World Resources Determine World Prices

Guo, Baoping

Individual Researcher

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Baoping Guo[1]

Abstract –This paper derived a general equilibrium of the Heckscher-Ohlin model for the context of two-factor, multiple-commodity, and multiple-country (2 x N x M model). The equalized factor price (PFE) and world commodity price at the equilibrium are just the prices Dixit and Norman predicted, that the prices from equilibriums remains the same when the allocation of factor endowments changes within (the 2 x N x M) IWE box. The study shows how price-trade equilibriums reached for multiple countries and how world prices was determined. The equilibrium and world price structure shows that world (factor endowments) resources determine the world price. This is another core understanding of international trade.

Keywords:
IWE; Factor price equalization; Heckscher-Ohlin; Equilibrium price; equalized factor price; Dixit-Norman

1. Introduction

There are many important literatures on general equilibriums of trade and the factor price equalizations. Samuelson's factor-price equalization theorem is a remarkable result. McKenzie (1955)'s cone of diversification of factor endowments is an important concept to understand the necessary condition of general equilibrium.

Vanek(1968)'s HOV model linked prices with trade and consumption by using shares of GNP. It also resulted in the application issue how to convert the assumption of homothetic taste into consumption balance.

Jones(1965) provided the popular system equations for the Heckscher-Ohlin model, which allowed costs going with productions smoothly and lightly.

[1] Former faculty member of College of West Virginia, corresponding address: 8916 Garden Stone Lane, Fairfax, VA 22031, USA. Email address: bxguo@yahoo.com.

Either (1984) discussed the generalization of the Heckscher-Ohlin theories in higher-dimensional contexts. David Gale and H. Nikaio (1965) provided conditions to guarantee global univalence of factor price and commodity price by an input-output matrix that is a matrix with all positive principal minors.

Dixit and Norman (1980) provided a strong clue for what a price-trade equilibrium should be and what an equalized factor price is. It draws out unique characteristic of equalized factor price under IWE diagram.

Wu (1987) discussed the equalization of factor prices in general equilibrium when commodities outnumber factors. Woodland (2011) mentioned that from a theory perspective, factor price equalization is a prediction of the model rather than an assumption. He summarized all-important achievements in the studies of general equilibriums.

Helpman and Krugman (1985) provide a formal definition of FPE set in higher dimensions. Deardroff (1994) studied the possibility of factor price equalizations. His endowment lenses with six goods and five countries are very impressive to understand the issues.

Guo (2005) started his researches of the structure of FPE price by using the term of trade and shares of GNP. Guo (2018) presented a solution of general equilibrium of trade on the Heckscher-Ohlin 2 x 2 x 2 model. He demonstrated that the equalized factor price is quantitatively available for a giving IWE box. This is an interesting result to understand where trade equilibriums reach and what a structure an equalized factor price is.

This study generalized the Guo (2018) result. It demonstrates that general equilibriums and equalized factor prices are available for the Heckscher-Ohlin 2 x N x M frameworks. The equilibriums are just also the Dixit-Norman's integrated world equilibrium. The equalized factor price and common commodity price will remain the same when the allocations of factor endowments change in the IWE box.

The study of this paper illustrates some new understandings of world trade equilibrium: 1. World resource determines world prices; 2. relative factor price equals inversely to the ratio of their factor endowments. 3. When the cone of the diversifications (technologies) of factor endowments changes simultaneously for all countries (still identical technologies), the factor price will remain same.

This paper is divided into five sections. Section 2 reviews the general equilibrium of trade (Guo 2018) and the price structure at the equilibrium. Section 3 derives a general equilibrium of trade for models with two factors, two commodities, and multiple countries (2 x 2 x M). Section 4 examines the 2 x N x M model. It provides equilibrium solutions for more general cases. Section 5 discusses a new understanding of international economics that world resources determine world price, under the market mechanism of commodity trade.

