}(q)] \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha Y_t \). Combining it with (1), (D.2), and (D.3), we obtain \( \frac{\bar{X}}{N_t} = \Omega(\tilde{\rho}) \). Substituting \( \frac{\bar{X}}{N_t} = \Omega(\tilde{\rho}) \) into (D.5) and (D.6) and dividing (D.6) by (D.5) leads to (20).

## E Proof of Proposition 2

Let us define the right-hand side (RHS) of (20) as \( \vartheta(z_t) = \frac{\eta^{1/\sigma} \{ [1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha \Omega(\tilde{\rho}) / (1 + \rho)]^{1/\sigma} [1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] \Omega(\tilde{\rho}) \}}{[1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] \Omega(\tilde{\rho}) - z_t} \). Here, note that the denominator of \( \vartheta(z_t) \): \( [1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] \Omega(\tilde{\rho}) - z_t \) must be positive, that is, \( z_t < \bar{z} \equiv [1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] \Omega(\tilde{\rho}) \), otherwise \( N_t \) eventually equals to zero from (D.5):

\[
\frac{N_{t+1}}{N_t} = \frac{1}{\eta} \left\{ [1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] \Omega(\tilde{\rho}) - z_t \right\}.
\]

When \( N_t = 0 \), both output and consumption equal to zero, which violates the first order condition of the representative household.

The properties of \( \vartheta(z_t) \) for \( z_t \in [0, \bar{z}] \) are as follows:

\[
\vartheta(0) = 0, \quad \lim_{z_t \to 2} \vartheta(z_t) = +\infty,
\]

\[
\vartheta'(z_t) = \eta^{1/\sigma} \frac{\{ [1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha \Omega(\tilde{\rho}) / (1 + \rho)]^{1/\sigma} [1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] \Omega(\tilde{\rho}) \}}{[1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] \Omega(\tilde{\rho}) - z_t} > 0,
\]

\[
\vartheta''(0) = \eta \left[ \frac{[1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] (1 + \rho)^{-1} \Omega^{-1}(\tilde{\rho}) \]^{1/\sigma} \right], \quad \lim_{z_t \to 2} \vartheta''(z_t) = +\infty
\]

\[
\vartheta''(z_t) = 2\eta^{1/\sigma} \frac{\{ [1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha \Omega(\tilde{\rho}) / (1 + \rho)]^{1/\sigma} [1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] \Omega(\tilde{\rho}) \}}{[1 - \tilde{\rho}(1 - \alpha \Theta^{-1}) \alpha] \Omega(\tilde{\rho}) - z_t} > 0. \tag{E.1}
\]

(E.1) indicates that \( \vartheta(z_t) \) is monotonically increasing and convex function of \( z_t \) and takes zero when \( z_t = 0 \).
On the other hand, the left-hand side (LHS) of (20) represents 45° line. Thus, we find that a unique steady state \( \tilde{\tau} \in (0, \bar{\tau}) \), which is unstable exists if and only if \( \vartheta'(0) < 1 \) holds. From (D.5), (D.6) and \( Y_t/N_t = \Omega(\tau) \), \( C_t \), \( N_t \), and \( Y_t \) grow at the same constant rate, \( C_{t+1}/C_t = N_{t+1}/N_t = Y_{t+1}/Y_t = \hat{g} \) in the steady state.

The rest of this appendix shows that the TVC (16) ensures \( \vartheta'(0) < 1 \). (23) and the asset market clearing condition, \( W_t = \eta N_{t+1} \) together with the assumption \( N_0 = 1 \) transform the TVC (16) into \( \lim_{t \to \infty} \frac{\hat{g}^{(1-\sigma)}}{(1+\rho)^t} = 0 \). To satisfy the TVC, \( 1 > (1 + \rho)^{-1} \hat{g}^{1-\sigma} \) must holds. \( 1 > (1 + \rho)^{-1} \hat{g}^{1-\sigma} \) and (23) together with \( \frac{(1-\bar{\tau})(1-\alpha \Theta^{-1})}{\alpha_{\tau\alpha}} < 1 \) lead to \( \vartheta'(0) < 1 \).

