Evolutionary stability of behavioural rules in bargaining

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Abstract

I study the evolutionary stability of behavioural rules in a bargaining game. Individuals draw random samples of strategies used in the past and respond to it by using a behavioural rule Even though individuals actually respond to historical demands, a necessary condition for stability is the existence of a state such that it is as-if the individuals are hardwired to make the same demand. Furthermore, the state where all individuals demand half of the pie is the unique neutrally stable state; all other states are unstable in the face of an invasion by a mutant behavioural rule.

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1 Introduction

Consider a situation where two parties can potentially generate a surplus of fixed size by means of a transaction, or alternatively, by some form of cooperation. This transaction or cooperation will be undertaken only if the two parties can first agree on how the surplus will be divided. In order to reach an agreement, they bargain with each other by simultaneously claiming a share of the surplus (or pie) for himself. They receive their respective claims if the two demands are compatible, i.e. they sum up to no greater than unity. Otherwise, there is an impasse – the transaction (or cooperation) is not undertaken, the surplus is not generated, and neither party receives anything. While this represents a very basic and arguably pervasive situation, its resolution is not obvious. In this paper, I posit that individuals try to resolve this situation of strategic uncertainty by paying attention and responding to the antecedents (i.e. the recent history of demands) in a manner that is determined by a behavioural rule. The objective of this paper is to analyse the evolutionary stability of a population comprised of such behavioural rules, and explore the implication of stability for the manner in which the pie is divided in this bargaining game.

I consider a population of individuals (which may be finite or be described by individuals distributed over a continuum of fixed mass), where each individual interacts with all other individuals in a pair-wise fashion to play the bargaining game. The primary component of the decision-process which governs the demand that an individual will make is his behavioural rule: individuals draw a random sample of demands that have been made in the previous time period, and respond to the distribution of demands in the sample in a manner dictated by his behavioural rule. I impose a weak restriction, namely 'mild responsiveness', on each behavioural rule. (I argue in Section 2 that this is the weakest condition that should be satisfied by any reasonable behavioural rule that responds to the information about demands made in the past.) The demands that the individuals make determines the outcome of the bargaining game. The fitness of a behavioural rule is a function of the average share of the pie received by individuals that follow that particular behavioural. I differentiate between two cases: one, where the population is homogeneous, i.e. all individuals are described by the same (mildly responsive) behavioural rule, and the other when the population is heterogeneous, i.e. not all individuals are described by the same (mildly responsive) behavioural rule. Then, a population comprised of the incumbent behavioural rules is said to be stable if there exists a state (or configuration of demands made by individuals in the population) such that the following two conditions are satisfied:
(i) internal stability: the fitness of all incumbent behavioural rules must be the same; if not, then it is possible that a fitter incumbent behavioural rule displaces an incumbent behavioural rule that is less fit, leading to the incumbent population composition not being stable, and,
(ii) external stability: suppose that an individual from a particular behavioural rule mutates

to, or experiments with, another (mildly responsive) behavioural rule (in case of population defined by a continuum of individuals, I suppose that a fraction of individuals from a particular behavioural rule mutate to, or experiment with, another (mildly responsive) behavioural rule); external stability requires the incumbent behavioural rule(s) to be fitter than any such mutant behavioural rule. Here, I appeal to an un-modelled dynamic process that selects in favour of fitter behavioural rules, and against less fit behavioural rules – then, external stability is a necessary condition for reversion to the initial population composition.

While both these conditions need to be satisfied for stability of a heterogeneous population, it is self-evident that only the external stability condition is relevant for a homogeneous population. I also define the weaker notion of neutral stability, which requires, in addition to internal stability (in case of heterogeneous population), that whenever an individual (in case of a finite population) or a fraction of individuals (in case of an infinite population) from a particular behavioural rule mutates to, or experiments with, another (mildly responsive) behavioural rule, then the incumbent behavioural rules should not be less fit than the mutant behavioural rule. In particular, a population is neutrally stable if there exists a state where the incumbent and mutant behavioural rules are equally fit: the interpretation is that here, all the behavioural rules co-exist without displacing one another.

Interestingly, I show that internal stability has a striking implication: a necessary condition for a state to be stable is that an individual must always choose the same demand, and this holds for all individuals. Thus, even though each individual actually responds to the demands made by in the past, a necessary condition for stability is that the configuration of demands is such that it is *as-if* individuals are hard-wired to play the same strategy. I call this the hardwired behaviour-responsive behaviour equivalence theorem. While this result is not relevant for a homogeneous population, it significantly simplifies the analysis of external stability for a heterogeneous population.

The application of the external stability criterion gives the result that there does not exist any state for any combination of incumbent behavioural rule(s) that is stable against all mutant behavioural rules. This implies that a population made up of any particular mildly responsive behavioural rule, or any combination of mildly responsive behavioural rules, is unstable. However, any homogeneous or heterogeneous incumbent population of behavioural rules may be 'neutrally stable', i.e. the incumbent behavioural rule(s) may be as fit as the mutant, but this happens if and only if each individual in the incumbent population asks for exactly half of the pie. Thus, the state where all individuals split the surplus equally is the unique neutrally stable state for any particular mildly responsive behavioural rule/combination of mildly responsive behavioural rules, and all other states are neutrally unstable.

The primary contribution of the paper, as alluded to above, is to propose a notion and develop a framework for analysing the evolutionary stability of behavioural rules (rather than evolutionary stability of strategies), which is then applied to the bargaining game. The conventional notion of evolutionary stability, introduced in Smith and Price (1973), assumes that each individual is hard-wired to play a fixed strategy, and the analysis is thus restricted to stability of a particular strategy (in case of a 'monomorphic' population), or to a particular invariant mix of strategies (in case of a 'polymorphic' population). However, a more reasonable assumption in socio-economic contexts is that individuals, instead of being hard-wired to play the same strategy over time, do in fact respond to information by using a behavioural rule. Then, the object of interest is the stability of a population comprised of either of a particular behavioural rule or a mix of different behavioural rules, and the consequent implication of stability for how the surplus is shared.

Related Literature

A variety of approaches have been proposed to resolve the bargaining problem. Nash (1950) and Kalai and Smorodinsky (1975) put forth axiomatic solutions, while Rubinstein (1982) examines the equilibrium of the alternating offer bargaining game. In contrast, this paper follows an evolutionary approach, and analyses the stability of a population comprising of various behavioural rules, and examines the implication of stability of the population for the division of the pie. Other papers which also adopt an evolutionary approach to the bargaining problem can be broadly classified into two categories.

(a) Firstly, papers such as Skyrms (1994) and Ellingsen (1997) examine evolutionary stability of strategies in the bargaining game. Skyrms (1994) argues that the state where all individuals demand exactly half of the pie is the only stable 'monomorphic' population configuration (i.e. the only evolutionary stable state when all individuals in the population choose an identical time-invariant demand). There may exist stable polymorphic configurations (where different individuals choose different but time-invariant strategies), but with random shocks, the population would tend to gravitate towards the stable equal split monomorphic state. Ellingsen (1997) studies a framework where there are some obstinate individuals who are hard-wired to make the same demand, and some responsive agents who are able to identify their opponent and choose their strategy accordingly. Importantly, Ellingsen assumes that responsive agents are not actually responsive against each other but only against the obstinate agents. Using the term 'fair' for obstinate agents who demand exactly half of the pie, Ellingsen shows that any stable population will comprise only of fair agents and responsive agents, with the fair agents forming the majority of the population. Further, with some perturbation in the size of the pie, a stable population profile consists only of fair agents.

(b) Secondly, Young (1993) studies an adaptive play model of the bargaining game where individuals respond to the recent history of play by best-responding to the distribution of demands that is obtained by drawing a random sample from the history of play. Young (1993) goes on to demonstrate that the stochastically stable division of the pie corresponds to an asymmetric Nash bargaining solution. This framework is extended by Binmore, Samuelson and Young (2003) to consider different best-response protocols, and by Saez-Marti and Weibull (1998) and Khan and Peeters (2014) to the case where individuals may exhibit 'higher-order' best responses.

This paper differs from both these strands in the literature. As has been discussed earlier, the primary difference with Skyrms (1994) or Ellingsen (1997) is that here, individuals respond to the manner in which the bargaining game has unfolded in the past, and this leads me to develop and analyse the stability of behavioural rules rather than stability of strategies. This impetus on evolutionary stability of behavioural rules, where I look at a large class of such rules rather than very specific rules, differentiates this paper from Young (1993) and the other related papers referred to above – while the decision-makers in these papers respond to the way in which the game has unfolded, the only behavioural rule considered is best-responding, or some variant of it; further, their objective is not to analyse the stability of the behavioural rule but to determine the stochastically stable state of the game for a specific choice of a behavioural rule. In this context, the behavioural rule approach used in this paper also generalises the sampling best response studied in Oyama, Sandholm and Tercieux (2015). Existing papers that examine the interaction between individuals who use different behavioural rules include Kaniovski, Kryazhimskii and Young (2000), Juang (2002), Josephson (2009) and Khan (2018). However, as in Young (1993), the aim of these papers is not the stability of the behavioural rules but on the stochastically stable set when individuals behave according to disparate behavioural rules.

Plan of the paper

In section 2, I develop the notion of stability of behavioural rules for the finite population version of the bargaining game, and present the result of the analysis of this framework. In Section 3, motivated by the observation in Schaffer (1988) that evolutionary stable strategies for a finite population may differ from the evolutionary stable strategies when the population is described by individuals distributed over a continuum of fixed mass, I extend the finite population framework to the case of a continuum of individuals, and show that results obtained for a finite population carry over to this case as well. Section 4 concludes.

2 The bargaining game: a finite population

Consider a finite population comprised of n individuals. Each individual interacts in pairs with all the $n-1$ other individuals, and bargains over a pie/surplus of unit size by simultaneously announcing the share of the pie that they claim for themselves. The individuals in a pair receive their respective claims if the two claims are compatible, i.e. their sum does not exceed unity; otherwise they receive nothing. In a given time period t, each individual makes the same demand in his interaction with all other individuals, and cannot update his demand between interactions in that period. Let $x_{i,t} \in (0,1)$ be the demand made by individual i in period t; I assume that individuals do not demand the entire pie, and neither do they demand nothing at all. $x_t = (x_{1,t}, \ldots, x_{n,t})$ is the vector of demands made the n individuals in period t. I denote individual i's period t payoff from his interaction with another individual j, who chooses $x_{j,t} \in (0,1)$, by $\pi(x_{i,t}, x_{j,t})$. $\pi(x_{i,t}, x_{j,t})$ equals $x_{i,t}$ if $x_{i,t} + x_{j,t} \leq 1$, and 0 otherwise. The total payoff of individual i in period t is $\pi_{i,t}(x_t) = \frac{1}{n-1} \Sigma_{j \in N \setminus \{i\}} \pi(x_{i,t}, x_{j,t})$, i.e. the average share of the pie he receives in his bargaining interactions in that period. $\pi_t(x_t) = (\pi_{1,t}(x_t), \ldots, \pi_{n,t}(x_t))$ is the vector of payoffs received by the n individuals in period t.

I now describe the manner in which an individual i chooses his demand. This decision process has two components, each of which is described below: a response set that is derived from a behavioural rule, and an inward-looking set; each element in these two sets has strictly positive probability of being chosen.

