Antoine Augustin Cournot: The Pioneer of Modern Economic Ideas

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Abstract

Augustin Cournot, an unsung pioneer of many economic ideas during his time, who has written the book, Theory of Wealth, where he developed many economic ideas including the oligopoly theory, doupoly model, the ideas of function and probability into economic analysis. This paper discusses the way he thought the economics should be analyzed as well as his life cycle.

Introduction

Antoine Augustin Cournot (28 August 1801 – 31 March 1877) was a French philosopher and mathematician who also has great contribution to the development of economic theory [4]. His theories on monopolies and duopolies are still famous in economics. In 1838 the book Researches on the Mathematical Principles of the Theory of Wealth was published, in which he used the applications of the formulas and symbols of mathematics in economic analysis. This

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book was highly criticized and not very successful during Cournot's lifetime, and he did try to rewrite it twice, but it still has influence in economics today. Today many economists believe this book to be the point of departure for modern economic analysis. Cournot introduced the ideas of functions and probability into economic analysis. He derived the first formula for the rule of supply and demand as a function of price and in fact was the first to draw supply and demand curves on a graph, anticipating the work of Alfred Marshall by roughly thirty years. The Cournot duopoly model developed in his book also introduced the concept of a (pure strategy) Nash equilibrium, the Reaction function and best-response dynamics. Cournot believed that economists must utilize the tools of mathematics only to establish probable limits and to express less stable facts in more absolute terms. He further held that the practical uses of mathematics in economics do not necessarily involve strict numerical precision. Today, Cournot's work is recognized in econometrics. He was also a teacher of political economy and mathematics to Auguste Walras, who was the father of Léon Walras. Cournot and Auguste Walras persuaded Léon Walras to try political economics. Cournot is also credited to be one of the sources of inspiration for Léon Walras and his equilibrium theory. By the time Cournot died in 1877, he was nearly blind. In the field of economics he is best known for his work in the field of oligopoly theory—Cournot competition which is named after him.

Cournot's life and his contribution

Of Franche-Comté peasant stock, Cournot's family had belonged for two generations to the petite bourgeoisie of Gray. In his Souvenirs he says very little about his parents but a great deal about his paternal uncle, a notary to whom he apparently owed his early education [5]. Cournot was deeply impressed by the conflict that divided the society in which he lived into the adherents of the ancien régime and the supporters of new ideas, especially in the realm of religion. One of his uncles was a conformist priest, the other a faithful disciple of the Jesuits, having been educated by them.

Between 1809 and 1816 Cournot received his secondary education at the collège of Gray and showed a precocious interest in politics by attending the meetings of a small royalist club. He spent the next four years idling away his time, working "en amateur" in a lawyer's office. Influenced by reading Laplace's Système du monde and the Leibniz-Clarke correspondence, he became interested in mathematics and decided to enroll at the École Normale Supérieure in Paris. In preparation, he attended a course in special mathematics at the Collège Royal in Besançon (1820–1821) and was admitted to the École Normale after competitive examinations in August 1821. However, on 6 September 1822 the abbé Frayssinous, newly appointed grand master of the University of France, closed the École Normale. Cournot found himself without a school and with only a modest allowance for twenty months. He remained in Paris, using this free time which he called the happiest of his life to prepare at the Sorbonne for the licence in mathematics (1822–1823). His teachers at the Sorbonne were Lacroix, a disciple of Condorcet, and Hachette, a former colleague of Monge. A fellow student and friend was Dirichlet.

In October 1823, Cournot was hired by Marshal Gouvion-Saint-Cyr as tutor for his small son. Soon Cournot became his secretary and collaborator in the editing and publishing of his Mémoires Thus, for seven years, until the death of the marshal, Cournot had the opportunity to meet the many important persons around the marshal and to reflect on matters of history and politics. Nevertheless, Cournot was still interested in mathematics. He published eight papers in the baron de Férussac's Bulletin des sciences, and in 1829 he defended his thesis for the doctorate in science, "Le mouvement d'un corps rigide soutenu par un plan fixe." The papers attracted the attention of Poisson, who at that time headed the teaching of mathematics in France. When, in the summer of 1833, Cournot left the service of the Gouvion-Saint-Cyr family, Poisson immediately secured him a temporary position with the Academy of Paris. In October 1834 the Faculty of Sciences in Lyons created a chair of analysis, and Poisson saw to it that Cournot was appointed to this post. In between, Cournot translated and adapted John Herschel's Treatise on Astronomy and Kater and Lardner's A Treatise on Mechanics, both published, with success, in 1834.

From then on, Cournot was a high official of the French university system. He taught in Lyons for a year. In October 1835 he accepted the post of rector at Grenoble, with a professorship in mathematics at the Faculty of Sciences. Subsequently he was appointed acting inspector general of public education. In September 1838, Cournot married and left Grenoble to become inspector general. In 1839 he was appointed chairman of the Jury d'Agrégation in mathematics, an office he held until 1853. He left the post of inspector general to become rector at Dijon in 1854, after the Fortoul reform, and served there until his retirement in 1862.

Cournot produces several mathematical, statistical, philosophical as well as economical work through out his life. He also translated some English mathematical books, including Sir John Herschel's Astronomy, and edited in two volumes Euler's celebrated Letters to a Princess, etc. For nearly forty years the Principes mathematiques was completely ignored by economists.

