Incomplete Contracts, Shared Ownership, and Investment Incentives

Schmitz, Patrick W.

2017
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Abstract

Consider a partnership consisting of two symmetrically informed parties who may each own a share of an asset. It is ex post efficient that tomorrow the party with the larger valuation gets the asset. Yet, today the parties can make investments to enhance the asset’s productivity. Contracts are incomplete, so today only the ownership structure can be specified, which may be renegotiated tomorrow. It turns out that shared ownership is often optimal. If the investments are embodied in the physical asset, it may be optimal that party B has a larger ownership share even when party A has a larger valuation and a better investment technology. When shared ownership is taken into account, joint ownership in the sense of bilateral veto power cannot be optimal, regardless of whether the investments are in human capital or in physical capital.

Keywords: property rights, incomplete contracts, investment incentives, partnership dissolution, shared ownership

JEL Classification: D23; D86; C78; L24; O32

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1 Introduction

Consider a partnership consisting of two risk-neutral parties each of whom owns a share of an asset that can be traded tomorrow. Clearly, it is ex post efficient that the party with the larger valuation of the asset will get 100 percent of the asset tomorrow. Yet, suppose that today the parties can make non-contractible investments in order to enhance the productivity of the asset. We analyze who should initially own the asset, given that tomorrow negotiations between the symmetrically informed parties will result in the ex post efficient ownership allocation.

The problem that we study is relevant in many fields. For instance, temporary partnerships between firms are often formed in the context of R&D activities.\(^1\) As has been pointed out by Aghion and Tirole (1994, p. 1205), “managing innovation properly is one of the most important challenges faced by developed economies.” They explore the relationship between a customer and a research unit who can both invest in order to increase the probability of making an innovation. Only the customer can commercialize the innovation; i.e., the customer’s valuation is positive, while the research unit’s valuation is zero. Even though the innovation should thus be owned by the customer ex post, it can be optimal to initially allocate ownership to the research unit in order to improve its investment incentives. We generalize their model, such that both parties may have a positive valuation, and we study in detail under what circumstances ownership by the low-valuation party can be optimal. Moreover, we allow for shared ownership, as it is typically observed in research joint ventures.\(^2\) Another example where our analysis is very relevant in practice are public-private partnerships. Such partnerships between the public sector and a private firm are often formed to realize a specific infrastructure project such as a road, a hospital, an airport, or a prison.\(^3\) Who should own the facility used to supply a public service? The

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\(^1\)On R&D alliances, see e.g. Pisano (1989) and Oxley and Sampson (2004). Bleeke and Ernst (1995) find that in their sample the median life span of an alliance is seven years, and Chan et al. (1997) report an average life span of about five years.

\(^2\)Cf. Santamaría et al. (2009), who point out that technological joint ventures are legal entities in which equity ownership is shared between firms.

\(^3\)Akintoye (2009) points out that public-private partnerships “can be described as a con-
public sector and the private firm may have different ex post valuations. Again, while the large-valuation party should be the owner ex post, it may be optimal to give (a share of) the initial ownership to the low-valuation party to enhance its investment incentives. We analyze how the nature of investments (i.e., whether or not the investments are embodied in the physical asset) influences the optimal ownership arrangement.

We build on the property rights approach to the theory of the firm, which is the leading application of the incomplete contracting paradigm developed by Grossman and Hart (1986), Hart and Moore (1990), and Hart (1995). The incomplete contracts approach has turned out to be very helpful to discuss the pros and cons of ownership structures, and it is by now widely regarded as one of the most important advances made in microeconomics in the past three decades. In the property rights approach to the theory of the firm, most contributions make the plausible assumptions that partners who work together tend to be symmetrically informed, investment incentives are important, and real-world contracts are typically incomplete. We also make these assumptions. Yet, so far the incomplete contracts literature is usually focused on sole ownership by one party (the “make-or-buy” decision), while it remains rather silent on the issue of shared ownership. This is a major shortcoming, as was already pointed out by Holmström (1999, p. 86), who emphasized that “joint ventures (and shared ownership, more generally) have always been an important part of the corporate landscape.” Moreover, in most papers in the incomplete contracting literature, tractive agreement of shared ownership between a public agency and a private company.” For recent work on public-private partnerships, see e.g. Desrieux (2009), Saussier et al. (2009), Hoppe and Schmitz (2013), Greco (2015), Li et al. (2015), Deng et al. (2016), and Buso et al. (2017). Sarmento and Renneboog (2016) report that infrastructure construction often takes four to five years.

For a vivid discussion of the incomplete contracting methodology, see Tirole (1999).

The incomplete contracts approach has been successfully applied in fields such as organizational economics, corporate finance, industrial organization, and privatization theory. Andrei Shleifer has recently emphasized that the “Grossman-Hart incomplete contracts approach represents perhaps the most influential advance in economic theory in the last thirty years” (see the back cover of Aghion et al., 2016).

In their recent literature survey, Gattai and Natale (2017) point out that worldwide and in many sectors, joint ventures are a very common form of inter-firm collaborations. On the
the *ex post* allocation of ownership does not matter. Furthermore, the analysis is usually focused on investments in human capital, while investments in physical capital are typically mentioned only briefly. It is our aim to broaden the property rights theory in order to cover these omissions. In particular, we demonstrate that central elements of the property rights approach (contractual incompleteness, relationship-specific investments, and renegotiation) can be fruitfully combined with the definition of ownership employed in the partnership dissolution literature.\(^7\)

The main novel insights of the present paper are as follows. First, we clarify the role that the nature of investments plays for the optimal ownership arrangement. When (in line with the standard property rights approach) we consider only sole ownership by one of the two parties, then the party with the larger valuation should already have initial ownership if the investments are in human capital, while the party with the *smaller* valuation should be the initial owner if the investments are in physical capital, given that the parties have the same investment costs. Furthermore, in the case of physical capital investments, initial ownership by the low-valuation party can be optimal even when its investment technology is *less efficient* than that of the high-valuation party. Second, when we allow for *shared ownership*, then sole ownership by one party is never optimal if the parties have the same valuations or the same investment technologies. Moreover, if the parties have the same investment costs but differ in their valuations, then the initial ownership share of the large-valuation party should be relatively large in the case of human capital investments, but relatively small in the case of physical-capital investments. Third, it is well-known that joint ownership in the sense of bilateral veto power is never optimal in the case of

\(^7\)See the discussion of the related literature below. The definition of ownership in Grossman and Hart (1986) is very useful in some contexts such as vertical integration, but it has turned out to be too narrow for other purposes. As pointed out, the traditional property rights approach cannot account for shared ownership, which is discussed in the present paper. Moreover, the traditional approach has problems explaining publicly traded corporations. Specifically, as has recently been emphasized by Zingales (2016, p. 147), “Starting from the original version of the theory, it is a bit hard to explain what it means to separate ownership from control, since ownership is defined as control.”
human capital investments, while in standard property rights models it can be optimal when investments are in physical capital.\textsuperscript{8} We show that the latter result crucially relies on the usual assumption that there is only sole ownership by one of the parties. When we allow for shared ownership, then bilateral veto power \textit{cannot} improve investment incentives, regardless of the nature of investments.

