Information Trading in Social Networks

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This paper considers information trading in fixed networks of economic agents who can only observe and trade with other agents with whom they are directly connected. We study the nature of price competition for information in this environment. The linear network, when the agents are located at the integer points of the real line, is a specific example I completely characterize. For the linear network there always exists a stationary equilibrium, where the strategies do not depend on time. I show that there is an equilibrium where any agent has a nonzero probability of staying uninformed forever. Under certain initial conditions this equilibrium is a limit of equilibria of finite-horizon games. The role of a transversality condition is emphasized, namely that the price in the transaction should not exceed the expected utility of all the agents who get the information due to the transaction. I show that the price offered does not converge to zero with time.

Key Words and Phrases: networks, information trading, information diffusion.
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1. Introduction

In this paper, we consider a network of agents, with each agent only able to observe and communicate with his direct neighbors. The social network is fixed. Initially, each agent becomes informed with a probability p, independently of other agents. The informed agents then offer to sell the information to their uninformed neighbors who decide to accept the offer or wait. The uninformed agents who buy the information can in turn sell it to their neighbors, if these neighbors are uninformed. We analyze the equilibria of this game.

"Neighbors" and "networks" need not be interpreted spatially. One can think of firms in similar markets as "neighbors" and the discovery of how to solve the problem of miniaturizing electronics, as in the 1970s, as the "information". Firms in similar industries become aware that their neighbors have solved a problem and might want to buy the solution. Similarly, prices need not be in terms of money but could be reciprocal exchange. Eric von Hippel [11] discusses a network of steel mini-mills, whose managers exchanged information on how to solve common problems, with the implicit contract being that each member would tell the others of relevant information. Exchange of gossip also falls into the category of such reciprocal exchange.

There have been many recent studies of learning through observing the actions or strategies of the neighbors. Boyd and Richerson [5] consider this learning a fundamental way of behavior pattern diffusion and call it cultural evolution. Empirical studies such as those of Banerjee and Munshi [2] show that the structure of the social network is especially important when the markets function imperfectly. These authors consider the effect of the social network on lending. In particular, they demonstrate that migrants prefer to be in places close to their community's lending resources. This serves as evidence that there are benefits to being in proximity to the social network. The authors show that those who migrate to places with no access to the lending network are characterized by higher production ability. The relative independence of these migrants emphasizes the importance of the network for all the others who are less productive and therefore rely more on the lending network's benefits.

Foster and Rosenzweig [9] show that the structure of the social network plays an important role in spreading information about new technologies. They demonstrate that for farmers in India, imperfect knowledge about the management of high-yielding seed varieties is a significant barrier to their adoption, and neighbors' familiarity with these seed varieties significantly increases profitability. This neighbor effect indicates that farmers rely not only on official directions provided by the producers of the seed varieties but also on the experience of the people they know, which reveals the importance of the social network structure in information diffusion. Conley and Udry [7] also argue that the learning process about new technology in agriculture (they consider pineapple growing in Ghana) is rather social, and depends on one's neighbors' experience. The social nature of adopting new technology is explained by different conditions (soil, temperature, and so on) for different regions. The people paying someone they know to do research on financial markets (which stock to invest in) is one more example of information diffusion in the social environment.

This "network effect" — the people learning from their direct neighbors in accordance with the established connections — is an evidence of market failure because the agents do not communicate with the rest of the group, and therefore information diffusion among the population is not socially optimal. This failure can be corrected only through the involvement of the government or related organizations; as Belli [3] observes, the agents themselves can not achieve a good level of communication.

In this paper we combine three main theoretical strains in the literature: information diffusion, exogenously given network structure, and rational agents who trade the information. Muto [12] discusses the sale of information but does not model a network structure. He addresses the question of diffusion of an information good from a monopolistic owner to a finite number of demanders, and a seller being allowed to charge a price for the information. Although this problem was considered within some community, the structure of the connections was not taken into account. The assumption is that everyone is connected to everyone. Muto stresses the role of information resale, and analyzes the monopolist and resellers behavior. If the resales are prohibited, then the outcome is always Pareto optimal (and therefore the society reaches maximum welfare), but if resales are allowed, then the outcome is not Pareto optimal. The author finds the number of final possessors of the information good.

Irrational agents whose response to the neighbors' actions is predetermined are studied in numerous papers. Chatterjee and Xu [6] consider myopic agents and place them at the integer points of the real line, i.e. everyone has exactly two neighbors. There are two types of technology, R(ed) and B(lue). Technology R is better than B because it provides a higher probability of success. Every period the agents decide on which technology to use. If there was a success in the technology the agent used during the last period, then he continues to use it. If there was a failure, then the agent chooses better technology based on his own and his neighbors' experience during the current period. The important finding of the paper is that sooner or later all the agents switch to the best technology.

Bala and Goyal [1] advance by taking into account an arbitrary structure. There is a finite connected social network of myopic agents who, without knowing actual payoffs, try to figure it out from their own and their neighbors' current and past experience. The agents do not have any beliefs about their neighbors. The information about the right technology is not traded: for every agent the result his action immediately becomes known to the neighbors. The authors show that an agent beliefs converges to some limit with probability one; consequently, the utilities of all the agents are the same at infinity. As the network is finite, there is a chance that all the agents would not choose the right action (what would not happen in an infinite network).

Polanski [13] considers information good pricing in a network for the bargaining process at which only one pair of agents can trade at each period of time. The seller makes an offer, and the buyer either accepts it or rejects. In the case of rejection, the pair may be allowed to trade next period of time. There is no discounting. The author studies the role of cycles in the trading process. The infirmation always diffuses completely, and the price does not exceed the utility of those who can get the information good only due to this transaction: the price is zero if the buyer and seller are connected in more than one way. This result is explained by the absence of discounting which increases the patience of the agents, and the special trading structure which decreases the competition in the case of several connections between the seller and the buyer.

This paper investigates information trading and information diffusion in the social network. The focus is on how the people trade, the equilibrium strategies and prices, and the final information distribution across the agents.

By "*information*" we mean a good that has the following properties (see Muto [12]):

- It delivers some level of utility to a person who has it (commodity);
- It is possible to duplicate it without any loss in the utility (free replication);
- Once a person knows the information, it is impossible to prohibit him from knowing it (irreversibility); and
- It is impossible to get utility from a fraction of the information (indivisibility).

For example, some financial information, technology, political news, or even gossip might be considered as the information.

The important property of information is everyone's ability to trade it. It can be paid for by barter or money — we do not distinguish between the two. Again, one may argue that it is difficult to trade gossip for money. In this case by price here we mean an obligation to provide another gossip next time — we can hardly imagine a person with whom other people want to share gossip and who never gives anything back.

"*Social network*" ("*social environment*"), in which the information diffusion is considered, is a set of agents with the following properties:

- Some agents are connected to each other (these agents are called "*neighbors*");
- The agents are able to trade only with their neighbors.

This social network is conveniently represented by a graph, where the agents are located at the nodes, and the connections of the agents are represented by the edges.

There is only one sort of information in the model. At the beginning, every agent independently with the same probability learns this information. At every consequent period (time is discrete) the informed agents make offers to their uninformed agents by setting the prices in exchange for providing the information. If the buyer accepts an offer, he becomes informed and can resell the information in the following periods of time.

We make the following assumptions:

1. **Everlasting offers.** Once made, the offer stays forever and the seller can not change it later. This assumption is made for the sake of simplicity of proofs to avoid dealing with evolving prices.

2. **Limited observability.** Any agent knows if his neighbors have the information or not, and all the offers made to him during previous periods of time. The agents, however, have only general knowledge about the rest of the network and the game — who is whose neighbor, and what are the strategies, probabilities, distributions, and so on. No agent knows who, besides his neighbors, has the information, and the offers made to other agents.

The linear network is considered in the paper. Our main results are the following: it is shown that for any initial parameters there is a stationary equilibrium where the strategies do not depend on time, although the fraction of informed agents increases every period. This equilibrium is possible because the agents' beliefs about the distribution of the uninformed agents prior to the next informed agent do not change with time. The price in the stationary equilibria does not converge to zero as it does in the random network.

The research demonstrates that for a small probability of learning the information at the beginning, the sellers' strategy always includes a mass point above the value of the information. The existence of this mass point above the personal valuation of the information leads to the possibility of a "low probability trap," when some agents never get the information because both their neighbors make high enough offers at the same time, and each of these offers requires reselling in order to get a non-negative payoff. Moreover, the probability for the agent with two uninformed neighbors to stay uninformed forever does not change over time. For some initial parameters, this equilibrium is a limit of equilibria of finite-horizon games.

For a high probability of learning the information at the beginning, the strategy of the stationary equilibrium has continuous distribution below the personal valuation of the information, which means that every agent gets the information.

The rest of the paper is organized as follows: in Section 2 the model is described and the equilibrium concept is defined. The importance of *the transversality condition* in an equilibrium is emphasized: the price an agent pays for the information does not exceed the expected discounted utility of all the agents who get the information due to the transaction. The linear model, where the agents are placed in the integer points of the real line, is considered in Section 4. The random network is considered in Section 5, where at every period the agents randomly meet each other.

2. The Model

In this section we define the game, describe the strategies, and establish the existence of the symmetric equilibrium.

2.1. The Game

Consider a network of agents without cycles, where every agent has exactly M neighbors. Because of the same number of neighbors for each agent, the network looks the same way no matter which agent we place at the center. An example of such a network for $M = 4$ is given in Figure 1.

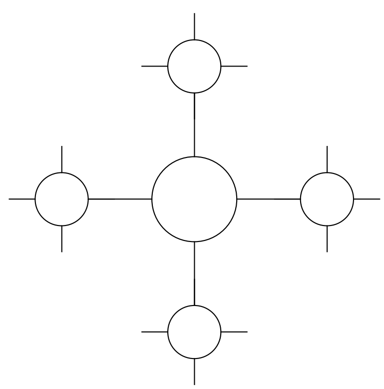

FIGURE 1. An example of a network for $M = 4$.