**F Proof of Proposition 3**

**F.1 Proof of 1**

In this case, since each firm does not evade CIT, \( \Theta = 1 \) and \( \tilde{\tau} = \tau \). Then, by (21) and (23), we obtain \( \tilde{\tau}^{GM} = \tau^{GM} = \alpha \) immediately.

**F.2 Proof of 2**

In the beginning, note that the decision of optimal announced CIT rate is equivalent to that of the optimal effective CIT rate. This is because \( \tilde{\tau} \) is the function of \( \tau \) from (B.5), and \( \tilde{\tau} \) is strictly increasing in \( \tau \), \( d\tilde{\tau}/d\tau > 0 \), for any \( \tau \in (0, 1] \), as shown in Appendix C.

Because the growth rate converges to 0 as \( \tau \) goes to 0 by the construction of the model, the growth rate is maximized at 1 or some interior point in \((0, 1)\).

First, we prove that the growth-maximizing effective CIT is higher than \( \alpha \) when it is an interior point in \((0, 1)\). From the definition of \( \Omega(\tilde{\tau}) \) and (D.6), growth maximization with respect to \( \tilde{\tau} \) is equivalent to max \( \tilde{\tau} f(\tilde{\tau}) = \ln(1 - \tilde{\tau}) \tilde{\tau}^{\frac{\alpha}{1-\alpha}} [1 - \alpha \gamma(\tau)] \frac{\gamma(\tau)}{1-\alpha + \alpha \gamma(\tau)} \) subject to (B.5): \( \frac{\tilde{\tau}}{\tau} = [1 - q(1 + s)] \frac{1-\alpha}{1-\alpha \gamma(\tau)} + q(1 + s) \). The first derivative of \( f(\tilde{\tau}) \) is

\[
\frac{df(\tilde{\tau})}{d\tilde{\tau}} = \left[ -\frac{1}{1 - \tilde{\tau}} + \frac{\alpha}{1 - \alpha \tilde{\tau}} \right] + \frac{\alpha}{1 - \alpha \tilde{\tau}} \frac{d\tilde{\tau}}{d\tau} \gamma'(\tau) \left[ \frac{1}{\gamma(\tau)} - \frac{1}{1 - \alpha \gamma(\tau)} - \frac{\alpha}{1 - \alpha + \alpha^2 \gamma(\tau)} \right].
\]

\[\text{(F.1)}\]

It is obvious that \( \Psi_1(\tilde{\tau}) = -\frac{1}{1 - \tilde{\tau}} + \frac{\alpha}{1 - \alpha \tilde{\tau}} = \frac{\tilde{\tau} - \tilde{\tau}}{(1 - \tilde{\tau})(1 - \alpha)} \geq 0 \) for \( \tilde{\tau} \leq \alpha \). Next, we show that the
sign of $\Psi_2(\tau)$ is negative for $\hat{\tau} \leq \alpha$.

$$\text{sign}\Psi_2(\tau) = [1 - \alpha \gamma(\tau)][1 - \alpha + \alpha^2 \gamma(\tau)] - \gamma(\tau)[1 - \alpha + \alpha^2 \gamma(\tau)] - \alpha \gamma(\tau)[1 - \alpha \gamma(\tau)]$$

$$= -\alpha^3 \gamma(\tau)^2 - (1 - \alpha)[(1 + 2\alpha)\gamma(\tau) - 1]$$  \hspace{1cm} (F.2)