2. Review of the Equalized Factor Price from Integrated World Equilibrium (IWE)

With the normal assumptions of the Heckscher-Ohlin theory, Guo (2018) denoted a standard 2 x 2x 2 model in the following way:

a. The production constraint of full employment of resources are

$$AX^h = V^h \qquad (h = H, F) \qquad \text{(2-1)}$$

where A is the 2x2 technology matrix, X^h is the 2 x1 vector of commodities of country h, V^h is the 2x1 vector of factor endowments of country h. The elements of matrix A is $a_{ki}(W), k = K, L, i = 1,2$.

b. The zero-profit unit cost condition is

$$A'W^* = P^* \qquad \text{(2-2)}$$

where $W^*$is the 2x1 vector of factor prices, P^* is the 2x1 vector of commodity prices. Both P^* and W^* are world price when factor price equalization happened, its elements are r^* rental and w^* wage.

c. The definition of the share of GNP of country h to world GNP,

$$s^h = P'\,X^h / P'\,X^W \qquad (h = H, F) \qquad \text{(2-3)}$$

d. The trade balance condition is

$$P'\,T^h = 0 \qquad (h = H, F) \qquad \text{(2-4)}$$

or

$$W'\,F^h = 0 \qquad (h = H, F) \qquad \text{(2-5)}$$

where $T^h$is a 2x1 vector of commodity export, F^h is a 2x1 vector of factor content of trade.

e. The constraint of the cone of diversification of factor endowments

$$\frac{a_{K1}}{a_{L1}} > \frac{K^H}{L^H} > \frac{a_{K2}}{a_{L2}} \qquad \frac{a_{K1}}{a_{L1}} > \frac{K^F}{L^F} > \frac{a_{K2}}{a_{L2}} \qquad \text{(2-6)}$$

f. The constraint of commodity price limits[2]

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \qquad \text{(2-7)}$$

[2] This condition will guarantee all possible factor prices are positive. We may refer (7) to the constraint of cone of commodity prices.

The model takes the normal Heckscher-Ohlin assumptions as (1) identical technology across countries, (2) identical homothetic taste, (3) perfect competition in the commodities and factors markets, (4) no cost for international exchanges of commodities, (5) factors are completely immobile across countries but that can move costlessly between sectors within a country, (6) constant return of scale and no factor intensity reversals, and (7) full employment of factor resources.

By using a simple competitive home country's GNP share as[3]

$$s^h = \frac{1}{2}\frac{K^H L^w + K^w L^H}{K^w L^w} \tag{2-8}$$

where superscript w indicates the factor endowment is the world factor endowment and s^h is country home's share of GNP to world GNP.

Guo (2018) obtained the general equilibrium of trade of the Heckscher-Ohlin model as

$$\frac{r^*}{w^*} = \frac{L^w}{K^w} \tag{2-9}$$

$$w^* = 1 \tag{2-10}$$

$$p_1^* = a_{k1}\frac{L^w}{K^w} + a_{L1} \tag{2-11}$$

$$p_2^* = a_{k2}\frac{L^w}{K^w} + a_{L2} \tag{2-12}$$

$$F_K^H = \frac{1}{2}\frac{K^H L^w - K^w L^H}{L^w} \tag{2-13}$$

$$F_L^H = \frac{1}{2}\frac{K^H L^w - K^w L^H}{K^w} \tag{2-14}$$

All the endogenous price variables p_1^*, p_2^*, w^*, r^* in the model are expressed by exogenous variables (K^w, L^w).

It shows that Samuelson's equalized factor price at the equilibrium is just the Dixit-Norman's IWE factor price. Dixit and Norman (1980) illustrated that if the allocation of the factor endowments in the IWE box changes, the factor price and the commodity price will remain same. The price solution just reflects that.

The factor content of trade in (2-13) and (2-14) restates the Heckscher-Ohlin theorem. It says that if $\frac{K^H}{L^H} > \frac{K^w}{L^w}$, then $F_K^H > 0$.

For a giving IWE box, the solution of the equilibrium is unique since there is only one trade equilibrium point in IWE diagram.

[3] Guo used a simple utility function to maximize each country benefits with trade box, to achieve this share of GNP, we will use the logic at section 3.

We will use the same approach in next section to address the multi-nation issue.

3. General Equilibrium of trade for the case of two factors, two commodities, and multiple countries

Figure 1 draws an IWE diagram for two factors, two commodities, and three countries. In this diagram, $V^h(L^h, K^h)$ represents the vector of factor endowments of country h, h=1, 2, and 3. We can observe three trade points C^1, C^2, and C^3 for each countries accordingly.

