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# Inequality and Asset Fire Sales\*

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## Abstract

It is widely acknowledged that fire sales were a critical factor in inducing and exacerbating the financial crises of 2007–2008. The leverage of financial intermediaries, which is defined as the ratio of total assets to capital, is a key factor in causing fire sales. Why do financial intermediaries expand their balance sheets despite subsequently having to sell their assets at discounted prices? To examine this question, we incorporate financial intermediaries into a three-period incomplete market economy model, in which households face countercyclical and uninsured idiosyncratic income shocks. We demonstrate that countercyclical income inequality and market incompleteness result in leveraged investment and subsequent asset fire sales by financial intermediaries in equilibrium. The first contribution of this paper is that we demonstrate that the mechanism between asset prices and leverages could successfully solve the famed asset-pricing puzzles. The second contribution is that we analyze the impact of financial regulation on the welfare of ex ante homogeneous households.

**Keywords:** arbitrage opportunities; fire sales; income inequality; incomplete markets; leveraged investments; precautionary demand for assets.

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# 1 Introduction

It is widely acknowledged that fire sales were a critical factor in inducing and exacerbating the financial crises of 2007–2008. Fire sales are forced sales of an asset that disconnect the asset price from its potential fundamental value; they occur when the seller cannot pay his/her debt without selling the asset. The leverage of financial intermediaries, which is defined as the ratio of total assets to capital, is a particularly important cause of fire sales. There is a vast literature on the limits of arbitrage that investigates how fire sales by financial intermediaries affect equilibrium asset prices. Recently, however, some researchers have begun to pay serious attention to the potential role of income inequality in the recent financial crises.

Why do financial intermediaries expand their balance sheets even though they subsequently have to sell their assets at discounted prices? To answer this question, we incorporate financial intermediaries into a three-period incomplete market economy model, in which households face countercyclical and uninsured idiosyncratic income shocks. Households, who are not specialists in trading risky assets, can hold risky assets in their portfolios but incur costs in changing them. Financial intermediaries are owned by households and their values are determined by a competitive market. They have a comparative advantage in trading risky assets, as they can trade them at no cost. Thus, financial intermediaries finance their positions by using the capital that they provide themselves and by borrowing from households. This borrowing takes the form of noncontingent short-term risk-free bonds, which households can trade at no cost. That is, financial intermediaries make leveraged investments. There are two important additional sources of frictions. First, a margin requirement regulation or a leverage constraint limits the use of leverage by financial intermediaries in relation to some multiple of their capital. Second, financial intermediaries can only build their capital by retaining earnings from their trading profits. That is, once a negative aggregate shock hits the economy, the margin requirement regulation may require financial intermediaries to sell their assets instead of raising new capital. In this case, they have to sell their risky assets to households, who require that the prices of risky assets should be sufficiently discounted. As a result, fire sales by the financial intermediaries occur.

We demonstrate that an equilibrium occurs as a result of the linkages between countercyclical income inequality, market incompleteness, and the leverage of financial intermediaries. In particular, this paper explains why financial intermediaries prefer investing in risky assets and borrowing by issuing risk-free bonds. Countercyclicity implies that dividends from risky assets, which are affected by aggregate economic conditions, decrease when many households suffer from

the loss of their idiosyncratic labor endowments.<sup>1</sup> As is well known, such risky assets generate high-risk premiums. In addition, because market incompleteness cannot exclude arbitrage opportunities, the financial intermediaries can gain profits from the leveraged investment. This paper demonstrates that the countercyclical income inequality and market incompleteness generate both high-risk premiums and arbitrage opportunities for the financial intermediaries.

The first contribution of this paper is to demonstrate that the mechanism between asset prices and leverage could solve the famed asset-pricing puzzles. Countercyclical idiosyncratic endowment shocks generate lower risk-free rates and higher equity premiums compared with the representative agent model, as long as households have a precautionary demand for assets. In addition, asset fire sales by financial intermediaries generate higher volatility of equity prices than those of rational for dividend fluctuations. That is, our model is not only consistent with the evidence of recent financial crises but also simultaneously solves the equity premium puzzle and the excess volatility puzzle.

The second contribution of this paper is to analyze the impact of financial regulation on the welfare of ex ante homogeneous households. The Basel III international regulatory framework for banks, finalized in December 2017, regulates the portfolio construction of financial intermediaries. Many researchers and policy makers regard such financial regulation as an important instrument to prevent financial crises. Our model enables us to examine the role of the financial intermediaries as the providers of liquidity or safe assets. As labor endowments correlate with the aggregate dividends, the households prefer holding risk-free bonds to risky assets. In other words, the rise in the financial intermediaries' leverage can be considered as a rise in the self-insurance opportunities for the households. Therefore, the high leverage and the following fire sales are not always harmful for welfare. In fact, we demonstrate that there exists an optimal rate for the margin requirement in terms of the welfare of ex ante homogeneous households.

This paper is organized as follows. Section 2 relates our paper to the existing literature. Section 3 describes the model setup and section 4 characterizes the competitive equilibrium. Section 5 solves the equilibrium numerically and discusses the results. Finally, Section 6 provides the conclusion.

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<sup>1</sup>We can interpret such assets as subprime mortgage-backed securities because subprime borrowers default, leading to the simultaneous collapse of the mortgage-backed securities during recessions.

## 2 Related Literature

The leverage of financial intermediaries, which is defined as the ratio of total assets to equity, is key to understanding fire sales and financial crises.<sup>2</sup> Adrian and Shin (2010a, b) point out that the balance sheets of the financial intermediaries expanded significantly prior to the 2007–2008 financial crisis. They emphasize the role of the balance sheets of investment banks, which were continuously marked to market. In addition, Brunnermeier (2009) describes events during the crisis in detail and emphasizes the role of financial intermediaries' balance sheets in causing fire sales. These studies indicate that the balance sheet expansions of the financial intermediaries, associated with leveraged investment, and the subsequent shrinkage of the balance sheets, associated with fire sales, triggered the financial crisis.

There is a vast theoretical literature emphasizing the role of the leverage of financial intermediaries or specialized investors in causing fire sales. For example, Shleifer and Vishny (1997, 2011), Aiyagari and Gertler (1999), Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009), He and Krishnamurthy (2012, 2013), and Dow and Han (2017) study a financial economy and consider limited arbitrage to be the source of fire sales.<sup>3</sup> In these models, financial securities, which are properly understood only by the specialized investors, are traded. If these investors become severely capital-constrained, they have to sell these financial securities to the nonspecialist investors at discounted prices. Although these models emphasize the role of specialized investors, they treat households in a simple manner. For example, Aiyagari and Gertler (1999) and He and Krishnamurthy (2012) consider the representative household. He and Krishnamurthy (2013) consider two types of households that differ in their portfolio choices, but pool wealth and distribute it equally in each period. Thus, the wealth distribution of the households plays no role in solving the equilibrium.

Some studies propose an alternative view in which the recent financial crises are attributed to the heterogeneity of households. Traditionally, such heterogeneity has been considered a potential resolution to the various asset-pricing puzzles. For example, Mankiw (1986) considers an economy in which there is income inequality and the insurance market is incomplete. He demonstrates that high equity premiums are generated when aggregate shocks affect all individuals *ex ante*

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<sup>2</sup>Leverage and fire sales are important in financial markets even during normal periods. For example, Phillipon (2015) demonstrates that the quantity of intermediated assets and the income of the financial sectors in relation to GDP have risen rapidly since 1980. Phillipon also shows that the unit cost of financial intermediation has remained stable around 2%. This implies that the continuous balance sheet expansions explain the continuous increase in the income shares of the financial sectors in relation to GDP. Coval and Stafford (2007) demonstrate that fire sales are actually observed in equity markets even in normal periods. They find that financially distressed mutual funds are forced to sell securities in response to withdrawals by their investors.

<sup>3</sup>Lorenzoni (2008) and Gertler and Kiyotaki (2015) study a production economy in which the specialized investors, which possess linear technology, need outside funds to invest in capital. They demonstrate that the fire sales occur when these investors have to sell their capital to consumers who are not specialists in managing it.

but are concentrated among a few ex post.<sup>4</sup> Rajan (2010) argues that income inequality played a potential role in causing and exacerbating the recent financial crises. Some empirical studies find a significant relationship between rising income inequality, credit growth, and the frequency of financial crises.<sup>5</sup> Kumhof et al. (2015) present a general equilibrium model in which the growing income share of high-income households results in higher leverage for low-income households and subsequent financial crises. However, in their model, high-income households are assumed to have an inherent preference for holding assets and the role of financial intermediaries is abstracted away.

This literature review provides the background context for the research question that we address, namely, is there any link between income inequality and the leverage and fire sales of financial intermediaries? In other words, we ask, are the two hypotheses in the literature regarding the causes of financial crises and fire sales—one emphasizing the leverage of the financial intermediaries and the other focusing on income inequality—independent of each other? In this paper, we investigate this issue by introducing financial intermediaries into a model of heterogeneous households.

Considering heterogeneous households naturally involves exploring the financial constraints that limit risk sharing among the households. This study assumes that assets markets are exogenously incomplete and that households face liquidity constraints. In contrast, some studies assume that households' borrowings are limited endogenously. For example, Kehoe and Levine (1993, 2001), Alvarez and Jermann (2000), and Kruger and Perri (2006) consider an economy where household borrowings are endogenously limited due to enforcement problems. Chien and Lustig (2010), Araújo et al. (2012), Geanakaplos and Zame (2014), Gottardi and Kubler (2015), and Fostel and Geanakaplos (2015) consider the role of collateral in limited enforcement environments. In these models, although asset markets are complete, the enforcement problem limits risk sharing among the households. As demonstrated in Alvarez and Jermann (2000) and Chien and Lustig (2010), stochastic discount factors (SDFs) used to price the securities are unique in these complete market economies. The uniqueness of SDFs implies that there are

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<sup>4</sup>Storesletten et al. (2004), Krueger and Perri (2006), and Guvenen et al. (2014) provide evidence regarding income inequality in the US. In particular, Storesletten et al. (2004) and Guvenen et al. (2015) provide evidence that idiosyncratic income inequality is countercyclical in the US. Constantinides and Duffie (1996) and Constantinides and Ghosh (2017) demonstrate that a model with uninsured idiosyncratic income shocks provides a good fit for the time-series averages of asset prices.

<sup>5</sup>Bordo and Meissner (2012) use cross-country OECD data and find a significant relationship between credit growth and the frequency of financial crises. However, they demonstrate that there is no significant relationship between credit growth (the change in the log ratio of bank loans to the price index) and changes in the share of income for the top 1% of the income distribution. However, subsequent studies, such as Gu and Huang (2014) and Perugini et al. (2016), examine the robustness of their results and demonstrate that there is indeed a significant relationship. In addition, Yamarik (2016) finds a positive relationship between rising income inequality and real estate lending across the US.

no arbitrage opportunities in the financial markets. Although these models are tractable, the no-arbitrage environments make it impossible to examine the role of arbitrageurs. This paper assumes that markets are exogenously incomplete and that households are subject to liquidity constraints, as assumed by Bewley (1983), Hugget (1992), and Aiyagari (1994). In this paper, market incompleteness violates the uniqueness of SDFs and generates arbitrage opportunities, which results in leveraged investment by financial intermediaries.

This paper is related to the literature investigating the role of margins or collateral requirements in borrowing and lending, where the leverage and the asset prices are simultaneously determined in an equilibrium. For example, Chien and Lustig (2010), Araújo et al. (2012), Geanakoplos and Zame (2014), Gottardi and Kubler (2015), and Fostel and Geanakoplos (2015) consider the collateral general equilibrium, where one security is used as a collateral for other securities. In these models, the possibility of future default limits the capacity for current borrowing. In addition, the margin requirement constraints in Gromb and Vayanos (2002) require that arbitrageurs hold enough in their current margin account to ensure that risky assets take a positive value in the next period. On the other hand, based on the institutional features in actual financial markets, Brunnermeier and Pedersen (2008) and Daw and Han (2018) assume that the total margin on the financial intermediaries' current position cannot exceed their current capital. In a similar fashion to Brunnermeier and Pedersen (2008) and Daw and Han (2018), we assume that regulation places a lower limit on the margin ratio required, which is defined as the ratio of current capital to the current holdings of equity shares. In our model, margin requirement constraints always bind due to the arbitrage opportunities and the margin ratio is fixed in equilibrium.<sup>6</sup>

Finally, this paper is also related to the literature that provides disaster explanations for the equity premium puzzle, including Rietz (1988) and Barro (2006). In the disaster models, both per capita real consumption (or GDP) and equity prices fall sharply when disasters occur; the negative correlation between equity returns and SDFs or the intertemporal marginal rates of substitution (IMRS) of the representative agent is magnified, and thus, large equity premiums arise. Apart from investigations of empirical plausibility, such as Julliard and Ghosh (2012) and Nakamura et al. (2013), the disaster explanation of equity premiums is theoretically controversial. For example, Gourio (2008) and Saito and Suzuki (2014) demonstrate that, depending on the assumption regarding the time-series properties of the disaster and the intertemporal elasticities

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<sup>6</sup>Brumn et al. (2015) construct a model with heterogeneous agents, collateral constraints, and multiple collateralized assets. Then, they explain why margin requirement regulations on leverage have an insignificant impact on stock price volatility in US. On the other hand, with the fixed margin ratio in our model, a change in margin requirement regulations directly causes a change in asset prices. Although this result is contrary to Brumn et al. (2015), we employ this specification because of its tractability.

of substitution, the equity prices rise sharply when disasters occur.<sup>7</sup> Leverage and fire sales played a critical role in decreasing asset prices during the Great Depression and other financial crises. Nevertheless, they play no role in the disaster models. Therefore, the consumption-based asset-pricing model with leverage and fire sales considered in this paper enables us to examine the question: does a decline in equity prices due to fire sales properly explain asset prices, including the equity premium?

### 3 The Model

There are three periods, 0, 1, and 2, and two types of agents of equal mass, households and financial intermediaries. There are two types of assets, risk-free bonds and risky shares of a single “Lucas tree”.

Figure 1 shows the timing of events. A Lucas tree produces dividends in each period, which are denoted by  $\{d_0, d_1^j, d_2^j\}$ .  $j$  is an aggregate state and it is common across households and financial intermediaries. The Lucas tree pays a certain dividend,  $d_0$ , in period 0. In period 1, a normal state,  $j = n$ , occurs with a probability of  $1 - \phi$  and the Lucas tree produces dividends  $d_1^n$  and  $d_2^n$  in periods 1 and 2, respectively. On the other hand, a bad state,  $j = b$ , occurs with a probability of  $\phi$  and the Lucas tree produces dividends  $d_1^b$  and  $d_2^b$  in periods 1 and 2, respectively. Note that the dividend in the bad state,  $d_1^b$ , is lower than that in the normal state,  $d_1^n$ .

Households receive labor endowments in each period, which are denoted by  $\{e_0, e_1^n, e_2^n, e_{1i}^b, e_{2i}^b\}$ .  $i$  is an idiosyncratic state that affects the labor endowments. The households receive a certain labor endowment,  $e_0$ , in period 0. If a normal state,  $j = n$ , occurs in period 1, all households receive the same labor endowments,  $e_1^n$  and  $e_2^n$ , in periods 1 and 2, respectively. If a bad state,  $j = b$ , occurs in period 1, a fraction of households,  $1 - \psi$ , receives high endowments,  $e_{1h}^b$  and  $e_{2h}^b$ , in periods 1 and 2, respectively, whereas the remaining fraction,  $\psi$  households, receives low endowments,  $e_{1l}^b$  and  $e_{2l}^b$ , in periods 1 and 2, respectively. As the population of households is normalized to unity, the fractions of households  $1 - \psi$  and  $\psi$  correspond to the probability of drawing idiosyncratic states  $i = h$  and  $l$ .

Suppose that there is no uncertainty between periods 1 and 2.

[Figure 1]

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<sup>7</sup>Such sharp rises in equity prices at the beginning of disasters were actually observed during World War II (Jorion and Goetzmann 1999, Oosterlinck 2010, and Suzuki 2012.)