Here, $\text{sign}\Psi_2(\tau) < 0$ for $\gamma(\tau) > \frac{1}{1+2\alpha}$. Furthermore, (B.4) and (B.5) indicate that $\gamma(\tau)$ is increasing in $q(1+s)$ and when $q(1+s) = 0$, $\gamma(\tau) = 1 - (1-\alpha)\tau$, $\tau = \frac{\hat{\tau}}{1-\alpha \tau}$ and $\gamma(\frac{\hat{\tau}}{1-\alpha \tau}) = \frac{1-\hat{\tau}}{1-\alpha \tau}$ hold. From $\frac{1-\hat{\tau}}{1-\alpha \tau} - \frac{1}{1+2\alpha} = \frac{\alpha - \hat{\tau} + \alpha(1-\hat{\tau})}{(1-\alpha \tau)(1+2\alpha)} > 0$, we obtain $\gamma(\frac{\hat{\tau}}{1-\alpha \tau}) = \frac{1-\hat{\tau}}{1-\alpha \tau} > \frac{1}{1+2\alpha}$ for $\hat{\tau} \leq \alpha$. Thus, $\text{sign}\Psi_2(\tau) < 0$ for $\hat{\tau} \leq \alpha$. Combining $\Psi_1(\hat{\tau}) \geq 0$ and $\Psi_2(\hat{\tau}) < 0$ for $\hat{\tau} \leq \alpha$ with $\gamma'(\tau) < 0$ ((B.5)) and $\frac{dt}{d\tau} > 0$, we obtain $f'(\hat{\tau}) > 0$ for $\hat{\tau} \leq \alpha$. From the discussion so far, we find that $\hat{\tau}^{GM} > \alpha$ holds.

Next, we consider the case of the corner solution of growth-maximization: $\tau^{GM} = 1$. Assuming $q(1+s) > \frac{\alpha(1+\alpha)-1}{\alpha}$ additionally, we ensure that $\hat{\tau}^{GM} > \alpha$ because $\hat{\tau}|_{\tau=1} = \frac{1+\alpha q(1+s)}{1+\alpha}$.

\section{Proof and intuition of Proposition 4}

(i) Proof of 1

The maximization condition of social welfare is $\frac{\partial U}{\partial \tau} = 0$. By (26), this is equivalent to

$$[1 - (1 + \rho)^{-1} \hat{g}^{1-\sigma}] \frac{\partial \hat{z}}{\partial \tau} + (1 + \rho)^{-1} \hat{z} \hat{g}^{-\sigma} \frac{\partial \hat{g}}{\partial \tau} = 0. \hspace{1cm} (G.1)$$

From (22) and (23), with $\Theta = 1$, we obtain $\hat{z} = \frac{1-\alpha(1-\alpha)\tau}{\eta^{-1}(1+\rho)^{-1}a(1-\alpha)(1-\tau)} \hat{g}^{\sigma} - \eta \hat{g}$. Differentiating it with respect to $\tau$ yields $\frac{\partial \hat{z}}{\partial \tau} = \frac{\eta(1+\rho)}{a(1-\alpha)} \left[ \frac{1-\alpha(1-\alpha)\tau}{(1-\tau)^2} \frac{1-\alpha(1-\alpha)\tau}{1-\tau} \hat{g}^{\sigma} + \frac{1-\alpha(1-\alpha)\tau}{1-\tau} \sigma \hat{g}^{\sigma-1} \frac{\partial \hat{g}}{\partial \tau} \right] - \eta \frac{\partial \hat{g}}{\partial \tau}$. Substituting it into (G.1) and rearranging it using $\hat{z} = \frac{1-\alpha(1-\alpha)\tau}{\eta^{-1}(1+\rho)^{-1}a(1-\alpha)(1-\tau)} \hat{g}^{\sigma} - \eta \hat{g}$, we have

$$\frac{\partial \hat{g}}{\partial \tau} = -\frac{[1 - (1 + \rho)^{-1} \hat{g}^{1-\sigma}] \hat{g}^{\sigma} \frac{1-\alpha(1-\alpha)\tau}{a(1-\alpha)(1-\tau)^2} \hat{g}^{-\sigma}}{K}, \hspace{1cm} (G.2)$$