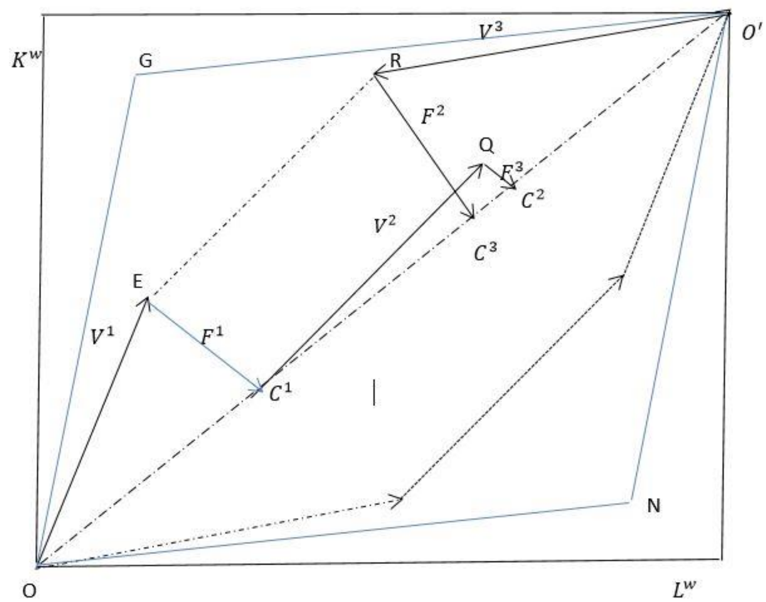

Figure.1 IWE diagram for two factors, two commodities, and three countries

3.1 trade partner for a country

In a two-country system, home and foreign, they are trade partners each other. In a 3-country system, country 1, country 2, and country 3, who is the trade partner for whom? We specify that trades here are one that a country trade with the rest of the world. We do not talk about the trade in which country 1 trade with country 2 or with country 3. We talk about that country 1 trade with the rest of the world. The trade relations now are very simple; it just likes the scenario of the two-country system.

Leamer (1984) in the preface of his book mentioned, "This theorem, in its most general form, states that a country's trading relations with rest of the world depend on its endowments of productive factors. Usually identified in theory textbooks as land, labor, and capital." He stated what a trade means in a multiple-nation system, from the view of the Heckscher-Ohlin model.

Figure 2. is an IWE diagram which shows how a country trades with the rest of the world. In addition, the boundaries of shares of GNP are added to the diagram.

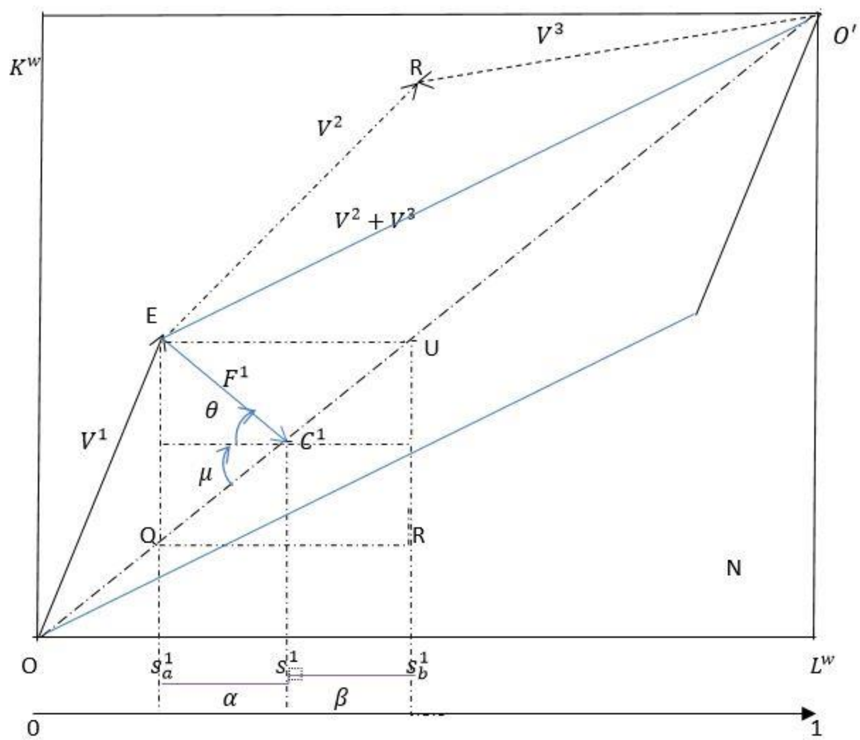

Figure.2 IWE diagram for 2x2x3 model with GNP limits

The factor endowment vector V^1 of country 1 is arranged to start at origin point O. The rest of the world factor endowment is $V^2 + V^3$, they starts at the origin point O'.