### 3.1 Households

Households maximize their expected utility function subject to budget and liquidity constraints, as follows:

$$\max_{\{c_0, c_1^n, c_2^n, c_{1i}^b, c_{2i}^b, s_1, s_2^n, s_{2i}^b, f_1, f_2^n, f_{2i}^b\}_{i \in \{h, l\}}} u(c_0) + \beta E_0 \left[ u(c_{1i}^j) - a_i^j p_1^j \frac{(s_{2i}^j - s_1)^2}{2} + \beta u(c_{2i}^j) \right], \quad (1)$$

$$\begin{aligned} \text{subject to} \quad & c_0 + p_0 s_1 + q_0 f_1 = (p_0 + d_0) s_0 + f_0 + D_0 + e_0, \\ & c_1^n + p_1^n s_2^n + q_1^n f_2^n = (p_1^n + d_1^n) s_1 + f_1 + D_1^n + e_1^n, \\ & c_2^n = d_2^n s_2^n + f_2^n + D_2^n + e_2^n, \\ & c_{1i}^b + p_1^b s_{2i}^b + q_1^b f_{2i}^b = (p_1^b + d_1^b) s_1 + f_1 + D_1^b + e_{1i}^b, \\ & c_{2i}^b = d_2^b s_{2i}^b + f_{2i}^b + D_2^b + e_{2i}^b, \\ & f_1 \geq 0, \quad f_2^n \geq 0, \quad \text{and} \quad f_{2i}^b \geq 0. \end{aligned} \quad (2)$$

Let  $\{c_0, c_1^n, c_2^n, c_{1i}^b, c_{2i}^b\}$ ,  $\{s_0, s_1, s_2^n, s_{2i}^b\}$ , and  $\{f_0, f_1, f_2^n, f_{2i}^b\}$  denote consumption, holdings of Lucas tree shares, and holdings of risk-free bonds, respectively, in periods 0, 1, and 2, in the aggregate states  $j = n$  and  $b$ , and in the idiosyncratic states  $i = h$  and  $l$ . Let  $\{p_0, p_1^j\}$  and  $\{q_0, q_1^j\}$  denote the prices of the Lucas tree and the prices of risk-free bonds, respectively, in periods 0 and 1 and in states  $j = n$  and  $b$ . Let  $\{D_0, D_1^j, D_2^j\}$  denote the dividend payments from financial intermediaries in periods 0, 1, and 2 and states  $j = n$  and  $b$ .  $\beta \in (0, 1)$  is a subjective time discount factor. Households are risk averse and the periodic utility function is twice differentiable:  $u'(c) > 0$  and  $u''(c) < 0$ .  $E_0[\cdot]$  denotes the expectation operator, conditional on the time 0 information.

As households are not specialists in trading equity shares, they derive disutility from trading equity shares in period 1. We use a quadratic cost function because of its tractability. However, it also captures the following idea. Suppose that the household has a fixed amount of time to either trade securities or enjoy leisure. As more securities are sold or bought, households must sell or buy increasingly illiquid assets. Large trading volumes require a large amount of time. Diminishing marginal utility of leisure could lead to convex costs of trading securities, which could be approximated by a quadratic cost function.  $a_i^j$  denotes the coefficient of the cost arising from trading in shares. Note that, in equation (1), the cost arising from trading in shares is different due to their idiosyncratic state. In what follows, we will discuss the details of this

assumption.

Because idiosyncratic labor endowments cannot be observed and seized by the other agent, households cannot trade state-contingent contracts against idiosyncratic endowment shocks. Therefore, the only traded assets are risky equity shares of a Lucas tree and noncontingent bonds. In addition, this lack of pledgeability of the labor endowment prevents households from issuing risk-free bonds. Thus, the inequalities (2) are the liquidity constraints.

### 3.2 Financial Intermediaries

There are homogeneous and competitive financial intermediaries owned by the households. They maximize the expected discounted value of dividend payments subject to the budget constraints, the margin requirement or the leverage constraints, and the dividend constraints:

$$\max_{\{s_1^{*j}, s_2^{*j}, f_1^{*j}, f_2^{*j}, D_0, D_1^j, D_2^j\}_{j \in \{n, b\}}} D_0 + E_0 \left[ M_1^j (D_1^j + M_2^j D_2^j) \right], \quad (3)$$

$$\text{subject to} \quad (p_0 + d_0)s_0^* + f_0^* = p_0 s_1^* + q_0 f_1^* + D_0, \quad (4)$$

$$(p_1^j + d_1^j)s_1^{*j} + f_1^{*j} = p_1^j s_2^{*j} + q_1^j f_2^{*j} + D_1^j, \quad (5)$$

$$d_2^j s_2^{*j} + f_2^{*j} = D_2^j, \quad (6)$$

$$[(p_0 + d_0)s_0^* + f_0^* - D_0] \geq \kappa p_0 s_1^*, \quad (7)$$

$$[(p_1^j + d_1^j)s_1^{*j} + f_1^{*j} - D_1^j] \geq \kappa p_1^j s_2^{*j}, \quad (8)$$

$$D_0 \geq 0, D_1^j \geq 0, \text{ and } D_2^j \geq 0. \quad (9)$$

Let  $\{s_0^*, s_1^{*j}, s_2^{*j}\}$  and  $\{f_0^*, f_1^{*j}, f_2^{*j}\}$  denote the intermediaries' holdings of shares and risk-free bonds, respectively, in periods 0, 1, and 2 in the aggregate states  $j = n$  and  $b$ . Let  $\{M_0^j, M_1^j, M_2^j\}$  denote the SDFs used for the market value of the financial intermediaries.

The equations (4), (5), and (6) are the budget constraints. Based on their capital holdings at the beginning of each period (the left-hand side), the intermediaries choose their financial positions and the dividends paid to the households (the right-hand side).

The inequalities (7) and (8) are margin requirement constraints or leverage constraints. Hereafter, we use leverage constraints throughout the paper. Although the intermediaries can exchange securities at no cost, the leverage constraints restrict how they can construct their portfolios. In particular, the fraction  $\kappa \in (0, 1)$  of the intermediaries' holding capital (the right-hand side) should be backed by their own ex-dividend capital (the left-hand side).

We refer to the inequalities (9) as dividend constraints, which require that the dividend payments to the households should not be negative. In other words, issuing new equity is prohibitively expensive, and it follows that dividend payments cannot be negative.

### 3.3 Market-Clearing Conditions

The market-clearing conditions for goods are as follows:

$$c_0 = y_0 \equiv e_0 + d_0, \quad c_1^n = y_1^n \equiv e_1^n + d_1^n, \quad c_2^n = y_2^n \equiv e_1^n + d_1^n$$

$$(1 - \psi)c_{1h}^b + \psi c_{1l}^b = y_1^b \equiv (1 - \psi)e_{1h}^b + \psi e_{1l}^b + d_1^b, \quad (10)$$

$$\text{and} \quad (1 - \psi)c_{2h}^b + \psi c_{2l}^b = y_2^b \equiv (1 - \psi)e_{2h}^b + \psi e_{2l}^b + d_2^b, \quad (11)$$

where  $y_0$ ,  $y_1^n$ ,  $y_2^n$ ,  $y_1^b$ , and  $y_2^b$  denote the aggregate endowments.

The total number of shares is normalized to unity. The market-clearing conditions for shares are as follows:

$$s_0 + s_0^* = 1, \quad s_1 + s_1^* = 1, \quad s_2^n + s_2^* = 1, \quad \text{and} \quad (1 - \psi)s_{2h}^b + \psi s_{2l}^b + s_2^{*b} = 1. \quad (12)$$

There are no external bonds. The market-clearing conditions for risk-free bonds are as follows:

$$f_0 + f_0^* = 0, \quad f_1 + f_1^* = 0, \quad f_2^n + f_2^* = 0, \quad \text{and} \quad (1 - \psi)f_{2h}^b + \psi f_{2l}^b + f_2^{*b} = 0.$$

### 3.4 Competitive Equilibrium

Our definition of the competitive equilibrium is as follows.

**Definition 1** For  $j = n$  and  $b$  and  $i = h$  and  $l$ , and for the given endowments,  $\{e_0, e_1^n, e_2^n, e_{1i}^b, e_{2i}^b\}$ , the dividend of the Lucas tree,  $\{d_0, d_1^j, d_2^j\}$ , and the initial level of equity shares and risk-free bonds,  $\{s_0, f_0, s_0^*, f_0^*\}$ , the competitive equilibrium is defined as the consumption allocation,  $\{c_0, c_1^n, c_2^n, c_{1i}^b, c_{2i}^b\}$ , the households' portfolio,  $\{s_1, s_2^n, s_{2i}^b\}$  and  $\{f_1, f_2^n, f_{2i}^b\}$ , the financial intermediaries' dividend payments,  $\{D_0, D_1^j, D_2^j\}$ , the financial intermediaries' portfolios,  $\{s_1^*, s_2^{*j}\}$  and  $\{f_1^*, f_2^{*j}\}$ , and the asset prices  $\{p_0, p_1^j\}$  and  $\{q_0, q_1^j\}$  that simultaneously solve the households' and intermediaries' optimization problems, and satisfy the market-clearing conditions.

### 3.5 Assumptions

Three additional assumptions will be useful in the analysis. First, we make an assumption regarding the coefficient of equity adjustment costs,  $a_i^j$ .

**Assumption 1**  $a_i^n$  equals  $au'(c_1^n)$  and  $a_i^b$  equals  $au'(c_{1i}^b)$ , which the households take as given when they solve the utility maximization problem. Suppose that  $a$  is a constant parameter.

The specification for the adjustment costs of equity transactions follows Heaton and Lucas (1996) and Aiyagari and Gertler (1999). In particular, it implies that the disutility of low-endowment households is higher than that of high-endowment households if their trading volumes are the same. This is because the consumption of low-endowment households is lower than that of high-endowment households in period 1 and state  $b$ . The logic behind this assumption is as follows: low-endowment households may be unemployed or paid relatively low wages and they must spend many hours seeking a new job or working harder, given their low wages. Therefore, they incur more severe disutility from trading securities, which requires large amounts of time.<sup>8</sup>

Second, we make an assumption of idiosyncratic endowments.

**Assumption 2** *Low-endowment households have lower endowments than those of high-endowment households and relatively upward-sloping income profiles compared with those of high-endowment households:  $e_{1h}^b > e_{1l}^b$ , and  $\frac{e_{1l}^b}{e_{1h}^b} < \frac{e_{2l}^b}{e_{2h}^b}$ . Moreover, the income profiles of low-endowment households are sufficiently upward sloping that the liquidity constraints of low-endowment households always bind.*

The assumption states that idiosyncratic income risk is countercyclical because state  $b$  represents a recession in our setting. Mankiw (1986) demonstrates that this countercyclical income risk can resolve the equity premium puzzle. From an empirical point of view, Storesletten et al. (2004) and Guvenen et al. (2014) provide evidence that idiosyncratic income risks are countercyclical in the US. Our endowment structure model incorporates the countercyclical income risks in a simple manner.

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<sup>8</sup>Our assumption draws on that of Aiyagari and Gertler (1999). They introduce adjustment costs into the utility function, and assume that the coefficient equals the marginal utility of ex post aggregate consumption. However, adjustment costs are not heterogeneous because they employ the representative agent framework. On the other hand, Heaton and Lucas (1999) consider a heterogeneous household model and introduce quadratic adjustment costs in budget constraints. They also assume that the coefficient is constant. However, when households solve the utility maximization problem subject to the budget constraints, the Lagrangian multiplier, which equals the marginal utility of income, is associated with the budget constraints. That is, heterogeneous households evaluate the adjustment costs in terms of their own marginal utility of income. As a result, their adjustment costs are the same as in our paper. Our specification that the heterogeneous adjustment cost enters the utility function rather than the budget constraints enables us to discuss the existence of the equilibrium analytically.

Finally, we make an assumption regarding the SDF of the financial intermediaries.

**Assumption 3**  $M_2^b$  is bounded:

$$\frac{1}{q_1^b} \leq \frac{1}{M_2^b} \leq \frac{1}{\kappa} \frac{d_2^b}{p_1^b} - \left(\frac{1}{\kappa} - 1\right) \frac{1}{q_1^b}. \quad (13)$$

The assumption indicates that the inverse of the financial intermediaries' SDF,  $M_2^b$ , lies between the risk-free rate and the rate of return on the leveraged portfolio. The financial intermediaries use the household IMRS as the SDF because they are owned by the households. However, there are no unique and proper candidates for  $M_2^b$  because the financial markets are incomplete between periods 1 and 2 in state  $b$ . We do not replace  $M_2^b$  with one of those candidates but make an assumption regarding the upper and lower bounds of the candidate. The term on the left-hand side of the inequality (13) is the risk-free rate, whereas the term on the right-hand side is the rate of return on the leveraged portfolio. Because the rates of return on the leveraged portfolio are higher than the risk-free rates in general, this assumption requires that the  $M_2^b$  takes a reasonable value.<sup>9</sup>

## 4 Equilibrium

### 4.1 Asset Prices

Using the households' first-order conditions, we can derive the relationship between asset prices and the households' IMRS. Although households face liquidity constraints, they do not incur costs when adjusting their holdings of risk-free bonds. The unconstrained household IMRS determine the risk-free prices in the standard manner:  $q_0 = (1 - \phi)\beta_1^n + \phi\beta_1^b$  and  $q_1^n = \beta_2^n$ , where  $\beta_1^n \equiv \beta \frac{u'(c_1^n)}{u'(c_0)}$ ,  $\beta_2^n \equiv \beta \frac{u'(c_2^n)}{u'(c_1^n)}$ , and  $\beta_1^b \equiv \beta \frac{(1-\psi)u'(c_{1h}^b) + \psi u'(c_{1l}^b)}{u'(c_0)}$ . Because Assumption 1 ensures that high-endowment households have no incentive to borrow, the IMRS of the high-endowment households, which have the strongest motivation to save, determine the risk-free prices in state  $b$ . That is:

$$q_1^b = \beta_{2h}^b \geq \beta_{2l}^b, \quad (14)$$

where  $\beta_{2h}^b \equiv \beta \frac{u'(c_{2h}^b)}{u'(c_{1h}^b)}$  and  $\beta_{2l}^b \equiv \beta \frac{u'(c_{2l}^b)}{u'(c_{1l}^b)}$ .

<sup>9</sup>Note that we do not assume that the leveraged portfolio rates of return are higher than the risk-free rates:  $\frac{1}{q_1^b} \leq \frac{1}{\kappa} \frac{d_2^b}{p_1^b} - \left(\frac{1}{\kappa} - 1\right) \frac{1}{q_1^b}$ . This inequality actually occurs in an equilibrium, as demonstrated later.

By using the first-order conditions with respect to shares, we can derive the following conditions:  $p_0 = (1 - \phi)\beta_1^n(d_1^n + \beta_2^n d_2^n) + \phi\beta_1^b(d_1^b + \tilde{\beta}_2^b d_2^b)$ , where  $\tilde{\beta}_2^b \equiv \beta \frac{(1-\psi)u'(c_{2h}^b) + \psi u'(c_{2l}^b)}{(1-\psi)u'(c_{1h}^b) + \psi u'(c_{1l}^b)}$ .  $\tilde{\beta}_2^b$  denotes the ex ante homogeneous households' expected IMRS between periods 1 and 2 in state  $b$ . Note that  $\tilde{\beta}_2^b$  is used for pricing the dividends  $d_2^b$  in period 0.

The equity prices in period 1 can be characterized by using the households' first-order conditions and the financial intermediaries' constraints.

**Lemma 1** For  $\bar{p}_1^j \equiv \beta_2^j d_2^j$ , where  $\beta_2^b \equiv \beta \left[ (1-\psi) \frac{u'(c_{2h}^b)}{u'(c_{1h}^b)} + \psi \frac{u'(c_{2l}^b)}{u'(c_{1l}^b)} \right]$ , the following inequality holds:  $\bar{p}_1^j \geq p_1^j$ . That is,  $\bar{p}_1^j$  is the highest possible value of equity prices. If the leverage constraints bind in period 1 and state  $j$  under the equity prices  $p_1^j = \bar{p}_1^j$ , then the equity prices can be written as follows:

$$p_1^j = \frac{\kappa \bar{p}_1^j + a(d_1^j s_1^* + f_1^*)}{\kappa - a(1 - \kappa)s_1^*}. \quad (15)$$

It is unique if and only if the following condition is satisfied:

$$\frac{\bar{p}_1^j}{1 + a s_1^*} > \frac{d_1^j s_1^* + f_1^*}{s_1^*}. \quad (16)$$

**Proof.** See Appendix A.1.

This asset-pricing mechanism was originally analyzed by Aiyagari and Gertler (1999). The intuition behind Lemma 1 is as follows. If the equity prices are  $\bar{p}_1^j = \beta_2^j d_2^j$ , households have no incentive to trade equity shares with the financial intermediaries. For instance, in state  $b$ ,  $p_1^b = \beta_2^b d_2^b$  implies that the supply of equity shares of the low-endowment households equals the demand for equity shares by the high-endowment households. There is no supply for the financial intermediaries. If the financial intermediaries can maintain their current equity shares,  $s_2^{*j} = s_1^*$ , under  $p_1^j = \bar{p}_1^j$ , they would be realized as equilibrium equity prices. On the other hand, if the leverage and dividend constraints prevent them from maintaining the current equity shares, they have to shed securities, to the point where they are just able to satisfy the constraints. As it is costly for the households to absorb the equity shares,  $p_1^j$  will drop below  $\bar{p}_1^j$ .

The following inequality is convenient for investigating the optimal portfolio choice of the financial intermediaries.

**Lemma 2** When the liquidity constraints bind, the following inequality holds:

$$q_1^b \geq \beta_2^b \geq \tilde{\beta}_2^b. \quad (17)$$

If the liquidity constraints do not bind, and the IMRS are equalized between the two types of households, then  $q_1^b = \beta_2^b = \tilde{\beta}_2^b$  holds.

