Continuous Modeling of Foreign Exchange Rate of USD versus TRY

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Abstract—This study aims to construct continuous-time autoregressive (CAR) model and continuous-time GARCH (COGARCH) model from discrete time data of foreign exchange rate of United States Dollar (USD) versus Turkish Lira (TRY). These processes are solutions to stochastic differential equation Lévy-driven processes. We have shown that CAR(1) and COGARCH(1,1) processes are proper models to represent foreign exchange rate of USD and TRY for different periods of time February 2002- June 2010.

Keywords: Continuous modeling; Continuous AR; COGARCH; USD/TRY

I. INTRODUCTION

The modelling and forecasting the foreign exchange rates are one of the most important subject for the international financial markets. After 2001 crisis, The Government of Turkey changed the foreign exchange policy, anyway the government set up the currency control and the foreign exchange rates are set by the free market The forecasting becomes difficult because of the fluctations in supply and demand for the foreign currency.Hence, a succesful model is important for the investors, Turkish financial system and derivative pricing.

Many forecasting methods have been developed in the last few decades Time series forecasting is highly utilised in predicting economic and business trends.. The Box-Jenkins method [5] is one of the most widely used time series forecasting methods in practice. It is also one of the most popular models in traditional time series forecasting and is often used as a benchmark model for comparison with other time series methods. The method uses a systematic procedure to select an appropriate model from a rich family of models, namely, Autoregressive Moving Average (ARMA) models.

In fact, empirical investigations of various financial data Show that an ARMA models, combined with a GARCH model and a heavy-tailed assumption for the errors, gives superior in-sample and out-of-sample fits.

There are many studies that use the ARMA and GARCH models for the foreign exchange rates data in discrete time. But the continuous ARMA and COGARCH models for foreign exchange rate of USD versus TRY are not usual methods.

II. METHODOLOGY

A. *Discrete Time Modelling*

The discrete time models help us to obtain continuous models by following the ways of Brockwell for continuous ARMA and Klüppelberg for COGARCH.

After making the data stationary, the best candidate ARIMA model for the conditional mean is chosen according to Akakike Information Criteria (AIC) [1]. Then, ARCH effect, serial correlation in error terms and squared error terms are investigate by the ARCH-LM test[3] and Ljung Box test. If there is a serial correlation between residuals and ARCH effect occurs, then we estimate variance equation with GARCH model. The model verifying is the most important case in discrete modeling. The residual analysis for the GARCH model is the usual method. The probability density functions of the financial data and simulated data could be used for model verifying.

B. *Continuous ARMA Models*

ARIMA model is used to obtain continuous ARMA model. The parameters of CARMA model is found by using the autocovariance function of ARIMA [2]. CARMA(p,q) process with the condition $0 \le p < q$ is

$$a(D)Y_t = b(D)DW_t, t \ge 0 \quad (1)$$

Where D denotes the differentiation with respect to t, and

$$a(z) = z^p + a_1 z^{p-1} + ... + a_p$$

$$b(z) = b_0 + b_1 z + ...b_q z^q \quad (2)$$

Brockwell shows the observation and the state equations as the following

$$Y_t = b^T X_t \qquad (3)$$

$$dX_t = AX_t dt + edW_t \quad (4)$$

The solution for state equation is

$$X_t = e^{At} X_0 + \int_0^t e^{A(t-u)} dW_u \quad (5)$$

$$\Sigma = E[X_o X_o^T] = \int_0^w e^{Ay} e e^T dy \quad (6)$$

The real parts of the eigenvalues of matrix A must be negative for stationarity conditions where

$$E[X_t^T] = 0, t \geq 0$$

$$E[X_{t+h} X_t^T] = e^{Ah}\Sigma \quad , \; h \geq 0$$

The autocovariance function of CARMA(p,q) process is

$$\gamma_y(h) = \Sigma_{\lambda i} \frac{b(\lambda)b(-\lambda)}{a^T(\lambda)a(-\lambda)} e^{\lambda(h)} \quad (7)$$

The autocovariance function of ARMA(p,q) process [6] is

$$\gamma_y(h) = \sigma^2 \sum_{j=1}^{p} \frac{\lambda_j^{|h|+1}\theta(\lambda)\theta(-\lambda)}{\phi(\lambda_j)\phi^{\cdot}(\lambda_j^{-1})} \quad (8)$$

The parameters of CARMA(p,q) process are found comparing the equations (7) and (8) where $a_0,...,a_p,b_0,...,b_q$ are CARMA model parameters and $\phi_0,...,\phi_p,\theta_0,...,\theta_q$ are ARMA model parameters.

