How Persistent and Dependent are Pricing of Bitcoin to other Cryptocurrencies Before and After 2017/18 Crash?

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23 December 2018

Online at https://mpra.ub.uni-muenchen.de/91253/
MPRA Paper No. 91253, posted UNSPECIFIED
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Abstract

The present paper investigates persistence and dependence of Bitcoin on other popular alternative coins. We employ fractional integration approach in our analysis of persistence while a more recent fractional cointegration technique in VAR set-up, proposed by Johansen and co-authors is used to investigate dependency of the paired variables. Having segregated the series into periods before crash and those after the crash as determined by Bitcoin pricing, we obtain results of interests. Higher persistence of shocks is expected after the crash due to speculations in the mind of cryptocurrency traders, and more evidences of non-mean reversions, implying chances of further price fall in cryptocurrencies. Cointegration analysis between Bitcoin and alternative coin exists during both periods, with weak correlation observed mostly in the post-crash period. We hope the findings will serve as guide to investors in cryptocurrency.

Keywords: Cointegration; Cryptocurrency; Fractional integration; Fractional cointegration; Vector autoregression

JEL Classification: C22

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1. Introduction

Cryptocurrency, a digital currency that uses cryptography to ensure its security, is becoming popular as another alternative source of investment globally. The introduction of this new investment option is a result of failure of financial agencies such as central banks and the government to control economic activities, particularly after the 2008/09 crash in stock markets and 2010/2013 European sovereign debts (Balcilar et al., 2017). Instead, marketers and portfolio managers have lost quite much interests in stocks and other volatile assets. Despite the fact that cryptocurrency is yet to be accepted by government, still its popularity as a mean of investment and exchange continues to grow. Another noticeable trading strategy about cryptocurrency is that it is traded throughout the seven calendar days in the week, unlike stocks and other assets that are traded only in five business days and in non-public holidays.

The first cryptocurrency is the Bitcoin and it is the most valuable and highly capitalized (Corbet et al., 2018). This cryptocurrency was introduced in 2009. Since the inception, there was a calm increase in the price, even till 2012 when the price fluctuated around $12.0. In the early 2013, the price tripled, and at the end of 2013, the price of Bitcoin had skyrocketed to around $750. The pricing was steadily maintained at low increase for up to 2 years after and it skyrocketed again till it reached around $19,000 before it crashed. Currently, as at 25 November 2018, a Bitcoin was valued at $4141.0. Still, investors and financial expert see cryptocurrency as alternative to other assets that can be traded. As alternative to Bitcoin, there are other cryptocurrency such as Ethereum, Litecoin, Ripple, Dash, etc. Actually, there are 2074 cryptocurrency types as at 28 November 2018 (see Cointelegraph, 2018) of which Ethereum is next highly priced to Bitcoin.

As a result of the growth in cryptocurrency industry, and its global interest by traders and those interested in knowing more about it, it is of interest by financial time series expert to
study its time dependent property, particularly how Bitcoin pricing relates to pricing of other cryptocurrency known as altcoin.\(^1\)

In this paper, we investigate price dynamics of other cryptocurrency types to movement in price of Bitcoin by means of fractional cointegration technique. This technique allows one to first determine the order of integration of each time series x and y in the cointegrating test. The fractional orders, which are expected to be homogeneous, that is each time series is I(d) where d is the common fractional integration parameter. We estimate these orders via semi-parametric estimation approach using Geweke and Porter-Hudak (GPH) (see Robinson, 1995a) and Exact Local Whittle (ELW) estimation method (see Robinson, 1995b). Then, we consider Fractional Cointegrating Vector AutoRegressive (FCVAR) model, which allows for multiple of time series to come into the cointegrating matrix at the same time. In our case, it is bivariate; that is we consider Bitcoin price series with each of the other altcoin price series. This model allows to examine the long run equilibrium process, since it actually gives the stationary model linking both series together. The FCVAR strategy is recently proposed in Johansen and Nielsen (2012, 2014).

The rest of the paper is structured as follows: we conduct extensive literature review on modelling cryptocurrency in Section 2 of the paper. In Section 3, we present our dataset and conduct some preliminary analysis such as the descriptive statistics and cross correlation matrices. In section 4, we present FCVAR statistical framework, including fractional persistence estimation via semi-parametric approaches. We also present the results of the analysis obtained here. Finally in Section 5, we conclude the paper.

\(^1\) Altcoin means alternative to Bitcoin
2. Literature Review

The literature on virtual, digital or crypto-currencies is fast gaining popularity among researchers, both in the academia and in industries, such as, the banking and information technology (IT), among others. These virtual currencies, which are based on cryptographic proofs, built on the blockchain technology (Cheung et al., 2015) are fast becoming fiat currencies, as evidenced by their wide range of usage, which include means of payment for research grants by the UK government; stockpiling of cryptocurrency (specifically, Bitcoin) by IT companies in defence of ransomware; intra- and inter-bank transactions (e.g., the People’s Bank of China, Bank of Canada, Monetary Authority of Singapore, among others); investment or hedging strategies/options or diversification opportunities (Yi et al., 2018; Liu, 2018; Zhang et al., 2018; Borri, 2018; among others).