There is one kind of information (for example, some particular technology) every agent can use to extract a one-time utility u. Time is discrete, $t \in \mathbb{N}$. At $t = 0$ the agents obtain independent realizations of a $\{0, 1\}$ random variable. If an agent gets a realization of 1 (this happens with exogenous probability p), he becomes "informed," otherwise "uninformed." Once an agent has the information, he remembers it forever. Every agent always knows who of his neighbors is informed; however, no one knows anything about his neighbors' neighbors.

At each period starting from $t = 1$, the informed agents (sellers) decide on making offers to their uninformed neighbors (buyers). If made, the offer is a price at which the seller agrees to share the information with a buyer. The sellers who decide to wait with an offer can make it next period of time, if the neighbor is still uninformed. The sellers make the decision about the offers and set the prices separately for each of their uninformed neighbors. At the end of the period, the uninformed agents who have at least one offer can accept one of them, or wait.

The discount factor equals $\delta \in (0, 1)$. All the agents are risk neutral. The agent's utility at $t = 0$ equals

$$U = \begin{cases} 0, & \text{the agent is never informed;} \\ \delta^t(u - v) + W, & \text{the agent gets the information at period } t, \end{cases}$$

where v is the price the agent pays for the information, and W is the total discounted revenue from selling the information to the neighbors. The agents maximize their expected utility.

2.2. The Strategies

At every period t agent α has history

$$H_t^\alpha = (\{s_{tn}^\alpha\}_{n=1}^M, \{(s_{tn}^{\alpha B}, v_{tn}^{\alpha B})\}_{n=1}^M, \{(s_{tn}^{\alpha S}, v_{tn}^{\alpha S}, \tilde{s}_{tn}^{\alpha S})\}_{n=1}^M, (s_t^\alpha, m_t^\alpha)),$$

where

s_{tn}^α — the time when neighbor n got informed;

$s_{tn}^{\alpha B}, v_{tn}^{\alpha B}$ — the time when neighbor n made an offer, and the price offered;

$s_{tn}^{\alpha S}, v_{tn}^{\alpha S}, \tilde{s}_{tn}^{\alpha S}$ — the time of the offer to neighbor n, the price and the time of acceptance, if any;

s_t^α, m_t^α — the time when the agent got the information, and the neighbor from whom he got it.

All the histories are consistent across the agents and across time.[3] Denote $\mathbf{H}_t$ — the set of all possible histories at time t. The state of the world at time t is the set of all histories for all agents $\{H_t^\alpha\}_\alpha$.

The buyer pure strategy is the decision to buy the information from one of the neighbors, or to wait (0 corresponds to waiting):

$$R_t^{\alpha B} : \mathbf{H}_t \to \{0, 1, \ldots, M\}.$$

The sellers pure strategy for each of the uninformed neighbors is the decision to wait (represented by $\emptyset$, which is also played for the informed neighbors) or a price:

$$R_t^{\alpha S} : \mathbf{H}_t \to \{\mathbb{R}_+, \emptyset\}^M.$$

Denote the sets of pure strategies by $\mathbf{R}_t^B$ and $\mathbf{R}_t^S$ respectively. We allow mixed strategies, i.e. some probability measures $\mu_t^B(\cdot) \in \Delta(\mathbf{R}_t^B)$ and $\mu_t^S(\cdot) \in \Delta(\mathbf{R}_t^S)$.

2.3. Equilibrium Definition

To find the equilibrium strategies we use Perfect Bayesian Equilibrium concept. This means that the agents play the best response to their histories in accordance with the beliefs, even if the histories are not achievable under the given equilibrium strategies. This equilibrium concept shares with the Perfect Bayesian Equilibrium the idea that every agent maximizes the expected utility in every state, given the system of beliefs consistent with all the other players' strategies. Since there are infinitely many agents in the game, we can not directly apply the PBE concept, but need to generalize it in order to use it in our context. This generalization is similar in spirit to the local perfect equilibrium in Fudenberg, Levine, and Maskin [10] and is possible because at every period of time only a finite number of agents can influence the agent's history.

Definition. A symmetric equilibrium of the game is a set of strategies

$$\{\mu_t^B(\cdot), \mu_t^S(\cdot)\}_{t \geq 0}$$

such that no agent with any history (on or off the equilibrium path) can get extra payoff by deviating from his strategy given that all the other agents play the equilibrium strategies.

[3] By "consistent" we mean that the agents can not have contradictory histories. For example, if agent α got the information at some time t', then at all the consequent periods of time $s_t^\alpha = t'$.

The seller strategy $\mu_t^S(\cdot)\}_{t\geq 0}$ is a product of identical distribution functions towards each of the neighbors.

We consider symmetric equilibria, i.e. equilibria in which all the agents use the same strategies, and the strategies are symmetric with respect to different neighbors. The network does not contain cycles; therefore, the agent's action towards one neighbor can not influence the decision of another neighbor, and based on this we assume that the agents act independently towards their different neighbors.

In the definition of an equilibrium the strategies depend on the history. The knowledge of the whole history is excessive and the decision might depend on a smaller number of parameters than the history contains. Also, we want to reduce the number of equilibria in the game by introducing the concept of equivalence between the equilibria.

The equivalence of two equilibria is understood in the following way. For the sellers, their expected revenue from selling the information to a particular uninformed neighbor does not change. What we change is when the offer is made. However, the offer itself (or a distribution of the offers) is the same and, although it is made at different period of time, the time of the acceptance does not change. For the buyers, if an informed neighbor does not make an offer, it means that the future offer is such that, made at the current period of time, it would not change the buyer decision on buying the information: the earlier offer does not change the buyer behavior. Consequently, the buyers in the equivalent equilibrium face the same distribution of the offers and the sellers make the offers with the same distribution as before. The expected utility of the agents is the same, although in the original equilibrium we need to take expectation with respect to the offers of the informed neighbors who wait with the decision to make their offers. The information diffuses in the same manner, and the fraction of the informed agents as well as the spacial structure of the informed/uninformed agents also stays the same.

The following proposition describes the necessary parameters and reduces the number of equilibria by introducing an equivalent equilibrium in which the sellers make their offers immediately after acquiring the information.

Proposition 1. *For any equilibrium there exists an equivalent one, in which all the informed agents make their offers immediately, the seller strategy at time t is a distribution* *function of offer prices $F_t(v)$, and the buyer strategy is a function*

$$K_t : \{1, 2, \ldots, M\} \to \mathbb{R}_+,$$

which determines the reservation price for a given number of informed neighbors.[4]

We concentrate on the equilibria from proposition 1 at which the offers are made immediately, and the offers to different neighbors are independently drawn from distribution $F_t(\cdot)$. Denote

$$V_t = \sup(\operatorname{supp} F_t(v))$$

— the highest possible price offered at time period t. The buyers with l informed neighbors accept the lowest offer if this offer does not exceed $K_t(l)$.

The game has infinitely many agents and infinite horizon. Therefore, the equilibria may have the property of the Ponzi game, where the prices are not consistent with the utility the agents get from knowing the information. To exclude such equilibria from consideration, we use a *transversality condition*. We require that for any period of time t and for any l

$$K_t(l) \leq u * \mathbf{E}\, A_{tl}, \tag{1}$$

where A_{tl} stands for the random variable representing the discounted number of the uninformed agents who will get the information due to the transaction between the agent and the seller, if the buyer has exactly l informed neighbors. This condition requires that the price does not exceed the expected discounted utility of all the agents who will get the information due to the transaction.

In equilibrium a buyer with all informed neighbors prefers to buy the information for any price not exceeding u. At the same time, buying the information for a price above u results in a negative payoff, therefore

$$K_t(M) = u. \tag{2}$$

[4]We described here the strategies on the equilibrium path. The only deviation these strategies do not take into account is the one when a neighbor gets the information and then does not make an offer. In this case, we assume that the agent believes that the neighbor will make an offer next period of time, and uses corresponding best response. This happens with probability zero, therefore we should not worry about the effect of such a deviation except for that this kind of a deviation should not be profitable.

A buyer immediately agrees on price $v \leq (1-\delta)u$ because the loss in the expected utility from waiting is $u - \delta u = u(1-\delta)$. Therefore,

$$K_t(l) \geq (1-\delta)u \qquad \forall l \in \{1, 2, \ldots, M-1\}.$$

3. Linear Network

This section considers a special case of an infinite symmetric network without cycles — the infinite linear network (see Figure 2), where every agent has exactly two neighbors.

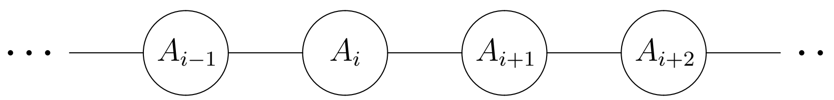

FIGURE 2. Infinite linear network.

3.1. General Results

The linear network, along with its plain structure, has the advantage of simple beliefs of the agents, which we can formulate using the following notation. Denote event "agent i is informed at time t" by $\overline{A_i^t}$, and event "agent i is uninformed at time t" by A_i^t.

The following proposition describes the agent belief about the distance till the next informed agent. Although the fraction of the informed agents increases over time, this belief does not change as long as the agent himself and his neighbor stay uninformed.