C. Continuous GARCH Modelling

Nelson[9] introduce COGARCH model that includes two independent Brownian motions B(1) and B(2)

$$dG_t = \sigma_t dB_t^{(1)}, \; t \geq 0 \quad (9)$$

$$\sigma_t^2 = (\beta - \eta\sigma_t^2)dt + \varphi\sigma_t^2 dB_t^{(2)}, \; t \geq 0 \quad (10)$$

where $\beta > 0$, $\eta \geq 0$, and $\phi \geq 0$ are constants.

Klüppelberg [4] shows that COGARCH model is analogue of the discrete time GARCH model, based on a single background driving Lévy process. COGARCH model has the basic properties of discrete time GARCH process.

The COGARCH process $(G_t)_{t\geq 0}$ is defined in terms of its stochastic differential dG, such that

$$dG_t = \sigma_t dL_t \quad t\geq 0 \quad (11)$$

$$d\sigma_t^2 = (\beta - \eta\sigma_{t-}^2)dt + \phi\sigma_{t-}^2 d[L,L]_t, \; t > 0 \quad (12)$$

where $\beta > 0$, $\eta \geq 0$, and $\phi \geq 0$ are constants.

The solution for the stochastic equation is

$$\sigma_i^2 = \sigma_{i-1}^2 - \beta + \int_0^t \sigma_s^2 ds + \varphi \sum_{0<s\leq t} \sigma_s^2 (\Delta L_t)^2 + \sigma_0^2 \quad (13)$$

Euler approximation is used for the integral $\int_0^t \sigma_s^2 ds \approx \sigma_{t-1}^2$ and $\sum_{0<s\leq t} \sigma_s^2(\Delta L_t)^2 \approx (G_t - G_{t-1})^2$ since ΔL_s is usually not observable. So, we get

$$\sigma_i^2 = \beta + (1-\eta)\sigma_{i-1}^2 + \varphi(G_t - G_{t-1})^2 \quad (14)$$

III. DATA

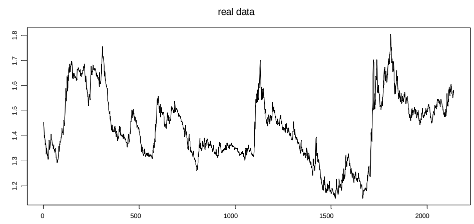

Figure1:TCMB USD versus TRY Exchange Rates
Source: Turkish Republic Central Bank's Web Page http://www.tcmb.gov.tr

In order to realize the model, for the period of years Februrary 2002- June 2010, the daily real foreign exchange rate data have been used related to Turkish Lira versus United States Dolar. The datas have ben got from the Central Bank of Turkey. The data is the selling prices of the end of the daily exchange rates**.**

IV. RESULTS AND DIAGNOSTICS

A. Unitroot And Stationary Tests

TABLE I. ADF AND P-P TEST RESULTS

t-Test	Test for Unit Root		
	Null Hypothesis	***Test Statistic***	***P-value***
ADF	There is a unit root	11.81	1.842e-24
P-P	There is a unit root	-42.84	9.262e-4

The real data is not stationary. After taking the first difference of the data, it becomes stationary according to ADF and Philips-Perron tests are shown in Table I. Here after the difference data is called DFX.

B. ARIMA Modeling

TABLE II. THE ESTIMATION OF ARIMA(1,1,0) VIA GAUSSIAN MAXIMUM LIKELIHOODS

	ARIMA(1,1,0) Results		
	Value	***Standard Error***	***t-value***
C	0.02857	0.01487	1.921
AR(1)	0.00005797	NA	NA

The best candidate model is ARIMA(1,1,0) pure AR(1) process for DFX data according to AIC value -12181.1576. The coefficient of AR(1) in Table II. is statistically significant. Although the constant of the model is not equal to zero, the coefficient will not used for simulations and in CAR(1) process. AR(1) model for DFX is

$$DFX_t = 0.000058 + 0.02857 DFX_{t-1} + a_t \quad (15)$$

where a_t is the error term.

