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Abstract

This paper revisits the "race to the bottom" phenomenon in a simple game theoretic framework. We consider two countries and one multinational firm, which requires two inputs that are imperfect substitutes. In the benchmark model the labour of each country specializes in a distinct input. Seeking to maximize their labour incomes, countries simultaneously announce wages following which the firm chooses its labour employment in each country. We show that "race to the bottom" (countries setting minimum possible wages) is never an equilibrium. Moreover there are equilibria with "race to the top", that is, countries set maximum possible wages. This result is robust in an extended model where prior to competing in wages, each country can make input-specific investments to make its labour available for one or both inputs. Provided the production function of the firm is not asymmetrically intensive in either one of the two inputs, there are equilibria of the extended game with specialization (that is, countries invest in distinct inputs) as well as "race to the top".

Keywords: race to the bottom; race to the top; labour policy; multinational; constant elasticity of substitution

JEL Classification: F23, J42, O12

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1 Introduction

The phenomenon of competitive dilution of labour standards—"race to the bottom"—by the governments of developing countries for attracting multinational corporate economic activities to one's country, especially for direct investment, is a crucial current issue in the political economy of such countries. We are tempted to quote from an editorial piece in the Economic and Political Weekly (June, 2014) providing a succinct but illuminating description of the phenomenon:

> "The strategy is basically one of global labour arbitrage, which is a means whereby transnational business earns a higher rate of profit by shifting production to businesses in countries of the global South to take advantage of the significant wage difference...In the process, the countries of the South are driven to engage in a "race to the bottom" in order to emerge globally competitive as far as unit labour cost is concerned...Basically, business's need for labour market flexibility is being rationalised in terms of the supposed interests of workers as a whole—such flexibility is claimed to be in the latter's interest, for them to accept casual labour, variable wages and working hours linked to product market demand...The workers will now be told to conduct themselves in a manner that maintains and enhances flexibility "in their own interests"."

In this paper we revisit the phenomenon of "race to the bottom" in a simple game theoretic framework. We explore whether drastic strategic undercutting of labour's bargaining power, as reflected in the expected wage labourers can get, is an *inevitable* outcome of strategic competition between policy-makers of different countries. Toward this end, first we model the strategic behaviour of two countries as a two-player game with simultaneous moves where the action of each country is to choose a wage for its labourers (or a labour policy that might generate such a price as the expected wage). The payoffs to the players are determined by the production decision of a multinational monopsonist availing the labour inputs provided by these countries which may be substitutes but not *perfect* substitutes. Using the properties of supermodular games (see Milgrom and Roberts, 1990), we show that this game has a unique Nash equilibrium in pure strategies. Exploring the properties of this equilibrium, we find that "race to the bottom" (countries setting minimum possible wages) never emerges as an equilibrium outcome while the complete opposite—"race to the top" (countries setting maximum possible wages)—is possible for a range of relevant parameters of our model.

We extend this benchmark model to incorporate the feature that at the very first stage, both countries have to decide simultaneously whether to make an input-specific fixed investment with the assumption that if such an investment is undertaken for a specific input, only then the country gets that input available for selling to the monopsonist. We show that provided the production function of the firm is not asymmetrically intensive in either one of the two inputs, in this augmented multi-stage game there are subgame perfect equilibria at which there is specialization as well as "race to the top", that is, each country specializes in getting an input different from the other and then sets the maximum possible wage for that labour-input.

While anecdotal descriptions of "race to the bottom" in labour standards or wages are quite common (see, again, EPW, 2014), concrete identification of the phenomenon often proved elusive (see Singh and Zammit, 2004; Potrafke, 2013). Recently Davies and Vadlamannati (2013) and Olney (2013) have provided empirical evidence in favour of such phenomena.

There has been consideration of whether the phenomenon of "race to the bottom" is inevitable and some channels through which this can be endogenously counteracted have been identified. Of course, generating externality of increased demand through increased wages is one well-known channel. This prevalent theme of development economics has been studied again by Cueto (2017) in the context of labour standards. In the context of tax competition, Baldwin and Krugman (2004) analyze how agglomeration effect can counteract "race to the bottom". Dorsch et al. (2014) see how pressure of getting re-elected in a democracy may induce a government to adopt ways to woo foreign investments other than lowering labour standards. Our research is along this line: that of identifying factors which can endogenously counteract this phenomenon.

Our work is motivated by the following simple observations. Race to the bottom in labour markets is akin to Bertrand competition: competitive undercutting of prices to increase demand for one's product. However, the possibility of relaxing such competition in presence of product differentiation is well-known (from onwards at least Shaked and Sutton, 1982). A multinational firm (MNC) often organizes production in more than one country for intermediate products to create its final product which is sold all over the world. This implies that for such an MNC, productive inputs obtainable in different countries may not be *perfect* substitutes although near-perfect substitution is still possible. As an example one might think of a car manufacturer obtaining mineral ore or processed metal from one country and having the assembly line in another country; in the first country it may obtain labour experienced in mining activities and in the second the labour skilled in works related to a modern automobile industry. We explore the implication of such production processes involving imperfectly substitutable labour inputs.

To model the imperfect substitutability of the inputs in the MNC's production process, we adopt one usual approach of taking its production function to be of the CES type. Then we analyze the policy-setting game played by the two countries (as outlined above and described in detail in the next section). We look at the properties of pure strategy Nash equilibria as parameters in this benchmark model—especially the elasticity of substitution—change. However, our focus is on ascertaining whether "race to the bottom" is feasible (and we find the answer to be negative) and on identifying situations for which the completely opposite phenomenon, "race to the top", rather, is possible. Next we study the extended model focussing on the possibility of "race to the top" when countries first decide about making specific labour-inputs being available in the country.

There have been discussions on the channels through which foreign direct investment may *better* rights of the labourers in LDCs (see Mosley and Uno, 2007, especially pp.925-926 for a review they made on this issue). However, the channels identified were rather non-central and exogenous to the central choice problems of the firms and the governments. In contrast, the channel through which "race to the top" emerges in our model is through the core strategic choices in presence of imperfect substitutability of the factors of production. Moreover, while analyses of MNC's decisions of input choices are profuse (see the survey by Antras and Yeaple, 2014; also see Sly and Soderbery, 2014, whose work is close to the theme of this paper), our finding of the possibility of "race to the top" even when inputs are substitutes seems novel. We would like to mention here that in a recent paper Fukumura and Yamagishi (2018) have explored the possibility of "race to the top" in a model of minimum wage competition. However, our set-up is quite different, in fact somewhat opposite: in their model labour is partially mobile whereas in our model, as is natural in our context, labour is confined to its own country or geographical location.

The main ingredients of our setup and the benchmark model is described and analyzed in the following section. Section 3 gives the analysis of the extended model. Section 4 provides some concluding remarks. Most proofs are collected in the Appendix.

2 The benchmark model

There are two countries a, b. There is a multinational firm that requires two different labour inputs $1, 2$ to carry out production of a final commodity which we take as the numeraire. The two inputs are imperfect substitutes. Let x_1, x_2 be the amount of inputs $1, 2$. The firm has a constant elasticity of substitution (CES) production function (Arrow et al., 1961) given by

$$F(x_1, x_2) = \left[\alpha x_1^{-\rho} + (1-\alpha) x_2^{-\rho}\right]^{-1/\rho} \tag{1}$$

where $0 < \alpha < 1$ and $\rho \in (-1, 0) \cup (0, \infty)$, i.e., $\rho > -1$ and $\rho \neq 0$. As we assume that the product is the numeraire which is sold in the rest of the world, the function F is called the firm's profit function.[1]

Let w_1, w_2 represent the wages for inputs $1, 2$ which the firm has to pay. For any input i, the firm can purchase any quantity of that input from either of the countries, at the wages available in that country if the input is indeed available in the country. However, for both the inputs, the firm has an outside option for getting the input. For any input, the outside option has a unit wage $\overline{w}$.

Assume that the firm has a fixed amount of capital $K > 0$ that it uses to pay for the inputs. The budget constraint of the firm is given by

$$w_1 x_1 + w_2 x_2 = K \tag{2}$$

For any $w_1, w_2 > 0$, the firm's constrained profit maximization problem (to maximize (1) subject to (2)) has a unique solution $\left(x_1^*(w_1, w_2), x_2^*(w_1, w_2)\right)$. Let $\pi(w_1, w_2, K) = F(x_1^*, x_2^*)$ be the maximized value of the profit of the firm.

The total labour population in country $j \in \{a, b\}$ is denoted by $\overline{x}^j$. It is assumed that $\overline{x}^a, \overline{x}^b$ are sufficiently large so that for the firm's problem, the labour constraint is never binding. If the labour of a country is available as an input for the firm, then the government or the decision-maker of that country sets a fixed wage for that input. At that wage, the firm may employ any labour for that input. Any labour that is not employed by the firm gets the reservation wage prevailing in that country.

Notice that it is not necessary that the government in each country has to *actually* administer a fixed wage. Think of a more realistic scenario that the labourers in each country and the management of the firm get into a bilateral conflict over the wages to be paid and where the probability of a party's winning depends on the labour policy adopted by the government. Then the wage set up by the government can be thought of as the expected price to be paid to the labourers of that country in the face of this possible conflict. However, in what follows, we shall adopt the simple convention as if the government in each country administers its wage for each labour input available in the country and offers the wages to the firm.

2.1 The benchmark model: further ingredients

In the benchmark model we consider the case where each country specializes in *one* (and only one) of the inputs. Specifically we assume country a specializes in input 1 and country b in input 2. Therefore, for the remainder of this section, we simply denote $x_a = x_1$ and $x_b = x_2$. Likewise, the wages for the inputs are denoted simply by w_a, w_b.

The strategic interaction is modeled as a simultaneous-move game G where two countries simultaneously set wages (or, as we remarked above, equivalently, set policies resulting in effective wages) w_a, w_b. Note that if w_i exceeds $\overline{w}$ (wage at firm's outside option), the firm will not employ any labour from country i. So there is no loss of generality in restricting $w_a, w_b \leq \overline{w}$. Given any such wage pair (w_a, w_b), the firm employs $x_i^*(w_a, w_b)$ units of labour from[2] country i. Let $\psi_i^M(w_a, w_b, K) = w_i x_i^*$ be the labour income accruing from the firm in country i. Any labour unit that is not employed by the firm earns the reservation wage $\underline{w}^i$ of country i (for example, by working in a traditional sector). So the income for the labour that is not employed by the firm is $\psi_i^T(w_a, w_b, K) = \underline{w}^i(\overline{x}^i - x_i^*)$. Consequently the total labour income in country i is

$$\psi_i(w_a, w_b, K) = \psi_i^M(w_a, w_b, K) + \psi_i^T(w_a, w_b, K) = w_i x_i^* - \underline{w}^i x_i^* + \underline{w}^i \overline{x}^i \tag{3}$$

For any (w_a, w_b), the payoff of the decision-maker in each country is its total labour income given by (3). We assume $0 < \underline{w}^i < \overline{w}$ for $i \in \{a, b\}$. So there is no loss of generality in taking $w_i \in [\underline{w}^i, \overline{w}]$. We look for *Nash Equilibria in pure strategies* (abbreviated simply as NE) for the game G.

