Is an unfunded social security system good or bad for growth? A theoretical analysis of social security systems financed by VAT

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Is an unfunded social security system good or bad for growth? A theoretical analysis of social security systems financed by VAT*

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Abstract

This study investigates (i) how unfunded public pensions financed by VAT, as discussed in Japan, affect economic growth, and (ii) whether payroll tax or VAT is the more growth-friendly tax structure for the finance of public pensions. We examine these issues in overlapping generations (OLG) models with parental altruism and find the following results. A public pension system financed by VAT itself may increase economic growth when bequests are operative. By contrast, when bequests are inoperative, public pensions hinder growth unless agents are sufficiently patient. Finally, public pensions financed by VAT have turned out to be more growth-friendly than those financed by payroll tax when bequests are operative.

JEL classification: D64, H20, H55, I20, O40

Keywords: Public pensions financed by VAT, Altruism, Education, Bequests, Growth

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1 Introduction

Population aging driven by increasing longevity and low fertility rates has been making it difficult to sustain funded social security systems.[1] In addition to population aging, many countries have been suffering from lower growth and weak fiscal conditions, both of which put pressure on the finance of pension systems. In these situations, many OECD countries have implemented reforms of public pension systems in order to improve the financial stability of pension systems. According to OECD (2015), many countries have increased revenue for financing public pensions by raising payroll tax rates or contributions. In contrast to these countries, in Japan, the consumption tax (VAT) rate has been increased from 5% into 8% in order to improve the financial stability of social security systems including public pensions. Therefore, the Japanese pension system was reformed by introducing VAT financing for the public pension(hereafter VAT-public pension).

Motivated by these policy efforts to stabilize public pensions, this study investigates the following research questions. First, is a VAT-public pension good or bad for growth? An increase in VAT would increase revenues for public pensions in the short run; however it might hinder economic growth and in turn be bad for financing public pensions in the long run. Therefore, investigating the effect of a VAT-public pension on growth is important. Second, is a VAT-public pension or a public pension financed by payroll tax (hereafter PT-public pension) better for economic growth? In the literature on fiscal policy, many studies investigate the relationship between tax structures and economic growth (see the survey by Arnold (2008) for example). However, in the literature on intergenerational public expenditure such as unfunded social security systems, little attention has been paid to whether payroll tax or VAT is the more growth-friendly tax structure.[2] Therefore, it is important to examine the relationship between the differences in financing of public pensions and economic growth.

This study tackles these problems by using endogenous growth models with overlapping-

[1] OECD (2015) state as follows: “The share of individuals aged 65 and above will increase from 8% of the total world population in 2015 to almost 18% by 2050 and from 16% to 27% in the OECD. In the OECD, the share of the population older than 75 years will be similar in 2050 to the share older than 65 years today. Ageing directly affects the financing of pay-as-you-go (PAYG) pension schemes, as a decreasing number of working-age people has to sustain pension levels for an increasing number of elderly.”

[2] The exception is Naqib and Stolley (1985). They show the following. First, a public pension system reduces capital accumulation regardless of whether its finance is based on payroll tax or VAT. Second, the reduction in capital accumulation with a VAT-public pension is less than that with a PT-public pension.

generations. The models have the following three features. First, human capital accumulation associated with parental altruism drives economic growth. Second, altruistic parents face a trade-off between leaving bequests and investing in their children's human capital taking account of these relative returns. Finally, public pension benefits are financed by VAT and payroll tax.

In this study, we consider two kinds of parental altruism. One is the family altruism developed by Lambrecht et al. (2005) under which agents experience the warm-glow of giving with either education or bequests to their closest children. The other is a perfect altruism under which agents care about all future descendants when they decide about educational spending and bequests for them. This type of family is also known as a dynasty.

Under these frameworks, there are opposite intergenerational transfers in effect, in the sense that public pension provision is a transfer from children to parents, while educational investment and bequest from parents affects the disposable income of children. The burden of pension on the younger generations deters saving and educational spending, which hinders economic growth. By contrast, public pension benefits increase disposable lifetime income and stimulate transfers (educational investment and bequests) to children, which enhance economic growth. Hence, public pensions exert opposite effects on growth through intergenerational transfers.

The difference in financing the public pension also affects economic growth through the trade-off of altruistic transfers between education and bequest. Under the PT-public pension system, payroll tax lowers returns from educational investment (wage rate) and has negative effects on growth as indicated in Lambrecht et al. (2005). Conversely, under the VAT-public pension system, an increase in VAT substitutes consumption when young into educational spending for children, while it substitutes consumption when aged into leaving bequests. The influence of the VAT-public pension on growth depends on how it affects relative returns between education and bequests through these substitution effects.

The main results from our study are summarized as follows. First, in a family altruism model, we obtain the following results.

(i) When bequests are operative, a VAT-public pension increases growth unambiguously if individuals' utility function takes the log-linear. This is opposite to the case of the PT-public pension examined by Lambrecht et al. (2005).

(ii) When bequests are inoperative, a VAT-public pension does not always increase growth. It is not positive for growth unless individuals are sufficiently patient. This result is qualitatively similar to the case of the PT-public pension examined by Lambrecht et al (2005).

(iii) We also check whether a VAT-public pension increases growth when bequests are operative even under the constant relative risk aversion (CRRA) and constant elasticity of substitution (CES) utility functions in a reduced from of endogenous growth model with Romer's (1986) AK production technology.

Second, in a perfect altruism model (dynasty model), a VAT-public pension is neutral to growth while a PT-public pension is bad for growth.

Our results lead to the following implications. First, whether a VAT-public pension is good for growth depends on the country's type of altruism. If parents have family altruism and they are altruistic enough to educate children and leave bequests, introducing a VAT-public pension may enhance economic growth. By contrast, if parents are not sufficiently altruistic such that bequests are inoperative, large burdens of VAT-public pension may be bad for growth. If parents have perfect altruism, a VAT-public pension is neutral to growth. Second, a VAT-public pension is more growth-friendly than a PT-public pension when bequests are operative.

The key mechanisms that explain how a VAT-public pension can promote economic growth and be growth-friendly are as follows. As mentioned above, VAT generates the substitution effect between educational spending and consumption in youth and between leaving bequests and consumption in old age. When bequests are operative, these two substitution effects offset each other because of the trade-off between leaving bequests and educational investment. Therefore VAT has no distortionary effect on education. This is different from the case of a PT-public pension under which payroll tax lowers returns from educational investment (wage rate) and has a negative effect on growth as indicated in Lambrecht et al. (2005). In addition to this, VAT does not distort saving because it becomes neutral to the intertemporal decision of consumption. Thus, tax can be neutral to both educational spending and saving. Furthermore, VAT-public pension benefits are partly transferred to children as a bequest, which increases disposable income of children and promotes saving, educational spending, and economic growth.

Related Literature

To our knowledge, there are some studies on VAT-public pension systems in exogenous growth models (e.g., Naqib and Stolley 1985; Okamoto 2010, 2013). These studies simulate the policy reform into VAT-public pension system. Naqib and Stolley (1985) show that VAT-public pension has a negative effect on capital accumulation, but the negative effect of VAT-public pension on capital accumulation is weaker than PT-public pension. Okamoto (2010, 2013) demonstrates that replacing a PT-public pension with a VAT-public pension stimulates capital accumulation in Japanese economy. In contrast to these studies, our study shows that the VAT-public pension system itself may be good for growth analytically.

This study is also related to other studies that address the growth-enhancing effect of public pension (e.g., Zhang 1995; Sala-i-Martin 1996; Kaganovich and Zilcha 1999; Sanchez-Losada 2000; Lambrecht et al. 2005).[3] Sanchez-Losada (2000) and Lambrecht et al. (2005) are closer to our study. Sanchez-Losada (2000) derives the result that public pensions can increase growth in an economy of operative bequest. However, he/she does not consider the trade-off of altruistic transfer between bequest and education. Lambrecht et al. (2005) incorporate this trade-off and find the opposite result to Sanchez-Losada (2000): A public pension is bad for growth when bequests are operative, while positive effects from pensions on growth occur when bequests are inoperative. Our study differs from these studies as follows. First, while they focus only on a PT-public pension, we consider a VAT-public pension. Second, a public pension can be good for growth under operative bequests in contrast to Lambrecht et al. (2005).

To sum up, this paper is the first study that addresses the growth-enhancing effect of a VAT-public pension and analytically compares it with a PT-public pension.