C. Continuous AR Modeling

The state equation of CAR(1) is

$$dX_t = -aX_t dt + bdW_t \quad (16)$$

Which is the analogue of AR(1). In (16) we replaced Wiener process with Lévy process Benth et.al [10] and Brockwell [11], since the error terms are not normally distributed. Consequently (16) comes to that

$$dX_t = -aX_t dt + bdL_t$$

Where

$a = -\log\phi$, $\phi =$ 0.02857 AR(1) coefficient,

$$b = -\left(\frac{2\sigma^2}{1-\phi^2}\right)^{\frac{1}{2}} \log\phi \qquad \sigma = 1.000226$$

As a result. The continuous model for DFX is

$$dX_t = -1.5441 X_t dt - 1.0812 dL_t \quad (17)$$

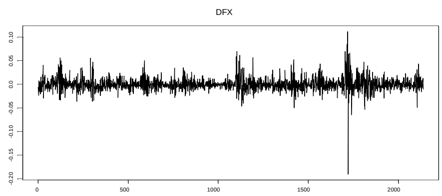

Figure2:First Difference of USD versus TRY Exchange Rates (DFX)

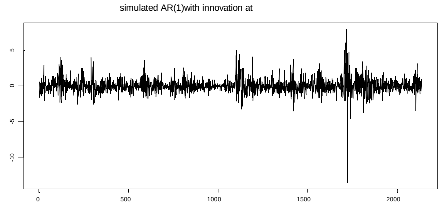

Figure3:Simulated AR(1)

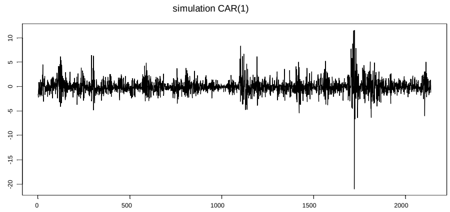

Figure4:Simulated CAR(1)

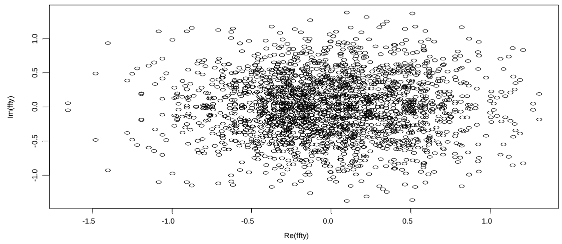

Figure5:Fast Fourier Transform of DFX

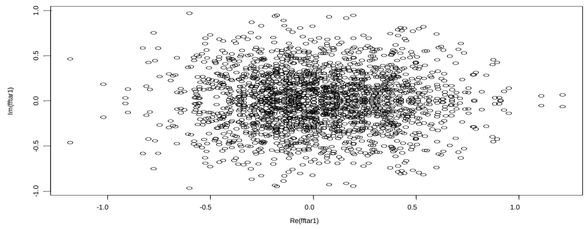

Figure6: Fast Fourier Transform of AR(1)

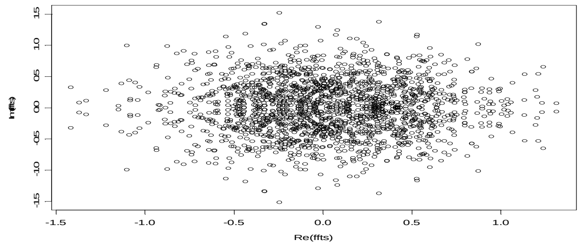

Figure7: Fast Fourier Transform of CAR(1)

The Figure3 and Figure4 show that the difference data in Figure2 and the simulated data from AR(1)and CAR(1) have same manner. Another approach to verify our model is Fast Fourier Transform (FFT) of DFX and simulated data. FFT is used for changing time domain data to frequency domain. FFT of DFX and simulated data from discrete and continuous models show common allocation.

A. Discrete Volatility Modeling

We fail to reject the null hypothesis of Ljung Box that is no autocorrelation between residuals and between squared residuals since the p-values of the test are 0.1215 and 0.8493 test. This situation is strengthened by ARCH-LM test with p-values 0.8463 and 0.9150. In other word ARCH effect occurs in residuals. ARCH-LM test The best candidate model is AR(1)~GARCH(1,1) according to AIC.