\textit{Related literature.} Our main research question is somewhat reminiscent of the partnership dissolution problem that was first studied by Cramton et al. (1987), who consider a variant of Myerson and Satterthwaite’s (1983) well-known contribution.\textsuperscript{9} However, in this literature the parties’ valuations are assumed to be private information and unrestrainedly complex mechanisms are allowed, which contrasts with the incomplete contracts approach that we follow in the present paper. Myerson and Satterthwaite (1983) have demonstrated that ex post efficiency cannot always be attained by voluntary bargaining under asymmetric information when initially one of the parties is the sole owner of the asset, while the partnership dissolution literature shows that ex post efficiency can be achieved if initial ownership is more evenly shared between the parties.\textsuperscript{10} The present paper is complementary to this literature, since we give a different explanation for the optimality of shared ownership.

In the complete contracting literature, some papers such as Bhattacharyya and Lafontaine (1995) explain the optimality of revenue sharing contracts in the context of a double-sided moral hazard problem (i.e., while the parties are symmetrically informed at the contracting stage, there are post-contractual hidden actions). This literature is different from the incomplete contracts approach, because in a complete contracting world the Coase Theorem applies and ownership

\textsuperscript{8}Investments in physical capital were briefly discussed by Hart and Moore (1990). See also the recent contribution by Gattai and Natale (2016).

\textsuperscript{9}See also Segal and Whinston (2013) for a recent survey on property rights encompassing both the partnership dissolution literature and the incomplete contracting literature. Note that they also use a broader definition of ownership than Grossman and Hart (1986).

\textsuperscript{10}See also Schmitz (2002) for a variant of the partnership dissolution problem where a party’s privately known valuation is influenced by prior investments made by this party. In contrast, in the present paper we follow the incomplete contracting literature in assuming that the partners are symmetrically informed and we allow investments to have external effects on the other party’s payoff.
does not matter (see Hart, 1995). Furthermore, Tamada and Tsai (2007, 2014) have also studied the termination of projects, albeit in a complete contracting framework with moral hazard and limited liability where ownership does not play a role.

Organization of the paper. The remainder of the paper is organized as follows. In the following section, the model is introduced and the first-best benchmark is presented. In Section 3, the second-best solution is derived. In Section 4, only sole ownership by one of the parties can be specified ex ante, so we analyze whether or not the large-valuation party should already be the owner at the ex ante stage. In Section 5 we allow for shared ownership. Section 6 studies joint ownership in the sense of bilateral veto power. Concluding remarks follow in Section 7. Some technical details are relegated to the Appendix.

2 The model

Consider a partnership between two risk-neutral parties, $A$ and $B$. Initially, party $A$ owns the share $r_0 \in [0, 1]$ of an asset that can be traded at date 2, while party $B$ owns the share $1 - r_0$. The valuation of party $i$ for the entire asset is given by $v_i > 0$. At date 1, the two parties simultaneously choose observable but non-contractible investment levels $a \geq 0$ and $b \geq 0$ that can improve the productivity of the asset.$^{12}$ The investment costs are given by $\frac{1}{2}c_A a^2$ and $\frac{1}{2}c_B b^2$.

$^{11}$In contrast, in the partnership dissolution literature ownership determines the parties’ default payoffs (just as in the property rights theory), and the parties are privately informed at the contracting stage. As has been pointed out by Laffont and Martimort (2002, p. 58), the fact that the parties did not write a contract before the information was realized can actually be interpreted as a form of incomplete contracting. Indeed, it is well-known that Myerson and Satterthwaite’s (1983) impossibility theorem does not hold if a contract can be signed before the private information is realized (see d’Aspremont and Gérard-Varet, 1979).

$^{12}$Note that alternatively one might consider investments that increase the valuations. Such a model has been studied by Schmitz (2002) and is closer to the hold-up literature that does not address asset ownership (cf. Rogerson, 1992). In contrast, the current model is focused on property rights. Note that in Aghion and Tirole’s (1994) management of innovation model and in Besley and Ghatak’s (2001) public goods model the parties’ valuations are also independent of the investments.
respectively, where $c_i > 0$. Following the incomplete contracting approach, we assume that at date 0 only the initial ownership structure $r_0$ is contractible. However, the ownership structure may be renegotiated at date 2.

If the parties reach an agreement at date 2, then they dissolve their partnership in an ex post efficient way, such that the party with the larger valuation gets the entire asset. Moreover, there is an “amicable divorce,” i.e., the party with the larger valuation will be able to make full use not only of its own investment, but also of the other party’s investment. Thus, the joint date-2 surplus is $\max\{v_A, v_B\}(a + b)$.

Following the property rights approach, there is symmetric information and the date-2 negotiations are modeled by the regular Nash bargaining solution, such that a date-2 agreement will always be reached. Yet, the division of the date-2 surplus depends on the default payoffs that the parties would get in the absence of an agreement. Party A’s and party B’s date-2 default payoffs are $d_A(a, b|r_0) = v_A r_0(a + \lambda_B b)$ and $d_B(a, b|r_0) = v_B(1 - r_0)(b + \lambda_A a)$, respectively.

Specifically, suppose that party A is the sole owner of the asset ($r_0 = 1$). When no agreement is reached, party B’s payoff is zero, since party B does not have the asset. Party A can make full use of its own investment $a$, but it can use only a fraction $\lambda_B$ of party B’s investment. In the wording of the property rights theory, this means that the fraction $\lambda_B$ of party B’s investment is embodied in

\[ v_A - v_B = b(1 - \lambda_B) + a\lambda_B. \]

13 As it is often done in the related literature, we consider quadratic investment costs in order to simplify the exposition. Yet, qualitatively our results also hold when the investment costs are linear and the investments’ returns are concave.

14 In addition, the parties can agree on a lump-sum up-front payment in order to distribute the anticipated total surplus according to their ex ante bargaining powers.