Following the origination and introduction of the Bitcoin in 2009 by the “pseudonymous” Satosh Nakamoto, more than 2,000 different cryptocurrencies (extract from CoinMarketCap website on Nov. 28, 2018) have been developed. This is however not surprising, given the observed popularisation of cryptocurrencies, catalysed by recent innovations. Thus, in reliance on, and in conformity to the recent innovations, the transfer of these virtual currencies, without any intervention from the prevailing governments and/or monetary authorities, is not only made feasible but also easy (Dwyer, 2015). These virtual currencies all have similar features in some respect, however; Bitcoin differs, in terms of design, from all others, with the former designed to function as a global digital currency (CoinDesk, 2016; Zhang et al., 2018). This informs the prominence of Bitcoin related studies in extant literature, over the other alternatives (see Ciaian et al., 2017; Cheah et al., 2018). Wallace (2011) notes that it is partly due to its usage on websites where purchases of some legal goods can be done anonymously. Consequently, studies involving two or more
cryptocurrencies adopt the Bitcoin as a reference or benchmark currency (see Ammous, 2018; Koutmos, 2018; among others).

Koutmos (2018) finds high degrees of contagion risk emanating from interdependencies among the eighteen (18) major cryptocurrencies therein considered, which is suggestive that the cryptocurrencies are becoming more integrated. The author’s results further reveals steady rise in return and volatility spillovers over time, with Bitcoin being a dominant contributor. However, these spillovers are time-varying in nature, exhibiting certain dimension of uncertainty about the future these digital currencies. In the same vein, Baur and Dimpfl (2018) show that volatility associated with cryptocurrencies (the 20 largest by market capitalization) is increased more by positive than negative shocks, a departure from the equity market stance. However, in another research, Yi et al. (2018) applied the spillover index approach to examine both static and dynamic volatility connectedness among eight typical cryptocurrencies. Their results revealed that their connectedness fluctuates cyclically and showed an obvious rise trend. In their analysis, they further construct a volatility connectedness network linking 52 cryptocurrencies using VARs, and they found out that the 52 cryptocurrencies are tightly interconnected with cryptocurrencies with high market capitalization (e.g., Bitcoin, Litecoin and Dogecoin) propagating large volatility shocks, while small-cap cryptocurrencies are more likely to receive volatility shocks from others.

Borri (2018), while estimating the conditional tail-risk for four prominent cryptocurrencies (bitcoin, ether, ripple and litecoin) using CoVaR, observed high exposure of these cryptocurrencies to tail-risk within the cryptomarket; however, in relation to other global assets (equity market or gold), these cryptocurrencies are not exposed to tail-risk. The returns on these cryptocurrencies are not only highly correlated but exhibit idiosyncratic risk, which can be significantly reduced. The portfolio of these currencies (see also, Brauneis and Mestel, 2018), rather than individual currencies, offer better risk-adjusted and conditional returns,
hence, their attractive returns and inherent hedging properties when included in investors’ portfolios. Bouri et al. (2018) show evidence of gradual erosion of the dominance of bitcoin by other alternative cryptocurrencies and multi-directional co-explosivity behaviour. Cryptocurrencies are further observed to exhibit seasonality (Kaiser, 2018); persistence (Caporale et al., 2018); Granger causality (Bouri et al., 2018)

Cheah et al., (2018) studied the dynamic interdependence under a fractionally co-integrated VAR framework, while modelling the cross-market bitcoin prices as long-memory processes in the individual markets and system of markets. The authors find bitcoin markets to be fractionally co-integrated, with uncertainty negatively affecting the co-integration relationship, as well as characterized by non-homogeneous informational inefficiency. Also, in an empirical study, Ciaian et al. (2017) examined and found interdependencies between bitcoin and sixteen alternative virtual currencies (altcoin) markets in the short- and long-run. A stronger bitcoin-altcoin price relationship is observed in the short-run, compared to the long-run, however, in the long-run, macro-financial indicators tend to better determine the altcoin price formation than bitcoin.