Unassuming and shy, Cournot was considered an exemplary civil servant by his contemporaries. His religious opinions seem to have been very conservative. In politics he was an enthusiastic royalist in 1815, only to be disappointed by the restoration of the monarchy. In the presidential elections following the 1848 Revolution, he voted for Louis Eugène Cavaignac, a moderate republican. In 1851, sharply disapproving the organization of public instruction as directed by Louis Napoleon, he decided to become a candidate in the legislative elections in Haute-Saône; this election, however, was prevented by the coup d'état of 2 December.

Cournot's background and his education made him a member of the provincial petite bourgeoisie of the ancien régime. But as a civil servant of the July monarchy and the Second Empire, he became integrated into the new bourgeoisie of the nineteenth century. Of certainly mediocre talents as far as pure mathematics was concerned, he left behind work on the philosophy of science, remarkably forceful and original for its period, that foreshadowed the application of mathematics to the sciences of mankind. Nobody could express better and more humorously Cournot's importance than he himself when he reported Poisson's appreciation of his first works: "He [Poisson] discovered in them a philosophical depth—and, I must honestly say, he was not altogether wrong. Furthermore, from them he predicted that I would go far in the field of pure mathematical speculation but (as I have always thought and have never hesitated to say) in this he was wrong".

Cournot's mathematical work amounts to very little: some papers on mechanics without much originality, the draft of his course on analysis, and an essay on the relationship between algebra and geometry. Thus, it is mainly the precise idea of a possible application of mathematics to as yet unexplored fields that constitutes his claim to fame. With the publication in 1838 of his Recherches sur les principes mathématiques de la théorie des rishesses he was a third of a century ahead of Walras and Jevons and must be considered the true founder of mathematical economics. By reducing the problem of price formation in a given market to a question of analysis, he was the first to formulate the data of the diagram of monopolistic competition, thus defining a type of solution that has remained famous as "Cournot's point." Since then, his arguments have of course been criticized and amended within a new perspective. Undoubtedly, he remains the first of the important pioneers in this field.

Cournot's work on the "theory of chance occurrences" contains no mathematical innovation. Nevertheless, it is important in the history of the calculus of probability, since it examines in an original way the interpretation and foundations of this calculus and its applications. According to Cournot, occurrences in our world are always determined by a cause. But in the universe there are independent causal chains. If at a given point in time and space, two of these chains have a common link, this coincidence constitutes the fortuitous character of the event thus engendered. Consequently, there would be an objective chance occurrence that would nevertheless have a cause. This seeming paradox would be no reflection of our ignorance.

This objective chance occurrence is assigned a certain value in a case where it is possible to enumerate—for a given event—all the possible combinations of circumstances and all those in which the event occurs. This value is to be interpreted as a degree of "physical possibility." However, one must distinguish between a physical possibility that differs from 0 (or 1)only by an infinitely small amount and a strict logical impossibility (or necessity).

On the other hand, Cournot also insisted on the necessary distinction between this physical possibility, or "objective probability," and the "subjective probability" that depends on our ignorance and rests on the consideration of events that are deemed equiprobable1 since there is not sufficient cause to decide otherwise. Blaise Pascal, Fermat, Huygens, and Leibniz would have seen only this aspect of probability. Jakob I Bernoulli, despite his ambiguous vocabulary, would have been the first to deal with objective probabilities that Cournot was easily able to estimate on the basis of frequencies within a sufficiently large number of series of events.

To these two ideas of probability Cournot added a third that he defined as "philosophical probability." This is the degree of rational, not measurable, belief that we accord a given scientific hypothesis. It "depends mainly upon the idea that we have of the simplicity of the laws of nature, of order, and of the rational succession of phenomena". Of course, Cournot neither solved nor even satisfactorily stated the problem of the logical foundation of the calculus of probability. But he had the distinction of having been the first to dissociate—in a radical way—various ideas that still were obscure, thus opening the way for deeper and more systematic research by more exact mathematicians. He also was able to show clearly the importance of the applications of the calculus of probability to the scientific description and explanation of human acts. He himself—following Condorcet and Poisson—attempted to interpret legal statistics. But he also warned against premature and abusive applications" that might discredit this ambitious project.

More than for his mathematical originality, Cournot is known for his views on scientific knowledge. He defined science as logically organized knowledge, comprising both a classification of the objects with which it deals and an ordered concatenation of the propositions it sets forth. It claims neither the eternal nor the absolute: "There can be nothing more inconsistent than the degree of generality of the data with which the sciences deal—data susceptible to the degree of order and the classification that constitute scientific perfection". Therefore, the fundamental characteristic of the scientific object must be defined differently. “What strikes us first of all, what we understand best, is the form,” Cournot wrote at the beginning of the Traite de l’enchaînement des idées, adding, scientifically we shall always know only the form and the order.” Thus, it was from this perspective that he interpreted scientific explanation and stressed the privilege of mathematics—the science of form par excellence. Even though establishing himself as forerunner of a completely modern structural concept of the scientific object, Cournot did not go so far as to propose a reduction of the process of knowledge to the application of logical rules. On the contrary, he insisted upon the domination of strictly formal and demonstrative logic by “another logic, much more fruitful, a logic which separates appearance from reality, a logic which connects specific observations and infers general laws from them, a logic which ranks truth and fact”.