Remark 1: Generally, the payoff of the decision-maker in each country can be a weighted sum of the firm's profit and its total labour income as follows

$$\begin{aligned} \Pi_a(w_a, w_b) &= \lambda_a \pi(w_a, w_b, K) + (1 - \lambda_a)\psi_a(w_a, w_b, K) \\ \Pi_b(w_a, w_b) &= \lambda_b \pi(w_a, w_b, K) + (1 - \lambda_b)\psi_b(w_a, w_b, K) \end{aligned} \tag{4}$$

where $\lambda_a, \lambda_b \in [0, 1]$ and $\lambda_a + \lambda_b < 1$. The interpretation of this general payoff function would be that a country's decision-maker, a priori, may have two kinds of incentives. It can get a share of the firm's profit which may be thought of a pecuniary gain of it or bribe paid to it by the firm. However, the decision-maker may also have some incentive for increasing the labourers' income (perhaps so that it does not get too unpopular). For our analysis we take $\lambda_a = \lambda_b = 0$. Later, at the end of this section we make a remark on the implication of having $\lambda_i \neq 0$ (see Remark 4).

Remark 2: We outline a few notable features of this set-up. First, we focus on the labour policy and extract away from the other general equilibrium features of international trade: e.g., in our model the firm presumably sells its output in a third country.

Next, we endow the firm with maximum market power. Also, we allow substitutability of inputs for the firm apart from the single-point of *perfect* substitutability. And finally, with $\lambda_i = 0$, our model is equivalent to a variant of Bertrand duopoly with differentiated products.

Lemma 1 *The following hold for $i, j \in \{a, b\}$ and $i \neq j$.*

(i) *x_i^* is decreasing in w_i.*

(ii) *x_i^* is increasing in w_j if $\rho \in (-1, 0)$ and decreasing in w_j if $\rho \in (0, \infty)$.*

(iii) *π is decreasing in w_i.*

(iv) *ψ_i^M is decreasing in w_i if $\rho \in (-1, 0)$ and increasing in w_i if $\rho \in (0, \infty)$.*

Proof See the Appendix. ∎

We explore the properties of NE of G. In particular, we are interested in whether the equilibria show strategic undercutting or otherwise. Therefore, we introduce the following definitions.

Definitions A NE of G has

(i) *race to the bottom* property if $w_i = \underline{w}^i$ for some $i \in \{a, b\}$;

(ii) *complete race to the bottom* property if $w_i = \underline{w}^i$ for both $i \in \{a, b\}$;

(iii) *race to the top* property if $w_i = \overline{w}$ for some $i \in \{a, b\}$;

(iv) *complete race to the top* property if $w_i = \overline{w}$ for both $i \in \{a, b\}$.

2.2 The equilibria in the benchmark model

Proposition 1 below characterizes best responses of the players in G and shows that G has a unique NE. It also identifies some initial properties of the NE.

Proposition 1

(I) *The best responses of countries in the game G have the following properties.*

(i) *If $\rho \in (-1, 0)$, then for any $w_j \in [\underline{w}^j, \overline{w}]$, country i has a unique best response $B_i(w_j)$. The best response function B_i is non-decreasing in w_j and $B_i(w_j) > \underline{w}^i$ for any $w_j \in [\underline{w}^j, \overline{w}]$.*

(ii) *If* $\rho \in (0, \infty)$*, then for any* $w_j \in [\underline{w}, \overline{w}]$*, country* i *has a unique best response* $\overline{w}$*.*

(II) *The game* G *has a unique NE. The NE has the following properties.*

(i) *If* $\rho \in (-1, 0)$*, then at the NE,* $w_i > \underline{w}^i$ *for* $i \in \{a, b\}$*, i.e., the NE does not have the race to the bottom property.*

(ii) *If* $\rho \in (0, \infty)$*, then the NE has the complete race to the top property.*

(III) *The NE value of* w_i *is increasing in* $\underline{w}_i$ *for* $i \in \{a, b\}$*.*

Proof See the Appendix. ■

Since the case where $\rho \in (0, \infty)$ is immediately clear, next we get on to the case where $\rho \in (-1, 0)$ and identify the property of the equilibrium as the parameters affecting demands vary.

Proposition 2 *Consider the game* G*. Let* $\rho \in (-1, 0)$ *and* $\delta \equiv -\rho \in (0, 1)$*. Let* $\tau_a \equiv \alpha$*,* $\tau_b \equiv 1 - \alpha$ *and for* $i, j \in \{a, b\}$*,* $i \neq j$*, define*

$$\widetilde{\delta}_i \equiv \delta / [1 + (1 - \delta)(\tau_i / \tau_j)^{1/(1-\delta)}] \in (0, \delta) \tag{5}$$

(i) (**not race to the top**) *If* $\underline{w}^i < \widetilde{\delta}_i \overline{w}$ *for* $i \in \{a, b\}$*, then at the NE,* $w_i < \overline{w}$ *for both* i*.*

(ii) (**partial race to the top**) *If* $\underline{w}^i < \widetilde{\delta}_i \overline{w}$ *and* $\underline{w}^j \geq \delta \overline{w}$ *for* $i, j \in \{a, b\}$*,* $i \neq j$*, then at the NE,* $w_i < \overline{w}$ *and* $w_j = \overline{w}$*.*

(iii) (**complete race to the top**) *If* $\underline{w}^i \geq \widetilde{\delta}_i \overline{w}$ *for both* $i \in \{a, b\}$*, then the NE has complete race to the top property.*

(iv) *If* $\widetilde{\delta}_i \overline{w} \leq \underline{w}^i < \delta \overline{w}$ *and* $\underline{w}^j < \widetilde{\delta}_j \overline{w}$ *then the NE has either has "not race to the top" or shows partial race to the top.*

Proof See the Appendix. ■

Notice that the most interesting aspect of our results so far is the possibility of having an equilibrium with "race to the top"—opposite to "race to the bottom"—even when $\rho \in (-1, 0)$, that is, even when the labour inputs can be said to be substitutes in production. In the case where the two countries are symmetric with respect to reservation wages, we explore whether we can provide a bound on ρ for which the equilibrium possesses this property. Let $\underline{w}^a = \underline{w}^b = \underline{w}$ and denote $\overline{w}/\underline{w} \equiv \theta$. Also denote $m \equiv \min\{\alpha/(1-\alpha), (1-\alpha)/\alpha\}$. Since $0 < m \leq 1$ and $\theta > 1$, we have

$$0 < 2/(1+\theta) \leq (m+1)/(m\theta+1) < 1$$

Proposition 3 *Suppose* $\underline{w}^a = \underline{w}^b = \underline{w}$. *If* $\delta \in (0, (m+1)/(m\theta+1)]$, *then the unique NE of* G *has race to the top property. Moreover if* $\delta \in (0, 2/(1+\theta)]$, *then the unique NE of* G *has complete race to the top property.*

Proof See the Appendix. ■

To summarize, the unique equilibrium of G does not have "race to the bottom" property. When inputs are complementary in production, the equilibrium has "complete race to the top" property. However, even when the labour inputs can be said to be substitutes in production (but when they are not perfect substitutes) it is possible to have equilibria showing "race to the top", the opposite feature of "race to the bottom". We provide characterization of such equilibria in terms of the parameters affecting demand.

Given the nature of the conditions in Proposition 2, the central proposition characterizing the equilibrium in this section, the economics behind the proposition seems as follows. At an equilibrium $(\overline{w}, \overline{w})$ if a country reduces wage, then indeed the demand for its labour goes up. However, if the substitution of labour in its favour, driven by the parameters controlling the firm's demand for labour is low enough in magnitude, then the total labour income resulting from such a unilateral lowering of wage, may, however, go down owing to the lowering of the wage. Moreover, as the reservation wage in the country goes up, the volume of incremental labour income of the labour units shifting from the reservation sector to the monopsonist would also be low enough. Therefore, as $\underline{w}^i$ increases or as the degree of plausible substitution of labour goes down, the propensity of "race to the top" being an equilibrium phenomenon goes up.

Remark 3: Note that our model is equivalent to a variant of Bertrand duopoly with differentiated products. Therefore, this exercise can also be seen as a contribution to the literature on oligopoly pricing without invoking any connection to the economics of developing countries. Analyses of price-setting duopoly with differentiated products (with consumer utility function being of linear-quadratic form) are available (see, e.g., Zanchettin, 2006). However, we consider our framing of the model to be helpful in throwing light in analyzing inter-country competition for getting multinational investments. Moreover, the feature of "race to the top" in equilibrium choice seems novel.

Remark 4: Our specification of payoff for the countries—consisting of labour income only—is crucial for our result. By Lemma 1, π is decreasing in w_i. It follows from (4) that in the general case if λ_i (the weight put by the decision-maker of country i to the profit of the firm) can be made large enough, then it would be optimal for the decision-maker of country i to push the effective wages down and thus, "race to the bottom" may re-appear as an equilibrium outcome. Looking at the case $\lambda_i = 0$ enables us to concentrate on the issue of endogenous counteracting of "race to the bottom" and to explore the rather counter-intuitive possibility of the "race to the top".

3 An extension: endogenous specialization

The benchmark model assumed that each of the two countries is endowed with one distinct country-specific labour input. In this section we drop this assumption and extend the model by making the choices of specialization endogenous. In the extended model, to make a specific labour input available in a country the government of that country has to carry out an initial input-specific investment. This investment can be conceptualized as a training program which enriches the input-specific human capital of the labour-force or it can correspond to the fixed cost associated with building an infrastructure. Specifically, we assume that a country has to make investment $c > 0$ for every input, so that it has to invest $2c$ if it wishes that both kinds of labour-inputs are available within the country.

Note that thematically the analysis following our assumption of input-specific investment is quite close to analyzing price competition in a differentiated duopoly following a quantity precommitment. There is a literature that has looked at this problem with quadratic utility functions (e.g., Yin and Ng, 1997; 2000). However, our focus is a little different. First, we do not concentrate on the precise quantity of capacity creation: we assume that once the specific investment for an input is made in a country, any labour unit of that country can be used for that input. Further, our goal is to verify whether the equilibrium behaviours of wage-setting as observed in the benchmark model—especially the "race to the top" behaviour—continue even when the countries choose on the decision to generate the inputs at a prior stage.

The strategic interaction in this extended framework is modeled as the extensive-form game Γ. In stage 1 of Γ, the two countries simultaneously decide to make one of the following investment choices: (i) invest only for input 1 by incurring cost c (choice

1), (ii) invest only for input 2 by incurring cost c (choice 2), (iii) make investment for both inputs by incurring cost $2c$ (choice 12) and (iv) invest for none of the inputs (choice 0). The investment choices become commonly known at the end of stage 1. In stage 2, countries simultaneously announce wages for inputs in which they have made investment. For $j \in \{a, b\}$ and $i \in \{1, 2\}$, if country j has invested in only input i, it announces $w_i^j > 0$ (wage of input i); if it has invested in both inputs, it announces wage pair (w_1^j, w_2^j) such that $w_i^j > 0$ for both $i \in \{1, 2\}$; if it has not invested at all, there is no wage announcement. In stage 3, the firm chooses how much of each input to buy and use from each country. We continue to assume that the government of each country seeks to maximize its total labour income. We restrict to pure strategies to determine subgame perfect Nash equilibria (SPNE) of Γ.