3. Data and Pretests
The data used in this paper was extracted from an open source cryptoasset analytics database, Coin Metrics (https://coinmetrics.io/data-downloads/), on the 29th November, 2018. The data comprise prices for 13 core samples of cryptocurrencies, spanning between 7 August 2015 and 28 November 2018, collected on a daily frequency and amounting to a total of 1,210 data points. The investigated cryptocurrencies include Bitcoin, Dash, Digibyte, Doge, Ethereum, Litecoin, Maidsafecoin, Monero, Nem, Ripple, Stellar, Verge and Vertcoin, selected based on data availability, high pricing and capitalization.
As noted in Figure 1, we observed a great crash in cryptocurrency at the end of 2017 (17 December), where Bitcoin reached an all-time high of $19,475.8 before crashing down from the next day following, and other cryptocurrencies responded to the crash within one month, that is till end of January 2018. As observed on the graph, Litecoin and Vertcoin responded to the crash in Bitcoin price faster than other cryptocurrency. The vertical line then divide the sample into sample before crash and sample after the crash.

**INSER FIGURE 1 ABOUT HERE**

Next, we look at the statistical distribution of the cryptocurrency prices, first, for the full sample period and then, for sub-samples - periods before and after the global cryptocurrency crash of 2017/18, separately, given in Table 1. The essence of the three different sample is to see if the observed statistical properties are not dependent on the sample periods chosen. In all three sample periods considered, we find Bitcoin to be the most valuable cryptocurrency with average price of $3596.6 for the full sample, $1495 before crash and $8194 after crash, while Doge is the least valuable cryptocurrency with prices $0.001, $0.002 and $0.002 for sample before crash, after crash and full sample, respectively, when compared to the other cryptocurrencies considered in this study. Cryptocurrency prices seem to be more volatile after the crash compared to the prices before the crash as observed in higher standard deviation values in the sample observed after crash. This is however, expected as the crash increases speculations in the cryptocurrency market. We also find the cryptocurrencies to be leptokurtic in all three considered sample periods except for Ethereum and Monero in the periods after crash, while all the cryptocurrencies are found to be positively skewed and consequently non-normal, as expected of price series.

**INSERT TABLE 1 ABOUT HERE**

As part of the pretests, we investigate linear dependency by means of cross correlations and correlogram. Cross-correlation analysis is used to analyse the lead-lag relationships
between two time series of interests. This is achieved by using the cross-correlation matrices (CCM). The cross-covariance matrix is given as,

$$\Gamma_i = Cov(z_t, z_{t-i}) = E[(z_t - \mu)(z_{t-i} - \mu)^\prime]$$  \hspace{1cm} (1)

where $z_t = (z_{1t}, z_{2t})$ is a 2-dimensional time series vector of consisting of two time series variables of Bitcoin and other cryptocurrency, $\mu$ is the mean of each time series. Following Tsay (2014), univariate linear correlations are symmetric while cross correlations are asymmetric for lead and lags. The autocovariances of lead ($+i$) and lag ($-i$) are identical, since negative-lag cross-covariance matrix is obtained by taking the transpose of the positive-lag cross-covariance matrix. The lag $i$ CCM is obtained as

$$\rho_i = D^{-1}\Gamma_i D^{-1} = [\rho_{ij}(i)]$$  \hspace{1cm} (2)

where $D = diag(\sigma_1, ..., \sigma_k)$ is the diagonal matrix of the standard deviations of the time series.

For $i = 0$, equation (2) obtains the contemporaneous correlations, while for $i > 0$, $\rho_i$ gives the linear dependence of $z_t$ and $z_{t-i}$, prior to period $t$. Therefore, cryptocurrency is said to lead Bitcoin at lag $i$ provided $\rho_i$ is significantly different from zero. If the two series are interchanged, then Cryptocurrency is said to lag Bitcoin. Details about CCM analysis and its cut off bounds are given in Tsay (2014). Table 4 presents the results of the linear dependency for samples before crash, after crash and for the full sample. We present results for no lag ($i=0$), $i=1,20$ and 30, where these are in days. It is observed that correlations decayed slowly and the cut-off bounds are very low implying long lags for significant correlations. Meanwhile, all correlations presented in Table 4 are significant. We have considered up to lag 30 in order to investigate the effect of Bitcoin price shocks on cryptocurrency after 30 trading days. There is no clear judgment on the extent of dependencies before crash compared to after crash samples, since in some cases, correlations before crash are higher, while in other cases correlation after crash are higher. But in each case, correlations decrease slowly. Dash, Litecoin, Monero and
Vertcoin present high correlations with Bitcoin at lag 0 compared to other cryptocurrencies, while Digibyte and Doge present low correlations in the pre-crash sample.

**INSERT TABLE 2 ABOUT HERE**

4. **Fractional Cointegrating VAR model**

Granger’s cointegration definition is not restricted to unit integration as applied in the classical Augmented Dickey-Fuller (ADF) test (Engle and Granger, 1987), and due to the introduction of fractional integration, the scope of cointegration has been widened (see Cheung and Lai, 1993; Gil-Alana, 2003).