Book: Researches on the Mathematical Principles of the Theory of Wealth

A distinguished scholar, a skillful writer, endowed with an original and lofty intellect, Cournot was a master in the art of education. According to Reghinos D. Theocharis [6] the long and slow road of Cournot’s Recherches sur les Principes Mathematiques de la Theorie des Richesses towards recognition of its value for economic theory. Until the appearance of the second edition of W.S. Jevons’s Theory of Political Economy, Cournot, when not ignored, was generally the object of adverse and even derisory criticism; but we also find solitary and appreciative comments on aspects of his work, including those by Leon Walras, whose key role in establishing the importance of the Recherches is stressed. Mr. Walras does himself credit by being his disciple. " Mr. Cournot," he says, " is the first person to have seriously attempted the application of mathematics to political economy". It is also pointed out that this process was particularly slow in the English-speaking world.

C. Esmenard du Mazet, who in a book published in 1849 attempted some appli- cations of mathematics to economic theory and gave some formulas in introducing a rudimentary equation of exchange of doubtful significance, chose to refer to Cournot and his Recherches in a critical and definitely derisory way. Referring to what he considered the causes of price variations, Esmenard accused economists in general: 'without analysing the general phenomenon, without ever starting from clear and distinct principles, they have built a kind of a tower of Babel, they have lost themselves in the confusion of expressions, in which some want to retrench by using the famous formula of supply and demand. A fine formula indeed! From which it is radically impossible to draw any serious inferences, but people fortu- itously put it forward so as not to be left behind.' And Cournot's Recherches was the only work singled out by Esmenard as the typical example of writings by the authors declaimed [6].

The first real appreciation of Cournot's contribution was made in 1857 by a Canadian, J.B. Cherriman, who was a Cambridge-trained professor of mathematics and natural philosophy at the University of Toronto. He published an extensive review article on the Recherches, full of praise for Cournot's work. This review has already been cited by Goodwin and Baumol and Goldfeld and has been extensively analysed by Dimand (1988)[6].

Coumot's long road to recognition finally came to an end with the publication of the second edition of Jevons's Theory. In his preface to that edition, Jevons brought all the weight of his authority to bear in support of the need for the long-delayed recognition of Coumot. He described the Recherches as a work that 'must occupy a remarkable position' in the development of mathematical economics, and he spoke of Coumot's 'wonderful analysis of the laws of supply. and demand, and of the relations of prices, production, consumption, expenses and profits', adding that he was quite convinced that Cournot's investigation was of 'high economic importance, and that, when the parts of political economy to which the theory relates come to be adequately treated, as they never have yet been, the treatment must be based upon the analysis of Cournot, or at least must follow his general method'. From then onwards, recognition of Cournot becomes more widespread. In the years that follow, his analysis was utilized further or commented upon by many authors, including A. Marshall, who appears to have read Coumot as early as 1868. Finally in 1897 an English translation of the Recherches was published [6].

Cournot model of Monopoly

Not surprisingly, Cournot's work is one of the classics of game theory; it is also one of the cornerstones of the theory of industrial organization. Cournot describes his model of monopoly in his book, Researches on the Mathematical Principles of the Theory of Wealth, as follows [2]: suppose that a man finds himself proprietor of a mineral spring which has just been found to possess salutary properties possessed by no other. He could doubtless fix the price of a liter of this water at 100 francs; bu he would soon see by the scant demand, that this is not the way to make the most of his property. He will therefore successively reduce the price of the liter to the point which will give him the greatest possible profit; i.e. if $F(p)$ denotes the law of demand, he will end, after various trials, by adopting the value of p which renders the product $pF(p)$ a maximum, or which is determined by the equation

$$F(p) + pF'(p) = 0 \tag{1}$$

The product

$$pF(p) = \frac{\left[F(p)\right]^2}{-F'(p}$$

will be the annual revenue of the owner of the spring, and this revenue will only depend on the nature of function F.

To make equation (1) applicable, it must be supposed that for the value of p obtained from it, there will be a corresponding value of D which the owner of the spring can deliver, or which does not exceed the annual flow of this spring; otherwise the owner could not, without damage to himself, reduce the price per liter as low as would be for his interest where the spring more abundant. I the spring produces annually a number of liters expressed by Δ, by deducing p from the relation $F(p) = \Delta$, we necessarily obtain the price per liter which must finally be fixed by the competition of customers.

Then Cournot discussed another monopoly game in his book as, let suppose that of a man who possesses the secret of a medical preparation or an artificial mineral water, for which the material and labor must paid for. It will no longer be the function $pF(p)$, or the annual gross receipts, which the producer should strive to carry to its maximum value, bu the net receipts, or the function $pF(p) - \phi(D)$, in which $\phi(D)$ denotes the cost of making a number of liters equal to D. Since D is connected with p by the relation $D + F(p)$, the complex function $pF(p) - \phi(D)$ can be regarded as depending implicitly, on the single variable p, although generally the cost of production is an explicit function, not of the price of the article produced, but of the quantity produced. Consequently the price to which the producer should bring his article will be determined by t he equation

$$D + \frac{dD}{dp}\left[p - \frac{d\left[\phi(D)\right]}{dD}\right] \qquad (2)$$

This price will fix in turn the annual net receipts or the revenue of the inventor, and the capital value of his secret, or his productive property, the ownership of which is guaranteed by law and can have commercial circulation as well as that of a piece of land or any material property. If this values is nil or insignificant, the owner of the property will obtain no pecuniary profit from it; he will abandon it gratis, or for a very small payment, to the first comer who seek to develop it. The value of a liter will only represent the value of the raw materials, the wages or profits of the agents who cooperate in making and and marketing it, and the interest on the capital necessary for development. The terms for our example prevent our admitting in this case a limitation of the productive forces, which would hinder the producer from lowering the price to th rate which would give the maximum net receipts, according to the law of demand. But in a great many other case there may be such limitation, and if Δ, expresses the limit which the production or the demand cannot exceed, the price will be fixed by th relation $F(p) = \Delta$, as if there were no cost of production. The cost, in this case, is not borne by the consumers at all; it only diminishes the income of the producer. It fall not exactly on the proprietor (who, unless the inventor or first holder,- a question of original conditions with which theory has nothing to do,- acquired the property, himself or through his agents, for a value proportioned to its revenue), but on the property itself. A decrease of this cost will only be to the advantage of he producer, so far as it does not result in the possibility of increasing his producing power.