Recall that there is an external source—the outside option of the firm—at which the firm can obtain any input at wage $\overline{w}$. So the firm will not purchase an input in a country that sets a wage higher than $\overline{w}$ for that input. Since the profit of the firm is decreasing in the wage of any input, for each input it is optimal for the firm to employ labour from a location that offers the lowest wage. We retain the tie-breaking assumption that if the lowest wage for an input offered by the countries is $\overline{w}$, then the firm does not use the external source. Furthermore, for ease of presentation we carry out the analysis of the extended game Γ under the assumption that the countries are symmetric with respect to reservation wages, that is, $\underline{w}^a = \underline{w}^b = \underline{w}$.

For $t_a, t_b \in \{1, 2, 12, 0\}$, let $\Gamma(t_a, t_b)$ be the subgame of Γ that follows the investment choice t_j by country j. Observe that the cost of investment of any country is incurred prior to this subgame. So this cost can be ignored to determine NE outcomes of the subgames $\Gamma(t_a, t_b)$. If for $i = 1, 2$, the firm employs x_i^j units of input i at wage w_i from country j, then given the assumption $\underline{w}^a = \underline{w}^b = \underline{w}$, the total labour income of country j is

$$w_1 x_1^j + w_2 x_2^j + \underline{w}(\overline{x}^j - x_1^j - x_2^j) = (w_1 - \underline{w})x_1^j + (w_2 - \underline{w})x_2^j + \underline{w}\overline{x}^j \tag{6}$$

Note if $x_1^j > 0$ and $x_2^j = 0$, then the labour income in (6) is $(w_1 - \underline{w})x_1^j + \underline{w}\overline{x}^j$ whereas if $x_1^j = 0$ and $x_2^j > 0$, it is $(w_2 - \underline{w})x_2^j + \underline{w}\overline{x}^j$. If $x_1^j = x_2^j = 0$, the labour income is $\underline{w}\overline{x}^j$. Noting that the constant term $\underline{w}\overline{x}^j$ is common in the labour income of country j for any w_i, x_i^j, we can consider the payoff of country j in $\Gamma(t_a, t_b)$ to be simply the "incremental labour income" above the reservation labour income $\underline{w}\overline{x}^j$.

Denote by $\phi_i^j(w_1, w_2)$ the incremental labour income of country j when (i) the only labour input that the firm employs from country j is input i and (ii) the firm employs $x_i^*(w_1, w_2)$ units of input i (its optimal amount of input i at (w_1, w_2)) at wage w_i from country j, that is,

$$\phi_i^j(w_1, w_2) := (w_i - \underline{w})x_i^*(w_1, w_2) \tag{7}$$

Denote by $\phi_{12}^j(w_1, w_2)$ the incremental labour income of country j when for both $i = 1, 2$, the firm employs $x_i^*(w_1, w_2)$ units of input i at wage w_i from country j. That is,

$$\phi_{12}^j(w_1, w_2) := (w_1 - \underline{w})x_1^*(w_1, w_2) + (w_2 - \underline{w})x_2^*(w_1, w_2) = \phi_1^j(w_1, w_2) + \phi_2^j(w_1, w_2)$$

Since $w_1 x_1^*(w_1, w_2) + w_2 x_2^*(w_1, w_2) = K$, we have

$$\phi_{12}^j(w_1, w_2) = K - \underline{w}(x_1^*(w_1, w_2) + x_2^*(w_1, w_2)) \tag{8}$$

Note from (7) and (8) that $\phi_i^a(w_1, w_2) = \phi_i^b(w_1, w_2)$ and $\phi_{12}^a(w_1, w_2) = \phi_{12}^b(w_1, w_2)$. Henceforth we denote the expressions in (7) and (8) simply by $\phi_i(w_1, w_2)$ and $\phi_{12}(w_1, w_2)$ dropping the superscript j. Also observe that

$$\phi_{12}(w_1, w_2) = \phi_1(w_1, w_2) + \phi_2(w_1, w_2) \tag{9}$$

Lemma 2 (i) *If $w_1 \leq w_2$, then $\phi_{12}(w_1, w_2)$ is increasing in w_1 and if $w_2 \leq w_1$, then $\phi_{12}(w_1, w_2)$ is increasing in w_2.*
(ii) *For $w_1, w_2 \in (0, \overline{w}]$, the unique maximum of $\phi_{12}(w_1, w_2)$ is attained at $w_1 = w_2 = \overline{w}$.*

Proof See the Appendix. ■

3.1 SPNE of Γ

Recall that for $\rho \in (-1, 0)$, we denote $\delta \equiv 1 - \rho$ so that $\delta \in (0, 1)$. Also recall that for the case $\underline{w}^a = \underline{w}^b = \underline{w}$, we denote $\theta \equiv \overline{w}/\underline{w}$. We have shown in Proposition 3 that if $\delta \leq 1/(1+\theta)$, then the unique NE of the game G has complete race to the top property.

The next proposition shows this result is robust under endogenous choices of specialization when the production function of the firm is not asymmetrically intensive in either one of the two inputs. Specifically, when α (the *distribution parameter* of the CES function (1)) is close to 1/2, there is always an interval of magnitudes of c for which the extended game Γ has an SPNE that has specialization as well as complete race to the top.

Proposition 4 *Suppose $\underline{w}^a = \underline{w}^b = \underline{w}$ and let $\delta \leq 2/(1+\theta)$. Then $\exists\ \varepsilon \in (0, 1/2)$ such that for any $\alpha \in (1/2 - \varepsilon, 1/2 + \varepsilon)$:*

$\exists\ 0 \leq \underline{c}(\alpha) < \overline{c}(\alpha)$ such that if $c \in (\underline{c}(\alpha), \overline{c}(\alpha))$, then Γ has an SPNE that has specialization in inputs as well as complete race to the top. Specifically, for $i, j \in \{1, 2\}$

and $i \neq j$, the following outcome is an SPNE: country a invests only in input i, country b invests only in input j and the wage set for each input is $\overline{w}$. Furthermore there is no other SPNE where at least one of the countries invests in only one input.

Proof To determine SPNE of Γ we note that for any $t_a, t_b \in \{0, 1, 2, 12\}$, either (a) the subgame $\Gamma(t_a, t_b)$ has multiple NE, all of which give the same input wages and payoffs, or (b) it has a unique NE (see Observations 1-6, Section 5.2, Appendix). Using these results, the interaction between countries a, b in stage 1 of Γ can be presented as the following "reduced form" game of investment choices in Table 1 where the payoff of a country in the cell (t_a, t_b) is its NE payoff at $\Gamma(t_a, t_b)$ net of its cost of investment. For $i \in \{1, 2\}$, $\widetilde{w}_i(\alpha) \in (\underline{w}, \overline{w}]$ is a function of α. We write a payoff $\phi_i(.\,,\,.|\alpha)$ to indicate that ϕ_i is a function of α. Note that $\phi_{12}(w, w)$ is independent of α.

Table 1: The reduced form game Γ^*_α of investment choices

	0	1	2	12
0	$0, 0$	$0,$ $\phi_1(\overline{w}, \overline{w}\|\alpha) - c$	$0,$ $\phi_2(\overline{w}, \overline{w}\|\alpha) - c$	$0,$ $\phi_{12}(\overline{w}, \overline{w}) - 2c$
1	$\phi_1(\overline{w}, \overline{w}\|\alpha) - c,$ 0	$-c, -c$	$\phi_1(\overline{w}, \overline{w}\|\alpha) - c,$ $\phi_2(\overline{w}, \overline{w}\|\alpha) - c$	$-c,$ $\phi_2(\underline{w}, \widetilde{w}_2(\alpha)\|\alpha) - 2c$
2	$\phi_2(\overline{w}, \overline{w}\|\alpha) - c,$ 0	$\phi_2(\overline{w}, \overline{w}\|\alpha) - c,$ $\phi_1(\overline{w}, \overline{w}\|\alpha) - c$	$-c, -c$	$-c,$ $\phi_1(\widetilde{w}_1(\alpha), \underline{w}\|\alpha) - 2c$
12	$\phi_{12}(\overline{w}, \overline{w}) - 2c,$ 0	$\phi_2(\underline{w}, \widetilde{w}_2(\alpha)\|\alpha) - 2c,$ 0	$\phi_1(\widetilde{w}_1(\alpha), \underline{w}\|\alpha) - 2c,$ 0	$-2c, -2c$

Observe that for any $\alpha \in (0, 1)$, a pair of investment choices (t_a, t_b) constitutes an SPNE of Γ if and only if (t_a, t_b) is an NE of the reduced form game Γ^*_α.

By (8), $\phi_{12}(w_1, w_2|\alpha) = \phi_1(w_1, w_2|\alpha) + \phi_2(w_1, w_2|\alpha)$. Since $\phi_1(\underline{w}, w_2|\alpha) = 0$ and $\phi_2(w_1, \underline{w}|\alpha) = 0$, we have $\phi_2(\underline{w}, w_2|\alpha) = \phi_{12}(\underline{w}, w_2|\alpha)$ and $\phi_1(w_1, \underline{w}|\alpha) = \phi_{12}(w_1, \underline{w}|\alpha)$. Then by Lemma 2 it follows that

$$\phi_1(w_1, \underline{w}|\alpha) < \phi_{12}(\overline{w}, \overline{w}) \text{ and } \phi_2(\underline{w}, w_2|\alpha) < \phi_{12}(\overline{w}, \overline{w}) \tag{10}$$

Consider $\alpha = 0.5$. Note that $\phi_1(\overline{w}, \overline{w}|0.5) = \phi_2(\overline{w}, \overline{w}|0.5) = (1/2)\phi_{12}(\overline{w}, \overline{w})$. By (10), we have $\phi_1(w_1, \underline{w}|0.5) < 2\phi_1(\overline{w}, \overline{w}|0.5)$ and $\phi_2(\underline{w}, w_2|0.5) < 2\phi_2(\overline{w}, \overline{w}|0.5)$. Hence

$$\phi_1(w_1, \underline{w}|0.5) - \phi_1(\overline{w}, \overline{w}|0.5) < \phi_1(\overline{w}, \overline{w}|0.5) = \phi_2(\overline{w}, \overline{w}|0.5) \text{ and}$$

$$\phi_2(\underline{w}, w_2|0.5) - \phi_2(\overline{w}, \overline{w}|0.5) < \phi_2(\overline{w}, \overline{w}|0.5) = \phi_1(\overline{w}, \overline{w}|0.5).$$

Taking $w_i = \widetilde{w}_i(0.5)$, these inequalities imply

$$\phi_1(\widetilde{w}_1(0.5), \underline{w}|0.5) - \phi_1(\overline{w}, \overline{w}|0.5) < \phi_i(\overline{w}, \overline{w}|0.5) \text{ and}$$

$$\phi_2(\underline{w}, \widetilde{w}_2(0.5)|0.5) - \phi_2(\overline{w}, \overline{w}|0.5) < \phi_i(\overline{w}, \overline{w}|0.5) \text{ for } i = 1, 2 \tag{11}$$

Since the functions ϕ_1, ϕ_2 are continuous in α, from (11) we conclude that $\exists\, \varepsilon \in (0, 0.5)$ such that for all $\alpha \in [0.5 - \varepsilon, 0.5 + \varepsilon]$:

$$\phi_1(\widetilde{w}_1(\alpha), \underline{w}|\alpha) - \phi_1(\overline{w}, \overline{w}|\alpha) < \phi_i(\overline{w}, \overline{w}|\alpha) \text{ and}$$

$$\phi_2(\underline{w}, \widetilde{w}_2(\alpha)|\alpha) - \phi_2(\overline{w}, \overline{w}|\alpha) < \phi_i(\overline{w}, \overline{w}|\alpha) \text{ for } i = 1, 2 \tag{12}$$

For $\alpha \in [0.5 - \varepsilon, 0.5 + \varepsilon]$, denote $\overline{c}(\alpha) := \min\{\phi_1(\overline{w}, \overline{w}|\alpha), \phi_2(\overline{w}, \overline{w}|\alpha)\}$ and

$$\underline{c}(\alpha) := \max\{\phi_1(\widetilde{w}_1(\alpha), \underline{w}|\alpha) - \phi_1(\overline{w}, \overline{w}|\alpha), \phi_2(\underline{w}, \widetilde{w}_2(\alpha)|\alpha) - \phi_2(\overline{w}, \overline{w}|\alpha), 0\}.$$

Then by (12), $0 \leq \underline{c}(\alpha) < \overline{c}(\alpha)$.