The standard form of fractional persistence is given as

\[(1 - L)^d y_t = x_t\]  \( \text{ (3)}\)

where \( y_t \) is the time series under investigation, that is cryptocurrency price in this case and \( x_t \) is the fractionally differenced series expected to be covariance stationary. The exponent number \( d \) is the fractional persistence parameter that can assume any real number (including thus integer and fractional values), \( L \) is the lag-operator that is \( L y_t = y_{t-1}. \) By expanding the polynomial \( (1 - L)^d \) in (3) binomially, we have the expansion,

\[\sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j y_t = y_t - dL y_t + \frac{d(d-1)}{2} L^2 y_t - ... = x_t\]

and thus

\[x_t = (1 - L)^d y_t = y_t - d y_{t-1} + \frac{d(d-1)}{2} y_{t-2} - ... \]  \( \text{ (4)}\)

As observed in (4), it is obvious to see that parameter \( d \) plays a very important role in determining the degree of association between time series \( y_t \) and its lagged values. The values

\(^2\)Fractional persistence is synonymous to fractional integration originally introduced in Granger and Joyeux (1980) and Hosking (1981).
of fractional $d$ has appealing economic meaning which unit root $d$ lacks. If $0 < d < 0.5$ with upper bound not approaching 0.5, the process is stationary long memory with persistent autocorrelations. If $d$ lies in the confidence interval $0.5 < d < 1$ with upper bound not approaching 1, the time series process is no longer covariance stationary but mean reverting. Though, persistence increased compared to long memory case and also, shocks to the series tends to disappear in the long run. If $d \geq 1$, the time series is non-stationary non-mean reverting, implying that the effect of the shocks on the series persists indefinitely. Thus, the higher the $d$ value, the more the persistence of shocks on the time series.

A survey of different fractional cointegration estimation methods is given in Gil-Alana and Hualde (2009). Meanwhile, each method requires the determination of fractional integration order $d_1$ and $d_2$ of time series $y_{1t}$ and $y_{2t}$, respectively. Specifically in this paper, we applied semi-parametric approaches of Geweke and Porter-Hudak (GPH) and Exact Maximum Local Whittle (ELW) estimation methods to estimate integration order of the series (see Robinson (1995a,b). The methods are developed in the frequency domain which uses Fourier functions and ordinates of periodogram that can be varied. Thus, the methods allow one to obtain robust results to those ordinate numbers.

Once integration order is determined, we next determine cointegration by employing the Fractionally Cointegrated Vector Autoregression (FCVAR) model, proposed in Johansen (2008), and applied in Johansen and Nielsen (2010; 2012; 2016). The FCVAR methodology is a built up on the unit CVAR model of Johansen (2005) that assumed unit root for the time series to be cointegrated which is too restrictive and opposes Granger’s assertion. The FCVAR allows the flexibility of any integration orders, in stationary or nonstationary fractional integration range. The methodology allows for more than two variables to be cointegrated, thus it makes it appealing approach compared to other fractional cointegration method in the literature.
Given a \((k + 1)\)-dimensional vector of time series \(y_t, t = 1, 2, ..., T\), each of fractional integration order \(d\). By setting \(\Pi = \alpha \beta'\), and using the lag operator differencing \(L y_t = y_{t-1}\), one obtains,

\[
\Delta y_t = \alpha \beta' Ly_t + \sum_{i=1}^{k} \Gamma_i \Delta L^i y_t + \varepsilon_t, \tag{5}
\]

where \(\alpha\) and \(\beta\) are matrices of constant and regressors in the long run equation. By replacing the difference and lag operator \(\Delta\) and \(L = 1 - \Delta\) in (5) as in unit root with their fractional unit root counterparts, \(\Delta^b\) and \(L_b = 1 - \Delta^b\), respectively, we have,

\[
\Delta^b y_t = \alpha \beta' L_b y_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b^i y_t + \varepsilon_t \tag{6}
\]

with \(y_t = \Delta^{d-b} x_t\), equation (4) becomes:

\[
\Delta^d x_t = \alpha \beta' \Delta^{d-b} L_b x_t + \sum_{i=1}^{k} \Gamma_i \Delta^d L_b^i x_t + \varepsilon_t, \tag{7}
\]

where \(\Delta^d\) is the fractional operator, \(b\) is the cointegrating factor and \(L_b\) is the fractional lag operator. The d-b (with \(b > 0\)) is the degree of fractional cointegration, that is the fractional integration order of \(\beta' x_t\), which is expected to be lower compared to that of \(x_t\) itself for fractional cointegration in VAR framework to exist. The elements of \(\beta' x_t\) gives the cointegrating relationships in the system, where \(k\) determines the number of long-run equilibrium relationships, i.e. the cointegration or co-fractional rank. \(\Gamma = \Gamma_1, ..., \Gamma_k\) governs the short-run dynamics. The coefficients in matrix \(\alpha\) represent the speed of adjustment towards equilibrium for each of the variables in response to shocks. For the estimation of FCVAR model, reference is made to the Matlab programming code of Nielsen and Popiel (2018) that
estimates the model using Maximum Likelihood Estimation (MLE) approach. Meanwhile, the authors noted the case of $0 < b < 0.5$ as the case of weak cointegration, while strong cointegration is obtained when $0.5 < b < d$. The asymptotic distributions in each case are found in Johansen and Nielsen (2012).