(a) With positive probability $p_i \in (0,1)$ (which may vary by individual), at the end of period t, individual i draws a random sample (without replacement) of strategies of exogenously fixed size S (with $S \leq n$) from x_t (i.e. the strategies used in period t). A *feasible sample* is defined as any sample of size S that can be drawn without replacement from the demands made in the previous period. Depending on the value of S, he may obtain the entire distribution of strategies (when $S = n$), or a strict subset of the strategies used (when $S < n$). In case of the latter, any feasible sample has a strictly positive probability of being drawn; however, it is possible that particular samples comprised of demands made by specific individuals have a higher probability of being drawn. Let $s_{i,t}$ denote the sample drawn by individual i. $supp(s_{i,t}) = \{x : x \in s_{i,t}\}$ is the support of sample $s_{i,t}$, and represents the strategies that appear in the randomly drawn sample $s_{i,t}$. Individual i associated with a time-invariant *behavioural rule* $R_i : (0,1)^S \rightrightarrows (0,1)$ that goes from the space of feasible samples to a set of demands in the interval $(0,1)$. $R_i(s_{i,t})$ is the *response set* of individual i corresponding to the sample $s_{i,t}$, and each element of the response set has strictly positive probability of being played by him.

I define individual i to be *mildly responsive*, or alternatively his behavioural rule R_i to be *mildly responsive*, if $supp(s_{i,t}) = \{x\}$ implies $R_i(s_{i,t}) \subset \{x, 1-x\}$, for any time period t and any $x \in (0,1)$. A few salient comments on mildly responsive behavioural rules are as follows. Firstly, mild responsiveness does impose any restriction whatsoever on the response set when the support of the sample is non-singleton. Secondly, mild responsiveness allows for a variety of possibilities for the behavioural rule, even when the support of the drawn sample is a singleton; for example if $supp(s_{i,t}) = \{x\}$, then:

(i) the behavioural rule might imply that the individual assumes that the his co-players'

demands will be drawn from the same distribution as the sample; since $supp(s_{i,t}) = \{x\}$, he expects his co-players to demand x, and so he demands $R_i(s_{i,t}) = \{1-x\}$ for all $x \in (0,1)$,
(ii) the behavioural rule might result in the individual expecting that his co-players will respond by demanding $1-x$; so anticipating this, he demands $R_i(s_{i,t}) = \{x\}$ for all $x \in (0,1)$,
(iii) the behavioural rule might cause the individual to have higher order beliefs about how the co-players might respond, and depending on the degree of higher order belief, he demands x or $1-x$; hence, either $R_i(s_{i,t}) = \{x\}$ or $R_i(s_{i,t}) = \{1-x\}$ for all $x \in (0,1)$
(iv) it might be that $R_i(s_{i,t}) = \{x, 1-x\}$ for all $x \in (0,1)$, implying that due to reasons outlined above, the individual probabilistically demands x or $1-x$
(v) there exists some values of $x \in (0,1)$ for which $R_i(s_{i,t}) = \{x\}$, other values of x for which $R_i(s_{i,t}) = \{1-x\}$ and yet other values of x for which $R_i(s_{i,t}) = \{x, 1-x\}$, or some combination thereof.

I argue that mild responsiveness is the weakest condition that must be satisfied by any behavioural rule that responds to past information – after all, conditional on individual's response depending on the obtained sample, the situation where a sample comprises of only one particular strategy is the simplest decision making situation for an individual; if the $supp(s_i(t)) = \{x\}$ does not imply $R_i(s_{i,t}) \subset \{x, 1-x\}$, then the individual is not responsive to the obtained information at all. Hence, it is the weakest condition that should be satisfied by any behavioural rule that responds to information obtained from the sample. Pertinently, while the mild responsiveness is motivated by how a behavioural rule would induce an individual to respond to a sample whose support is a singleton, it is also consistent with imitative behavioural or imitation of demands chosen by others in the past (in case of imitation, $R_i(s_{i,t}) = \{x\}$ for all $x \in (0,1)$).

Now that the behavioural rules have been defined, let the population of n individuals comprises of K disparate mildly responsive behavioural rules: two behavioural rules R_I and R_J are disparate if there exists at least one random sample of demands $s \in (0,1)^S$ such that $R_I(s) \neq R_J(s)$, where $I, J \in \{1, \ldots, K\}$ and $I \neq J$. I use lower case letters such as i to denote an individual in the population, and upper case letters such as I to denote a behavioural rule without reference to the individual. So, if $R_i(s) = R_I(s)$ for all samples s, then the behavioural rule of the i^{th} individual is R_I; I use the notation $i \in I$ to denote that individual i belongs to behavioural rule R_I, and $PBR(i)$ to denote the *parent behavioural rule* of individual i (i.e. the behavioural rule to which individual i belongs). Let $n_I(t)$ be the number of individuals who belong to the behavioural rule R_I in period t, with $I \in \{1, \ldots, K\}$. Then $(n_1(t), \ldots, n_K(t))$, with $\Sigma_{I=1}^K n_I = n$, is the *composition vector* – it describes the composition of the population in time period t in terms of the number of individuals corresponding to each behavioural rule. $\pi_{I,t}(x_t)$ is the n_I dimensional vector of payoffs of individuals belonging to behavioural rule R_I, and $\min \pi_{I,t}(x_t)$ and $\max \pi_{I,t}(x_t)$ are the minimal and maximal element of $\pi_{I,t}(x_t)$. When $K = 1$, the population is homogeneous in that it comprises of only one behavioural rule; in this case I will denote the behavioural rule simply by R.

(b) At the end of period t, each individual knows about the demands made and payoffs received by the individuals who follow the same behavioural rule as him. Each individual i has an *inward-looking correspondence* $I_i : (0,1)^N \times [0,1]^N \rightrightarrows (0,1)$ that maps from the space of demands made and payoffs received by individuals of the same behavioural rule to an *inward-looking set* $I_i(x_t, \pi_t(x_t)) \subset \{x_{j,t} : j \in PBR(i)\}$, which is a subset of the strategies used by the individuals who belong to the same parent behavioural rule. With strictly positive probability $1 - p(i) \in (0,1)$, individual i's demand comes from the set $I_i(x_t, \pi_t(x_t))$. This inward-looking correspondence is very general in nature, and allows for a wide variety of protocols. Prominent example includes (i) inertia: an individual does not change his strategy and makes the same demand he had made, (ii) imitation of the strategy that resulted in the highest payoff in some subset of the individuals who follow the same behavioural rule as him, (iii) imitation of the most commonly used strategy in some subset of the individuals who follow the same behavioural rule as him.

Summarily, the state at the end of period t is described by $x_t = (x_{1,t}, \ldots, x_{n,t})$ i.e. the demands made by the n individuals. At the end of this period, with probability $p(i) \in (0,1)$, individual i draws a sample $s_{i,t}$ of size S from x_t, and his demand comes from his response set $R_i(s_{i,t})$, which is derived from the operation of the individual's behavioural on the sample, and each element of $R_i(s_{i,t})$ having strictly positive probability of being chosen; with probability $1 - p(i) \in (0,1)$, the individual's demand comes from his inward-looking set $I_i(x_t, \pi_t(x_t))$. Dropping the time sub-script, a state $x' = (x'_1, \ldots, x'_n)$ is denoted to be a *successor state* of $x = (x_1, \ldots, x_n)$ if $x'_i \in R_i(s_i) \cup I_i(x, \pi(x))$ for some feasible sample $s_i \subset x$, and this holds for all $i = 1, \ldots, n$. Let $Suc(x) = \{x' \in (0,1)^N : x'_i \in R_i(s_i) \cup I_i(x, \pi(x))$, for all $i = 1, \ldots, n\}$ denote the set of successor states of x, and let $Suc^z(x)$ be defined recursively as $Suc^z(x) = \{y \in (0,1)^N : y \in Suc(q)$ for all $q \in Suc^{z-1}(x)\}$ for all natural numbers $z > 2$. A state x is said to be *absorbing* if $Suc^\infty(x) = \{x\}$: this happens if and only if x_i is the only element in individual i's inward-looking set as well as his response set for any feasible sample that may be drawn from x, and this holds for all $i = 1, \ldots, n$; hence, in an absorbing state, an individual always makes the same demand, and this holds for all individuals. Thus, if a state is absorbing, it is not possible to transition from it to any other distinct state.

A state is defined to be a *convention* if a fixed demand $\bar{x}$ has been chosen by all n individuals. Here, I make the observation that once a convention, where all individuals demand $\bar{x}$, is reached, then the inward-looking set for any inward-looking correspondence comprises only of $\bar{x}$; it follows that if an individual chooses a demand $\tilde{x} \neq \bar{x}$ from the said convention, then it must be because $\tilde{x}$ belongs to the response set of the individual. Because I assume that the responsive behavioural rules to be mildly responsive, the equal-split convention (i.e. when $\bar{x} = \frac{1}{2}$) is always an absorbing state; in this state, both the inward-looking set (see argument above) and the response set of any mildly responsive individuals are singletons with $\bar{x} = \frac{1}{2}$ being their only element. If, on the contrary, the convention is characterized by $\bar{x} \neq \frac{1}{2}$, then the convention is absorbing if and only if the behavioural rule of all the individuals dictates that the any individual's response to a sample that is formed only of $\bar{x}$ is only $\bar{x}$ itself; if $1 - \bar{x}$ is an element of the response set of at least one individual, then such a convention is not absorbing.

The stability of a population comprising of K behavioural rules depends on the *fitness* of the incumbent behavioural rules in face of an invasion by a mutant behavioural rule. The fitness of a behavioural rule at the state x_t, $F_I(\pi_{I,t}(x_t)) : [0,1]^N \rightrightarrows [0,1]$, is a function of the vector of payoffs obtained by individuals of the behavioural rule R_I in that period. The fitness function may vary by behavioural rule, and the only restriction I impose is $F_I(\pi_{I,t}(x_t)) \in [\min \pi_{I,t}(x_t), \max \pi_{I,t}(x_t)]$ (where $\min \pi_{I,t}(x_t)$ and $\max \pi_{I,t}(x_t)$ are the minimal and maximal elements of the vector $\pi_{I,t}(x_t)$ of payoffs received by individuals following behavioural rule R_I). Thus, the fitness of a behavioural rule must lie in between the maximum and minimum payoff in the sub-population of individuals following the behavioural rule. When the population is homogeneous (i.e. $K = 1$), I use the terms fitness of the incumbent behavioural rule and fitness of the incumbent population interchangeably, and denote the fitness of the homogeneous population by $F(\pi_t(x_t)) \in [\min \pi_t(x_t), \max \pi_t(x_t)]$.

Now, suppose that one individual m becomes a mildly responsive mutant, with the mutant's mildly responsive behavioural rule being denoted by R_m. The particular behavioural rule from which the mutant emerges is termed the *source behavioural rule*, and the mutation may be interpreted as an individual experimenting with another behaviour rule. Suppressing the time-index, the payoff of the mutant when it chooses $x_m \in (0,1)$ and the state is x (suppressing the time subscript) is $\pi_m(x) = \frac{1}{n-1} \Sigma_{i \in N \setminus \{m\}} \pi(x_m, x_i)$, i.e the average share of the pie the mutant receives in its $n-1$ bargaining interactions. I equate the fitness of the mutant behavioural rule, $F_M(x)$, with $\pi_m(x)$. I assume that $n_I \geq 2, \forall i \in \{1, \ldots, K\}$ – since I posit that mutation involves an individual from an existing behavioural rule mutating to another behavioural rule, this assumption ensures that each of the incumbent behavioural rules is still represented in the population after the mutation; as a result, the fitness of the mutant behavioural rule can be compared to the fitness of all incumbent behavioural rules.