2 Model

2.1 individuals

Consider an OLG model in which each individual lives through three periods: childhood, adulthood and old age. During adulthood each individual gives birth to $1 + n$ children. We as-

[3] Zhang (1995) investigates the issue by exploring a perfect altruism model (dynasty model) with endogenous fertility and education choices. He shows that an unfunded program may enhance growth by reducing fertility and increasing human capital investment. Sala-i-Martin (1996) shows that a public pension induces retirement of aged workers whose obsolete skill exert a negative externality effect on the young workers' human capital and therefore promotes economic growth. Kaganovich and Zilcha (1999) consider the role of government's allocation of revenue between public spending on education and public pension, in which altruistically motivated educational spending from parents to children exists. They show that if agents are sufficiently patient and with a large altruism toward children, a public pension promotes economic growth.

sume population growth rate n is constant over time. Accordingly, a new cohort consisting of $N_t(= (1+n)N_{t-1})$ identical individuals is born in each period t. During childhood individuals do not make any economic decisions but are educated by their parents. When individuals born at $t-1$ become adult, they supply h_t efficiency units of labor (human capital) which depends on the educational spending by their parents. They receive a market wage (w_t) per one unit of efficient labor supply and a bequest (x_t) from their parents. Their disposable current income is $(1-\tau)w_t h_t + x_t$, where τ is the payroll contribution rate to the unfunded public pension scheme. Each individual spends his/her current disposable income on consumption (c_t), private education ($(1+n)e_t$), and savings, (s_t):

$$I_t \equiv (1-\tau)w_t h_t + x_t = (1+\tau_c)c_t + (1+n)e_t + s_t, \tag{1}$$

where τ_c is the VAT contribution rate to the unfunded VAT-public pension scheme. (1) indicates the following assumptions. When both consumption goods and expenditure on private education services are subject to VAT (i.e., $(1+\tau_c)(c_t + (1+n)e_t)$), each individual can receive a subsidy on educational spending through their VAT payments ($(1+n)\tau_c e_t$).[4] This means that, letting η denote the subsidy rate, we assume that $\eta = \tau_c$. Alternatively, we can simply assume that education services are exempt from VAT.[5]

During old-age, individuals born in $t-1$ are retired and receive the proceeds of their savings ($R_{t+1}s_t$), and public pension benefits (θ_{t+1}). They allocate their total revenue to old-age consumption (d_{t+1}), VAT payment ($\tau_c d_{t+1}$), and a non-negative bequest ($x_{t+1}(\geq 0)$) to each of their $1+n$ children. Thus, the budget constraint when aged is as follows:

$$(1+\tau_c)d_{t+1} = R_{t+1}s_t + \theta_{t+1} - (1+n)x_{t+1}. \tag{2}$$

The human capital of each individual born at t (h_{t+1}) is a function of his/her parents' private

[4] This assumption may not be so unrealistic in the future because this is in line with Japanese Prime Minister Shinzo Abe's plan to use sales tax revenue for education.

[5] In the EU, education services are exempt from VAT. According to the council of the EU (2006), the EU member states shall exempt VAT from the provision of children's or young people's education, school or university education, vocational training or retraining, including the supply of services and of goods closely related thereto, by bodies governed by public law having such as their aim or by other organizations recognized by the Member State concerned as having similar objects.

educational spending (e_t), and his/her parents' human capital (h_t):

$$h_{t+1} = De_t^{\delta} h_t^{1-\delta}, \tag{3}$$

where $D(>0)$ is a scale parameter and $\delta \in (0,1)$ is the elasticity of education technology with respect to private educational spending.

Here, we consider the family altruism developed by Lambrecht et al. (2005) under which agents have the warm-glow of giving between education and bequests to their closest children. As in Lambrecht et al. (2005), we assume that individuals who have family altruism derive utility from the disposable income of their adult children.[6] Each individual born at $t-1$ has the following logarithmic utility function (hereafter LUF):

$$U_t = (1-\beta)\ln c_t + \beta \ln d_{t+1} + \gamma \ln I_{t+1}, \tag{4}$$

where $\beta \in (0,1)$ is the intertemporal preference parameter and $\gamma(>0)$ is the degree of altruism towards one ' s own children and

$$I_{t+1} = (1-\tau)w_{t+1}h_{t+1} + x_{t+1} = (1-\tau)w_{t+1}De_t^{\delta}h_t^{1-\delta} + x_{t+1}. \tag{5}$$

Each individual maximizes utility (4) under constraints (1), (2), and (5) and the non-negativity of bequests $x_{t+1} \geq 0$ by choosing c_t, e_t, s_t, d_{t+1}, and x_{t+1}. The first order conditions (FOCs) are

$$\frac{\partial U_t}{\partial s_t} = -\frac{1-\beta}{c_t} + \frac{\beta R_{t+1}}{d_{t+1}} = 0, \tag{6}$$

$$\frac{\partial U_t}{\partial e_t} = -\frac{(1-\beta)(1+n)}{(1+\tau_c)c_t} + \frac{(1-\tau)\gamma w_{t+1} D\delta e_t^{\delta-1}h_t^{1-\delta}}{I_{t+1}} = 0, \tag{7}$$

$$\frac{\partial U_t}{\partial x_{t+1}} = -\frac{\beta(1+n)}{(1+\tau_c)d_{t+1}} + \frac{\gamma}{I_{t+1}} \leq 0 \quad (= 0 \text{ if } x_{t+1} > 0). \tag{8}$$

First, (6) indicates that VAT is neutral to the intertemporal decision of saving. Second, (7) shows that VAT reduces the marginal cost of educational spending and boosts investment in education: *the substitution effect* of τ_c on e_t. By contrast, payroll tax (τ) decreases the marginal benefit of

[6]This assumption is common to some studies that investigate intergenerational transfers or fiscal policies in a family altruism model (e.g., Lambrecht et al. 2005; Lambrecht et al. 2006; Kunze 2010, 2012, 2014; Alonso-Carrera et al. 2012).

educational spending and retards investment in education: *the negative distortionary effect* of τ on e_t. Finally, (8) indicates that VAT increases the incentive to leave bequests because τ_c reduces the marginal cost of leaving bequests: *the substitution effect* of τ_c on x_{t+1}.

Substituting (6) and (7) into (8) yields

$$(1-\tau)w_{t+1}D\delta e_t^{\delta-1}h_t^{1-\delta} \geq R_{t+1}. \tag{9}$$

The left hand side (LHS) of (9) represents the rate of return on educational spending whereas the right hand side (RHS) represents the rate of interest (the rate of return on saving for bequeathing). When bequests are operative, (9) holds with equality. In contrast, when bequests are inoperative, the rate of return on educational spending is strictly higher than the rate of interest.

2.2 Firms

In every period t, firms produce a single output under perfect competition by using physical capital (K_t), and human capital (H_t). The production function is assumed to take the Cobb-Douglas form:

$$Y_t = AK_t^{\alpha}H_t^{1-\alpha}, \tag{10}$$

where $A(>0)$ is the scale parameter and $\alpha \in (0,1)$ denotes the capital share. Profit maximization yields the following marginal productivity conditions:

$$w_t = (1-\alpha)\frac{Y_t}{H_t} = (1-\alpha)Ak_t^{\alpha}, \tag{11}$$

$$R_t = \alpha\frac{Y_t}{K_t} = \alpha Ak_t^{\alpha-1}, \tag{12}$$

where, $k_t = K_t/H_t$ is the physical to human capital ratio.

2.3 public pension system

The unfunded public pension system works as follows. The government in period t collects contributions from the wage income of the adults ($\tau w_t h_t N_{t-1}$), and from consumption both by adults and the aged ($\tau_c(c_t N_{t-1}+d_t N_{t-2})$), and transfers its total revenue ($\tau w_t h_t N_{t-1}+\tau_c(c_t N_{t-1}+d_t N_{t-2})$), to the aged in period t as a pension benefit ($\theta_t N_{t-2}$). Thus, the government constraint in each period t is given by

$$(1+n)\tau w_t h_t + (1+n)\tau_c c_t + \tau_c d_t = \theta_t. \tag{13}$$

2.4 Intertemporal Equilibrium

In equilibrium, the capital market clearing condition in any period $t \geq 0$ becomes

$$K_t = s_{t-1} N_{t-2}. \tag{14}$$

With the initial stock of physical capital (K_0), given, K_0 belongs to the N_{-2} individuals who are aged in period 0. That is each of the initial aged owns $s_{-1} = K_0/N_{-2}$. Furthermore, the market clearing conditions for the labor and goods market are:

$$H_t = h_t N_{t-1}, \tag{15}$$

$$Y_t = [c_t + (1+n)e_t] N_{t-1} + d_t N_{t-2} + K_{t+1}. \tag{16}$$

Here, let us define $c_t + d_t/(1+n) \equiv \varphi_t$ for tractability. By using φ_t, equations (13) and (16) are rewritten as

$$\tau_c \varphi_t + \tau w_t h_t = \frac{\theta_t}{1+n}, \tag{17}$$

$$Y_t = \varphi_t N_{t-1} + (1+n) e_t N_{t-1} + K_{t+1}. \tag{18}$$

3 Dynamics

3.1 Operative bequests

When bequests are operative, (9) holds with equality: $(1-\tau) w_{t+1} D \delta e_t^{\delta-1} h_t^{1-\delta} = R_{t+1}$. Substituting (11), (12), and (15) into this condition gives

$$(1+n) e_t N_{t-1} = \frac{(1-\alpha)\delta(1-\tau)}{\alpha} K_{t+1}. \tag{19}$$

Because of the trade-off between educational spending and bequest motives, the substitution effect of τ_c on e_t (see (7)) and that on x_{t+1} (see (8)) offset each other. Therefore, educational spending (e_t) is independent of the burden from VAT (τ_c). By contrast, payroll tax (τ) has a negative distortionary effect on e_t as seen from (7).