The results of fractional integration are presented in Tables 3 and 4 for GPH and ELW estimation methods, respectively. Cryptocurrencies are prices and as expected, prices are assumed to be non-stationary. For both GPH and ELW, after varying the results for different bandwidth, $m$, we obtained consistent results for $T^m$ ($m = 0.6, 0.7$). Results based on GPH approach are presented in Table 3. For corresponding bandwidth, the persistence estimated for sample before price crash are lower than those obtained for samples after price crash. We observe evidences of mean reversion during pre-crisis period in almost all the cryptocurrency except in MaidSafeCoin and VertCoin. During this period, low persistence are observed in the case of Litecoin, Ripple and Verge. Based on post-crash samples, evidence of mean reversion is found in few cases. Mean reversions are only found in Bitcoin and Litecoin. Due to the fact that cryptocurrencies are in the downward price trend currently, higher persistence after the crash implies the possibility of further drop in prices. Results found here support the conclusion of other authors on bull and bear financial markets who found that during post-crash market (bear), volatility is always higher than during pre-crisis (bull) market (Maheu and McCurdy, 2000; Gomez et al., 2004; Gonzalez et al, 2005; Yaya et al., 2015; Gil-Alana et al., 2018). This is due to market speculations by the traders and portfolio managers. By looking at the results rendered by ELW Gaussian semi-parametric approach presented in Table 3, we also found persistence after the price crash to be higher than persistence before the price crash across all cryptocurrency.

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\(^3\) We are grateful to the following program contributors: Morten Orregaard Nielsen and Michal Ksawery Popiel both of Queen’s University for making freely available the FCVAR Matlab code; Jurgen A. Doornik and Marius Ooms for the OxMetrics-ARFIMA code.
Since I(d=1) is confirmed in the results above, next we investigate long run relationships between Bitcoin and any other cryptocurrency during pre-crisis, post crisis and for the full sample. The results are given in Table 5. In each case, 3 was selected as the appropriate lag length \( k \) on the basis of standard information criteria, and since our case is bivariate, cointegrating rank test is not considered since rank is expected to be 1. We have results for the joint fractional persistence parameter for the paired variables (Bitcoin and cryptocurrency), estimates of cointegrating factor, resulting fractional persistence for the long run equation \((d-b)\) and parameters of the long run equation in squared brackets. Starting with results for pre-crash period, for the joint fractional parameter \( d \), we obtain consistent results with those obtained in Tables 3 and 4. We observe both strong \((b > 0.5)\) and weak cointegration \((b < 0.5)\), and in each \( b < d \) indicating fractional cointegration. In the sample before crash, evidence of weak cointegration is only found for cointegration with each of Dash and Vertcoin, while in sample after crash, weak cointegration is found for more cases, that is in cointegration of Bitcoin with Digibyte, Ethereum, Litecoin, Maidsafecoin, Nem, Ripple, Stellar, Verge and Vertcoin. By looking at the full sample, weak cointegration is observed in the case of Ethereum, Nem, Stellar, Verge and Vertcoin.

## 5. Conclusion

We have considered persistence and dependence of Bitcoin with other cryptocurrency in this paper. The research focus is necessitated since market players find Bitcoin to drive other cryptocurrency, though Bitcoin is the most valuable and highly capitalized coin, taking about 40% of 2074 cryptocurrency market share (Cointelegraph, 2018). We included 13 highly priced and data available cryptocurrency in our findings, equally sampled from
7 August 2015 and 28 November 2018. Having identified late 2017 to early 2018 price crash in cryptocurrency, we divided the series into two subsamples. We found lesser price shock persistence in the pre-crash sample than in the post-crash sample with post-crash sample indicating more evidence of mean reversion due to further speculation of price fall. This is in support of assertions of researchers on bear and bull market that during post-crash market (bear), volatility is always higher than during pre-crisis (bull) market. (Maheu and McCurdy, 2000; Gomez et al., 2004; Gonzalez et al, 2005; Yaya et al., 2015; Gil-Alana et al., 2018). Evidence of cointegration between Bitcoin and each of other cryptocurrency is found in all cases, where cointegration is either weak or strong. Weak cointegration is observed in most cases in the post-crash sample. This further support evidence of price divergence from bitcoin and market disintegration. We cointegration observed also signal future market lows to market players.