Since the population, in general, comprises of multiple behavioural rules, I impose two conditions, namely internal stability and external stability, for the population to be stable. The stability notions to be presented are motivated by an un-modelled selection dynamic whereby a fitter behavioural rules grow in representation at the expense of less fit behavioural rules. The notion of stability used here is an adaptation of the standard notion of evolutionary stable strategies (introduced in Smith and Price (1973)), which requires that for a strategy to be stable, a mutant (which plays another strategy) must always receive lower payoff/fitness than the incumbents. I adapt the standard stability definition to account for two features.

Firstly, the population here is finite whereas the more conventional framework assumes a population defined over continuum (of unit mass, for example), which is then invaded by a small proportion of mutants.[1] Secondly, and more importantly, in the more conventional model of evolutionary stability, it is assumed that each individual in the population is hard-wired to play a particular action/strategy. While this may be more relevant in a biological context, it is probably less so in situations where individuals do in fact deliberate over their action and follow a rule or a heuristic to choose an action. In this paper, it is the behavioural rule (or the corresponding response correspondence) that is the primitive, and individuals may change their strategy over time even as they follow their behavioural rule. This has two implications:

(i) In the standard model of evolutionary stability, since each individual's strategy remains invariant, the distribution of strategies used in the incumbent population is invariant as well. However, here, this may not hold as individuals may change their strategies in response to the history of play. Hence, in the definition of stability, I will require that a mutant should not be fitter than an incumbent not only in the particular state in question, say x, but also in any other state that the state may transition into with positive probability (i.e. all states in $Suc^{\infty}(x)$); only then can one say with certainty that starting from that particular state x, the mutant behavioural rule will never successfully invade an incumbent population.

(ii) In the standard model of evolutionary stability, all individuals in the incumbent population receive the same payoff (as all individuals play an identical strategy), due to which the fitness of the incumbent population can be equated with the identical payoff received by an individual in the incumbent population. Here, two individuals belonging to the same incumbent population may have different payoffs because they may use different strategies – this may happen either because they draw different samples or because the response correspondence is not single-valued. This necessitates the use of a fitness function.

A state x is said to be *internally stable* if $F_I(\pi_I(y)) = F_J(\pi_J(y))$ for all behavioural rules $I, J = 1, \ldots, K$ and for all $y \in x \cup Suc^{\infty}(x)$, i.e. all behavioural rules are equally fit, not only in the state x in question but also in all states that the particular state may transition to with positive probability. Recall that since individuals may respond to historical demands, the state may change over time. Thus, internal stability is a pre-mutant-entry condition, and the equal fitness of all behavioural rules, not only in the particular state in question but in all the states that may succeed the particular state, is meant to suggest that the composition vector of the incumbent population (i.e. the number of individuals following each behavioural rule) remains stable/unchanged over time. Since a homogeneous population has only one behavioural rule, the internal stability condition is relevant only for a heterogeneous population.

The external stability condition, on the other hand, compares the fitness of incumbent behavioural rules and the mutant behavioural rule. A particular state x is *externally stable* for

[1] Schaffer (1988) also adapts the notion of evolutionary stable strategies to account for a finite population.

a particular mix of K incumbent behavioural rules if $F_m(y) < F_I(\pi_{I,t}(y))$ for all $I = 1, \ldots, K$ and for all states $y \in x \cup Suc^{\infty}(x)$, i.e. the fitness of each incumbent behavioural rule is strictly higher than the fitness of the mutant behavioural rule not only in that state but all states that the bargaining game may transition to with positive probability. I consider this to be a *necessary condition* for a reversion to the initial population composition by virtue of the un-modeled selection process selecting against the mutant behavioural rule.

I will also say a state is *neutrally externally stable* (i) if $F_m(y) \leq F_I(\pi_{I,t}(y))$ for all $I = 1, \ldots, K$ and for all states $y \in x \cup Suc^{\infty}(x)$, and (ii) if $F_m(y) = F_J(\pi_{J,t}(y))$ for some $J \in \{1, \ldots, K\}$, then $F_I(\pi_{J,t}(y)) = F_J(\pi_{J,t}(y))$ for $I, J \in \{1, \ldots, K\}$ That is, (i) the fitness of each incumbent behavioural rule is not lower than the fitness of the mutant behavioural rule, and (ii) in case the fitness of the mutant behavioural rule is equal to the fitness of at least one incumbent behavioural rule, then the fitness of all incumbent behavioural rules is the same. In absence of the latter condition, one incumbent behavioural rule might displace another incumbent behavioural rule via the selection dynamic if the lowest fitness is obtained by both the mutant behavioural rule and by an incumbent behavioural rule, and there exists another incumbent behavioural rule that obtains a higher fitness than these two rules – this is the motivation for condition (ii) above.

A state is said to be *stable (neutrally stable)* for the population under consideration if the internal stability and external stability (neutral external stability) conditions are satisfied for all mildly responsive mutant behavioural rules. As mentioned earlier, a state being stable is a necessary requirement for the population to revert to its initial composition after the mutant behavioural rule appears, while the interpretation of a neutrally stable state is that in such a state, a the mutant behavioural rule cannot further displace incumbent behavioural rule(s). On the other hand, if the internal stability and external stability (neutral external stability) conditions are not satisfied for at least one mildly responsive mutant behavioural rule, then the state is an *unstable state (neutrally unstable state)*. It follows that if a state is stable (neutrally unstable), it is also neutrally stable (unstable). I will say that a population of incumbent behavioural rules is stable (neutrally stable) if there exists a state that is stable (neutrally stable); otherwise the population is said to be unstable (neutrally unstable). In the particular case of a homogeneous population, a state is stable for a particular behavioural rule if the internal stability condition and external stability condition hold for all sizes of the finite population (i.e. for all n); if all states are unstable (neutrally unstable), then the incumbent behavioural rule is said to be an *unstable (neutrally unstable) behavioural rule*, and I will also say that the incumbent and mutant behavioural rules *co-exist* when $\pi_m = F((\pi_i)_{i \in N \setminus \{m\}})$, where $F((\pi_i)_{i \in N \setminus \{m\}})$ is the fitness of the incumbent behavioural rule after the entry of the mutant in the homogeneous population.

I denote the set of states where all individuals of a behavioural rule make the same demand by $\bar{\Omega}$, and make the observation that *(i)* stability (neutral stability) of at least one state in $\bar{\Omega}$ is a necessary and sufficient condition for the existence of a stable state (neutrally stable state), and *(ii)* instability (neutral instability) of all states in $\bar{\Omega}$ is a necessary and sufficient condition for the non-existence of a stable state (neutrally stable state). The reason is as follows. Consider a state that does not belong to $\bar{\Omega}$; with positive probability, all individuals of a particular behavioural rule choose the same sample, in response to which all of them make the same demand, and so, the bargaining process reaches a state $x_t \in \bar{\Omega}$. In case of a homogeneous population, this implies that, with positive probability, the bargaining process transitions into a convention from any other state. Now, by the definition of stability (neutral stability), if x_t is not stable (neutrally stable), then the initial state cannot be stable (neutrally stable) either; this follows from the fact that initial state leads to x_t with positive probability, and so, for the former to be stable (neutrally stable), x_t has to be stable (neutrally stable) as well. Hence instability (neutrally instability) of all states in $\bar{\Omega}$ is sufficient to conclude that all states are unstable (neutrally unstable); it is also trivially true that the instability of all states in $\bar{\Omega}$ is necessary for all states to be unstable. So, stability of at least one state in $\bar{\Omega}$ is necessary for a stable state (neutrally stable) to exist; but at the same time, if a state in $\bar{\Omega}$ is stable (neutrally stable), then a stable state (neutrally stable) exists. Thus, stability (neutrally stability) of at least one state in $\bar{\Omega}$ is necessary and sufficient for a stable state (neutrally stable) to exist. Consequently, for the analysis of stability, it suffices to examine states in the set $\bar{\Omega}$.

There are two other implications of states in the set $\bar{\Omega}$. Firstly, for any such state, the inward-looking set is a singleton and comprises only of the demand chosen by the individual himself. Hence, even though the inward-looking correspondence is very general, in states in the set $\bar{\Omega}$, it is *as-if* the inward-looking correspondence is equivalent to inertia. Secondly, in case of heterogeneous populations, all individuals belonging to the same behavioural rule receive the same payoff in a state in $\bar{\Omega}$ (as they make the same demand). Furthermore, because $F_I(\pi_{I,t}(x_t)) \in$[min $\pi_{I,t}(x_t)$, max $\pi_{I,t}(x_t)$], for all $I \in \{1, \ldots, K\}$, the fitness of behavioural rule R_I equals the identical payoff received by each of its constituent individuals. Hence, in any state in the set $\bar{\Omega}$, the payoff of an individual belonging to a particular behavioural rule, and the fitness of that behavioural rule itself can be used as interchangeable terms. In addition, if internal stability is satisfied, then all behavioural rules have equal fitness. This implies that if a state in the set $\bar{\Omega}$ is internally stable, then the payoff received by all individuals of the population is the same. I summarise this in the following observation.

Observation 1. *(i) With positive probability, from any (other) state, the bargaining process transitions into a state where all individuals of a behavioural rule make the same demand. This implies that for a homogeneous population, with positive probability, the bargaining game reaches a convention from any other state.*

(ii) In context of stability (neutral stability) of heterogeneous populations, stability (neutral stability) of at least one state where all individuals of a behavioural rule make the same demand is a necessary and sufficient condition for the existence of a stable state (neutrally stable state), and instability (neutral instability) of all states where all individuals of a behavioural rule make the same demand is a necessary and sufficient condition for the non-existence of a stable state (neutrally stable state). In such states, where all individuals of a behavioural rule make the same demand, if the internal stability condition is satisfied, then all individuals in the population receive the same payoff, and this payoff is also equal to the fitness of each behavioural rule.

(iii) In context of stability of a homogeneous population, stability (neutral stability) of at least one convention is a necessary and sufficient condition for the existence of a stable state (neutrally stable state), and instability (neutral instability) of all conventions is a necessary and sufficient condition for the non-existence of a stable state (neutrally stable state).

(iv) In any state where all individuals belonging to a particular behavioural rule make the same demand, any inward-looking correspondence gives the same inward-looking set as would be obtained by assuming inertia.

I will first analyse the implication of internal stability for a heterogeneous population, and because of the above observation, I will focus on states in the set $\bar{\Omega}$. Since all individuals of a behavioural rule make the same demand in a state in $\bar{\Omega}$, let X_I denote the demand made by individuals of the behavioural rule R_I, where $I \in \{1, \ldots, K\}$. Then the *demand vector* $(X_1, \ldots, X_K)$ gives the demands made by individuals of each of the K behavioural rule. Let $x(1) < \ldots < x(\bar{k})$ represent the distinct demands made in the state (belonging to set $\bar{\Omega}$) whose stability is under consideration. So, $x(1)$ and $x(\bar{k})$ represent the minimum and maximum demands made in that state respectively. Since the vector $(x(1), \ldots, x(\bar{k}))$ can be derived from the demand vector $(X_1, \ldots, X_K)$, I call the former the *reduced demand vector*. Whenever it is more convenient, I also use $x_{min} = x(1)$ and $x_{max} = x(\bar{k})$. Since all individuals belonging to a behavioural rule make the same demand, and because more than one behavioural rule may make the same demand, it must be that $1 \leq \bar{k} \leq K$.