**Proof.** See Appendix A.2.

The left inequality  $q_1^b \geq \beta_2^b$  has the following implications. If the equity prices equal  $q_1^b d_2^b$ , there are no arbitrage opportunities between the risk-free bonds and the equities. We refer to  $q_1^b d_2^b$  as the fundamental equity prices. The inequalities (17) in Lemma 2 indicate that  $q_1^b d_2^b \geq \beta_2^b d_2^b$ , which implies that the actual equity prices are lower than the fundamental equity prices. This is because the low-endowment households sell their equity shares to raise their consumption in period 1 and state  $b$ . The high-endowment households, which would like to save, buy the equity shares but require that the prices are discounted because of the adjustment cost. In other words, the decrease in equity prices from the fundamental level in period 1 and state  $b$  is caused by the fire sales conducted by the low-endowment households. Note that  $q_1^b$  equals  $\beta_2^b$  if there are no liquidity constraints or any market incompleteness.

The right inequality  $\beta_2^b \geq \tilde{\beta}_2^b$  has the following implications.  $\tilde{\beta}_2^b$  is the ex ante homogeneous households' expected IMRS between periods 1 and 2; it is used for pricing  $d_2^b$  in the determination of  $p_0$ . On the other hand,  $\beta_2^b$  is the weighted average of the ex post heterogeneous households' IMRS; it is used for pricing  $d_2^b$  in the determination of  $\bar{p}_1^b$ .  $\beta_2^b \geq \tilde{\beta}_2^b$  is derived from the heterogeneity in the adjustment costs from trading equity shares. The high-endowment households have higher IMRS because they have lower adjustment costs. Thus, the right inequality holds. Note that  $\beta_2^b$  equals  $\tilde{\beta}_2^b$  if the adjustment costs are homogeneous among households.

## 4.2 Financial Intermediaries' Portfolio

Based on the asset price relationship derived above, we can derive the financial intermediaries' portfolio. Figure 2 displays the portfolio that the financial intermediaries could choose in period 0, on a plane where the horizontal axis is  $s_1^*$  and the vertical axis is  $f_1^*$ .

The downward-sloping line passing through the origin labeled *Leverage constraint in period 0* in Figure 2 illustrates the following relationship: Incorporating the budget constraints of the financial intermediaries (4) into the leverage constraints (7) and rearranging them, we obtain the inequality  $q_0 f_1^* \geq -(1 - \kappa) p_0 s_1^*$ . This indicates the upper limit of the risk-free bonds that the intermediaries can issue is equal to the fraction  $1 - \kappa$  of their equity holdings. This can be reformulated as follows:

$$f_1^* \geq -(1 - \kappa) \frac{p_0}{q_0} s_1^*, \quad (18)$$

That is, the more equity holdings that the financial intermediaries choose, the more risk-free bonds they can issue.

The downward-sloping line labeled *Dividend constraint in period 0* in Figure 2 illustrates the following relationship: By substituting the budget constraints (4) into the dividend nonnegativity constraints,  $D_0 \geq 0$ , we can derive the following inequality:  $(p_0 + d_0)s_0^* + f_0^* - p_0s_1^* \geq q_0f_1^*$ . That is, the holdings of the risk-free bonds cannot exceed the initial capital holdings minus the equity purchases. This can be reformulated as follows:

$$f_1^* \leq -\frac{p_0}{q_0}s_1^* + \frac{(p_0 + d_0)s_0^* + f_0^*}{q_0}. \quad (19)$$

Therefore, the more equity holdings that the financial intermediaries choose, the fewer risk-free bonds they can hold.

Now, we can determine the range of the portfolio  $s_1^*$  and  $f_1^*$ , where the leverage constraints bind in period 1 and state  $j$ . Suppose that the financial intermediaries' net assets evaluated at the equity prices  $\bar{p}_1^j$  are not sufficient to permit them to satisfy the leverage constraint, pay dividends, and maintain their current holding of equity,  $s_2^{*j} = s_1^*$ . In this case, they must reduce dividends to zero because the inequality  $[(\bar{p}_1^j + d_1^j)s_1^* + f_1^{*j} + D_1^j] < \kappa\bar{p}_1^j s_1^*$  holds. Furthermore, suppose that the leverage constraints under the equity prices still bind  $[(\bar{p}_1^j + d_1^j)s_1^* + f_1^{*j}] < \kappa\bar{p}_1^j s_1^*$ . In this case, they must shed securities to the point where they are just able to satisfy the leverage constraints. This can be reformulated as follows:

$$f_1^* < -[d_1^j + (1 - \kappa)\bar{p}_1^j]s_1^*. \quad (20)$$

If the intermediaries' portfolios satisfy the inequality (20) in period 0, the leverage constraints in period 1 in state  $j$  bind. That is, fire sales by the financial intermediaries occur.

For concreteness, we focus on the case where the leverage constraints bind in state  $b$ . The following Lemma 3 concerns this case.

**Lemma 3** *If the leverage constraint could bind in state  $b$ , it could not bind in state  $n$  as long as  $s_1^* \geq 0$ . That is:*

$$\begin{aligned} (1 - \kappa)\frac{p_0}{q_0} &> d_1^b + (1 - \kappa)\bar{p}_1^b \\ \Rightarrow d_1^n + (1 - \kappa)\bar{p}_1^n &\geq (1 - \kappa)\frac{p_0}{q_0}. \end{aligned} \quad (21)$$

**Proof.** See Appendix A.3.



[Figure 2]

The downward-sloping lines passing through the origin labeled *Leverage constraint in period 1 and state b* and *Leverage constraint in period 1 and state n* in Figure 2 illustrate the inequality (20). If the inequality (21) holds, *Leverage constraint in period 1 and state b* is located above *Leverage constraint in period 0* and below *Dividend constraint in period 0*. At the same time, *Leverage constraint in period 1 and state n* is located below *Leverage constraint in period 0*. Thus, as long as the financial intermediaries choose a positive  $s_1^*$ , the leverage constraints would never bind in state  $n$ .

Now, we can determine the optimal portfolio that the financial intermediaries choose in period 0.

**Lemma 4** *If inequality (21) holds, the optimal portfolio that the financial intermediaries choose in period 0 is written as:  $s_1^* = \tilde{s}_1^*$  and  $f_1^* = \tilde{f}_1^*$ , where,*

$$\tilde{s}_1^* \equiv \frac{1}{\kappa} \frac{(p_0 + d_0)s_0^* + f_0^*}{p_0}, \quad (22)$$

$$\tilde{f}_1^* \equiv -\frac{1 - \kappa}{\kappa} \frac{(p_0 + d_0)s_0^* + f_0^*}{q_0}. \quad (23)$$

**Proof.** See Appendix A.4.

The intuition behind Lemma 4 is as follows. To determine the portfolio choices of the financial intermediaries, we must analyze how their market value changes as  $s_1^*$  and  $f_1^*$  change. Whereas a change in the risk-free bonds,  $f_1^*$ , does not affect the market value, a change in the equity shares,  $s_1^*$ , does. For a given  $f_1^*$ ,  $s_2^{*j}$ ,  $f_2^{*j}$ , an increase in the equity shares,  $s_1^*$ , by one decreases the dividend payment in period 0,  $D_0$ , by  $p_0$  but increases the dividend payment in period 1,  $D_1^j$ , by  $p_1^n + d_1^n$  in state  $n$  and by  $p_1^b + d_1^b$  in state  $b$ . Therefore, the expected present market value of the increase in the dividend payment in period 1 is  $(1 - \phi)\beta_1^n(p_1^n + d_1^n) + \phi\beta_1^b(p_1^b + d_1^b)$ . Subtracting  $p_0$  from  $(1 - \phi)\beta_1^n(p_1^n + d_1^n) + \phi\beta_1^b(p_1^b + d_1^b)$  yields  $\phi\beta_1^b(p_1^b - \tilde{\beta}_2^b d_2^b)$ . As explained in Appendix A.4 we can prove that  $p_1^b > \tilde{\beta}_2^b d_2^b$ . That is, the financial intermediaries will increase their market value by raising their equity shares as much as possible. However, the dividend and leverage constraints limit the holding of equity shares,  $s_1^*$ . As a result, the intermediaries choose the portfolio at the point of intersection of *Leverage constraint in period 0* and *Dividend constraint in period 0* shown in Figure 2.

Following a similar procedure to the derivations of (18) and (19), we can determine the range of portfolio  $s_2^{*j}$  and  $f_2^{*j}$  that the financial intermediaries can choose in period 1 and state  $j$ .

Figure 3 displays this portfolio on a plane, where the horizontal axis is  $s_2^{*j}$  and the vertical axis is  $f_2^{*j}$ . By substituting the budget constraints, (5), into the leverage constraints, (8), and the dividend constraints, (9), we can derive the following constraints:

$$f_2^{*j} \geq -(1 - \kappa) \frac{p_1}{q_1} s_2^{*j}. \quad (24)$$

$$f_2^{*j} \leq -\frac{p_1^j}{q_1^j} s_2^{*j} + \frac{(p_1^j + d_1^j) s_1^* + f_1^*}{q_1^j}. \quad (25)$$

The downward-sloping lines labeled *Leverage constraint in period 1 and state j* and *Dividend constraint in period 1 and state j* in Figure 3 illustrate the relationship.

The downward-sloping line passing through the origin labeled *Dividend constraint in period 2* in Figure 3 illustrates the following relationship: Incorporating the budget constraint in period 2, (6), into the dividend constraint in period 2, (9), we acquire the following constraints:

$$f_2^{*j} \geq -d_2^j s_2^j. \quad (26)$$

Note that  $q_1^j d_2^j \geq p_1^j$  implies that  $d_2^j \geq \frac{p_1^j}{q_1^j} > (1 - \kappa) \frac{p_1^j}{q_1^j}$ . Figure 3 illustrates how the constraints (24) (25) and (26) are related.

[Figure 3]

Now, we can determine the portfolio in period 1,  $s_2^{*n}$ ,  $f_2^{*n}$ ,  $s_2^{*b}$ , and  $f_2^{*b}$ .

**Lemma 5** *If the inequality (21) holds, the optimal portfolios that the financial intermediaries choose in period 1 are  $s_2^{*n} = \tilde{s}_1^*$  and  $f_2^{*n} = 0$  in state  $n$  and  $s_2^{*b} = \tilde{s}_2^b$  and  $f_2^{*b} = \tilde{f}_2^{*b}$  in state  $b$ , where*

$$\tilde{s}_2^{*b} = \frac{1}{\kappa p_1^b} \left( \frac{p_1^b + d_1^b}{\kappa p_0} - \frac{1 - \kappa}{\kappa q_0} \right) [(p_0 + d_0) s_0^* + f_0^*], \quad (27)$$

$$\tilde{f}_2^{*b} = -\frac{1 - \kappa}{\kappa q_1^b} \left( \frac{p_1^b + d_1^b}{\kappa p_0} - \frac{1 - \kappa}{\kappa q_0} \right) [(p_0 + d_0) s_0^* + f_0^*]. \quad (28)$$

**Proof.** See Appendix A.5.

The intuition behind Lemma 5 is as follows. A one unit increase in  $f_2^{*b}$  brings about a decrease in the dividend payment in period 1 by  $q_1^b$ , but an increase in the dividend payment in period 2 by one, which is translated into present market value using the SDF,  $M_2^b$ . The financial intermediaries would like to lower  $f_2^{*b}$  because Assumption 3 ensures that  $q_1^b \geq M_2^b$ .

For a given  $s_2^{*b}$ , the financial intermediaries choose the portfolio on the line of the *Leverage constraint in period 1* in Figure 3. Suppose that the financial intermediaries reduce dividend payments by one unit in period 1, issue  $\frac{1}{\kappa} - 1$  units of the risk-free bonds  $f_2^{*b}$ , and buy  $\frac{1}{\kappa}$  units of the equity shares,  $s_2^{*b}$ . They can raise  $\frac{1}{\kappa} \frac{d_2^b}{p_1^b} - (\frac{1}{\kappa} - 1) \frac{1}{q_1^b}$  units of dividend payment in period 2. If the present market value from such leveraged investment is higher than one, the financial intermediaries will raise their holdings of equity shares as much as possible. They prefer such a leveraged investment portfolio because Assumption 3 ensures that  $M_2^b \left[ \frac{1}{\kappa} \frac{d_2^b}{p_1^b} - (\frac{1}{\kappa} - 1) \frac{1}{q_1^b} \right] \geq 1$ . As a result, the intermediaries choose the investment portfolio at the point of intersection of the *Leverage constraint in period 1* and the *Dividend constraint in period 1*, as shown in Figure 3. The inequality (21) ensures that the leverage constraints in period 1 bind only in state  $b$ . In this case, the financial intermediaries would not change their portfolio in period 1 and state  $n$ . Thus, the financial intermediaries set  $s_2^{n*}$  equal to  $\tilde{s}_1^*$ . Because the prices of the risk-free bonds are determined by  $q_1^n = \beta \frac{u'(c_2^n)}{u'(c_1^n)} = \beta \frac{u'(y_2^n)}{u'(y_1^n)}$ , there is no demand for safe assets from the households or the financial intermediary. Thus,  $f_2^{n*} = 0$  must hold.

### 4.3 Equilibrium

Using the asset prices and portfolio derived above, we can characterize the competitive equilibrium. From the market-clearing conditions for equity shares and risk-free bonds in period 0 and the financial intermediaries' portfolios, (22) and (23), we use  $\tilde{s}_1$  and  $\tilde{f}_1$  to denote the equilibrium equity shares and risk-free bonds, respectively, held by the households in period 1, where  $\tilde{s}_1 = 1 - \tilde{s}_1^{*b}$  and  $\tilde{f}_1 = -\tilde{f}_1^{*b}$ . From the first-order conditions for the equity shares of the households, we let  $\tilde{s}_{2h}^b$  and  $\tilde{s}_{2l}^b$  denote the equilibrium equity shares for the high- and low-endowment households, respectively, where  $\tilde{s}_{2h}^b = \tilde{s}_1 + \frac{1}{a} \left( \frac{\beta_{2h}^b d_2^b}{p_1^b} - 1 \right)$  and  $\tilde{s}_{2l}^b = \tilde{s}_1 + \frac{1}{a} \left( \frac{\beta_{2l}^b d_2^b}{p_1^b} - 1 \right)$ . Note that the equity prices,  $p_1^b$ , defined in (15) are derived from the market-clearing conditions for the equity shares in period 1 and state  $b$ . From the market-clearing conditions for risk-free bonds in period 1 and state  $b$  and the financial intermediaries' portfolio, (28), we use  $\tilde{f}_{2h}^b$  and  $\tilde{f}_{2l}^b$  to denote the equilibrium risk-free bonds for high- and low-endowment households, respectively, where  $\tilde{f}_{2h}^b = -\frac{\tilde{f}_2^{*b}}{1-\psi}$  and  $\tilde{f}_{2l}^b = 0$ . In addition, note that  $D_0 = 0$ ,  $D_1^b = 0$ , and  $D_2^b = d_2^b \tilde{s}_2^{*b} + \tilde{f}_2^{*b}$ .<sup>10</sup>

Using the equilibrium portfolio choices and dividend payments from the financial intermediaries defined above, we can transform the budget constraints for high-endowment households as

<sup>10</sup>  $D_1^n$  and  $D_2^n$  are also determined as follows:  $D_1^n = \left[ \frac{1}{\kappa} \frac{d_1^n}{p_0} - (\frac{1}{\kappa} - 1) \frac{1}{q_0} \right] [(p_0 + d_0)s_0^* + f_0^*]$  and  $D_2^n = \frac{d_2^n}{\kappa p_0} [(p_0 + d_0)s_0^* + f_0^*]$ . However, they are irrelevant in solving the competitive equilibrium.

follows:

$$c_{1h}^b - d_1^b - e_{1h}^b + \frac{1}{a}(q_1^b d_2^b - p_1^b) + d_1 \tilde{s}_1^* + \tilde{f}_1^* - \frac{q_1^b \tilde{f}_2^{*b}}{1 - \psi} = 0, \quad (29)$$

$$\text{and} \quad c_{2h}^b - e_{2h}^b - d_2^b + \frac{d_2^b}{ap_1^b}(q_1^b d_2^b - p_1^b) + d_2^b(\tilde{s}_1^* - \tilde{s}_2^{*b}) + \frac{\psi}{1 - \psi} \tilde{f}_2^{*b} = 0. \quad (30)$$

It is convenient to use the following consumption shares for household  $h$ , which are derived from the goods-market-clearing conditions in periods 1 and 2 in state  $b$ , (10) and (11):  $\theta_1 \equiv \frac{(1-\psi)c_{1h}^b}{y_1^b}$  and  $\theta_2 \equiv \frac{(1-\psi)c_{2h}^b}{y_2^b}$ . Note that consumption is written as follows:  $c_{1h}^b = \theta_1 y_1^b (1 - \psi)^{-1}$ ,  $c_{1l}^b = (1 - \theta_1) y_1^b \psi^{-1}$ ,  $c_{2h}^b = \theta_2 y_2^b (1 - \psi)^{-1}$ , and  $c_{2l}^b = (1 - \theta_2) y_2^b \psi^{-1}$ . According to Lemmas 4 and 5,  $\tilde{s}_0^*$ ,  $\tilde{f}_0^*$ ,  $\tilde{s}_2^{*b}$ , and  $\tilde{f}_2^{*b}$  depend on  $p_0$ ,  $q_0$ ,  $q_1^b$ , and  $p_1^b$ . According to Lemma 1,  $p_1^b$  depends on  $\bar{p}_1^b$ . Because  $p_0$ ,  $q_0$ ,  $q_1^b$ ,  $p_1^b$ , and  $\bar{p}_1^b$  depend on  $c_{1h}^b$ ,  $c_{1l}^b$ ,  $c_{2h}^b$ , and  $c_{2l}^b$ , these prices depend on the consumption shares,  $\theta_1$  and  $\theta_2$ . Therefore, the left-hand sides of (29) and (30) depend on  $\theta_1$  and  $\theta_2$ . Hereafter, we denote the left-hand sides of (29) and (30) as  $\Omega_1(\theta_1, \theta_2)$  and  $\Omega_2(\theta_1, \theta_2)$ .<sup>11</sup> Furthermore, we denote asset prices as functions of  $\theta_1$  and  $\theta_2$ :  $p_0(\theta_1, \theta_2)$ ,  $q_0(\theta_1, \theta_2)$ ,  $q_1^b(\theta_1, \theta_2)$ , and  $p_1^b(\theta_1, \theta_2)$ ,  $\bar{p}_1^b(\theta_1, \theta_2)$ .

