Dynamic Growth Rate of U.S. Economy

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Abstract

This paper reports the estimates of the dynamic growth rate of U.S. economy using exponential growth model, Cob-Douglas production function with a regression framework. The estimates indicates that 100% output growth is broken down into 58% technology growth, 19.10% labor growth, and 22.90% capital growth. Growth rates of U.S. production, capital, and employment are decreasing by 0.4%, 0.6%, and 0.01% respectively for each additional year regardless of recession while growth rate of technological changes in U.S. economy has been changing in a systematic way. It also shows that forecasted growth rate of U.S. output with restricted elasticity is lower than that with unrestricted elasticity.

Key Words: Elasticity, Cobb-Douglas production function, exponential growth model.

1. **Introduction**

The U.S. has the largest Gross Domestic Product (GDP) over the world, and GDP of this developed nation is increasing exponentially for a long tme. Mostly the growth of this largest economy is driven by the growth rate of employment, technology, and capital. Economists are trying to find the key catalysts of tremendous growth of this economy. Shanker, M. C., & Astrachan, J. H.[12] shows that family business is one of the components of the growth of GDP as well as employment. One of the empirical research have done by Kim, C. J., & Nelson, C. R.[7] which shows that there is a break in GDP growth toward stabilization. They have also found a narrowing gap between growth rates during recessions and booms that is at least as important as any decline in the volatility of shocks. Barro, R. J. [1] finds that the growth rate of real per capita GDP is tightly related with initial human capital and initial real per capita GDP. Barro also shows that developed nations put more physical investment to the growth of GDP. Another empirical reseach done by King, R. G., & Levine, R.[8] which shows that the financial mechanism can promote economic growth, and future economic growth, physical capital accumulation, and economic efficiency improvement is driven by the predetermined component of financial development. Corrado, C., Hulten, C., & Sichel, D. [2] show that the rate of change of output per worker increases more rapidly when intangibles are counted as capital, and capital deepening becomes

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the unambiguously dominant source of growth in labor productivity. They also show the role of multifactor productivity is correspondingly diminished, and labor's income share is found to have decreased significantly over the last 50 years. Research have done by Groot, W., & Van Den Brink, H. M. [5] have shown that growth rate of the labor force has a positive impact on the incidence of overeducation, while the unemployment rate has a negative impact on the rate of return to education. Fagerberg, J. [3] finds that the level of economic development, measured as GDP per capita, and the level of technological development, measured through R& D or patent statistics are closely related. The empirical evidence of Friedberg, R. M., & Hunt, J. [4] shows that a 10 percent increase in the fraction of immigrants in the population decreases native wages by 0-1 percent. Their findings also show that those natives who are the closest substitutes with immigrant labor do not suffer significantly as a result of increased immigration. Their findings also suggest that there is no evidence of economically significant reductions in native employment, the impact on natives' per capita income growth depends crucially on the immigrants' human capital levels. Hall, R. E.[6] finds that fluctuations in the natural unemployment rate and the fluctuations in the observed unemployment rate are not quite correlated. The research of Phillips, B. D., & Kirchhoff, B. A. [9] shows that two out of five new firms survive at least six years and over half of the survivors grow.

2. **Description of data**

Table 1 presents the particular data series we will use. We will use Real GDP to measure economic performance. The other series will be used to explain the performance also.

TABLE 1. Raw Data Series

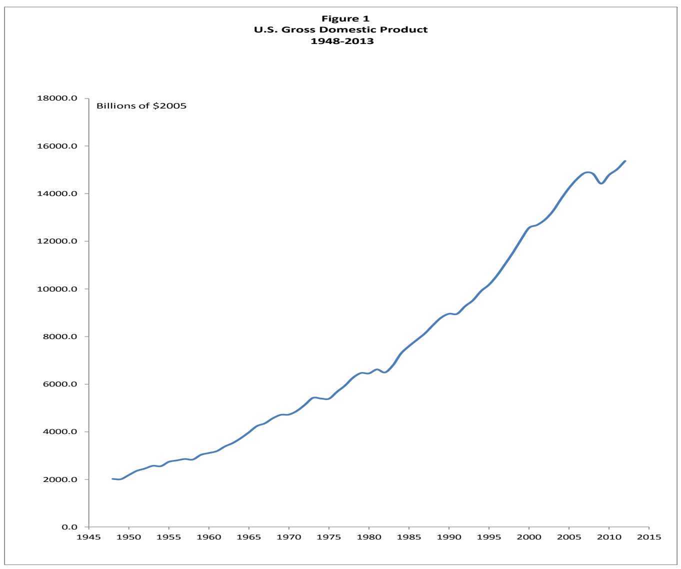
Variable	Variable Name	Units	Source
Real GDP	Gross domestic product	(Billions of $ 2005)	Bureau of Economics Analysis
GDP	Gross domestic product	(Billions of current dollars)	Bureau of Economic Analysis
COMP	Compensation of employees	(Billions of current dollars)	Bureau of Economic Analysis
EMP	Full time equivalent employees	(Thousands of Employees)	Bureau of Economic Analysis
CAP	Net Stock of Fixed Assets and Consumer Durables	(Billions of $ 2005)	Bureau of Economic Analysis
UTIL	Capacity utilization rate	Percent of total capacity	Federal Reserve Board of Governors

Real Gross Domestic product (GDP) indicates the total value of the product in base year 2005.
Nominal GDP indicates the current value of the product. It is obviously better to compare the real GDP rather than nominal GDP because the real GDP does not include the inflation of any product whereas the nominal GDP includes the current prices of the products which include the inflation.
Compensation of employees (COMP) is a payment which workers are getting paid if they are injured on the job.