The *composite behavioural rule* $\hat{R}(i)$ comprises of the set of behavioural rules such that all individuals of the behavioural rules in $\hat{R}(i)$ demand $x(i)$. The number of individuals who demand $x(i)$, i.e. the number of individuals of the composite behavioural rule $\hat{R}(i)$, is denoted by $n(i)$. Let $n^{-1}(i)$ be the number of individuals in the population who demand no more than $1 - x(i)$; hence, $n^{-1}(i)$ is the number of successful bargains made by individuals who make the i^{th} lowest demand equal to $x(i)$. By definition, $n^{-1}(1) \geq \ldots \geq n^{-1}(\bar{k})$. Furthermore, the payoff of an individual demanding $x(i)$ is $\frac{n^{-1}(i)}{n-1} x(i)$; this is the fitness not only of his parent behavioural rule but also the fitness of each behavioural rule in $\hat{R}(i)$ as well. Equality of payoff/fitness of a behavioural rule in $\hat{R}(i)$ and a behavioural rule in $\hat{R}(j)$

(recall internal stability) implies $\frac{n^{-1}(i)}{n-1}\, x(i) = \frac{n^{-1}(j)}{n-1}\, x(j)$. Thus, if a behavioural rule that demands a lower share of the pie is as fit as a behavioural rule that asks for a higher share, then former must strike sufficiently greater number of bargains than the latter. Hence, in an internally stable state, the inequality $n^{-1}(1) \geq \ldots \geq n^{-1}(\bar{k})$ above must hold strictly, i.e. $n^{-1}(1) > \ldots > n^{-1}(\bar{k})$. I call the vectors $(n(1), \ldots, n(\bar{k}))$ and $(n^{-1}(1), \ldots, n^{-1}(\bar{k}))$ the *reduced composition vector* and the *reduced matching vector* respectively.

In the lemmata to follow, I will assume that at least two distinct demands are made in any state in the set $\bar{\Omega}$ (i.e. $x_{min} \neq x_{max}$) – this is because I will show in Proposition 3 (to follow later) that any state where individuals from all behavioural rules make the same demand (i.e. $x_{min} = x_{max}$) is neutrally unstable for any heterogenous population. In addition to $x_{min} \neq x_{max}$, I will also assume that there is at least one behavioural rule that obtains positive fitness – an implication of this is that there is at least one behavioural rule whose individuals choose a demand less than one-half (i.e. $x_{min} \leq \frac{1}{2}$); otherwise, i.e. if $x_{min} \neq x_{max}$ and $x_{min} \leq \frac{1}{2}$, then I show in Observation 2 (to follow) that a mutant behavioural rule can obtain higher fitness than an incumbent behavioural rule, leading to the state being neutrally unstable for any heterogenous population. I will now show that internal stability substantially narrows down the candidate states that may be stable. In particular:

- In Lemma 1, I demonstrate that in an internally stable state belonging to the set $\bar{\Omega}$, the lowest demand must be strictly less than one-half, and this lowest demand is always compatible with the demand made by any other individual.

- In Lemma 2, I show that the latter part of the previous sentence is true only for the individuals who make the lowest demand, i.e. in an internally stable state in the set $\bar{\Omega}$, any demand that is not the lowest demand is incompatible with the demand made some individuals. In fact, the individuals who make the i^{th} lowest demand find that their demand is compatible only with the individuals who make the less than $(i-1)^{th}$ highest demand; the corollary of this is that individuals who make the i^{th} highest demand find that their demand is compatible only with the individuals whose demand is at most equal to the i^{th} lowest demand. In Lemma 2, I also prove by construction that if an internal stable state in the set $\bar{\Omega}$ exists, then there is at most one reduced composition vector for a given reduced demand vector.

- In Lemma 3, I show that in an internally stable state belonging to the set $\bar{\Omega}$, the demand of half of the pie acts as a divider in the sense that the 'median' demand (median defined suitably for the case when $\bar{k}$ is even) in the reduced demand vector is less than half but the demand just above the median demand (in the reduced demand vector) is greater than half.

These lemmata culminate in Lemma 4, which establishes a *hardwired behaviour-responsive behaviour equivalence theorem* which says that even though individuals respond to past information, a *necessary condition* for a state in the set $\bar{\Omega}$ to be internally stable is that each individual should always make the same demand in response to any sample that he may draw from the vector of demand, and this holds for all individuals. This implies that individuals play *as-if* they were hardwired to make the same demand, and so, the stability analysis when individuals are responsive to the demands made by individuals in the population is similar to the stability analysis when they are hardwired to make a particular demand.

Lemma 1. *Consider any state where all individuals of a particular behavioural rule make the same demand. If* $\frac{1}{2} \geq x_{min} \neq x_{max}$ *and the internal stability condition holds, then* $x_{max} \leq 1 - x_{min}$ *and* $x_{min} < \frac{1}{2} < x_{max}$.

Proof. First, suppose by contradiction that $x_{max} > 1 - x_{min}$. Then, because $x_{max} > x_{min}$ (since $x_{min} \neq x_{max}$), $x_{max} > 1 - x_{min}$ (by supposition) and $x_{max}, x_{min} \in (0, 1)$, it must be that $x_{min} \leq \frac{1}{2}$ and $x_{max} > \frac{1}{2}$. Then, the individuals who demand x_{min} obtain a positive payoff (when they play against each other, or when they play against individuals who demand no more than $1 - x_{min}$).[2] But because $x_{max} > 1 - x_{min}$, it must be that $x_{max} > 1 - x(i)$ $\forall i \in [1, \ldots, \bar{k}]$, i.e. individuals who play x_{max} never reach an agreement, and hence receive a payoff of zero. It follows that the behavioural rules who demand x_{max} are less fit than the behavioural rules who demand x_{min}. Since this contradicts internal stability, it must be that $x_{max} \leq 1 - x_{min}$. But if $x_{max} \leq 1 - x_{min}$ and $x_{max} > x_{min}$ both hold, then $x_{min} = \frac{1}{2}$ is not possible; hence, $x_{min} < \frac{1}{2}$. This proves that $x_{min} < \frac{1}{2}$ and $x_{max} \leq 1 - x_{min}$ must hold.
Now, I will show that $x_{max} > \frac{1}{2}$ must also hold. So, if (by contradiction) $x_{min} < x_{max} \leq \frac{1}{2}$, then all individuals in the population make compatible demands in all their bargaining interactions. This implies $n^{-1}(1) = n^{-1}(\bar{k}) = n - 1$. Consequently, $\frac{n^{-1}(\bar{k})}{n-1} x_{max} > \frac{n^{-1}(1)}{n-1} x_{min}$, i.e. the behavioural rules who demand x_{max} have a higher payoff/ fitness than individuals who demand x_{min}, contradicting internal stability. Thus, $x_{min} < \frac{1}{2} < x_{max}$. ■

Corollary 1. *Consider any state where all individuals of a particular behavioural rule make the same demand,* $\frac{1}{2} \geq x_{min} \neq x_{max}$*, and internal stability holds. Then the fitness of each behavioural rule equals* x_{min}.

Lemma 2. *Consider any state where all individuals of a particular behavioural rule make the same demand. If* $\frac{1}{2} \geq x_{min} \neq x_{max}$ *and internal stability holds, then, for a population of fixed size (of n individuals), and for a given vector of reduced demands* $(x(1), \ldots, x(\bar{k}))$*:*
(i) $1 - x(\bar{k} - i) \geq x(i+1) > 1 - x(\bar{k} - i + 1), \forall i \in \{1, \ldots, \bar{k} - 1\}$
(ii) there exists at most one reduced composition vector $(n(1), \ldots, n(\bar{k}))$.

[2]Since I assume that there exists at least two individuals of a given behavioural rule, an individual who demands x_{min} meets at least another individual who demands x_{min}.

Proof. Because of internal stability, $\frac{n^{-1}(i)}{n-1}x(i) = \frac{n^{-1}(j)}{n-1}x(j)$ for any two composite behavioural rules $\hat{R}(i), \hat{R}(j)$. By Lemma 1, $x_{max} \leq 1 - x_{min}$; so, $n^{-1}(1) = n - 1$, and the payoff received by the behavioural rules demanding $x(1)$ is $x(1)$ itself. Then, $\frac{n^{-1}(j)}{n-1}x(j) = x(1)$ holds for all $j \in \{1, \ldots, \bar{k}\}$, and unique values of $n^{-1}(j)$ can be obtained.

Now, it must be that $x_{max} > 1-x(2) \geq x(\bar{k}-1)$, or alternatively, $1-x(\bar{k}) < x(2) \leq 1-x(\bar{k}-1)$. Firstly, if $x_{max} > 1-x(2)$ does not hold, then $x_{max} \leq 1-x(2)$ implies $n^{-1}(2) = n^{-1}(1) = n-1$, and so, $\frac{n^{-1}(j)}{n-1}x(j) > x(1)$ for $j = 2$, thereby violating internal stability. But $x_{max} > 1 - x(2)$ along with $x_{max} \leq 1 - x_{min}$ (from Lemma 1) implies $n^{-1}(\bar{k}) = n(1)$; since I have already shown that $n^{-1}(-j)$ is unique for all $j \in \{1, \ldots, \bar{k}\}$, this gives the unique value of $n(1)$.

Secondly, if $x(\bar{k} - 1) \leq 1 - x(2)$ does not hold, then $x(\bar{k} - 1) > 1 - x(2)$, and so $n^{-1}(\bar{k} - 1) = n^{-1}(\bar{k})$, thereby implying $\frac{n^{-1}(\bar{k}-1)}{n-1}x(\bar{k} - 1) < \frac{n^{-1}(\bar{k})}{n-1}x(\bar{k})$; but this violates internal stability. This shows that $1 - x(\bar{k}) < x(2) \leq 1 - x(\bar{k} - 1)$. Furthermore, $x(\bar{k} - 1) \leq 1 - x(2)$ implies $n^{-1}(\bar{k} - 1) \geq n^{-1}(\bar{k}) + n(2)$.

For the same reason, $1 - x(\bar{k} - 1) < x(3)$ must hold; for if not, $1 - x(\bar{k} - 1) \geq x(3)$ gives $n^{-1}(2) = n^{-1}(3)$, or, $\frac{n^{-1}(2)}{n-1}x(2) < \frac{n^{-1}(3)}{n-1}x(3)$; but this violates internal stability. Again, as before, $1 - x(\bar{k} - 1) < x(3)$ implies $n^{-1}(\bar{k} - 1) < n^{-1}(\bar{k}) + n(2) + n(3)$. Combining $n^{-1}(\bar{k} - 1) < n^{-1}(\bar{k}) + n(2) + n(3)$ and $n^{-1}(\bar{k} - 1) \geq n^{-1}(\bar{k}) + n(2)$ (from above), and taking note of the fact that $n^{-1}(\bar{k} - 1)$ cannot take any value strictly in between $n^{-1}(\bar{k}) + n(2)$ and $n^{-1}(\bar{k}) + n(2) + n(3)$, I get $n^{-1}(\bar{k} - 1) = n^{-1}(\bar{k}) + n(2)$; since I have shown that $n^{-1}(j)$ is unique for all $j \in \{1, \ldots, \bar{k}\}$, a unique value of $n(2)$ is obtained.