The algebra expression for the 2 x 2 x M model is as same as equation (1)-(6); the only difference is the country number. The country number now goes from 1 to 3.

3.2 Identifying the trade box

Trades will redistribute national welfares, which are measured by GNPs. The GNP is a function of price.

The trade in the IWE box has boundaries, which are determined by price limits (7) through the share of GNP.

Using $p_1^* = a_{K1}$ and $p_2^* = a_{K2}$ in the definition of GNP (2-3), we obtain the first boundary of country 1' share of GNP as

$$s_b^1 = \frac{K^1}{K^w} \tag{3-1}$$

Similarly, using $p_1^* = a_{L1}$ and $p_2^* = a_{L2}$, we obtain another boundary of country 1's share of GNP boundary as

$$s_a^1 = \frac{L^1}{L^W} \tag{3-2}$$

Assuming country 1 is capital abundant as

$$\frac{K^1}{L^1} > \frac{K^2+K^3}{L^2+L^3} \tag{3-3}$$

The boundaries of country 1's share of GNP satisfy

$$s_b^1 = \frac{K^1}{K^w} > s^1 > s_a^1 = \frac{L^1}{L^w} \tag{3-4}$$

They are indicated on the horizontal unity axis.
When $s \to \frac{K^1}{K^w}$ there are

$$\frac{p_1^*}{p_2^*} = \frac{a_{K1}}{a_{K2}}, \qquad r^* = \frac{1}{a_{K2}} \quad , \qquad w^* = 0 \tag{3-5}$$

and when $s \to \frac{L^1}{L^w}$ there are

$$\frac{p_1^*}{p_2^*} = \frac{a_{L1}}{a_{L2}}, \qquad r^* = 0\,, \qquad w^* = \frac{1}{a_{L2}} \tag{3-6}$$

So the commodity price limits are

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L1}} \tag{3-7}$$

And the wage and rental limits are:

$$\frac{1}{a_{K2}} > r^* > 0 \tag{3-8}$$

$$0 < w^* < \frac{1}{a_{L2}} \tag{3-9}$$

Prices[4] from equilibriums should be limited by (3-7) through (3-9)

The factor endowment allocation is at point E, where country 1 is capital abundant.

We refer to the area EUQR as the trade box. Line EC^1 indicates a trade. It can measure the factor contents of trade as

$$F_K^1 = K^1 - s^1 K^w \tag{3-10}$$

$$F_L^1 = L^1 - s^1 L^w \tag{3-11}$$

By trade balance (3-10), the term of factor content of trade is the relative factor price as

$$\frac{r}{w} = -\frac{L^1 - s^1 L^w}{K^1 - s^1 K^w} \tag{3-12}$$

[4] Prices here are relative prices referring $p_2^* = 1$.

If we know s^1 , we can obtain the relative factor price and trade amounts.

3.3 Settling trade equilibrium point

The country 1' s share of GNP s^1 divides the trade box into two parts. Their lengths are α and β, separately. When α increases, the countr 1's share of GNP increases and the rest of the world's share of GNP decreases, and vice versa. In trade competitions, the both sides want to reach their maximum GNP shares through free trade.

We need to ware that only trade box is the redistribution area of shares of GNP for the economy. Outside the box, they are not redistributable. There are no right commodity prices reaching them. This is an essential meaning of boundaries of shares of GNPs.

From Figure. 3, the length of β can be expressed as

$$\beta = s_b^1 - s_a^1 - \alpha \tag{3-13}$$

When $\alpha = \beta$, both countries reach their maximum values of GNP shares in the trade box; the country 1's share of GNP now is

$$s^1 = \alpha + s_a^1 = \frac{1}{2}(s_b^1 + s_a^1) \tag{3-14}$$

Substituting (3-1) and (3-2) into (3-14) yields

$$s^1 = \frac{1}{2}\frac{K^1 L^W + K^W L^1}{K^W L^W} \tag{3-15}$$

We can also interpret the result as that the best welfares of two countries should avoid the hurts of extreme trade points by s_b^1, s_a^1 as far as possible. When taking the share of GNP as s_b^1, then $w^* = 0$; and when taking the share of GNP as s_a^1, then $r^* = 0$. The middle point is the best position to reward both factors fairly based on existing factor endowment supplies.