Full-time equivalent employees (EMP) is the ratio of total number of paid hours during a period (part time, full time, contracted) by the number of working hours in that period Mondays through Fridays.
Net Stock of Fixed Assets and Consumer Durables (CAP) defined by BEA to consist of private fixed assets (excluding durable goods owned by consumers) and government fixed assets.
Capacity utilization rate (UTIL) is a metric used to measure the rate at which potential output levels are being met or used which displayed as a percentage, capacity utilization levels given insight into the overall slack that is in the economy or a firm at a given point in time.

3. **The Path of U.S. economy**

Figure 1 presents U.S. Real GDP from 1948 to 2013.



In the most recent year 2013 we can see the real GDP is 15,710.3 billion dollar, whereas in the year of 1948 the total real GDP is 2020 billion dollar on the base year 2005. Compare to the real GDP of 1948 the real GDP of 2013 was 13,690.3 billion dollar higher. From Figure 1 we can see that the real GDP is increasing.

3.1. **First Order Exponential Model:** The "first order" exponential model generates a path with a constant growth rate [2]. Letting r represent this constant rate and letting y denote the value of the variable that is growing, the value of the variable at a point in time t is given by

$$y_t = y_o e^{rt}, \tag{1}$$

2

If $y_t = y_o e^{rt}$ then $\frac{dy_t}{dt} = r y_o e^{rt} = r y_t$. Therefore $\frac{1}{y_t}\frac{dy_t}{dt} = r$ is the growth rate of y_t.

where y_o is the initial value of the variable at point in time $t = 0$.

Given data on the variable y_t, the least squares regression technique can be used to obtain an estimate of the constant growth rate r. This estimate provides a characterization of the trend followed by the variable, in particular the best fit constant growth rate. In equation (1) the fact that r is an exponent implies that there is a non-linear relationship between r and y_t. That is, we would not get a straight line if we plotted the relationship between y_t and r. Because least squares is a "linear regression" technique, we must "linearize" the model before we can apply least squares. This can be accomplished by taking the natural log of both sides of equation (1). Doing so, we obtain

$$\ln(y_t) = \ln(y_o) + rt. \tag{2}$$

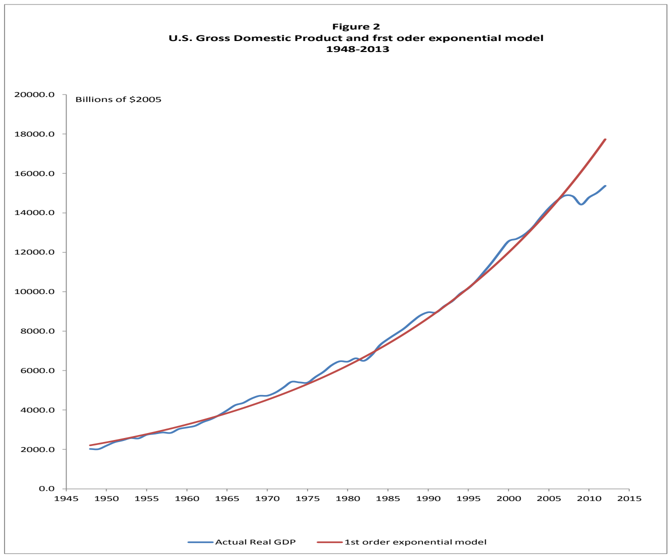
The fact that r is a coefficient on t implies that there is a linear relationship between r and $\ln(y_t)$. Because of this linear relationship, we can obtain a least squares estimate for r by regressing $\ln(y_t)$ on t. Doing so, the estimated log-linear equation is

$$\begin{array}{cccccc} \widehat{\ln(y_t)} & = & 7.698 & + & 0.033t, & R^2 = 0.993 \\ & & (0.0132)^{***} & & (0.0004)^{***} & \end{array} \tag{3}$$

The numbers inside the parentheses with asterisks indicate the standard errors of the corresponding estimates. Statistically at 1% level of significant [3] the estimated constant growth rate $r = 0.033$, indicates that the real GDP is increasing by 3.3% each year, and the intercept $\ln(y_0) = 7.698$ indicates that the initial real GDP was $y_0 = 2203.94$ billions of dollar (base year 2005) in 1948. The $R^2 = 0.993$ indicates that 99.3% of the variance of y_t is explained by this log-linear model.

If we plot the fitted first order exponential model with the actual real GDP data as in Figure 2 then we can see how the first order exponential model fits the actual data.

[3] Level of significance, α%, indicates that we are $(100-\alpha)$% confident that the test statistic is in the rejection region in the favor of alternative hypothesis. In our analysis always we will consider the null hypothesis as the estimates are statistically not other than zero. If we reject the null hypothesis at α% level of significance then we are $(100-\alpha)$% confident that the estimates are other than zero. We will express the level of significance by asterisks raised on standard errors. Single $\star$, double $\star$, and triple $\star$, indicates that the estimate is significant at 10% level, at 5% level, and at 1% level respectively.