Proposition 2. *Suppose that all the agents in the linear network act independently and use the same (even non-equilibrium) strategies. Then for any uninformed agent with an uninformed neighbor his belief that there are exactly k other uninformed agents beyond the uninformed neighbor has a geometric distribution with parameter p:*

$$\mathbf{P}(A_{i-1}^t A_i^t \ldots A_{i+k}^t \overline{A_{i+k+1}^t} | A_{i-1}^t A_i^t) = p(1-p)^k. \tag{3}$$

Consider uninformed Agent A_{i-1}, whose neighbor A_i is uninformed (Figure 3). We do not need to assume that the agents use equilibrium strategies; the only assumption

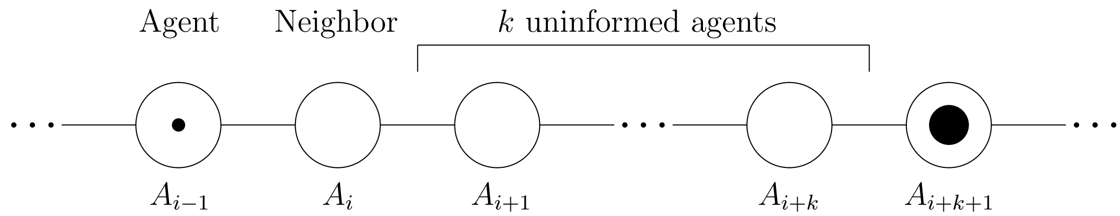

FIGURE 3. Illustration for Proposition 2.

necessary is that the strategies are the same (mixed or pure) for all the agents. Then Agent A_{i-1} believes that the probability of k agents $A_{i+1}, A_{i+2}, \ldots, A_{i+k}$ to be uninformed and agents A_{i+k+1} to be informed (this agent is marked with a black circle) equals $p(1-p)^k$. Probability that the next k agents are uninformed

$$\begin{aligned} \mathbf{P}(A_{i-1}^t A_i^t \ldots A_{i+k}^t | A_{i-1}^t A_i^t) &= \sum_{l=0}^{\infty} \mathbf{P}(A_{i-1}^t A_i^t \ldots A_{i+k}^t \overline{A_{i+k+l}^t} | A_{i-1}^t A_i^t) \\ &= \sum_{l=0}^{\infty} p(1-p)^{k+l} = (1-p)^k. \end{aligned}$$

This result of Proposition 2 holds because of the following reasoning. First, the belief is calculated conditionally on the fact that the agent himself and his neighbor are uninformed. In particular, Agent A_{i-1} does not know anything about agents A_l for $l \geq 1$. Second, the initial distribution of the number of uninformed agents preceding the first informed one is geometric with parameter p because at the beginning everyone learns the information independently. And finally, the geometric distribution has the property similar to the constant hazard rate of the exponential distribution: the distribution of the difference of a geometrically distributed random variable and a constant (which models the diffusion of the information towards the agent) is the same as the distribution of the random variable if the difference is non-negative.

Suppose that the strategies are such that an uninformed agent with one offer always buys the information, i.e. $K_t(1) \geq V_t$. Then the probability of acquiring the information by an uninformed neighbor of an uninformed agent equals p. (The product of $\mathbf{P}(A_{i-1}^t A_i^t \overline{A_{i+1}^t} | A_{i-1}^t A_i^t) = p$ and the probability that the information will be transferred, which equals one.) In other words,

$$\mathbf{P}\{A_{i-1}^t, A_i^t, \overline{A_i^{t+1}} | A_{i-1}^t, A_i^t\} = \mathbf{P}\{A_{i-1}^t, A_i^t, \overline{A_{i+1}^t} | A_{i-1}^t, A_i^t\} = p. \tag{4}$$

Consider a pure strategy equilibrium. In this equilibrium all the informed agents offer the information for the same price V_t at time t. The following proposition characterizes all such equilibria that satisfy the transversality condition.

Proposition 3. *For any $p \in (0,1)$ there exists at most one pure strategy equilibrium satisfying the transversality condition; for this equilibrium*

$$K_t(1) = V_t = \frac{u}{1-\delta(1-p)^2}.$$

As the buyers and sellers use the same strategy every period of time in this pure strategy equilibrium, the information always diffuses from an informed agent to his uninformed neighbor if this neighbor has only one offer. The range for the initial parameter p when pure equilibria exists will be found in the next subsection.

3.2. Stationary Equilibria

Proposition 3 showed that in all pure strategy equilibria the strategies do not depend on time. Such equilibria, in which the strategies do not depend on time, $F_t(\cdot) = F(\cdot)$, $K_t(1) = K$, we will call *stationary equilibria*. Equation 4 shows that if an agent with one only offer always buys the information, then the probability of an uninformed agent's uninformed neighbor becoming informed equals p. This argument allows us to guess that there might be other stationary equilibria except for pure strategy equilibria. In this subsection we characterize all such equilibria.

All the possible strategies of stationary equilibria can be characterized using the following proposition.

Proposition 4. *In any stationary equilibrium $K(1) = V$. For any $p \in (0,1)$ there exists exactly one stationary equilibrium. All stationary equilibria satisfy the transversality condition. The type of the equilibrium depends on p:*

1. For $p \in \left(0, \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}\right]$

$$F(v) \equiv F^p(v) = \begin{cases} 0, & v < V^p; \\ 1, & v \geq V^p, \end{cases}$$

and $V^p = \frac{u}{1-\delta(1-p)^2}$.

2. For $p \in \left(\frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}, p^*\right)$

$$F(v) \equiv F_1^m(v) = \begin{cases} 0, & v < (1-p)V_1^m; \\ \frac{1}{p} - \frac{(1-p)V_1^m}{pv}, & (1-p)V_1^m \le v \le u; \\ \frac{1}{p} - \frac{(1-p)V_1^m}{pu}, & u < v < V_1^m; \\ 1, & v \ge V_1^m, \end{cases}$$

and $V_1^m > u$ *is uniquely determined by equation*

$$\frac{u}{(1-p)V_1^m} + \delta(1-p) - \frac{1}{1-p} = \frac{\delta}{1-\delta(1-p)}\left(\frac{u}{(1-p)V_1^m} - 1 - \ln\left(\frac{u}{(1-p)V_1^m}\right)\right) \quad (5)$$

and decreases with p.

3. For $p \in [p^*, 1)$

$$F(v) \equiv F_1^m(v) = \begin{cases} 0, & v < (1-p)V_1^m; \\ \frac{1}{p} - \frac{V_1^m(1-p)}{pv}, & (1-p)V_1^m \le v \le V_1^m; \\ 1, & v > V_1^m, \end{cases}$$

and

$$V_1^m = \frac{u(1-\delta)}{(1-\delta(1-p))(1-\delta(1-p)^2) + \delta(1-p)\ln(1-p)} \in (0,1) \quad (6)$$

is a decreasing function of p *for* $p \ge p^*$.

Constant p^* *is the unique solution of equation*

$$p - (1-\delta(1-p))(1-p)^2 + (1-p)\ln(1-p) = 0 \quad (7)$$

from interval $\left(\frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}, 1\right)$.

Different strategies $F(v)$ for all three types of stationary equilibria are depicted at Figure 4.

For a small p strategy $F(v)$ is a degenerate distribution with the mass point at $V^p > u$, for a medium p strategy $F(v)$ has both continuous part on $[(1-p)V_1^m, u]$ and mass point at $V_1^m > u$, and for a high p strategy $F(v)$ is an absolutely continuous distribution on $[(1-p)V_2^m, V_2^m]$, where $V_2^m \le u$.

Strategy $F(v)$ for stationary equilibria evolves in the following way as p increases (see Figure 5). For small p strategy $F(v) = F^p(v)$ is a degenerate distribution with a mass point at $V^p > u$, and this mass point decreases with p. After $p = \frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}$, an absolutely continuous segment on $[(1-p)V_1^m, u]$ appears in $F(v) = F_1^m(v)$; this segment grows ((1 −

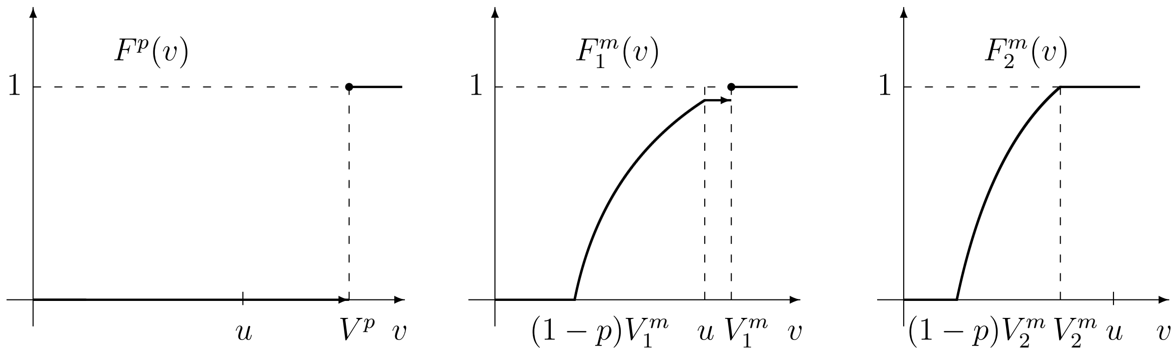

FIGURE 4. Stationary equilibra strategies in the infinite linear network. Left graph: pure strategy $F^p(v)$; Center graph: strategy $F_1^m(v)$; Right graph: strategy $F_2^m(v)$.

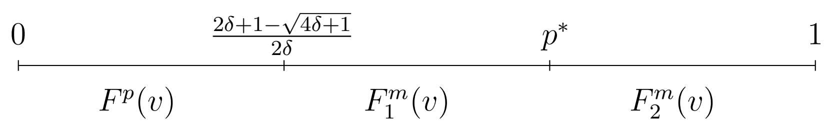

FIGURE 5. Stationary Equilibria Regions.