The following proposition states the conditions under which fire sales occur in an equilibrium.

**Proposition 1** *Fire sales occur in equilibrium if consumption shares,  $\theta_1$  and  $\theta_2$ , satisfy the following conditions:*

$$\Omega_1(\theta_1, \theta_2) = 0, \quad (31)$$

$$\Omega_2(\theta_1, \theta_2) = 0, \quad (32)$$

$$\begin{aligned} \frac{p_0(\theta_1, \theta_2)}{q_0(\theta_1, \theta_2)} - \frac{d_1^b}{1 - \kappa} &> \bar{p}_1(\theta_1, \theta_2) \\ &> (1 - \kappa) \left\{ 1 + \frac{a}{\kappa} [(p_0(\theta_1, \theta_2) + d_0) s_0^* + f_0^*] \right\} \left[ \frac{p_0(\theta_1, \theta_2)}{q_0(\theta_1, \theta_2)} - \frac{d_1^b}{1 - \kappa} \right], \end{aligned} \quad (33)$$

$$\text{and } 1 > \theta_1 > \theta_2 > 1 - \psi. \quad (34)$$

*Consumption shares, prices, and portfolio choices associated with such values of  $\theta_1$  and  $\theta_2$  constitute the competitive equilibrium where the leverage constraints bind in period 1 and state  $b$ .*

**Proof.** The left-hand sides of (31) and (32) represent excess demand functions for the goods market in periods 1 and 2, as defined on the left-hand sides of (29) and (30). The first inequality in (33) is derived from (21) in Lemma 3; it ensures that the leverage constraints bind in state  $b$  but not in state  $n$  in period 1. The second inequality in (33) is derived from (16) in Lemma 1; it

<sup>11</sup> $\Omega_1(\theta_1, \theta_2)$  and  $\Omega_2(\theta_1, \theta_2)$  are listed in Appendix A.6.

ensures that the equilibrium equity price in period 1 and state  $b$  is unique. The inequality (34), derived from Assumption 2, ensures that the high-endowment households have higher IMRS between periods 1 and 2 in state  $b$  than do the low-endowment households. (Q.E.D.)

**The case of quadratic utility** Proposition 1 argues that the competitive equilibrium in which the financial intermediaries conduct fire sales exists under certain conditions. However, the conditions are quite complex. Thus, we examine the case of quadratic utility to acquire closed-form conditions for the existence of the competitive equilibrium. In the case where the utility function is specified as a quadratic function,  $u(c) = -\frac{\alpha}{2}c^2 + \gamma c$ , where  $\alpha$  and  $\gamma$  are parameters.

There is no precautionary demand for assets and the households' consumption distributions do not affect the asset prices ex ante. That is,  $q_0$  and  $p_0$  are independent of  $\theta_1$  and  $\theta_2$ . Therefore, the inequality (33) indicates that  $\bar{p}_1^b(\theta_1, \theta_2)$  has constant upper and lower bounds, as follows:

$$\frac{p_0}{q_0} - \frac{d_1^b}{1 - \kappa} > \bar{p}_1^b(\theta_1, \theta_2) > (1 - \kappa) \left\{ 1 + \frac{a}{\kappa} [(p_0 + d_0)s_0^* + f_0^*] \right\} \left[ \frac{p_0}{q_0} - \frac{d_1^b}{1 - \kappa} \right].$$

The market-clearing conditions for goods in period 1,  $\Omega_1(\theta_1, \theta_2) = 0$ , are written as:

$$A_1\theta_1^3 + B_1\theta_2^2 + \Gamma_1\theta_1 + \Delta_1\theta_1\theta_2 + E_1\theta_2 + Z_1 = 0, \quad (35)$$

where the coefficients,  $A_1$ ,  $B_1$ ,  $\Gamma_1$ ,  $\Delta_1$ ,  $E_1$ , and  $Z_1$  are defined in the Appendix A.7. Note that these coefficients do not depend on either  $\theta_1$  or  $\theta_2$ . The market-clearing conditions for the goods market in period 2 and state  $b$ ,  $\Omega_2(\theta_1, \theta_2) = 0$ , are written as follows:

$$A_2(\theta_1)\theta_2^3 + B_2(\theta_1)\theta_2^2 + \Gamma_2(\theta_1)\theta_2 + \Delta_2(\theta_1) = 0, \quad (36)$$

where the coefficients  $A_2(\theta_1)$ ,  $B_2(\theta_1)$ ,  $\Gamma_2(\theta_1)$ , and  $\Delta_2(\theta_1)$  are defined in Appendix A.7.

We can derive sufficient conditions for the existence of the competitive equilibrium where the financial intermediaries' leverage constraints bind in period 1 and state  $b$ .

**Proposition 2** *In the case of quadratic utility, the competitive equilibrium exists under certain conditions.*

**Proof.** See Appendix A.8.

**The case of constant relative risk aversion utility** In the case of constant relative risk aversion (CRRA) utility,  $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ , where  $\sigma$  is the coefficient of the relative risk aversion or the reciprocal of the intertemporal elasticities of substitution. The marginal utility is written as  $u'(c) = c^{-\sigma}$ . Because the market-clearing conditions are quite complex, it is difficult to discuss the existence of the competitive equilibrium using closed-form conditions. Thus, we compute the competitive equilibrium numerically, and confirm that the equilibrium uniquely exists.

## 5 Results and Discussion

We conduct numerical computations to understand the qualitative implications of our model, rather than as an attempt to match data quantitatively. The computational algorithm is based on Proposition 1. We confirm the existence and uniqueness of the competitive equilibrium. To aid understanding of the model implications, we present the results from the representative agent (RA) model. In this case, the idiosyncratic endowment shocks are perfectly shared:  $\theta_1 = \theta_2 = 0.5$ .

### 5.1 Calibration Parameters

Table 1 presents the basic calibration parameters: the time discount factor,  $\beta$ , is 0.94; the coefficient of adjustment costs of equity transactions,  $a$ , is 1; the probability of the aggregate economy being in a bad state,  $\phi$ , is 0.1; and the probability of a household having a low endowment,  $\psi$ , is 0.5. Idiosyncratic shocks drive period 1's endowment down to  $e_{1l}^b = 0.6$ , but otherwise labor endowments are 0.7. Dividends from the Lucas trees fall to  $d_1^b = 0.03$  in period 1 and state  $b$ , but otherwise they are 0.3. That is, the aggregate endowments,  $y_0, y_1^n, y_2^n$ , and  $y_2^b$  are set to 1 but  $y_1^b$  is set to 0.68. In state  $b$  and period 1, the dividends from Lucas trees decrease by 90% and the endowments of the low-endowment household decrease by 14%. The initial holdings of equity shares by the financial intermediaries,  $s_0^*$ , are set to 0.01, and the risk-free bonds,  $f_0^*$ , are set to 0. The utility functions are specified in terms of quadratic utility,  $u(c) = -\frac{c^2}{2} + 1.75c$ , or CRRA,  $u(c) = \frac{c^{1-1.1}}{1-1.1}$ . These values are chosen so that the risk-free rates and the equity premiums in the RA model with CRRA utility equal about 0.01 and 0.03, respectively.

[Table 1]

## 5.2 Asset Prices and Portfolio

Figure 4 presents the asset prices and portfolio results in the case of quadratic utility and Figure 5 presents those in the case of CRRA utility. In each figure, Panel A presents “the equity price to fundamental price ratio”, which is calculated as  $p_1^b/q_1^b d_2^b$ ; Panel B presents the risk-free rates,  $R_f \equiv 1/q_0$ ; Panel C presents the equity premiums,  $R_e - R_f$ , where the expected equity returns,  $R_e$ , are calculated as  $R_e \equiv (1 - \phi)(p_1^n + d_1^n)/p_0 + \phi(p_1^b + d_1^b)/p_0$ ; and Panel D presents the volatilities in equity prices, which are calculated as  $\{(1 - \phi)[(p_1^n - \bar{p}_1)/p_0]^2 + \phi[(p_1^b - \bar{p}_1)/p_0]^2\}^{\frac{1}{2}}$ , where  $\bar{p}_0 \equiv (1 - \phi)p_1^n + \phi p_1^b$ . Panel E presents the risk-free bonds issued by the financial intermediaries in periods 0 and 1, which equal  $f_1^*$  and  $f_2^{*b}$ , respectively; and Panel F presents the flow of the equity shares traded by financial intermediaries in periods 0 and 1,  $s_1^* - s_0^*$  and  $s_2^{*b} - s_1^*$ , and those traded by the high- and low-endowment households, respectively, in period 1,  $(1 - \psi)(s_{2h}^b - s_1)$  and  $\psi(s_{2l}^b - s_1)$ . Each panel reports results where the margin requirement ratio,  $\kappa$ , takes a value between 0.6 and 0.9.

Figure 4 reports results in the case of quadratic utility. First, Panel A demonstrates that the equity price to fundamental price ratio takes values lower than one, whereas it takes a value of one in the RA. The decrease in  $\kappa$  enables the financial intermediaries to issue risk-free bonds and buy more shares of Lucas trees. Thus, such leveraged investment by the financial intermediaries causes a severe fall in equity prices in period 1 and state  $b$  associated with fire sales. Second, Panel B demonstrates that the risk-free rates in our model are the same as those in the RA model. Due to the absence of the precautionary saving motive, the expected IMRS between periods 0 and 1 are equal to those of the representative agent. As a result,  $p_0$  and  $q_0$  are independent of the consumption distribution. Finally, Panels C and D demonstrate that the equity premiums decrease and the expected volatilities increase as  $\kappa$  decreases. In particular, the equity premiums in our model are lower than those in the RA when  $\kappa$  is below 0.85. Because  $q_0$  and  $p_0$  are constant, a decrease in  $p_1^b$  directly decreases the expected equity returns and the equity premiums. This is consistent with the arguments by Mankiw (1986): If investors do not have a precautionary saving motive, the concentration of aggregate risks cannot generate large equity premiums. In our model, a decrease in the equity returns due to fire sales reduces the equity premiums. Note that Panels E and F are similar to the case of CRRA utility. We will explain them later.

Figure 5 shows the results in the case of CRRA utility, where asset prices are affected by the precautionary saving motive. We begin by explaining the results of asset prices using Panels A, B, C, and D. First, Panel A demonstrates that the decrease in  $\kappa$  decreases the equity price to the

fundamental price ratio, as in the case of quadratic utility. Second, Panel B demonstrates that the risk-free rates increase as  $\kappa$  decreases from 0.9 to 0.75, but they decrease as  $\kappa$  decreases from 0.75 to 0.6. Such an inverse U-shape occurs because of the following two opposite effects. On the one hand, a decrease in  $\kappa$  enables the financial intermediaries to issue more risk-free bonds. The risk-free bonds serve as self-insurance for households against their idiosyncratic endowment shocks. Therefore, a decrease in  $\kappa$  mitigates precautionary demands for assets and raises the risk-free rates. On the other hand, a decrease in  $\kappa$  exaggerates a decline in equity prices because of fire sales. The low-endowment households suffer from fire sales because they have to sell their equity shares to raise their period 1 consumption. Hence, fire sales increase the consumption disparity between high- and low-endowment households in period 1 and state  $b$ , they magnify the precautionary demands for assets, and lower the risk-free rates. As the latter effects dominate the former, the risk-free rates decrease as  $\kappa$  decreases from 0.75 to 0.6. Then, Panel C demonstrates that the equity premium decreases as  $\kappa$  decreases from 0.9 to 0.6. These values are higher than those of the RA model as long as  $\kappa$  is higher than about 0.7. Labor endowments are exposed to uninsurable idiosyncratic shocks and such uninsured idiosyncratic shocks correlate with the aggregate shocks. As explained by Mankiw (1986), because such correlation implies that holding equity shares is risky, the equity premium is higher than that in the RA model. However, equity premiums are lower than those of the RA model when  $\kappa$  is lower than 0.7. This is because fire sales induce a decrease in the expected equity returns that is much more severe than the decrease in the risk-free rates. Panel D demonstrates that the expected volatilities of the equity prices increase as  $\kappa$  decreases, as in the case of quadratic utility.

Panels E and F show the portfolio results and confirm that asset prices are actually related to the balance sheets of financial intermediaries. Panel E shows the risk-free bonds issued by the financial intermediaries. Note that the holdings of risk-free bonds by households in period 0 are  $f_1 = -f_1^*$ ; in period 1, high-endowment households in period 1 hold  $(1 - \psi)f_{2h}^b = -f_2^*$  and low-endowment households hold  $\psi f_{2l}^b = 0$ . Both  $f_1^*$  and  $f_2^{*b}$  take negative values because financial intermediaries issue risk-free bonds,  $f_1^*$  monotonically decreases as  $\kappa$  decreases,  $f_2^{*b}$  decreases as  $\kappa$  decreases from 0.9 to 0.75, but increases as  $\kappa$  decreases from 0.75 to 0.6, and  $f_2^{*b}$  is higher than  $f_1^*$ . Panel F shows that the equity shares purchased by the financial intermediaries take positive values in period 0 but negative values in period 1. In addition, the absolute values of equity shares purchased (sold) increase as  $\kappa$  decreases. These results confirm that the balance sheets of financial intermediaries expand in period 0 but shrink in period 1 and state  $b$ . Such a shrinkage in the balance sheets is considered to indicate fire sales because it is associated with a decline in



equity prices from their fundamental values, as shown in Panel A. Note that the equity shares purchased by the high-endowment households increase as  $\kappa$  decreases. On the other hand, the equity shares sold by the low-endowment household decrease as  $\kappa$  decreases.

These results are insightful in terms of the disaster explanations of asset prices. The model with CRRA utility successfully generates low risk-free rates, large equity premiums, and high expected volatilities in equity returns or prices compared with the RA model as long as  $\kappa$  is higher than about 0.7. The original disaster models proposed by Rietz (1988) and Barro (2006) argue that high equity premiums occur even in the RA framework, where the equity prices maintain their fundamental values. Our results demonstrate that not only equity premiums but also the volatilities of equity returns can take higher values than those of the representative agent. Therefore, the noted link between leverage and fire sales during financial crises may, at least qualitatively, resolve both the equity premium puzzles and the excess volatility puzzles.

[Figure 4]

[Figure 5]

### 5.3 Consumption Risk Sharing

Figure 6 presents the consumption risk sharing and welfare based on quadratic utility and Figure 7 presents those based on CRRA utility. These figures demonstrate that there is no large difference in the qualitative features of the quadratic and CRRA utility models.

Panel A of each figure shows the consumption shares of high-endowment households in state  $b$  in period 1,  $\theta_1$ , and period 2,  $\theta_2$ , when  $\kappa$  equals 0.6, 0.625,  $\dots$ , 0.9.  $\theta_1$  decreases but  $\theta_2$  increases as  $\kappa$  declines from 0.9 to 0.75, whereas  $\theta_1$  increases but  $\theta_2$  decreases as  $\kappa$  declines from 0.75 to 0.6. The relationship between  $\theta_1$  and  $\theta_2$  is drawn as a curve. In particular, the slope of the curve between  $\kappa = 0.9$  and 0.75 is steeper than that between  $\kappa = 0.75$  and 0.6. It is convenient to separate the effects of a decrease in  $\kappa$  on  $\theta_1$  and  $\theta_2$  into two components. First, a decrease in  $\kappa$  allows the financial intermediaries to issue more risk-free bonds, which are held by households for self-insurance in period 0. It enables the low-endowment households to successfully raise their consumption and reduce the amount of equity shares sold in period 1. Thus,  $\theta_1$  decreases and  $\theta_2$  increases. Second, a decrease in  $\kappa$  implies a sharp decline in equity prices due to fire sales. Such fire sales prevent low-endowment households from raising their consumption in period 1. That is,  $\theta_1$  increases and  $\theta_2$  decreases. In particular, because fire sales enable the high-endowment households to buy equity shares at discounted prices, they could raise  $\theta_2$  at less compensations

in  $\theta_1$ . Thus, the more sharply that equity prices fall, the more the second effect dominates the first. As a result,  $\theta_2$  increases at less compensations in  $\theta_1$  when  $\kappa$  is lower than 0.75.