While the first order exponential model assumes the growth rate is constant, the trend growth rate need not be constant. Higher order exponential model allows for a non-constant growth rate. The second order exponential model is

$$y_t = y_o e^{rt + rt^2}, \tag{4}$$

which after linearizing becomes

$$\ln(y_t) = \ln(y_o) + r_1 t + r_2 t^2. \tag{5}$$

The growth rate implied by the model (4) is

$$\frac{1}{y_t}\frac{dy_t}{dt} = r_1 + 2r_2 t. \tag{6}$$

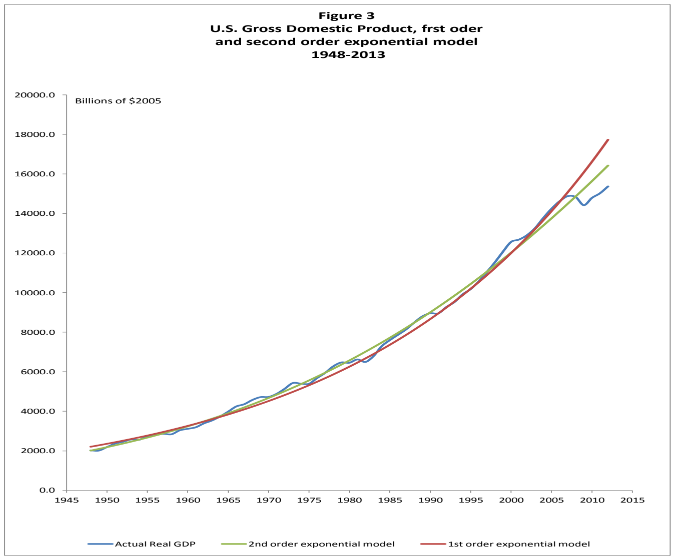
If $r_2 < 0$ then growth rate is decreasing, if $r_2 > 0$ then growth rate is increasing, and if $r_2 = 0$ then growth rate reduces to constant rate. Regressing $\ln(y_t)$ on t and t^2 the estimated model is

$$\widehat{\ln y_t} = \quad 7.609 \quad + \quad 0.041t \quad - \quad 0.00013t^2 \qquad R^2 = 0.997 \tag{7}$$

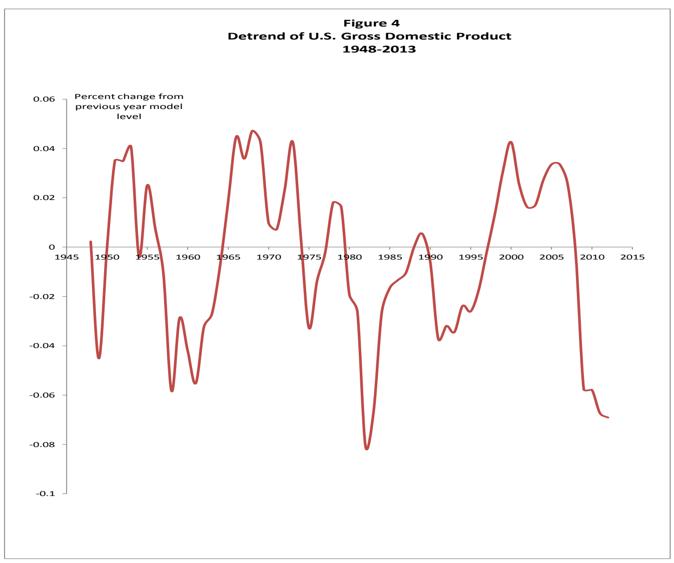
$$(0.012)^{\star\star\star} \qquad (0.0009)^{\star\star\star} \quad (0.00001)^{\star\star\star} \tag{8}$$

The coefficient on t^2 is statistically significant at 1% level of significance. Growth rate is decreasing since this estimate is negative. The estimated growth rate is $r(t) = 0.041 - .00026t$. For example, in 1970(t=22) the growth rate is $r(22) = 0.035$ whereas in 2005(t=57) the growth rate is $r(57) = 0.026$.

Figure 3 presents second order exponential model along with actual real GDP and fitted first order exponential model. The second order exponential model fits the data better.



3.2. **Business Cycles:** We can detrend the data by subtracting the second order exponential model (the best model) from the real GDP. We can avoid heteroskedasticity by dividing this detrend data by the model levels. Doing so, we get the following graph.



From the Figure 4, and the table 2, we can see there are six business cycles in the U.S. economy from 1949 to 2007. The highest cycle duration is 18 years, which occurs two times, from 1957 to 1974 as well as from 1990 to 2007. Whereas the the lowest cycle duration is 3 years, which occurs only once from 1954 to 1956. The highest trough duration is 9 years, which occurs from 1980 to 1988 with the highest amplitude, 0.081, in 1982 over the whole economic period. And the lowest trough duration is 1 year takes place in 1954 with the lowest amplitude, 0.004. On the other hand the highest peak duration is 10 years, which takes place twice, from 1965 to 1974 as well as from

1998 to 2007 with the highest amplitude, 0.045 in 1966 over the whole economic period. And the lowest peak duration is 1 year, which occurs once in 1989 with the lowest amplitude, 0.006. If we observe the Figure 4 carefully we can see the U.S. economy is in trough since 2008. Since 9 years is the maximum trough period since 1949, we can expect the U.S. economy will come back to peak very soon.

TABLE 2. Business cycles observed since 1948

Cycle duration	trough	amplitude(year)	peak	amplitude(year)
1949-1953	1949-1950	0.045(1949)	1951-1953	0.041(1953)
1954-1956	1954-1954	0.004(1954)	1955-1956	0.025(1955)
1957-1974	1957-1964	0.058(1958)	1965-1974	0.045(1966)
1975-1979	1975-1977	0.033(1975)	1978-1979	0.018(1978)
1980-1989	1980-1988	0.081(1982)	1989-1989	0.006(1989)
1990-2007	1990-1997	0.037(1991)	1998-2007	0.043(2000)

4. Modeling and Forecasting Growth Rates of U.S. Production, Employment, and Capital

Here we will discuss the modeling and forecasting of growth rates of U.S. production, employment, and capital. And we will also examine whether the growth rates are slowing or not.

