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Pihnastyi, Oleh and Bondarenko, Kristina

National Technical University "Kharkov Polytechnic Institute",
National Technical University "Kharkov Polytechnic Institute"

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ABOUT THE PROBLEM OF SELECTING MODELS OF PRODUCTION LINES

Pihnastyi O.M., Bondarenko K.M.

Resume: The article presents comparative characteristic of models of industrial production lines operating in transition mode, their advantages and disadvantages are analyzed. For these models the accuracy of calculations of aggregated parameters of industrial production lines is evaluated. The principle of the PDE-models is showed using the statistical theory of describing production systems. The prospects of developing models of industrial production lines that are used in the design of in-line production control systems.

Keywords: models of production processes, production line transition mode, PDE-model, a model of statistical dynamics of control systems.

Introduction

The effectiveness of the control system of production flow lines is determined by many factors, among which the choice of the model of controlled manufacturing processes and supervised algorithm take an important place. This selection is extensively determined by the structure of the life cycle of producible products, the dynamics of change of in time main parameters that characterize the state of the life cycle phase and the duration of its separate stages, which directly relate to the production process. Characteristic feature, which determines the choice of control systems of production lines for the majority of manufacturing enterprises in the last century, was the presence in the structure of the life cycle of a duration period (Figure 2), associated with the production process, which is characterized by a slow change in the time of production. This allowed us to use quasi-static model of production processes to describe the production process.
One of the main trends in the development of modern industrial production is a constant reduction in the duration of the life cycle of producible products. The duration of the life cycle is limited, and is determined by a time interval that is required to replace a technology with another production technology [4].

This leads to the fact that a) on the one hand the production line operates in a significant space of time at a transient rate; b) on the other hand time devoted to the search of the mode of production line’s technological areas management is reduced, estimated minutes or even seconds [2, c. 139]. Shortening of product life cycle leads to a strong deformation portion of the life cycle duration, this was characterized in Figure 2 with a slow variation during the production [5]. Production is forced to operate in a mode of transition from a state with one volume of production to the state with the changed volume of production. Plateau (2), corresponding to a constant volume of production disappears (Fig. 4). In connection with that the design of control systems of production lines for transition regimes in the past decade focuses on the use of brand new types of models of controlled production processes, as well as programs and management algorithms. Application of quasi-static model becomes widespread unacceptable [1, 2]. The high relevance of the problem is illustrated by the fact that the leading global companies (Intel Corporation, Volkswagen AG, Royal Philips) along with funding numerous research grants, created an experimental laboratory of transient study. Modern production requires reliable, not requiring a lot of processing power models that enable to describe the behavior of the parameters of the production lines.
for both quasi-static and transient for transients in order to solve appropriate management problems. The process of managing a modern manufacturing is complicated by the fact that production lines companies are diversified, consist of a large number of technological operations, in interoperable backlogs, in which there is a large number of subjects of labour, use a variety of different types of technological resources. Routes of different nomenclatures products overlap. Many manufacturing operations are performed on the same equipment that requires prioritization of processing and resource consumption. To control the production line multidimensional control systems are used, the complexity of which has reached its limit. Review [2] of recent publications showed that for describing the operation of production lines three basic types of models are involved.

The first type of model is a model of queuing theory (TQ-model). They are used to describe the simultaneous production lines of small number of operations, functioning as a rule in the steady quasi-static mode. The use of single-step description of the production lines for transient simulation of unsteady modes leads to excessive complexity of models and high costs of computing and time resources. The second type of models is discrete-event model (DES-model). It requires large expenditures of time and computational resources in connection with multiple simulated processes, consisting of a large number of technological operations in interoperable backlogs which contain a large number of items of work. Many years of experience with the DES-models by Intel show that the calculation results are in good agreement with the experimental results [6], but the estimated time for the characteristic values of the parameters of the production line is about a day. This makes it impossible to use them effectively. Not suitable for the construction of analytical relationships between the parameters of the projected lines. They require the use of stable computational algorithms.
The third type of models is models of fluid (Fluid-model). Their variety is widely known, provided the direction of system dynamics. It focused on a small number of intervals of partitioning process flow and linear stationary solutions. The main reasons that don’t allow the effective use of the data model are a high dimension and complexity of building a many closed nonlinear system of equations, which reduces the accuracy of the description. Each type of model has its advantages, but none of them fully fits for a full description of the production lines operating in transient transition mode [5].