$p)V_1^m$ decreases) and the mass point V_1^m decreases to u with the mass at V_1^m decreasing to zero. At $p = p^*$, the mass point disappears, and the absolutely continuous segment starts moving towards zero. The lower bound decreases to 0, and the upper bound decreases to $u(1-\delta)$. Distribution $F(v)$ weakly converges to the degenerate distribution with mass point at 0.

For $p \in (0, p^*)$ there exists a mass point at $V > u$. Because this mass point is above the agent's personal valuation of the information, there is a non-zero probability that the agent will get two offers V at the same time, and therefore will stay uninformed forever.

Proposition 5. *In the stationary equilibrium with $p \in (0, p^*)$ probability that an uninformed agent with two uninformed neighbors will stay uninformed forever equals $\frac{p(1-F(u))^2}{2-p} > 0$ and does not depend on time.*

Probability that a randomly chosen agent will stay uninformed forever can be calculated as the sum of two probabilities: (1) the probability that the agent and his neighbors were initially uninformed multiplied by the probability that the agent will stay uninformed forever, and (2) the probability that the agent has two informed neighbors each of which offers price above u:

$$(1-p)^3\frac{p(1-F(u))^2}{2-p}+(1-p)p^2(1-F(u))^2=(1-p)\frac{p(1-F(u))^2}{2-p}.$$

For $p \geq p^*$ every agent in the network will get informed. This threshold p^* divides interval (0,1) into the areas of efficient and non-efficient equilibria. In order to achieve efficiency, the central planner does not need to give the information to everyone; it is enough to give the information randomly to a sufficient fraction of the population.

The equilibria of the game, in particular the stationary equilibria, might not be robust with respect to some modifications of the game. The question is what happens with the strategies if we consider the same game with a finite horizon instead of the infinite one. Take a sequence of equilibria in the games with the time limited by T. We want to investigate how close are the equilibria in such finite horizon games to the infinite horizon game equilibria, i.e. the limit of the equilibria of the games with finite horizons.

Proposition 6. *For small enough p the equilibria for the finite horizon games converge to the pure strategy stationary equilibrium for the infinite horizon game.*

3.3. Equilibria with Unbounded Price

The transversality condition restricts the prices. In this subsection we construct an example with a family of strategies in which this condition is not satisfied. The prices offered exceed some level and increase to infinity with time. What the agents pay for the information is not justified by the utility of the agents who get the information due to the transaction; the current price is supported by the expectations of the higher prices in the future.

Consider the linear network. For simplicity, we restrict our attention to the equilibria with pure strategies only, in which a buyer with two offers will not buy the information because these offers exceed his personal valuation of the information. By V_t we denote the offer/acceptance price at period t. A buyer with one informed neighbor only should be indifferent between buying the information and waiting, therefore the following equation holds for any t:

$$u - V_t + \delta(1-p)^2 V_{t+1} = 0,$$

i.e. buying the information and offering it to the uninformed agent gives zero expected utility. After rearranging the terms, one can get

$$V_{t+1} - \frac{u}{1-\delta(1-p)^2} = \frac{V_t - \frac{u}{1-\delta(1-p)^2}}{\delta(1-p)^2}.$$

Taking into account that in the stationary pure strategy equilibrium price always equals $\bar{V} = \frac{u}{1-\delta(1-p)^2}$, we get

$$V_{t+1} - \bar{V} = \frac{V_t - \bar{V}}{\delta(1-p)^2}. \tag{8}$$

As $\delta(1-p)^2 < 1$, difference $V - \bar{V}$ grows exponentially if initial V_0 exceeds $\bar{V}$:

$$V_t = \bar{V} + (V_0 - \bar{V}) \left(\frac{1}{\delta(1-p)^2} \right)^t.$$

The only additional requirement for V_0 is that a seller does not deviate to offering u at $t = 0$, i.e. $V_0(1-p) > u$ (if the prices increase, it will also be true for arbitrary t). Therefore, for any

$$V_0 > \max\left(\frac{u}{1-p}, \frac{u}{1-\delta(1-p)^2} \right)$$

the equilibrium we get is a pure strategy equilibrium for which the transversality condition fails, and the prices increases to infinity with time.

4. Random Networks

The analysis of the fixed networks showed that some equilibria in such networks possess some properties, like the price does not converge to zero. In this section we want to consider random networks, and find the properties of equilibria in these random networks to compare them with the properties of equilibria of the fixed networks.

Suppose that every period of time the agents are randomly matched with exactly M other agents[5], and the network formed does not contain cycles. It means that at every

[5]Random network is a controversial issue, although it is used in many models. In this paper we do not discuss the question of existence of such networks (although we believe that it is possible to construct a formal justification). We rather use some assumptions about such networks, namely that no two current

period of time a new M-network or a set of them is formed, and no past history can influences the agents' current decisions. Therefore, the agents' actions are independent across the time and neighbors.

As before, all the informed agents can simultaneously make their offers to their uninformed neighbors, and the uninformed agents decide to accept one of the available offers or to wait. As we deal with the random network, the informed agents make their offers to uninformed neighbors every period of time, and the offers made expire at the end of each period with the abortion of the connections.

At the beginning, every agent independently with probability p learns the information. The seller strategy is a distribution function of offers $F_t(v)$. The buyer strategy is a threshold K_t — the maximal price at which he is ready to buy the information. As new network is randomly formed each period of time, K_t does not depend on the number of informed neighbors. We consider only symmetric equilibria, i.e. the agents use the same strategies.

As before, denote $V_t = \sup \operatorname{supp} F_t(\cdot)$. Threshold $K_t \geq V_t$ because otherwise offer $V_t > K_t$ will never be accepted. Distribution function $F_t(\cdot)$ is absolutely continuous because $K_t \geq V_t$ and the agents will try to avoid the competition from other agents at the mass points. Also, $K_t \leq V_t$ because otherwise the agents selling the information for price $V_t < K_t$ will be able to increase their offer to K_t without decreasing the probability of the deal. Therefore, K_t coincides with V_t, and later in this section V_t will represent both constants.

From $K_t = V_t$ follows that an agent becomes informed once he has at least one informed neighbor. Denote the probability of being informed at the beginning of period t by p_t, with $p_1 = p$. Then

$$p_{t+1} = p_t + (1-p_t)(1-(1-p_t)^M) = 1-(1-p_t)^{M+1};$$
$$1-p_{t+1} = (1-p_t)^{M+1},$$

and p_t monotonically approaches 1.

neighbors can have any influence on each other in the future. In particular, the probability of being matched with the same partner twice is assumed to be zero.

Denote

$$g_t = (1-p_t)^{M-1}(M + (1-p_t)). \tag{9}$$

As p_t monotonically approaches 1, g_t monotonically approaches 0.

As before, by the transversality condition we understand that the price in the transactions does not exceed the discounted expected utility of all the agents who get the information due to this transaction. The following proposition completely characterizes equilibria satisfying the transversality condition.

Proposition 7. *For any initial probability $p \in (0,1)$ there is only one equilibrium that satisfies the transversality condition. In this equilibrium the seller strategy*

$$F_t(v) = \frac{1}{p_t} - \frac{1-p_t}{p_t}\left(\frac{V_t}{v}\right)^{\frac{1}{M-1}}; \tag{10}$$

$$\operatorname{supp} F_t(\cdot) = [V_t(1-p_t)^{M-1}, V_t]. \tag{11}$$

The highest price possible at period t

$$V_t = V_1 \prod_{i=2}^{t} \frac{1}{\delta g_i} - u(1-\delta)\sum_{i=2}^{t}\prod_{j=i}^{t}\frac{1}{\delta g_j}; \tag{12}$$

$$V_1 = u(1-\delta)\left(1 + \sum_{i=3}^{\infty}\prod_{j=2}^{i-1}\delta g_i\right) < \infty. \tag{13}$$

The highest possible price V_t monotonically decreases to $u(1-\delta)$, and the expected price $\mathbf{E}_{F_t} v$ converges to 0.

As we see, V_t is uniquely determined by constants M, p, u, and δ. $F_t(\cdot)$ weakly converges to the degenerate distribution with the mass point at zero.

5. Conclusion

In this paper we show that the structure of the social connections plays an important role in information diffusion. It determines the price pattern the sellers charge for the information and the buyers strategy. In particular, the price asked does not always converge to zero. The agents making an offer might believe that the probability of an uninformed neighbor getting another acceptable offer is small enough, therefore they do not decrease the price. In the case of many uninformed agents at the beginning, this belief leads to the price exceeding the personal valuation of the information.

Not all the agents might learn the information at the end if the price exceeds the personal valuation; it happens if the information is a scarce resource. The information diffuses to all the agents if the fraction of the initially informed agents is large enough. Therefore, if the government wants everyone to have the information, it does not need to give it to all the agents; it is enough to exceed some threshold, and after this the agents will successfully trade the information with each other.

The linear network considered in many papers does not constitute a representative example. It has the property which is particular only for such a network: the belief about the number of uninformed agents till the first informed one, conditional on the fact that the agent himself and his neighbor are uninformed, does not depend on time. Due to this there exists the stationary equilibrium where the strategies the agents use do not depend on time.

The equilibrium for the random network differs from the fixed network in the following aspects. The uninformed agents buy the information as soon as they get at least one offer. The average price offered at period t converges to 0; however, the upper bound of the price converges to $u(1-\delta)$. In the random network, every agent becomes informed with probability 1.

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Appendix

Lemma 1. The differential equation

$$af'(x)x = 1 - bf(x)$$

for $b \neq 0$, $a \neq 0$ has solution

$$f(x) = \frac{1}{b} - Cx^{-b/a}. \tag{14}$$

Proof of lemma 1.

The solution is verified by substituting formula (14) for $f(x)$ into the original equation and the fact that the first-order differential equation has only one undetermined constant.