15 Note that if we normalize the model such that $0 \leq a + b \leq 1$, then we can interpret $a + b$ as the probability that an innovation is made. If party A is a research unit and party B is a customer and $\lambda_A = 1$, our model somewhat resembles Aghion and Tirole’s (1994) management of innovation model, where the parties’ valuations of the innovation are given by $v_A = 0$ and $v_B = V > 0$. Observe that in contrast to their setup, we allow for horizontal research joint ventures where each party may have a positive valuation. Moreover, our model differs from Aghion and Tirole’s (1994) work because our focus is on shared ownership and we allow for investments in human capital as well as in physical capital. In contrast, Aghion and Tirole (1994) study the implications of their assumption that party A is cash-constrained, which we do not make in the present paper.
the physical capital (so that it can always be used by whoever is the owner of the asset), while the fraction \(1 - \lambda_B\) of party B’s investment is embodied in his human capital. In particular, the human capital investment can be interpreted as acquisition of asset-specific know-how that party B will not disclose to party A if no agreement at date 2 is reached.\(^{16}\) Analogously, if party B is the sole owner of the asset \((r_0 = 0)\), then in the absence of a date-2 agreement party A’s payoff is zero, while party B can make use of his own investment and of a fraction \(\lambda_A\) of party A’s investment. In the case of shared ownership \((0 < r_0 < 1)\), when no agreement is reached party A owns the share \(r_0\) and party B owns the share \(1 - r_0\) of the asset.\(^{17}\)

Note that following the incomplete contracts approach we assume throughout that the investments are asset-specific and non-contractible, regardless of whether they are embodied in the investor’s human capital or in the physical asset. In particular, the investments can be interpreted as effort exerted in order to improve the profitability of the asset. Technological improvements of the asset are embodied in the physical capital, while ideas how to best commercialize the asset are examples of investments that are embodied in the parties’ human capital.\(^{18}\) The investment costs \(c_A\) and \(c_B\) reflect the abilities of the two parties. Observe also that in our model profits are non-contractible, while asset ownership provides parties with income streams.\(^{19}\)

\(^{16}\)Thus, the acquired know-how is of the “eye-opener” nature discussed by Tirole (2015); i.e., when an agreement is reached then disclosure immediately enables the other party to make full use of the know-how. In contrast, when no agreement is reached, the parties do not cooperate and thus no know-how is transferred (cf. Besley and Ghatak, 2001, for a related discussion). Investments in transferable know-how have also been studied by Rosenkranz and Schmitz (2003). Yet, in their model the asset is a public good that can be used by both parties simultaneously. On know-how transmission in a different setup, see also Tsai and Kung (2011).

\(^{17}\)Observe that given risk-neutrality, shared ownership is equivalent to stochastic ownership; i.e., when no agreement is reached at date 2, then party A will be the owner with probability \(r_0\) and party B will be the owner with probability \(1 - r_0\). Note that when the valuations are uncertain ex ante, stochastic ownership can be implemented by giving one of the parties the right to buy the asset at an ex ante determined price.

\(^{18}\)Of course, in addition to the non-contractible investments in practice there may also be contractible (e.g., financial) investments, which we do not model explicitly in order to simplify the exposition.

\(^{19}\)While this assumption was not made in Grossman and Hart (1986), similar assumptions
The first-best benchmark. In a first-best world, the two parties would always agree to give the asset at date 2 to the party with the larger valuation. Moreover, the first-best investment levels $a^{FB}$ and $b^{FB}$ which maximize the total surplus $\max\{v_A, v_B\} (a + b) - \frac{1}{2} c_A a^2 - \frac{1}{2} c_B b^2$ are characterized by

$$
a^{FB} = \max\{v_A, v_B\}/c_A, \\
b^{FB} = \max\{v_A, v_B\}/c_B.
$$

Note that in line with the standard property rights approach, initial ownership $r_0$ does not matter in a first-best world.

3 The second-best solution

Following the property rights theory as synthesized by Hart (1995), the outcome of the date-2 negotiations is given by the regular Nash bargaining solution, where the default payoffs constitute the threatpoint. Thus, the parties agree on a transfer payment such that at date 2 each party gets its default payoff (which it would get in case of disagreement) plus half of the renegotiation surplus (i.e., the additional surplus that is generated by an “amicable divorce”).

Hence, given the initial ownership structure $r_0 \in [0, 1]$, party A’s date-2 payoff reads

$$u_A(a, b|r_0) = d_A(a, b|r_0) + \frac{1}{2} \Delta(a, b|r_0)$$

and party B’s date-2 payoff is

$$u_B(a, b|r_0) = d_B(a, b|r_0) + \frac{1}{2} \Delta(a, b|r_0),$$

have often been made in the subsequent literature, see e.g. Holmström and Milgrom (1994), Stein (1997), Aghion et al. (2016, Part 2), and the recent work by Su (2017). Note that our definition of ownership is the same as in the partnership dissolution literature discussed above.

$^{20}$This property of our model is similar to the partnership dissolution literature. As a consequence, when it turns out that ex ante shared ownership is strictly better than sole ownership in a second-best world, then this must be due to incentive considerations only. Alternatively, one might also consider a model in which it is ex post efficient to share ownership due to complementarities between the two parties. Yet, in such a model it would be less surprising to find that the parties might agree on shared ownership already at the outset of their relationship.

$^{21}$See Muthoo (1999) for an excellent exposition of bargaining theory.
where the renegotiation surplus $\Delta(a, b|r_0)$ is given by

$$\Delta(a, b|r_0) = \max\{v_A, v_B\}(a + b) - d_A(a, b|r_0) - d_B(a, b|r_0).$$

Let us now analyze the parties’ date-1 investment incentives. Given the initial ownership structure $r_0$, the parties anticipate that at date 2 party $A$’s payoﬀ will be $u_A(a, b|r_0)$ and party $B$’s payoﬀ will be $u_B(a, b|r_0)$. Thus, at date 1 party $A$ chooses the investment level

$$a(r_0) = \arg \max_a \{u_A(a, b|r_0) - \frac{1}{2} c_A a^2\}$$

$$= \arg \max_a \{v_A r_0(a + \lambda_B b) + \frac{1}{2} [\max\{v_A, v_B\}(a + b) - v_A r_0(a + \lambda_B b) - v_B(1 - r_0)(b + \lambda_A a)] - \frac{1}{2} c_A a^2\},$$

while party $B$ chooses the investment level

$$b(r_0) = \arg \max_b \{u_B(a, b|r_0) - \frac{1}{2} c_B b^2\}$$

$$= \arg \max_b \{v_B(1 - r_0)(b + \lambda_A a) + \frac{1}{2} [\max\{v_A, v_B\}(a + b) - v_A r_0(a + \lambda_B b) - v_B(1 - r_0)(b + \lambda_A a)] - \frac{1}{2} c_B b^2\}.$$

As a consequence, the investment levels that the two parties choose at date 1 are given by

$$a(r_0) = \frac{1}{2c_A} [\max\{v_A, v_B\} + v_A r_0 - v_B(1 - r_0)\lambda_A],$$

$$b(r_0) = \frac{1}{2c_B} [\max\{v_A, v_B\} - v_A r_0 \lambda_B + v_B(1 - r_0)].$$

Inspection of these investment levels immediately leads to the following result.