Cournot describes that let us return to the case where where this possibility exists, and where the price p is determined according to equation (2). We shall observe that the coefficient $\frac{d[\phi(D)]}{dD}$, though it may increase or decrease as D increases, must be supposed to be positive, for it would be absurd that absolute expense of production should decrease as production increases. We shall call attention also to the fact that necessarily $p > \frac{d[\phi(D)]}{dD}$, for dD being the increase of production, $d\left[\phi(D)\right]$ is the increase in the cost, pDD is the increase of the gross receipts, the producer will always stop when the increase in expenses exceeds the increase in receipts. This is also abundant evident from the form of equation (2), since D is always a positive quantity, and $\frac{dD}{dp}$ a negative quantity. In the course of our investigations we shall seldom have occasion to consider $\phi(D)$ directly, but only its differential coefficient $\frac{d[\phi(D)]}{dD}$, which we will denote by $\phi'(D)$. This differential coefficient is a new function of D, the form of which exerts very great influence on the principal problems of economic science. The function $\phi'(D)$ is capable of increasing or decreasing as D increases, according to the nature of the producing forces and of the articles produced. For what are properly called manufactured articles, it is generally the case that the cost becomes proportionally less as production increases, or, in other words, when D increases $\phi'(D)$ is decreasing function. This comes from better organization of the work, form discounts on the price of raw materials for large purchases, and finally from the reduction of what is known to producers as general expense. It may happen, however, even in exploiting products of this nature, that when the exploitation is carried beyond certain limits, it induces higher prices for raw materials and labor, to the point where $\phi'(D)$ again begins to increases with D. Whenever it is a question of working agricultural lands, of mines, or of quarries, i.e. of what is essentially real estate, the function $\phi'(D)$ increases with D; and as we shall soon see, it is consequence of this fact alone that farms, mines, and quarries yield a net revenue to their owners, long before all has been extracted from the soil which it is physically able to produce, and notwithstanding the great subdivision of these properties, which causes between producers a competition which can be considered as unlimited. On the contrary, investments made under the condition that as D increases $\phi'(D)$ decreases, can only yield a net income or a rent in the case of a monopoly properly so-called, or of a competition sufficiently limited to allow the effects of a monopoly collectively maintained to be still perceptible.

Then he describes between the two case where the function $\phi'(D)$ increasing and decreasing, there falls naturally the one where this function reduces to a constant, the cost being constantly proportional to the production, and where equation (2) takes the form

$$D + \frac{dD}{dp}(p - g) = 0$$

The case must also be pointed out where $\phi(D)$ is a constant, and $\phi'(D) = 0$, so that the price is the same as if there were no cost. This case occurs more frequently than would be suspected at fist glace, especially where we have to do with production under a monopoly, and where the value of the number D receives the extension of which it admits. For instance, in a theatrical enterprise D denotes the number of tickets sold, and the cost of the enterprise remains practically the same, without reference to the number of spectators. for the tills of a bridge, which is another monopolistic investment, D denotes the number passengers; and the cost for repairs, watching, and bookkeeping will be the same, whether the crossing is much or little used. In such cases the constant g disappears, equation (2) be comes the same as equation (1), and the price p is determined in the same manner as if there were no costs.

Cournot describes it seems a matter of course that when the cost of production increases, the price fixed by the monopolist, according to equation (2), will increase likewise; but, on consideration, it will appear that so important a proposition should be supported by a rational demonstration; and furthermore, this demonstration will lead us to an equally important observation, which only mathematics can incontestably establish.

Let p_0 be the root of equation (2) which we will put in the form

$$F(p) + F'(p)\left[p - \psi(p)\right] = 0, \qquad (3)$$

as $\phi'(D) = \phi'\left[F(p)\right]$ can be more simply replaced by $\psi(p)$; and suppose that, as the function $\psi(p)$ varies by a quantity u, and becomes $\psi(p) + u$, p becomes $p_0 + \delta$. If we neglect the squares and higher powers of the increments u and δ, equation (3) will establish the following relation between these two increments:

$$\left\{F'(p_0)\left[2 - \psi'(p_0)\right] + F''(p_0)\left[p_0 - \psi(p_0)\right]\right\}\delta - uF'(p_0) = 0; \qquad (4)$$

The coefficient of δ in this expression being the derivative with respect to p of th fist member of equation (3), in which derivative the values p_0 has been given to p.

But this coefficient of δ is necessarily negative, according to the well-known theory of maxima and minima; for if it were positive, the root p_0 of equation (3) would correspond to the minimum of the function $pF(p) - \phi(D)$, and not to the maximum of his function, as it should. Moreover $F(p)$ is by its nature a negative quantity. In general, therefore, the increment δ is of the same sign as the increment u.