□

Proof of proposition 1.

The neighbors are connected only through the agent, therefore the seller strategy can be independent for each of his uninformed neighbors; if the offer is made, it follows some distribution function $F_t(v)$, which depends only on time.

Suppose that a buyer with exactly l informed neighbors accepts offer v. Then accepting offer $v' < v$ increases the buyer's expected payoff by $v - v'$ without changing his expectations of the future resales. The expected utility of waiting with the lowest offer $v' < v$ increases by less than $v - v'$ because the best difference is $v - v'$ and the discount factor decreases it. Therefore, the strategy of a buyer with l informed neighbors is to accept an offer either from interval $[0, K_t(l))$ or $[0, K_t(l)]$ for some $K_t(l) \geq 0$. The buyer is indifferent to accept offer $K_t(l)$ or to wait.

If for a buyer there is no mass of offers at $K_t(l)$, then these two options (to buy immediately and to wait) do not differ, and we can choose the closed interval. If there is a mass point, then the sellers who create the mass point ($F_t(\cdot)$ has a mass point) would prefer to deviate to $K_t(l) - \epsilon$, which means that this is not an equilibrium and $K_t(l)$ can not be a mass point of offers. Therefore, we can always assume that a buyer with l neighbors accepts any offer not exceeding $K_t(l)$.

To prove the existence of an equivalent equilibrium in which all the sellers make their offers immediately, consider one informed agent A and his uninformed neighbor B. By waiting agent A can observe only the fact that B gets the information from his other neighbor (what makes impossible selling the information to B). Agent A makes such offer v that maximizes his expected payoff.

There are 2 options:

Option 1. Agent B with non-zero probability may accept offer v earlier than agent A normally makes it. Then agent A is strictly better off by making the offer earlier, and therefore this is not an equilibrium to delay with making this offer.

Option 2. Agent B would not accept offer v earlier than agent A normally makes it. Then by making offer v earlier Agent A does not change his own payoff and the rest of his strategy. Suppose that Agent B has other lowest offer and making offer v earlier changes B's behavior. As B does not accept offer v (we excluded option 1) then the other offer he has is better, and B knows it because A does not make an offer. Consequently, revealing v does not change B's decision to accept other offers. Therefore, making offer v earlier does not change anything and making the offers as soon as possible is a new equivalent equilibrium.

□

Proof of proposition 2.

The proof has the following structure. First, we consider the following modification of the game: agents A_i, A_{i-1},... are always uninformed at the beginning (see Figure 3). Second, we demonstrate that random variables ξ_1 and $\xi_1 - \xi_t$ are independent for any t, where ξ_t is the number of uninformed agents A_{i+1}, A_{i+2},..., A_{i+k} till the first informed agent A_{i+k+1} at time t. Third, we show that ξ_t conditional on $\xi_t \geq 0$ has the same geometric distribution as ξ_1. And last, we return to the original game, and prove formula 3 from the Proposition.

Step 1. Defining the game and random variables.

Suppose that A_i, A_{i-1},... are always uninformed at the beginning. Define random variable $\xi_t \in \mathbb{Z}$ in the following way:

$$\xi_t = \min\{k : A_{i+k+1} \text{ is informed at the beginning of period } t\}.$$

Random variable $\xi_t \in \mathbb{Z}$ stands for the first informed agent in the network.

Step 2. Independence of ξ_1 and $\xi_1 - \xi_t$.

Consider agent l who acquires the information at period t. Let η_{lt} be the number of periods it takes for agent l to transfer the information to his left neighbor, if this neighbor has only one offer. The agents act independently, therefore all random variables $\{\eta_{lt}\}$ are independent of each other and ξ_1. The agents use the same strategies, therefore $\{\eta_{lt}\}_l$ are identically distributed for each t.

Let η_l stands for the number of agents the information diffused to the left by time t if initially agent l is the first informed agent. Variables η_l are determined by $\{\eta_{l't'}\}_{l't'}$ and therefore independent of ξ_1 for any l (but not from each other). As $\{\eta_{lt}\}_l$ are identically distributed for each t, η_l are identically distribute for every l. Denote this distribution by η.

Note that

$$\begin{aligned}
\mathbf{P}(\xi_1 = m, \xi_1 - \xi_t = l) &= \mathbf{P}(\xi_1 = m, \eta_{\xi_1} = l) = \mathbf{P}(\xi_1 = m, \eta_m = l) \\
&= \mathbf{P}(\xi_1 = m)\,\mathbf{P}(\eta = l);
\end{aligned}$$

$$\mathbf{P}(\xi_1 - \xi_t = l) = \sum_m \mathbf{P}(\xi_1 = m, \xi_1 - \xi_t = l) = \sum_m \mathbf{P}(\xi_1 = m)\,\mathbf{P}(\eta = l) = \mathbf{P}(\eta = l),$$

i.e. random variables ξ_1 and $\xi_1 - \xi_t$ are independent.

Step 3. Geometric distribution of ξ_t conditional on $\xi_t \geq 0$.

We want to prove

$$\mathbf{P}\{\xi_t = k | \xi_t \geq 0\} = p(1-p)^k, \qquad \forall t, k \in \mathbb{N}. \tag{15}$$

Note that this formula holds for $t = 1$ because the agents independently with probability p get the information at the beginning.

$$\begin{aligned}
\mathbf{P}\{\xi_t = k | \xi_t \geq 0\} &= \frac{\sum\limits_{l\geq 0} \mathbf{P}(\xi_1 = k + l, \xi_1 - \xi_t = l)}{\sum\limits_{l\geq 0, k'\geq 0} \mathbf{P}(\xi_1 = k' + l, \xi_1 - \xi_t = l)} = \frac{\sum\limits_{l\geq 0} p(1-p)^{k+l}\,\mathbf{P}(\xi_1 - \xi_t = l)}{\sum\limits_{l\geq 0, k'\geq 0} p(1-p)^{k'+l}\,\mathbf{P}(\xi_1 - \xi_t = l)} \\
&= \frac{(1-p)^k \sum\limits_{l\geq 0} p(1-p)^l\,\mathbf{P}(\xi_1 - \xi_t = l)}{\frac{1}{p} \sum\limits_{l\geq 0} p(1-p)^l\,\mathbf{P}(\xi_1 - \xi_t = l)} = p(1-p)^k,
\end{aligned}$$

i.e. formula 15 holds for any t.

Step 4. Proof of formula 3 from the Proposition.

Consider the original game. In this game, agents A_{i-1}, A_i, can get the information by time t either at the beginning, from A_{i-2}, or from A_{i+1}. We considered the process from the right. We can make the same analysis from the left, and consider corresponding random variable ζ_t — the distance from the right informed agent to A_{i-1} in the hypothetical network where all the agents $A_{i-1}, A_i, \ldots$ are uninformed at the beginning, Then ξ_t and ζ_t are independent, and for any $k \geq 0$

$$\begin{aligned}
\mathbf{P}(A^t_{i-1} A^t_i \ldots A^t_{i+k} \bar{A}^t_{i+k+1} | A^t_{i-1} A^t_i) &= \mathbf{P}(\zeta_t \geq 0, \xi_t = k | \zeta_t \geq 0, \xi_t \geq 0) = \frac{\mathbf{P}(\zeta_t \geq 0, \xi_t = k)}{\mathbf{P}(\zeta_t \geq 0, \xi_t \geq 0)} \\
&= = \frac{\mathbf{P}(\xi_t = k)}{\mathbf{P}(\xi_t \geq 0)} = p(1-p)^k.
\end{aligned}$$

□

Proof of proposition 3.

Consider first t such that $V_t \leq u$. Consider an agent who makes an offer at time t to his uninformed neighbor. There is a non-zero probability that the neighbor has the same offer V_t from his another neighbor, and will be choosing the best one. Then the agent will benefit by decreasing his offer to $V_t - \epsilon$: the probability of selling the information increases, and the payment stays almost the same. Therefore, $V_t > u$ for all t.

Suppose that there exists t such that $V_t < K_t(1)$. The agent accepts offer V_t only if this is the only offer, and another neighbor is uninformed. By increasing the offer to $K_t(1)$ the seller does not decrease the chance of the deal, but increases the payment. Therefore, V_t can not be less than $K_t(1)$.

At every period of time there is either no trade or all the agents with one informed neighbor only buy the information.

Consider first t such that $V_t = K_t(1)$, $V_{t+1} > K_{t+1}(1)$. Suppose that $t > 1$ (the proof with slight modification works for $t = 1$, too.) The agents with one offer V_{t+1} only at time period $t+1$ wait with the purchase until some period $\tau > t+1$ with $V_{t+1} \leq K_\tau(1)$, and there is no trade in periods $t+1, t+2, \ldots, \tau - 1$.

Any agent who buys the information at period t has utility zero because the offer from other neighbors will exceed u, and the seller has all the power. Therefore,

$$-V_t + u + \delta^{\tau-t}(1-p)^2 V_{t+1} = 0. \tag{16}$$

Suppose that some agent with one the only offer V_t at period t does not buy the information immediately, but waits till period $t+1$. If his neighbor still stays uninformed, then he pays V_t at period $t+1$, and offers it to his uninformed neighbor at time $t+2 \leq \tau$ for V_{t+1}. Then, using equation 16, his utility

$$\begin{aligned} \delta(1-p)(-V_t + u + \delta^{\tau-t-1}(1-p)V_{t+1}) &= \delta(1-p)(-V_t + u) + V_t - u \\ &= (V_t - u)(1 - \delta(1-p)) > 0 \end{aligned}$$

because $V_t > 0$, which means that this is not an equilibrium. The intuition behind the fact the the utility increases if the agent waits is the following: by waiting the agent decreases the uncertainty about the possibility of reselling the information.