**Lemma 1** (i) Regardless of the ownership structure $r_0 \in [0, 1]$, there is never overinvestment with regard to the first-best benchmark; i.e., $a(r_0) \leq a^{FB}$, $b(r_0) \leq b^{FB}$.

(ii) A party’s investment is always increasing in its ownership share; i.e., $a(r_0)$ is an increasing function and $b(r_0)$ is a decreasing function.

At date 0, the anticipated total surplus level given ownership structure $r_0 \in [0, 1]$ is given by

$$S(a(r_0), b(r_0)) = \max\{v_A, v_B\}[a(r_0) + b(r_0)] - \frac{1}{2} c_A a(r_0)^2 - \frac{1}{2} c_B b(r_0)^2.$$
In the following sections, we analyze how the initial ownership structure \( r_0 \) should be chosen in order to maximize the total surplus \( S(a(r_0), b(r_0)) \). Note that since there is always underinvestment with regard to the first-best solution and the total surplus is concave in the investments, a change of the ownership structure that induces both parties to invest more is always desirable.

4 A-ownership versus B-ownership

In this section, following most contributions to the property rights theory, we assume that at date 0 the parties can agree on \( r_0 \in \{0, 1\} \) only. Thus, we confine our attention to sole ownership, either by party \( A \) (i.e., \( r_0 = 1 \)) or by party \( B \) (i.e., \( r_0 = 0 \)). Shared ownership \((0 < r_0 < 1)\) will be analyzed in the next section.\(^{22}\)

Suppose first that for efficiency reasons it does not matter who is the final owner of the asset, because both parties have the same valuation. Then the following result holds.

**Proposition 1** Suppose only sole ownership is feasible, \( r_0 \in \{0, 1\} \). Consider \( v_A = v_B \) and \( \lambda_A = \lambda_B \). Then at date 0 the party with the more efficient investment technology should be the owner; i.e., party \( A \) should be the owner if \( c_A < c_B \) and party \( B \) should be the owner if \( c_B < c_A \).

**Proof.** Suppose \( v_A = v_B =: v \) and \( \lambda_A = \lambda_B =: \lambda \). Then under \( A \)-ownership the investment levels are \( a(1) = v/c_A \) and \( b(1) = (1 - \lambda)v/2c_B \), while under \( B \)-ownership the investment levels are \( a(0) = (1 - \lambda)v/2c_A \) and \( b(0) = v/c_B \). It is straightforward to check that \( A \)-ownership is better than \( B \)-ownership, i.e. \( S(a(1), b(1)) \geq S(a(0), b(0)) \), whenever \( c_A \leq c_B \) holds.

In other words, everything else equal, the party with the better investment technology should be the owner. Proposition 1 thus replicates a central insight of the property rights approach to the theory of the firm (cf. Hart, 1995) in our

\(^{22}\)Following prominent contributions to the property rights theory such as Hart (1995) and Hart et al. (1997), in the propositions below we focus on specific cases that best convey the economic intuition. See the Appendix for the general conditions under which \( A \)-ownership is strictly better than \( B \)-ownership. These conditions underlie the four figures displayed in this section.
setup. Intuitively, recall from Lemma 1(ii) that ownership increases a party’s investment incentives. Hence, in order to make the most of the superior investment technology, the party with the smaller investment costs should be the owner. Note that this result holds regardless of whether the investments are in human capital ($\lambda_A = \lambda_B = 0$) or in physical capital ($\lambda_A = \lambda_B = 1$).

Next, let us explore the impact of the parties’ valuations.

Proposition 2 Suppose only sole ownership is feasible, \( r_0 \in \{0, 1\} \).

(i) Suppose \( c_A = c_B \) and let \( \lambda_A = \lambda_B = 0 \), so that the investments are in human capital. Then the party who is the ex post efficient final owner of the good should also be the initial owner; i.e., at date 0 A-ownership is optimal if \( v_A > v_B \) and B-ownership is optimal if \( v_B > v_A \).

(ii) Suppose \( c_A = c_B \) and let \( \lambda_A = \lambda_B = 1 \), so that the investments are embodied in the physical capital. Then the party who is the ex post efficient final owner of the good should not be the initial owner; i.e., at date 0 B-ownership is optimal if \( v_A > v_B \) and A-ownership is optimal if \( v_B > v_A \).

Proof. (i) Suppose \( c_A = c_B =: c \) and \( \lambda_A = \lambda_B = 0 \). Then under A-ownership the investment levels are \( a(1) = \frac{\max\{v_A, v_B\} + v_A}{2c} \) and \( b(1) = \frac{\max\{v_A, v_B\}}{2c} \), while under B-ownership the investment levels are \( a(0) = \frac{\max\{v_A, v_B\}}{2c} \) and \( b(0) = \frac{\max\{v_A, v_B\} + v_B}{2c} \). One can easily verify that A-ownership is better than B-ownership, i.e. \( S(a(1), b(1)) \geq S(a(0), b(0)) \), whenever \( v_A \geq v_B \) holds.

(ii) Suppose \( c_A = c_B =: c \) and \( \lambda_A = \lambda_B = 1 \). Under A-ownership the investments are \( a(1) = \frac{\max\{v_A, v_B\} + v_A}{2c} \) and \( b(1) = \frac{\max\{v_A, v_B\} - v_A}{2c} \), while under B-ownership the investment levels are \( a(0) = \frac{\max\{v_A, v_B\} - v_B}{2c} \) and \( b(0) = \frac{\max\{v_A, v_B\} + v_B}{2c} \). One can now check that A-ownership is better than B-ownership whenever \( v_B \geq v_A \).}

Part (i) of Proposition 2 is not surprising. Given that both parties have the same investment technology, the party with the larger valuation should be the owner. Yet, note that this result holds when the investments are in human capital (which is the case most often studied in the property rights literature).