3.4 General equilibrium

Substituting equation (3-15) into (3-12), we can get the relative factor price ratio as

$$\frac{r^{*1}}{w^{*1}} = \frac{L^W}{K^W} \tag{3-16}$$

We can also get commodity price and export vector accordingly by equation (2-10) through (2-12).

3.5 The price solution is the same for all countries

If we process the analyses above on country 2, we can get the same factor price as

$$\frac{r^{*2}}{w^{*2}} = \frac{L^w}{K^w} \qquad (3\text{-}17)$$

This means that the relative factor price is the same for all countries.

$$\frac{r^{*1}}{w^{*1}} = \frac{r^{*2}}{w^{*2}} = \frac{L^w}{K^w} = \frac{r^*}{w^*} \qquad (3\text{-}18)$$

In addition, trade angle θ is the same for all countries. The shares of GNP will be different from country to country.

By assuming $w^* = 1$ to drop one market-clearing condition by Walras's equilibrium, we obtain

$$s^1 = \frac{1}{2}\frac{K^h L^w + K^w L^h}{K^w L^w} \qquad (h = 1,2,3) \qquad (3\text{-}19)$$

$$\frac{r^*}{w^*} = \frac{L^w}{K^w} \qquad (3\text{-}20)$$

$$w^* = 1 \qquad (3\text{-}21)$$

$$p_1^* = a_{k1}\frac{L^w}{K^w} + a_{L1} \qquad (3\text{-}22)$$

$$p_2^* = a_{k2}\frac{L^w}{K^w} + a_{L2} \qquad (3\text{-}23)$$

$$F_K^h = \frac{1}{2}\frac{K^h L^w - K^w L^h}{L^w} \qquad (h = 1,2,3) \qquad (3\text{-}24)$$

$$F_L^h = -\frac{1}{2}\frac{K^h L^w - K^w L^h}{K^w} \qquad (h = 1,2,3) \qquad (3\text{-}25)$$

Those are the equilibrium solution for the 2 x 2 x 3 model.

3.5 A numerical example

Let see a numerical example for a three-country economy. The technology matrix is

$$\begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L1} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Commodity 1 is capital intensive. The factor endowments for three countries are

$$\begin{bmatrix} K^1 \\ L^1 \end{bmatrix} = \begin{bmatrix} 5000 \\ 4500 \end{bmatrix}, \qquad \begin{bmatrix} K^2 \\ L^2 \end{bmatrix} = \begin{bmatrix} 6000 \\ 4500 \end{bmatrix}, \qquad \begin{bmatrix} K^3 \\ L^3 \end{bmatrix} = \begin{bmatrix} 4500 \\ 5500 \end{bmatrix}$$

By using factor abundance definition $\frac{K^h}{K^w} > \frac{L^h}{L^w}$, country 1 and country 2 is capital abundant; country 3 is labor abundant. Country 1 and country 2 will export commodity 1 and country 3 will export commodity 2. The commodity outputs of three countries by the full employment of factor resources separately as

$$\begin{bmatrix} x_1^1 \\ x_2^1 \end{bmatrix} = \begin{bmatrix} 1100 \\ 1700 \end{bmatrix}, \qquad \begin{bmatrix} x_1^2 \\ x_2^2 \end{bmatrix} = \begin{bmatrix} 1500 \\ 1500 \end{bmatrix}, \qquad \begin{bmatrix} x_1^3 \\ x_2^3 \end{bmatrix} = \begin{bmatrix} 700 \\ 2400 \end{bmatrix}$$

The correspondent limits of share of GNP of each country separately are

$$s_1^b = 0.3225 > \quad s_1 \ > \ s_1^a = 0.2903$$

$$s_2^b = 0.3870 > \quad s_2 \ > \ s_2^a = 0.3225$$

$$s_3^b = 0.2903 > \quad s_3 \; > \; s_3^a = 0.3870$$

The shares of GNP of three countries separately are

$$s_1 = 0.3164$$
$$s_2 = 0.3487$$
$$s_3 = 0.3348$$

The sum of the three countries' share of GNP is 1.