We can show that the "log difference" $\ln(y_t) - \ln(y_{(t-)})$, where $y_t = e^{rt}$, is the growth rate [4] of the variable y_t that is growing at a constant rate with continuous compounding, that is,

$$\ln(y_t) - \ln(y_{(t-)}) = r \tag{9}$$

4.1. Growth Rate of U.S. Production:

Suppose growth rate $g_{yt} = \ln(y_t) - \ln(y_{(t-)})$ is defined for the real GDP of the U.S economy. Now regressing g_{yt} on t we get

$$\begin{array}{cccccc} \widehat{g}_{yt} & = & 0.04 & - & 0.0003t, & R^2 = 0.0753 \\ & & (0.006)^{***} & & (0.0002)^{***} & \end{array} \tag{10}$$

The estimates are statistically significant at 1% level of significance. The intercept 0.04 indicates that the initial growth rate of U.S. production is 4%, and the coefficient of t, -0.0003, indicates that the growth rate of U.S. production is decreasing by 0.03% for each additional year. So $\widehat{g}_{yt}$ indicates that the growth rate of U.S. production is slowing.

[4] The log difference $\ln(y_t) - \ln(y_{(t-)})$ is a measure of instantaneous growth rate comes from the exponential model $y_t = y_o e^{rt}$. If we substitute t by $t-1$ then we have $y_{t-1} = y_o e^{r(t-1)}$. We can show that the log difference of these two equations is the constant growth rate r, that is,

$$\ln(y_t) - \ln(y_{(t-)}) = \ln(y_o) + rt\ln(e) - (\ln(y_o) + r(t-1)\ln(e))$$
$$= \ln(y_o) + rt - \ln(y_o) - r(t-1) = \ln(y_o) + rt - \ln(y_o) - rt + r = r$$

We can measure whether the growth rate of U.S. production is slowing or not regardless of recession by using dummy variable. Suppose a dummy variable is defined as follows for real GDP

$$D = \begin{cases} 1, & \text{if recession} \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

Now if we regress the growth rate g_{yt} on t and dummy variable D then we get

$$\begin{array}{lllllll} \hat{g}_{yt} = & 0.052 & - & 0.0004t & - & 0.05D & R^2 = 0.6276 \\ & (0.004)^{\star\star\star} & & (9.6E-05)^{\star\star\star} & & (0.005)^{\star\star\star} & \end{array} \tag{12}$$

All estimates are statistically significant at 1% level of significant. The intercept 0.052 indicates that the initial growth rate of U.S. production is 5.2% if the economy is not in recession (D=0). However, if the economy is in recession (D=1) then the initial growth rate is 0.2%. (5.2%-0.5%=0.2%). The coefficient of t, -0.0004, indicates that the growth rate of U.S. production is decreasing by 0.04% for each additional year

From the above regression we can see that the estimated growth rate of U.S. production is slowing regardless of recession.

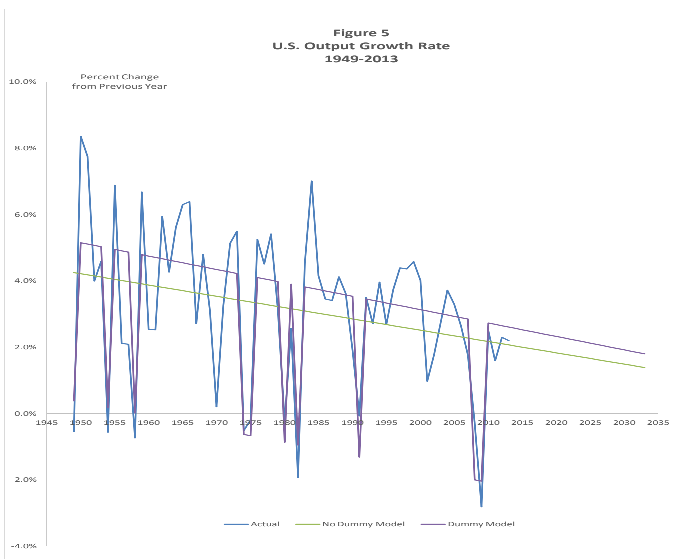


Figure 5 indicates the actual growth rate ("log difference"), the fitted growth rates without dummy (10), and with dummy (12) with 20 years forecasting. It is obvious from the graphs that the growth rate of the U.S. production is slowing regardless of recession. It looks trend of the growth rate towards zero even negative.

4.2. **Growth Rate of U.S. Capital:** Similarly we can discuss the growth rate of the U.S. capital. Suppose the growth rate $g_{kt} = \ln(k_t) - \ln(k_{(t-)})$ is defined for the capital of the U.S economy.

Now regressing g_{kt} on t we get

$$\begin{array}{cccccc} \widehat{g}_{kt} & = & 0.04 & - & 0.0004t & R^2 = 0.0152 \\ & & (0.01)^{***} & & (0.0004)^{***} & \end{array} \tag{13}$$

The estimates are statistically significant at 1% level of significance. The intercept 0.04 indicates that the initial growth rate of U.S. capital is 4%, and the coefficient of t, -0.0004, indicates that the growth rate of U.S. capital is decreasing by 0.04% for each additional year. So $\widehat{g}_{kt}$ indicates that the growth rate of U.S. capital is slowing.

We can measure whether the growth rate of U.S. capital is slowing or not regardless of recession by using dummy variable. Suppose a dummy variable is defined as follows for the U.S. capital

$$\mathrm{K} = \begin{cases} 1, & \text{if recession} \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

Now if we regress the growth rate g_{kt} on t and dummy variable K then we get

$$\begin{array}{cccccccc} \widehat{g}_{kt} = & 0.07 & - & 0.0006t & - & 0.11\mathrm{K} & R^2 = 0.652 \\ & (0.009)^{\star\star\star} & & (0.0002)^{\star\star\star} & & 0.01)^{\star\star\star} & \end{array} \tag{15}$$

All estimates are statistically significant at 1% level of significant. The intercept 0.07 indicates that the initial growth rate of U.S. capital is 7% if the economy is not in recession (K=0). However, if the economy is in recession (K=1) then the initial growth rate is -4%. (7%-11%=-4%). The coefficient of t, -0.0006, indicates that the growth rate of U.S. capital is decreasing by 0.06% for each additional year