\[31]\text{The parameter restriction } q(1+s) > \frac{\alpha(1+\alpha)-1}{\alpha}\text{ holds in a plausible parameter region because the valid range of } \alpha\text{, the output elasticity of public service is around } 0.1\text{, according to empirical studies. See the calibration in Section 5.2. The corner point } \tau = 1\text{ can be optimal because the effective CIT rate is lower than 1 when firms evade CIT. If } q(1+s) \text{ is small and } \alpha \text{ is large, the effective CIT rate becomes low, even though the growth-maximization CIT rate is high. Thus, the corner solution } \tau = 1\text{ is optimal in such a case.}
where \( K = [1 - (1 + \rho)^{-1}\hat{g}^{1-\sigma} \frac{(1+\rho)[1-\alpha(1-\alpha)\tau]}{\alpha(1-\alpha)(1-\tau)} \hat{g}^{\sigma-1} + \frac{1-\alpha(1-\alpha)}{\alpha(1-\alpha)(1-\tau)}] > 0 \). Therefore, by (G.2), we find that \( \frac{\partial \hat{g}}{\partial \tau} \bigg|_{\tau=\tau_{WM}} < 0 \). This implies \( \tau^{GM} < \tau^{WM} \) because \( \hat{g} \) is a single-peaked function of \( \tau \) (see (23) and the definition of \( \Omega(\tilde{\tau}) \) with \( \Theta = 1 \)).

(ii) Proof of 2

Since \( \text{sign}\{ \frac{dU}{d\tau} \big|_{\tau=\tau_{GM}} \} = \text{sign}\{ \frac{\partial \hat{g}}{\partial \tau} \big|_{\tau=\tau_{GM}} \} \), we show \( \frac{\partial \hat{g}}{\partial \tau} \big|_{\tau=\tau_{GM}} > 0 \) for \( q = 0 \). From (23) and (22), with \( \Theta = \Gamma(\tau) = \gamma(\tau)^{-1} \), we obtain \( \hat{z} = \frac{1-\alpha[1-\alpha\gamma(\tau)]}{\eta^{-1}(1+\rho)^{-1}(1-\tilde{\tau})[1-\alpha\gamma(\tau)]\alpha} \hat{g}^\sigma - \eta \hat{g} \). Differentiating it with respect to \( \tau \), we obtain

\[
\frac{\partial \hat{z}}{\partial \tau} \bigg|_{\tau=\tau_{GM}} = \hat{g}^\sigma \frac{(1 - \alpha\gamma(\tau))}{\eta^{-1}(1+\rho)^{-1}\alpha} \frac{d\hat{g}}{d\tau} + \alpha(1 - \tilde{\tau}) \gamma'(\tau)
\]

(G.3)

Here, let us define the numerator of (G.3) as \( J \). Through simple algebra, we have \( \gamma(\tau) = 1 - (1 - \alpha)\tau \) and \( \tilde{\tau} = \left[ \frac{1-\alpha}{1-\alpha\gamma(\tau)} \right] \tau \). Hence, utilizing these, we have

\[
J = \frac{[1 - \alpha(1 - \alpha)](1 - \alpha) \left( 1 - \frac{1}{\alpha\gamma(\tau)} \right)}{(1 - \alpha\gamma(\tau)) \left[ 1 - \alpha(1 - \alpha)(1 - \alpha) \right] [1 - \alpha + \alpha(1 + \alpha)\tau].
\]

(G.4)

Equation (G.4) ensures that \( J > 0 \) for any \( \tau \in (0, 1] \). This completes the proof.