In contrast, if the investments are in physical capital, then according to part (ii) of Proposition 2, the party with the smaller valuation should be the initial owner. At first sight, this result might be surprising. The intuition behind
the result can be explained as follows. If the investments are embodied in the physical capital and the party with the larger valuation is the initial owner, then it can make full use of the other party’s investment without reaching a date-2 agreement. Thus, the high-valuation party has no reason to negotiate with the other party at date 2. In anticipation, the low-valuation party will not make any investments. Yet, if the party with the smaller valuation is the initial owner, then renegotiation of the ownership structure will take place at date 2. Since the party with the smaller valuation will get half of the renegotiation surplus, it will make a strictly positive investment. As a consequence, even though ownership by the high-valuation party is ex post efficient, at date 0 ownership should be given to the low-valuation party.

As an illustration, consider Figures 1 and 2. Observe that the figures show the optimal initial ownership structure depending on \( v_A \) and \( c_A \), given that \( v_B = 1 \) and \( c_B = 1 \). Hence, in the region above the dashed line, party \( B \) has the better investment technology. Figure 1 depicts the case of investments in human capital, while Figure 2 depicts the case of investments in physical capital.

Moreover, note that the total investments are always equal to \( \max\{v_A, v_B\}/2c \). Thus, given convexity of the investment cost functions, the total surplus is larger if the investments are more evenly distributed between the two parties (i.e., if the low-valuation party makes a strictly positive investment). An analogous argument would hold if the investment costs were linear and the investments’ returns were concave.

See Huberman and Kahn (1988) for other examples where parties ex ante could sign an ex post efficient contract, but instead prefer to agree on a (seemingly) suboptimal contract that will be renegotiated later on. For instance, bank loans often stipulate that assets will be taken over by the bank if the borrower does not repay. Yet, banks are usually less efficient as managers of the assets, hence the threat of takeover is not carried out and loans are renegotiated instead. See also Maskin and Moore’s (1999, Section 4) result according to which in a setting with ex ante contractible trade it may be optimal to write no contract when the seller’s investment improves the buyer’s valuation and there is ex post efficient renegotiation.
Figure 1. The optimal initial ownership structure when the investments are in human capital ($\lambda_A = \lambda_B = 0$).

If $v_A = v_B$, then in line with Proposition 1, in both figures $A$-ownership is optimal when $c_A < c_B$ and $B$-ownership is optimal when $c_A > c_B$. However, consider now values of $v_A$ that are smaller than $v_B$, so that ownership by party $B$ is ex post efficient. In the case of human capital investments (Figure 1), initial
$B$-ownership is optimal when $c_A = c_B$.\textsuperscript{25} In contrast, in the case of physical capital investments (Figure 2), initial ownership by party $A$ is optimal when $c_A = c_B$. Note that initial $A$-ownership is even optimal for some values of $c_A$ that are larger than $c_B$. Thus, in the case of physical capital investments, $A$-ownership can be optimal at date 0 even when party $A$ has both the smaller valuation and the less efficient investment technology.

So far, we have focused on cases in which either all investments were in human capital or all investments were in physical capital. Let us now study the role played by the nature of the investments in more detail.

As long as $\lambda_A = \lambda_B$, the previous results generalize in a straightforward way to the intermediate case in which both parties’ investments are partially in human capital and partially in physical capital (i.e., $0 < \lambda_A = \lambda_B < 1$). This case is illustrated in Figure 3.

\textbf{Figure 3.} The optimal initial ownership structure when the investments are partially in human capital and partially in physical capital ($\lambda_A = \lambda_B = 1/2$).

Next, let us consider the case where one party can invest in human capital, while the other party can invest in physical capital.

\textsuperscript{25}Of course, $B$-ownership remains optimal when $c_A > c_B$. However, note that $B$-ownership is also optimal for some values of $c_A$ that are smaller than $c_B$; i.e., the fact that party $B$ has the larger valuation can overcompensate the fact that party $A$ has the better investment technology.
Proposition 3 Suppose only sole ownership is feasible, \( r_0 \in \{0, 1\} \). Consider \( c_A = c_B \) and let \( \lambda_A = 0, \lambda_B = 1 \), so that party A’s investment is in human capital and party B’s investment is in physical capital. Then initial ownership by party B is always optimal, regardless of the parties’ valuations.

Proof. Suppose \( c_A = c_B =: c \) and \( \lambda_A = 0, \lambda_B = 1 \). Then under A-ownership the investment levels are \( a(1) = [\max\{v_A, v_B\} + v_A] / 2c \) and \( b(1) = [\max\{v_A, v_B\} - v_A] / 2c \), while under B-ownership the investment levels are \( a(0) = \max\{v_A, v_B\} / 2c \) and \( b(0) = [\max\{v_A, v_B\} + v_B] / 2c \). It is straightforward to verify that B-ownership is always better than A-ownership, i.e. \( S(a(1), b(1)) < S(a(0), b(0)) \) holds.

Intuitively, a party that makes investments in physical capital needs more protection from hold-up than a party that invests in human capital.\(^{26}\) Suppose that party A (who invests in human capital) is the initial owner. Then party A can make full use of party B’s investment even when no agreement at date 2 is reached, since party B’s investment is embodied in the physical capital owned by party A. For this reason, it is particularly important to improve party B’s investment incentives, so initial ownership by party B turns out to be optimal.

Proposition 3 has been derived under the assumption that both parties’ investment technologies are equally efficient (\( c_A = c_B \)). As is illustrated in Figure 4, ownership by party A will become optimal if party A’s investment technology is much more efficient than party B’s investment technology, i.e., if \( c_A \) is much smaller than \( c_B \).

\(^{26}\)Zhao and Zhu (1998) study international joint ventures in China and find that foreign entrants tend to have a large ownership share in industries with a high skill intensity. To the extent that the foreign entrant mainly provides monetary investments in the physical capital, the empirical finding is consistent with our result that foreign ownership is desirable when it is important that local workers invest in their human capital.
Figure 4. The optimal initial ownership structure when party A invests in human capital and party B invests in physical capital ($\lambda_A = 0$, $\lambda_B = 1$).

5 Shared Ownership

While the property rights approach to the theory of the firm is usually focused on sole ownership of an asset by one party, the literature on partnership dissolution considers shared ownership, where each party initially owns a fraction of the asset. Of course, also in the latter literature sole ownership by the high-valuation party is ex post efficient; yet, shared initial ownership facilitates reaching an agreement under asymmetric information. We now investigate whether shared ownership ($0 < r_0 < 1$) may also be beneficial in the present context, where in line with the property rights approach information is symmetric, but where the provision of investment incentives is a major concern.

Proposition 4 Suppose shared ownership is feasible.