Panel B shows the ex post welfare measured by the certainty equivalent consumption of high-endowment households and demonstrates that it monotonically deteriorates as  $\kappa$  decreases. As shown in Panel A, a decrease in  $\kappa$  from 0.9 to 0.75 mitigates the dispersion in consumption of the two types of households in period 1. Note that the marginal utility of the high-endowment households is lower than that of low-endowment households in period 1. This reduction in the disparity in consumption levels between low- and high-endowment households has a relatively negligible impact on the welfare of the high-endowment households. On the other hand, as shown in Panel F of Figure 4 and 5, the high-endowment households incur the costs of the fire sales by the financial intermediaries. As a result, the welfare of the high-endowment households monotonically decreases.

Panel C shows the ex post welfare of the low-endowment households, which improves as  $\kappa$  decreases from 0.9 to 0.75 but deteriorates as  $\kappa$  decreases from 0.75 to 0.6. As shown in Panel F in Figure 4 and 5, a decrease in  $\kappa$  enables the low-endowment households to avoid paying adjustment costs in period 1. As shown in Panel A, the disparity in consumption for the two types of households in period 1 is mitigated as  $\kappa$  decreases from 0.9 to 0.75. Thus, the low-endowment households' welfare is improved as  $\kappa$  decreases from 0.9 to 0.75. On the other hand, the dispersion is significantly enlarged as  $\kappa$  decreases from 0.75 to 0.6. Because the loss from the decrease in period 1 consumption dominates the gain from avoiding the adjustment costs, welfare deteriorates as  $\kappa$  decreases from 0.75 to 0.6.

Panel D demonstrates that the ex ante welfare displays an inverse U-shape; it increases as  $\kappa$  declines from 0.9 to 0.75 but decreases as  $\kappa$  declines from 0.75 to 0.6. This result is naturally derived from the interaction of the different ex post welfare results for the high- and low-endowment households.

[Figure 6]

[Figure 7]

Taken together, the numerical results demonstrate that there is a trade-off for the financial intermediaries between the ex ante self-insurance opportunities derived from expanding their balance sheets and the costly ex post financial instabilities arising from fire sales. If  $\kappa$  is in the moderate range (from 0.9 to 0.75), then mitigating the leverage constraints raises the ex ante welfare due to improvements in the welfare of low-endowment households. On the other hand,

if  $\kappa$  takes a sufficiently low value ( $\kappa$  from 0.75 to 0.6), the ex ante welfare worsens as the welfare of both household types deteriorates. In other words, there is an optimal level for the margin requirement ratio to maximize ex ante welfare. However, note that it is only high-endowment households who absorb the costs of fire sales.

## 6 Conclusion

The contribution of our paper is to demonstrate that countercyclical idiosyncratic endowment risks and market incompleteness give rise to leverage by financial intermediaries and cause fire sales in a competitive equilibrium. If the margin requirement constraint is in the moderate range specified, our model successfully generates lower risk-free rates, larger equity premiums, and higher expected volatilities in equity returns compared with the RA model. At the same time, if the margin requirement constraint is in the moderate range, the mitigation of the leverage constraints raises the ex ante welfare. However, if the margin requirement constraint takes already sufficiently mitigated, it worsens the ex ante welfare.

## A Appendix

### A.1 Proof of Lemma 1

The first-order conditions with respect to the equity shares,  $s_2^n$ ,  $s_{2h}^b$ , and  $s_{2l}^b$  are  $s_2^n = s_1 + \frac{1}{a} \left( \frac{\beta_2^n d_2^b}{p_1^b} - 1 \right)$  and  $s_{2i}^b = s_1 + \frac{1}{a} \left( \frac{\beta_{2i}^b d_2^b}{p_1^b} - 1 \right)$ , where  $i = h$  and  $l$ . From the market-clearing condition (12), we can derive the following “equity supply function” for the financial intermediaries.

$$s_2^{*j} = s_1^* - \frac{1}{a} \left( \frac{\bar{p}_1^j}{p_1^j} - 1 \right) \quad (37)$$

If equity prices are determined as  $p_1^j = \bar{p}_1^b$ , there is no supply of equity shares for the financial intermediaries, that is,  $s_2^{*j} = s_1^*$  holds.

Suppose that the financial intermediaries’ net assets evaluated at the equity prices  $\bar{p}_1^j$  are not sufficient to enable them to satisfy the leverage constraint, pay dividends, and maintain their current holding of equity,  $s_2^{*j} = s_1^*$ ; that is, when  $\frac{1}{\kappa} [(\bar{p}_1^j + d_1^j)s_1^* + f_1^{*j} + D_1^j] \leq \bar{p}_1^j s_1^*$ , they must reduce their dividends to zero. Nevertheless, the leverage constraints under the equity prices still bind, that is, when  $\frac{1}{\kappa} [(\bar{p}_1^j + d_1^j)s_1^* + f_1^{*j}] \leq \bar{p}_1^j s_1^*$ , they must shed securities to the point where

they are just able to satisfy the leverage constraints:

$$\frac{1}{\kappa} [(p_1^j + d_1^j)s_1^* + f_1^{*j}] = p_1^j s_2^{*j}, \quad (38)$$

where  $s_2^{*j} < s_1^{*j}$ . As it is costly for the households to absorb the equity shares,  $p_1^j$  will drop below  $\bar{p}_1^j$ . By inverting (38), we obtain the following “equity demand function” for the financial intermediaries in the regime where the leverage constraints bind.

$$s_2^{*j} = \frac{1}{\kappa p_1^j} [(p_1^j + d_1^j)s_1^* + f_1^{*j}] \quad (39)$$

For the leverage constraints to bind, the financial intermediaries must be unable to cover their debt obligations simply with their dividend earnings. Therefore, the term  $d_1^j s_1^j + f_1^*$  is negative. In this instance, equation (39) implies that the financial intermediaries’ demand for equity is upward sloping in prices.

Figure 8 illustrates the equilibrium. From equation (37), the supply curve is written as  $p_1^j = \frac{\bar{p}_1^j}{1 - a(s_2^{*j} - s_1^*)}$ . It is positioned so that  $p_1^j = \bar{p}_1^j$  when  $s_2^{*j} = s_1^*$ . The figure also portrays the upward-sloping demand curve,  $p_1^j = -\frac{d_1^j s_1^* + f_1^*}{s_1 - \kappa s_2^{*j}}$  derived from equation (39). When the leverage constraints bind, the demand curve is to the left of the supply curve at  $p_1^j = \bar{p}_1^j$ . In this instance, when  $p_1^j = \bar{p}_1^j$ , the maximum feasible number of shares that the financial intermediaries can hold is less than  $s_1^*$ . Given the position of the demand curve, the equilibrium lies at a point where  $p_1^j$  is below  $\bar{p}_1^j$  and  $s_2^{*j}$  is below  $s_1^*$ . Equation (15) follows directly from equations (37) and (39). As displayed in Figure 8, there is a unique intersection of the supply and demand curves ((37 and 39, respectively) if and only if the condition (16) holds. (Q.E.D.)

[Figure 8]

## A.2 Proof of Lemma 2

From (14) the following inequality holds:  $q_1^b = \beta \frac{u'(c_{2h}^b)}{u'(c_{1h}^b)} \geq \beta \left[ (1 - \psi) \frac{u'(c_{2h}^b)}{u'(c_{1h}^b)} + \psi \frac{u'(c_{2l}^b)}{u'(c_{1l}^b)} \right] = \beta_2^b$ . Proving  $\beta_2^b \geq \tilde{\beta}_2^b$  is straightforward. We can demonstrate that  $\beta \frac{u'(c_{2h}^b)}{u'(c_{1h}^b)} \geq \beta \frac{u'(c_{2l}^b)}{u'(c_{1l}^b)} \Rightarrow \beta_2^b \geq \tilde{\beta}_2^b$ . We define the following notation:  $m_{1h} \equiv u'(c_{1h}^b)$ ,  $m_{1l} \equiv u'(c_{1l}^b)$ ,  $m_{2h} \equiv u'(c_{2h}^b)$ , and  $m_{2l} \equiv u'(c_{2l}^b)$ .  $\beta \frac{u'(c_{2h}^b)}{u'(c_{1h}^b)} \geq \beta \frac{u'(c_{2l}^b)}{u'(c_{1l}^b)}$  implies that  $\frac{m_{2h}}{m_{1h}} \geq \frac{m_{2l}}{m_{1l}} \Leftrightarrow \frac{m_{2h}}{m_{1h}}(m_{1l} - m_{1h}) \geq \frac{m_{2l}}{m_{1l}}(m_{1l} - m_{1h})$   
 $\Leftrightarrow \frac{m_{2h}}{m_{1h}}(m_{1l} - m_{1h}) + \frac{m_{2l}}{m_{1l}}(m_{1h} - m_{1l}) \geq 0 \Leftrightarrow m_{2h}m_{1l}(m_{1l} - m_{1h}) + m_{2l}m_{1h}(m_{1h} - m_{1l}) \geq 0$   
 $\Leftrightarrow m_{2h}(m_{1l})^2 + m_{2l}(m_{1h})^2 \geq (m_{2h} + m_{2l})m_{1h}m_{1l} \Leftrightarrow m_{2h} \frac{m_{1l}}{m_{1h}} + m_{2l} \frac{m_{1h}}{m_{1l}} \geq m_{2h} + m_{2l} \Leftrightarrow$   
 $-\psi(1 - \psi)m_{2h} - \psi(1 - \psi)m_{2l} + \psi(1 - \psi)m_{2h} \frac{m_{1l}}{m_{1h}} + \psi(1 - \psi)m_{2l} \frac{m_{1h}}{m_{1l}} \geq 0 \Leftrightarrow (1 - \psi)^2 m_{2h} + \psi(1 -$

$$\begin{aligned} & \psi)m_{2h} \frac{m_{1l}}{m_{1h}} + \psi(1-\psi)m_{2l} \frac{m_{1h}}{m_{1l}} + \psi^2 m_{2l} \geq (1-\psi)m_{2h} + \psi m_{2l} \Leftrightarrow (1-\psi) \frac{m_{2h}}{m_{1h}} [(1-\psi)m_{1h} + \psi m_{1l}] + \\ & \psi \frac{m_{2l}}{m_{1l}} [(1-\psi)m_{1h} + \psi m_{1l}] \geq (1-\psi)m_{2h} + \psi m_{2l} \Leftrightarrow \beta [(1-\psi) \frac{m_{2h}}{m_{1h}} + \psi \frac{m_{2l}}{m_{1l}}] \geq \beta \frac{(1-\psi)m_{2h} + \psi m_{2l}}{(1-\psi)m_{1h} + \psi m_{1l}} \\ & \text{(Q.E.D.)} \end{aligned}$$

### A.3 Proof of Lemma 3

To prove that  $(1-\kappa) \frac{p_0}{q_0} > d_1^b + (1-\kappa) \bar{p}_1^b \Rightarrow d_1^n + (1-\kappa) \bar{p}_1^n \geq (1-\kappa) \frac{p_0}{q_0}$  holds, it is sufficient to demonstrate that  $(1-\kappa) \frac{p_0}{q_0} > d_1^b + (1-\kappa) \bar{p}_1^b \wedge (1-\kappa) \frac{p_0}{q_0} > d_1^n + (1-\kappa) \bar{p}_1^n$  is a contradiction. Assume that  $\beta_2^b \equiv \tilde{\beta}_2^b + \delta$  ( $\delta > 0$ ). If both inequalities hold, the following inequality must hold:  $\phi \beta_1^b (1-\kappa) \frac{p_0}{q_0} > \phi \beta_1^b \{d_1^b + (1-\kappa)(\tilde{\beta}_2^b + \delta) d_2^b\} \wedge (1-\phi) \beta_1^n (1-\kappa) \frac{p_0}{q_0} > (1-\phi) \beta_1^n \{d_1^n + (1-\kappa) \beta_2^n d_2^n\} \Leftrightarrow \phi \beta_1^b (1-\kappa) \frac{p_0}{q_0} > \phi \beta_1^b \{(1-\kappa) d_1^b + \kappa d_1^b + (1-\kappa)(\tilde{\beta}_2^b + \delta) d_2^b\} \wedge (1-\phi) \beta_1^n (1-\kappa) \frac{p_0}{q_0} > (1-\phi) \beta_1^n \{(1-\kappa) d_1^n + \kappa d_1^n + (1-\kappa) \beta_2^n d_2^n\}$ . However, the sums of the inequalities imply that  $(1-\kappa) p_0 > (1-\kappa) p_0 + \delta(1-\kappa) \beta_1^b d_2^b + \kappa [\phi \beta_1^b d_1^b + (1-\phi) \beta_1^n d_1^n] \Leftrightarrow 0 > \delta(1-\kappa) \beta_1^b d_2^b + \kappa [\phi \beta_1^b d_1^b + (1-\phi) \beta_1^n d_1^n]$ . Because the right-hand side is positive, the above inequality is a contradiction. (Q.E.D.)

Note that  $(1-\kappa) \frac{p_0}{q_0} < d_1^b + (1-\kappa) \bar{p}_1^b \wedge (1-\kappa) \frac{p_0}{q_0} < d_1^n + (1-\kappa) \bar{p}_1^n$  is not a contradiction. Therefore,  $(1-\kappa) \frac{p_0}{q_0} > d_1^b + (1-\kappa) \bar{p}_1^b \Leftarrow d_1^n + (1-\kappa) \bar{p}_1^n \geq (1-\kappa) \frac{p_0}{q_0}$  does not hold.

### A.4 Proof of Lemma 4

Using the budget constraints (4), (5), and (6), the financial intermediaries' value, (3), can be written as follows:

$$\begin{aligned} V_{FI} & \equiv (p_0 + d_0) s_0^* + f_0^* - \left\{ p_0 - [(1-\phi) \beta_1^n (p_1^n + d_1^n) + \phi \beta_1^b (p_1^b + d_1^b)] \right\} s_1^* \\ & - \left\{ q_0 - [(1-\phi) \beta_1^n + \phi \beta_1^b] \right\} f_1^* - (1-\phi) \beta_1^n (p_1^n - \beta_2^n d_2^n) s_2^{*n} \\ & - (1-\phi) \beta_1^n (q_1^n - \beta_2^n) f_2^{*n} - \phi \beta_1^b (p_1^b - M_2^b d_2^b) s_2^{*b} - \phi \beta_1^b (q_1^b - M_2^b) f_2^{*b}. \quad (40) \end{aligned}$$

The effects of marginal changes of  $s_1^*$  and  $f_1^*$  on the financial intermediaries' values,  $V_{FI}$ , can be written as follows:  $\frac{\partial V_{FI}}{\partial s_1^*} = (1-\phi) \beta_1^n (p_1^n - \beta_2^n d_2^n) + \phi \beta_1^b (p_1^b - \tilde{\beta}_2^b d_2^b)$  and  $\frac{\partial V_{FI}}{\partial f_1^*} = 0$ . If the inequality (21) holds, we can prove that  $p_1^b < \tilde{\beta}_2^b d_2^b$  is a contradiction. If  $p_1^b < \tilde{\beta}_2^b d_2^b$ , the financial intermediaries would like to decrease the holdings of equity shares. In this case,  $s_1^*$  take a negative value. As displayed in Figure 2, the leverage constraints do not bind in state  $b$  and period 1 if  $s_1^* < 0$ . Then, the equity prices must be  $p_1^b = \beta_2^b d_2^b$ . However, Lemma 2 argues that  $\beta_2^b \geq \tilde{\beta}_2^b$ ,  $p_1^b < \tilde{\beta}_2^b d_2^b$  is a contradiction in equilibrium. Therefore, the equity prices in state  $b$  and period 1 should be  $p_1^b \geq \tilde{\beta}_2^b d_2^b$ . Thus, it implies that  $\frac{\partial V_{FI}}{\partial s_1^*} \geq 0$  holds and the financial intermediaries raise their equity shares as high as they can. As a result, the intermediaries choose the point

of intersection of the *Leverage constraint in period 0* and the *Dividend constraint in period 0* displayed in Figure 2. (Q.E.D.)

## A.5 Proof of Lemma 5

From (40), the effects of marginal changes of  $s_2^{*j}$  and  $f_1^{*j}$  on the financial intermediaries' values,  $V_{FI}$ , can be written as follows:  $\frac{\partial V_{FI}}{\partial s_2^{*n}} = (1 - \phi)\beta_1^n(\beta_2^n d_2^n - p_1^n)$ ,  $\frac{\partial V_{FI}}{\partial f_2^{*n}} = 0$ ,  $\frac{\partial V_{FI}}{\partial s_2^{*b}} = \phi\beta_1^b(M_2^b d_2^b - p_1^b)$ , and  $\frac{\partial V_{FI}}{\partial f_2^{*b}} = \phi\beta_1^b(M_2^b - q_1^b)$ .