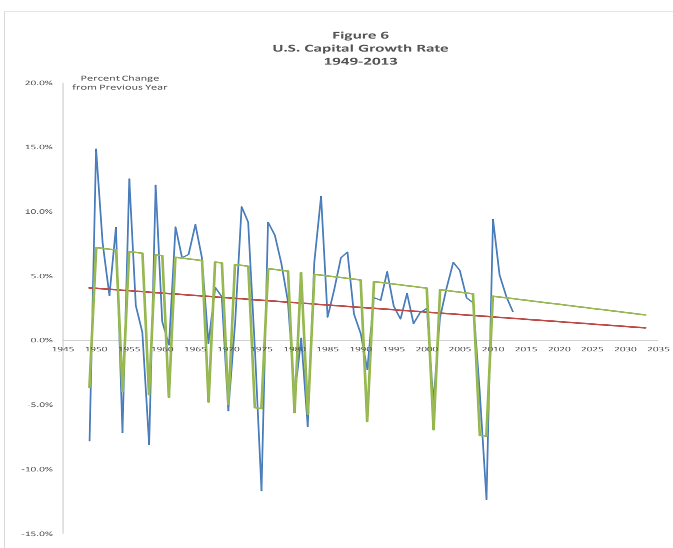


Figure 6 indicates the actual growth rate ("log difference"), the fitted growth rates without dummy (13), and with dummy (15) with 20 years forecasting. It is obvious from the graphs that the growth rate of the U.S. capital is slowing regardless of recession. It looks trend of the growth rate towards zero even negative.

4.3. **Growth Rate of U.S. Employment:** Similarly we can discuss the growth rate of the U.S. employment. Suppose the growth rate $g_{lt} = \ln(l_t) - \ln(l_{(t-)})$ is defined for the employment of the U.S economy. Now regressing g_{lt} on t we get

$$\begin{array}{lccccc} \widehat{g}_{lt} & = & \underset{(0.005)^{***}}{0.02} & - & \underset{(0.0002)^{***}}{0.0002t} & R^2 = 0.0293 \end{array} \tag{16}$$

The estimates are statistically significant at 1% level of significance. The intercept 0.02 indicates that the initial growth rate of U.S. employment is 2%, and the coefficient of t, -0.0002, indicates that the growth rate of U.S. employment is decreasing by 0.02% for each additional year. So $\widehat{g}_{lt}$ indicates that the growth rate of U.S. employment is slowing.

We can measure whether the growth rate of U.S. employment is slowing or not regardless of recession by using dummy variable. Suppose a dummy variable is defined as follows for the U.S. employment

$$\mathrm{L} = \begin{cases} 1, & \text{if recession} \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

Now if we regress the growth rate g_{kt} on t and dummy variable L then we get

$$\begin{array}{lccccc} \widehat{g}_{lt} = & \underset{(0.003)^{\star\star\star}}{0.03} & - & \underset{(9.04E-05)^{\star\star\star}}{0.0001t} & - & \underset{0.004)^{\star\star\star}}{0.04\mathrm{L}} & R^2 = 0.6401 \end{array} \tag{18}$$

All estimates are statistically significant at 1% level of significant. The intercept 0.03 indicates that the initial growth rate of U.S. employment is 3% if the economy is not in recession (L=0). However, if the economy is in recession (L=1) then the initial growth rate is -1%. (3%-4%=-1%). The coefficient of t, -0.0001, indicates that the growth rate of U.S. employment is decreasing by 0.01% for each additional year

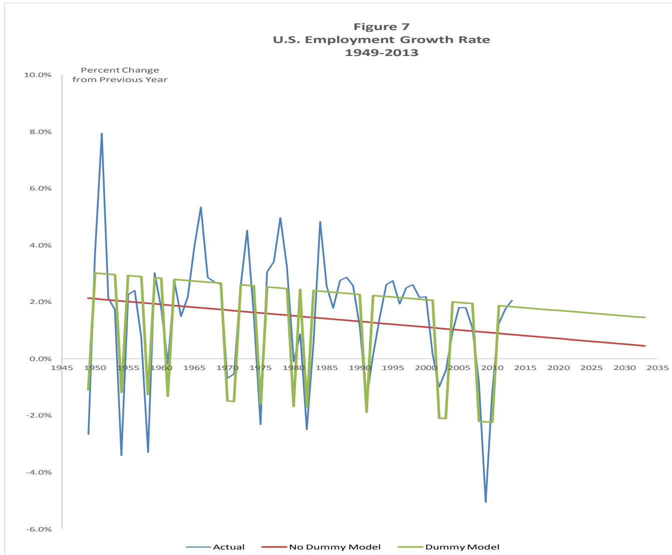


Figure 7 indicates the actual growth rate ("log difference"), the fitted growth rates without dummy (16), and with dummy (18) with 20 years forecasting. It is obvious from the graphs that the growth rate of the U.S. employment is slowing regardless of recession. It looks trend of the growth rate towards zero even negative.

Above analysis indicates that the growth rates of U.S. Production, Employment, and Capital are slowing down regardless of recession.

5. **Explaining and Forecasting Economic Growth**

In this section we will explain and forecast the economic growth of U.S. economy using the following Cobb Douglas production function.

$$Y = (AL)^{\beta} K^{\alpha} \tag{19}$$

We can convert this Cobb Douglas production function to a model in the growth rates [5]. Doing so we get

$$g_Y = \beta g_A + \beta g_L + \alpha g_K \tag{20}$$

Where $g_Y, g_A, g_L,$ andg_K represent the growth rate of output Y, technology A, labor L, and capital K respectively .