(iii) Intuition of Proposition 4

For an intuitive interpretation, we focus on the marginal effect of raising the tax rate on the growth-maximizing rate, \( \tau_{GM} \). By (26), the lifetime utility, \( U_0 \), depends on the long-run growth rate, \( \hat{g} \), and the initial consumption, \( \hat{z} \). At \( \tau = \tau_{GM} \), the marginal effect of raising \( \tau \) on \( \hat{g} \) disappears. Then, the CIT rate affects welfare only through the effect on initial consumption: \( \text{sign} \{ \frac{dU}{d\tau} \big|_{\tau=\tau_{GM}} \} = \text{sign} \{ \frac{\partial \hat{g}}{\partial \tau} \big|_{\tau=\tau_{GM}} \} \). Note that by (22) and (D.4) in Appendix D, the initial consumption is decomposed into

\[
\hat{z} = \left[ 1 - \tilde{\tau} \left( 1 - \alpha \Theta^{-1} \right) \alpha \right] \left( \Omega(\tilde{\tau}) - \eta \left[ (1 - \tilde{\tau} \left( 1 - \alpha \Theta^{-1} \right) \alpha(1 + \rho)^{-1} \eta^{-1} \Omega(\tilde{\tau}) \right]^{1/\sigma} \right.
\]

Because the second term depends on the interest rate, this is a Slutsky decomposition of the initial consumption. Since the interest rate is also maximized at \( \tau = \tau_{GM} \), the marginal effect on the second term disappears here. Consequently, the marginal effect of raising \( \tau \) on \( \hat{z} \) at \( \tau = \tau_{GM} \)
equals that on the disposable income of household.

The disposable income of a household increases by raising \( \tau \) from \( \tau^{GM} \) marginally. Thus, \( \tau^{GM} < \tau^{WM} \). Raising \( \tau \) increases before tax income \( \Omega(\tilde{\tau}) \). While \( \Omega(\tilde{\tau}) \) includes both labor and asset income, CIT is imposed only on the source of asset income, that is, the firms’ profits. Thus, the disposable income of a household increases as a whole. It suggests that the break in the coincidence of growth- and welfare-maximization tax rates is because the tax base is CIT.\(^{32}\)

**H Proof of Remark**

The counterpart of Proposition 4 is proved the same way as Proposition 4 because our calculations do not depend on the expression of the growth rate, \( \hat{g} \), which is the unique difference between the baseline and extended model. We can prove the counterpart of Proposition 3 in the Remark by modifying Appendix F as follows.

Under the production technology of the final goods, (27) and (28), the wage rate, (2) and the price of intermediate goods, (3) are rewritten into

\[
\text{\( w_t = (1 - \alpha)L_{Y,t}^{-\alpha} \int_0^{N_t}(a(G_t, N_t)x_{t,i})^\alpha di = (1 - \alpha)\frac{Y_t}{L_{Y,t}} \) and \( p_{i,t} = \alpha AL_{Y,t}^{1-\alpha} a(G_t, N_t)^{\alpha} x_{i,t}^{\alpha-1} \), respectively. Other equations remain unchanged with the following exceptions. The definition of \( \Omega(\tilde{\tau}) \) changes into}
\]

\[
\Omega(\tilde{\tau}) \equiv A \frac{1}{1-\beta} \left\{ \left[ 1 - (1 - Q(q))\tilde{\tau} \right] (1 - \alpha \Theta^{-1}) \right\}^{\frac{\beta}{1-\beta}} \left\{ (1 - \alpha)\Theta \right\}^{\frac{1-\alpha}{1-\beta}} \left\{ \frac{L}{(1 - \alpha)\Theta + \alpha^2} \right\}^{\frac{1}{1-\beta}}
\]

\[
\times \left[ \int b t^{-\alpha} dF(b) \right]^{\frac{1}{1-\beta}} \quad (H.1)
\]

By (23) and (H.1), we have

\[
\tilde{\tau}^{GM} = \text{argmax} \left( 1 - \tilde{\tau} \right) \tilde{\tau}^{\frac{\beta}{1-\beta}} (1 - \alpha \Theta^{-1})^{\frac{1-\alpha}{1-\beta}} \tilde{\tau}^{\frac{1-\alpha}{1-\beta}} \left( \frac{\Theta^{-1} \tilde{\tau}^{1-\alpha \Theta^{-1}}}{1-\alpha + \alpha \Theta^{-1}} \right)^{\frac{1}{1-\beta}}.
\]