Using (2) and (8) with equality leads to $(1+\tau_c)d_{t+1} = [\beta(1+n)/\gamma]I_{t+1}$. Substituting it into (2), we obtain:

$$I_{t+1} = \frac{\gamma}{\beta(1+n)}[R_{t+1}s_t + \theta_{t+1} - (1+n)x_{t+1}]. \tag{20}$$

Combining (5) with (20) and using (17) yield:

$$x_{t+1} = \frac{\gamma}{\beta+\gamma}\left(\frac{R_{t+1}s_t}{1+n} + \tau_c\varphi_{t+1}\right) + \left(\tau - \frac{\beta}{\beta+\gamma}\right)w_{t+1}h_{t+1}. \tag{21}$$

(21) indicates the following. First, bequest motive (x_{t+1}) is independent of the burden from VAT(τ_c).This is because the positive substitution effect of τ_c on x_{t+1} (see (8)) and the negative income effect of τ_c on x_{t+1} (see (1)) offset each other. [7] Second, public pension benefits ($\tau_c\varphi_{t+1}$ and $\tau w_{t+1}h_{t+1}$) increase bequests (x_{t+1}) because they increase consumption in old age (d_{t+1}) and have *positive income effects* on bequests.

Next, from (14) and (19), educational spending is proportional to saving: $e_t = \alpha^{-1}(1-\alpha)\delta(1-\tau)s_t$. Substituting it into (1) and using (2), (6), and (17), we obtain

$$\begin{aligned}\left[1 + \frac{\beta(1-\alpha)\delta(1-\tau)}{\alpha}\right]s_t =&\beta[(1-\tau)w_t h_t + x_t] \\ &+ (1-\beta)(1+n)\left(\frac{x_{t+1}}{R_{t+1}} - \frac{\tau_c\varphi_{t+1} + \tau w_{t+1}h_{t+1}}{R_{t+1}}\right).\end{aligned} \tag{22}$$

(22) indicates the following. First, saving (s_t) is independent of the burden from VAT (τ_c) as we have seen in (6). On the contrary, a payroll tax (τ) has a negative distortionary effect on saving (s_t). Second, the bequest income from parents (x_t) and altruistic bequests to children

[7] This is attributed to the LUF under which the income and substitution effects offset each other. Therefore, if we apply more general preferences such as the CRRA and CES utility functions, x_{t+1} can depend on τ_c. However, this generalization complicates the analyses and makes it difficult to derive policy implications. Here, applying the LUF is somewhat more useful for the analytical investigations. In addition, the LUF is supported by some theoretical and empirical studies. In the literature of business cycle models, Kydland and Prescott (1982) and Jones et al. (2000) argue that the LUF leads the solutions from the model to fit the real data very well. Dalgaard and Jensen (2007) also justify the LUF, observing that the empirical saving elasticity is more or less constant. Furthermore, Guvenen (2006) shows that the elasticity of intertemporal substitution of 1 (the case of LUF) can be precisely estimated if the non-constant variance term is included in a regression of the log-linearized version of the Euler equation.

(x_{t+1}) positively affect s_t. Finally, public pension benefits have a negative effect on s_t in line with Feldstein (1974), which is attributed to the fact that public pension benefits lead to less incentive to save.

From (18) and (19), we obtain:

$$K_{t+1} = \frac{\alpha(1-\Delta_t)}{\alpha + (1-\alpha)\delta(1-\tau)} Y_t, \tag{23}$$

where we define the ratio of consumption to output as $\Delta_t \equiv (\varphi_t N_{t-1})/Y_t (< 1)$. Substituting (21) into (22) and using (11), (12), (14), (15), (23), and the definition of Δ_t, we obtain:

$$\frac{1-\beta+\beta[\alpha+(1-\alpha)\delta(1-\tau)]}{1+\tau_c} + \frac{(1-\beta)\tau_c}{1+\tau_c}\Delta_{t+1} = [\alpha+(1-\alpha)\delta(1-\tau)]\frac{\gamma\Delta_t}{1-\Delta_t}. \tag{24}$$

Please see Appendix A for the derivation of (24). Here, let us define the LHS and RHS of (24) as $\Gamma(\Delta_{t+1})$ and $\Psi(\Delta_t)$, respectively. By substituting the definition of Δ_t into (21), we can rewrite the non-negative bequest condition, $x_{t+1} \geq 0$ as follows.[8]

$$\tau + \frac{\gamma\tau_c\Delta_t}{(1-\alpha)(\beta+\gamma)} \geq 1 - \frac{\gamma}{(1-\alpha)(\beta+\gamma)} \equiv \chi. \tag{25}$$

(25) is satisfied for any $\Delta_t \in (0,1)$ if $\chi - \tau \leq 0$. Examining (25) and (24), we arrive at the following:

[Figure 1]

Lemma 1 *$\Gamma(\Delta_{t+1})$ and $\Psi(\Delta_t)$ intersect at a unique point E in which Δ_t has a unique equilibrium value $\Delta^* \in (0,1)$ if $\chi - \tau \leq 0$, as shown in Figure 1. Δ^* is given by:*

$$\frac{1-\beta+\beta[\alpha+(1-\alpha)\delta(1-\tau)]}{1+\tau_c} + \frac{(1-\beta)\tau_c}{1+\tau_c}\Delta^* = [\alpha+(1-\alpha)\delta(1-\tau)]\frac{\gamma\Delta^*}{1-\Delta^*}. \tag{26}$$

Δ_t jumps to its equilibrium value, Δ^*, initially.

Proof: See Appendix B.

[8] By adding $w_{t+1}h_{t+1}$ to both sides of (21), we can rewrite it into

$$x_{t+1} = \frac{\gamma}{\beta+\gamma}\left(\frac{R_{t+1}s_t}{1+n} + w_{t+1}h_{t+1} + \tau_c\varphi_{t+1}\right) - (1-\tau)w_{t+1}h_{t+1}.$$

Substituting it into $x_{t+1} \geq 0$ leads to (25).

We next move onto characterizing the intertemporal equilibrium path and the long-run growth rate of the economy with operative bequests, when the value of Δ^* is given by (26). Substituting (23) into (19) and using (10), (15), and $k_t = K_t/H_t$, we obtain:

$$e_t = \frac{\delta(1-\alpha)(1-\tau)(1-\Delta^*)}{(1+n)[\alpha+(1-\alpha)(1-\tau)\delta]} A k_t^{\alpha} h_t. \tag{27}$$

From (3) and (27), the accumulation of each individual's human capital is:

$$\frac{h_{t+1}}{h_t} = D\left\{\frac{\delta(1-\alpha)(1-\tau)(1-\Delta^*)A}{(1+n)[\alpha+(1-\alpha)(1-\tau)\delta]}\right\}^{\delta} k_t^{\alpha\delta}. \tag{28}$$

We rewrite (23) by using (10) and $k_t = K_t/H_t$ into $k_{t+1}\frac{H_{t+1}}{H_t} = \frac{\alpha(1-\Delta^*)}{\alpha+(1-\alpha)\delta(1-\tau)} A k_t^{\alpha}$. Substituting $H_{t+1}/H_t = (1+n)(h_{t+1}/h_t)$ and (28) into it, we obtain

$$k_{t+1} = \frac{\alpha}{D\delta(1-\alpha)(1-\tau)}\left\{\frac{\delta(1-\alpha)(1-\tau)(1-\Delta^*)A}{(1+n)[\alpha+(1-\alpha)(1-\tau)\delta]}\right\}^{1-\delta} k_t^{\alpha(1-\delta)}. \tag{29}$$

These dynamics of k_t show that a unique intertemporal equilibrium with operative bequests exists in each period, for $\chi - \tau \leq 0$, given initial values for physical, $K_0 > 0$, and human capital, $H_0 = h_0 N_{-1} > 0$. This is in line with Lambrecht et al. (2005) because we can confirm that when $\tau_c = 0$, (29) is consistent with the dynamics of k_t in Lambrecht et al. (2005).

From (28) and (29), the physical to human capital ratio in the steady state k^* and the long-run growth in individual's human capital are:

$$k^* = \left\{\frac{\alpha}{D\delta(1-\alpha)(1-\tau)}\right\}^{\frac{1}{1-\alpha(1-\delta)}}\left\{\frac{\delta(1-\alpha)(1-\tau)(1-\Delta^*)A}{(1+n)[\alpha+(1-\alpha)(1-\tau)\delta]}\right\}^{\frac{1-\delta}{1-\alpha(1-\delta)}}, \tag{30}$$

$$\frac{h_{t+1}}{h_t} = D\left\{\frac{\delta(1-\alpha)(1-\tau)(1-\Delta^*)A}{(1+n)[\alpha+(1-\alpha)(1-\tau)\delta]}\right\}^{\delta} (k^*)^{\alpha\delta}, \tag{31}$$

and Y_t, K_t, and H_t grow at the same rate of $H_{t+1}/H_t = (1+n)(h_{t+1}/h_t)$ in the steady state.

Policy effect of an increase in τ_c

Consider now the effect of an increase in τ_c on Δ^* and economic growth. An increase in τ_c leads to a downward shift in $\Gamma(\Delta_{t+1})$ because $\frac{d\Gamma(\Delta_{t+1})}{d\tau_c} = \frac{1-\beta}{(1+\tau_c)^2}\left\{\Delta_{t+1} - 1 - \frac{\beta[\alpha+(1-\alpha)(1-\tau)\delta]}{1-\beta}\right\} < 0$ holds from $\Delta_{t+1} < 1$. The new equilibrium is shown in point E' in Figure 1. Accordingly, we find that an increase in τ_c reduces the ratio of consumption to output from Δ^* to Δ^{**}. Because the individual's human capital in the long run (h_{t+1}/h_t) is decreasing in Δ^* from (31), an increase in τ_c encourages the accumulation of individual's human capital and fosters economic growth. We summarize these results in the following proposition.