Let $c \in (\underline{c}(\alpha), \overline{c}(\alpha))$. In what follows we show that in this case both $(1, 2)$ and $(2, 1)$ are NE of the game Γ^*_α. Since $c > \underline{c}(\alpha)$, we have $\phi_1(\overline{w}, \overline{w}|\alpha) - c > \phi_1(\widetilde{w}_1(\alpha), \underline{w}|\alpha) - 2c$ and since $c < \overline{c}(\alpha)$, we have $\phi_1(\overline{w}, \overline{w}|\alpha) - c > 0 > -c$. Then by Table 1, for $i, j \in \{a, b\}$, investment choice 1 is the unique best response of country j to country i's investment choice 2.

Similarly, since $c > \underline{c}(\alpha)$, we have $\phi_2(\overline{w}, \overline{w}|\alpha) - c > \phi_2(\underline{w}, \widetilde{w}_2(\alpha)|\alpha) - 2c$ and since $c < \overline{c}(\alpha)$, we have $\phi_2(\overline{w}, \overline{w}|\alpha) - c > 0 > -c$, which shows for $i, j \in \{a, b\}$, investment choice 2 is the unique best response of country j to country i's investment choice 1. This shows that both $(1, 2)$ and $(2, 1)$ are NE of Γ^*_α.

Noting that for any country, 2 is the unique best response when the other country chooses 1 and 1 is the unique best response when the other country chooses 2, we conclude that $(1, 2)$ and $(2, 1)$ are the only SPNE where at least one country invests in only one input. ■

4 Concluding remarks

In this paper we considered a simple game theoretic framework with two countries and one multinational corporation (MNC) to show that drastic strategic undercutting of labour's bargaining power is not an inevitable outcome of strategic competition between policy-makers of different countries. Notice that our setup is not meant to provide a comprehensive model of MNCs' competition and input choice decisions involving all its multifarious complexities. We focussed on *one* aspect of an MNC's organization of production and looked into the possibility of this feature generating a counteracting effect to the "race to the bottom". We find that indeed this feature of imperfect substitutability of productive inputs can act as a possible counteracting factor to the "race to the bottom".

Our analysis suggests that facing the problem of attracting multinational investment and consequent possible necessity of undercutting labour standards, a way-out for the developing countries could be to specialize on different types of labour-skills. We provide some modellings to illustrate this possibility analytically. In fact, in our setup we have rather deliberately left out some additional factors such as any demand externality for producers from increased wage of labourers or any agglomeration effects, which can act as additional countervailing factors to "race to the bottom".

As we noted in Remark 4 earlier, our specification of payoffs in which the decision maker of each country maximizes its labour income, is crucial for our result. What emerges for the labourers in richer environments (e.g., when the payoff of decision-maker in a country is a weighted sum of labour income and the firms's profit in a dynamic situation) is a matter for future research.

5 Appendix

5.1 Proofs of results

Proof of Lemma 1 (i)-(ii) Solving the firm's problem, optimal labour inputs for the firm are given by

$$x_a^* = \frac{K}{w_a + [(1-\alpha)w_a/\alpha w_b]^{1/(1+\rho)} w_b}, x_b^* = \frac{K}{w_b + [\alpha w_b/(1-\alpha)w_a]^{1/(1+\rho)} w_a} \tag{13}$$

Using the expressions above, standard reasoning proves (i)-(ii).

(iii) Recall that $\pi(w_a, w_b, K)$ is the maximized value of the profit of the firm, i.e., $\pi(w_a, w_b, K) = F(x_a^*, x_b^*)$. Invoking Roy's identity (see, e.g., Kreps, 1990, p.57) we have

$$\partial\pi(w_a, w_b, K)/\partial w_i = -x_i^*[\partial\pi(w_a, w_b, K)/\partial K] \text{ for } i \in \{a, b\} \tag{14}$$

By (13), both x_a^*, x_b^* are increasing in K. Since F is increasing in both x_a, x_b, we conclude that $\pi(w_a, w_b, K)$ is increasing in K. As $x_i^* > 0$, from (14) it follows that $\pi(w_a, w_b, K)$ is decreasing in w_i for $i \in \{a, b\}$.

(iii) The labour income in the monopoly sector for countries a, b, are given by

$$\psi_a^M(w_a, w_b, K) = w_a x_a^* = \frac{K}{1 + [(1-\alpha)/\alpha]^{1/(1+\rho)}(w_b/w_a)^{\rho/(1+\rho)}}$$

$$\psi_b^M(w_a, w_b, K) = w_b x_b^* = \frac{K}{1 + [\alpha/(1-\alpha)]^{1/(1+\rho)}(w_a/w_b)^{\rho/(1+\rho)}} \tag{15}$$

If $\rho \in (-1, 0)$, we have $\rho/(1+\rho) < 0$ and hence $w_a^{\rho/(1+\rho)}$ is decreasing in w_a. Consequently the denominator of ψ_a^M is increasing in w_a and hence ψ_a^M is decreasing in w_a for any $w_b \geq 0$. By the same reasoning, ψ_b^M is decreasing in w_b for any $w_a \geq 0$.

If $\rho \in (0, \infty)$, we have $\rho/(1+\rho) > 0$ and hence $w_a^{\rho/(1+\rho)}$ is increasing in w_a. Consequently the denominator of ψ_a^M is decreasing in w_a and hence ψ_a^M is increasing in w_a for any $w_b \geq 0$. By the same reasoning, ψ_b^M is increasing in w_b for any $w_a \geq 0$. ■

Proof of Proposition 1 Throughout let $i, j \in \{a, b\}$ and $i \neq j$. First we prove parts (I)(ii) and (II)(ii), then parts (I)(i) and (II)(i).

(I)(ii), (II)(ii): Let $\rho \in (0, \infty)$. Then ψ_i^M is increasing in w_i (Lemma 1(iv)). As x_i^* is decreasing in w_i (Lemma 1(i)), so is ψ_i^T. Then from (3), it follows that ψ_i is increasing in w_i for any w_j, so the unique best response of country i to any w_j is to choose $w_i = \overline{w}$ which proves (I)(ii). It is immediate from (I)(i) that G has a unique NE $(\overline{w}, \overline{w})$, proving (II)(ii).

(I)(i), (II)(i): Let $\rho \in (-1, 0)$ and $\delta \equiv -\rho \in (0, 1)$. Denote $\tau_a \equiv \alpha$ and $\tau_b \equiv 1 - \alpha$. For this case ψ_i^M is decreasing in w_i (Lemma 1(iii)(a)). As x_i^* is decreasing in w_i (Lemma 1(i)), so is ψ_i^T. Define

$$g_i(w_i) := \tau_j^{1/(1-\delta)}(\delta w_i - \underline{w}^i) w_i^{\delta/(1-\delta)} \text{ and } h_i(w_j) := (1-\delta)\tau_i^{1/(1-\delta)} \underline{w}^i w_j^{\delta/(1-\delta)} \tag{16}$$

Observe that $g_i(w_i)$ is increasing in w_i, $h_i(w_j)$ is increasing in w_j and $\lim_{w_i \to \infty} g_i(w_i) = \lim_{w_j \to \infty} h_i(w_j) = \infty$. Note from (3) that $\partial\psi_i/\partial w_i \gtreqless 0 \Leftrightarrow g_i(w_i) \lesseqgtr h_i(w_j)$.

To prove (I)(i), we consider the following two cases.

Case 1 If $\underline{w}^i \geq \delta\overline{w}$, then for any w_j, we have $g_i(w_i) \leq 0 < h_i(w_j)$ and hence $\partial\psi_i/\partial w_i > 0$ for all $w_i \in [\underline{w}^i, \overline{w}]$. So the unique best response of country i to any w_j is to choose $w_i = \overline{w}$.

Case 2 If $\underline{w}^i < \delta\overline{w}$, then for $w_i \in [\underline{w}^i, \underline{w}^i/\delta]$, we have $g_i(w_i) \leq 0 < h_i(w_j)$ and hence ψ_i is increasing in w_i in this interval. So for any w_j, best response of country i is to choose $w_i \in [\underline{w}^i/\delta, \overline{w}]$. As $g_i(\underline{w}^i/\delta) = 0 < h_i(w_j) < \lim_{w_i \to \infty} g_i(w_i) = \infty$, by the monotonicity of g_i, $\exists$ a unique $w_i = b_i(w_j) \in (\underline{w}^i/\delta, \infty)$ such that $g_i(w_i) \lesseqgtr h_i(w_j) \Leftrightarrow \partial\psi_i/\partial w_i \gtreqless 0 \Leftrightarrow w_i \lesseqgtr b_i(w_j)$. Therefore the unique best response of country i to any $w_j \in [\underline{w}^j, \overline{w}]$, is $B_i(w_j) = \min\{b_i(w_j), \overline{w}\}$. As h_i is increasing in w_j, it follows that $b_i(w_j)$ is increasing and $B_i(w_j)$ is non-decreasing in w_j. This completes the proof of (I)(i).

To prove (II)(i), first we show that G has a unique NE for $\rho \in (-1, 0)$. Note from the proof of (I)(i) that for $i \in \{a, b\}$, $\exists\ 0 < \varepsilon_i < \overline{w} - \underline{w}^i$ such that $B_i(w_j) \in [\underline{w}^i + \varepsilon_i, \overline{w}]$

for any[3] w_j. Also observe that the constant term $\underline{w}^i\overline{x}^i$ in (3) does not play any role in determining NE outcomes of G. Consider the two-person "transformed" game H in which countries a, b choose w_a, w_b, where the strategy set of i is $[\underline{w}^i + \varepsilon_i, \overline{w}]$ and its payoff is

$$\Phi_i(w_a, w_b) = \log[(w_i - \underline{w})x_i^*(w_a, w_b)] = \log(w_i - \underline{w}^i) + \log(x_i^*(w_a, w_b)) \qquad (17)$$

The log function is well defined for the game H. Note that the set of NE of G coincides with the set of NE of H.