The findings have policy implications to market players since the new regime of cryptocurrency dynamics calls for speculations, higher volatility and possible further downward price trend. An in-depth study on market efficiency of cryptocurrency is recommended.
References


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http://www.wired.com/magazine/2011/11/mf_bitcoin/all/1


Figure 1: Plots of Cryptocurrency pricing (US dollar/coin)
<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>Before Crash</th>
<th>After Crash</th>
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<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Kurtosis</td>
</tr>
<tr>
<td>N</td>
<td>N = 1210</td>
<td>N = 864</td>
<td>N = 346</td>
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<td>Bitcoin</td>
<td>3,596.646</td>
<td>3,961.201</td>
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<td>Dash</td>
<td>177.316</td>
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<td>0.013</td>
<td>0.018</td>
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<td>Doge</td>
<td>0.002</td>
<td>0.002</td>
<td>1.99</td>
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<td>Ethereum</td>
<td>213.463</td>
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<td>Litecoin</td>
<td>47.745</td>
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<td>0.234</td>
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<td>Monero</td>
<td>73.391</td>
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<td>Nem</td>
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<td>Vertcoin</td>
<td>0.996</td>
<td>1.740</td>
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Table 2: Cross Correlation estimates (Bitcoin, Cryptocurrency(-i) & (Bitcoin, Cryptocurrency(+i))

<table>
<thead>
<tr>
<th>Cryptocurrency</th>
<th>Full Sample</th>
<th>Before Crash</th>
<th>After Crash</th>
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</table>

Note: Bartlett standard error bounds for correlations are given by ±2/sqrt(N), where N is 864, 346 and 1210 for before crash sample, after crash sample and full sample, respectively. Thus, standard error bounds are ±0.068, ±0.108 and ±0.057, respectively. All correlations are significant in that case.
Table 3: Fractional integration estimation based on GPH log-periodogram regression

<table>
<thead>
<tr>
<th>Cryptocurrency</th>
<th>Full Sample</th>
<th>Before Crash</th>
<th>After Crash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_m = T^{0.6}$</td>
<td>$d_m = T^{0.7}$</td>
<td></td>
</tr>
<tr>
<td>Bitcoin</td>
<td>1.141***</td>
<td>1.128***</td>
<td>0.767***</td>
</tr>
<tr>
<td>Dash</td>
<td>1.318***</td>
<td>1.137***</td>
<td>0.874***</td>
</tr>
<tr>
<td>Digibyte</td>
<td>0.874***</td>
<td>0.862***</td>
<td>0.792***</td>
</tr>
<tr>
<td>Doge</td>
<td>0.949***</td>
<td>0.885***</td>
<td>0.896***</td>
</tr>
<tr>
<td>Ethereum</td>
<td>1.238***</td>
<td>0.980***</td>
<td>0.795***</td>
</tr>
<tr>
<td>Litecoin</td>
<td>0.996***</td>
<td>1.068***</td>
<td>0.492***</td>
</tr>
<tr>
<td>Maidsafecoin</td>
<td>1.019***</td>
<td>1.041***</td>
<td>1.043***</td>
</tr>
<tr>
<td>Monero</td>
<td>1.147***</td>
<td>1.063***</td>
<td>0.770***</td>
</tr>
<tr>
<td>Nem</td>
<td>1.149***</td>
<td>1.031***</td>
<td>0.644***</td>
</tr>
<tr>
<td>Ripple</td>
<td>0.835***</td>
<td>0.951***</td>
<td>0.654***</td>
</tr>
<tr>
<td>Stellar</td>
<td>1.141***</td>
<td>0.970***</td>
<td>0.688***</td>
</tr>
<tr>
<td>Verge</td>
<td>0.869***</td>
<td>0.900***</td>
<td>0.592***</td>
</tr>
<tr>
<td>Vertcoin</td>
<td>1.122***</td>
<td>1.230***</td>
<td>0.924***</td>
</tr>
</tbody>
</table>

Note: $m$ is the number of bandwidth, while $T$ is the sample size. *** indicates significance of persistence parameter at 5% level. In bold, evidence of mean reversion, while unbold, evidence of non-mean reversion.
Table 4: Fractional integration estimation based on ELW log-periodogram regression