This result has been demonstrated on the supposition that the variation u, δ are very small quantities, of which the squares and products can be neglected without sensible error, but by a very simple argument this restriction can be removed. In fact, whatever the increase of cost denoted by u, it can be supposed that that the function $\psi(p)$ passes from the value $\psi(p)$ to the value $\psi(p) + u$ by a series of very small increments, u_1, u_2, u_3, etc., all of the same sign. At the same time p will pass from the value p_0 to the value $p_0 + \delta$ by a series of corresponding increments, also very small, $\delta_1, \delta_2, \delta_3$, etc.; δ_1 will be (according to the preceding paragraph) of the same as u_1, δ_2 as u_2, etc.

Therefore,

$$\delta = \delta_1 + \delta_2 + \delta_3 + etc.,$$

will be of the same sign as

$$u = u_1 + u_2 + u_3 + etc.$$

This method of demonstration should be borne in mind, as it will be frequently recurred to.

From equation 4) we obtain

$$\frac{\delta}{u} = \frac{F'(p_0)}{\left\{F'(p_0)\left[2 - \psi'(p_0)\right] + F''(p_0)\left[p_0 - \psi(p_0)\right]\right\}}$$

and since both terms of the fraction in the second number are negative, we conclude that δ will be numerically greater or less than u accordingly as we have

$$-F'(p_0) > -\left\{F'(p_0)\left[2 - \psi'(p_0)\right] + F''(p_0)\left[p_0 - \psi(p_0)\right]\right\}$$

or

$$-F'(p_0) < -\left\{F'(p_0)\left[2 - \psi'(p_0)\right] + F''(p_0)\left[p_0 - \psi(p_0)\right]\right\}$$

or in other words,

$$F'(p_0)\left\{F'(p_0)\left[1 - \psi'(p_0)\right] + F''(p_0)\left[p_0 - \psi(p_0)\right]\right\} > 0$$

or

$$F'(p_0)\left\{F'(p_0)\left[1 - \psi'(p_0)\right] + F''(p_0)\left[p_0 - \psi(p_0)\right]\right\} < 0$$

which, by replacing $p_0 - \psi(p_0)$ by its value as deduced from equation (3), becomes

$$\left[F'(p_0)\right]\left[1-\psi'(p_0)\right]-F(p_0)F''(p_0)<0$$

or

$$\left[F'(p_0)\right]\left[1-\psi'(p_0)\right]-F(p_0)F''(p_0)>0.$$

To make this more obvious by numerical applications, let us take a fictitious case. Suppose that function $\phi'(D)$ were at first=0, and that it subsequently reduces to a constant g. The first value of p or p_0 will be given by the equation

$$F(p)+pF'(p)=0;$$

the second value of p, which we will call p', will be given by another equation

$$F(p)+(p-g)F'(p)=0 \tag{5}$$

Suppose, in the first place, that $F(p)=\frac{a}{b+p^2}$; the values of p_0 and p', according ot the preceding equations, will be respectively
$p_0=\sqrt{b}$, and $p'=g+\sqrt{b+g^2}=g+\sqrt{p_0^2+g_0^2}$

(the root of equation (5) which would give a negative value of p' is greater than p_0 by a quantity greater than g, i.e. greater than the amount of the new cost imposed on the production. If, for instance, the new cost is one-tenth of the original price, or if $g=\frac{1}{10}p_0$, we shall have $p'=1.1488p_0$; the increase in price will be very nearly one and one-half tenths; the old price being 20 francs and the cost 2 francs; the new price will be 23 francs, or more exactly, 22 francs 97 centimes.

Suppose, in the second place, $F(p)=\frac{1}{b+p^3}$, we shall have $p_0=\sqrt[3]{\frac{b}{2}}$; and equation (5) will become

$$2p^3 - 3gp^2 - b = 0$$

or

$$2p^3 - \frac{3}{2}gp^2 - p_0^3 = 0$$

which be th ordinary method of solution will give

$$p' = \frac{1}{2}\left\{g + \sqrt[3]{g^3 + 4p_0^3 + 2\sqrt{2p_0^3(g^3 + 2p_0^3}} + \sqrt[3]{g^2 + 4p_0^3 + 2\sqrt{2p_0^3(g^3 + 2p_0^3} +}\right\}$$

In this case the excess of p' over p_0 will be less than g.

If $g = \frac{1}{10}p_0$, we shall have $p' = 1.0505p_0$. Thus if the new cost is one-tenth of the original price, the increase in price will only be one-half of one-tenth of that price. The old price being 20 francs, and the added cost 2 francs, the new price will be only 21 francs, or, more exactly 21 francs .01 centime.

Then result which we have just reached is well worth attention: it shows us that, according to the form of the function $F(p)$, or according to the law of demand, and increase in the cost of production augments the price of a commodity, for which there exists a monopoly, sometimes much more and sometimes much less than the increase in cost; and that, in the same manner, there is no equality between the reduction of cost and fall in the price of the commodity.

It results from this, that if the new cost were not met by the produced himself, but by hte consumer, or by an intermediate agent, who would be reimbursed by the consumer, this increase in cost, which would always make the article dearer for the consumer, and which would always diminish the net income of the producer, might, according to circumstances, produce and advance or a decline in the price paid to the producer.

Reciprocally, a fall in he cost of transmission, or in that of passing the commodity from the possession of the produced to that of the consumer, may have the effect at one time of increasing the price paid to that producer, and at another time of diminishing it; but in all case it will diminish the final price paid by the consumer, and will cause an increase n the net income of the producer.