We can estimate the growth rate version of he Cobb Douglas production function (20) by regressing g_Y on $g_L,$and g_K to obtain estimates of $\alpha, \beta,$and βg_A assuming the production of U.S. economy follows Cobb Douglas production function. Doing so we get

[5]Taking natural log from both sides of the equation (19) we get $\ln Y = \beta \ln A + \beta \ln L + \alpha K$. Now taking derivative with respect to time we get $\frac{Y'}{Y} = \beta \frac{A'}{A} + \beta \frac{L'}{L} + \alpha \frac{K'}{K}$ which implies $g_Y = \beta g_A + \beta g_L + \alpha g_K$.

$$
\begin{array}{rclll}
\widehat{g}_Y & = & 0.018 \quad + & 0.41\, g_L + & 0.25 g_K \qquad\qquad R^2 = 0.8561 \\
 & & (0.0013)^{***} & (0.077)^{*} & (0.031)^{**}
\end{array}
\tag{21}
$$

All estimates are statistically significant at 5% level of significance except the estimate $\hat{\beta}$. The estimate $\hat{\beta}$ is significant at 10% level of significance. $R^2 = 0.8561$ indicates 85.61% variation is explained by this model. From above result we have $\widehat{\beta g_A} = 0.018$, $\hat{\beta} = 0.41$, and $\hat{\alpha} = 0.25$. Therefore $\hat{g}_A = \frac{\widehat{\beta g_A}}{\hat{\beta}} = \frac{0.018}{0.41} = 0.044$.

Setting $g_A = g_L = 0$ in equation (20) we get $\alpha = \frac{g_Y}{g_K} = \epsilon_{YK}$, which is the elasticity of output with respect to capital. Again setting $g_A = g_K = 0$ in the same equation we get $\beta = \frac{g_Y}{g_L} = \epsilon_{YL}$, which is the elasticity of output with respect to labor.

Therefore the regression result in equation (21) is telling us that the elasticity of output with respect to capital is 0.25, and the elasticity of output with respect to labor is 0.41.

We can measure the growth accounting using the estimated model (21) as follows.

$$
\begin{array}{ccccccc}
\frac{.0316}{.0316} & = & \frac{0.018}{.0316} & + & \frac{(0.41)(.0148)}{.0316} & + & \frac{(0.25)(.0290)}{.0316} \\
100\% & & 58\% & & 19.1\% & & 22.9\%
\end{array}
\tag{22}
$$

Therefore growth accounting is telling us 100% output growth is broken down into 58% technology growth, 19.1% labor growth, and 22.9% capital growth.

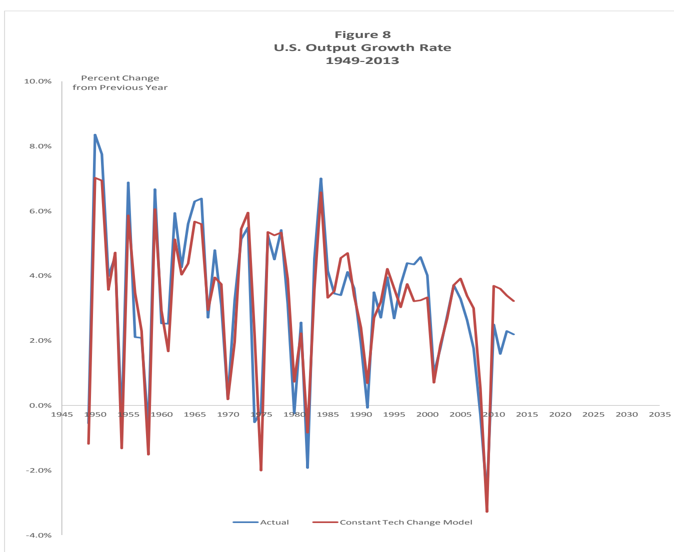


Figure 8 indicates that the actual growth rate, and the growth rate using the Cobb-Douglas production function, where technological change is constant. It looks there is no big difference between these growth rates of output.

Using the first order exponential model, $A = a_0 e^{r_1 t}$, for technology we can show that the variable r_1 gives the constant rate [6] at which technology is improving. Similarly we can show that the technology model $A = a_0 e^{r_1 t + r_2 t^2}$ has the growth rate $g_A = r_1 + 2r_2 t = \gamma_0 + \gamma_1$, where $r_1 = \gamma_0$ and $2r_2 t = \gamma_1$. However this growth rate is no more constant. It depends on time. We can make this growth rate constant by setting $r_2 = 0$. Now suppose production is given by the Cobb Douglas production function (19) and the level of technology is given by the exponential model $A = a_0 e^{r_1 t + r_2 t^2}$. We can convert these two models into the economy's output and technology growth rates model.

Plugging the exponential model for technology in the Cobb Douglas production function we get

$$g_Y = \beta g_A + \beta g_L + \alpha g_K, \tag{23}$$

$$\text{where } \beta g_A = \gamma_0 + \gamma_1 t \tag{24}$$

Regressing g_Y on t, g_L, and g_K to obtain the estimates of $\gamma_0, \gamma_1, \beta,$ and α we have

$$\begin{array}{lccccccc} \widehat{g}_Y = & .02 & - & 0.0002t & + & 0.38\,g_L & + & 0.25 g_K \quad R^2 = .8746 \\ & (0.0023)^{***} & & (5.7E-05)^{***} & & (0.07)^{*} & & (0.03)^{**} \end{array} \tag{25}$$

$$\widehat{g}_A = 0.05 - .0005t \tag{26}$$

Now suppose production is given by the Cobb Douglas production function (19) and the level of technology is given by the exponential model $A = a_0 e^{r_1 t + r_2 t^2 + r_3 t^3 + r_4 t^4}$. We can convert these two models into the economy's output and technology growth rates model.