Proposition 1

When bequests are operative, an increase (introduction) of an unfunded VAT-public pension decreases the ratio of consumption to output (Δ^*) and enhances economic growth.

This is opposite to the result in the case of a PT-public pension system shown by Lambrecht et al (2005). They show that a PT-public pension system (τ) unambiguously reduces economic growth when bequests are operative.

Here, we consider the intuition behind the result from Proposition 1 and the difference between a VAT-public pension and PT-public pension. As we have seen in (19) and (22), both saving (s_t) and educational spending (e_t) are independent of the burden from VAT (τ_c).

However, τ_c affects both s_t and e_t through VAT-public pension benefits. Some opposite effects of a VAT-public pension on s_t exist. First, the positive income effect of public pension on bequests to children (x_{t+1}) increases s_t (see (22) and (22)). Second, VAT-public pension counters s_t because it reduces incentive to save (see (22)). These are also the case with e_t because e_t is proportional to $s_t(= K_{t+1}/N_{t-1})$ (see (19)). Finally, the positive income effect of a public pension on bequests (x_t) increases disposable income when young (I_t) and increases both s_t and e_t. These positive effects of a VAT-public pension on both e_t and s_t dominate the negative effect, which encourages physical and human capital accumulation and fosters economic growth.

In contrast to the case of a VAT-public pension, a PT-public pension reduces economic growth because the direct negative distortionary effect of τ on e_t has a detrimental effect on growth (see (19)).

3.2 Inoperative bequests

When bequests are not operative ($\chi - \tau > 0$ and $x_{t+1} = 0$), (5) becomes $I_{t+1} = (1-\tau)w_{t+1}h_{t+1}$. Substituting $I_{t+1} = (1-\tau)w_{t+1}h_{t+1}$ into (7), we obtain

$$e_t = \frac{\gamma\delta(1+\tau_c)c_t}{(1-\beta)(1+n)}. \tag{32}$$

Furthermore, applying inoperative bequests, $x_t = x_{t+1} = 0$, into both (1) and (2), we obtain the budget constraint in adulthood, $(1-\tau)w_t h_t = (1+\tau_c)c_t + (1+n)e_t + s_t$, and that in old age, $(1+\tau_c)d_{t+1} = R_{t+1}s_t + \theta_{t+1}$, respectively. From these two budget constraints (6) and (32), saving (s_t), and educational spending (e_t) of each adult are obtained as follows:

$$s_t = \frac{\beta(1-\tau)}{1+\gamma\delta}w_t h_t - \frac{1-\beta+\gamma\delta}{1+\gamma\delta}\frac{\theta_{t+1}}{R_{t+1}}, \tag{33}$$

$$e_t = \frac{\gamma\delta}{(1+\gamma\delta)(1+n)}\left[(1-\tau)w_t h_t + \frac{\theta_{t+1}}{R_{t+1}}\right]. \tag{34}$$

Substituting (11), (12), (14), (15), and (17) into (33) and using $\Delta_t \equiv (\varphi_t N_{t-1})/Y_t$, we obtain:

$$K_{t+1} = \frac{\alpha\beta(1-\alpha)(1-\tau)}{\alpha\beta + (1-\beta+\gamma\delta)[\alpha + (1-\alpha)\tau + \tau_c\Delta_{t+1}]}Y_t \tag{35}$$

Substituting (34) into (18) and using (11), (12), (14), (17), (33), and (35), we obtain

$$\frac{(1-\alpha)(1-\tau)[\alpha\beta + \gamma\delta\{\alpha + (1-\alpha)\tau + \tau_c\Delta_{t+1}\}]}{\alpha\beta + (1-\beta+\gamma\delta)\{\alpha + (1-\alpha)\tau + \tau_c\Delta_{t+1}\}} = 1 - \Delta_t. \tag{36}$$

Please see Appendix C for the derivation of this equation. Here, let us define the LHS and RHS of (36) as $\tilde{\Gamma}(\Delta_{t+1})$ and $\tilde{\Psi}(\Delta_t)$, respectively. Examining (36), we arrive at the following:

[Figure 2]

Lemma 2 $\tilde{\Gamma}(\Delta_{t+1})$ *and* $\tilde{\Psi}(\Delta_t)$ *intersect at a unique point* E *in which* Δ_t *has a unique steady state value* Δ^* *as shown in Figure 2.* Δ^* *is given by:*

$$\frac{(1-\alpha)(1-\tau)[\alpha\beta + \gamma\delta\{\alpha + (1-\alpha)\tau + \tau_c\Delta^*\}]}{\alpha\beta + (1-\beta+\gamma\delta)\{\alpha + (1-\alpha)\tau + \tau_c\Delta^*\}} = 1 - \Delta^*. \tag{37}$$

Δ_t *jumps to its equilibrium value,* Δ^**, initially.*

Proof: See Appendix D

We next move to characterize the intertemporal equilibrium path and the long-run growth rate of the economy with inoperative bequests, when the value of Δ^* is given by (37). Substituting (17) into (34) and using (10), (11), (12), (15), (35), $1+n = N_t/N_{t-1}$, $k_t = K_t/H_t$, and the definition of Δ_t, we obtain

$$e_t = \frac{\gamma\delta[\alpha + \tau_c\Delta^* + \tau(1-\alpha)](1-\alpha)(1-\tau)}{(1+n)\{\alpha\beta + [\alpha + \tau_c\Delta^* + \tau(1-\alpha)](1-\beta+\gamma\delta)\}} A k_t^{\alpha} h_t. \tag{38}$$

See Appendix E for the derivation of (38) in detail. From (3) and (38), the accumulation of each individual's human capital is

$$\frac{h_{t+1}}{h_t} = D\left\{\frac{\gamma\delta[\alpha + \tau_c\Delta^* + \tau(1-\alpha)](1-\alpha)(1-\tau)A}{(1+n)\{\alpha\beta + [\alpha + \tau_c\Delta^* + \tau(1-\alpha)](1-\beta+\gamma\delta)\}}\right\}^{\delta} k_t^{\alpha\delta}. \tag{39}$$

Now rewrite (35) in equilibrium by using (10) and $k_t = K_t/H_t$ into

$$k_{t+1}\frac{H_{t+1}}{H_t} = \frac{\alpha\beta(1-\alpha)(1-\tau)}{\alpha\beta + [\alpha + \tau_c\Delta^* + \tau(1-\alpha)](1-\beta+\gamma\delta)} A k_t^{\alpha}.$$

Substituting $H_{t+1}/H_t = (1+n)(h_{t+1}/h_t)$ and (39) into it, we obtain:

$$k_{t+1} = \frac{\alpha\beta D^{-1} z(\Delta^*)^{1-\delta}}{\{\gamma\delta[\alpha + \tau_c\Delta^* + \tau(1-\alpha)]\}^{\delta}} k_t^{\alpha(1-\delta)}, \tag{40}$$

$$z(\Delta^*) \equiv \frac{(1-\alpha)(1-\tau)A}{(1+n)\{\alpha\beta + [\alpha + \tau_c\Delta^* + \tau(1-\alpha)](1-\beta+\gamma\delta)\}}.$$

These dynamics of k_t converge monotonically towards a steady state.[9] From (39) and (40), the physical to human capital ratio in the steady state k^* and the long-run growth in individual's human capital are

$$k^* = \left\{D^{-1}\alpha\beta(\gamma\delta)^{-\delta}[\alpha + \tau_c\Delta^* + \tau(1-\alpha)]^{-\delta} z(\Delta^*)^{1-\delta}\right\}^{\frac{1}{1-\alpha(1-\delta)}}, \tag{41}$$

$$\frac{h_{t+1}}{h_t} = D\left\{\gamma\delta[\alpha + \tau_c\Delta^* + \tau(1-\alpha)] z(\Delta^*)\right\}^{\delta} (k^*)^{\alpha\delta}. \tag{42}$$

[9]Here, we can confirm that when $\tau_c = 0$, (40) is consistent with the dynamics of k_t in Lambrecht et al (2005), when we use $\alpha + \tau(1-\alpha) = 1 - (1-\alpha)(1-\tau)$.

In this steady state with inoperative bequests, Y_t, K_t, and H_t grow at the same constant rate: $H_{t+1}/H_t = (1+n)(h_{t+1}/h_t)$.

Policy effect of an increase in τ_c

The rest of this subsection examines the effects of a public pension system financed by τ_c. An increase in τ_c leads to a downward shift in $\tilde{\Gamma}(\Delta_{t+1})$ because of

$$sign\frac{d\tilde{\Gamma}(\Delta_{t+1})}{d\tau_c} = -\frac{\alpha\beta(1-\beta)\Delta_{t+1}}{\{\alpha\beta + (1-\beta+\gamma\delta)[\alpha + (1-\alpha)\tau + \tau_c\Delta_{t+1}]\}^2} < 0$$

Thus, an increase in τ_c raises the long–run ratio of consumption to output from Δ^* to Δ^{**}. It is noticeable that this result is opposite to the case when the bequests are operative.