We have proved that for any period t holds $V_t = K_t(1)$. Equation 16 for $\tau = t+1$ gives us the the law of motion for V_t:

$$-V_t + u + \delta(1-p)^2 V_{t+1} = 0.$$

Fixed point

$$V = \frac{u}{1 - \delta(1-p)^2}.$$

Therefore,

$$V_t - V = \delta(1-p)^2(V_{t+1} - V),$$

which means that this fixed point is unstable: if $V_1 \neq V$ then V_t converges either to $-\infty$ or to $+\infty$. The first option contradicts $V_t \geq 0$, and the second one contradicts the transversality condition (the price is limited by some constant).

□

Proof of proposition 4.

First, we want to show that $V = K(1)$. We already know that $K(2) = u$. Distribution function $F(v)$ is continuous for $v \leq u$ because $K_t(2) = u$ and a seller offering the mass point price would better off by decreasing his offer by small ϵ to avoid the tie.

Suppose that $V \neq K(1)$. As $V \leq \max(K(1), u)$, the following four options are possible: $V < K(1) \leq u$, $K(1) < V \leq u$, $V < u \leq K(1)$, and $u \leq V < K(1)$.

Options $V < K(1) \leq u$ and $V < u \leq K(1)$ can not be an equilibrium because $F(v)$ is continuous below u, and a seller offering the information for price V is better off by asking $K(1)$ and u correspondingly.

Consider option $K(1) < V \leq u$. There is a non-zero probability of offers $v \in [0, K(1)]$ and $v \in (K(1), u]$ because otherwise an agent with offer $K(1)$ will not get be able to get a better offer in the future. An offer from $[0, K(1)]$ is always accepted, and an offer from $(K(1), u]$ is accepted if and only if there are two offers; in the case of one offer the agent always waits for the second one. Because of the waiting the agents change their belief about event "the first informed agent behind the uninformed neighbor got an offer above $K(1)$," which is impossible in stationary equilibrium. Therefore, $K(1) < V \leq u$ is not an equilibrium.

Consider option $u \leq V < K(1)$. Offers V and $K(1)$ have the same chance to be accepted (the neighbor's neighbor is uninformed and stays uninformed till the next round), but $K(1)$ delivers a higher payoff. Therefore, this is also not an equilibrium.

We have proved that either $V = K(1) < u$ or $u < V = K(1)$. Later in the proof we will always use V instead of $K(1)$, The support of $F(v)$ below u constitutes a connected set; if not, an agent can increase his expected payoff by increasing the offer in the gap as the probability of the deal does not change.

There are 3 cases: $F(u) = 0$, $F(u) \in (0, 1)$, and $F(u) = 1$. If $F(u) < 1$, then the distribution of prices $F(v)$ has mass $1 - F(u)$ at $V > u$. If $F(u) > 0$, then the expected payoff maximization problem for $v \in [0, \min(u, V)$ gives

$$(1-p)v + pv(1 - F(v)) \to \max; \tag{17}$$

$$1 - pF(v) - pvf(v) = 0.$$

Applying Lemma 1,

$$F(v) = \frac{1}{p} - \frac{C}{v} \qquad \text{for } v \in [pC, \min(u, V)].$$

Offer pC is always accepted, and offer V is accepted only of there is the neighbor does not have other offer. Both these prices are in the support of $F(\cdot)$ and deliver the same expected payoff, therefore $pC = (1-p)V$, and

$$F(v) = \frac{1}{p} - \frac{(1-p)V}{pv} \qquad \text{for } v \in [(1-p)V, \min(u, V)].$$

Now we want to find $F(v)$ for each of the three cases.

Case 1. $F(u) = 0$, pure strategy with mass 1 at $V > u$.

Denote this distribution function of offers by $F^p(v)$. In accordance with Proposition 3, $V = \frac{u}{1-\delta(1-p)^2}$. This equilibrium exists if and only if the agents do not want to offer price u which is always accepted, i.e.

$$V(1-p) \geq u;$$

$$p \leq \delta(1-p)^2; \tag{18}$$

Case 2. $F(u) \in (0, 1)$, some mass at $V > u$ and a continuous part on $[(1-p)V, u]$.

Denote this distribution function of offers by $F_1^m(v)$.

An agent with offer V and one uninformed neighbor is indifferent between accepting the offer and waiting for another one. The expected payoff from buying the information immediately equals

$$-V + u + \delta(1-p)^2 V. \tag{19}$$

If the agent waits for another offer, he gets the information if and only if his another neighbor offers the information for a price $v \le u$. The expected payoff from waiting is

$$\begin{aligned}
\mathbf{E}\left(\sum_{t\ge 1}\delta^t p(1-p)^{t-1}(u-v)I_{\{v\le u\}}\right) &= \frac{p\delta}{1-\delta(1-p)}\left(F(u)u - \int\limits_{(1-p)V}^{u} v\,dF(v)\right)\\
&= \frac{(1-p)V\delta}{1-\delta(1-p)}\left(\frac{u}{(1-p)V} - 1 - \int\limits_{(1-p)V}^{u}\frac{dv}{v}\right)\\
&= \frac{(1-p)V\delta}{1-\delta(1-p)}\left(\frac{u}{(1-p)V} - 1 - \ln\left(\frac{u}{(1-p)V}\right)\right).
\end{aligned}$$

Equating the expected payoff of from buying the information immediately (formula 19) and waiting, one gets equation 5:

$$\frac{u}{(1-p)V} + \delta(1-p) - \frac{1}{1-p} = \frac{\delta}{1-\delta(1-p)}\left(\frac{u}{(1-p)V} - 1 - \ln\left(\frac{u}{(1-p)V}\right)\right). \tag{20}$$

The condition for the existence of such equilibrium $F(u) \in (0,1)$ is equivalent to $x \equiv \frac{u}{(1-p)V} \in \left(1, \frac{1}{1-p}\right)$. Rewriting equation 20 using x gives

$$x - \frac{\delta}{1-\delta(1-p)}\left(x - 1 - \ln x\right) = \frac{1}{1-p} - \delta(1-p). \tag{21}$$

Denote the left-hand side of equation 21 by $h(x,\delta,p)$. For any $x \in \left[1, \frac{1}{1-p}\right]$ the derivative

$$\frac{\partial h(x,\delta,p)}{\partial x} = 1 - \frac{\delta(1-1/x)}{1-\delta(1-p)} \ge 1 - \frac{\delta(1-(1-p))}{1-\delta(1-p)} = \frac{1-\delta}{1-\delta(1-p)} > 0;$$

Therefore, there exists $x \in \left(1, \frac{1}{1-p}\right)$ satisfying equation 21 if and only if

$$h(1,\delta,p) < \frac{1}{1-p} - \delta(1-p) < h\left(\frac{1}{1-p},\delta,p\right); \tag{22}$$

$$1 < \frac{1}{1-p} - \delta(1-p) < \frac{1}{1-p} - \frac{\delta}{1-\delta(1-p)}\left(\frac{p}{1-p} - \ln\frac{1}{1-p}\right).$$

Therefore, this equilibrium exists if and only if the following two inequalities hold:

$$p > \delta(1-p)^2; \tag{23}$$

$$-(1-\delta(1-p))(1-p)^2 + p + (1-p)\ln(1-p) < 0. \tag{24}$$

Case 3. $F(u) = 1$, $\mathrm{supp}(F(v)) = [(1-p)V, V]$, $V \le u$.

Denote this distribution function of offers by $F_2^m(v)$. A buyer is indifferent between buying at the maximal price V (formula 19) and waiting:

$$
\begin{aligned}
-V+u+\delta(1-p)^2V &= \sum_{t\geq 1}\delta^t p(1-p)^{t-1}\int (u-v)\,dF(v) = \frac{(1-p)V\delta}{1-\delta(1-p)}\int\limits_{(1-p)V}^{V}\frac{u-v}{v^2}\,dv \\
&= \frac{(1-p)V\delta}{1-\delta(1-p)}\left(\left(-\frac{u}{V}+\frac{u}{(1-p)V}\right)+\ln(1-p)\right) \\
&= \frac{p\delta u}{1-\delta(1-p)}+\frac{(1-p)V\delta\ln(1-p)}{1-\delta(1-p)};
\end{aligned}
$$

$$
V=\frac{u(1-\delta)}{(1-\delta(1-p))(1-\delta(1-p)^2)+\delta(1-p)\ln(1-p)}.
$$

This equilibrium exists if and only if $V\leq u$, or

$$
-(1-\delta(1-p))(1-p)^2+p+(1-p)\ln(1-p)\geq 0. \tag{25}
$$

We want to show that for any $\delta\in(0,1)$ interval $(0,1)$ is divided into three parts by $p'\in(0,1)$ and $p''\in(p',1)$. On $(0,p']$ inequality 18 holds (Case 1), on (p',p'') inequalities 23 and 24 hold (Case 2), and on $[p'',1)$ inequality 25 holds (Case 3).

Inequality 18 holds on $(0,p']$ and inequality 23 holds on $(p',1)$, where

$$
p'=\frac{2\delta+1-\sqrt{4\delta+1}}{2\delta}.
$$

Denote left-hand side of inequalities 24 and 25 as $g(p,\delta)$. The second derivative

$$
\begin{aligned}
\frac{\partial^2}{\partial p^2}\left(\frac{g(p,\delta)}{1-p}\right) &= \frac{\partial}{\partial p}\left((1-\delta(1-p))-\delta(1-p)+\frac{1}{1-p}+\frac{p}{(1-p)^2}-\frac{1}{1-p}\right) \\
&= 2\delta+\frac{1}{(1-p)^2}+2\frac{p}{(1-p)^3}>0,
\end{aligned}
$$

therefore $\frac{g(p,\delta)}{1-p}$ either increases or first decreases and then increases. As $g(0,\delta)<0$ and $g(0,\delta)>0$, equation $g(p,\delta)=0$ has exactly one solution $p''\in(0,1)$, and on $(0,p'')$ inequality 24 holds, and on $[p'',1)$ inequality 25 holds. The only fact we have to prove is that $p''>p'$. To do this, it is enough to show that there exists p satisfying both inequalities in 22.