Plugging the exponential model for technology in the Cobb Douglas production function we get

$$Y = \left(a_0 e^{r_1 t + r_2 t^2 + r_3 t^3 + r_4 t^4} L\right)^{\beta} K^{\alpha} \tag{27}$$

We can also convert this Cobb Douglas production function to a model in the growth rates [7]. Doing so we get

$$g_Y = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3 + \beta g_L + \alpha g_K, \tag{28}$$

$$\text{where } \beta g_A = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3 \tag{29}$$

[6] Taking natural log from both sides of the model $A = a_0 e^{r_1 t}$ we get $\ln A = \ln a_0 + r_1 t$. Now taking derivative of this equation with respect to time we get $\frac{A'}{A} = r_1$, which is the growth rate of the model.

[7] Taking natural log from both sides of the equation (27) we get $\ln Y = \beta \ln a_0 + \beta(r_1 t + r_2 t^2 + r_3 t^3 + r_4 t^4) + \beta \ln L + \alpha K$. Now taking derivative with respect to time we get $\frac{Y'}{Y} = \beta(r_1 + 2r_2 t + 3r_3 t^2 + 4r_4 t^3) + \beta \frac{L'}{L} + \alpha \frac{K'}{K} \Rightarrow g_Y = \beta g_A + \beta g_L + \alpha g_K$, where $g_A = r_1 + 2r_2 t + 3r_3 t^2 + 4r_4 t^3$. Setting $\gamma_0 = \beta r_1, \gamma_1 = 2\beta r_2, \gamma_2 = 3\beta r_3,$ and $\gamma_3 = 4\beta r_4$ we get $g_Y = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3 + \beta g_L + \alpha g_K$, where $\beta g_A = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3$.

Regressing g_Y on $t,\ t^2,\ t^3, g_L,$ and g_K to obtain the estimates of $\gamma_0,\ \gamma_1,\ \gamma_2,\ \gamma_3,\ \beta,$ and α we get

$$\begin{array}{lllllllll} \widehat{g}_Y = & .03 & - & 0.001t & + & 3.8E-05t^2 - & 4.12E-07t^3 & + \; 0.36\,g_L & + \; 0.25g_K & R^2 = 0.8834 \\ & (0.004)^{***} & & (0.0005)^{***} & & (1.9E-05)^{***} & (2E-07)^{***} & (0.07)^{*} & (0.03)^{**} & \end{array} \tag{30}$$

All estimates are statistically significant at 5% level of significance except the estimate $\hat{\beta}$. The estimate $\hat{\beta}$ is significant at 10% level of significance. $R^2 = 0.8834$ is telling us 88.34% variance are explained by this model. From the above estimated model we can see that the rate of technical change in the U.S. economy has been changing in a systematic way since the estimates $\gamma_0, \gamma_1, \gamma_2,$ and γ_3 are statistically significant at 1% level of significance.

Now using the equation (29) we can easily estimate the technology growth rate as follows.

$$\widehat{g}_A = .08 - 2.8E-03t + 1.1E-04t^2 - 1.1E-06t^3 \tag{31}$$

Plotting the first order (26) and the third order (31) estimated rate of technological changes in the same graph we get the Figure 9. That figure indicates that the first order technological growth rate is decreasing linearly, however, the third order technological growth rate is oscillating in a decreasing trend .

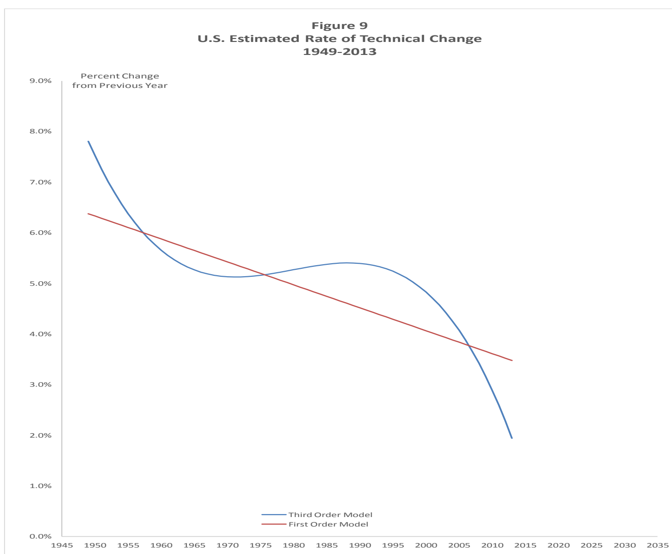


Figure 10 shows the plots of the estimated growth rates of U.S. production with dummy variable model (12) and no dummy variable model (10), and the estimated structural model (25) with 20 years forecast. For the structural model we use the estimated first order technological model (26). For the structural optimistic graph we use the estimated values of U.S capital and employment with dummy variable models. For the structural pessimistic graph we use the estimated values of the U.S. capital, and employment with no dummy variable models.