With those shares of GNP, the consumptions and the export will be

$$\begin{bmatrix} c_1^1 \\ c_2^1 \end{bmatrix} = \begin{bmatrix} 1044.32 \\ 1772.19 \end{bmatrix}, \qquad \begin{bmatrix} c_1^2 \\ c_2^2 \end{bmatrix} = \begin{bmatrix} 1150.77 \\ 1952.83 \end{bmatrix}, \qquad \begin{bmatrix} c_1^3 \\ c_2^3 \end{bmatrix} = \begin{bmatrix} 1104.89 \\ 1874.97 \end{bmatrix}$$
$$\begin{bmatrix} T_1^1 \\ T_2^1 \end{bmatrix} = \begin{bmatrix} 55.67 \\ -72.19 \end{bmatrix}, \qquad \begin{bmatrix} T_1^2 \\ T_2^2 \end{bmatrix} = \begin{bmatrix} 349.22 \\ -452.83 \end{bmatrix}, \qquad \begin{bmatrix} T_1^3 \\ T_2^3 \end{bmatrix} = \begin{bmatrix} -404.89 \\ 525.02 \end{bmatrix}$$
$$\begin{bmatrix} F_K^1 \\ F_L^1 \end{bmatrix} = \begin{bmatrix} 94.82 \\ -88.70 \end{bmatrix}, \qquad \begin{bmatrix} F_K^2 \\ F_L^2 \end{bmatrix} = \begin{bmatrix} 594.82 \\ -556.452 \end{bmatrix}, \qquad \begin{bmatrix} F_K^3 \\ F_L^3 \end{bmatrix} = \begin{bmatrix} -669.65 \\ 645.16 \end{bmatrix}$$

In addition, the common commodity price and the equalized factor price at trade equilibrium are

$$\begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} 3.8064 \\ 2.9354 \end{bmatrix}, \qquad \begin{bmatrix} r^* \\ w^* \end{bmatrix} = \begin{bmatrix} 0.9354 \\ 1.000 \end{bmatrix}$$

4. The case of two factors, multiple commodities, and multiple countries

We cannot determine commodity outputs by factor endowments if the technology matrix is not square (not even). With multiple commodities (N>2), we need to know the commodity output quantities and the amounts of the factor endowments in advance if we start to process any equilibrium analyses. Deardorff (1994) illustrates that when the allocation of factor endowment is inside the cones, but above the lenses, the factor-equalization is not available. To avoid this situation, we assume in the following analysis that outputs be positive, that factor resources be of full employment, and that allocations of factor endowments are inside the lenses. We also emphasize that equations (2-6) and (2-7) is the necessary condition for a price-trade equilibrium.

We still use the basic notations in section 2. The technology matrix for multiple commodities and two factors now is

$$A = \begin{bmatrix} a_{k1} & a_{k2} & \cdots & a_{kn} \\ a_{L1} & a_{L2} & \cdots & a_{Ln} \end{bmatrix} \qquad n = (1,2,\cdots,N) \qquad (4\text{-}1)$$

where n is the number of commodities (n>=2). The commodity vector is the N x1 vector, and the price vector is the N × 1 vector as

$$P = \begin{bmatrix} p_1^h \\ p_2^h \\ \vdots \\ p_n^h \end{bmatrix}, \quad X^h = \begin{bmatrix} x_1^h \\ x_2^h \\ \vdots \\ x_n^h \end{bmatrix} \quad h = (1,2,\cdots,M), n = (1,2,\cdots,N) \tag{4-2}$$

where h indicates countries.

And factor endowments and factor prices are the 2×1 vectors

$$V^h = \begin{bmatrix} K^h \\ L^h \end{bmatrix}, \quad W^* = \begin{bmatrix} r^* \\ w^* \end{bmatrix} \quad , \quad h = (1,2,\cdots,M) \tag{4-3}$$

To establish the trade equilibrium, we start at finding the boundaries of shares of GNP of country h.

We cannot use equation (2-7) directly to obtain the boundaries of commodity price. Let see it in another way. We rewrite unit cost as the following,

$$\begin{bmatrix} a_{k1} \\ a_{k2} \\ \vdots \\ a_{kn} \end{bmatrix} r^* + \begin{bmatrix} a_{L1} \\ a_{L2} \\ \vdots \\ a_{Ln} \end{bmatrix} w^* = \begin{bmatrix} p_1^h \\ p_2^h \\ \vdots \\ p_n^h \end{bmatrix} \tag{4-4}$$

When $w^* = 0$, price reaches at its one boundary as

$$p^b = \begin{bmatrix} a_{k1} \\ a_{k2} \\ \vdots \\ a_{kn} \end{bmatrix} r^* \tag{4-5}$$