All those expenses which are incurred with a view to preparing for final consumption an crude commodity as it leaves the producers' hands, must in this respect be considered in the same light as costs of transmission.

However, this calculation is only applicable in the case where the producer can meet the demand which gives him the greatest net return, and reduce his price as much as is necessary to attain this maximum return. In other cases, he will produce all he can, before as well as after the change in the cost, whether of production or of transmission, and the cost price to the consumer will remain invariable, because in a condition of equilibrium, and on a large scale, there cannot be two different prices for the same quantity marketed. The increase of cost, therefore, from whatever source, must finally be wholly borne by the producer.

Cournot model of Duopoly

We consider a very simple version of Cournot's model here. Illustration of the model [7] (a) the translation of an informal statement of a problem into a normal-form representation of a game; (b). The computations involved in solving for the game's Nash equilibrium; and (c) iterated elimination of strictly dominated strategies.

Let q_1 and q_2 denote the quantities (of a homogeneous product) produced by firms 1 and 2, respectively. Let $P(Q) = a - Q$ be the market-clearing price when the aggregate quantity on the market is $Q = q1 + qz$. (More precisely, $P(Q) = a - Q$ for $Q < a$, and $P(Q) = 0$ for $Q > a$.) Assume that the total cost to firm i of producing quantity q_i; is $C_i(q_i) = cq_i$. That is, there are no fixed costs and the marginal cost is constant at c, where we assume $c < a$. Following Cournot, suppose that the firms choose their quantities simultaneously. In order to find the

Nash equilibrium of the Cournot game, we first translate the problem into a normal-form game. (1) the players in the game, (2) the strategies available to each player, and (3) the payoff received by each player for each combination of strategies that could be chosen by the players. There are of course two players in any duopoly game-the two firms. In the Cournot model, the strategies available to each firm are the different quantities it might produce. We will assume that output is continuously divisible. Naturally, negative outputs are not feasible. Thus, each firm's strategy space can be represented as $S_i = [0, \infty)$, the non-negative real numbers, in which case a typical strategy s_i; is a quantity choice, $q_i > 0$. One could argue that extremely large quantities are not feasible and so should not be included in a firm's strategy space. Because $P(Q) = 0$ for $Q > a$, however, neither firm will produce a quantity $q_i > a$. It remains to specify the payoff to firm i as a function of the strategies chosen by it and by the other firm, and to define and solve for equilibrium. We assume that the firm's payoff is simply its profit. Thus, the payoff $u_i(s_i, s_j)$ in a general two-player game in normal form can be written here as

$$\pi_i(q_i, q_j) = q_i[P(q_i + q_j) - c] = q_i[a - (q_i + q_j) - c].$$

In a two-player game in normal form, the strategy pair $(s_i^\star, s_j^\star)$ is a Nash equilibrium if, for each player i,

$$u_i(s_i, s_j) \geq u_i(s_i, s_j)$$

for every feasible strategy s_i in S_i. Equivalently, for each player i, $s_i^\star$ must solve the optimization problem.

$$\max_{(s_i) \in S_i} u_i(s_i, s_j^\star)$$

In the Cournot duopoly model, the analogous statement is that the quantity pair (q_j, q_i) is a

Nash equilibrium if, for each firm i, q_i solves

$$\max_{(0 \le q_i < \infty)} \pi_i \left(q_i, q_j^\star\right) = \max_{(0 \le q_i < \infty)} q_i \left[a - \left(q_i + q_j^\star\right) - c\right]$$

Assuming $q_j^\star < a - c$ (as will be shown to be true), the first-order condition for firm i's optimization problem is both necessary and sufficient; it yields

$$q_i = \frac{1}{2}\left(a - q_j^\star - c\right) \tag{6}$$

Thus, if the quantity pair (q_i, q_j) is to be a Nash equilibrium, the firms' quantity choices must satisfy

$$q_1^\star = \frac{1}{2}(a - q_2^\star - c)$$

$$q_2^\star = \frac{1}{2}(a - q_1^\star - c)$$

Solving this pair of equations yields

$$q_1^\star = q_2^\star = \frac{a-c}{3}$$

which is indeed less than $a - c$, as assumed. The intuition behind this equilibrium is simple. Each firm would of course like to be a monopolist in this market, in which case it would choose q_i to maximize $\pi_i(q_i, 0)$-it would produce the monopoly quantity $q_m = (a-c)/2$ and earn the monopoly profit $\pi_i\left(q_m, 0\right) = \frac{(a-c)^2}{4}$. Given that there are two firms, aggregate profits for the duopoly would be maximized by setting the aggregate quantity $q1 + q2$ equal to the monopoly quantity q_m, as would occur if $q_i = \frac{q_m}{2}$ for each i, for example. The problem with this arrangement is that each firm has an incentive to deviate: because the monopoly quantity is low, the associated price $P(q_m)$ is high, and at this price each firm would like to increase its quantity, in spite of the fact that such an increase in production drives down the market-clearing price. (To see this formally, use (6) to check that $\frac{q_m}{2}$ is not firm 2's best response to the choice of $\frac{q_m}{2}$ by firm 1.) In the Cournot equilibrium, in contrast, the aggregate quantity is higher, so the associated price is lower, so the temptation to increase output is reduced-reduced by just enough that each firm is just deterred from increasing its output by the realization that the market-clearing price will fall.