We know that $h(1,\delta,p')<h\left(\frac{1}{1-p'},\delta,p'\right)$, $h(1,\delta,p)=1$, $h(1,\delta,p')=\frac{1}{1-p'}-\delta(1-p')$, and function $h(x,\delta,\tilde{p})$ is continuous in all arguments. The middle part of 22 increases with p because

$$
\frac{1}{1-p}-\delta(1-p)=(1-\delta)+p(1+\delta)+p^2+p^3+\ldots.
$$

Therefore, in some neighborhood of p' for $p>p'$ both inequalities in 22 hold, and therefore $p''>p'$.

Value V_2^m decreases with p because $p > \delta(1-p)^2$ and therefore

$$\begin{aligned} \frac{d}{dp}\left(\frac{u(1-\delta)}{\delta V_2^m}\right) &= 1-\delta(1-p)^2+2(1-p)(1-\delta(1-p))-1-\ln(1-p) \\ &\geq 1-p-1-\ln(1-p) \geq 0. \end{aligned}$$

Value V_1^m decreases with p because $p > \delta(1-p)^2$ and therefore

$$\begin{aligned} \frac{d}{dp}\left(\frac{u(1-\delta)}{\delta V_2^m}\right) &= 1-\delta(1-p)^2+2(1-p)(1-\delta(1-p))-1-\ln(1-p) \\ &\geq 1-p-1-\ln(1-p) \geq 0. \end{aligned}$$

□

Proof of proposition 5.

The agent will get two offers simultaneously only if the informed agents on the opposite sides are located on the same distance. Therefore, the probability of staying uninformed forever equals

$$\sum_{t\geq 0}((1-p)^t p)^2(1-F(u))^2 = p^2(1-F(u))^2\frac{1}{1-(1-p)^2} = \frac{p(1-F(u))^2}{2-p}.$$

□

Proof of proposition 6.

We will denote all the strategies in the game with horizon T by upper index T. We are looking for the finite horizon equilibria at which the agents with one informed neighbor only always buy the information, i.e. $V_t^T = K_t^T(1)$.

At the last period $K(1) = u$ and therefore $F_T^T(u) = 1,$. The seller's problem is the same as problem 17, which means that the solution is also the same:

$$F_T^T(v) = \frac{1}{p}\left(1-\frac{u(1-p)}{v}\right), \qquad v \in [(1-p)u, u].$$

The expected payoff of the agent who gets only one offer is

$$\pi \equiv \int_{(1-p)u}^{u} (u-v)\, dF_T^T(v) \leq \int_{(1-p)u}^{u} (u-(1-p)u)\, dF_T^T(v) = pu.$$

Suppose that for any $t < T$ distribution function $F_t^T(v)$ has mass 1 at $V = K_t^T(1) > u$ and does not have the continuous part below u. Then to make the buyer indifferent between buying and waiting till the last period the following equation should hold:

$$-V_t^T + u + \delta(1-p)^2 V_{t+1}^T = \delta^{T-t}(1-p)^{T-t-1}p\pi \tag{26}$$

Note that

$$V_{T-1}^T = -\delta p\pi + u + \delta(1-p)(1-p)u > u(1+\delta(1-p)^2-p).$$

Note that $V_T^T < \frac{u}{1-\delta(1-p)^2}$. By induction,

$$V_t^T = -\delta^{T-t}(1-p)^{T-t-1}p\pi + u + \delta(1-p)^2 V_{t+1}^T < u + \frac{\delta(1-p)^2 u}{1-\delta(1-p)^2} = \frac{u}{1-\delta(1-p)^2}.$$

Note that $V_{T-1}^T > V_T^T$. By induction, $V_{t-1}^T > V_t^T$ because

$$\begin{aligned} V_{t-1}^T &= -\delta^{T-(t-1)}(1-p)^{T-(t-1)-1}p\pi + u + \delta(1-p)^2 V_t^T \\ &> -\delta^{T-t}(1-p)^{T-t-1}p\pi + u + \delta(1-p)^2 V_{t+1}^T = V_t^T. \end{aligned}$$

No seller will deviate from V_t^T because the expected payoff from V_t^T is greater than the expected payoff from u:

$$(1-p)V_t^T \geq (1-p)V_{T-1}^T > (1-p)(u(1+\delta(1-p)^2) - pu) > u,$$

where the last inequality holds for small enough p.

Finally, for any t values V_t^T converge as T increases because V_t^T are limited, increase, and $V_{t+1}^{T+1} = V_t^T$. Denote $V_t = \lim_{T\to\infty} V_t^T$. Then

$$V - V_t^T = \delta(1-p)^2(V - V_{t+1}^T) + \delta^{T-t}(1-p)^{T-t-1}p\pi;$$

$$V - V_t = \delta(1-p)^2(V - V_{t+1}).$$

V_t are limited, and have the same law of motion as V_t for the pure strategy equilibrium, therefore $V_t = \frac{u}{1-\delta(1-p)^2}$ for any t.

□

Proof of proposition 7.

The structure of the proof is the following. First, we show that $F_t(\cdot)$ does not have mass points and has a connected support for any t. Second, we prove that

$$F_t(v) = \frac{1}{p_t} - C_t v^{-\frac{1}{M-1}}. \tag{27}$$

and find formula for the support (formulas 10 and 11). Third, we prove the law of motion for V_t:

$$\frac{1}{\delta}(V_{t-1} - u) = V_t g_t - u. \tag{28}$$

Forth, based on the law of motion for V_t we establish formulas 12 and 13 for V_t. And last, we show monotonicity and convergence of V_t and convergence of $\mathbf{E}_{F_t}\, v$.

Step 1. Properties of $F_t(\cdot)$.

No offer above V_t will be accepted, therefore $F_t(V_t) = 1$. The distribution function $F_t(\cdot)$ does not have mass points because otherwise a seller would prefer to decrease his offer from these mass points by some small ϵ.

The support $\text{supp}(F_t(\cdot))$ is connected because by increasing the offer in the gap, a seller will increase his expected payoff as the acceptance probability of the offer stays the same, and the price increases.

Step 2. The proof of formulas 10 and 11 for $F_t(\cdot)$ and for its support.

The expected payoff from one uninformed neighbor is equal to

$$\begin{aligned}
\pi_t(v) &= v\,\mathbf{P}\{v \le \text{other offers }\} \\
&= v \prod_{i=1}^{M-1} \left(\mathbf{P}\{v \le \text{offer from neighbor } i\} + \mathbf{P}\{\text{no offer from neighbor } i\}\right) \\
&= v \prod_{i=1}^{M-1} \left((1 - F_t(v))p_t + (1 - p_t)\right) = (1 - p_t F_t(v))^{M-1}\, v.
\end{aligned}$$

All the points in the support of $F_t(\cdot)$ should deliver the same utility π, we have

$$\pi_t'(v) \equiv (1 - p_t F_t(v))^{M-1} - p_t f_t(v) v (1 - p_t F_t(v))^{M-2}(M-1) = 0;$$

$$p_t f_t(v) v (M-1) = 1 - p_t F_t(v).$$

Applying Lemma 1,

$$F_t(v) = \frac{1}{p_t} - C_t v^{-\frac{1}{M-1}}$$

for some constant $C_t > 0$ (formula 27).

One can verify that

$$V_t \equiv \sup \operatorname{supp} F_t(\cdot) = \left(\frac{p_t C_t}{1 - p_t}\right)^{M-1},$$

therefore $C_t = \frac{1-p_t}{p_t} V_t^{\frac{1}{M-1}}$ and we have proved formula 10 for $F_t(\cdot)$ and formula 11 for the support of $F_t(\cdot)$.

Step 3. Law of motion for V_t (formula 28).

Let U_t^i be the expected payoff of an informed agent at the beginning of period t, and let U_t^u be the expected payoff of an uninformed agent at the beginning of period t. Then

$$U_t^i = M(1 - p_t)\pi_t + \delta U_{t+1}^i = \sum_{i=t}^{\infty} \delta^{i-t} M (1 - p_i)\pi_i; \tag{29}$$

$$U_t^u = (u - \mathbf{E}\, v_t + \delta U_{t+1}^i)(1 - (1 - p_t)^M) + \delta U_{t+1}^u (1 - p_t)^M, \tag{30}$$

where $\mathbf{E}\, v_t$ stands for the expected price an agent pays for acquiring the information at period t, conditional on the fact that there is at least one offer.

In an equilibrium the buyer with the highest possible offer V_t is indifferent between accepting the offer and waiting, therefore

$$u - V_t + \delta U^i_{t+1} = \delta U^u_{t+1}. \tag{31}$$

Substituting expressions for U^i_t (formula 29) and U^u_t (formula 30) into 31 one can get

$$\begin{aligned}
\frac{1}{\delta}(V_{t-1} - u) &= M(1-p_t)\tilde{V}_t + \delta U^i_{t+1} \\
&\qquad -(u - \mathbf{E}\, v_t + \delta U^i_{t+1})(1-(1-p_t)^M) - \delta U^u_{t+1}(1-p_t)^M \\
&= (V_t - u)(1-p_t)^M + M(1-p_t)^M V_t + (\mathbf{E}\, v_t - u)(1-(1-p_t)^M) \\
&= (M+1)V_t(1-p_t)^M + \mathbf{E}\, v_t(1-(1-p_t)^M) - u.
\end{aligned} \tag{32}$$

In order to simplify expression 32, we need formula for $\mathbf{E}\, v_t$.