Continuing in this manner, it must be that $\forall i \in \{1, \ldots, \bar{k} - 1\}, x(\bar{k} - i) \leq 1 - x(i + 1) < x(\bar{k} - i + 1)$, or alternatively $1 - x(\bar{k} - i) \geq x(i + 1) > 1 - x(\bar{k} - i + 1)$, and so, the value of $n(i + 1)$ is obtained from $n^{-1}(\bar{k} - i) = n^{-1}(\bar{k} - i + 1) + n(i + 1)$. Hence, the solution of $n^{-1}(1), \ldots, n^{-1}(\bar{k})$ can be mapped back to obtain a unique solution for $n(1), \ldots, n(\bar{k})$. However, due to the population being finite, there may not exist a integer valued solution, thereby completing the proof. ■

Lemma 3. *Consider any state where all individuals of a behavioural rule make the same demand. Suppose $\frac{1}{2} \geq x_{min} \neq x_{max}$ and the internal stability condition holds. If $\bar{k}$ is even, then $x(\frac{\bar{k}}{2}) < \frac{1}{2}$ and $x(\frac{\bar{k}}{2} + 1) > \frac{1}{2}$, while if $\bar{k}$ is odd, then $x(\frac{\bar{k}+1}{2}) \leq \frac{1}{2}$ and $x(\frac{\bar{k}+1}{2} + 1) > \frac{1}{2}$.*

Proof. I will use the result $1 - x(\bar{k} - i) \geq x(i + 1) > 1 - x(\bar{k} - i + 1), \forall i \in \{1, \ldots, k - 1\}$ from Lemma 2.

Suppose $\bar{k}$ is even. Using $i = \frac{\bar{k}}{2}$ in $x(i + 1) > 1 - x(\bar{k} - i + 1)$ gives $x(\frac{\bar{k}}{2} + 1) > \frac{1}{2}$. Using $i = \frac{\bar{k}}{2}$ in $1 - x(\bar{k} - i) \geq x(i + 1)$ gives $x(\frac{\bar{k}}{2}) + x(\frac{\bar{k}}{2} + 1) \leq 1$; combined with $x(\frac{\bar{k}}{2} + 1) > \frac{1}{2}$, it implies $x(\frac{\bar{k}}{2}) < \frac{1}{2}$. This proves the part of the lemma for k even.

Now, suppose k is odd. Using $i = \frac{\bar{k}-1}{2}$ in $1 - x(\bar{k} - i) \geq x(i + 1)$ gives $x(\frac{\bar{k}+1}{2}) \leq \frac{1}{2}$. Using $i = \frac{\bar{k}-1}{2}$ in $x(i+1) > 1-x(\bar{k}-i+1)$ gives $x(\frac{\bar{k}+1}{2})+x(\frac{\bar{k}+1}{2}+1) > 1$; combined with $x(\frac{\bar{k}+1}{2}) \leq \frac{1}{2}$, it yields $x(\frac{\bar{k}+1}{2} + 1) > \frac{1}{2}$. This proves the part of the lemma for k odd. ■

Lemma 4. *Consider any state where all individuals of a behavioural rule make the same demand. Let X_I be the demand made by individuals of behavioural rule R_I. If $\frac{1}{2} \geq x_{min} \neq x_{max}$ and the internal stability condition holds, then $R_I(s) = \{X_I\}$ for any feasible sample s from the demand vector describing the state, and this holds for all $I \in \{1, 2, \ldots, K\}$.*

(Proof in the appendix)

Now, to put things together, Observation $1(i)$ informs that, with positive probability, the bargaining game transitions to a state in $\bar{\Omega}$. According to Observation $1(ii), (iii)$, for stability, it is sufficient to examine states in this set. By Observation $1(iv)$, in any state in $\bar{\Omega}$, the only element of the inward-looking set is the demand made by that individual in that state. Lemma 4 demonstrates that if a state in $\bar{\Omega}$ is internally stable, then the configuration of demands must be such that the only element in the response set of any individual to any random sample of demands must be the demand made by that individual in that state. This implies that if a state in $\bar{\Omega}$ is internally stable, then it must be an absorbing state. Hence, for the analysis of stability, it is sufficient to examine states in $\bar{\Omega}$ that are absorbing, and in these states, even though the individuals are actually (mildly) responsive, it is *as-if* they are hard-wired to play a particular strategy. I summarise this in the following *hardwired behaviour-responsive behaviour equivalence* theorem.

Theorem 1. *In the analysis of evolutionary stability of behavioural rules, even though individuals respond to demands made by other individuals, internal stability implies that it is sufficient to examine the absorbing states of the bargaining process. Hence, even though the individuals are responsive and may change the demand they make, a necessary condition for a state to be stable is that the demands in the state should be such that it is as-if individuals are hard-wired to play a particular strategy.*

It is easy to construct examples of internally stable states with different behavioural rules such that all individuals of each behavioural rule always make the same demand. For instance, suppose that a heterogeneous population comprises of three mildly responsive behaviorial rules, namely 'extreme optimism', 'extreme pessimism', and 'mode-responsiveness'. These behavioural rules are described below:

(a) when extremely optimistic individuals draw a sample of demands s, they assess that their co-players will claim the lowest demand in the sample (i.e. $min(s)$) , and hence, they respond by demanding $1 - min(s)$,

(b) when extremely pessimistic individuals draw a sample of demands s, they assess that their co-players will claim the highest demand in the sample (i.e. $max(s)$) , and hence, they respond by demanding $1 - max(s)$,

(c) when mode-responsive individuals draw a sample of demands s, they estimate that it is optimal to demand $1 - x_{mode}$, where x_{mode} is a modal demand in the sample

Suppose that the population size n is such that $\frac{2}{3}(n-1) > \frac{n}{2}$, and each individual samples the entire history (i.e. $S = n$). Also suppose that the extremely pessimistic individuals demand two-fifths of the pie and let the number of such individuals be $\frac{2}{3}(n-1)$. The rest of the population comprises of the extremely optimistic individuals and the mode-responsive individuals referred to above in some arbitrary proportion, and suppose that these individuals demand three-fifths of the pie. Then, it can be easily verified that each of the three mildly responsive behavioural rules continue to make the same demand in all ensuing states, and that the payoff/fitness of each behavioural rule equals two-fifths – hence, this state is internally stable.

As a consequence of the above theorem, I re-label and re-classify the different behavioural rules that make an identical demand (in response to any sample of past period's demand) as a single composite behavioural rule. Specifically, the behavioural rules which always choose $x(i)$ are grouped together into a single composite behavioural rule, namely $\hat{R}(i)$. Thus, in context of the example above, the re-labelling implies that there are only two distinct composite behavioural rules – one comprising of the extremely pessimistic individuals, and a composite behavioural rule comprising of the extremely optimistic individuals and the mode-responsive individuals.

At this point, I re-iterate that the internal stability condition and its derived implications are relevant only for a heterogeneous population. I now continue with the stability analysis making use of the external stability condition. I will first present the results relating to stability of a homogeneous population.

Proposition 1. *A homogeneous population described by any mildly responsive behavioural rule is unstable in the bargaining game.*

Proof. By Observation 1, a convention is reached (with positive probability) from any other state. So, suppose a mutant behavioural rule appears when the state is a convention where all individuals have demanded $\bar{x}$. First, suppose the incumbent behavioural rule is such that $supp(s) = \{\bar{x}\}$ implies $\bar{x} \in R(s)$. If the mutant behavioural rule is such that $supp(s) = \{\bar{x}\}$ implies $\bar{x} \in R_m(s)$, then all individuals (incumbent or mutant) demand $\bar{x}$ with positive probability, and so, the fitness of the mutant behavioural rule equals that of the incumbent behavioural rule. On the other hand, if the incumbent behavioural rule is such that $supp(s) = \{\bar{x}\}$ implies $1 - \bar{x} \in R(s)$, and the mutant behavioural rule is such that $supp(s) = \{\bar{x}\}$ implies $1 - \bar{x} \in R_m(s)$, then all individuals (incumbent or mutant) demand $1 - \bar{x}$ with positive probability, and so, the fitness of the mutant behavioural rule equals that of the incumbent behavioural rule. Since this situation occurs with positive probability from any state x, it follows that there does not exist any state and any mutant behavioural rule such that whenever a mutant enters, $\pi_m(x) < F((\pi_i(x))_{i \in N \setminus \{m\}})$ always holds. Hence, any behavioural rule is unstable. ■

Proposition 2. *(i) Any state apart from the equal split convention is a neutrally unstable state for a homogeneous population comprised of any mildly responsive behavioural rule.*
(ii) The equal split convention is a neutrally stable state for a homogeneous population comprised of any mildly responsive behavioural rule.

Proof. (i) By Observation 1, with positive probability, the bargaining process reaches a convention where all individuals demand $\bar{x}$. By the supposition in statement (i), $x \neq \frac{1}{2}$. Firstly, suppose that $\bar{x} < \frac{1}{2}$, and a mildly responsive mutant behavioural rule enters the population now. Consider a mutant behavioural rule R_m such that $supp(s) = \bar{x}$ implies $1 - \bar{x} \in R_m(s)$. Then, with positive probability, all incumbent individuals demand $\bar{x} < \frac{1}{2}$ (from the inward-looking set) while the mildly responsive mutant demands $1 - \bar{x} > \frac{1}{2}$. This results in the incumbents receiving $\bar{x} < \frac{1}{2}$ of the pie in every interaction (and hence, also on average), while the mutant receives $1 - \bar{x} > \frac{1}{2}$ of the pie in every interaction (and thus, also on average). As a result, the mutant's fitness (which equals $1-\bar{x} > \frac{1}{2}$) is strictly greater than that of any incumbent (the fitness of the incumbents being $\bar{x} < \frac{1}{2}$). Consequently, all such states are neutrally unstable states for any mildly responsive behavioural rule.
Secondly, suppose that $\bar{x} > \frac{1}{2}$, and a mildly responsive mutant behavioural rule enters the population now. Consider a mutant behavioural rule R_m such that $supp(s) = \bar{x}$ implies $1 - \bar{x} \in R_m(s)$. Then, with positive probability, all incumbent individuals demand $\bar{x} > \frac{1}{2}$ (from the inward-looking set) while the mildly responsive mutant demands $1 - \bar{x} < \frac{1}{2}$. This results in each of the incumbent individuals receiving $\bar{x} > \frac{1}{2}$ of the pie in their only interaction with the mutant while the mutant receives $1 - \bar{x} < \frac{1}{2}$ of the pie in every interaction. So, the payoff (i.e. average share received across all $n-1$ interactions) of an incumbent individual is $\frac{1}{n-1}\,\bar{x}$, while the payoff of the mutant is $1 - \bar{x}$. The fitness of the mutant behavioural rule is strictly greater than that of any incumbent behavioural rule when $1 - \bar{x} > \frac{1}{n-1}\,\bar{x}$, or $n > 1 + \frac{\bar{x}}{1-\bar{x}}$. Consequently, such states are neutrally unstable states for any mildly responsive behavioural rule.