When  $M_2^b = q_1^b$ ,  $\frac{\partial V_{FI}}{\partial f_2^{*b}} < 0$  and  $\frac{\partial V_{FI}}{\partial s_2^{*b}} > 0$  hold. That is, both selling the risk-free bonds and buying the equity shares raises the financial intermediaries' value. In this case, the optimal portfolio occurs at the point of intersection between the *Leverage constraint in period 1* and the *Dividend constraint in period 1* in Figure 3.

When  $M_2^b < q_1^b$ ,  $\frac{\partial V_{FI}}{\partial f_2^{*b}} < 0$  holds. That is, because selling the risk-free bonds increases the financial intermediaries' value and the leverage constraint in period 1 binds. On the one hand, if  $M_2^b d_2^b \geq p_1^b$ , the optimal portfolio is at the point of intersection between the *Leverage constraint in period 1* and the *Dividend constraint in period 1* in Figure 3. On the other hand, if  $M_2^b d_2^b \leq p_1^b$  holds, selling both the risk-free bonds and the equity shares raises the financial intermediaries' value. If the following inequality holds, the financial intermediaries prefer selling the risk-free bonds to selling the equity shares:  $\frac{\partial V_{FI}}{\partial s_2^{*b}} - (1 - \kappa)\frac{p_1^b}{q_1^b}\frac{\partial V_{FI}}{\partial f_2^{*b}} > 0 \Rightarrow \phi\beta_1^b p_1^b \left[ \frac{M_2^b d_2^b}{p_1^b} - 1 - (1 - \kappa)\frac{M_2^b - q_1^b}{q_1^b} \right] > 0$ . The term in parentheses,  $[\cdot]$ , is positive if the following inequality holds:  $M_2^b > \underline{\beta}_2^b \equiv \left( \frac{d_2^b}{\kappa p_1^b} - \frac{1 - \kappa}{\kappa q_1^b} \right)^{-1}$ . In this case, the optimal portfolio is at the point of intersection of the *Leverage constraint in period 1* and the *Dividend constraint in period 1* in Figure 3. Assumption 3 ensures that the SDF,  $M_2^b$ , is higher than  $\underline{\beta}_2^b$ .

The above arguments demonstrate that, regardless of the sign of  $M_2^b d_2^b - p_1^b$ , the optimal portfolio is at the point of intersection between the *Leverage constraint in period 1* and the *Dividend constraint in period 1* in Figure 3. Therefore, the optimal portfolio that the financial intermediaries choose in period 1 and state  $b$  is written as  $s_2^{*b} = \tilde{s}_2^b$  and  $f_2^{*b} = \tilde{f}_2^{*b}$ , where

$$\tilde{s}_2^b \equiv \frac{1}{\kappa} \frac{(p_1^b + d_1^b)s_1^* + f_1^*}{p_1^b} \quad (41)$$

$$\tilde{f}_2^{*b} \equiv -\frac{1 - \kappa}{\kappa} \frac{(p_1^b + d_1^b)s_1^* + f_1^*}{q_1^b}. \quad (42)$$

The inequality (21) ensures that the leverage constraints in period 1 bind only in state  $b$ . In this case, the financial intermediaries would not change their portfolio in period 1 and state  $n$ . Thus, the financial intermediaries set  $s_2^{n*}$  equal to  $\tilde{s}_1^*$ . Because the prices of the risk-free

bonds are determined by  $q_1^n = \beta \frac{u'(c_2^n)}{u'(c_1^n)} = \beta \frac{u'(y_2^n)}{u'(y_1^n)}$ , there is no demand for safe assets from the households or the financial intermediary. Thus,  $f_2^{n*} = 0$  must hold. Using (41), (42), (22), and (23),  $\tilde{s}_2^{*b}$  and  $\tilde{f}_2^{*b}$  can be written as in (27) and (28). (Q.E.D.)

## A.6 Derivation of (29) and (30)

Using the optimal holdings of risk-free bonds, (22), (23), and (28), the sum of the fifth, sixth, and seventh terms on the left-hand side of equation (29) is written as follows:

$$d_1^b s_1^{*b} + f_1^* - q_1^b \frac{1}{1-\psi} f_2^* = \left[ \frac{(1-\kappa)p_1^b + (1-\psi\kappa)d_1^b}{(1-\psi)\kappa^2 p_0} - \frac{(1-\kappa)(1-\kappa\psi)}{(1-\psi)\kappa^2 q_0} \right] [(p_0 + d_0)s_0^* + f_0^*]. \quad (43)$$

Using (43) and  $c_{1h}^b = \frac{\theta_1 y_1^b}{1-\psi}$ , and replacing  $p_0$ ,  $p_1^b$ ,  $q_0$ ,  $q_1^b$  with  $p_0(\theta_1, \theta_2)$ ,  $p_1^b(\theta_1, \theta_2)$ ,  $q_0(\theta_1, \theta_2)$ ,  $q_1^b(\theta_1, \theta_2)$ , we can define the left-hand side of equation (29) as follows:

$$\begin{aligned} \Omega_1(\theta_1, \theta_2) &\equiv \frac{\theta_1 y_1^b}{1-\psi} - d_1^b - e_{1h}^b + \frac{1}{a} [q_1^b(\theta_1, \theta_2) d_2^b - p_1^b(\theta_1, \theta_2)] \\ &\quad + \left[ \frac{1-\psi\kappa}{(1-\psi)\kappa^2} d_1^b + \frac{1-\kappa}{(1-\psi)\kappa^2} p_1^b(\theta_1, \theta_2) - \frac{(1-\kappa)(1-\kappa\psi)}{(1-\psi)\kappa^2} \frac{p_0(\theta_1, \theta_2)}{q_0(\theta_1, \theta_2)} \right] \left( s_0^* + \frac{d_0 s_0^* + f_0^*}{p_0(\theta_1, \theta_2)} \right). \end{aligned}$$

Using the optimal level of risk-free bonds, (22), (27), and (28), the sum of the fifth and sixth terms on the left-hand side of equation (30) is written as follows:

$$d_2^b (s_1^{*b} - s_2^{*b}) + \frac{\psi}{1-\psi} f_2^{*b} = \left[ \frac{d_2^b}{\kappa p_0} - \left( \frac{d_2^b}{\kappa p_1^b} + \frac{\psi(1-\kappa)}{\kappa(1-\psi)q_1^b} \right) \left( \frac{p_1^b + d_1^b}{\kappa p_0} - \frac{1-\kappa}{\kappa q_0} \right) \right] [(p_0 + d_0)s_0^* + f_0^*]. \quad (44)$$

Using (44) and  $c_{2h}^b = \frac{\theta_2 y_2^b}{1-\psi}$ , and replacing  $p_0$ ,  $p_1^b$ ,  $q_0$ ,  $q_1^b$  with  $p_0(\theta_1, \theta_2)$ ,  $p_1^b(\theta_1, \theta_2)$ ,  $q_0(\theta_1, \theta_2)$ ,  $q_1^b(\theta_1, \theta_2)$ , we can define the left-hand side of equation (30) as follows:

$$\begin{aligned} \Omega_2(\theta_1, \theta_2) &\equiv \frac{\theta_2 y_2^b}{1-\psi} + \left( \frac{1-a}{a} \right) d_2^b - e_{2h}^b - \frac{q_1^b(\theta_1, \theta_2)(d_2^b)^2}{a p_1^b(\theta_1, \theta_2)} \\ &\quad - \left\{ \left( \frac{d_2^b}{\kappa p_1^b(\theta_1, \theta_2)} + \frac{\psi(1-\kappa)}{\kappa(1-\psi)q_1^b(\theta_1, \theta_2)} \right) \left[ p_1^b(\theta_1, \theta_2) + d_1^b - (1-\kappa) \frac{p_0(\theta_1, \theta_2)}{q_0(\theta_1, \theta_2)} \right] - d_2^b \right\} \\ &\quad \times \frac{1}{\kappa} \left( s_0^* + \frac{d_0 s_0^* + f_0^*}{p_0(\theta_1, \theta_2)} \right). \end{aligned}$$

Note that the equity prices in period 1 and state  $b$  can be written as follows:

$$p_1^b(\theta_1, \theta_2) = \frac{\kappa \bar{p}_1^b(\theta_1, \theta_2) + \frac{a}{\kappa} \left[ d_1^b - (1-\kappa) \frac{p_0(\theta_1, \theta_2)}{q_0(\theta_1, \theta_2)} \right] \left( s_0^* + \frac{d_0 s_0^* + f_0^*}{p_0(\theta_1, \theta_2)} \right)}{\kappa - a \frac{1-\kappa}{\kappa} \left( s_0^* + \frac{d_0 s_0^* + f_0^*}{p_0(\theta_1, \theta_2)} \right)}.$$

## A.7 Definition of the Coefficients of (35) and (36)

The market-clearing condition for the goods market in period 1,  $\Omega_1(\theta_1, \theta_2) = 0$ , is written as (35), where the coefficients are as follows:

$$\begin{aligned}
A_1 &\equiv -\frac{\alpha^2 y_1^{b3}}{\psi(1-\psi)^2}, \\
B_1 &\equiv \left( \kappa m \left\{ \alpha [y_1^b + (1-\psi)(d_1^b + e_{1h}^b)] + \gamma(1-2\psi) \right\} - \alpha(1-\kappa)\psi(\kappa + ax)wx \right) \frac{\alpha y_1^{b2}}{\kappa(1-\psi)^2 m}, \\
\Gamma_1 &\equiv \left( -(1-\psi)m\alpha\kappa\gamma(\alpha y_1^b - \psi\gamma) + \beta\alpha\psi d_2^b \kappa \left\{ [(1-\kappa)x - (1-\psi)\kappa^2](\alpha y_2^b - \psi\gamma) + \kappa^2\gamma(1-\psi)^2 \right\} \right. \\
&\quad \left. - a\alpha \left[ \kappa(1-\psi)m(d_1^b + e_{1h}^b) - (1-\kappa)\psi(\kappa + ax)wx \right] [\alpha y_1^b + \gamma(1-2\psi)] \right) \frac{y_1^b}{\kappa\psi(1-\psi)^2 am}, \\
\Delta_1 &= -\frac{(1-\kappa)x\beta d_2^b \alpha^2 y_1^b y_2^b}{am(1-\psi)^2}, \\
E_1 &= \left[ \kappa^2(\alpha y_1^b - \gamma) + (1-\kappa)\gamma x \right] \frac{\beta d_2^b \alpha y_2^b}{am(1-\psi)}, \\
\text{and } Z_1 &= \left\{ \kappa\psi\beta d_2^b \left[ \kappa^2(1-\psi)\alpha(y_2^b - y_1^b) - (1-\kappa)x(\alpha y_2^b - \psi\gamma) \right] \right. \\
&\quad \left. + \left[ \kappa(1-\psi)am(d_1^b + e_{1h}^b) - a(1-\kappa)\psi(\kappa + ax)wx \right] (\alpha y_1^b - \psi\gamma) \right\} \frac{\gamma}{\kappa\psi(1-\psi)am}.
\end{aligned}$$

Note that  $w$ ,  $x$ , and  $m$  are parameters defined as follows:  $w \equiv d_1^b - (1-\kappa)\frac{p_0}{q_0}$ ,  $x \equiv s_0^* + \frac{d_0 s_0^* + f_0^*}{p_0}$ , and  $m \equiv \kappa^2 - a(1-\kappa)x$ .

The market-clearing condition for the goods market in period 2 and state  $b$ ,  $\Omega_2(\theta_1, \theta_2) = 0$ , is written as (36), where the coefficients  $A_2(\theta_1)$ ,  $B_2(\theta_1)$ ,  $\Gamma_2(\theta_1)$ , and  $\Delta_2(\theta_1)$  are defined as follows:

$$\begin{aligned}
A_2(\theta_1) &\equiv \frac{\beta^2 \alpha^3 y_1^b y_2^{b3} \kappa^2 d_2^b}{(1-\psi)^2 \psi m} \left( \frac{\theta_1}{1-\psi} - 1 \right) \left( \frac{\alpha y_1^b}{\psi} \theta_1 - \frac{\alpha y_1^b}{\psi} + \gamma \right), \\
B_2(\theta_1) &\equiv \frac{\beta \alpha y_2^b}{(1-\psi)^2 m} \left[ -wax \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right) \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right)^2 \right. \\
&\quad \left. - (1-\psi)\psi(1-\kappa)\kappa^2 d_2^{b2} x \beta \alpha \frac{1}{m} \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right)^2 \right. \\
&\quad \left. - \beta d_2^b \left( 2(1-\psi)\kappa^2\gamma - \alpha \left\{ (1-\psi)\kappa^2 \left[ n - \frac{\psi(1-\kappa)x d_2^b}{m} \right] - \frac{m d_2^b}{a} \right\} \right) \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right)^2 \right. \\
&\quad \left. + \beta \kappa^2 d_2^b \left\{ y_2^b [\alpha y_2^b + \gamma(1-2\psi)] - (1-\psi) \left[ n - \frac{2\psi(1-\kappa)x d_2^b}{m} \right] \right\} \right. \\
&\quad \left. \times \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right) \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right) \right],
\end{aligned}$$



$$\begin{aligned}
\Gamma_2(\theta_1) \equiv & \frac{\beta y_2^b}{(1-\psi)m} \left[ \beta \gamma d_2^b \left( \gamma(1-\psi)\kappa^2 - 2\alpha \left\{ (1-\psi)\kappa^2 \left[ n - \frac{\psi(1-\kappa)xd_2^b}{m} \right] \right\} \right) \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right)^2 \right. \\
& + \frac{2\alpha\beta\kappa^2\psi^2(1-\kappa)xd_2^b}{m} \left( \frac{\alpha y_2^b}{\psi} - \gamma \right) \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right)^2 \\
& - \beta\psi\kappa^2 d_2^b \left\{ \left( \frac{\alpha y_2^b}{\psi} - \gamma \right) \gamma - \frac{\alpha y_2^b + \gamma(1-2\psi)}{\psi} \alpha \left[ n - \frac{2\psi(1-\kappa)xd_2^b}{m} \right] \right. \\
& \times \left. \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right) \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right) \right. \\
& + wx \left( \gamma a - \alpha \left[ na - \psi(1-\kappa)d_2^b \left( 1 + \frac{2ax}{m} \right) - \frac{md_2^b}{\kappa^2} \right] \right) \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right) \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right)^2 \\
& \left. - \alpha\psi(1-\kappa)wx d_2^b \left( 1 + \frac{2ax}{m} \right) \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right)^2 \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right) \right],
\end{aligned}$$

$$\begin{aligned}
\text{and } \Delta_2(\theta_1) \equiv & \frac{1}{m} \left[ \beta^2 \gamma^2 d_2^b \left\{ (1-\psi)\kappa^2 \left[ n - \frac{\psi(1-\kappa)xd_2^b}{m} \right] - \frac{md_2^b}{a} \right\} \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right)^2 \right. \\
& - \frac{\beta^2 \gamma^2 \psi^3 (1-\kappa) x d_2^b}{(1-\psi)m} \left( \frac{\alpha y_2^b}{\psi} - \gamma \right)^2 \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right)^2 \\
& - \frac{\psi(1-\kappa)\omega^2 x^2 a}{(1-\psi)\kappa^2} \left( 1 + \frac{xa}{m} \right) \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right)^2 \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right)^2 \\
& + \beta\gamma wx \left\{ na - \psi(1-\kappa)d_2^b \left( 1 + \frac{2ax}{m} \right) - \frac{md_2^b}{\kappa^2} \right\} \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right) \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right)^2 \\
& + \frac{\beta\psi^2(1-\kappa)wx d_2^b}{1-\psi} \left( 1 + \frac{2ax}{m} \right) \left( \frac{\alpha y_2^b}{\psi} - \gamma \right) \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right)^2 \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right) \\
& \left. - \beta^2 \psi \kappa^2 d_2^b \gamma \left\{ n - \frac{2\psi(1-\kappa)xd_2^b}{m} \right\} \left( \frac{\alpha y_2^b}{\psi} - \gamma \right) \left( -\frac{\alpha y_2^b}{1-\psi} \theta_1 + \gamma \right) \left( \frac{\alpha y_2^b}{\psi} \theta_1 - \frac{\alpha y_2^b}{\psi} + \gamma \right) \right].
\end{aligned}$$

Note that  $n$  is defined as:

$$n \equiv \left( \frac{1-a}{a} + \frac{x}{\kappa} - \frac{x}{\kappa^2} \right) d_2^b - e_{2h}^b.$$

## A.8 Proof of Proposition 2

We propose sufficient conditions for the existence of the competitive equilibrium under quadratic utility. Equation (35) can be reformulated as follows:

$$\theta_2 = -\frac{A_1\theta_1^3 + B_1\theta_1^2 + \Gamma_1\theta_1 + Z_1}{\Delta_1\theta_1 + E_1}. \quad (45)$$

Because equation (45) is a rational function, it is continuous except at the point where the denominator equals 0, that is,  $\theta_1 = -\frac{E_1}{\Delta_1}$ . In addition, differentiating equation (35) with respect to  $\theta_1$  yields  $\frac{\partial\theta_2}{\partial\theta_1} = \frac{-2A_1\Delta_1\theta_1^3 - (2B_1\Delta_1 + 3A_1E_1)\theta_1^2 - 2B_1E_1\theta_1 + Z_1\Delta_1 - Z_1E_1}{(\Delta_1\theta_1 + E_1)^2}$ . Suppose that  $\frac{\partial\theta_2}{\partial\theta_1} > (<) 0$  holds within the range of  $1 - \psi \leq \theta_1 \leq 1$ , it ensures that the excess demand function in period 1 is a monotonically increasing (decreasing) function.