**The production lines operating in transition mode**

Problematic is the question of building control systems for production lines operating in transition mode. As stated above, in the current conditions the life cycle of the product for the company with in-line type of organization of production is short enough. For a semiconductor product, the life cycle is less than one year, while the duration of the production cycle is 40-60 days, and the introduction of a new product and its launch can take several months. In this connection, the production line functions only a small part of the time in the steady state [6, c. 4589]. In the design of modern control systems of production lines, it is given special attention to building models for unsteady transients when systems’ parameters are in an unstable state. [7] DES-resources models, requiring considerable computing, are successfully applied for a detailed description of stable transition modes of operation of the production line [2, 8, 9]. If the production lines stability criteria are not met, then the simulation result becomes largely dependent on the selected computational scheme. Using clearing functions for transients requires a special approach [1,2]. Clearing function allows you to close the system of PDE-equations, but the application of the model is sufficiently accurate only for systems in quasi-stationary stable states. Attempts to create TQ-model for transients, describing the behavior of queuing networks are presented in Riano G. (2003) [10]. Selçuk B., Fransoo J., Gok A. (2007) [11] used to describe transients clearing stochastic function. Armbruster D., Ringhofer S., Bramson M., Kempf K took up creating models of transition processes. [6]. However, resentment,
arose within the considered planning horizon was usually making invalid calculated by the model for transient optimal plan of production (Armbruster D., Marthaler D, Ringhofer C) [12]. Parameters stability of production lines was examined by Lefeber E. (2004) [13], Missbauer H. (2009) [14]. It was stressed that the stability of the parameters defined by the criteria that determine the relationship between the density of objects of labor and the value of the rate of movement of path flow. Armbruster D., Kempf K.G. (by the production of company Intel) showed (2012) [15], in order to ensure the sustainability of production capacity deviation from the standard reaches 20%. Parameters stability of product lines represented TQ- and Fluid-models was in detail examined by M. Bramson (2008) [16] and Dai (2004) [17,18].

One of approaches to building control systems of production lines in unsteady modes is the use of dispatching control theory (The supervisory control theory, SCT, Ramadge P., Wonham W., 1987) [19]. Currently, this theory based on the discrete event specification production lines, is used by many authors. When the management of large industrial systems, the complexity of the SCT-control model excessively increases because of the high level of details of DES-model. Cassandras C. (2002) [20], Harrison J. (1995) [21], Kimemia J. (1983) [8] proposed Fluid-use model with a lower level of details to build systems of production management. However, considered in the works of Cassandra's S., Harrison J. and Kim Mya J. Fluid-model although had less computation time, but unlike DES-models have not given a precise describe the dynamics of movement of products on transactions [22], what doesn’t enable to determine with sufficient accuracy the duration of the production cycle. By their nature Fluid-models can take into account the effects of the transition using the correction empirical coefficients for equations rates [23]. At the same time, PDE-class models [3] based on partial differential equations, allow to obtain a sufficient degree of accuracy balance equations describing the rate of movement of objects of labor, for transition and steady state, which makes it possible to build effective model control systems with modern production lines. Considerable efforts have been directed towards the formulation of optimization problems PDE-simultaneous parameter models. La Marca
considered in the one-stage approach the task of optimizing the value stream of orders for the final time interval $T$ [24]:

$$J(\rho(t,x), \lambda(t)) = \frac{1}{2} \int_0^T (d(t) - F(t,1))^2 \, dt \rightarrow \min, \quad F(t, x) = \frac{\mu_0}{1 + \int_0^t \rho(t, z) \, dz}$$  \hspace{1cm} (1)

$$\frac{\partial \rho(t, x)}{\partial t} + \frac{\partial \rho(t, x) \cdot v(t, x)}{\partial x} = 0, \quad \rho(0, x) = \rho_0(x), \quad \rho(t, 0) \cdot v(t, 0) = \lambda(t)$$  \hspace{1cm} (2)

where $d(t)$ - the instantaneous velocity demand, $F(t, 1)$ - the departure rate of production from the plant. Minimization cost is determined by (1) the presence of constraints on the phase variables (2). The results of calculations using the model (1), (2) showed that the departure rate as a time function repeats the behavior of demand function, displaced in the phase by the amount certain amount. Control parameters of production line are reduced, generally to manage technological parameters of the modules along the technological route, consisting of inter-operation of equipment and storage [25]. The main parameters of process module are their productivity and the value of insurance stocks in interoperational storages.