In contrast to the case of operative bequests, the introduction of a VAT-public pension system does not always enhance economic growth. More specifically, if $\beta > (<)\bar{\beta} \equiv \frac{(1+\gamma\delta)[1-(1-\alpha)(1-\tau)]}{1+\tau(1-\alpha)}$, the introduction of a small VAT-public pension is good (bad) for growth. When $\beta > \bar{\beta}$, the growth-maximizing size of a VAT-public pension (τ_c^{GM}) exists from some numerical examples.[10] These results are qualitatively similar to those in Lambrecht et al. (2005) who examine public pensions financed both by a lump sum tax and payroll tax.

These results are a result of the following reasons. There are two opposite effects of public pension benefits on growth because public pension benefits lead to less incentive to save (see (33)), but have a positive income effect on education (see (34)). In contrast to the case of operative bequests, the young do not benefit from bequests from their parents, and then disposable income for both saving and educational spending becomes smaller. Therefore, the growth-enhancing effects are weaker than in the case of operative bequests, and thus a VAT-public pension does not always foster economic growth.

4 Case studies under more general utility functions

Thus far, we have assumed the LUF because this is beneficial to analyzing the social security policies in the standard OLG model with complementarity between physical and human capital. However, the LUF offsets the income and substitution effects through VAT. Then, the objective

[10] When $(\alpha, \beta, \gamma, \delta, \tau) = (0.25, 0.4, 0.2, 0.6, 0.1)$, we obtain the growth-maximizing size of a VAT-public pension system τ_c^{GM} of 0.12.

in this section is to take these two effects into account. To do so, we introduce the CRRA and CES utility functions and focus on the case of operative bequests in which policy implication is largely different from Lambrecht et al. (2005). In this experiment, we simplify the model by ignoring human capital and using the AK model as in Romer (1986). This is because, in the case of operative bequests, investment in human capital (e_t) is proportional to that in physical capital (s_t) and these two play similar role in growth.[11]

Here, we specify the production function of firm j as $Y_{j,t} = AK_{j,t}^{\alpha}\left(a_t L_{j,t}\right)^{1-\alpha}$ $(0 < \alpha < 1)$, where $a_t = K_t/L_t$, $K_{j,t}$, $L_{j,t}$, K_t, and L_t represent labor productivity, physical capital of firm j, labor input of firm j, the aggregate stock of private capital, and the aggregate labor input, respectively. This specification follows Romer (1986). We assume that the population size is normalized to 1 without loss of generality. In the equilibrium, $K_{j,t} = K_t$ and $L_{j,t} = L_t = 1$ hold for all j, and thus the factor prices and aggregate output in period t can be written as $w_t = A(1-\alpha)K_t$, $R = \alpha A$,, and $Y_t = AK_t$, respectively.

Without educational spending, the budget constraints of each individual are $(1-\tau)w_t + x_t = (1+\tau_c)c_t + s_t$ and $(1+\tau_c)d_{t+1} = R_{t+1}s_t + \theta_{t+1} - x_{t+1}$, where we assume that the population growth rate (n) is zero for simplicity.

4.1 A time-additive CRRA preference

Here, we consider the case of time–additive CRRA preference. Then, let us change (4) into

$$U_t = (1-\beta)\frac{c_t^{1-1/\sigma}-1}{1-1/\sigma} + \beta\frac{d_{t+1}^{1-1/\sigma}-1}{1-1/\sigma} + \gamma\frac{I_{t+1}^{1-1/\sigma}-1}{1-1/\sigma}, \tag{43}$$

where σ represents the elasticity of substitution between c_t and d_{t+1} and I_{t+1}.[12] The case of (4) (i.e., the LUF) realizes when $\sigma = 1$. The FOCs for the households with respect to s_t and x_{t+1} are as follows:

$$d_{t+1} = (\rho R)^{\sigma} c_t, \tag{44}$$

$$I_{t+1} = (1+\tau_c)^{\sigma}(\gamma/\beta)^{\sigma} d_{t+1}. \tag{45}$$

[11] Remember that when bequests are operative, e_t is determined by the relative returns of physical and human capital (see (9) and (19)).

[12] This specification is in line with Kunze (2010) who investigates the effects of increasing capital income tax on economic growth.

where $\rho \equiv \beta/(1-\beta)$. Because we restrict our attention to the case of operative bequest, equation (45) holds with equality.

From (44), (45), aggregate output: $Y_t = AK_t$, factor prices: $w_t = A(1-\alpha)K_t$ and $R = \alpha A$, the households' budget constraints: $(1-\tau)w_t + x_t = (1+\tau_c)c_t + s_t$ and $(1+\tau_c)d_{t+1} = R_{t+1}s_t + \theta_{t+1} - x_{t+1}$, the asset market clearing condition: $K_{t+1} = s_t$, and the resource constraint of the economy: $Y_t = c_t + d_t + K_{t+1}$, we obtain:

$$\frac{A(1+\tau_c\Delta_{t+1})}{1+\tau_c} + \frac{(\rho R)^\sigma}{1+\tau_c} = (\rho R)^\sigma(\gamma/\beta)^\sigma(1+\tau_c)^{\sigma-1}\frac{\Delta_t}{1-\Delta_t}, \tag{46}$$

where $\Delta_t \equiv (c_t + d_t)/Y_t$.[13] (46) indicates the following. First, the steady–state value, Δ^*, is uniquely determined as represented by Figure 3. Second, as we have expected (see the first paragraph of this section), the properties of (46) are qualitatively similar to those of (24). Finally, in the case of a VAT-public pension, how an increase in τ_c affects Δ^* depends largely on the value of σ. The relationship between σ and the effect of an increase in τ_c on Δ^* is summarized in the following Lemma 3.

[Figure 3]

Lemma 3 *Under the CRRA utility function, AK production technology, and operative bequest, the following results hold. (I) An increase in τ_c shifts the LHS of* (46) *downward. (II) The effects of increasing τ_c on the RHS of* (46) *are classified into the following two cases.*

1. *If $\sigma \geq 1$, an increase in τ_c shifts the RHS of* (46) *leftward, and therefore Δ^* decreases as represented in Figure 3.*

2. *If $0 < \sigma < 1$, an increase in τ_c shifts the RHS of* (46) *rightward, and therefore, whether Δ^* decreases or increases is ambiguous. However, if σ is small enough to be close to 0, an increase in τ_c raises Δ^* as represented in Figure 3.*

Next, we rewrite the resource constraint: $K_{t+1} = Y_t - (c_t + d_t)$ into $K_{t+1}/K_t = A(1-\Delta_t)$ by using $(c_t + d_t)/Y_t \equiv \Delta_t$ and $Y_t = AK_t$. Combining Lemma 3 with it, we arrive at the following proposition:

[13] Without investment on human capital, payroll tax (τ) does not affect Δ^*, and then Ricardian equivalence holds with respect to a PT-public pension in line with Lambrecht et al. (2006).

Proposition 2

1. *If* $\sigma \geq 1$*, an increase in* τ_c *enhances economic growth.*

2. *If* $0 < \sigma < 1$*, the effect of an increase in* τ_c *on economic growth is ambiguous . However, if* σ *is small enough to be close to 0, an increase in* τ_c *hinders economic growth.*

From (44) and (45), we obtain

$$I_{t+1} = (\rho R)^{\sigma}(\gamma/\beta)^{\sigma}(1+\tau_c)^{\sigma}c_t.$$

A large σ means that individuals save more for bequeathing, and then the substitution effect between c_t and I_{t+1} becomes large. A higher VAT (τ_c) leads to a higher (lower) price of consumption (bequeathing) and increases bequests. This positive substitution effect of τ_c on bequests is represented as $(1+\tau_c)^{\sigma}$. By contrast, there is a negative income effect of τ_c on bequests, which is represented as $(1+\tau_c)^{-1}$. From (46), we can confirm that without the case of LUF ($\sigma \neq 1$), both the substitution and income effects do not offset each other, and therefore τ_c is not neutral to bequests.

When $\sigma \geq 1$, the positive substitution effect dominates the negative income effect, and then VAT increases bequest motives and savings. This stimulus adds to the positive income effect of a VAT-public pension on bequests as we have seen in subsection 3.1. Thus, increases in a VAT-public pension promote capital accumulation and economic growth.

In contrast, when $\sigma < 1$, the income effect dominates the substitution effect, and then VAT decreases bequest motives and savings. In this case, whether a VAT-public pension system is good for growth is ambiguous and depends on the relative magnitude of the negative burden effect of VAT and the positive income effect of the public pension on bequests. When $\sigma \to 0$, the former is stronger than the latter. Thus, increases in VAT-public pension retard economic growth.