Because equation (35) is a cubic equation of  $\theta_1$  for a given  $\theta_2$ , we can acquire the three closed-form solutions of  $\theta_1$  for a given  $\theta_2$  by employing Cardano's formula.<sup>12</sup> In particular, (35) can be written as:

$$A_1\theta_1^3 + B_1\theta_1^2 + [\Gamma_1 + (1 - \psi)\Delta_1]\theta_1 + (1 - \psi)E_1 + Z_1 = 0, \quad (46)$$

if  $\theta_2 = 1 - \psi$ , and

$$A_1\theta_1^3 + (B_1 + \Delta_1)\theta_1^2 + [\Gamma_1 + E_1]\theta_1 + E_1 + Z_1 = 0, \quad (47)$$

if  $\theta_2 = \theta_1$ . We denote the three roots of equation (46) and (47) as  $\underline{\theta}_1^k$  and  $\bar{\theta}_1^k$  for  $k = 1, 2$ , and  $3$ , respectively. The closed forms of  $\underline{\theta}_1^k$  and  $\bar{\theta}_1^k$  are listed in the Appendix A.9. Suppose that a root takes a real value within the interval  $[1 - \psi, 1]$  and let  $\underline{\theta}_1$  and  $\bar{\theta}_1$  denote such a real root. Note that when  $\frac{\partial\theta_2}{\partial\theta_1} > (<)0$  holds and  $\underline{\theta}_1$  and  $\bar{\theta}_1$  exists, the excess demand function in period 1 is continuous and monotonically increasing (decreasing) in the region where  $\theta_1 \in [1 - \psi, 1]$  and  $\theta_2 \in [1 - \psi, \theta_1]$ , as plotted in Figure 9.

Because equation (36) is a cubic equation of  $\theta_2$  for a given  $\theta_1$ , we can acquire the three closed-form solutions of  $\theta_2$  as a function of  $\theta_1$  by employing Cardano's formula. Note that the equation has at least one real solution and that  $\theta_2^l(\theta_1)$  is an almost continuous function within the interval  $[\underline{\theta}_1, \bar{\theta}_1]$  because  $\theta_2^l(\theta_1)$  can be expressed as the polynomial of  $\theta_1$ . We denote the three solutions of equation (36) as  $\theta_2^l(\theta_1)$ , for  $l = 1, 2$ , and  $3$ . The closed forms of  $\theta_2^l(\theta_1)$  are listed in the Appendix A.9. Then, we compute  $\theta_2^l(\underline{\theta}_1)$  and  $\theta_2^l(\bar{\theta}_1)$  and denote them as  $\underline{\theta}_2^l$  and  $\bar{\theta}_2^l$ , respectively, if  $\theta_2^l(\underline{\theta}_1)$  and  $\theta_2^l(\bar{\theta}_1)$  take the real values.

Finally, if  $(\theta_2 - \underline{\theta}_1)(\bar{\theta}_2 - \bar{\theta}_1) > 0$  holds, the equilibrium consumption shares  $\theta_1^*$  and  $\theta_2^*$  exist. Figure 9 illustrates a case where  $\bar{\theta}_1 > \underline{\theta}_1$ ,  $\bar{\theta}_1 > \bar{\theta}_2$ , and  $\underline{\theta}_1 > \underline{\theta}_2$ . (Q.E.D.)

[Figure 9]

## A.9 Roots of (35) and (36)

Because equation (35) is a cubic equation of  $\theta_1$  for a given  $\theta_2$ , we can acquire the three closed-form solutions of  $\theta_1$  by employing Cardano's formula. Note that it has at least one real solution. In particular, when  $\theta_2 = 1 - \psi$ , equation (35) can be written as follows:

$$A_1\theta^3 + B_1\theta^2 + [\Gamma_1 + (1 - \psi)\Delta_1]\theta + (1 - \psi)E_1 + Z_1 = 0.$$

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<sup>12</sup>See Jacobson (2009).

Using Cardano's formula, the roots of the above cubic equations can be written as follows:

$$\theta_1^1 = \frac{\{(Q_1^2 + 4R_1^3)^{\frac{1}{2}} + Q_1\}^{\frac{1}{3}}}{3 \times 2^{\frac{1}{3}} A_1} - \frac{2^{\frac{1}{3}} R_1}{3A_1 \{(Q_1^2 + 4R_1^3)^{\frac{1}{2}} + Q_1\}^{\frac{1}{3}}} - \frac{B_1}{3A_1},$$

$$\theta_1^2 = \frac{-(1 - i\sqrt{3})\{(Q_1^2 + 4R_1^3)^{\frac{1}{2}} + Q_1\}^{\frac{1}{3}}}{6 \times 2^{\frac{1}{3}} A_1} + \frac{(1 + i\sqrt{3})2^{\frac{1}{3}} R_1}{3 \times 2^{\frac{2}{3}} A_1 \{(Q_1^2 + 4R_1^3)^{\frac{1}{2}} + Q_1\}^{\frac{1}{3}}} - \frac{B_1}{3A_1},$$

$$\text{and } \theta_1^3 = \frac{-(1 + i\sqrt{3})\{(Q_1^2 + 4R_1^3)^{\frac{1}{2}} + Q_1\}^{\frac{1}{3}}}{6 \times 2^{\frac{1}{3}} A_1} + \frac{(1 - i\sqrt{3})2^{\frac{1}{3}} R_1}{3 \times 2^{\frac{2}{3}} A_1 \{(Q_1^2 + 4R_1^3)^{\frac{1}{2}} + Q_1\}^{\frac{1}{3}}} - \frac{B_1}{3A_1},$$

where  $Q_1 \equiv -27A_1^2[(1 - \psi)E_1 + Z_1] + 9A_1B_1[\Gamma_1 + (1 - \psi)\Delta_1] - 2B_1^3$  and  $R_1 \equiv 3A_1[\Gamma_1 + (1 - \psi)\Delta_1] - B_1^2$ .

On the other hand, when  $\theta_2 = \theta_1$ , equation (35) can be written as follows:

$$A_1\theta^3 + (B_1 + \Delta_1)\theta^2 + [\Gamma_1 + E_1]\theta + E_1 + Z_1 = 0.$$

Using Cardano's formula, the roots of the above cubic equations can be written as follows:

$$\bar{\theta}_1^1 = \frac{\{(\bar{Q}_1^2 + 4\bar{R}_1^3)^{\frac{1}{2}} + \bar{Q}_1\}^{\frac{1}{3}}}{3 \times 2^{\frac{1}{3}} A_1} - \frac{2^{\frac{1}{3}} \bar{R}_1}{3A_1 \{(\bar{Q}_1^2 + 4\bar{R}_1^3)^{\frac{1}{2}} + \bar{Q}_1\}^{\frac{1}{3}}} - \frac{B_1 + \Delta_1}{3A_1},$$

$$\bar{\theta}_1^2 = \frac{-(1 - i\sqrt{3})\{(\bar{Q}_1^2 + 4\bar{R}_1^3)^{\frac{1}{2}} + \bar{Q}_1\}^{\frac{1}{3}}}{6 \times 2^{\frac{1}{3}} A_1} + \frac{(1 + i\sqrt{3})2^{\frac{1}{3}} \bar{R}_1}{3 \times 2^{\frac{2}{3}} A_1 \{(\bar{Q}_1^2 + 4\bar{R}_1^3)^{\frac{1}{2}} + \bar{Q}_1\}^{\frac{1}{3}}} - \frac{B_1 + \Delta_1}{3A_1},$$

$$\bar{\theta}_1^3 = \frac{-(1 + i\sqrt{3})\{(\bar{Q}_1^2 + 4\bar{R}_1^3)^{\frac{1}{2}} + \bar{Q}_1\}^{\frac{1}{3}}}{6 \times 2^{\frac{1}{3}} A_1} + \frac{(1 - i\sqrt{3})2^{\frac{1}{3}} \bar{R}_1}{3 \times 2^{\frac{2}{3}} A_1 \{(\bar{Q}_1^2 + 4\bar{R}_1^3)^{\frac{1}{2}} + \bar{Q}_1\}^{\frac{1}{3}}} - \frac{B_1 + \Delta_1}{3A_1},$$

where  $\bar{Q}_1 \equiv -27A_1^2[E_1 + Z_1] + 9A_1(B_1 + \Delta_1)[\Gamma_1 + E_1] - 2(B_1 + \Delta_1)^3$  and  $\bar{R}_1 \equiv 3A_1[\Gamma_1 + E_1] - (B_1 + \Delta_1)^2$ .

Because equation (36) is also a cubic equation of  $\theta_1$  for a given  $\theta_2$ , we can acquire the three closed-form solutions of  $\theta_2$  as a function of  $\theta_1$  by employing Cardano's formula. Note that the equation has at least one real solution. The  $\theta_2$  such that meet equation (36) for a given  $\theta_1$  can

be written as follows:

$$\theta_2^1(\theta_1) = \frac{\{(Q_2(\theta_1)^2 + 4R_2(\theta_1)^3)^{\frac{1}{2}} + Q_2(\theta_1)\}^{\frac{1}{3}}}{3 \times 2^{\frac{1}{3}} A_2(\theta_1)} - \frac{2^{\frac{1}{3}} R_1(\theta_1)}{3A_2(\theta_1)\{(Q_2(\theta_1)^2 + 4R_2(\theta_1)^3)^{\frac{1}{2}} + Q_2(\theta_1)\}^{\frac{1}{3}}} - \frac{B_2(\theta_1)}{3A_2(\theta_1)},$$

$$\theta_2^2(\theta_1) = \frac{-(1 - i\sqrt{3})\{(Q_2(\theta_1)^2 + 4R_2(\theta_1)^3)^{\frac{1}{2}} + Q_2(\theta_1)\}^{\frac{1}{3}}}{6 \times 2^{\frac{1}{3}} A_2(\theta_1)} + \frac{(1 + i\sqrt{3})2^{\frac{1}{3}} R_1(\theta_1)}{3 \times 2^{\frac{2}{3}} A_2(\theta_1)\{(Q_2(\theta_1)^2 + 4R_2(\theta_1)^3)^{\frac{1}{2}} + Q_2(\theta_1)\}^{\frac{1}{3}}} - \frac{B_2(\theta_1)}{3A_2(\theta_1)},$$

$$\text{and } \theta_2^3(\theta_1) = \frac{-(1 + i\sqrt{3})\{(Q_2(\theta_1)^2 + 4R_2(\theta_1)^3)^{\frac{1}{2}} + Q_2(\theta_1)\}^{\frac{1}{3}}}{6 \times 2^{\frac{1}{3}} A_2(\theta_1)} + \frac{(1 - i\sqrt{3})2^{\frac{1}{3}} R_1(\theta_1)}{3 \times 2^{\frac{2}{3}} A_2(\theta_1)\{(Q_2(\theta_1)^2 + 4R_2(\theta_1)^3)^{\frac{1}{2}} + Q_2(\theta_1)\}^{\frac{1}{3}}} - \frac{B_2(\theta_1)}{3A_2(\theta_1)},$$

where  $Q_2(\theta_1)$  and  $R_2(\theta_1)$  are given as follows:  $Q_2(\theta_1) \equiv -27A_2(\theta_1)^2\Delta_2(\theta_1) + 9A_2(\theta_1)B_2(\theta_1)\Gamma_2(\theta_1) - 2B_2(\theta_1)^3$  and  $R_2(\theta_1) \equiv 3A_2(\theta_1)\Gamma_2(\theta_1) - B_1(\theta_1)^2$ . Because  $A_2(\theta_1)$ ,  $B_2(\theta_1)$ ,  $\Gamma_2(\theta_1)$ , and  $\Delta_2(\theta_1)$  are polynomial with respect to  $\theta_1$ ,  $Q_2(\theta_1)$  and  $R_2(\theta_1)$  are polynomial and differentiable with respect to  $\theta_1$ . Then,  $\theta_2$  expressed above is continuous and differentiable except at the point where  $\theta_1$  meets the following conditions:  $A_2(\theta_1) = 0$  and  $(Q_2(\theta_1)^2 + 4R_2(\theta_1)^3)^{\frac{1}{2}} + Q_2(\theta_1) = 0$ .

## References

- [1] Adrian T., and H. Shin, (2010a) "The Changing Nature of Financial Intermediation and the Financial Crisis of 2007-2009," *Annual Review of Economics*, Vol. 2, pp. 603-618.
- [2] Adrian T., and H. Shin, (2010b) "Liquidity and Leverage," *Journal of Financial Intermediation*, Vol. 19, pp. 418-437.
- [3] Aiyagari, S. R. and M. Gertler, (1999) "'Overreaction' of Asset Prices in General Equilibrium," *Review of Economic Dynamics* Vol. 121, pp. 823-866.
- [4] Alvarez, F., and U. J. Jermann (2000) "Efficiency, Equilibrium and Asset Pricing with Risk of Default" *Econometrica*, Vol. 68, pp. 775-797.

- [5] Araújo, A., F. Kubler, and S. Schommer (2012) “Regulating Collateral-requirements When Markets Are Incomplete,” *Journal of Economic Theory* Vol. 147, pp. 450-476.
- [6] Barro, R. J. (2006) “Rare Disasters and Asset Markets in the Twentieth Century,” *Quarterly Journal of Economics* Vol. 121, pp. 823-866.
- [7] Bewly, T. (1983) “A Difficulty with the Optimum Quantity of Money” *Econometrica* Vol. 51, pp. 1485-1504.
- BM Bordo, M. D., and C. M. Meissner (2012) “Does Inequality Lead to a Financial Crisis?” *Journal of International Money and Finance*, Vol. 31, pp. 2147-2161.
- [8] Brumm, J., M. Grill, F. Kubler, and K. Schmedders (2015) “Margin Regulation and Volatility,” *Journal of Monetary Economics*, Vol. 75, pp. 54-68.
- [9] Brunnermeier, M. (2009) “Deciphering the Liquidity and Credit Crunch 2007-2008,” *Journal of Economic Perspectives*, Vol. 23, pp. 77-100.
- [10] Brunnermeier, M., and L. H. Pedersen, (2009), “Market Liquidity and Funding Liquidity,” *Review of Financial Studies*, Vol. 22, pp. 2201-2238.
- [11] Constantinides, G. M., and A. Ghosh, (2017) “Asset Pricing with Countercyclical Household Consumption Risk” *Journal of Finance*, Vol. 72, pp. 415-459.
- [12] Coval, J. and E. Stafford, (2007) “Asset Fire Sales (and Purchases) in Equity Markets,” *Journal of Financial Economics*, Vol. 86, pp. 479-512.
- [13] Dow, J., and J. Han, (2018) “The Paradox of Financial Fire Sales: The Role of Arbitrage Capital in Determining Liquidity,” *The Journal of Finance*, Vo. 73, pp. 229-274.
- [14] Fostel, A., and J. Geanakoplos (2015) “Leverage and Default in Binomial Economies: A Complete Characterization,” *Econometrica*, Vol. 83, pp. 2193-2229.
- [15] Geanakoplos, J., and W. R. Zame (2014) “Collateral Equilibrium, I: A Basic Framework,” *Economic Theory*, Vol. 56, pp. 443-492.
- [16] Gertler, K., and N. Kiyotaki, (2015) “Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy,” *American Economic Review*, Vol. 105, pp.2011-2043.
- [17] Gottardi, P., and F. Kubler (2015) “Dynamic Competitive Economies with Complete Markets and Collateral Constraints,” *Review of Economic Studies*, Vol. 82, 1119-1153.

- [18] Gourio, F., (2008) “Disasters and Recoveries,” *American Economic Review*, Vol. 98(2), pp. 68-73.
- [19] Gromb, D., and D. Vayanos, (2002) “Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs,” *Journal of Financial Economics*, Vol. 66, pp.361-407.
- BM Gu, X., and B.Huang, (2014) “Does Inequality Lead to a Financial Crisis? Revisited” *Review of Development Economics*, Vol. 18, pp.502-516.
- [20] Guvenen, F., S. Ozkan, and J. Song, (2014) “The Nature of Countercyclical Income Risk,” *Journal of Political Economy*, Vol. 122, pp. 621-660.
- [21] He, Z., and A. Krishnamurthy (2012) “A Model of Capital Crises,” *Review of Economic Studies*, Vol. 79, pp. 735-777.
- [22] He, Z., and A. Krishnamurthy (2013) “Intermediary Asset Pricing,” *American Economic Review*, Vol. 103, pp. 732-770.
- [23] Heaton, J. and D. Lucas, (1996) “Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing,” *Journal of Political Economy*, Vol. 104, pp. 443-87.
- [24] Huggett, M. (1993) “The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies,” *Journal of Economic Dynamics and Control* Vol. 17, pp. 953-969.
- [25] Jacobson, N., (2009) *Basic Algebra I: Second Edition*, Dover.
- [26] Jorion, P., and W. N. Goetzmann (1999) “Global Stock Markets in the Twentieth Century,” *Journal of Finance*, Vol. 54, pp. 953-980.
- [27] Julliard C., and A. Ghosh (2012) “Can Rare Events Explain the Equity Premium Puzzle?” *Review of Financial Studies* Vo. 25, pp.3037-3076.
- [28] Kehoe, T., and D. Levine (1993) “Debt Constrained Asset Markets,” *Review of Economic Studies*, Vol. 60, pp.856-888.
- [29] Kehoe, T., and Levine (2001) “Liquidity Constrained vs. Debt Constrained Markets,” *Econometrica*, 69, 575-598.
- [30] Kruger, D., and F. Perri (2006) “Does Income Inequality Lead to Consumption Inequality? Evidence and Theory,” *Review of Economic Studies*, Vol. 73, 163-193.
- [31] Kumhof, M., R. Rancière, and P. Winant (2015) “Inequality, Leverage, and Crises,” *American Economic Review*, Vol. 105, pp.1217-1245.