Thus, management of $U(t, x)$ should be a function of how the time $t$, and the coordinates $x$, which determines the position of the objects of labor and equipment in the technological route. However, the distribution function along the control process route $U(t, x)$ for the production lines is not defined [26]. Special interest it is to develop a method of constructing models for problems of stabilization of stream line parameters, which control $U(t, x)$ is determined by using the Lyapunov functions [26].

**Evaluation of the accuracy of calculation models managed production processes**

The evaluation of the accuracy of the model is made by PDE-Berg RA, Lefeber E., Rooda JE with using DES-models. The analysis of the results of calculations of parameters production line obtained using DES models, $M / M / 1$ model and PDE-models [26]:

$$\frac{\partial \rho(t, x)}{\partial t} + \frac{\partial F(t, x)}{\partial x} = 0, \quad v(t) = \frac{\mu}{M + W(t)}, \quad (3)$$
\[
\frac{\partial p(t, x)}{\partial t} + \frac{\partial q(t, x)}{\partial x} = 0, \quad \frac{\partial v(t, x)}{\partial t} + \frac{1}{2} \frac{\partial v(t, x)^2}{\partial x} = 0, \quad v(t, 0) = \frac{\mu}{M + W(t)}, \quad (4)
\]

\[
\frac{\partial p(t, x)}{\partial t} + \frac{\partial F(t, x)}{\partial x} = 0, \quad v(t, x) = \frac{\mu}{M + p(t, x)}, \quad (5)
\]

\[
\frac{\partial p(t, x)}{\partial t} + \frac{\partial F(t, x)}{\partial x} = 0, \quad F(t, x) = C(t) \cdot \rho(t, x) - D(t) \frac{\partial \rho(t, x)}{\partial x}, \quad C(t) = \frac{1}{\tau(t, x)} \quad (6)
\]

where \( F(t, x) = p(t, x) \cdot v(t, x) \) from stream of object of labor, \( M \) - number of pieces of equipment along technological route to be used for batch processing of objects of labor, \( \tau(t, x) \) - effective processing time, \( D(t) \) - the diffusion coefficient is determined as a result of experimental studies of technological trajectories of objects of labor. As is shown the calculation results obtained by the PDE-model equations in cross-approximation, with sufficient degree of accuracy correspond to the results of calculations, obtained in the stationary case by DES and model \( M/M/1 \) model. Model (3) (Armbruster D.) is based on the LWR-model (Lighthill MJ, Whitham JB, Richards PI) [27] used to describe the transfer of traffic. The model includes the continuity equation, representing the law of conservation of objects of labor, and the static relation between the speed of the objects of labor \( v(t) = \mu/(M + W(t)) \) and their number in goods-in-process inventory \( W(t) \), where \( \mu \) is the intensity of processing the objects of labor in one of the \( M \) units of the same equipment. The velocity of the objects of labor \( v(t) \) does not depend on coordinates, is uniform in every place of the production line. Model (2) is used for rough calculations [12]. Model (4) (Armbruster D.) [12] is supplemented with the Burgers equation for speed with the initial \( v(0, x) = \vartheta_0(x) \) and the boundary conditions. The speed \( v(t, 0) \) is used only as a left boundary condition, and not for the whole line, as for the model (3). Traverse speed of objects of labor \( v(t, x) \) is variable along the process route and does not depend on density. The model (5) includes the continuity equation and the static ratio between the speed \( v(t, x) \) and density \( p(t, x) \) (LWR-model). Static ratio, based on \( M/M/1 \) model [28], shows resemblance to the equation in the model (3). However, the speed \( v(t, x) \) does not
depend on work in process $W(t)$ of the entire system, and relates only to the local density. Thus, the speed $v(t, x)$ may vary along the process route. The right boundary is supposed to be free. The combination of the first two equations of the model (5) gives equation

$$\frac{\partial \rho(t, x)}{\partial t} + \frac{\mu \cdot m}{(m + \rho(t, x))^2} \frac{\partial \rho(t, x)}{\partial x} = 0. \tag{7}$$