<table>
<thead>
<tr>
<th>Cryptocurrency</th>
<th>Full Sample (d_m = T^{0.6})</th>
<th>Full Sample (d_m = T^{0.7})</th>
<th>Before Crash (d_m = T^{0.6})</th>
<th>Before Crash (d_m = T^{0.7})</th>
<th>After Crash (d_m = T^{0.6})</th>
<th>After Crash (d_m = T^{0.7})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bitcoin</td>
<td>1.078***</td>
<td>1.102***</td>
<td>0.762***</td>
<td>0.896***</td>
<td>0.800***</td>
<td>0.773***</td>
</tr>
<tr>
<td>Dash</td>
<td>1.216***</td>
<td>1.026***</td>
<td>0.884***</td>
<td>0.849***</td>
<td>0.924***</td>
<td>1.003***</td>
</tr>
<tr>
<td>Digibyte</td>
<td>0.923***  <strong>0.885</strong>*</td>
<td>0.777***  0.917***</td>
<td>1.071***</td>
<td>0.972***</td>
<td>1.075***</td>
<td>0.906***</td>
</tr>
<tr>
<td>Doge</td>
<td>0.990***  0.946***</td>
<td>0.857***  0.814***</td>
<td>0.999***</td>
<td>0.981***</td>
<td>0.999***</td>
<td>0.981***</td>
</tr>
<tr>
<td>Ethereum</td>
<td>1.227***  0.969***</td>
<td>0.815***  0.803***</td>
<td>0.999***</td>
<td>0.981***</td>
<td>0.857***  0.836***</td>
<td>0.857***</td>
</tr>
<tr>
<td>Litecoin</td>
<td><strong>0.900</strong>*  0.986***</td>
<td>0.512***  0.617***</td>
<td>1.133***  0.961***</td>
<td>1.133***  0.961***</td>
<td>1.133***  0.961***</td>
<td></td>
</tr>
<tr>
<td>Maidasafecoin</td>
<td>1.025***  1.072***</td>
<td>1.009***  0.889***</td>
<td>0.926***  0.905***</td>
<td>0.926***  0.905***</td>
<td>0.926***  0.905***</td>
<td></td>
</tr>
<tr>
<td>Monero</td>
<td>1.103***  1.029***</td>
<td>0.779***  0.836***</td>
<td>1.085***  1.058***</td>
<td>1.085***  1.058***</td>
<td>1.085***  1.058***</td>
<td></td>
</tr>
<tr>
<td>Nem</td>
<td>1.259***  1.057***</td>
<td>0.706***  0.853***</td>
<td>1.182***</td>
<td>1.104***</td>
<td>1.182***  1.104***</td>
<td></td>
</tr>
<tr>
<td>Ripple</td>
<td><strong>0.800</strong>*  0.947***</td>
<td>0.684***  0.616***</td>
<td>0.849***  1.020***</td>
<td>0.849***  1.020***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stellar</td>
<td>1.063***  0.920***</td>
<td>0.743***  0.732***</td>
<td>1.083***  0.985***</td>
<td>1.083***  0.985***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verge</td>
<td><strong>0.861</strong>*  0.917***</td>
<td>0.620***  0.543***</td>
<td>0.877***</td>
<td>0.886***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertcoin</td>
<td>1.077***  1.090***</td>
<td>0.918***  0.942***</td>
<td>0.877***</td>
<td>0.886***</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: \(m\) is the number of bandwidth, while \(T\) is the sample size. *** indicates significance of persistence parameter at 5% level. In bold, evidence of mean reversion, while unbold, evidence of non-mean reversion.
Table 5: Fractional Cointegration VAR of Bitcoin on each other Cryptocurrency