(F.1) changes as follows:

\[
f'(\tilde{\tau}) = \left[ \frac{-1}{1 - \tilde{\tau}} + \frac{\beta}{1 - \beta \tilde{\tau}} \right] + \frac{\alpha}{1 - \beta} \frac{d\tau}{d\tilde{\tau}} \gamma'(\tau) \Psi_2(\tau).
\]

(H.2)

Immediately, \( \tilde{\Psi}_1 = \frac{\beta - \tilde{\tau}}{(1 - \tilde{\tau})(1 - \alpha)^{\tilde{\tau}}} \geq 0 \) for \( \tilde{\tau} \leq \beta \). As in Appendix F, sign \( \Psi_2(\tau) < 0 \) for \( \tilde{\tau} \leq \beta \) because of \( \beta < \alpha \). This indicates that \( \tilde{\tau}^{GM} \geq \beta \) in the case of the interior solution. Besides, we

\(^{32}\text{When the tax system is household total income tax, as in Barro (1990) and Kafkalas et al. (2014), this effect vanishes. In fact, we find that the first term of \( \hat{\tau} \) is replaced by \( (1 - \hat{\tau})\Omega(\hat{\tau}) \), and hence, welfare-maximization is equivalent to growth-maximization; see (23).} \)
discuss the case of the corner solution \( \tau^{GM} = 1 \) in the same way as Appendix F.\(^{33}\)

## I Details of calibration

We seek to obtain quantitative implications for tax evasion in OECD countries.

- The distribution of productivity is set in such a way that the distribution of firm size is set to the Pareto distribution, which is estimated by Axtell (2001).

By (D.1) and (D.2), letting \( N_0 = 1 \), we have

\[
\frac{x_i}{b_i} = \left[ \frac{\alpha^2}{(1 - \alpha)\Gamma(\tau) + \alpha^2} \right] \frac{L}{\int b^{\frac{\psi}{\psi - 1}} dF(b)} b_i^{\alpha - 1},
\]

where \( \Gamma(\tau) = \frac{1-q(1+s)\tau}{1-q(1+s)(1-\alpha)} \). This is the size of intermediate good firms. To make its distribution a Pareto distribution, we set the distribution of \( b^{\frac{\psi}{\psi - 1}} \) to Pareto distribution with scale parameter \( \phi > 0 \) and shape parameter \( \psi > 1 \). Then, letting \( \text{SCALE} = B\phi \), because \( \int b^{\frac{\psi}{\psi - 1}} dF(b) = \frac{\psi - 1}{\psi} \phi \) in \( B \), the distribution of firm size is the Pareto distribution with scale parameter

\[
\text{SCALE} = \frac{\alpha^2}{(1 - \alpha)\Gamma(\tau) + \alpha^2} \frac{\psi - 1}{\psi} L,
\]

and shape parameter \( \psi \). Since the shape parameter is the most important factor of firm distribution, we set \( \psi = 1.059 \) according to the estimate in Axtell (2001), which uses US data.

- Next, we control the markup rate of intermediate good firms. Letting the markup rate be \( \mu \), by (11),

\[
\frac{\Gamma(\tau)}{\alpha} = 1 + \mu.
\]

As the benchmark value of \( \mu \), we adopt 0.2, a usual value of the macroeconomic model

---

\(^{33}\)An additional parameter restriction is \( q(1+s) > \frac{2(1+\alpha)-1}{\alpha} \). This is a quantitatively reasonable assumption, as we explain at the end of Appendix F.
with imperfect competition. ³⁴

- To determine the value of the remaining parameters, we set the benchmark value of \( \tau, q \) and \( s \) exogenously, as shown in Table 1. For their source, see Table 1 and the text.