Numerical calculations showed that the model (4) and (5) has the same productivity [26, c.33]. The model (5) (E. Lefeber [26,29]) describes the behavior of the velocity $v(t, x)$ as a function of density $p(t, x)$. Calculations show that the transition period time is understated. Adding diffusion addition [26, c. 32] made it possible to increase the accuracy of calculations. These models (3) - (5) describe well the stationary modes of production line operation. For a description of the transitional regimes it is attempted to supplement models (3) - (5) with correction diffusion term [26, c.23], [6, c.4593], allows on occasion to increase the accuracy of the description. In the model (6) obtained by Armbruster D., Ringhofer S. [6, c. 4593], the equation of state $F(t,x)$ includes $D(t) \frac{\partial \rho(t, x)}{\partial x}$ a correction term that characterizes the dispersion of the experimental technological trajectories with diffusion coefficient $D(t) = 0,6 * 10^4 \pm 35\%$, (1 / day).

Studied in detail the accuracy of calculating the capacity of the production line and batch processing time for transition objects of labor and fixed cases Berg (2008) [26]. Technological process consisted of 10 identical workstations ($M = 10$), having buffer and equipment. The buffer means more capacity without restrictions. Objects of labor went into processing in accordance with a FIFO rule with intensity $\lambda$, distributed by exponentially. Speed $\mu$ of processing objects of labor is distributed by the same law with an average value of processing time $\frac{1}{\mu} = 0.5$ (hours). Two types of experiments confirming the accuracy of the calculation using the PDE-model: experiments with an increase in the flow of objects of labor going to processing and the experiment with the decreasing flow of objects of labor $\lambda = 0.5, \lambda = 1.0, \lambda = 1.5$ and $\lambda = 1.9$ (pcs. / h). It was assumed that the system is initially at steady state. Four experiments were carried out
with the transition of the intensity values of $\lambda = 0.5, \lambda = 1.0, \lambda = 1.5$ and $\lambda = 1.9$ (pcs. / h) to value of $\lambda = 0.5$. After reaching steady state of technological process parameters the experiment is complete. To check compliance with PDE-models of Lefeber E., Berg R., Roda J. used a simulation model of DES-managed production process. This approach has worked well as a method for modeling and analysis of production systems. DES-model is built using programming language developed in Eindhoven University of Technology [26]. To calculate average values of production line parameters for each variant $\lambda$ one million independent simulations of processing kit of parts were carried out. PDE-model parameters were calculated using the method of Godunov. The coincidence was to be expected, because to close the PDE-model the stationary TQ-model $M / M / 1$-queue was used [26]. Studying the behavior of the parameters of the production line shows result mismatch of DES and PDE-models due to the fact that the speed of movement of objects of labor was represented by the equation of state, it holds true only for the stationary case.

Review of recent publications shows that the existing PDE-models [6, 12, 26] describe well steady-state regimes, but do not allow the same accuracy to describe transients of process lines.

To improve the accuracy of the description of unsteady modes it is necessary, on the one hand, to develop PDE-models containing a large number of balance equations, on the other hand, to develop methods for closure of the system of balance equations, which must be based on an elaborate mechanism of action of the equipment for object of labor.