Lemma A1 *The log labour demand* $\log(x_i^*(w_a, w_b))$ *of any country* $i \in \{a, b\}$ *has increasing differences in* (w_a, w_b)*, i.e., the following hold for* $w_a' > w_a$, $w_b' > w_b$.

$$[\log(x_i^*(w_a', w_b')) - \log(x_i^*(w_a, w_b'))] - [\log(x_i^*(w_a', w_b)) - \log(x_i^*(w_a, w_b))] > 0$$

Consequently the game H *is a supermodular game.*

Proof We prove the increasing difference result for $i = a$ (the proof is similar for $i = b$). Let $t_\alpha(w) := \alpha^{1/(1+\rho)}w^{\rho/(1+\rho)}$. Using (13) and simplifying, we have

$$[\log(x_a^*(w_a', w_b')) - \log(x_a^*(w_a, w_b'))] - [\log(x_a^*(w_a', w_b)) - \log(x_a^*(w_a, w_b))]$$
$$= \log \frac{[t_\alpha(w_a') + t_{1-\alpha}(w_b)][t_\alpha(w_a) + t_{1-\alpha}(w_b')]}{[t_\alpha(w_a') + t_{1-\alpha}(w_b')][(t_\alpha(w_a) + t_{1-\alpha}(w_b)]} > 0$$

Using the increasing difference result and the conclusions of Milgrom and Roberts (1990) (see page 1271, the paragraph before equation (5)), it follows that the game H is supermodular. ■

Lemma A2 *For* $i \in \{a, b\}$*, let* $y_i = \log(w_i)$*. The payoff of* i *in the game* H *has the following property:* $\partial^2\Phi_i/\partial y_a\partial y_b + \partial^2\Phi_i/(\partial y_a)^2 < 0$*. Consequently* H *has a unique NE.*

Proof We prove the inequality above for $i = a$ (similar reasoning applies for $i = b$). Denote $\widetilde{\alpha} \equiv [(1-\alpha)/\alpha]^{1/(1+\rho)}$. Using (13) in (17) and simplifying:

$$\Phi_a = \log[\exp(y_a) - \underline{w}^a] + \log K - y_a + y_b/(1+\rho) - \log[\nu(y_a, y_b)]$$

where $\nu(y_a, y_b) := \exp[y_b/(1+\rho)] + \widetilde{\alpha}\exp[y_a/(1+\rho)]$. For $i, j \in \{a, b\}$, let $\nu_i = \partial\nu/\partial y_i$ and $\nu_{ij} = \partial^2\nu/\partial y_i\partial y_j$. Note that $\nu_{ab} = 0$ and $\nu\nu_{aa} - (\nu_a)^2 - \nu_a\nu_b = 0$. Hence $\partial^2\log[\nu]/\partial y_a\partial y_b + \partial^2\log[\nu]/(\partial y_a)^2 = [\nu\nu_{aa} - (\nu_a)^2 - \nu_a\nu_b]/\nu^2 = 0$ implying that $\partial^2\Phi_a/\partial y_a\partial y_b + \partial\Phi_a/(\partial y_a)^2 = -\underline{w}^a w_a/(w_a - \underline{w}^a)^2 < 0$.

Since H is a supermodular game, the inequalities above imply that H has a unique NE (see equation (6), page 1271 of Milgrom and Roberts, 1990). ■

Since H has a unique NE, so does G. Having shown that G has a unique NE, observe from part (I)(i) that for any country i, the unique best response $B_i(w_j)$ to any

w_j has either $B_i(w_j) = \overline{w} > \underline{w}^i$, or $B_i(w_j) > \underline{w}^i/\delta > \underline{w}^i$. So the unique NE of G does not have $w_i = \underline{w}^i$ for any i. This completes the proof of part (II)(i).
(III) This is immediate from the facts that for any w_j, best response of i is increasing in $\underline{w}^i$ and that G possesses a unique NE. ■

Proof of Proposition 2 Note from the proof of Proposition 1 that if $\underline{w}^i \geq \delta\overline{w}$ for $i \in \{a, b\}$, then the unique NE is $(w_a = \overline{w}, w_b = \overline{w})$. To characterize NE for other cases, let $\underline{w}^i < \delta\overline{w}$ for some i. Then the best response function of i is $B_i(w_j) = \min\{b_i(w_j), \overline{w}\}$. By the monotonicity of g_i, we have $b_i(\overline{w}) \gtreqless \overline{w} \Leftrightarrow g_i(b_i(\overline{w})) = h_i(\overline{w}) \gtreqless g_i(\overline{w})$. Using the expressions of g_i and h_i from (16), we conclude that $\exists\ \widetilde{\delta}_i \in (0, \delta)$ (given by (5)) such that

$$b_i(\overline{w}) \gtreqless \overline{w} \Leftrightarrow \underline{w}^i \gtreqless \widetilde{\delta}_i\overline{w} \tag{18}$$

(i) If $\underline{w}^i < \widetilde{\delta}_i\overline{w}$, then $b_i(\overline{w}) < \overline{w}$ and hence $b_i(w_j) < \overline{w}$ for all $w_j \in [\underline{w}^j, \overline{w}]$. So $B_i(w_j) = b_i(w_j)$ for all $w_j \in [\underline{w}^j, \overline{w}]$. Therefore if $\underline{w}^i < \widetilde{\delta}_i\overline{w}$ for $i \in \{a, b\}$, then there is no NE where $w_i = \overline{w}$. Therefore, the unique NE has $w_i < \overline{w}$ for both i.

(ii) Let $\underline{w}^i < \widetilde{\delta}_i\overline{w}$ and $\underline{w}^j \geq \delta\overline{w}$. Then $B_j(w_i) = \overline{w}$ for all $w_i \in [\underline{w}^i, \overline{w}]$. So, the NE has $w_j = \overline{w}$ and $w_i = B_i(\overline{w})$. As $\underline{w}^i < \widetilde{\delta}_i\overline{w}$, we have $b_i(\overline{w}) < \overline{w}$ and hence $B_i(\overline{w}) = b_i(\overline{w}) < \overline{w}$. So the unique NE has $w_i < \overline{w}$ and $w_j = \overline{w}$.

(iii) By inequality (18) above, in this case we have $b_i(\overline{w}) \geq \overline{w}$ for both i and hence $B_i(\overline{w}) = \overline{w}$. So the unique NE has $(w_a = w_b = \overline{w})$.

(iv) The proof is exactly similar to the three cases above. ■

The following lemma will be useful for the proof of Proposition 3.

Lemma A3 *Let $\rho \in (-1, 0)$ and $\delta \equiv -\rho \in (0, 1)$. Denote $\overline{w}/\underline{w} \equiv \theta > 1$ and let $\delta \in (1/\theta, 1)$. Denote $\widetilde{\tau}_a \equiv \alpha/(1-\alpha)$, $\widetilde{\tau}_b \equiv (1-\alpha)/\alpha$ and for $t > 0$,*

$$\ell^{t,\delta}(w) := t^{1/\delta}[(\delta w - \underline{w})/(1-\delta)\underline{w}]^{(1-\delta)/\delta} w, r^{\delta}(w) := \underline{w}/\delta + (1-\delta)^2\underline{w}^2/\delta(\delta w - \underline{w}) \tag{19}$$

(i) *$\ell^{t,\delta}(w)$ is increasing and $r^{\delta}(w)$ is decreasing in w.*

(ii) *If G has an NE where $w_i < \overline{w}$ for both $i \in \{a, b\}$, then $\ell^{\widetilde{\tau}_a,\delta}(w_a) = r^{\delta}(w_a)$ and $\ell^{\widetilde{\tau}_b,\delta}(w_b) = r^{\delta}(w_b)$.*

(iii) *Let $m = min\{\widetilde{\tau}_a, \widetilde{\tau}_b\}$. If $r^{\delta}(\overline{w}) > \ell^{m,\delta}(\overline{w})$, then G cannot have an NE where $w_i < \overline{w}$ for $i \in \{a, b\}$.*

Proof Part (i) is immediate. For part (ii), let $\delta \in (1/\theta, 1)$. If G has an NE where $w_i < \overline{w}$ for both $i \in \{a, b\}$, then from the proof of Proposition 1, we have $g_a(w_a) = h_a(w_b)$ and

$g_b(w_a) = h_b(w_a)$ where the functions g_i, h_i are given in (16). Taking $\underline{w}^a = \underline{w}^b = \underline{w}$, we obtain $w_b = \ell^{\widetilde{\tau}_b,\delta}(w_a)$ from the first and $w_a = \ell^{\widetilde{\tau}_a,\delta}(w_b)$ from the second equation. Since $\widetilde{\tau}_a\widetilde{\tau}_b = 1$, from these two equations we have $(\delta w_a - \underline{w})(\delta w_b - \underline{w}) = (1-\delta)^2\underline{w}^2$ which implies that $w_b = r^\delta(w_a)$ and $w_a = r^\delta(w_b)$, proving (ii).

(iii) If $r^\delta(\overline{w}) > \ell^{m,\delta}(\overline{w})$, then there is at least one $i \in \{a,b\}$ such that $r^\delta(\overline{w}) > \ell^{\widetilde{\tau}_i,\delta}(\overline{w})$. Then by (i), $r^\delta(w_i) > \ell^{\widetilde{\tau}_i,\delta}(w_i)$ for all $w_i \in [\underline{w}, \overline{w}]$ and part (iii) follows by (ii).
■

Proof of Proposition 3 Note from (5) that (a) if $\alpha < 1/2$, then $\widetilde{\delta}_a < \widetilde{\delta}_b < \delta/(2-\delta)$, (b) if $\alpha > 1/2$, then $\widetilde{\delta}_b < \widetilde{\delta}_a < \delta/(2-\delta)$ and (c) if $\alpha = 1/2$, then $\widetilde{\delta}_a = \widetilde{\delta}_b = \delta/(2-\delta)$.

First we prove the second part of the proposition. Let $\underline{w}^a = \underline{w}^b = \underline{w}$ and $\delta \le 2/(1+\theta)$. Then $\underline{w} \ge \delta\overline{w}/(2-\delta)$ and by the last paragraph, $\underline{w} \ge \widetilde{\delta}_i\overline{w}$ for $i \in \{a,b\}$. Applying Proposition 2(iii), the result follows.

Now consider $\theta > 2/(1+\theta)$. Note that $2/(1+\theta) > 1/\theta$ (since $\theta > 1$), so in this case we can apply Lemma A3. Note that $r^\delta(\overline{w})$ is decreasing in δ. Denoting $\delta\theta - 1 \equiv \tau > 0$, we have

$$\partial\ell^{m,\delta}(\overline{w})/\partial\delta = m^{1/\delta}\tau^{(1-\delta)/\delta}[(\theta-1)\delta + \tau\log((1-\delta)/m\tau)]/(1-\delta)^{(1-\delta)/\delta}\delta^2\tau \qquad (20)$$

Let $\widehat{\delta} \equiv (m+1)/(m\theta+1)$. Observe that if $\delta \le \widehat{\delta}$, then $(1-\delta)/m\tau \ge 1$ and by (20), $\ell^{m,\delta}(\overline{w})$ is increasing in δ. Next observe that $r^{\widehat{\delta}}(\overline{w}) = [1+m(\theta-1)]\overline{w} > \overline{w} = \ell^{m,\widehat{\delta}}(\overline{w})$. As $\ell^{m,\delta}(\overline{w})$ is increasing and $r^\delta(\overline{w})$ is decreasing in δ, it follows that $r^\delta(\overline{w}) > \ell^{m,\delta}(\overline{w})$ for all $\delta \le \widehat{\delta}$. So by Lemma A3, for $\delta \in (2/(1+\theta), \widehat{\delta}]$, there is no NE of G where $w_i < \overline{w}$ for both $i \in \{a,b\}$, so the unique NE has race to the top property. ■

Proof of Lemma 2 (i) Using the budget constraint of the firm in (8) we have

$$\phi^j_{12}(w_1,w_2) = K - \underline{w}^j(1 - w_1/w_2)x^*_1(w_1,w_2) - K\underline{w}^j/w_2$$

Hence $\partial\phi^j_{12}/\partial w_1 = -\underline{w}^j(1-w_1/w_2)\partial x^*_1/\partial w_1 + \underline{w}^j x^*_1/w_2$. Since $\partial x^*_1/\partial w_1 < 0$, it follows that $\partial\phi^j_{12}/\partial w_1 > 0$ if $w_1 \le w_2$. Similar reasoning shows $\partial\phi^j_{12}/\partial w_2 > 0$ if $w_2 \le w_1$.