Substituting it into GNP function (2-2), we obtain the correspondence GNP boundary value as

$$s_b^h(p^b) = \frac{K^h}{K^w} \quad h = (1,2,\cdots,M) \tag{4-6}$$

Similarly, using another price boundary as the following,

$$, \quad p^a = \begin{bmatrix} a_{L1} \\ a_{L2} \\ \vdots \\ a_{Ln} \end{bmatrix} w^* \tag{4-7}$$

substituting it into the function of the share of GNP in equation (3) yields

$$s_a^h(p^a) = \frac{L^h}{L^w} \quad h = (1,2,\cdots,M) \tag{4-8}$$

The competitive share of GNP of country h, as we discussed in the last section, will be

$$s^h = \frac{1}{2}\left(s_b^h + s_a^h\right) = \frac{1}{2}\frac{K^h L^w + K^w L^h}{K^w L^w} \quad h = (1,2,\cdots,M) \tag{4-9}$$

With this share of GNP, we obtain factor contents of trade for country h as

$$F_K^h = \frac{1}{2}\frac{K^h L^w - K^w L^h}{L^w} \quad h = (1,2,\cdots,M) \tag{4-10}$$

$$F_L^h = \frac{1}{2}\frac{K^h L^w - K^w L^h}{K^w} \quad h = (1,2,\cdots,M) \tag{4-11}$$

The commodity exports for country h will be

$$T_1^h = x_1^h - \frac{1}{2}\frac{K^h L^w + K^w L^h}{K^w L^w} x_1^w \qquad h = (1,2,\cdots,M) \qquad (4\text{-}12)$$

$$T_2^h = x_2^h - \frac{1}{2}\frac{K^h L^w + K^w L^h}{K^w L^w} x_2^w \qquad h = (1,2,\cdots,M) \qquad (4\text{-}13)$$

The relative factor price for country h can be expressed as

$$\frac{r^{*h}}{w^{*h}} = -\frac{F_L^h}{F_K^h} = \frac{L^w}{K^w} = \frac{r^*}{w^*} \qquad h = (1,2,\cdots,M) \qquad (4\text{-}14)$$

Therefore, all countries' rental-wage ratios are same. Assume world common wage as numerica as 1,

$$w^* = 1 \qquad (4\text{-}15)$$

The common commodity price can be written as

$$\begin{bmatrix} p_1^* \\ p_2^* \\ \vdots \\ p_n^* \end{bmatrix} = \begin{bmatrix} a_{k1} & a_{L1} \\ a_{k2} & a_{L2} \\ \vdots & \vdots \\ a_{kn} & a_{Ln} \end{bmatrix} \begin{bmatrix} \frac{L^w}{K^w} \\ 1 \end{bmatrix} \qquad (4\text{-}16)$$

Equations (4-14) through (4-16) are the price solution of two factors, multiple commodities, and multiple countries. They are an analytical expression of the factor-price equalization theorem.

The solution shows that prices depend on world factor endowments directly, so it is Dixit-Norman IWE price.

We can observe that the sum of the shares of GNP (4-9) of all countries equals 1.

Equations (4-10) through (4-13) are an analytical expression of the Heckscher-Ohlin theorem generalized, in the case of multiple commodities and multiple countries of the two-factor economy.

For the scenario of the non-square matrix (not even), we can get trade amounts of commodities without issues. Once we know the share of GNP of a country, we can get its import and export amounts by using equation (4-10) and (4-13).

Let see a numerical example for two factors, three commodities, and two countries. In this example, the technology matrix is not a square matrix. Therefore, we need to know the output quantities in advance.

The technology matrix in this case is

$$A = \begin{bmatrix} a_{k1} & a_{k2} & a_{k3} \\ a_{L1} & a_{L2} & a_{L3} \end{bmatrix} = \begin{bmatrix} 3 & 1.3 & 1.2 \\ 0.9 & 2 & 3 \end{bmatrix}$$

Suppose the commodity outputs are

$$\begin{bmatrix} x_1^H \\ x_2^H \\ x_3^H \end{bmatrix} = \begin{bmatrix} 200 \\ 100 \\ 300 \end{bmatrix}, \begin{bmatrix} x_1^F \\ x_2^F \\ x_3^F \end{bmatrix} = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$$

By full employment, the factor endowments for the two countries are

$$\begin{bmatrix} K^H \\ L^H \end{bmatrix} = \begin{bmatrix} 2700 \\ 2500 \end{bmatrix}, \qquad \begin{bmatrix} K^F \\ L^F \end{bmatrix} = \begin{bmatrix} 2400 \\ 2600 \end{bmatrix}$$

The commodity prices are in 3-demission space.