**Models of the statistical dynamics of control systems**

Statistical theory of control systems is based on probability theory, mathematical statistics and dynamics of systems. Since patterns in random events are studied with the theory of probability, an essential part of the instrument of the statistical theory of systems represents the instrument of the theory of probability. Basic papers in the probability theory and mathematical statistics belong to A.N. Kolmogorov [30] A.Y. Khinchin [31], N. Wiener, V.I Romanovsky., E.E. Slutsky, I.V. Smirnov, B.V.
Gnedenko. Development of the theory of kinetic equations provided a powerful tool for solving practical problems [32, 33]. The kinetic equation of the Fokker-Planck-Kolmogorov (FPK-equation) applied for the study of random processes in nonlinear control systems, initiated the modern statistical theory of optimal control systems. Physical problem definition of the statistical analysis of dynamical systems associated with the FPK-equation, was published A. by Fokker (Netherlands), M. Planck (Germany), A.N. Kolmogorov, A.A. Andronov, L.S. Pontryagin [34]. V.A. Kotelnikov initiated the development of the theory of statistically optimal nonlinear systems. L.S. Pontryagin developed the principle of maximum on the basis of which the results of optimization of executive systems, reviewed in statistical formulation of the problem of optimal control [34]. The statement of the statistical theory of optimal dynamical systems is presented in papers [35, 36]. Methods of maximum principle by Pontryagin and dynamic programming by Bellamana had a significant influence on the development of the statistical theory of optimal control systems. The theory of stationary random processes in relation to the study of the statistical control systems is researched by A.Y. Khinchin. The results of the modern statistical theory of dynamical systems, which describe the influence of random impacts on the constant and variable system parameters, are stated by V.S. Pugachev [37]. V.S. Pugachev [242] and A.Y. Khinchin built application theory of random functions. The statement and solving problems of statistical optimization and synthesis of control systems belong to V.V. Solodovsky [35]. They developed the statistical theory of adaptive control systems, principles and the concept of computer-aided design of complex control systems of technological processes. V.V. Solodovsky systematized methods of statistical calculation of line dynamical systems with constant parameters steady states [35]. A significant contribution to the development of statistical methods for the study of dynamical systems belongs to Rosin MF belong, R. Fischer, E. Pearson, G. Kramer, D. Neumann, A. Wald, N.Wiener. The development of production systems takes place in an environment where thousands of created heterogeneous processes can’t exist without optimal control. The need for accurate and reliable models of optimal control passes far ahead of the development of a detailed mathematical instrument for
describing processes. Today's enterprises need not only the optimal, but adaptive optimal control algorithms that provide the best in terms of a certain quality criteria result in a state of incomplete knowledge of process parameters and random effects of environment on its parameters. The adaptability of industrial enterprises to unforeseen conditions is a feature of the development of modern industry. Statistical methods and building on the basis of their stable optimal control algorithms under modern conditions are of great importance [3,13]. Such algorithms can reduce the requirements for the accuracy of the description of controlled processes; simplify the design process of control systems. The universal algorithm evaluation, identification and control of process parameters are increasingly using the method of Lyapunov functions [3,13].

The task of creating optimal sustainable control systems seems so important that almost all areas of modern management theory set themselves this task. Often, the optimal program of changing modes of production is considered to be known, is determined at the design stage. The optimal control problem is to implement this program and stabilization of programmed motion [3, 38]. Here are allowed only small deviations from the target state program of process parameters.

Automation of production requires extensive implementation in the manufacturing process of computer technology and the use of the latest management principles without the participation of the employee. In constructing the parameter control model of production processes it’s accepted to distinguish between open and closed control system [3, 35]. Open control system provides state change of objects of labor by a given law, which doesn’t depend on the results of the previous administration. Closed loop control systems of process parameters use information about the management results and form control action depending on the state of the object of labor. The principle of feedback is the basis of parameter control of manufacturing processes in almost all automatic and semiautomatic production lines. This is due to the fact that during operation random, not previously known disturbances or hindrances bring influence to the control objects that may cause random changes in the managed production process that do not correspond to the programmed. In the theory of industrial process control objects and control systems are generally
considered from a functional point of view. Besides function relations in a system are important which characterize the quality of its work. The criteria for the production organization theory have quantitative characteristics. This approach enables to develop a unified theory of production process system control that is based on the commonality of equations describing technological phenomena [1, 2]. Accounting factors of energy, economy, reliability, complexity characterizing the process control system, is possible by the use of criteria and their mathematical formalization. Status of the production system is generally characterized by a large number of variables that determine the condition of the individual subject of labor [3]. Essential are a small number of aggregated variables. In statistical statement the process of parameter control depends on the presence of random factors and is characterized by the statistical characteristics.

Studying the behavior of parameters of the controlled production process is generally reduced to a study of transforming random input process parameters with the statistical control system into random output parameters. Transport processes of technological resources on the subject of work are described with the statistical characteristics. If you know the full statistical characteristics of the input parameters of the process and the dynamic properties of the production system, the studied process control would be defined in a statistical sense, i.e., it can be found corresponding to the statistical characteristics of the output parameters. Often, however, statistical information about the input parameters of the production process and the dynamic properties of the production system is not sufficient to determine the characteristics of the output parameters. In such a case management process in a statistical uncertainty is studied on the basis of the guaranteed result consisting in the fact that an output result will be provided for any possible outcome. Statistical control model determines the dependence of the statistical characteristics of output process parameters from the statistical characteristics of the input parameters by which the reliability, accuracy and throughput of a production line are evaluated [3, 256].