4.2 A CES preference

Next, we consider the following CES utility function:

$$U_t = \left[\zeta_1 c_t^{\eta} + \zeta_2 d_{t+1}^{\eta} + \zeta_3 I_{t+1}^{\eta}\right]^{\frac{1}{\eta}}, \quad \eta \leq 1, \tag{47}$$

where ζ_1, ζ_2, and ζ_3 are positive constants that satisfy $\zeta_1 + \zeta_2 + \zeta_3 = 1$, and $1/(1-\eta)$ is the elasticity of substitution between c_t and d_{t+1} and I_{t+1}. The FOCs for the households with respect to s_t and x_{t+1} are $d_{t+1} = \left(\frac{\zeta_2}{\zeta_1}\right)^{\frac{1}{1-\eta}} R^{\frac{1}{1-\eta}} c_t$ and $I_{t+1} = \left(\frac{\zeta_3}{\zeta_2}\right)^{\frac{1}{1-\eta}} (1+\tau_c)^{\frac{1}{1-\eta}} d_{t+1}$, respectively. As in the procedure to derive (46), we obtain

$$\frac{A(1+\tau_c\Delta_{t+1})}{1+\tau_c} + \left(\frac{\zeta_2}{\zeta_1}\right)^{\frac{1}{1-\eta}} \frac{R^{\frac{1}{1-\eta}}}{1+\tau_c} = \left(\frac{\zeta_3}{\zeta_1}\right)^{\frac{1}{1-\eta}} R^{\frac{1}{1-\eta}} (1+\tau_c)^{\frac{\eta}{1-\eta}} \frac{\Delta_t}{1-\Delta_t}. \tag{48}$$

We find that the properties of (48) are qualitatively similar to those of (46). (48) indicates that when $0 < \eta \leq 1$, the positive substitution effect of an increase in τ_c dominates the negative income effect, and then VAT increases bequest motives and savings, which fosters economic growth.

5 Growth effect of VAT-public pension system under a perfect altruism model with operative bequests

In this section, we examine the growth effect of a VAT-public pension system under a perfect altruism model with operative bequests (Barro's (1974) type model). In contrast to family altruism, a perfect altruism assumes that parents take all future generations' utility into account. That is the representative agent maximizes the following utility

$$V_0 = \sum_{t=0}^{\infty} \bar{\gamma}^t [(1-\beta)\ln c_t + \beta \ln d_{t+1}], \tag{49}$$

subject to the budget constraints (1): $(1-\tau)w_t h_t + x_t = (1+\tau_c)c_t + (1+n)e_t + s_t$ and (2): $(1+\tau_c)d_{t+1} = R_{t+1}s_t + \theta_{t+1} - (1+n)x_{t+1}$, human capital accumulation (3): $h_{t+1} = De_t^{\delta} h_t^{1-\delta}$, and nonnegative bequest constraint ($x_{t+1} > 0$), taking τ and τ_c as given. FOCs are given by

$$s_t: \qquad d_{t+1} = \frac{\beta}{1-\beta} R_{t+1} c_t, \tag{50}$$

$$e_t: \qquad \frac{1+n}{c_t} = \bar{\gamma}\delta \frac{1-\tau}{c_{t+1}} \frac{w_{t+1}h_{t+1}}{e_t}, \tag{51}$$

$$x_{t+1}: \qquad c_{t+1} = \frac{\bar{\gamma} R_{t+1}}{1+n} c_t. \tag{52}$$

Definition. *Given an initial state* (N_{-2}, K_0, H_0)*, a competitive equilibrium in the economy with public pensions is a sequence of allocations* $\{c_t, d_t, N_{t-1}, x_t, e_t, s_t, \theta_t, K_{t+1}, H_{t+1}, Y_t\}_{t=0}^{\infty}$ *and prices* $\{R_t, w_t\}_{t=0}^{\infty}$ *such that (i) taking prices and the tax and replacement rates* (τ, τ_c) *as given, firms and households optimize their solutions ((11), (12), (50), (51), and (52)) are feasible, (ii) the budget for public pensions is balanced: (13), and (iii) markets clear with (14) and (15).*

From these equilibrium conditions, Appendix F derives

$$(1+n)e_t N_{t-1} = \frac{(1-\alpha)\delta(1-\tau)}{\alpha} K_{t+1}, \tag{53}$$

$$K_{t+1} = s_t N_{t-1} = \bar{\gamma}\alpha Y_t, \tag{54}$$

and the aggregate growth rate in the steady state as

$$\frac{Y_{t+1}}{Y_t} = D(1+n)\left\{\frac{\bar{\gamma}\delta(1-\alpha)A}{1+n}\right\}^{\delta} \kappa^{\frac{\alpha\delta}{1-\alpha(1-\delta)}}(1-\tau)^{\frac{\delta(1-\alpha)}{1-\alpha(1-\delta)}}. \tag{55}$$

From (55), we immediately obtain the following proposition.

Proposition 3

a VAT-Public pension (τ_c) *is neutral to economic growth whereas a PT-public pension* (τ) *is bad for economic growth.*

Proposition 3 indicates that, in contrast to the case of family altruism, VAT-public pension (τ_c) is neutral to economic growth in that of perfect altruism, while the effect of a PT-public pension (τ) on economic growth is common to both cases.

This is as a result of the following reasons. First of all, (53) is attributed to the trade-off between educational spending and bequest motives and is the same as (19). Thus, like the family altruism model, τ has a negative distortionary effect on e_t whereas τ_c is neutral to e_t (see below (19)).

Furthermore, (54) indicates that public pensions are neutral to the saving ratio in line with the Ricardian hypothesis. Combining (54) with (53), we find that a VAT-public pension (τ_c) has no impact on both physical and human capital accumulation, and therefore it is neutral to growth. On the other hand, a PT-public pension (τ) has no impact on physical capital accumulation, but the negative distortionary effect in (53) remains, which hinders growth.

6 Conclusion

This study investigated (i) how a VAT-public pension affects economic growth, and (ii) whether payroll tax or VAT is a more growth-friendly tax structure for the finance of public pensions. We tackled these problems by endogenous growth models in which altruistic parents face a trade-off between leaving a bequest and investing in their children's human capital taking account of these relative returns. In the case of family altruism, we obtained the following results.

A VAT-public pension system itself can increase economic growth when bequests are operative. This result is opposite to the case of PT-public pension as in Lambrecht et al. (2005). By contrast, when bequests are inoperative, VAT-public pension does not enhance economic growth unless agents are sufficiently patient. This result is qualitatively similar to a PT-public pension.

In the case of the perfect altruism model, a VAT-public pension is neutral to growth while a PT-public pension is bad for growth.

Our results lead to the following implications. First, whether a VAT-public pension is good for growth depends on its country's type of altruism. If parents have family altruism and they are altruistic enough to educate children and leave bequests, introducing a VAT-public pension may enhance economic growth. By contrast, if parents are not sufficiently altruistic such that bequests are inoperative, the large burdens of a VAT-public pension may be bad for growth. If parents have perfect altruism, a VAT-public pension is neutral to growth. Second, a VAT-public pension is more growth-friendly than a PT-public pension when bequests are operative.

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Appendix

A Derivation of (24)

Substituting (21) into (22) gives

$$\left[1+\frac{\beta(1-\alpha)\delta(1-\tau)}{\alpha}\right]s_t = \frac{\beta\gamma}{\beta+\gamma}\left(w_t h_t + \frac{R_t s_{t-1}}{1+n} + \tau_c\varphi_t\right) \\ + \frac{1-\beta}{\beta+\gamma}\left(\gamma s_t - \beta(1+n)\frac{\tau_c\varphi_{t+1} + w_{t+1}h_{t+1}}{R_{t+1}}\right). \tag{A.1}$$

$w_t = (1-\alpha)Y_t/H_t$ ((11)), $H_t = h_t N_{t-1}$ ((15)), $R_t = \alpha Y_t/K_t$ ((12)), $K_t = s_{t-1}N_{t-2}$ ((14)), and $N_{t-1} = (1+n)N_{t-2}$ rewrite (A.1) into

$$\left[1+\frac{\beta(1-\alpha)\delta(1-\tau)}{\alpha}\right]\frac{K_{t+1}}{N_{t-1}} \\ = \frac{\beta\gamma}{\beta+\gamma}\left(\frac{Y_t}{N_{t-1}} + \tau_c\varphi_t\right) + \frac{1-\beta}{\beta+\gamma}\left[\left(\gamma - \frac{\beta(1-\alpha)}{\alpha}\right)\frac{K_{t+1}}{N_{t-1}} - \frac{\beta}{\alpha}\frac{\tau_c(1+n)\varphi_{t+1}K_{t+1}}{Y_{t+1}}\right], \\ \Leftrightarrow \left[\{\alpha+\beta(1-\alpha)\delta(1-\tau)\}(\beta+\gamma) - (1-\beta)\{\alpha(\beta+\gamma)-\beta\} + \beta(1-\beta)\frac{\tau_c\varphi_{t+1}N_t}{Y_{t+1}}\right]K_{t+1} \\ = \alpha\beta\gamma\left(1+\frac{\tau_c\varphi_t N_{t-1}}{Y_t}\right)Y_t. \tag{A.2}$$

Substituting (23) into (A.2) and using $\Delta_t \equiv \varphi_t N_{t-1}/Y_t$ yield

$$\frac{\left[1-\beta+\{\alpha+(1-\alpha)\delta(1-\tau)\}(\beta+\gamma)+(1-\beta)\tau_c\Delta_{t+1}\right](1-\Delta_t)}{\alpha+(1-\alpha)\delta(1-\tau)} = \gamma(1+\tau_c\Delta_t), \\ \Leftrightarrow \frac{1-\beta+\beta[\alpha+(1-\alpha)\delta(1-\tau)]}{1+\tau_c} + \frac{(1-\beta)\tau_c}{1+\tau_c}\Delta_{t+1} = [\alpha+(1-\alpha)\delta(1-\tau)]\frac{\gamma\Delta_t}{1-\Delta_t}.$$