(ii) I only have to show that the equal split convention is a neutrally stable state for any mildly responsive behavioural rule and any mildly responsive mutant behavioural rule. Once an equal-split convention is reached, it is absorbing, and so the incumbents never make any other demand. So, if a mildly responsive mutant appears in an equal-split convention, it also demands one-half. The incumbents and the mutant receive half of the pie on average. Hence, the fitness of the incumbent behavioural rule and the mutant behavioural rule is identical for any mildly responsive incumbent behavioural rule and for any mildly responsive mutant behavioural rule. Then, statement (ii) of the proposition follows. ■

Proposition 1 and Proposition 2 indicate that, in terms of stability, all mildly responsive behavioural rules are similar – sophisticated mildly responsive behavioural rules are as stable/unstable as relatively naive mildly responsive behavioural rules. Not only can a homogeneous population described any mildly responsive behavioural rule be invaded by a mildly responsive mutant rule but a mildly responsive mutant behavioural rule can further displace any mildly responsive incumbent behavioural rule unless the incumbent behavioural rule settles on the equal split convention. In the latter case (that the equal split convention is attained), any incumbent and mutant behavioural rules can co-exist without displacing one another. I summarise these results in the next theorem.

Theorem 2. *In context of a homogeneous population, all mildly responsive behavioural rules are unstable in the bargaining game. Furthermore, any state apart from the equal-split convention is neutrally unstable for any mildly responsive behavioural rule. However, the equal split convention is the unique neutrally stable state for any mildly responsive behavioural rule.*

The stability result presented above is based on the premise that a population comprises of a single behavioural rule – I consider this to be the analogue of the conventional notion of evolutionary stability of a monomorphic population. In what follows, I analyse the stability of a heterogeneous population, i.e. a population comprised of multiple behavioural rules – this may be considered to be the analogue of the conventional notion of evolutionary stability of a polymorphic population. In this analysis, I will only consider states that are internally stable, i.e. I will only consider states in $\bar{\Omega}$ that are absorbing.

First, consider a heterogeneous population with one composite behavioural rule. The arguments used for the case of a homogeneous population (recall Proposition 2) can be used to now argue that the equal-split state is the only neutrally stable. I summarise this in the next proposition (without proof, since the arguments have already been made in the proof of Proposition 2).

Proposition 3. *The equal split state, where all individuals demand half of the pie, is the only neutrally stable state for any heterogeneous population. Any other state where all individuals in a heterogeneous population make an identical demand is neutrally unstable.*

Now, suppose that there the heterogeneous population is comprised of at least two composite behavioural rules; so $x_{min} \neq x_{max}$. I now argue that in this case, a necessary condition for a state to not be neutrally unstable is $x(1) \leq \frac{1}{2}$ and $x(1) = 1 - x_{max}$ (by Lemma 1, internal stability requires $x(1) \leq 1 - x_{max}$):

(i) Firstly, suppose that there are at least two composite behavioural rules, and that $x(1) > \frac{1}{2}$. Then, none of the individuals obtain a positive share of the pie in any of their bargaining interactions, and the fitness of all behavioural rules equals zero. Now suppose that a mildly responsive mutant behavioural rule appears, the source behavioural rule being one where individuals demand a share greater $x(1)$, and the mutant demands $1 - x(1)$. Then, the mutant's fitness is strictly positive as it obtains $1 - x(1)$ share of the pie in all its bargaining interactions with individuals of the composite behavioural rule that demand $x(1)$; however, the fitness of all behavioural rules that demand a share greater than $x(1)$ is still zero. Since the fitness of the mutant behavioural rule is strictly higher than that of some of the incumbent behavioural rules, states where there exist at least two composite behavioural rules and $x(1) > \frac{1}{2}$ are neutrally unstable.

(ii) Secondly, if $x(1) \leq \frac{1}{2}$ but $x(1) < 1 - x(\bar{k})$, then a mildly responsive mutant may demand $x_m = 1 - x(1)$. Then, the number of successful bargains by the mutant is $n(1)$, which is also the number of successful bargains struck by individuals of the behavioural rules who demand $x(\bar{k})$; however, since the mutant obtains a higher share of the pie whenever he strikes a successful bargain, he obtains strictly higher fitness than the behavioural rules who demand $x(\bar{k})$. This results in the neutral instability of states comprised of at least two composite behavioural rules such that $x(1) < 1 - x(\bar{k})$. Thus, a necessary condition for neutral stability of a state is $x_{min} \leq \frac{1}{2}$ and $x(1) = 1 - x_{max}$. I summarise this in the following observation.

Observation 2. *In context of a heterogeneous population with at least two composite behavioural rules, a necessary condition for a state to not be neutrally unstable is* $x(1) \leq \frac{1}{2}$ *and* $x(1) = 1 - x(\bar{k})$.

Now, I first show in Proposition 4 that any state where there are exactly two composite behavioural rules in the heterogeneous population is unstable. In Proposition 5, I present the same result for a heterogeneous population with at least three composite behavioural rules.

Proposition 4. *All states are neutrally unstable for any heterogeneous population comprised of exactly two composite behavioural rules.*

Proof. Let the demand made by individuals of the two composite behavioural rules, $\hat{R}(1)$ and $\hat{R}(2)$, be $x_{min} = x(1)$ and $x_{max} = x(2)$ respectively. The population size is n. Because of Observation 2, a necessary condition for a state to not be neutrally unstable is $\frac{1}{2} \geq x_{min} = 1 - x_{max}$; so, I only consider such states. Internal stability implies that the payoff of each individual in the population must be $x(1)$ (recall Corollary 1). Each individual belonging to $\hat{R}(1)$ obtains a payoff of $x(1)$; in order for the same to hold for individuals of $\hat{R}(2)$, it must be that the number of individuals belonging to the behavioural rule who demand $x(1)$ is $\frac{1-x(2)}{x(2)}(n-1)$ – in this case (and only in this case), the payoff of an individual of composite behavioural rule $\hat{R}(2)$ equals $\frac{\frac{1-x(2)}{x(2)}(n-1)}{n-1}x(2) = 1 - x(2) = x(1)$. (The individuals belonging to the composite behavioural rule $\hat{R}(2)$ obtain a positive fraction of the pie when only they meet individuals belonging to $\hat{R}(1)$). Let me denote the fraction of individuals belonging to $\hat{R}(1)$ by q; so $q = \frac{\frac{1-x(2)}{x(2)}(n-1)}{n}$.

Suppose now that a mutant originates from the behavioural rule $\hat{R}(1)$, and demands $x_m = x(2)$. Then, the fitness of any individual in $\hat{R}(1)$ remains $1 - x(2)$, while the fitness of any individual in $\hat{R}(2)$ now is $\frac{(qn)-1}{n-1}x(2) < 1 - x(2)$. The mutant's fitness is $\frac{(qn)-1}{n-1}x(2)$. Since the mutant behavioural rule's fitness is at least as much as that of an incumbent behavioural rule, and because the fitness of all incumbent behavioural rules is not the same, all such states are neutrally unstable. ■

Proposition 5. *All states are neutrally unstable for any heterogeneous population comprised of at least three composite behavioural rules.*

Proof. If the state is not internally stable, then this is trivially true. So, in what follows, I will assume the state to be internally stable and I will use the external stability criterion to show that any internally stable state must be neutrally unstable.
Firstly, if $\bar{k} \geq 3$ and $\bar{k}$ is even, then, by Lemma 3, $x(\frac{\bar{k}}{2}) < \frac{1}{2}$, and $x(\frac{\bar{k}}{2} + 1) > \frac{1}{2}$. The payoff received by each behavioural rule equals $x(1)$ (by Corollary 1). Now, suppose an individual from the composite behavioural rule demanding $x(1)$ mutates and demands $x_m = x(\frac{\bar{k}}{2})$ instead. Then, the mutant's payoff is same as the payoff of the individuals demanding $x(\frac{\bar{k}}{2})$; since the number of successful bargains made by the individuals of the composite behavioural rule demanding $x(\frac{\bar{k}}{2})$ remains the same, this payoff remains $x(1)$. However, the payoff of the behavioural rules demanding $x(\bar{k})$ decreases. Prior to the appearance of the mutant, their payoff was $\frac{n^{-1}(\bar{k})}{n-1} x(\bar{k}) = x(1)$; now, due to the mutation, the number of successful bargains they make reduces by one, and so their payoff is less than $x(1)$. Hence, the fitness of this composite behavioural rule is strictly less than that of the mutant's, and the incumbent population is not neutrally stable.
Secondly, if $\bar{k} \geq 3$ and $\bar{k}$ is odd, then, by Lemma 3, $x(\frac{\bar{k}+1}{2}) \leq \frac{1}{2}$, and $x(\frac{\bar{k}+1}{2} + 1) > \frac{1}{2}$. More specifically, $x(1), x(2) \leq \frac{1}{2}$, and $x(\bar{k}) = 1 - x(1)$. The fitness/payoff of each behavioural rule equals $x(1)$ (by Corollary 1). Then, by internal stability, it must be that the fitness of the composite behavioural rule that demands $x(2)$ is $\frac{n^{-1}(2)}{n-1} x(2) = x(1)$. Now suppose an individual from the composite behavioural rule that demands $x(2)$ mutates and demands $x(\bar{k})$. Then, because $n^{-1}(\bar{k}) = n(1)$ does not change, the mutant's fitness (and the fitness of all individual demanding $x(\bar{k})$) is $\frac{n^{-1}(\bar{k})}{n-1} x(\bar{k}) = x(1)$. However, now, due to the mutation, the number of successful bargains made by the composite behavioural rule that demands $x(2)$ reduces by one, and so their payoff is less than $x(1)$. Thus, the source behavioural rule is less fit than the mutant behavioural rule. [More generally, suppose an individual from the composite behavioural rule that demands $x(i)$, where $i \in int(1, \frac{\bar{k}}{2}]$ if $\bar{k}$ is even or $i \in int(1, \frac{\bar{k}+1}{2})$ if k is odd, mutates and now demands $x_m \in \{x(\bar{k} - i + 1), \ldots, x(\bar{k})\}$. Then, the mutant's fitness is $x(i)$ but the source behavioural rule's fitness is strictly less than that.] Thus, any state is neutrally unstable. ■

The propositions presented above show that all states, apart from the equal split state where all individuals demand exactly one-half of the pie in their bargaining interactions, are neutrally unstable. Only in the equal split state is it possible to definitely say that each behavioural rule can stably co-exist, not only with each other, but with any mildly responsive mutant behavioural rule as well. I conclude this section by summarising these results in the following theorem.