To date, the most developed part of the statistical theory of dynamic systems is a section in which we study the properties of linear automatic control systems of production processes [3, 13, 25]. Many non-linear systems with sufficient accuracy can
be represented by linear. It should be noted that, although the general formulas of the statistical characteristics of the input and output parameters are written in a relatively simple manner, the use of these formulas for specific tasks is usually associated with considerable difficulties. These difficulties increase when determining the statistical characteristics of a dynamic system parameters for transients [35].

The basic unit of analysis and synthesis of statistical control systems are the kinetic equations [3, 13]. An extensive literature is devoted to the mathematical theory of continuous processes associated with these equations. Connection of the kinetic equation with the first integrals of dynamical control system of the production process parameters allows for some cases to obtain a particular solution. In general, the method of moments is used to solve the kinetic equation [3, 32, 33, 36]. The infinite system of differential equations in partial derivatives for the moments of the distribution function is closed by introducing the sequence of distribution functions containing a small parameter. The method is efficient, especially in the study of the initial phase of transient production process. The perturbation theory is widely used in the solution of the kinetic equation [3, 32, 33, 36]. Transients in dynamic production systems are called initial abnormalities or external disturbance. In many works, studying transients in dynamic production systems, the initial conditions for the parameters are considered to be deterministic.

Meanwhile, the real initial conditions and external influences are always more or less random. Statistical methods for the study of transients in control systems use the concepts of information theory, such as the entropy of the system [33]. With the use of kinetic equations [3, 26, 38] are formulated and solved the problems of constructing dynamic control systems in which statistical stability is important. Using the direct method by Lyapunov and the concept of entropy of a production system [35] allows you to obtain the necessary and sufficient conditions for the stability of the system. In this case a method for constructing equations for the moments is of great importance. Existing methods of statistical dynamics of control systems provide a powerful instrument that can be used to construct models of PDE-control systems and stabilization of parameters of the production line.
Conclusion

Under present-day conditions, the life cycle of the product for the company with in-line type of organization of production is short. In this connection, the production line operates most of the life cycle in the transient mode. If stability criteria are not met for the production line transient mode, using DES-model to describe the production line is difficult. The simulation result becomes largely dependent on the selected computing circuit. Arising within the planning horizon disturbances of stream parameters invalidate calculated by means of a model optimal production plan.

PDE-model is a new and promising direction of construction of models of production processes used in the design of control systems of production lines. However, although the presented results that use PDE-models in the planning and production management are promising, though more research is necessary, it allows finding out the true value of PDE-model for building control systems of production lines. PDE-model make possible more fully describe the behavior of the production line parameters in comparison with models TQ-considerably, they are less bulky and labor-intensive as compared with DES-models. Using for research of manufacturing processes instruments of the statistical theory of dynamical systems and methods of statistical physics allows us to study a particular type of laws that govern the behavior and properties of production systems, whose state is determined by the state of a large number of elements - objects of labor. Results of calculating the parameters of the flow line, obtained by PDE- and DES-models have shown relatively high matching accuracy calculations for steady state and unsatisfactory for transient. Thus, the above article overview and brief analysis of models of controlled production processes, operating under conditions of transient state, as well as an overview of the main types of models in the work[2] allows us to formulate the main directions of research of production systems with flow line method of organization of production, operating under transient conditions:

a) development of detailed subject-technological description of the controlled production process, based on the stochastic mechanism of the transfer of technological resources on the object of labor as a result of the impact of the equipment in the course
of technological operation, allows you to simulate production of several nomenclatures with different technological methods under conditions of sharing resources and production equipment;

b) development of analytical design methods in the phase space of trajectories of objects of labor to build effective models of technology subject-controlled production processes, describing the motion of objects of the labor batch in the technological production line, the foundation of which is the law of conservation, describing the process of transferring resources to the object of the labor;

c) development of stream description of controlled production process, based on the kinetic representation of the process;

d) development of design methods for control system of product line parameters based on a multimoment two-level PDE-model of description of controlled production process for transients.

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