(ii) Let $w, w_1, w_2 \in (0, \overline{w}]$. By part (i), we have $\phi^j_{12}(w,w) > \phi^j_{12}(w_1,w)$ for $w_1 < w$ and $\phi^j_{12}(w,w) > \phi^j_{12}(w,w_2)$ for $w_2 < w$. Consequently, a necessary condition for $\phi^j_{12}(w_1,w_2)$ to be maximized is that $w_1 = w_2$. When $w_1 = w_2 = w$, by (8), we have $\phi_{12}(w,w) = K - \underline{w}(x^*_1(w,w) + x^*_2(w,w))$. Denoting $\alpha/(1-\alpha)^{1/(1+\rho)} = t$, in this case we have $x^*_1(w,w) = K/w(1+t)$, $x^*_2(w,w) = tK/w(1+t)$, so that $(x^*_1(w,w) + x^*_2(w,w)) = K/w$. This shows that $\phi_{12}(w,w)$ is increasing in w and its unique maximizer for $0 < w \le \overline{w}$ is attained at $w = \overline{w}$. ■

5.2 Subgames of Γ

In what follows we determine NE outcomes of subgames $\Gamma(t_a, t_b)$ by classifying these subgames into different classes. We assume $\rho \in (-1, 0)$ and $\underline{w}^a = \underline{w}^b = \underline{w}$. Denote $\delta \equiv 1 - \rho$ and $\theta \equiv \overline{w}/\underline{w}$, so that $\delta \in (0,1)$ and $\theta > 1$. We also assume $\delta \leq 1/(1+\theta)$.

5.2.1 Each country invested in only one input, each invested in different inputs

Consider the game $\Gamma(1,2)$. This is the game in which country a has invested in input 1 and country b has invested in input 2 (the analysis is similar for the game $\Gamma(2,1)$). In the game $\Gamma(1,2)$, country a announces wage w_1^a and country b announces wage w_1^b. Let $w_1 = \min\{w_1^a, \overline{w}\}$ and $w_2 = \min\{w_1^b, \overline{w}\}$. The firm employs $x_i^*(w_1, w_2)$ units of input i. For any input, the firm employs input from a country if the country offers a wage for the input that does not exceed $\overline{w}$; otherwise the firm employs from the external source. Since $\delta \leq 1/(1+\theta)$, by the last part of Proposition 3, we conclude the following.

Observation 1 $\Gamma(1,2)$ *has a unique NE:* $w_1^a = w_2^b = \overline{w}$. *At the NE, country a obtains* $\phi_1(\overline{w}, \overline{w})$ *and country b obtains* $\phi_2(\overline{w}, \overline{w})$.

5.2.2 Each country invested in only one input, each invested in the same input

Consider the game $\Gamma(1,1)$. This is the game in which both countries have invested in the same input 1 (the analysis is similar for the game $\Gamma(2,2)$). In the game $\Gamma(1,1)$, country $j \in \{a,b\}$ announces wage w_1^j. Let $w_1 = \min\{w_1^a, w_1^b, \overline{w}\}$. The firm employs $x_i^*(w_1, \overline{w})$ units of input i. The firm employs input 2 from the external source at wage $\overline{w}$. If $\min\{w_1^a, w_1^b\} > \overline{w}$, the firm employs input 1 from the external source; otherwise it employs input 1 from countries that offer the lowest wage for input i.

Observation 2 $\Gamma(1,1)$ *has a unique NE:* $w_1^a = w_1^b = \underline{w}$. *At the NE, each country obtains zero payoff.*

Proof Denote $w_1 = \min\{w_1^a, w_1^b, \overline{w}\}$. We prove the result in following steps.

Step 1: We cannot have an NE where $w_1 < \underline{w}$. To see this, consider a strategy profile such that $w_1 < \underline{w}$. Then $\exists$ a country $j \in \{a,b\}$ such that $x_1^j = \lambda x_1^*(w_1, \overline{w})$ for some $\lambda \in (0,1]$ and j obtains $\lambda\phi_1(w_1, \overline{w}) < 0$. By unilaterally deviating to $w_1^j = \underline{w}$, country j obtains zero payoff, so such a deviation is gainful.

Step 2: We cannot have an NE where $w_1 > \underline{w}$. To see this, consider a strategy profile such that $w_1 > \underline{w}$. Then $\exists$ $j \in \{a,b\}$ such that $x_1^j = \lambda x_1^*(w_1, \overline{w})$ for some

$\lambda \in [0, 1)$ and j obtains $\lambda\phi_1(w_1, \overline{w})$. Let j unilaterally deviate to $w_1 - \varepsilon$ where $\varepsilon > 0$ and $w_1 - \varepsilon > \underline{w}$. Following this deviation, j obtains $\phi_1(w_1 - \varepsilon, \overline{w})$. Since $w_1 > \underline{w}$ and $\lambda < 1$, we have $(1 - \lambda)\phi_1(w_1, \overline{w}) > 0$. Let $0 < \delta < (1 - \lambda)\phi_1(w_1, \overline{w})$. Since $\phi_1(w_1, \overline{w})$ is continuous in w_1, for sufficiently small $\varepsilon > 0$ we have $\phi_1(w_1 - \varepsilon, \overline{w}) > \phi_1(w_1, \overline{w}) - \delta > \lambda\phi_1(w_1, \overline{w})$, showing gainful deviation for j.

Step 3: By Steps 1-2, at any NE, we must have $\min\{w_1^a, w_1^b\} = \underline{w}$. Now we show that at any NE, we must also have $\max\{w_1^a, w_1^b\} = \underline{w}$. Consider a strategy profile such that $\min\{w_1^a, w_1^b\} = \underline{w} < \max\{w_1^a, w_1^b\}$. Without loss of generality, let $w_1^a = \underline{w} < w_1^b$. Since $\min\{w_1^a, w_1^b\} = \underline{w}$, each country obtains zero payoff. Let country a unilaterally deviate to $\tilde{w}_1^a$ such that $\underline{w} < \tilde{w}_1^a < w_1^b$. Following this deviation, a would obtain $\phi_1(\tilde{w}_1^a, \overline{w}) > 0$, so the deviation is gainful. This shows that at any NE, we must have $w_1^a = w_1^b = \underline{w}$.

Step 4: Finally observe that the strategy profile (w_1^a, w_1^b) where $w_1^a = w_1^b = \underline{w}$ is indeed an NE. At this strategy profile, each country gets zero. There is no unilateral deviation that gives positive payoff to a country. ■

5.2.3 One country invested in only one input, another country invested in no input

Consider the game $\Gamma(1, 0)$. This is the game in which country a invested in only input 1 and country b invested in no input (the analysis is similar for the games $\Gamma(2, 0)$, $\Gamma(0, 1)$ and $\Gamma(0, 2)$). In the game $\Gamma(1, 0)$, country b always obtains zero payoff. In this game country a announces wage w_1^a. Let $w_1 = \min\{w_1^a, \overline{w}\}$. The firm employs $x_i^*(w_1, \overline{w})$ units of input i. It employs input 2 from the external source at wage $\overline{w}$. If $w_1^a = w_1$, the firm employs input 1 from country a; otherwise it employs input 1 from the external source at wage $\overline{w}$.

Observation 3 $\Gamma(1, 0)$ *has a unique NE. At the NE:* $w_1^a = \overline{w}$, *country* a *obtains* $\phi_1(\overline{w}, \overline{w})$ *and country* b *obtains zero payoff.*

Proof As country a obtains at most zero by setting $w_1^a \leq \underline{w}$ and zero payoff by setting $w_1^a > \overline{w}$, at any NE we must have $w_1^a \in (\overline{w}, \overline{w}]$. Since $\delta \leq 2/(1+\theta)$, by the last statement of Proposition 3 it follows that the unique maximizer of $\phi_1(w_1^a, \overline{w})$ over $w_1^a \in (\underline{w}, \overline{w}]$ is attained at $w_1^a = \overline{w}$. This proves the result. ■

5.2.4 One country invested in both inputs, another country invested in only one input

Consider the game $\Gamma(12,1)$. In this game country a invested in both inputs $1,2$ and country b invested in only input 1 (the analysis is similar for games $\Gamma(12,2)$, $\Gamma(1,12)$ and $\Gamma(2,12)$).

In the game $\Gamma(12,1)$, country a announces wage pair (w_1^a, w_2^a) and country b announces w_1^b (its wage for input 1). Let $w_1 = \min\{w_1^a, w_1^b, \overline{w}\}$ and $w_2 = \min\{w_2^a, \overline{w}\}$. The firm employs $x_i^*(w_1, w_2)$ units of input i. If $\min\{w_1^a, w_1^b\} > \overline{w}$, the firm employs input 1 from the external source; otherwise it employs input 1 from countries that offers the lowest wage for input 1. If $w_2^a > \overline{w}$, the firm employs input 1 from the external source; otherwise it employs input 2 from country a.

Observation 4 $\Gamma(12,1)$ *has multiple payoff-equivalent NE. Any NE has* $(w_1^a, w_2^a) = (\underline{w}, \widetilde{w}_2)$ *and* $w_1^b = \underline{w}$, *where* $\underline{w} < \widetilde{w}_2 \leq \overline{w}$. *At any NE, country a obtains* $\phi_2(\underline{w}, \widetilde{w}_2)$ *and country b obtains zero payoff.*

Proof Denote $w_1 = \min\{w_1^a, w_1^b, \overline{w}\}$. We prove the result in the following steps.

Step 1: At any NE of $\Gamma(12,1)$, we must have $w_2^a \leq \overline{w}$. To see this, consider (w_1^a, w_2^a) such that $w_a^2 > \overline{w}$. Then $x_2^a = 0$ and $x_1^a \leq x_1^*(w_1, \overline{w})$, so the payoff of country a is at most $\phi_1(w_1, \overline{w})$. If $w_1 \leq \underline{w}$, country a obtains at most zero. Let it unilaterally deviate to $(\underline{w}, \overline{w})$; then x_2^a would be positive and a would obtain positive payoff. If $w_1 > \underline{w}$, let a deviate to $(w_1 - \varepsilon, \overline{w})$ such that $\varepsilon > 0$ and $w_1 - \varepsilon > \underline{w}$. Following this deviation, country a would obtain

$$\phi_{12}(w_1 - \varepsilon, \overline{w}) = \phi_1(w_1 - \varepsilon, \overline{w}) + \phi_2(w_1 - \varepsilon, \overline{w}).$$

Note that $\phi_2(w_1, \overline{w}) > 0$. Let $0 < \delta < \phi_2(w_1, \overline{w})/2$. Since for $i = 1, 2$, $\phi_i(w_1, \overline{w})$ is continuous in w_1, for sufficiently small $\varepsilon > 0$, we have $\phi_i(w_1 - \varepsilon, \overline{w}) > \phi_i(w_1, \overline{w}) - \delta$. Hence

$$\phi_1(w_1 - \varepsilon, \overline{w}) + \phi_2(w_1 - \varepsilon, \overline{w}) > \phi_1(w_1, \overline{w}) + \phi_2(w_1, \overline{w}) - 2\delta > \phi_1(w_1, \overline{w})$$

showing that the deviation is gainful for country a.