Theorem 3. *In the bargaining game, any state apart from the equal-split state is neutrally unstable for any finite heterogeneous population. The equal-split state is the only state that is neutrally stable for any finite heterogeneous population, and hence, it is the only state where disparate behavioural rules can stably co-exist.*

3 The bargaining game with a continuum of individuals

The previous section establishes the instability of homogeneous and heterogeneous populations under the assumption that the population is finite. In this section, I will modify the finite population framework, and instead, consider a population of individuals distributed uniformly over a continuum of constant mass; without loss of generality, let individuals be uniformly distributed on the unit interval $[0, 1]$. The pertinence of this exercise comes from the observation in Schaffer (1988) that a strategy profile that is evolutionarily stable when the population is finite may not carry over to the case where individuals are distributed uniformly over a unit interval. Hence, the question here is: do the results for the finite population case carry over to this case? The answer to this question is in the affirmative, and in order to show this, I will cast the model in terms of the notation of the finite population case; then, it can be easily verified that the arguments made in the previous section hold, thereby proving the robustness of the results.

The state at a particular time is given by the demands made by the individuals. Each individual decides on the demand that he will make in each period in the same manner as described in the previous section:
(a) (suppressing the time-index) with probability $p(i) \in (0, 1)$, each individual $i \in [0, 1]$ samples fraction S ($S \in (0, 1]$ being fixed exogenously) of the demands. Let $s_i(x)$ be the cumulative distribution of demands derived from his randomly drawn sample. His behavioural rule R_i gives rise to a response set $R_i(s_i(x))$ and each element of $R_i(s_i(x))$ is chosen by individual i with positive probability. Suppose that the population comprises of K disparate behavioural rules. I assume that each of the extant behavioural rules is followed by a positive mass of individuals. Let the I^{th} behavioural rule be denoted by R_I, where $I \in \{1, \ldots, K\}$. Thus, individual i follows behavioural rule R_I if $R_i(s(x)) = R_I(s(x))$, for all cumulative distributions of demands $s(x)$ that may be generated from any sample of demands.
(b) (suppressing the time-index) with the complimentary probability $1 - p(i) \in (0, 1)$, individual i chooses a demand from his inward-looking set. As before, each individual is aware of the demands made in the previous period by individuals of the same behavioural rule, and the

inward-looking set is a subset of the set of these demands. Each element of the inward-looking set may be chosen by individual i.

The state of the bargaining game at a particular time t is given by the cumulative distribution function of demands $X_t(\cdot)$ defined over the interval $[0,1]$. So, $X_t(a)$ is the fraction of individuals who make a demand less than or equal to a, where $a \in (0,1)$. Now, consider any state. With positive probability, all individuals of a particular behavioural rule draw the same sample, and respond by choosing the same demand. Thus, starting from any state, with positive probability, the bargaining game transitions to the state where all individuals of a behavioural rule choose the same demand, and this holds for all behavioural rules. Let the set of such states be denote by $\bar{\Omega}$, and let X_I denote the demand made by all individuals who follow behavioural rule R_I. Thus, $(X_1, \ldots, X_K)$ is the demand vector corresponding to a state in $\bar{\Omega}$. Let $x(1) < x(2) < \ldots < x(\bar{k})$ be the distinct demands made in this state. The vector $(x(1), x(2), \ldots, x(\bar{k}))$ is the reduced demand vector, and the composite behavioural rule $\hat{R}(i)$ is the set of behavioural rules such that all individuals who follow a behavioural rule in this set demand $x(i)$. The share of individuals demanding $x(i)$, or alternatively, the fraction of individuals following a behavioural rule in the set $\hat{R}(i)$ is denoted by $n(i)$, and $n^{-1}(i)$ denotes the share of individual who demand no more than $1 - x(i)$. For any state in the set $\bar{\Omega}$, the average share of the pie received by any individual following a behavioural rule in the set $\hat{R}(i)$ is $\Sigma_{j=1}^{\bar{k}} n(j)\, \pi(x(i), x(j))$, where $\pi(x(i), x(j))$ equals $x(i)$ if $x(i) + x(j) \leq 1$, and equals zero otherwise. Since the average share of the pie received by all individuals of a behavioural rule is the same, the fitness of all behavioural rules in the composite set $\hat{R}(i)$ is also equal to $\Sigma_{j=1}^{\bar{k}} n(j)\, \pi(x(i), x(j))$.

A particular state of this population of K disparate behavioural rules is stable (neutrally stable) if it satisfies:
(i) internal stability: a state x is internally stable if $F_I(\pi_I(y)) = F_J(\pi_J(y))$ for all behavioural rules $I, J = 1, \ldots, K$ and for all $y \in x \cup Suc^{\infty}(x)$, i.e. all behavioural rules are equally fit if not only in that particular state but in all states that the bargaining game may transition to with positive probability, and,
(ii) external stability: a state x is externally stable (neutrally externally stable) if there exists $\bar{\varepsilon} > 0$, such that if ε fraction of individuals from a particular behavioural rule mutate and follow some other mildly responsive behavioural rule R_m, then $F_I(\pi_I(y)) > F_m(\pi_m(y))$ ($F_I(\pi_I(y)) \geq F_m(\pi_m(y))$ and if $F_I(\pi_I(y)) = F_m(\pi_m(y))$, then $F_I(\pi_I(y)) = F_J(\pi_m(y))$) for all behavioural rules $I, J = 1, \ldots, K$, for all $y \in x \cup Suc^{\infty}(x)$ and for all $\varepsilon \leq \bar{\varepsilon}$. A state is stable (neutrally stable) if it satisfies internal stability and external stability (neutrally external stability); otherwise, it is said to be unstable (neutrally unstable).

The notation in this section corresponds exactly with the corresponding notation in the previous section. It is then easily verified that all the results obtained earlier hold in toto.

Hence, I again obtain the hardwired behaviour-responsive behaviour equivalence theorem as well as the stability results for a homogeneous and heterogeneous populations. I summarise this in the next theorem.

Theorem 4. *The stability results in the bargaining game are identical, irrespective of the individuals in the population are finite or distributed over a continuum of fixed mass. This implies that:*
(i) when the population is heterogeneous, any state apart from the equal-split state is neutrally unstable; the equal-split state is the only state that is neutrally stable and thus the only state where disparate behavioural rules can stably co-exist.
(ii) when the population is homogeneous, all mildly responsive behavioural rules are unstable; furthermore, any state apart from the equal-split convention is neutrally unstable for any mildly responsive behavioural rule, and the equal split convention is a neutrally stable state for any mildly responsive behavioural rule.

4 Conclusion

In this paper, I develop the notion of evolutionary stability of behavioural rules, and apply it in the context of a bargaining game. The motivation for this comes from the supposition that in socio-economic contexts, it seems reasonable that individuals react and respond to the manner in which the strategic situation has played out in the past, and that the manner in which individuals respond is determined by a behavioural rule. I impose a weak restriction on the permissible behavioural rules, namely 'mild responsiveness', which serves to rule out situations where the behavioural rule is completely unresponsive to the history of the game. In general, the population may comprise of individuals who follow various disparate behavioural rules, and the fitness of each behavioural rule is a function of the average share of the pie/surplus received by the individuals following that behavioural rule. In order for a population to be stable, I impose two conditions: (i) internal stability, which requires all incumbent behavioural rules to be equally fit so as to preserve the composition of the incumbent population, and (ii) external stability, which requires each incumbent behavioural rules to be, at the very least, at least as fit as any mildly responsive mutant behavioural rule that may appear in the population.

The main message of the paper is that irrespective of the population being finite or infinite, (i) internal stability implies that a necessary condition for stability is that there exists a state where the configuration of demands made by individuals in the population must be such that an individual responds to the recent history of play by choosing the same demand, and this holds for all individuals, and (ii) external stability, in general, shows the instability of behavioural rules, with the state where all individuals choose to split the surplus/pie equally being the only neutrally stable state. Thus, this is the only state where different behavioural rules can co-exist without displacing one another – in this sense, the equal splitting norm may be interpreted as the only state where it is possible to sustain co-existence of diverse behavioural rules.

Appendix

Lemma 4. Consider any state where all individuals of a behavioural rule make the same demand. Let X_I be the demand made by individuals of behavioural rule R_I. If $\frac{1}{2} \geq x_{min} \neq x_{max}$ and the internal stability condition holds, then $R_I(s) = \{X_I\}$ for any feasible sample s from the demand vector describing the state, and this holds for all $I \in \{1, 2, \ldots, K\}$.

Proof. Suppose not, i.e. suppose that for a particular behavioural rule R_I, there exists at least one sample s of size S from the demand vector such that $X'_I \in R_I(s)$, and $X'_I \neq X_I$. I distinguish between three mutually exclusive and exhaustive possibilities: (i) $X'_I \in \{x(1), \ldots, x(\bar{k})\}$, (ii) $X'_I \notin \{x(1), \ldots, x(\bar{k})\}$ and either $X'_I < x(1)$ or $X'_I > x(\bar{k})$, and (iii) $X'_I \in (x(1), x(\bar{k}))$ but $X'_I \notin \{x(1), \ldots, x(\bar{k})\}$. I will prove the lemma by showing that internal stability is violated in each case.

In case of (i), I consider the positive probability event that some individuals of behavioural rule R_I choose the alternative response X'_I while other individuals of the same behavioural rule, due to the inward-looking set, choose X_I; because of the inward-looking set, individuals of all other behavioural rules choose the demand they made in the last period. (Since each behavioural rule has at least two individuals, this always happens with positive probability.) In case of (ii) and (iii), I consider the event – which occurs with positive probability – that all individuals of behavioural rule R_I choose the alternative response X'_I, while, because of the inward looking set, individuals of all other behavioural rules choose the demand they made in the last period. As a result of this, let three vectors of the previous state $(X_1, \ldots, X_K)$, $(x(1), \ldots, x(\bar{k}))$ and $(n(1), \ldots, n(\bar{k}))$ be transformed to the vectors $(\hat{X}_1, \ldots, \hat{X}_K)$, $(\hat{x}(1), \ldots, \hat{x}(\bar{k}'))$ and $(\hat{n}(1), \ldots, \hat{n}(\bar{k}'))$ respectively in the new state.

Case (i): Here, the reduced demand vectors are identical but the reduced composition vectors are distinct i.e. $(x(1), \ldots, x(\bar{k})) = (\hat{x}(1), \ldots, \hat{x}(\bar{k}'))$ but $(n(1), \ldots, n(\bar{k})) \neq (\hat{n}(1), \ldots, \hat{n}(\bar{k}'))$. By Lemma 2, if the reduced demand vector is $(x(1), \ldots, x(\bar{k}))$, then there is a unique reduced composition where all behavioural rules are equally fit; hence, if all the behavioural rules were equally fit when the reduced composition vector was $(n(1), \ldots, n(\bar{k}))$, they cannot be equally fit when the reduced composition vector changes to $(\hat{n}(1), \ldots, \hat{n}(\bar{k}'))$. Thus, I get a contradiction to internal stability.

Case (ii): First, I will prove by contradiction that $X'_I > x(1)$. So, suppose on the contrary, $X'_I = \hat{x}(1) < x(1)$ (note that $X'_I \neq x(1)$ by case (i) above). Then either $\hat{x}(2) = x(2)$ or

$\hat{x}(2) = x(1)$. $\hat{x}(2) = x(2)$ implies that second lowest demand in the current period was also the second lowest demand in the previous period. This happens only when $X_I = x(1)$ and $X_H \neq x(1)\ \forall H \in \{1, \ldots, K\}$ and $H \neq I$, i.e. in the previous period, the specific demand $x(1)$ was made only by the individuals belonging to the behavioural rule R_I but not by individuals of any other behavioural rule. Since all individuals of behavioural rule R_I now demand $X'_I \neq x(1)$ in the current period, $x(1)$ is not demanded by anyone in the current period, and so, the second lowest demand in the last period (i.e. $x(2)$) becomes the second lowest demand in this period (i.e. $\hat{x}(2)$). On the other hand, $\hat{x}(2) = x(1)$ implies that the second lowest demand in the current period was the lowest demand in the previous period; this happens when there is another behavioural rule that had demanded $x(1)$ in previous period, and by making a choice from the inward-looking set, they also demand it in the current period.