The average minimal price from l independent offers $v, v_2, \ldots, v_l$ is equal to

$$\begin{aligned}
\mathbf{E}_{F_{lt}}\, v \equiv \int v\, d\, \mathbf{P}(\min(v_1,\ldots,v_l) \le v) &= \int v d \left(1 - (1 - \mathbf{P}(v_1 \le v))^l\right) \\
&= \int v d \left(1 - \left(h_t(v) - \left(\frac{1}{p_t} - 1\right)\right)^l\right) \\
&= \int \frac{h_t(v) l}{M-1} \left(h_t(v) - \frac{1}{p_t} + 1\right)^{l-1} dv,
\end{aligned} \tag{33}$$

where $h_t(v) = \frac{1-p_t}{p_t}\left(\frac{V_t}{v}\right)^{\frac{1}{M-1}}$ for simplicity of notation. Note that

$$\begin{aligned}
\sum_{l=1}^{M} l x^{l-1} C^l_M p^l_t (1-p_t)^{M-l} &= M p_t \sum_{l=0}^{M-1} C^l_{M-1} x^l p^l_t (1-p_t)^{(M-1)-l} \\
&= M p_t \left(px + 1 - p_t\right)^{M-1}
\end{aligned} \tag{34}$$

Equation 34 for $x = h_t(v) - \frac{1}{p_t} + 1$ gives

$$\sum_{l=1}^{M} l \left(h_t(v) - \frac{1}{p_t} + 1\right)^{l-1} C^l_M p^l_t (1-p_t)^{M-l} = M p_t (h_t(v) p_t)^{M-1}. \tag{35}$$

Combining 33 and 35 and swapping the integral and the sum, one can get

$$\begin{aligned}
\sum_{l=1}^{M} C_M^l p_t^l (1-p_t)^{M-l}\, \mathbf{E}_{F_{lt}}\, v &= \int\limits_{V_t(1-p_t)^{M-1}}^{V_t} \frac{M}{M-1} (h_t(v) p_t)^M \, dv \\
&= \int\limits_{V_t(1-p_t)^{M-1}}^{V_t} \frac{M}{M-1} \left((1-p_t) \left(\frac{V_t}{v}\right)^{\frac{1}{M-1}} \right)^M dv \\
&= V_t \int\limits_{(1-p_t)^{M-1}}^{1} \frac{M}{M-1} (1-p_t)^M v^{-\frac{1}{M-1}-1} \, dv. \qquad (36)
\end{aligned}$$

Taking into account the fact that the probability of exactly l informed neighbors is equal to $C_M^l p_t^l (1-p_t)^{M-l}$ and applying equation 36, we have

$$\begin{aligned}
\mathbf{E}\, v_t (1-(1-p_t)^M) &= \sum_{l=1}^{M} C_M^l p_t^l (1-p_t)^{M-l}\, \mathbf{E}_{F_{lt}}\, v \\
&= V_t \int\limits_{(1-p_t)^{M-1}}^{1} \frac{M(1-p_t)^M}{M-1} v^{-\frac{1}{M-1}-1} \, dv \\
&= -V_t\, M(1-p_t)^M v^{-\frac{1}{M-1}} \Big|_{(1-p_t)^{M-1}}^{1} = V_t M p_t (1-p_t)^{M-1}. \qquad (37)
\end{aligned}$$

Substituting $\mathbf{E}\, v_t(1-(1-p_t)^M)$ (formula 37) into formula 32 and taking definition for g_t (formula 9), we have the law of motion for V_t (formula 28).

Stage 4. Finding expression for V_t (formulas 12 and 13).

Rearranging terms in formula 28, one can get

$$V_t = \frac{V_{t-1}}{\delta g_t} - \frac{u(1-\delta)}{\delta g_t}. \qquad (38)$$

Formula 38 for $t = 2$ corresponds to the expression for V_t (formula 12). Using formula 38 again,

$$\begin{aligned}
V_{t+1} &= \frac{V_1 \prod\limits_{i=2}^{t} \frac{1}{\delta g_i} - u(1-\delta) \sum\limits_{i=2}^{t} \prod\limits_{j=i}^{t} \frac{1}{\delta g_j}}{\delta g_{t+1}} - \frac{u(1-\delta)}{\delta g_{t+1}} \\
&= V_1 \prod_{i=2}^{t+1} \frac{1}{\delta g_i} - u(1-\delta) \sum_{i=2}^{t} \prod_{j=i}^{t+1} \frac{1}{\delta g_j} - \frac{u(1-\delta)}{\delta g_{t+1}} \\
&= V_1 \prod_{i=2}^{t+1} \frac{1}{\delta g_i} - u(1-\delta) \sum_{i=2}^{t+1} \prod_{j=i}^{t+1} \frac{1}{\delta g_j},
\end{aligned}$$

which by induction proves formula 12 for any $t > 2$. Now we want to prove formula 13 for V_1. Expressing V_1 through V_t using formula 12 , we get

$$\begin{aligned} V_1 &= \left(V_t + u(1-\delta) \sum_{i=2}^{t} \prod_{j=i}^{t} \frac{1}{\delta g_j} \right) \prod_{i=2}^{t} \delta g_i \\ &= \left(V_t \prod_{i=2}^{t} \delta g_i + u(1-\delta) \left(1 + \sum_{i=3}^{t} \prod_{j=2}^{i-1} \delta g_j \right) \right). \end{aligned} \tag{39}$$

Values V_t are limited by some constant because of the transversality condition, therefore

$$\lim_{t\to\infty} V_t \prod_{i=2}^{t} \delta g_i = 0$$

as $\lim_{t\to\infty} g_t \equiv \lim_{t\to\infty} (1-p_t)^{M-1}(M+1-p_t) = 0$. (Probabilities p_t converge to 1.) Therefore, taking limits both parts of 39 for $t \to \infty$, one gets formula 13 for V_1.

To prove $V_1 < \infty$ notice that as values $g_t = (1-p_t)^{M-1}(M+(1-p_t))$ converge to 0, for any $\epsilon \in (0,1)$ there exists t_0 such that $g_t < \epsilon$ for any $t > t_0$. Therefore,

$$\left| \frac{V_1}{u(1-\delta)} - \left(1 + \sum_{i=3}^{t_0} \prod_{j=2}^{i-1} \delta g_j \right) \right| = \sum_{i=t_0+1}^{\infty} \prod_{j=2}^{i-1} \delta g_j < \prod_{j=2}^{t_0} \delta g_j \sum_{i=t_0}^{\infty} \epsilon^{i-t_0} < \infty.$$

Step 5. Properties of V_t and $\mathbf{E}_{F_t} v$.

Find expression for V_t in terms of p_t and g_t:

$$\begin{aligned} V_t &= V_1 \prod_{i=2}^{t} \frac{1}{\delta g_i} - u(1-\delta) \sum_{i=2}^{t} \prod_{j=i}^{t} \frac{1}{\delta g_j} \\ &= u(1-\delta) \left(1 + \sum_{i=3}^{\infty} \prod_{j=2}^{i-1} \delta g_j \right) \prod_{i=2}^{t} \frac{1}{\delta g_i} - u(1-\delta) \sum_{i=2}^{t} \prod_{j=i}^{t} \frac{1}{\delta g_i} \\ &= u(1-\delta) \left(1 + \sum_{i=t+2}^{\infty} \prod_{j=t+1}^{i-1} \delta g_j \right). \end{aligned}$$

Values g_t decrease with time to zero. Therefore,

$$\sum_{i=t+2}^{\infty} \prod_{j=t+1}^{i-1} \delta g_j < \sum_{i=t+2}^{\infty} \prod_{j=t+1}^{i-1} \delta g_{j-1} = \sum_{i=(t-1)+2}^{\infty} \prod_{j=(t-1)+1}^{i-1} \delta g_j,$$

and V_t decreases with t. Also, if $g_j < g$ for any $j > t$, then

$$\sum_{i=t+2}^{\infty} \prod_{j=t+1}^{i-1} \delta g_j \le \sum_{i=t+2}^{\infty} (\delta g)^{i-(t+1)} = \frac{\delta g}{1-\delta g},$$

and $V_t \to u(1-\delta)$ as $g_t \to 0$.

The average price at period t for $M > 2$

$$
\begin{aligned}
\mathbf{E}_{F_t}\, v &= \int\limits_{\tilde{V}_t}^{V_t} v\, d\left(\frac{1}{p_t} - C_t v^{-\frac{1}{M-1}}\right) = \int\limits_{\tilde{V}_t}^{V_t} \frac{C_t}{M-1} v^{-\frac{1}{M-1}}\, dv = \frac{C_t}{M-2} v^{\frac{M-2}{M-1}}\Bigg|_{\tilde{V}_t}^{V_t} \\
&= \frac{V_t^{\frac{1}{M-1}}}{M-2} \frac{1-p_t}{p_t} \left(1 - (1-p_t)^{M-2}\right) V_t^{\frac{M-2}{M-1}} \overset{t\to\infty}{\longrightarrow} 0,
\end{aligned}
$$

and the average price for $M = 2$

$$
\begin{aligned}
\mathbf{E}_{F_t}\, v &= \int\limits_{\tilde{V}_t}^{V_t} \frac{C_t}{M-1} v^{-\frac{1}{M-1}}\, dv = \frac{C_t}{M-1} \ln v \Bigg|_{\tilde{V}_t}^{V_t} = \\
&= \frac{V_t^{\frac{1}{M-1}}}{M-2} \frac{1-p_t}{p_t} \left(1 - (M-1)\ln(1-p_t)\right) \ln V_t \overset{t\to\infty}{\longrightarrow} 0.
\end{aligned}
$$

□