Step 2: At any NE, we must have $w_1^a = w_1^b = \underline{w}$. To see this, first note by Step 1, at any NE we have $w_2^a \leq \overline{w}$. So $w_2 = w_2^a$ and $x_2^a = x_2^*(w_1, w_2^a)$. In what follows, we show that we cannot have $w_1 < \underline{w}$ or $w_1 > \underline{w}$ at any NE (recall $w_1 = \min\{w_1^a, w_1^b, \overline{w}\}$).

If $w_1 < \underline{w}$, then $\exists\ j \in \{a, b\}$ such that $w_1^j < \underline{w}$ and $x_1^j = \lambda x_1^*(w_1, w_2)$ for some $\lambda \in (0, 1]$. If $j = b$, then b obtains $\lambda \phi_1(w_1, w_2) < 0$ (since $w_1 < \underline{w}$ and $\lambda > 0$) and b is better off unilaterally deviating to $\tilde{w}_1^b = \underline{w}$ to obtain zero payoff. If $j = a$, then

a obtains $\lambda\phi_1(w_1, w_2) + \phi_2(w_1, w_2) < \phi_2(w_1, w_2)$. By unilaterally deviating to $(\underline{w}, w_2)$, country a would obtain $\phi_2(w_1, w_2)$, so the deviation is gainful.

If $w_1 > \underline{w}$, then $\exists\ j \in \{a, b\}$ such that $w_1^j > \underline{w}$ and $x_1^j = \lambda x_1^*(w_1, w_2)$ for some $\lambda \in [0, 1)$. If $j = b$, then b obtains $\lambda\phi_1(w_1, w_2)$. Let b unilaterally deviate to $\tilde{w}_1^b = w_1 - \varepsilon$ such that $\varepsilon > 0$ and $w_1 - \varepsilon > \underline{w}$. Following this deviation, b obtains $\phi_1(w_1 - \varepsilon, w_2)$. As $w_1 > \underline{w}$ and $\lambda < 1$, we have $(1-\lambda)\phi_1(w_1, w_2) > 0$. Let $0 < \delta < (1-\lambda)\phi_1(w_1, w_2)$. Since $\phi_1(w_1, w_2)$ is continuous in w_1, for sufficiently small $\varepsilon > 0$, we have $\phi_1(w_1 - \varepsilon, w_2) > \phi_1(w_1, w_2) - \delta > \lambda\phi_1(w_1, w_2)$ showing that the deviation is gainful for b.

If $j = a$, then a obtains $\lambda\phi_1(w_1, w_2) + \phi_2(w_1, w_2)$. Let a unilaterally deviate to $(w_1 - \varepsilon, w_2)$ such that $\varepsilon > 0$ and $w_1 - \varepsilon > \underline{w}$. Following this deviation, a obtains

$$\phi_{12}(w_1 - \varepsilon, w_2) = \phi_1(w_1 - \varepsilon, w_2) + \phi_2(w_1 - \varepsilon, w_2).$$

As $w_1 > \underline{w}$ and $\lambda < 1$, we have $(1-\lambda)\phi_1(w_1, w_2) > 0$. Let $0 < \delta < (1-\lambda)\phi_1(w_1, w_2)/2$. Since for $i = 1, 2$, $\phi_i(w_1, w_2)$ is continuous in w_1, for sufficiently small $\varepsilon > 0$ we have $\phi_i(w_1 - \varepsilon, w_2) > \phi_i(w_1, w_2) - \delta$. Hence

$$\phi_{12}(w_1 - \varepsilon, w_2) > \phi_1(w_1, w_2) + \phi_2(w_1, w_2) - 2\delta > \lambda\phi_1(w_1, w_2) + \phi_2(w_1, w_2)$$

showing that the deviation is gainful for a.

Thus, at any NE we must have $w_1 = \min\{w_1^a, w_1^b, \overline{w}\} = \underline{w}$ so that $\min\{w_1^a, w_1^b\} = \underline{w}$. If $\max\{w_1^a, w_1^b\} > \underline{w}$, then $\exists\ j \in \{a, b\}$ such that $w_1^j = \underline{w}$ and $x_1^j = x_1^*(\underline{w}, w_2)$. If $j = b$, then b obtains $\phi_1(\underline{w}, w_2) = 0$ and b is better off unilaterally deviating to $\tilde{w}_1^b$ such that $\underline{w} < \tilde{w}_1^b < \min\{w_1^a, \overline{w}\}$ to obtain positive payoff. If $j = a$, then a obtains $\phi_{12}(\underline{w}, w_2)$. Let a unilaterally deviate to $(\underline{w} + \varepsilon, w_2)$ such that $\varepsilon > 0$ and $\underline{w} + \varepsilon < \min\{w_1^b, \overline{w}\}$. Following this deviation, a obtains $\phi_{12}(\underline{w} + \varepsilon, w_2)$. Since $\underline{w} \leq w_2$, by Lemma 2 we have $\phi_{12}(\underline{w} + \varepsilon, w_2) > \phi_{12}(\underline{w}, w_2)$. This shows the deviation is gainful for a. Therefore we must have $w_1^a = w_1^b = \underline{w}$ at any NE.

Step 3: To complete the proof, consider a strategy profile $((w_1^a, w_2^a), w_1^b)$ such that $w_1^a = w_1^b = \underline{w}$. Let $w_2 = \min\{w_2^a, \overline{w}\}$. At this strategy profile, choosing $x_1^a = \lambda x_1^*(\underline{w}, w_2)$, $x_1^b = (1-\lambda)x_1^*(\underline{w}, w_2)$ is optimal for the firm for any $\lambda \in [0, 1]$. Since $\phi_1(\underline{w}, w_2) = 0$, at this strategy profile the payoff of country b is $(1-\lambda)\phi_1(\underline{w}, w_2) = 0$ and the payoff of country a is: (i) $\lambda\phi_1(\underline{w}, \overline{w}) = 0$ if $w_2^a > \overline{w}$, (ii) $\lambda\phi_1(\underline{w}, w_2^a) + \phi_2(\underline{w}, w_2^a) = \phi_2(\underline{w}, w_2^a) \leq 0$ if $w_2 \leq \overline{w}$ and (iii) $\lambda\phi_1(\underline{w}, w_2^a) + \phi_2(\underline{w}, w_2^a) = \phi_2(\underline{w}, w_2^a) > 0$ if $\underline{w} < w_2^a \leq \overline{w}$. By Proposition 1(I)(i), $\phi_2(\underline{w}, w_2^a)$ has a unique maximizer $\widetilde{w}_2$ over $w_2^a \in [0, \overline{w}]$. Note that $\widetilde{w}_2 \in (\underline{w}, \overline{w}]$. Thus at any NE, we must have $w_1^a = w_1^b = \underline{w}$ and $w_2^a = \widetilde{w}_2$.

Finally we show that the profile $((\underline{w}, \widetilde{w}_2), \underline{w})$ indeed constitutes an NE. To see this first observe that at this strategy profile, for input 1 the firm is indifferent between the two countries. For any $\lambda \in [0,1]$, choosing $x_1^a = \lambda x_1^*(\underline{w}, \widetilde{w}_2)$ and $x_1^b = (1-\lambda)x_1^*(\underline{w}, \widetilde{w}_2)$ is optimal for the firm. For any choice of λ by the firm, b obtains $(1-\lambda)\phi_1(\underline{w}, \widetilde{w}_2) = 0$. If b unilaterally deviates to $w_1^b \neq \underline{w}$, it would obtain negative payoff if $w_1^b < \underline{w}$ and zero if $w_1^b > \underline{w}$, so no such deviation is gainful.

For any choice of λ by the firm, a obtains $\lambda\phi_1(\underline{w}, \widetilde{w}_2) + \phi_2(\underline{w}, \widetilde{w}_2) = \phi_2(\underline{w}, \widetilde{w}_2) > 0$. Suppose a unilaterally deviates to (w_1^a, w_2^a). If $w_1^a \geq \underline{w}$, then a would obtain $\phi_2(\underline{w}, w_2^a)$, which cannot exceed $\phi_2(\underline{w}, \widetilde{w}_2)$. If $w_1^a < \underline{w}$ and either $w_2^a \leq \underline{w}$ or $w_2^a > \overline{w}$, then a would obtain at most zero. If $w_1^a < \underline{w} < w_2^a \leq \overline{w}$, then a would obtain $\phi_{12}(w_1^a, w_2^a)$, which is lower than $\phi_{12}(\underline{w}, w_2^a)$ (by Lemma 2). Since $\phi_{12}(\underline{w}, w_2^a) = \phi_1(\underline{w}, w_2^a) + \phi_2(\underline{w}, w_2^a) = \phi_2(\underline{w}, w_2^a) \leq \phi_2(\underline{w}, \widetilde{w}_2)$, this deviation is also not gainful for country a. This shows that the strategy profile $((\underline{w}, \tilde{w}_2^a), \underline{w})$ constitutes an NE.

This shows that the game $\Gamma(12,1)$ has multiple NE. Any NE has $(w_1^a, w_2^a) = (\underline{w}, \widetilde{w}_2)$ and $w_b^1 = \underline{w}$. Since both countries offer the same wage $\underline{w}$ for input 1, the firm is indifferent between the two countries for this input. This results in multiple NE, but all NE are payoff-equivalent. ■

5.2.5 One country invested in both inputs, another country invested in no input

Consider the game $\Gamma(12,0)$. This is the game in which country a invested in both inputs $1,2$ and country b invested in no input (the analysis is similar for the game $\Gamma(0,12)$). In the game $\Gamma(12,0)$, country b always obtains zero payoff. In this game country a announces wage pair (w_1^a, w_2^a). Let $w_i = \min\{w_i^a, \overline{w}\}$ for $i = 1,2$. The firm employs $x_i^*(w_1, w_2)$ units of input i. If $w_i^a = w_i$, the firm employs input i from country a; otherwise it employs input i from the external source at wage $\overline{w}$.