- [32] Lorenzoni, G. (2008) “Inefficient Credit Booms,” *Review of Economic Studies*, Vol. 78, pp. 809-833.
- [33] Mankiw, N. G. (1986) “The Equity Premium and the Concentration of Aggregate Shocks,” *Journal of Financial Economics*, Vol. 17, pp. 211-219.
- [34] Nakamura, E., J. Steinsson, R. J. Barro, and J. Ursúa (2013) “Crises and Recoveries in an Empirical Model of Consumption Disasters” *American Economic Journal: Macroeconomics*, Vol. 5, pp.35-74.
- [35] Oosterlinck K., (2010) “French Stock Exchanges and Regulation during World War II,” *Financial History Review*, Vol. 17, pp. 211-237.
- [36] Perugini, C., J. Hölscher, and S. Collie (2016) “Inequality, credit and financial crises” *Cambridge Journal of Economics*, Vol. 40, pp.227-257.
- [37] Philippon, T. (2015) “Has the US Finance Industry Become Less Efficient? On the Theory and Measurement of Financial Intermediation,” *American Economic Review*, Vol. 105, pp.1408-1438.
- [38] Rajan. R., (2010) *Fault Lines*. Princeton University Press, Princeton, NJ.
- [39] Rietz, T. A., (1988) “The Equity Risk Premium A Solution,” *Journal of Monetary Economics* Vol.22, pp. 117-131.
- [40] Saito, M., and S. Suzuki, (2014) “Persistent Catastrophic Shocks and Equity Premiums: A Note,” *Macroeconomic Dynamics*, Vol. 18, pp. 1161-1171.
- [41] Shleifer, A., and R. Vishny., (1997) “The Limits of Arbitrage,” *Journal of Finance*, Vol. 52, pp. 35-55.
- [42] Shleifer, A., and R. Vishny., (2011) “Fire Sales in Finance and Macroeconomics,” *Journal of Economic Perspectives*, Vol. 25, pp.29-48.
- [43] Storesletten, K., C. I. Telmer, and A. Yaron., (2004) “Cyclical Dynamics in Idiosyncratic Labor Market Risk,” *Journal of Political Economy*, Vol. 112, pp. 695-717.
- [44] Suzuki, S. (2012) “Stock-market Booms in Damaged Countries during the World War II,” *Rsearch in Economics* Vol 66, pp. 175-183.

- [45] Yamarik, S., M. El-Shagi, and G. Yamashiro, (2016) “Does Inequality Lead to Credit Growth? Testing the Rajan Hypothesis Using State-level Data” *Economics Letters*, Vol. 148, pp.63-67.



Table 1: The basic calibration parameters

time discount factor	$\beta$	0.94	$e_0$	0.70	$d_0$	0.30
			$e_1^n$	0.70	$d_1^n$	0.30
equity adjustment cost	$a$	1.0	$e_2^n$	0.70	$d_2^n$	0.30
aggregate prob.	$\phi$	0.1	$e_{1h}^b$	0.70	$d_1^b$	0.03
idiosyncratic prob.	$\psi$	0.5	$e_{2h}^b$	0.70	$d_2^b$	0.30
leverage ratio	$\kappa$	$0.6 \sim 0.9$	$e_{1l}^b$	0.60	$s_0^*$	0.01
			$e_{2l}^b$	0.70	$f_0^*$	0.00

Utility functions are specified as quadratic,  $u(c) = -\frac{c^2}{2} + 1.75c$ , or constant relative risk aversion,  $u(c) = \frac{c^{1-1.1}}{1-1.1}$ . Aggregate endowments,  $y_0, y_1^n, y_2^n, y_2^b$  are set to 1 but  $y_1^b$  is set to 0.68. In the state  $b$  and the period 1, the dividend from Lucas trees decrease by 90% and endowments of the low-endowment household decreases by 14%.

Figure 1: State

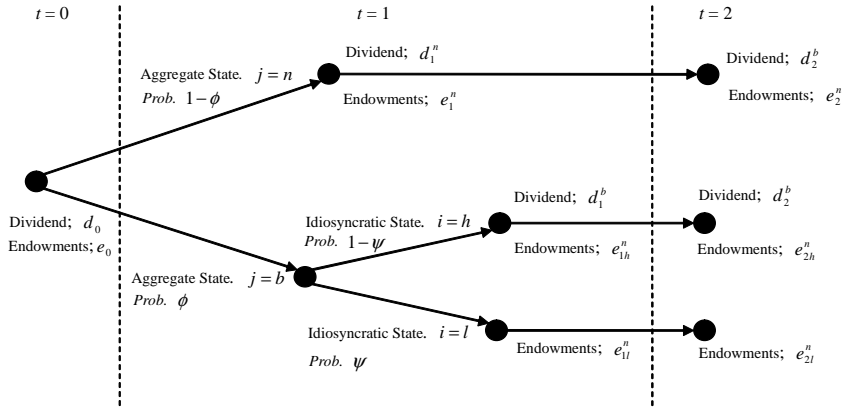


Figure 2: Feasible portfolio and leverage binding portfolio in period 0

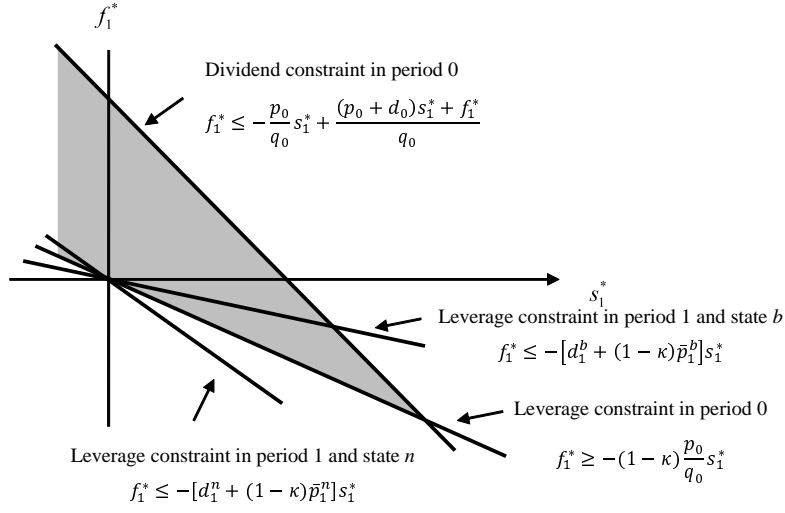


Figure 3: Feasible portfolio region in period 1 and state  $j$

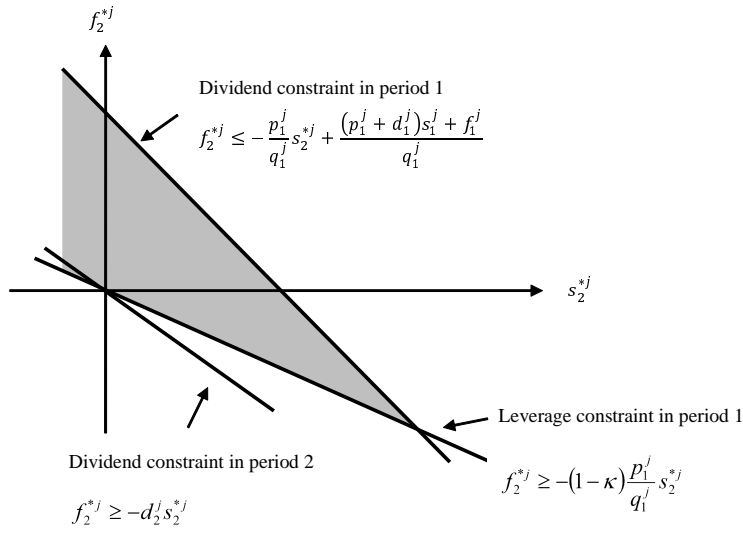


Figure 4: Asset prices in the quadratic utility

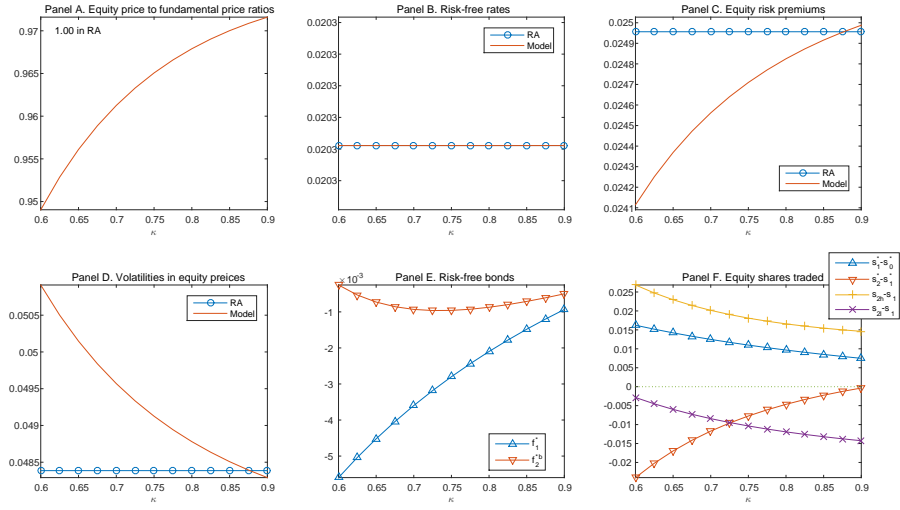


Figure 5: Asset prices in the CRRA utility

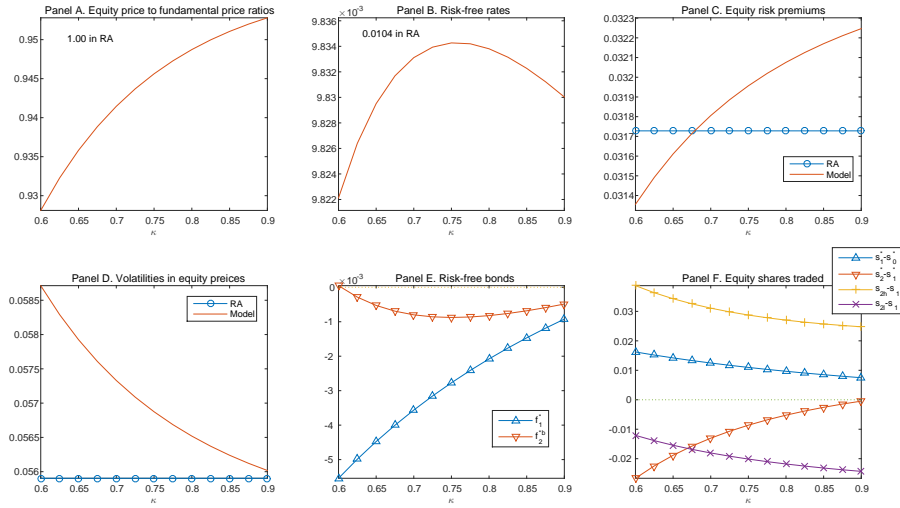


Figure 6: Consumption shares and the welfare in the quadratic utility

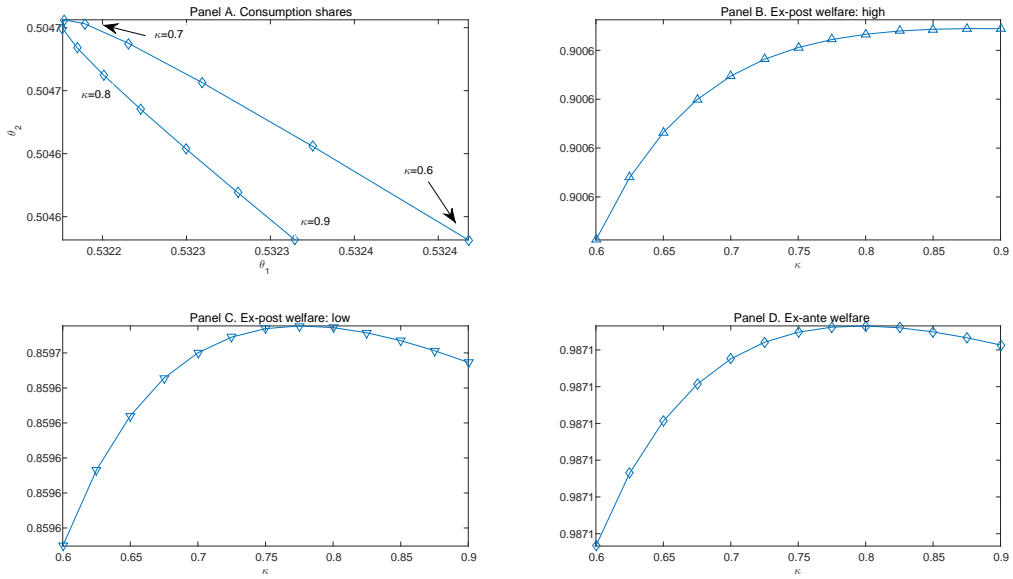


Figure 7: Consumption shares and the welfare in the CRRA utility

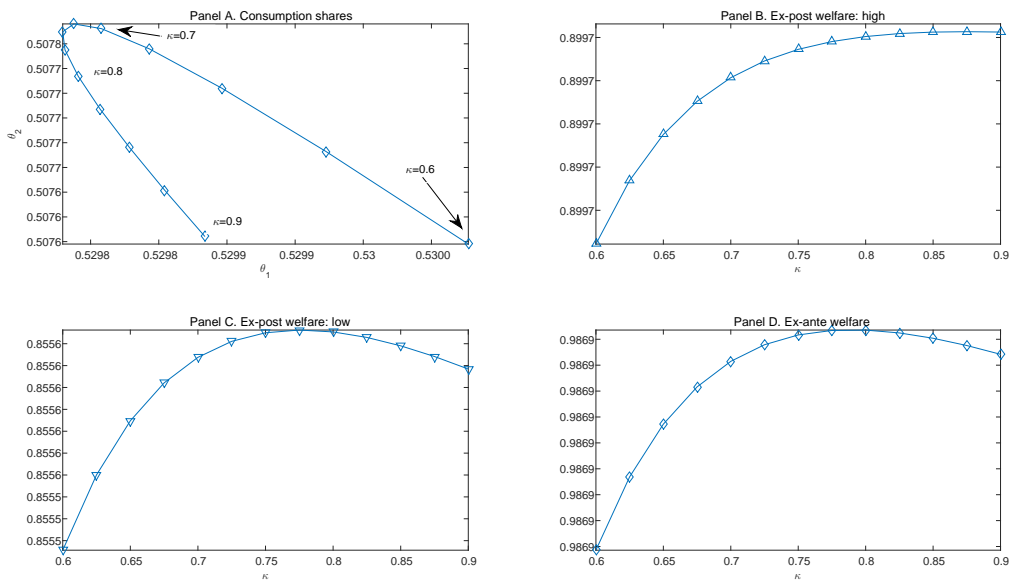


Figure 8: Determination of equilibrium equity prices in period 1 and state  $j$

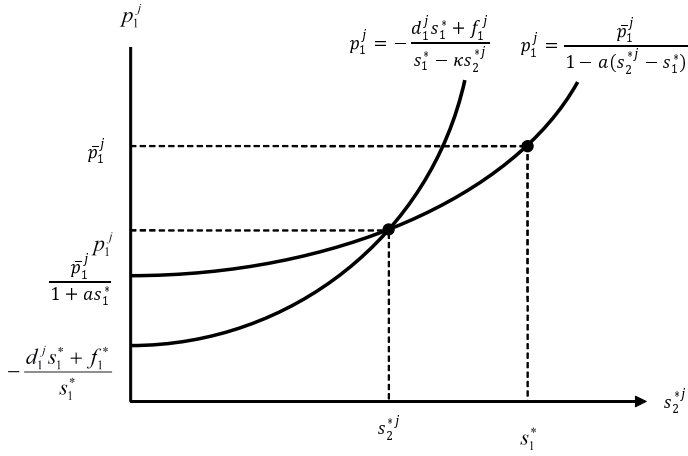


Figure 9: Existence of the competitive equilibrium