By Lemma 1, in the previous period, $n^{-1}(1) = n - 1$ and by Corollary 1, internal stability of the previous state gives that the fitness of each behavioural rule must have been $x(1)$. In the current state, since $X'_I < x(1)$, $\hat{n}^{-1}(1) = n - 1$ as well; as a result, the fitness of behavioural rule R_I is $X'_I < x(1)$. Now, first consider $\hat{x}(2) = x(2)$, which occurs only when $X_I = x(1)$ and $X_H \neq x(1)\ \forall H \in \{1, \ldots, K\}$ and $H \neq I$ (see explanation given in the earlier paragraph). Because individuals from all other behavioural rules apart from rule R_I make the same demand as in the last period (by choosing from the inward-looking set) and individuals from behavioural rule R_I make a lower demand in this period, the number of successful matches of individuals of the behavioural rules demanding $x(2)$ does not change between the two states; this implies that the fitness of this behavioural rule remains $x(1)$. On the other hand, if $\hat{x}(2) = x(1)$, then there exist behavioural rules that demand $x(1)$ in the current state. Since all individuals in each of these behavioural rules continue to make $n - 1$ successful bargains in the current state, the fitness of these behavioural rules remains $x(1)$. In either case, since the fitness of behavioural rule R_I in this period is $X'_I < x(1)$, where $x(1)$ is the fitness in the current state of at least one other behavioural rule, internal stability does not hold in this current state. Hence, $X'_i > x(1)$.

Now, I will prove by contradiction that $X'_I < x(\bar{k})$. So suppose, to the contrary, $X'_I > x(\bar{k})$ (note that $X'_I \neq x(\bar{k})$ by case (i) above). There are two possibilities: either (a) $x(2) = \hat{x}(1)$, or (b) $x(1) = \hat{x}(1)$.

(a) $x(2) = \hat{x}(1)$ implies that in this state, the demand $x(1)$ is not made by any behavioural rule. This happens if and only if no individual apart from the individuals belonging to the behavioural rule R_I demanded $x(1)$ in the previous state; since all these individuals choose $X'_I = \hat{x}(\bar{k}')$, the second lowest demand in the previous state becomes the lowest demand in this state. Now, if only two distinct demands were made in the previous state (i.e. $\bar{k} = 2$), then by Lemma 3, $x(2) > \frac{1}{2}$; since $x(2) = \hat{x}(1)$, $x(2) > \frac{1}{2}$ is equivalent to $\hat{x}(1) > \frac{1}{2}$; but $\hat{x}(1) > \frac{1}{2}$ contradicts internal stability of the current state (see Lemma 1). On the other hand, if at least

three distinct demands were made in the previous state, then $x(2) = \hat{x}(1) \leq \frac{1}{2}$ (see Lemma 3); but because $\hat{x}(1) = x(2) > 1 - x(\bar{k})$ (where the last inequality comes from Lemma 2), and because $X_I' = \hat{x}(\bar{k}') > x(\bar{k})$, individuals belonging to the behavioural rule R_I make zero successful bargains in the current state, and hence obtain zero fitness; however, the fitness of the behavioural rule(s) demanding $x(2) = \hat{x}(1)$ is strictly positive, thereby contradicting internal stability in the current state. Hence, $X_I' < x(\bar{k})$.

(b) Because of (a) above, I focus on the case where $x(1) = \hat{x}(1)$. Now there are two possibilities: either $1 - x(1) \geq X_I' > x(\bar{k})$, or $X_I' > 1 - x(1) \geq x(\bar{k})$, where the inequality $1 - x(1) \geq x(\bar{k})$ comes from the internal stability of the previous state and Lemma 1. If $1 - x(1) \geq X_I' > x(\bar{k})$ holds, then individuals of behavioural rule R_I make the same number of bargains as the individuals of the behavioural rules choosing $x(\bar{k})$.[3] Since $X_I' > x(\bar{k})$, the fitness of behavioural rule R_I is higher than the fitness of the behavioural rules that demand $x(\bar{k})$ – a violation of internal stability. If, on the other hand, $1 - x(1) < X_I'$, then the individuals of behavioural rule R_I never make a compatible demand and hence, obtain zero fitness; however, the fitness of the behavioural rule demanding $x(1) = \hat{x}(1)$ is positive – this again contradicts internal stability. Thus, in either case, $X_I' < x(\bar{k})$.

Case (iii): It follows from Case (*i*) and (*ii*) above that $X_I' \in (x(1), x(\bar{k}))$. I will now first establish (in Step 1) that in the new state, after the all individuals of behavioural rule R_I demand X_I', the lowest demand does not change, i.e. $x(1) = \hat{x}(1)$; because of Corollary 1, this implies that the fitness of any behavioural rule in an internally stable state must equal $x(1)$. Step 2 concludes the proof.

Step 1. Suppose (by contradiction) that $x(1) \neq \hat{x}(1)$. Then: either (a) $x(2) = \hat{x}(1)$, or (b) $X_I' = \hat{x}(1)$, which implies $X_I' \in (x(1), x(2))$ (note $X_I' \leq x(1)$ cannot hold because of Case (*i*) and Case (*ii*) above).

(a) First consider $x(2) = \hat{x}(1)$. Because of Lemma 2, in the previous period, $x(2) > 1 - x(\bar{k})$. But now, in the new state, because $X_I' < x(\bar{k})$ (by Case (*i*) and Case (*ii*) above), $x(\bar{k}) = \hat{x}(\bar{k}')$ must hold; again, by Lemma 1, if the new state is internally stable, then $\hat{x}(1) \leq 1 - \hat{x}(\bar{k}')$, i.e. using $\hat{x}(1) = x(2)$ and $\hat{x}(\bar{k}') = x(\bar{k})$, $x(2) \leq 1 - x(\bar{k})$ must hold. But this contradicts $x(2) > 1 - x(\bar{k})$; so $x(2) \neq \hat{x}(1)$.

(b) If $X_I' = \hat{x}(1)$, then by Lemma 1, if the new state is internally stable, it must be that $\hat{x}(1) \leq 1 - \hat{x}(\bar{k}')$; since $\hat{x}(\bar{k}') = x(\bar{k})$, $\hat{x}(1) \leq 1 - \hat{x}(\bar{k}')$ implies $\hat{x}(1) \leq 1 - x(\bar{k})$. But if $\hat{x}(1) \leq 1 - x(\bar{k})$, then all individuals who demand $\hat{x}(1)$ find that their demand is compatible with all other individuals; consequently, their fitness equals $\hat{x}(1)$, However, note that nothing changes for individuals belonging to other behavioural rules; they make the same demand,

[3] By Lemma 2 and internal stability of the previous state, the number of successful bargains made by individuals demanding $x(\bar{k})$ is $n(1)$; since $1 - x(1) \geq X_I' > x(\bar{k})$, the individuals demanding X_I' also make $n(1)$ successful bargains.

and also conclude the same number of successful bargains; consequently their payoff in the new state is the same as their payoff in the old state. By Corollary 1, their payoff in the old state was $x(1) < X'_I$; thus, their payoff in the new state is lower than the payoff of the individuals belonging to behavioural rule R_I – a contradiction to internal stability.
This proves that $x(1)$ remains the lowest demand in the new state; by Corollary 1, the fitness of each behavioural rule in an internally stable state must be $x(1)$.

Step 2. *(a)* From (the proof of) Lemma 2, $n^{-1}(j)$ is unique for all $j \in \{1, \dots, \bar{k}\}$; further $1-x(\bar{k}-j) \geq x(j+1) > 1-x(\bar{k}-j+1)$ for all $j \in \{1, \dots, \bar{k}-1\}$, and $x(1) \leq 1-x(\bar{k})$. Suppose that X_I, which is the demand made by all individuals of behavioural rule R_I in the previous period, is also equal to $x(h+1)$ for some $h+1 \in \{1, \dots, \bar{k}-1\}$; using the inequalities above, it lies in the interval $(1-x(\bar{k}-h+1), 1-x(\bar{k}-h)]$ if $h+1 \in \{2, \dots, \bar{k}-1\}$ or $(0, 1-x(\bar{k})]$ if $h+1 = 1$.
Importantly, in either case, by Step 1 above, the lowest demand does not change across the previous state and the current state. As a result, by Corollary 1, for internal stability in both the previous and current states, the fitness of each behavioural rule must equal $x(1)$. Since all other behavioural rules apart from R_I make the same demand, if all behavioural rules have to have the same level of fitness equal to $x(1)$ in both periods, $n^{-1}(j)$ cannot change for all $j \neq h+1$, i.e. $n^{-1}(j) = \hat{n}^{-1}(j)$ for all $j \neq h+1$. Further, since all demands $x(j)$, $j \neq h+1$, are made in both the previous and current states, it must be that the number of individuals making a demand in the interval $(1-x(\bar{k}-h+1), 1-x(\bar{k}-h)]$ if $h+1 \in \{2, \dots, \bar{k}-1\}$ or in the interval $(0, 1-x(\bar{k})]$ if $h+1 = 1$ does not change. This implies that X'_I must lie in the same interval as $X_I = x(h+1)$, i.e. if X_I lies in the interval $(1-x(\bar{k}-h+1), 1-x(\bar{k}-h)]$, then X'_I must lie in the same interval as well; otherwise, if X_I lies in the interval $(0, 1-x(\bar{k})]$, then X'_I lies in this interval as well. If, on the contrary, X'_I were to lie in a different interval, then it would cause a change in $n^{-1}(j)$ for some $j \neq h+1$, thereby resulting in a behavioural rule obtaining a level of fitness different from $x(1)$; this contradicts the observation that for internal stability to be maintained, the payoff of all behavioural rules must equal $x(1)$ in both the previous and the current states. Hence, X'_I lies in the same interval as X_I.
(b) As a result of (a) above, suppose now that X'_I lies in the same interval as X_I. Then, the lowest demand that is incompatible with X'_I is also the lowest demand in the last period that was incompatible with X_I; similarly, the highest demand that is compatible with X'_I is also the highest demand in the last period that was compatible with X_I. Hence, each individual of the behavioural rule R_I makes the same number of successful bargains in both the previous state and the current state (note that this rests on the argument that because of inertia, individuals from all other behavioural rules make the same demand in both the previous period and current period). However, since $X_I \neq X'_I$, the fitness of behavioural rule R_I differs between the two states; but I have argued (above) that for internal stability

to be satisfied, the fitness of each behavioural rule must equal $x(1)$. This contradicts internal stability; hence, X_I' cannot lie in the same interval as X_I.

It follows from the consideration of cases (i)-(iii) that if individuals of a behavioural rule R_I have another response $X_I' \neq X_I$ to any sample of demands in the last period, then with positive probability, there is a transition to a state where all behavioural rules do not have the same payoff, and so, the internal stability condition is violated, thus proving the lemma. ■

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