- There are four undetermined parameters for (I.1) and (I.2), \( \alpha, L, \text{SCALE}, \) and \( \phi \). Here, we put \( L = \phi = 1 \) and determine \( \alpha \) and \( \text{SCALE} \) by (I.1) and (I.2). Note that productivity \( b \) follows the Pareto distribution with scale parameter \( \phi^{\frac{\alpha}{1-\alpha}} \) and shape parameter \( \frac{1}{\alpha} \psi \), since we assume \( b^{\frac{\alpha}{1-\alpha}} \) follows the Pareto distribution with scale parameter \( \phi \) and shape parameter \( \psi \). Because the scale of productivity may be arbitrarily fixed whenever the distribution of firm size is properly controlled, we set \( \phi = 1 \). We simply normalize \( L = 1 \). Since the number of the intermediate good firms is a continuum, the minimum of firm size among them does not have to correspond to the minimal number of employees in actual data (and only the shape of the distribution, the curvature of the density function, matters). Thus, we do not care about the magnitude of parameter \( \text{SCALE} \), which is determined by (I.1) for given \( \alpha \) and the other parameters. Parameter \( \alpha \) is pinned down by the condition of the markup rate, (I.2).

- We explain the determination of \( \alpha \). Through long but straightforward algebra, (I.2) can be rearranged as the quadratic equation with respect to \( \alpha \) as follows: \( \Phi_2 \alpha^2 + \Phi_1 \alpha + \Phi_0 = 0 \), where \( \Phi_2 = (1 + \mu) \left[ 1 - q(1 + s) \right] \tau \), \( \Phi_1 = (1 + \mu)(1 - \tau) \) and \( \Phi_0 = - \left[ 1 - q(1 + s) \tau \right] \). By \( \Phi_2 > 0 \) and \( \Phi_1 > 0 \), a necessary and sufficient condition for a unique solution in \((0, 1)\) is \( \Phi_0 < 0 \) and \( \Phi_2 + \Phi_1 + \Phi_0 > 0 \), which holds for any parameter set. The solution is given by \( \alpha = \frac{-\Phi_1 + \sqrt{\Phi_1^2 - 4\Phi_2 \Phi_0}}{2\Phi_2} \).

- Finally, by (23), we choose the value of \( A^{\frac{1}{1-\alpha}} / \eta \) to fix the long-run growth rate to 2%. This completes the parameter specification for conducting the numerical exercises.

³⁴ For example, Rotemberg and Woodford (1999) adopt this value. However, because markup rate is a key parameter of our analysis, we conduct numerical exercises for alternative values. See Table 3.
Table 1: Baseline Parameter Value

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>1.059</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>.8620</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>.1160</td>
</tr>
<tr>
<td>$\tau$</td>
<td>.2706</td>
</tr>
<tr>
<td>$q$</td>
<td>.096</td>
</tr>
<tr>
<td>$s$</td>
<td>.5</td>
</tr>
<tr>
<td>$k$</td>
<td>.1667</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\rho$</td>
<td>.0204</td>
</tr>
<tr>
<td>$L$</td>
<td>1</td>
</tr>
<tr>
<td>$A^{\frac{1}{\eta}} / \eta$</td>
<td>34.2321</td>
</tr>
</tbody>
</table>

Table 2: The welfare- and growth-maximizing tax rates and the contributions to $\tau_{WM} - \beta$ for the benchmark

| $\tau_{WM}$ | .4025 |
| $\tau_{GM}$ | .3921 |
| $\tilde{\tau}_{GM}$ | .3133 |
| $\beta$     | .1000 |

the tax base effect: $\tau_{WM} - \tau_{GM}$ .0104
the difference in tax rates: $\tau_{GM} - \tilde{\tau}_{GM}$ .0787
the effect of CIT evasion: $\tilde{\tau}_{GM} - \beta$ .2133
total effect: $\tau_{WM} - \beta$ .3025
Table 3: The optimal CIT rates for alternative markup rates (benchmark: markup rate = .20)