Rather than solving for the Nash equilibrium in the Cournot game algebraically, one could instead proceed graphically, as follows. Equation (6) gives firm i's best response to firm j's equilibrium strategy, q_j. Analogous reasoning leads to firm 2's best response to an arbitrary strategy by firm 1 and firm 1's best response to an arbitrary strategy by firm 2. Assuming that firm l's strategy satisfies $q_1 < a - c$, firm 2's best response is likewise, if $q_2 < a - c$ then firm 1's best response is

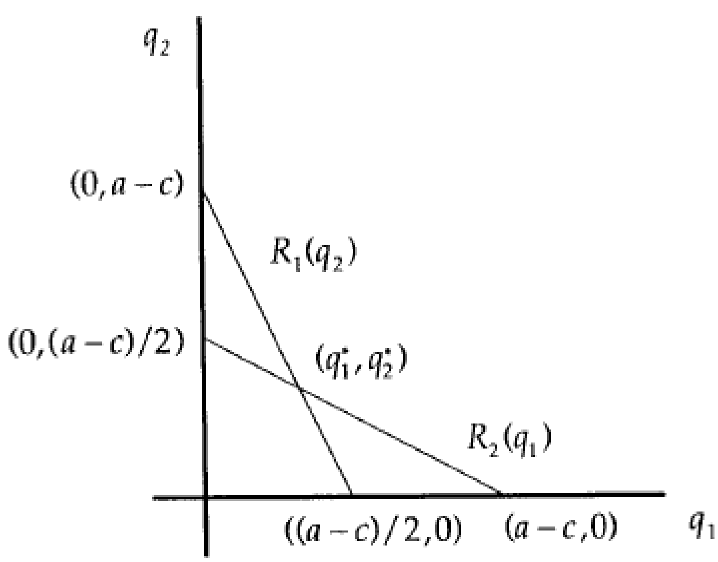

Figure 1

$$R_2(q_1) = \frac{1}{2}(a - q_1 - c)$$

likewise, if $q_2 < a - c$ then firm 1's best response is

$$R_1\left(q_2\right) = \frac{1}{2}\left(a - q_2 - c\right)$$

As shown in Figure 1, these two best-response functions intersect only once, at the equilibrium quantity (q_i, q_j).

Cournot model of Oligopoly

Suppose there are n firms in the Cournot oligopoly model. Let q_i denote the quantity produced by firm i, and Iet $Q = q_1 + , \ . \ . \ . + q_n$ denote the aggregate quantity on the market. Let P denote the market-clearing price and assume that inverse demand is given by $P(Q) = a - Q$ (assuming $Q < a$, else $P = 0$). Assume that the total cost of firm i from producing quantity q_i is $C_i\left(q_i\right) = cq_i$. That is, there are no fixed costs and the marginal cost is constant at c, where we assume $c < a$. Following Cournot, suppose that the firms choose their quantities simultaneously. According to the above explanation profit of firm i is given by

$$\pi_i = \pi_i\left(q_1, q_2, \ . \ . \ . q_N\right) = [P(Q) - c]\, q_i$$

First order condition:

$$\frac{\partial \pi}{\partial q_i} = a - Q_{-i} - 2q_2 - c = 0 \tag{7}$$

where

$$Q_i = q_1 + q_2 + ... + q_{i-1} + q_{i+1} + \ . \ . \ . + q_N$$

Equation (7) implies

$$q_i^{\star}\left(Q_{-i}^{\star}\right) = \frac{a - Q_{-i}^{\star} - c}{2} \tag{8}$$

Since function (8) is symmetric, we can write

$$q_1^{\star} = q_2^{\star} = q_3^{\star} = \ . \ . \ . = q_N^{\star}$$

which implies

$$Q_{-i} = (N-1)\,q_i^\star \tag{9}$$

plugging (9) into (8) we get

$$q_i^\star\left(Q_{-i}^\star\right) = \frac{a-(N-1)\,q_i^\star - c}{2} = q_i^\star$$

which implies

$$q_i^\star = \frac{a-c}{N+1}$$

which is the Nash equilibrium.

Hence

$$Q^\star = \frac{N}{N+1}(a-c)$$

$$\pi = \frac{(a-c)^2}{(N+1)^2}$$

$$P^\star\left(Q^\star\right) = a - \frac{N}{N+1}(a-c)$$

Now if $N \longleftrightarrow \infty$ then $Q^\star = a-c$, consequently $\pi^\star = 0$, and $p^\star = c$. Therefore no profit.

Tragedies in the life of Cournot

ANDREW CARNEGIE once mentioned, "It does not pay to pioneer." Certainly, if there is any truth in this statement, it finds ample illustration in the life and work of Antoine-Augustin Cournot. The pioneering which he did brought him no financial reward, little academic recognition, and scarcely any intellectual comradeship. Alfred Marshall in his old age received a substantial income from royalties on his books, particularly his Principles of Economics; but the book of which we celebrate the hundredth anniversary of original publication brought its author no such return. In a letter to Walras in 1873 Cournot confessed that he was very unpopular with his publishers because none of his books found any considerable number of readers until years after their first appearance [1, 8].