(i) Suppose $v_A = v_B$. Then at date 0 shared ownership ($0 < r_0^* < 1$) is optimal. Party A’s optimal initial ownership share $r_0^*$ is decreasing in $c_A$ and in $\lambda_B$, while it is increasing in $c_B$ and in $\lambda_A$. In particular, if $\lambda_A = \lambda_B$, then at date 0 party A’s optimal ownership share is $r_0^* = \frac{c_B}{c_A + c_B}$.

(ii) Suppose $c_A = c_B$ and let $\lambda_A = \lambda_B = 0$, so that the investments are in human capital. At date 0, shared ownership ($0 < r_0^* < 1$) is optimal. Party A’s
optimal initial ownership share \( r_0^* \) is increasing in \( v_A \) and decreasing in \( v_B \).

(iii) Suppose \( c_A = c_B \) and let \( \lambda_A = \lambda_B = 1 \), so that the investments are embodied in the physical capital. At date 0, shared ownership (\( 0 < r_0^* < 1 \)) is optimal. Party A’s optimal initial ownership share \( r_0^* \) is decreasing in \( v_A \) and increasing in \( v_B \).

**Proof.** (i) Let \( v_A = v_B =: v \). Then party A invests \( a(r_0) = [(1 + r_0)v - (1 - r_0)v\lambda_A]/2c_A \) and party B invests \( b(r_0) = [(2 - r_0)v - r_0v\lambda_B]/2c_B \). It is straightforward to check that the total surplus \( S(a(r_0), b(r_0)) \) is maximized when

\[
    r_0 = \frac{(1 + \lambda_A)^2c_B}{(1 + \lambda_B)^2c_A + (1 + \lambda_A)^2c_B}.
\]

The claims made in the proposition then follow immediately.

(ii) Let \( c_A = c_B =: c \) and \( \lambda_A = \lambda_B = 0 \). Then the investment levels are \( a(r_0) = [\max\{v_A, v_B\} + v_A r_0]/2c \) and \( b(r_0) = [\max\{v_A, v_B\} + v_B(1 - r_0)]/2c \). One can easily verify that the total surplus \( S(a(r_0), b(r_0)) \) is maximized when

\[
    r_0 = \begin{cases} 
        \frac{v_A v_B}{v_A + v_B} & \text{if } v_A \leq v_B \\
        1 - \frac{v_A v_B}{v_A + v_B} & \text{if } v_A > v_B,
    \end{cases}
\]

which implies the claims made in the proposition.

(iii) Let \( c_A = c_B =: c \) and \( \lambda_A = \lambda_B = 1 \). Then party A’s investment level is \( a(r_0) = [\max\{v_A, v_B\} + v_A r_0 - v_B(1 - r_0)]/2c \) and party B’s investment level is \( b(r_0) = [\max\{v_A, v_B\} - v_A r_0 + v_B(1 - r_0)]/2c \). One can easily check that the total surplus \( S(a(r_0), b(r_0)) \) is maximized when \( r_0 = \frac{v_B}{v_A + v_B} \). Thus, the proposition follows immediately. \( \square \)

Note that in each of the three cases in the proposition, shared ownership is better than sole ownership, since shared ownership allows to spread the investment incentives more evenly, which is desirable given the convexity of the investment costs.27 Other than that, Proposition 4 echoes the main insights that were gained in the preceding section. Specifically, Proposition 4(i) says that if the parties have the same valuation, such that it does not matter who is the final owner of the asset, then the party with the better investment technology (i.e.,

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27 It should be noted that “smoothing out” ownership across parties is not only desirable when investment costs are convex, but also when investment costs are linear and the returns of the investments are concave.
the party with smaller investment costs) should have a larger share of the date-0 ownership. In the symmetric case where both parties have the same investment costs and the same fractions of human capital investments, each party should own 50% of the asset. Moreover, when the parties have the same investment costs but differ in their valuations, then parts (ii) and (iii) of Proposition 4 show that party A’s share of the initial ownership increases in its valuation $v_A$ if the investments are in human capital, while it decreases in $v_A$ if the investments are in physical capital.

\[
c_A = c_B = 1 \\
c_A = 0.5, c_B = 1 \\
c_A = 2, c_B = 1
\]

**Figure 5.** Party A’s optimal initial ownership share $r_0^*$ when the investments are in human capital ($\lambda_A = \lambda_B = 0$).

Figure 5 illustrates the case of human capital investments, while Figure 6 depicts the case of investments in physical capital. The figures show party A’s optimal initial ownership share $r_0^*$ as a function of its valuation $v_A$, where $v_B$ is equal to 1. In addition to the case in which both parties have the same costs, the figures also illustrate that cost differences interact in a straightforward way with valuation differences in determining the optimal ownership structure.
Note also that sole ownership (i.e., a corner solution) can be optimal only in very asymmetric cases. For instance, in Figure 6 sole ownership by party A is optimal in the case $c_A = 1/2, c_B = 1$ when party A’s valuation is much smaller than party B’s valuation ($v_A \leq 1/3, v_B = 1$). More generally, one can show the following result.

**Proposition 5** Suppose shared ownership is feasible.

(i) Sole ownership by party A is optimal ($r_0^* = 1$) if $v_A < v_B$ and

$$\frac{c_A}{c_B} < \frac{(1 - \lambda_A)v_B}{v^2_B + 2\lambda_B v_A v_B + \lambda_B^2 v^2_A}.$$ 

(ii) Sole ownership by party B is optimal ($r_0^* = 0$) if $v_A > v_B$ and

$$\frac{c_A}{c_B} > \frac{(v_A + \lambda_A v_B)^2}{(v_A - v_B)(v_B + \lambda_B v_A)}.$$ 

**Proof.** See the Appendix.

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**6 Joint ownership vs. shared ownership**

In the property rights approach to the theory of the firm, “joint ownership” of an asset usually means that each party has veto power over the use of the asset (see
Hart and Moore, 1990, p. 1132); i.e., both parties’ default payoffs are zero. Hence, joint ownership is different from shared ownership.  

Hart and Moore (1990) have shown that when the parties invest only in their human capital, then joint ownership can never be optimal. In contrast, when the parties invest in physical capital, then in traditional property rights models there are circumstances under which joint ownership is better than sole ownership. However, we now show that joint ownership can never be better than shared ownership with a suitably chosen \( r_0 \in [0, 1] \).

**Proposition 6**  
(i) If we restrict attention to sole ownership, \( r_0 \in \{0, 1\} \), then joint ownership is never optimal when the investments are in human capital (\( \lambda_A = \lambda_B = 0 \)), while joint ownership is optimal when the investments are in physical capital (\( \lambda_A = \lambda_B = 1 \)) and \( c_A = c_B \).