B Proof of Lemma 1

The properties of $\Gamma(\Delta_{t+1})$ and $\Psi(\Delta_t)$ as follows. On the one hand, $\Gamma(\Delta_{t+1})$ is an upward-sloping line because of $\Gamma'(\Delta_{t+1}) = \frac{(1-\beta)\tau_c}{1+\tau_c} > 0$, and $\Gamma''(\Delta_{t+1}) = 0$, and it intersects the vertical axis at the positive value of $\Gamma(0) = \frac{1-\beta+\beta[\alpha+(1-\alpha)\delta(1-\tau)]}{1+\tau_c} > 0$. On the other hand, $\Psi(\Delta_t)$ is a strictly increasing and convex function of Δ_t because of $\Psi'(\Delta) = [\alpha+(1-\alpha)\delta(1-\tau)]\gamma\frac{2-\Delta_t}{(1-\Delta_t)^2} > 0$ and $\Psi''(\Delta) = [\alpha+(1-\alpha)\delta(1-\tau)]\gamma\frac{3-\Delta_t}{(1-\Delta_t)^3} > 0$. In addition, $\Psi(\Delta_t)$ equals to zero when $\Delta_t = 0$.

Thus, a unique equilibrium E that is unstable exists as represented by Figure 1. This implies that the forward-looking variable Δ must jump to Δ^* at the initial date. Otherwise, the monotonic dynamics would leads Δ_t to either 0 or 1. Neither $\Delta_t = 0$ nor $\Delta_t = 1$ are valid equilibria because of the following reasons. First, $\Delta_t = 0$ leads to $c_t = d_t = 0$ and violates the first order condition under the LUF of $\ln c_t$ and $\ln d_{t+1}$.[14] Second, $\Delta_t = 1$ leads to $K_{t+1} = 0$ from (23) and again to the unrational choice $c_{t+1} = d_{t+1} = 0$.

C Derivation of (36)

From (33), we obtain $(1-\tau)w_t h_t = \frac{1+\gamma\delta}{\beta} s_t + \frac{(1-\beta+\gamma\delta)\theta_{t+1}}{\beta R_{t+1}}$. Substituting it into (34), we obtain

$$(1+n)e_t = \frac{\gamma\delta}{\beta}\left(s_t + \frac{\theta_{t+1}}{R_{t+1}}\right). \tag{C.1}$$

Substituting (C.1) into (18) yields $Y_t - \varphi_t N_{t-1} = \frac{\gamma\delta}{\beta}\left(s_t N_{t-1} + \frac{\theta_{t+1}}{R_{t+1}}\right) + K_{t+1}$. By using (14) and (17), it is rewritten as

$$Y_t - \varphi_t N_{t-1} = \frac{\gamma\delta + \beta}{\beta} K_{t+1} + \frac{\gamma\delta}{\beta} \cdot \frac{\tau_c \varphi_{t+1} N_t + \tau w_{t+1} H_{t+1}}{R_{t+1}}. \tag{C.2}$$

Substituting (11) and (12) into (C.2), and using the definition of Δ_t, we obtain

$$1 - \Delta_t = \frac{\{\alpha\beta + \gamma\delta[\alpha + (1-\alpha)\tau + \tau_c \Delta_{t+1}]\}}{\alpha\beta} \cdot \frac{K_{t+1}}{Y_t}. \tag{C.3}$$

Substituting (35) into (C.3) yields

$$\frac{(1-\alpha)(1-\tau)[\alpha\beta + \gamma\delta\{\alpha + (1-\alpha)\tau + \tau_c \Delta_{t+1}\}]}{\alpha\beta + (1-\beta+\gamma\delta)\{\alpha + (1-\alpha)\tau + \tau_c \Delta_{t+1}\}} = 1 - \Delta_t. \tag{C.4}$$

D Proof of Lemma 2

It is obvious that $\tilde{\Gamma}(\Delta_{t+1}) \in (0,1)$ holds for any $\Delta_{t+1} \in (0,1)$ because of $(1-\alpha)(1-\tau) < 1$ and $1 - \beta > 0$. Furthermore, $sign\tilde{\Gamma}'(\Delta_{t+1}) = -\frac{(1-\beta)\tau_c\{\alpha\beta+\gamma\delta[\alpha+(1-\alpha)\tau+\tau_c\Delta_{t+1}]\}}{\{\alpha\beta+(1-\beta+\gamma\delta)[\alpha+(1-\alpha)\tau+\tau_c\Delta_{t+1}]\}^2} < 0$ holds, and therefore $\tilde{\Gamma}(\Delta_{t+1})$ is monotonically decreasing in Δ_{t+1}. By contrast, $\tilde{\Psi}(\Delta_t)$ is a line with a negative slope and takes the values, $\tilde{\Psi}(0) = 1$ and $\tilde{\Psi}(1) = 0$. From these properties of $\tilde{\Gamma}(\Delta_{t+1})$ and $\tilde{\Psi}(\Delta_t)$, a unique unstable equilibrium E exists as shown in Figure 2. This implies that the forward-looking variable Δ must jump to Δ^* at the initial date.

[14] The LUF, $u(y) = \ln y$, is a typical function that satisfies the Inada condition: $\lim_{y\to 0} u'(y) = +\infty$.

E Derivation of (38)

Substituting (17) into (34) and using (11), (12), we obtain

$$e_t = \frac{\gamma\delta}{(1+\gamma\delta)(1+n)} \left[(1-\tau)(1-\alpha)\frac{Y_t}{H_t}h_t + \frac{(1+n)[\tau_c\varphi_{t+1} + \tau(1-\alpha)\frac{Y_{t+1}}{H_{t+1}}h_{t+1}]}{\alpha\frac{Y_{t+1}}{K_{t+1}}} \right] \quad \text{(E.1)}$$

By using (15), $1+n = N_t/N_{t-1}$ and the definition of $\Delta_t(\equiv \varphi_t N_{t-1}/Y_t)$, (E.1) is rearranged as

$$e_t = \frac{\gamma\delta}{(1+\gamma\delta)(1+n)N_{t-1}} \left\{ (1-\tau)(1-\alpha)Y_t + \frac{[\tau_c\Delta^* + \tau(1-\alpha)]K_{t+1}}{\alpha} \right\}. \quad \text{(E.2)}$$

By substituting (35) into (E.2) and using (10), (15) and $k_t = K_t/H_t$, we obtain (38).

F Perfect altruism model with operative bequests

Substituting (52) into (51) and using (11), (12), and (15), we find that e_t is given by the same equation as (19): $(1+n)e_t N_{t-1} = \frac{(1-\alpha)\delta(1-\tau)}{\alpha}K_{t+1}$. In addtion, (23): $K_{t+1} = \frac{\alpha(1-\Delta_t)}{\alpha+(1-\alpha)\delta(1-\tau)}Y_t$ is obtained in the same procedures as in Section 3.

From (50) and (52), we obtain $d_t/(1+n) = \frac{\beta}{(1-\beta)\bar{\gamma}}c_t$. Adding c_t to both sides of this equation and using the definition of φ_t yields $c_t + d_t/(1+n) = \left[\frac{\beta}{(1-\beta)\bar{\gamma}} + 1\right]c_t \Leftrightarrow \varphi_t N_{t-1} = \frac{\beta+(1-\beta)\bar{\gamma}}{(1-\beta)\bar{\gamma}}c_t N_{t-1}$. Dividing both side of this equation by Y_t, we obtain

$$\frac{c_t N_{t-1}}{Y_t} = \frac{(1-\beta)\gamma}{(1-\beta)\gamma+\beta}\Delta_t. \quad \text{(F.1)}$$

Substituting (12) into (52) leads to $(1+n)c_{t+1}N_{t-1} = \bar{\gamma}\alpha\frac{Y_{t+1}}{K_{t+1}}c_t N_{t-1} \Leftrightarrow \frac{c_{t+1}N_t}{Y_{t+1}} = \bar{\gamma}\alpha\frac{c_t N_{t-1}}{Y_t}\frac{Y_t}{K_{t+1}}$. Combining this with (23) and (F.1) yields

$$\Delta_{t+1} = \bar{\gamma}[\alpha + (1-\alpha)\delta(1-\tau)]\frac{\Delta_t}{1-\Delta_t}. \quad \text{(F.2)}$$

The LHS of (F.2) is an upward-sloping $45°$ line and takes zero when $\Delta_t = 0$. On the other hand, the RHS of (F.2) is a monotonically increasing and convex function of Δ_t and takes zero when $\Delta_t = 0$. These properties of (F.2) indicate the following. First Δ_t has a unique steady value $\Delta^* = 1 - \bar{\gamma}[\alpha + (1-\alpha)\delta(1-\tau)]$. Second, this unique equilibrium is unstable so that the forward-looking variable Δ must jump to Δ^* at the initial date.