<table>
<thead>
<tr>
<th>Cryptocurrency</th>
<th>Full Sample</th>
<th></th>
<th></th>
<th></th>
<th>Before Crash</th>
<th></th>
<th></th>
<th></th>
<th>After Crash</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d$</td>
<td>$b$</td>
<td>$d - b$</td>
<td>long-run equation</td>
<td>$d$</td>
<td>$b$</td>
<td>$d - b$</td>
<td>long-run equation</td>
<td>$d$</td>
<td>$b$</td>
<td>$d - b$</td>
</tr>
<tr>
<td>Dash</td>
<td>1.200 (0.050)</td>
<td>0.848 (0.062)</td>
<td>0.352</td>
<td>$[1 , -1.4, 0 , -964.6]$</td>
<td>1.200 (0.093)</td>
<td>0.173 (0.023)</td>
<td>1.027</td>
<td>$[1 , -8.3 , -1.6]$</td>
<td>1.200 (0.052)</td>
<td>0.517 (0.036)</td>
<td>0.683</td>
</tr>
<tr>
<td>Digibyte</td>
<td>0.982 (0.039)</td>
<td>0.900 (0.033)</td>
<td>0.082</td>
<td>$[1 , -2.60483.4 , -32.3]$</td>
<td>0.800 (0.033)</td>
<td>0.567 (0.007)</td>
<td>0.233</td>
<td>$[1 , -11.5182.5 , -456.5]$</td>
<td>0.800 (0.126)</td>
<td>0.291 (0.019)</td>
<td>0.509</td>
</tr>
<tr>
<td>Doge</td>
<td>1.017 (0.033)</td>
<td>0.900 (0.037)</td>
<td>0.117</td>
<td>$[1 , -2.0359742 , 268.5]$</td>
<td>1.053 (0.029)</td>
<td>0.794 (0.024)</td>
<td>0.259</td>
<td>$[1 , -4.98897.0 , -591.5]$</td>
<td>1.082 (0.085)</td>
<td>0.729 (0.085)</td>
<td>0.353</td>
</tr>
<tr>
<td>Ethereum</td>
<td>0.800 (0.199)</td>
<td>0.346 (0.142)</td>
<td>0.454</td>
<td>$[1 , -1.47 , -300.8]$</td>
<td>1.082 (0.128)</td>
<td>0.900 (0.167)</td>
<td>0.182</td>
<td>$[1 , -9.1 , -399.6]$</td>
<td>0.800 (0.236)</td>
<td>0.480 (0.096)</td>
<td>0.320</td>
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<tr>
<td>Litecoin</td>
<td>1.200 (0.053)</td>
<td>0.668 (0.069)</td>
<td>0.532</td>
<td>$[1 , -52.0 , -459.2]$</td>
<td>1.200 (0.000)</td>
<td>0.900 (0.000)</td>
<td>0.300</td>
<td>$[1 , -67.1 , -315.6]$</td>
<td>1.200 (0.115)</td>
<td>0.125 (0.034)</td>
<td>1.075</td>
</tr>
<tr>
<td>MaidSafeCoin</td>
<td>1.126 (0.032)</td>
<td>0.852 (0.035)</td>
<td>0.274</td>
<td>$[1 , -2.04017 , 893.2]$</td>
<td>0.950 (0.043)</td>
<td>0.900 (0.033)</td>
<td>0.050</td>
<td>$[1 , -5629.2 , -106.1]$</td>
<td>0.800 (0.107)</td>
<td>0.100 (0.006)</td>
<td>0.700</td>
</tr>
<tr>
<td>Monero</td>
<td>1.028 (0.070)</td>
<td>0.900 (0.116)</td>
<td>0.128</td>
<td>$[1 , -39.3 , -699.0]$</td>
<td>1.102 (0.065)</td>
<td>0.900 (0.121)</td>
<td>0.202</td>
<td>$[1 , -27.6 , -508.7]$</td>
<td>0.946 (0.151)</td>
<td>0.900 (0.155)</td>
<td>0.046</td>
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<tr>
<td>Nem</td>
<td>1.200 (0.035)</td>
<td>0.100 (0.003)</td>
<td>1.100</td>
<td>$[1 , -12158.5 , -1.9]$</td>
<td>0.800 (0.033)</td>
<td>0.703 (0.019)</td>
<td>0.097</td>
<td>$[1 , -10974.8 , -458.0]$</td>
<td>0.800 (0.113)</td>
<td>0.205 (0.012)</td>
<td>0.595</td>
</tr>
<tr>
<td>Ripple</td>
<td>0.800 (0.036)</td>
<td>0.539 (0.013)</td>
<td>0.261</td>
<td>$[1 , -9916.0 , -594.8]$</td>
<td>1.200 (0.038)</td>
<td>0.418 (0.014)</td>
<td>0.782</td>
<td>$[1 , -7608.8 , -37.0]$</td>
<td>0.800 (0.101)</td>
<td>0.100 (0.007)</td>
<td>0.700</td>
</tr>
<tr>
<td>Stellar</td>
<td>0.800 (0.039)</td>
<td>0.399 (0.009)</td>
<td>0.401</td>
<td>$[1 , -23214.2 , -444.1]$</td>
<td>1.129 (0.034)</td>
<td>0.900 (0.018)</td>
<td>0.229</td>
<td>$[1 , 54420.0 , -1189.6]$</td>
<td>0.800 (0.119)</td>
<td>0.100 (0.007)</td>
<td>0.700</td>
</tr>
<tr>
<td>Verge</td>
<td>0.910 (0.036)</td>
<td>0.487 (0.014)</td>
<td>0.423</td>
<td>$[1 , -1371623 , -961.2]$</td>
<td>1.007 (0.024)</td>
<td>0.900 (0.008)</td>
<td>0.107</td>
<td>$[1 , -566015.2 , -605.4]$</td>
<td>0.839 (0.056)</td>
<td>0.279 (0.007)</td>
<td>0.560</td>
</tr>
<tr>
<td>Vertcoin</td>
<td>1.200 (0.033)</td>
<td>0.381 (0.013)</td>
<td>0.819</td>
<td>$[1 , -2003.9 , -60.2]$</td>
<td>1.200 (0.040)</td>
<td>0.176 (0.009)</td>
<td>1.024</td>
<td>$[1 , -1236.9 , -2.1]$</td>
<td>0.961 (0.071)</td>
<td>0.269 (0.012)</td>
<td>0.692</td>
</tr>
</tbody>
</table>

Note: estimates of standard error are given in parentheses.