TABLE III. ESTIMATED COEFFICIENTS OF AR(1)~GARCH(1,1)

	Value	***Std.Error***	***t value***	***Pr(>t)***
C	0.000045	2.150e-004	2.087	3.701e-002
AR(1)	0.0548	2.207e-002	2.482	1.313e-002
A	0.0000056	7.415e-007	7.528	7.572e-014
ARCH(1)	0.1749	1.306e-002	13.393	0.000e+000
GARCH(1)	0.8039	1.006e-002	79.932	0.000e+000

All the coefficients of AR(1)~GARCH(1,1) are statistically significant and the coefficients satisfy the covariance stationary conditions where totoal of the coefficienst of ARCH term and GARCH term is less than 1.

$$DFX_t = 0.000045 + 0.0548 DFX_{t-1} + a_t$$
$$a_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = 0.0000056 + 0.1749 a_{t-1}^2 + 0.8039 \sigma_{t-1}^2 \quad (18)$$

The Jarque-Bera normality test points out that data DFX is not normally distiributed. The distribution of DFX is Log-Logistic distribution with three parameter. The probability density function (pdf) of Log-Logistic distribution

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{-2} \quad (19)$$

Where α - continuous shape parameter (α > 0)

β - continuous scale parameter (β > 0) γ - continuous location parameter ($\gamma = 0$ yields the two-parameter Log-Logistic distribution)

Domain $\gamma < x \leq \infty$

TABLE IV. PARAMETERS OF THE DISTIRIBUTION

Data	Distribution	Parameters
DFX	Log-Logistic (3P)	α=76,857 β=0,53338 γ=-0,53399
ARIMA(1,1,0)-GARCH(1,1)	Log-Logistic (3P)	α=81,482 β=0,03397 γ=-0,034

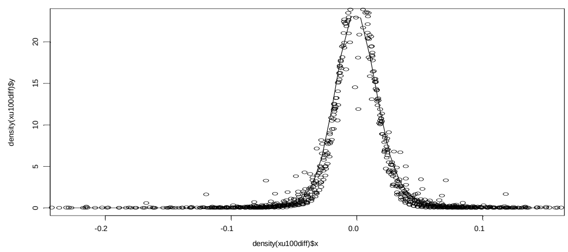

Figure8: PDF of DFX and PDF of Simulated Data

The pdf of the DFX and pdf of simulated data are closed. Also, Figure8 shows hundreds of simulations have same distribution shape with the real data. There is no autocorrelation between residuals of GARCH model with p-value 0.019.These evidences show that AR(1)~GARCH(1,1) is adequate model for discrete time volatility.

B. *Continuous Volatility Modeling*

The parameters of COGARCH model is obtained from the discrete GARCH model's parameters

$$\beta = \beta, \ \eta = \ln\delta, \ \phi = \lambda / \delta \quad (20)$$

Where β is the constant of GARCH model, δ is the coefficient of GARCH term and λ δ is the coefficient of ARCH term.The parameters' of COGARCH(1,1) model are

$$\eta = \ln 0.8039 \quad \phi = 0.1749 / 0.8039 \quad (21)$$

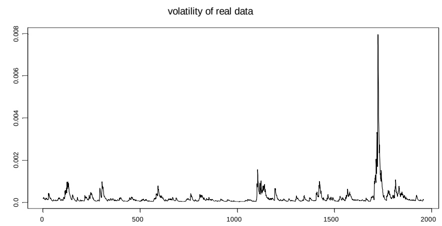

Figure9: Volatility of DFX

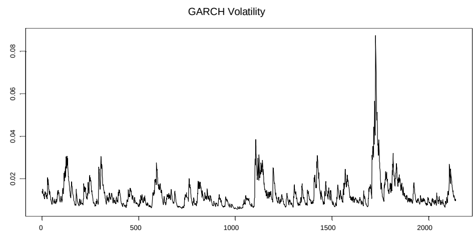

Figure10: GARCH Volatility

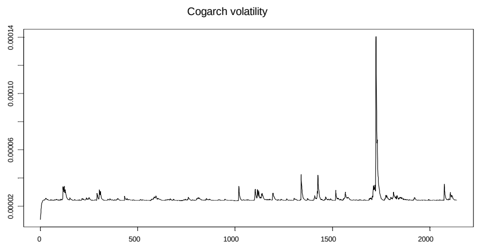

Figure10: COGARCH Volatility

The numerical solutions for dG_t and $d\sigma_t^2$ is done by using Lévy process driven by compound Poisson process. The above figures show that real data volatility, discrete time volatility and the continuous time volatility are closed to each other since jumps in the volatility plots almost have same pattern.

V. CONCLUSION

The exchange rate of USD versus TRY data was modeled two type continuous model. CAR and COGARCH models construct by the verified discrete models. Also CAR(1) and COGARCH(1,1) process was verified by comparing real data and simulated data from discrete models. The empirical results showed that continuous models for adequate models. This models should be used in studies about derivative pricing or Value at Risk calculations.

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