Inserting $\Delta_t = \Delta^* = 1 - \bar{\gamma}[\alpha + (1-\alpha)\delta(1-\tau)]$ into (23), we obtain $K_{t+1} = \bar{\gamma}\alpha Y_t$. This together with (14): $K_{t+1} = s_t N_{t-1}$ yields $s_t N_{t-1}/Y_t = \bar{\gamma}\alpha$. Substituting it into $(1+n)e_t N_{t-1} = \frac{(1-\alpha)\delta(1-\tau)}{\alpha}K_{t+1}$ yields $(1+n)e_t N_{t-1} = \bar{\gamma}\delta(1-\tau)(1-\alpha)Y_t$, and using (10), $k_t = K_t/H_t$, and (15), we obtain

$$e_t = \frac{\bar{\gamma}\delta(1-\tau)(1-\alpha)}{1+n}Ak_t^{\alpha}h_t. \tag{F.3}$$

From (3) and (F.3), we obtain

$$\frac{h_{t+1}}{h_t} = D\left\{\frac{\bar{\gamma}\delta(1-\tau)(1-\alpha)A}{1+n}\right\}^{\delta} k_t^{\alpha\delta}. \tag{F.4}$$

By using (10) and $k_t = K_t/H_t$, $K_{t+1} = \bar{\gamma}\alpha Y_t$ is rewritten into $k_{t+1}H_{t+1} = \bar{\gamma}\alpha A k_t^{\alpha} H_t$. Using it together with $H_{t+1}/H_t = (1+n)h_{t+1}/h_t$ and (F.4), we obtain

$$k_{t+1} = \frac{\bar{\gamma}\alpha A}{(1+n)D}\left\{\frac{1+n}{\bar{\gamma}\delta(1-\tau)(1-\alpha)A}\right\}^{\delta} k_t^{\alpha(1-\delta)}. \tag{F.5}$$

These dynamics of k_t converge monotonically towards a steady state value: $k^* = \kappa^{\frac{1}{1-\alpha(1-\delta)}}(1-\tau)^{\frac{-\delta}{1-\alpha(1-\delta)}}$, where $\kappa \equiv \frac{\bar{\gamma}\alpha A}{(1+n)D}\left[\frac{1+n}{\bar{\gamma}\delta(1-\alpha)A}\right]^{\delta}$. Inserting this steady state value (k^*) into (F.4) yields the accumulation of human capital per capita in the steady state as

$$\frac{h_{t+1}}{h_t} = D\left\{\frac{\bar{\gamma}\delta(1-\alpha)A}{1+n}\right\}^{\delta} \kappa^{\frac{\alpha\delta}{1-\alpha(1-\delta)}}(1-\tau)^{\frac{\delta(1-\alpha)}{1-\alpha(1-\delta)}}. \tag{F.6}$$

In the steady state, Y_t, K_t, and H_t grow at the same constant rate: $H_{t+1}/H_t = (1+n)(h_{t+1}/h_t) = D(1+n)\left\{\frac{\bar{\gamma}\delta(1-\alpha)A}{1+n}\right\}^{\delta}\kappa^{\frac{\alpha\delta}{1-\alpha(1-\delta)}}(1-\tau)^{\frac{\delta(1-\alpha)}{1-\alpha(1-\delta)}}$.

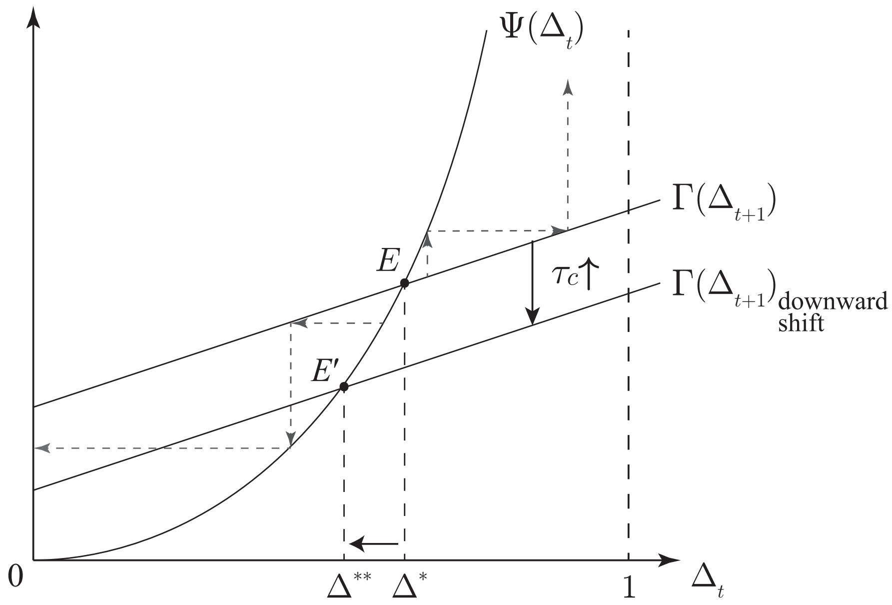

Figure 1: Δ^* when bequest is operative.

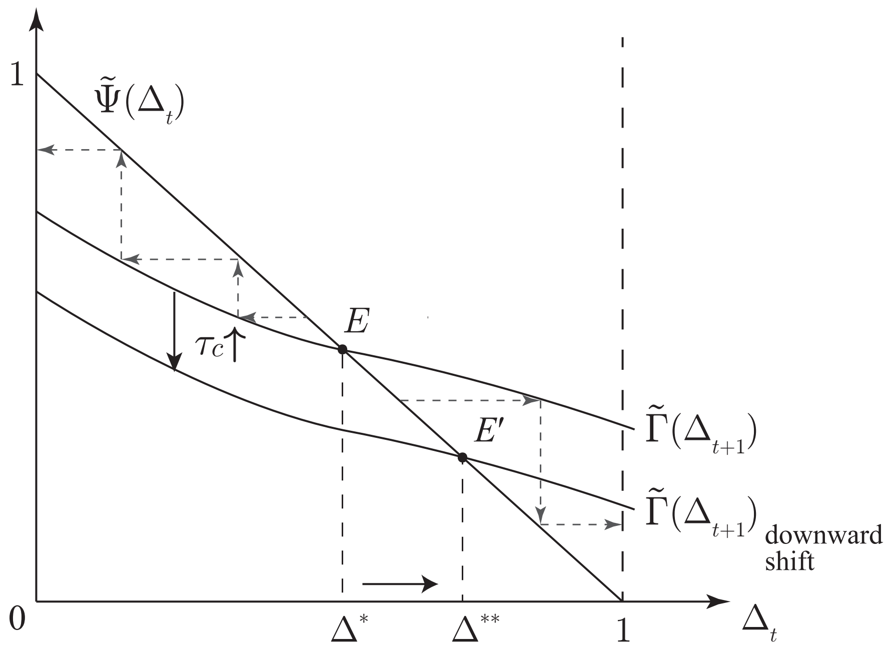

Figure 2: Δ^* when bequest is inoperative.

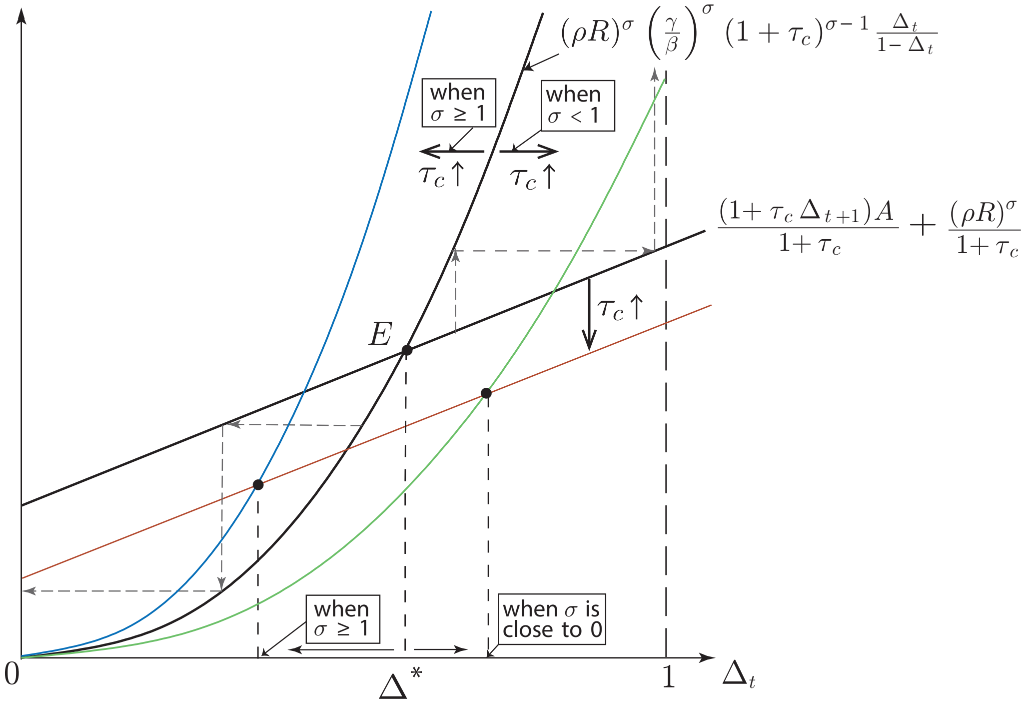

Figure 3: Δ^* when bequest is operative in an AK model with CRRA utility function.