(ii) If we allow for shared ownership, then the total surplus attained under \( r_0 = \frac{v_B}{v_A+v_B} \) is at least as large as the total surplus attained under joint ownership.

**Proof.** Under joint ownership the default payoffs are zero, hence the analysis in Section 3 immediately implies that the investment levels are given by

\[
a^J = \frac{1}{2c_A} \max\{v_A, v_B\}, \\
b^J = \frac{1}{2c_B} \max\{v_A, v_B\}.
\]

Note that there is always underinvestment with regard to the first-best benchmark.

(i) Suppose that the investments are in human capital (\( \lambda_A = \lambda_B = 0 \)). It is straightforward to see that then \( a(r_0) \geq a^J \) and \( b(r_0) \geq b^J \) must hold. Since \( a(1) > a^J \) and \( b(0) > b^J \), sole ownership is strictly better than joint ownership. Now suppose that the investments are in physical capital (\( \lambda_A = \lambda_B = 1 \)) and \( c_A = c_B =: c \). In this case, \( a(1) = \frac{\max\{v_A, v_B\}+v_A}{2c} > a^J \) and

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28 Observe that the notion of joint ownership cannot distinguish between, say, a joint venture in which both parties have equal ownership shares and a joint venture in which one party’s ownership share is 80 percent and the other party’s ownership share is 20 percent. Desai et al. (2004) argue that while common sense might suggest that joint ventures are typically 50% owned by each of two partners, in their empirical study of American multinational firms they find that in fact only 41% of all affiliates that were partially owned by American firms had between 40% and 60% American parent ownership in 1994.
\(b(1) = \frac{\max\{v_A, v_B\} - v_A}{2c} < a'\) What can easily verify that \(S(a', b') - S(a(1), b(1)) = v_A^2/4c > 0\), hence joint ownership is strictly better than A-ownership. Analogously, one can show that \(S(a', b') - S(a(0), b(0)) = v_B^2/4c > 0\), hence joint ownership is also strictly better than B-ownership.

(ii) Suppose that \(r_0 = \frac{v_B}{v_A + v_B}\). If the investments are in physical capital \((\lambda_A = \lambda_B = 1)\), then \(a(r_0) = a'\) and \(b(r_0) = b'\), so the total surplus levels under shared ownership and under joint ownership are equal. Otherwise, shared ownership yields a strictly larger total surplus than joint ownership, since \(a(r_0) - a' = \frac{v_A v_B}{v_A + v_B} (1 - \lambda_A)/2c_A > 0\) and \(b(r_0) - b' = \frac{v_A v_B}{v_A + v_B} (1 - \lambda_B)/2c_B > 0\) hold.

Part (i) of Proposition 5 replicates the well-known insights regarding the optimality of joint ownership in traditional property rights models where shared ownership is neglected. When the investments are in human capital, then moving from sole ownership to joint ownership simply reduces the investment incentives of the original owner, while it leaves the incentives of the other party unchanged. In contrast, when the investments are in physical capital, replacing sole ownership by joint ownership can increase the total surplus. The reason is that joint ownership reduces the original owner’s investment incentives, but it increases the other party’s incentives (since investments by the other party no longer improve the owner’s default payoff). However, part (ii) of the proposition shows that if we take shared ownership into account, then we can find an ownership share \(r_0\) such that joint ownership can never yield a larger total surplus, regardless of the nature of investments. Hence, our analysis casts doubt on explanations of joint ownership that are based on investments in physical capital.\(^{29}\) If we want to explain why parties might sometimes want to block each other from using an asset, it is necessary to introduce other elements such as asymmetric information into the property rights model.\(^{30}\)

\(^{29}\)See also Gattai and Natale (2016) for other reasons why investments in physical capital may not be convincing explanations of joint ownership.

\(^{30}\)Schmitz (2006) has extended the property rights approach to the case where a party may have private information about its default payoff. See also Gattai and Natale (2017) for a recent literature survey on joint ownership.
7 Conclusion

This paper provides an incomplete contracts perspective on shared ownership and the partnership dissolution problem. In line with the established property rights approach to the theory of the firm, the party with the better investment technology (i.e., the party with smaller investment costs) should be the initial owner if only sole ownership is feasible and the parties have the same valuation for the asset. However, when the parties’ valuations differ, then the optimal ownership structure crucially depends on the nature of investments. In particular, we have found that when the investments are in physical capital, then the party with the \textit{smaller} valuation should be the initial owner if only sole ownership is possible. Moreover, in line with the partnership dissolution literature, we have found that shared ownership (which is usually neglected in the property rights literature) is often optimal. Yet, the reason for the optimality of shared ownership is different. So far, the partnership dissolution literature was concerned with achieving ex post efficiency under asymmetric information, while we have instead focused on the provision of investment incentives in an incomplete contracting framework with symmetric information. Furthermore, we have shown that even when investments are (fully or partially) in physical capital, joint ownership in the sense of bilateral veto power is never optimal. Hence, to the extent that bilateral veto power is observed in practice, other explanations such as asymmetric information are required.

In future research, the simple model studied in the present paper could be extended in several directions. For example, one could introduce private information about the valuations, bringing the model closer to the partnership dissolution literature.\textsuperscript{31} Alternatively, one could retain the assumption that there is symmetric information but allow for other sources of ex post inefficiencies.\textsuperscript{32}

\textsuperscript{31}In the incomplete contracts literature, it is usually assumed that the parties are symmetrically informed. However, cf. Schmitz (2006), Goldlücke and Schmitz (2014), and Su (2017) who study implications of private information in the property rights approach to the theory of the firm. See also Lau (2008) and Goltsman (2011) on asymmetric information and the hold-up problem.

\textsuperscript{32}Hart (1995, p. 88) suggests two alternatives to asymmetric information. First, the parties might simply “not get along” at date 2, so the negotiations break down with a small probability. Cf. Schmitz (2015) for such a model of bargaining frictions in a public-good context. See also
Moreover, while following almost all contributions to the partnership dissolution literature we have considered the case of a single asset, it might be worthwhile to extend the model to the case of multiple assets.\textsuperscript{33} Finally, it seems to be an interesting avenue for future research to explore the relationship between the nature of investments and the allocation of ownership in field studies and to test the theoretical predictions in laboratory experiments.\textsuperscript{34}

\textsuperscript{33}Note that if there are two homogenous assets, then shared ownership with $r_0 = 0.5$ is equivalent to giving each party one asset. Figueroa and Skreta (2011) also stress the importance of whether or not assets are homogenous in a partnership dissolution model with multiple assets. Following Myerson and Satterthwaite (1983), their contribution is focused on inefficiencies due to asymmetric information, while they do not consider investment incentives.