Observation 5 $\Gamma(12,0)$ *has a unique NE:* $w_1^a = w_1^b = \overline{w}$. *At the NE country a obtains* $\phi_{12}(\overline{w}, \overline{w}) = (\overline{w} - \underline{w})K/\overline{w} > 0$ *and country b obtains zero payoff.*

Proof First observe that at any NE of $\Gamma(12,0)$, we must have $w_i^a \leq \overline{w}$ for both $i = 1,2$. If $w_i^a > \overline{w}$ for both $i = 1,2$, then country a obtains zero payoff. By unilaterally deviating to $(\overline{w}, w_2^a)$, it obtains payoff $\phi_1(\overline{w}, w_2^a) > 0$, so the deviation is gainful. If $w_i^a > \overline{w}$ for only one i, without loss of generality let $w_1^a \leq \overline{w} < w_2^a$. Then country a obtains $\phi_1(w_1^a, \overline{w})$. By unilaterally deviating to $(w_1^a, \overline{w})$, it would obtain $\phi_{12}(w_1^a, \overline{w}) = \phi_1(w_1^a, \overline{w}) + \phi_2(w_1^a, \overline{w}) > \phi_1(w_1^a, \overline{w})$ so the deviation is gainful.

Next we show that at any NE we must have $w_1^a = w_2^a$. Consider (w_1^a, w_2^a) such that $w_i^a \leq \overline{w}$ for $i = 1,2$. Then a obtains $\phi_{12}(w_1^a, w_2^a)$. If $w_1^a < w_2^a \leq \overline{w}$, then by unilaterally deviating to $(\tilde{w}_1^a, w_2^a)$ such that $w_1^a < \tilde{w}_1^a < w_2^a$, country a would obtain $\phi_{12}(\tilde{w}_1^a, w_2^a) > \phi_{12}(w_1^a, w_2^a)$ (by Lemma 2), so the deviation is gainful. By the same reasoning there is a gainful unilateral deviation if $w_2^a < w_1^a \leq \overline{w}$.

Finally consider (w_1^a, w_2^a) such that $w_1^a = w_2^a = w \leq \overline{w}$. By (8), a obtains $\phi_{12}(w, w) = K - \underline{w}(x_1^*(w, w) + x_2^*(w, w))$. Denoting $\alpha/(1-\alpha)^{1/(1+\rho)} = t$, from (13) we have $x_1^*(w, w) = K/w(1+t)$, $x_2^*(w, w) = tK/w(1+t)$, so that $x_1^*(w, w) + x_2^*(w, w) = K/w$. This shows that $\phi_{12}(w, w)$ is increasing in w and its unique maximizer for $0 < w \leq \overline{w}$ is attained at $w = \overline{w}$. This proves that this game has a unique NE: $w_1^a = w_1^b = \overline{w}$. The payoffs are immediate. ∎

5.2.6 Both countries invested in both inputs

Consider the game $\Gamma(12, 12)$ (the game in which both countries have invested in both inputs $1, 2$). In this game, country $j \in \{a, b\}$ announces wage pair (w_1^j, w_2^j). Let $w_i = \min\{w_i^a, w_i^b, \overline{w}\}$ for $i = 1, 2$. The firm employs $x_i^*(w_1, w_2)$ units of input i. If $\min\{w_i^a, w_i^b\} > \overline{w}$, the firm employs input i from the external source; otherwise it employs input i from countries that offer the lowest wage for input i.

Observation 6 $\Gamma(12, 12)$ *has a unique NE:* $w_i^a = w_i^b = \underline{w}$ *for both* $i = 1, 2$. *At the NE, each country obtains zero payoff.*

Proof For $i = 1, 2$, denote $w_i = \min\{w_i^a, w_i^b, \overline{w}\}$. We prove the result in following steps.

Step 1: We cannot have an NE where $w_i < \underline{w}$ for some $i = 1, 2$. To see this, take $i = 1$ (same reasoning applies for $i = 2$) and consider a strategy profile in which $w_1 < \underline{w}$. Then $\exists$ a country $j \in \{a, b\}$ such that $x_1^j = \lambda x_1^*(w_1, w_2)$ for some $\lambda \in (0, 1]$. If $w_2 \leq \underline{w}$, then country j obtains at most $\lambda \phi_1(w_1, w_2) < 0$. By unilaterally deviating to $(\underline{w}, \underline{w})$, country j would obtain zero payoff, so such a deviation is gainful.

Next suppose $w_2 > \underline{w}$. Then $\exists$ $j \in \{a, b\}$ such that $x_2^j = \lambda x_2^*(w_1, w_2)$ for some $\lambda \in [0, 1)$. Let $j = a$ (same reasoning applies if $j = b$). There are two possibilities: (i) $w_1 = w_1^a < w_1^b$ and (ii) $w_1 = w_1^b \leq w_1^a$.

Let $w_1 = w_1^a < w_1^b$. Then a obtains $\phi_1(w_1, w_2) + \lambda \phi_2(w_1, w_2)$. Let a unilaterally deviate to $(w_1, w_2 - \varepsilon)$ where $\varepsilon > 0$ and $w_2 - \varepsilon > \underline{w}$. Following this deviation, a would obtain $\phi_1(w_1, w_2 - \varepsilon) + \phi_2(w_1, w_2 - \varepsilon)$. Since $w_2 > \underline{w}$ and $\lambda < 1$, we have $(1 - \lambda)\phi_2(w_1, w_2) > 0$. Let $0 < \delta < (1 - \lambda)\phi_2(w_1, w_2)/2$. Since $\phi_i(w_1, w_2)$ is continuous in w_2, for sufficiently small $\varepsilon > 0$ we have

$$\phi_1(w_1, w_2 - \varepsilon) + \phi_2(w_1, w_2 - \varepsilon) > \phi_1(w_1, w_2) + \phi_2(w_1, w_2) - 2\delta > \phi_1(w_1, w_2) + \lambda \phi_2(w_1, w_2)$$

showing that the deviation is gainful for a.

Let $w_1 = w_1^b \leq w_1^a$. Since $w_1 < \underline{w}$, in this case a obtains at most $\lambda \phi_2(w_1, w_2)$. Let a unilaterally deviate to $(\underline{w}, w_2 - \varepsilon)$ where $\varepsilon > 0$ and $w_2 - \varepsilon > \underline{w}$. Following this deviation,

a would obtain $\phi_2(w_1, w_2 - \varepsilon)$. Since $w_2 > \underline{w}$ and $\lambda < 1$, we have $(1-\lambda)\phi_2(w_1, w_2) > 0$. Let $0 < \delta < (1-\lambda)\phi_2(w_1, w_2)$. Since $\phi_2(w_1, w_2)$ is continuous in w_2, for sufficiently small $\varepsilon > 0$ we have $\phi_2(w_1, w_2 - \varepsilon) > \phi_2(w_1, w_2) - \delta > \lambda\phi_2(w_1, w_2)$ showing that the deviation is gainful for a.

Step 2: We cannot have an NE where $w_i > \underline{w}$ for some $i = 1, 2$. To see this, take $i = 1$ (same reasoning applies for $i = 2$) and consider a strategy profile in which $w_1 > \underline{w}$. Then $\exists\ j \in \{a, b\}$ such that $x_1^j = \lambda x_1^*(w_1, w_2)$ for some $\lambda \in [0, 1)$. Let $j = a$ (same reasoning applies for $j = b$). There are two possibilities: (i) $w_2 = w_2^a \leq w_2^b$ and (ii) $w_2 = w_2^b < w_2^a$.

Let $w_2 = w_2^a \leq w_2^b$. Since $w_2 \geq \underline{w}$ (by Step 1), in this case a obtains at most $\lambda\phi_1(w_1, w_2) + \phi_2(w_1, w_2)$. Let a unilaterally deviate to $(w_1 - \varepsilon, w_2 - \varepsilon)$ where $\varepsilon > 0$ and $w_1 - \varepsilon > \underline{w}$. Following this deviation, a would obtain $\phi_1(w_1 - \varepsilon, w_2 - \varepsilon) + \phi_2(w_1 - \varepsilon, w_2 - \varepsilon)$. Since $w_1 > \underline{w}$ and $\lambda < 1$, we have $(1-\lambda)\phi_1(w_1, w_2) > 0$. Let $0 < \delta < (1-\lambda)\phi_1(w_1, w_2)/2$. Since $\phi_1(w_1, w_2)$ is continuous in w_1, w_2, for sufficiently small $\varepsilon > 0$ we have

$$\phi_1(w_1 - \varepsilon, w_2 - \varepsilon) + \phi_2(w_1 - \varepsilon, w_2 - \varepsilon)$$

$$> \phi_1(w_1, w_2) + \phi_2(w_1, w_2) - 2\delta > \lambda\phi_1(w_1, w_2) + \phi_2(w_1, w_2)$$

showing gainful deviation for a.

Let $w_2 = w_2^b < w_2^a$. In this case a obtains $\lambda\phi_1(w_1, w_2)$. Let a unilaterally deviate to $(w_1 - \varepsilon, w_2^a)$ where $\varepsilon > 0$ and $w_1 - \varepsilon > \underline{w}$. Following this deviation, a would obtain $\phi_1(w_1 - \varepsilon, w_2)$. Since $w_1 > \underline{w}$ and $\lambda < 1$, we have $(1-\lambda)\phi_1(w_1, w_2) > 0$. Let $0 < \delta < (1-\lambda)\phi_1(w_1, w_2)$. Since $\phi_1(w_1, w_2)$ is continuous in w_2, for sufficiently small $\varepsilon > 0$ we have $\phi_1(w_1 - \varepsilon, w_2) > \phi_1(w_1, w_2) - \delta > \lambda\phi_1(w_1, w_2)$ showing gainful deviation for a.

Step 3: By Steps 1-2, at any NE, for both $i = 1, 2$, we must have $\min\{w_i^a, w_i^b\} = \underline{w}$. Now we show that at any NE, we must also have $\max\{w_i^a, w_i^b\} = \underline{w}$ for both i. Consider a strategy profile such that $\min\{w_i^a, w_i^b\} = \underline{w}$ for both $i = 1, 2$ and there is some i (say $i = 1$) for which $\min\{w_1^a, w_1^b\} = \underline{w} < \max\{w_1^a, w_1^b\}$. Without loss of generality, let $w_1^a = \underline{w} < w_1^b$. Since $\min\{w_i^a, w_i^b\} = \underline{w}$ for both inputs $i = 1, 2$, each country obtains zero payoff. Let country a unilaterally deviate to $(\tilde{w}_1^a, w_2^a)$ such that $\underline{w} < \tilde{w}_1^a < w_1^b$. Following this deviation, the firm would employ $x_1^*(\tilde{w}_1^a, \underline{w})$ units of input 1 from country a. So a would obtain $\phi_1(\tilde{w}_1^a, \underline{w}) > 0$, making the deviation gainful. This shows that at any NE, for both $i = 1, 2$, we must have $w_i^a = w_i^b = \underline{w}$.

Finally observe that the strategy profile $((w_1^a, w_2^a), (w_1^b, w_2^b))$ such that $w_i^a = w_i^b = \underline{w}$ for both $i = 1, 2$ is indeed an NE. This is because at this strategy profile each country gets zero payoff and there is no unilateral deviation that gives positive payoff to a country. ■

Endnotes

1. In a recent paper, Cueto (2017) provides a convenient list of the dimensions along which such dilution has been noted.

2. Strictly speaking, the function F is the revenue of the firm, but revenue and profit are operationally equivalent in our model. Calling F the profit function enables us to better interpret our results.

3. We make the tie-breaking assumption that if a country sets a wage exactly equal to $\overline{w}$, the firm employs labour from that country rather than using its outside option.

4. For example, take $\varepsilon_i = (\underline{w}^i + \overline{w})/2$ in Case 1 and $\varepsilon_i = \delta\overline{w}$ in Case 2 of (I)(i).

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