<table>
<thead>
<tr>
<th>markup rate</th>
<th>.10</th>
<th>.20</th>
<th>.30</th>
<th>.40</th>
<th>.50</th>
<th>.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^{WM}$</td>
<td>.5258</td>
<td>.4025</td>
<td>.3324</td>
<td>.2868</td>
<td>.2549</td>
<td>.2058</td>
<td>.1783</td>
</tr>
<tr>
<td>$\tau^{GM}$</td>
<td>.5077</td>
<td>.3921</td>
<td>.3248</td>
<td>.2806</td>
<td>.2493</td>
<td>.2007</td>
<td>.1733</td>
</tr>
<tr>
<td>$\tilde{\tau}^{GM}$</td>
<td>.3761</td>
<td>.3133</td>
<td>.2718</td>
<td>.2422</td>
<td>.2201</td>
<td>.1835</td>
<td>.1617</td>
</tr>
<tr>
<td>$\tau^{WM} - \tau^{GM}$</td>
<td>.0181</td>
<td>.0104</td>
<td>.0075</td>
<td>.0062</td>
<td>.0056</td>
<td>.0050</td>
<td>.0050</td>
</tr>
<tr>
<td>$\tau^{GM} - \tilde{\tau}^{GM}$</td>
<td>.1315</td>
<td>.0787</td>
<td>.0529</td>
<td>.0383</td>
<td>.0291</td>
<td>.0171</td>
<td>.0116</td>
</tr>
<tr>
<td>$\tilde{\tau}^{GM} - \beta$</td>
<td>.2761</td>
<td>.2133</td>
<td>.1718</td>
<td>.1422</td>
<td>.1201</td>
<td>.0835</td>
<td>.0617</td>
</tr>
<tr>
<td>$\tau^{WM} - \beta$</td>
<td>.4258</td>
<td>.3025</td>
<td>.2324</td>
<td>.1868</td>
<td>.1549</td>
<td>.1058</td>
<td>.0783</td>
</tr>
</tbody>
</table>

Table 4: We provide $\tau^{WM}$, the ratio of $\tau^{WM}$ to $\beta$, and the shares of the effect of CIT evasion in the total effect for alternative output elasticities of public services (benchmark: $\beta = .10$).

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>.025</th>
<th>.05</th>
<th>.10</th>
<th>.15</th>
<th>.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^{WM}$</td>
<td>.0986</td>
<td>.1980</td>
<td>.4025</td>
<td>.6188</td>
<td>.8488</td>
</tr>
<tr>
<td>$\tau^{WM} / \beta$</td>
<td>3.94</td>
<td>3.96</td>
<td>4.03</td>
<td>4.13</td>
<td>4.24</td>
</tr>
<tr>
<td>$\frac{\tilde{\tau}^{GM} - \beta}{\tau^{WM} - \beta}$</td>
<td>.8833</td>
<td>.8225</td>
<td>.7025</td>
<td>.6000</td>
<td>.5084</td>
</tr>
</tbody>
</table>
Table 5: The optimal tax rates in the case of $q = .089$ for the benchmark

<table>
<thead>
<tr>
<th>Tax Rate</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{WM}$</td>
<td>.4162</td>
</tr>
<tr>
<td>$\tau_{GM}$</td>
<td>.4056</td>
</tr>
<tr>
<td>$\tilde{\tau}_{GM}$</td>
<td>.3203</td>
</tr>
<tr>
<td>$\beta$</td>
<td>.1000</td>
</tr>
</tbody>
</table>

the tax base effect: $\tau_{WM} - \tau_{GM}$
the difference in tax rates: $\tau_{GM} - \tilde{\tau}_{GM}$
the effect of CIT evasion: $\tilde{\tau}_{GM} - \beta$

total effect: $\tau_{WM} - \beta$

Figure 1: The growth- and welfare-maximizing tax rates ($\tau_{WM}$, $\tau_{GM}$, $\tilde{\tau}_{GM}$, and contributions to $\tau_{WM} > \beta$ for alternative values of $q$)