\textsuperscript{34}So far, surprisingly few experiments have been conducted on the allocation of ownership rights in incomplete contracting frameworks with an \textit{ex post} bargaining stage, see Sonnemans et al. (2001), Erlei and Siemer (2014), and Kusterer and Schmitz (2017).
Appendix

The analysis in Section 3 immediately implies that in general the total surplus \( S(0) := S(a(r_0), b(r_0)) \) can be written as\(^{35}\)

\[
S(r_0) = \begin{cases} 
\frac{1}{2}(\frac{1}{c_A} + \frac{1}{c_B})v_A^2 - \frac{1}{8c_A}(1 - r_0)^2(v_A + v_B\lambda A)^2 - \frac{1}{8c_B}[(1 + \lambda Br_0)v_A - (1 - r_0)v_B]^2 & \text{if } v_A \geq v_B, \\
\frac{1}{2}(\frac{1}{c_A} + \frac{1}{c_B})v_B^2 - \frac{1}{8c_A}[(1 + (1 - r_0)\lambda A)v_B - r_0v_A]^2 - \frac{1}{8c_B}r_0^2(v_B + \lambda Bu_A)^2 & \text{if } v_A < v_B.
\end{cases}
\]

Suppose first that following the traditional property rights approach to the theory of the firm only sole ownership is taken into account, \( r_0 \in \{0, 1\} \). It is straightforward to show that \( S(1) > S(0) \), i.e. \( A \)-ownership is strictly better than \( B \)-ownership, whenever

\[
\frac{c_A}{c_B} < \frac{(v_A + \lambda_A v_B)^2}{(2 + \lambda_B)\lambda_B v_A^2 + (2v_A - v_B)v_B} \quad \text{if } v_A \geq v_B,
\]

\[
\frac{c_A}{c_B} < \frac{(2 + \lambda_A)\lambda_A v_B^2 + (2v_B - v_A)v_A}{(v_B + \lambda_B v_A)^2} \quad \text{if } v_A < v_B.
\]

Figures 1, 2, 3, and 4 in the main text illustrate these general conditions for specific parameter constellations.

Observe that if \( \lambda_A = \lambda_B =: \lambda \), then an increase in \( \lambda \) increases the range of parameters for which ownership by the party with the smaller valuation outperforms ownership by the party with the larger valuation. To see this, note that

\[
\frac{d}{d\lambda} \frac{(v_A + \lambda v_B)^2}{(2 + \lambda)\lambda v_A^2 + (2v_A - v_B)v_B}
= -2\frac{v_A - v_B}{[(2 + \lambda^2) v_A^2 + 2v_A v_B - v_B^2]^2}[(v_A^3 - v_B^3)\lambda + (v_A^2 - v_B^2)v_A
+ (1 + \lambda^2) v_B^2 + \lambda v_A^2]^2
< 0
\]

if \( v_A > v_B \), while

\[
\frac{d}{d\lambda} \frac{(2 + \lambda)\lambda v_B^2 + (2v_B - v_A)v_A}{(v_B + \lambda v_A)^2}
= 2\frac{v_B - v_A}{(v_B + \lambda v_A)^2} \left(v_B^2 - v_A^2 + \lambda v_B^2 + v_A v_B\right) > 0
\]

\(^{35}\)Observe that \( S(r_0) \) is weakly decreasing in \( \lambda_A \) and in \( \lambda_B \). Hence, if at the outset the parties could endogenously choose which fraction of the investments should be embodied in the physical capital, they would prefer all investments to be in human capital.
if $v_A < v_B$.

Now suppose that shared ownership is feasible, so $r_0 \in [0, 1]$. Maximizing $S(r_0)$, we find that the optimal initial ownership share of party $A$ is given by

$$r_0^* = \begin{cases} \max\{(v_A + \lambda_A v_B)^2 c_B - (v_A - v_B)(v_B + \lambda_B v_A) c_A, 0\} & \text{if } v_A \geq v_B, \\ \min\{(1 + \lambda_A)(v_A + \lambda_A v_B) v_B c_B / (v_B + \lambda_B v_A)^2 c_A + (v_A + \lambda_A v_B)^2 c_B, 1\} & \text{if } v_A < v_B. \end{cases}$$

Figures 5 and 6 in the main text illustrate the optimal ownership share $r_0^*$ for specific parameter constellations.

Observe that when shared ownership is feasible, then the party with the larger valuation should not be the sole owner initially, since $r_0^* < 1$ if $v_A > v_B$, while $r_0^* > 0$ if $v_A < v_B$. Furthermore, sole ownership by party $B$ is optimal if $v_A > v_B$ and

$$c_A / c_B > \frac{(v_A + \lambda_A v_B)^2}{(v_A - v_B)(v_B + \lambda_B v_A)},$$

while sole ownership by party $A$ is optimal if $v_A < v_B$ and

$$c_A / c_B < \frac{\lambda_A v_B^2 + (1 - \lambda_A) v_A v_B - v_A^2}{v_B^2 + 2\lambda_B v_A v_B + \lambda_B^2 v_A^2},$$

which proves Proposition 5.

Note that $\frac{(v_A + \lambda_A v_B)^2}{(v_A - v_B)(v_B + \lambda_B v_A)} > 1$ when $v_A > v_B$, since

$$(v_A + \lambda_A v_B)^2 - (v_A - v_B)(v_B + \lambda_B v_A) = (1 - \lambda_B)(v_A - v_B) v_A + 2\lambda_A v_A v_B + (1 + \lambda_A^2) v_B^2 > 0.$$ 

Moreover, $\frac{\lambda_A v_B^2 + (1 - \lambda_A) v_A v_B - v_A^2}{v_B^2 + 2\lambda_B v_A v_B + \lambda_B^2 v_A^2} < 1$ when $v_A < v_B$, since

$$v_B^2 + 2\lambda_B v_A v_B + \lambda_B^2 v_A^2 - (\lambda_A v_B^2 + (1 - \lambda_A) v_A v_B - v_A^2) = (1 - \lambda_A)(v_B - v_A) v_B + 2\lambda_B v_A v_B + (1 + \lambda_B^2) v_A^2 > 0.$$ 

Hence, when shared ownership is feasible, then sole ownership by a party can be optimal only if the cost advantage of this party is sufficiently strong and the other party has a larger valuation.
References