When

$$p^b = \begin{bmatrix} a_{k1} \\ a_{k2} \\ a_{k3} \end{bmatrix} = \begin{bmatrix} 0.9 \\ 2 \\ 3 \end{bmatrix}$$

then

$$s_b^H = 0.4599$$

When

$$p^a = \begin{bmatrix} a_{L1} \\ a_{L2} \\ a_{L3} \end{bmatrix} = \begin{bmatrix} 3 \\ 1.3 \\ 1.2 \end{bmatrix}$$

then

$$s_a^H = 0.5688$$

The share of GNP of the home country is the average value of above two as 0.5144. With this share of GNP, the consumptions, the exports, factor contents of trade, and world prices will be

$$\begin{bmatrix} c_1^H \\ c_2^H \\ c_3^H \end{bmatrix} = \begin{bmatrix} 257.20 \\ 154.32 \\ 205.76 \end{bmatrix}, \qquad \begin{bmatrix} c_1^H \\ c_2^H \\ c_3^H \end{bmatrix} = \begin{bmatrix} 242.79 \\ 145.67 \\ 194.23 \end{bmatrix}, \qquad \begin{bmatrix} T_1^H \\ T_2^H \\ T_3^H \end{bmatrix} = \begin{bmatrix} -57.20 \\ -59.32 \\ 94.23 \end{bmatrix},$$

$$\begin{bmatrix} F_K^H \\ F_L^H \end{bmatrix} = \begin{bmatrix} -129.13 \\ 122.59 \end{bmatrix}, \qquad P^* = \begin{bmatrix} 3.772 \\ 3.244 \\ 4.148 \end{bmatrix}, \qquad \begin{bmatrix} r^* \\ w^* \end{bmatrix} = \begin{bmatrix} 0.9374 \\ 1.000 \end{bmatrix}$$

5. World resources determine world prices

Methodologically, we provide a simple way to solve the equilibrium of the 2 x N x N model. For a giving IWE box of the 2 x N x M model, we can present the corresponding equalized factor price and common commodity price by world factor endowments directly.

The equalized factor price does not relate to technological coefficients, not relate to country sizes. It only relates the world resources, which determine world price.

Equations (4-14) through (4-16) displays that no country in world trade stands a better position to determine or dominate world prices.

For the multiple country equilibria, each country has its trade box. Each country trades with its outside world with same trade angle θ. In the trade box of each country, there is only one equilibrium point fitting this angle, the equilibrium price at this point is world price. The beauty of multiple-country equilibrium is that trade amounts are localized for each country and prices are global (globally unique) for all countries.

For the 2 x 2 x M model, the world prices remain the same when the allocation of factor endowments changes across countries within IWE box.

The allocation of factor endowments in the 2 x N x M model (N>2) is slightly complex. Due to the no-even technological matrix, we do not know how a mobile factor can affect how much commodity output. We may image a mobile commodity output. It is similar to out-sourcing commodity production. The factor mobile is in a style that is a mobile of factor content of out-source commodity. Mobile factors are bundled in the propositions that are used to produce a commodity. This can guarantee the full employment of factor endowments for all countries. With the allocation of factor endowment by a mobile of factor content of out-source commodity in the IWE lenses, the world price remains the same.

The world prices by FPE (4-14) through (4-16) depend on world resources only. It displayed how the world resources decide the world commodity price and equalized factor price. It is a new logic in the field of international economics: world factor resources determine world price.

Conclusion

Deardrff (1994) mentioned the importance of PFE, “Much depends, in modern international trade theory, on whether prices of factors are equalized internationally. With factor price equalization (FPE), industries in different countries with identical constant-returns-to-scale technologies use in identical techniques of production, and the analysis of trade and production greatly simplified”. We moved the first step to identify PFE quantitatively.

We obtained the solutions of general equilibriums of the Heckscher-Ohlin 2 x N x M models. The factor price at the equilibriums is the equalized factor price. It is also the Dixit-Norman price.

The solution is a re-examining and representing for the Heckscher-Ohlin Theorem for multiple commodities and multiple countries. It also released the concerning of availabilities of Factor Price Equalization Theorem in the scenario of multiple commodities.

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