According to A. J. Nichol [1] the tragedies in the life of Cournot were not tragedies of stark physical misery such as have so often attended the lives of geniuses in literature, art, and philosophy. Cournot did not starve in a garret. He was not carried off by death in the springtime of youth as were Keats, Shelley, Byron, Abel, and Galois. He was not picked up out of a gutter to die as was Edgar Allan Poe. Economists generally do not do that sort of thing. The tragedies to which economists are usually subject are intellectual rather than physical. They are tragedies of the soul. So it was in the main with Cournot. Yet, as we shall see, a physical infirmity lay at the root of many of his intellectual disappointments. In America and England Cournot is known as an economist, but in France he is chiefly remembered as a philosopher. The article on Cournot in the Encyclopedia of the Social Sciences, written by one of his own countrymen, gives much more attention to his philosophical writings than to his work in economics. So it was also in the obituaries published in Paris after his death in 1877. published in Paris after his death in 1877. When a movement was started in 1905 for reconsideration of his work, the results were gathered together in no economic publication but in a philosophical journal. Among those with whom he mingled in day-to-day routine, however, Cournot was known as an administrative official of the French public schools. Not the least of the legacies left to France by Napoleon was its highly centralized educational system.

A. J. Nichol [1] discussed in his article, Cournot's friend, Poisson, died in 1840. After that Cournot made no further spectacular progress in the service of the University. Trouble with his eyes forced him in 1844 to take a year's leave of absence, and this he spent in Italy. He had experienced difficulties with vision while he was a student in the University and secretary to Marshal Gouvion Saint- Cyr. This condition had gradually grown worse through the years. After his return from Italy he does not seem to have been seriously handicapped in the performance of his regular duties as administrator and supervisor, but there was a very profound change in the character of his published work. To Walras in 1873 he wrote, "Thirty years ago I had to renounce all mathematics."

It became impossible for him to engage in any long-continued close work with his eyes. He did not give up his companionship with books. Secretaries read aloud to him regularly for hours at a time. He never found any one, however, who could read mathematics to him. Symbols, the meaning and possibilities of which he had once grasped so masterfully through the eye, he was never able to comprehend by ear. When Professor Irving Fisher edited the English translation of Cournot's Researches in the Mathematical Principles of the Theory of Wealth, he found the original French version replete with mistakes. Some of them were trivial and easily corrected; others were fundamental errors in mathematical reasoning. Thus Cournot's great work bore the appearance of gross carelessness in proofreading and original composition.3 Such perhaps there may indeed have been. Another very pertinent explanation of Cournot's errors of 1838, however, lies in the fact that even then he was approaching a state of partial blindness. To the academy (school district) of Dijon Cournot came as Rector (Superintendent) in 1854, having declined another more lucrative appointment because of political complications. He remained at Dijon until his retirement in 1862. Then, vigorous otherwise but eyesight worse than ever, almost Milton-like, he made his home again in Paris. A few years before retirement Cournot completed a very illuminating volume of personal memoirs. They remained in manuscript form, almost entirely unknown, for more than fifty years before they were published.9 These memoirs, and the voluminous philosophical works which Cournot had printed both before and after his retirement, were written in a very curious way. They were first jotted down piece- meal on scraps of paper, then collected and copied by an amanuensis, read back to him, and corrected. Outside of a few modest advertisements by his publisher the first printed reference to Courniot's work in economics came in a twenty- page review" in the Journal des fconomistes, August, 1864. In the first few months of his retirement Cournot had written a second book in economics. Its appearance turned the attention of the reviewer back to Cournot's first book, written twenty-five years previously; and to the earlier work the review was principally devoted. In conclusion Cournot was upbraided for lack of progress and lack of familiarity with other work in economics. Obstinate in his independence, Cournot in the preface to his third book in economics (published after his death) replied, saying in effect: "I am the only French economist who has not been cited by the others. I'll be d-d if I cite them." Fifteen years Cournot lived in retirement on the pension of an old employee of the University. These years were tinged with bitterness and disappointment. The news came that another young French professor, Leon Walras, was applying to economics the same technique which Cournot himself had used. Alas, it was a technique which had long before been renounced by its originator. As the shadows darkened, a possibility appeared of his election to the French Academy as an economist. To further his chances his friends urged him to write an- other book, and this he did; but death suddenly took him away without receipt of further honor. He did not live to see his last book in print. The supreme tragedy of Cournot's life without doubt was the tragedy of blindness. If he had found a good occulist, if he had been a well man, he might have advanced the progress of mathematical economics a generation. It would not then have been left to a Marshall, a Walras, and Irving Fisher to make his ideas known to the world. He might have done it himself.

Conclusion

It is hardly surprising that Cournot's road to a wider recognition of the significance of his contribution was so slow. His Recherches was a book before its time. Despite the appearance of other tendencies, mainstream economics in both England and on the continent of Europe was dominated, during the nineteenth century until the 1870s, by the classical tradition. Though not necessarily linked with this tradition, there was an aversion among most professional economists against the use of mathematics in economic analysis; any such attempt was treated with suspicion, if not hostility. This attitude was, perhaps, partly due to the fact that few of the academic economists of that time were able mathematicians. This was especially the case among academic economists in continental Europe, where political economy was in most cases one of the subjects taught in the law schools. On the other hand, most academic economists held the view that political economy was a branch of moral philosophy, for which the most appropriate method was the literary deductive method of analysing cause and effect. Characteristic in this respect was Caimes's thesis that 'economic truths are not discoverable through the instrumentality of mathematics' (1888, iv). It should be noted in this context that authors like Cher- riman, Fauveau, Newcomb, and Walras, who during this period had shown real appreciation of Coumot's contribution, had received formal training in theoretical and applied mathematics. Finally, one should not forget the language barrier, which made Cournot's Recherche practically inaccessible to English-speaking economists, before its trans- lation in 1897. It is interesting to note that the only English-speaking authors who had made references to Cournot in published writings before the second edition of Jevons's Theory in 1879 were Cherriman and Newcomb.

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