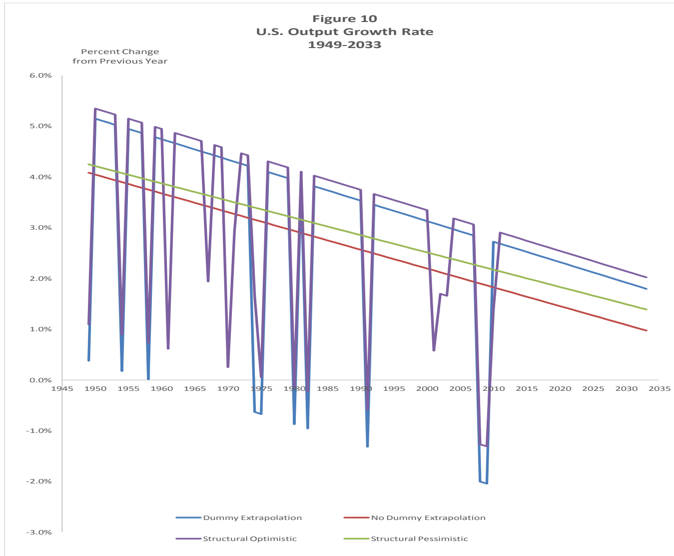


Figure 10 tells that the U.S. output growth rate is declining regardless of recession. Forecasted part of the graphs are telling that the model output growth rate is higher than the structural output growth rate for dummy variable, however, the model output growth rate is lower than the structural output growth rate for no dummy variable. So it clear that the structural forecast is different than the simply extrapolating the output growth rate. This is happening since we are taking care of technological growth rate. If we use the third order technological growth rate model to forecast the structural model then we will have different forecast.

Now suppose that the profits are given by

$$\Pi = PY - WL \tag{32}$$

where, WL = compensation of employees (COMP), and PY = nominal GDP. We can show that if the production function is given by the Cobb-Douglas function (19) then the labor share of output $\frac{WL}{PY}$ equal to the elasticity of output with respect to labor [8] β when profits are maximized.

From our data set we get $\beta = 0.55$. Using this value we can estimate the output growth rate model (28).

$$g_Y - .55g_L = .55g_A + \alpha g_K \tag{33}$$

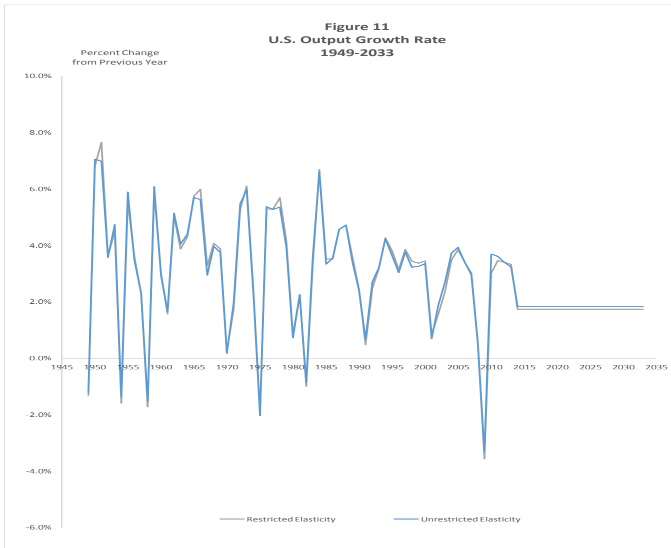
Regressing $g_Y - .55g_L$ on g_K to obtain the estimates of $.55g_A$ and α we get the following.

$$\begin{aligned} \widehat{g}_Y = 0.017 + \quad & 0.55g_L \quad + \quad 0.20g_K \qquad R^2 = 0.6122 \\ (0.001)^{***} & \qquad\qquad\quad (0.02)^{***} \end{aligned} \tag{34}$$

[8] Plugging Cobb-Douglas production function (19) in the profit function (32) we get $\Pi = P(AL)^{\beta}K^{\alpha} - WL$. Now taking the derivative of this function with respect to L and setting the derivative equal to zero we get, $P\beta A^{\beta}L^{\beta-1}K^{\alpha} - W = 0$, which implies $P\beta(AL)^{\beta}K^{\alpha} = WL$. Now substituting Cobb-Douglas function (19) we the elasticity of output with respect to labor $\beta = \frac{WL}{PY}$.

All estimates are statistically significant at 5% level of significance. The $R^2 = 0.6122$ indicates that 61.22% of the variance of $g_Y - .55g_L$ is explained by this model.

Figure 11 is showing the unrestricted elasticity model (21) and restricted elasticity model (34) for U.S. output growth rate. This figure indicates that the forecasted growth rate with restricted elasticity is lower than the growth rate with unrestricted elasticity. It looks both forecasted growth rates are constants.



6. **Conclusion**

Exponential growth model shows that the U.S. GDP is increasing with a decreasing trend regardless of recession. Log difference of growth models indicates that the growth rate of U.S. production is decreasing annually by 0.03% where initial growth rate is 4%. If the economy is not in recession then the initial growth rate of production is 5.2%. However, if the economy is in recession then the initial growth rate of production is 0.2%. Growth rate of U.S. production is decreasing annually by 0.4% for each additional year regardless of recession.

Growth rate of U.S. capital is decreasing by 0.04% for each additional year where the initial growth rate is 4%. Initial growth rate of capital is 7% if economy is not in recession. On the other hand, if economy is in recession then initial growth rate is -4%. Growth of capital is deceasing by 0.6% for each additional year regardless of recession.

Initial growth rate of U.S. employment is 2% and growth rate is deceasing by 0.02% for each additional year. Initial growth rate of employment is 3% if economy is not in recession. However, initial growth rate is -1% if economy is in recession. The growth rate of U.S. employment is decreasing by 0.01% for each additional year.

Estimates from Cob-Douglas production shows that the elasticity of output with respect to capital is 0.25, and elasticity of output with respect to labor is 0.41. Estimates from the Cob-Douglas production function also shows that the 100% output growth is broken down into 58% technology growth, 19.10% labor growth, and 22.90% capital growth. This analysis also shows that growth rate of technological changes in U.S. economy has been changing in a systematic way. Estimates from the economy's output and technology growth rates model indicate that the first order technological growth rate is decreasing while the third order technological growth rate is oscillating with a decreasing trend. It also shows that the output growth rate of U.S. economy is decreasing regardless of recession. Forecasted growth rate of U.S. output with restricted elasticity is lower than that with unrestricted elasticity.

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