Modeling rates of inflation in Nigeria: an application of ARMA, ARIMA and GARCH models

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Modeling Rates of Inflation in Nigeria: An Application of ARMA, ARIMA and GARCH models

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Abstract

Based on time series data on inflation rates in Nigeria from 1960 to 2016, we model and forecast inflation using ARMA, ARIMA and GARCH models. Our diagnostic tests such as the ADF tests indicate that NINF time series data is essentially I (1), although it is generally I (0) at 10% level of significance. Based on the minimum Theil's U forecast evaluation statistic, the study presents the ARMA (1, 0, 2) model, the ARIMA (1, 1, 1) model and the AR (3) – GARCH (1, 1) model; of which the ARMA (1, 0, 2) model is clearly the best optimal model. Our diagnostic tests also indicate that the presented models are stable and hence reliable. The results of the study reveal that inflation in Nigeria is likely to rise to about 17% per annum by end of 2021 and is likely to exceed that level by 2027. In order to address the problem of inflation in Nigeria, three main policy prescriptions have been suggested and are envisioned to assist policy makers in stabilizing the Nigerian economy.

Key Words: ARIMA, ARMA, Forecasting, GARCH, Inflation, Nigeria.

JEL Codes: C53, E31, E37, E47

I. INTRODUCTION

Inflation can be defined as the persistent and continuous rise in the general prices of commodities in an economy (Nyoni & Bonga, 2018a). In today's world, the knowledge of what helps forecast inflation is important (Duncan & Martínez-García, 2018). Policy makers can get prior indication about possible future inflation through inflation forecasting (Nyoni, 2018k). It is possible to attribute the high rate of inflation in Nigeria to factors such as, low output growth rate, high prices of imported products, depreciation in the exchange rate and probably external factors like crude oil price. Since, price stability is one of the key objectives of monetary policy (Hadrat *et al*, 2015), while another is to maintain a persistent economic growth along with low inflation (Islam, 2017), it is up to the policymakers to be forward – looking. Good forecasting ability is germane to achieve this objective (Hadrat *et al*, 2015). Inflation forecasting is not only a useful guide for policy discussion, it also plays a dominant role in a situation where a country is practicing an inflation targeting regime as it can alert policymakers to take drastic decision when inflation deviates from its target (Iftikhar & Iftikhar-ul-amin, 2013; Hadrat *et al*, 2015). Again, because monetary policy is associated with lags which are significant, it is ideal for policy to be designed in a forward – looking manner, this further stresses the importance of obtaining accurate forecasts for inflation (Mandalinci, 2017; Nyoni, 2018k). These and many other reasons make inflation modeling and forecasting sacrosanct for the monetary authority.

The history of high inflation rate in Nigeria could be traced to the Udoji Commission of 1974 that proposed an enhanced salary structure for civil servants, the so-called "Udoji Award"; without considering the aftermath, as well as, the unfortunate civil war of 1967 to 1970. Inflation has been one of the most persistent economic challenges in the world, especially in developing countries (Jere & Siyanga, 2016). Nigeria has been facing this challenge for so many years now. The monetary authorities in Nigeria are confronting two challenges- maintaining stable inflation and ensuring high growth in the economy. As a result of the political upheaval in the country, the inflation rate surged to 57.16% in 1993. It further increased to 72.83% in 1995. However, in 1997, it reduced by 64.33% to 8.5%. It remained on a single digit from 1997 to 2000. Having achieved a single digit inflation, the Nigerian government and the monetary authority couldn't sustain the trend as inflation increased to 19% in 2002. Between 2003 and 2009, the inflation rate averaged 11.42%. The country recorded its lowest inflation rate (5.38%) in 2007. The inflation rate was 8.47%, 8.05%, 9.01% and 15.69% in 2013, 2014, 2015 and 2016 respectively (WDI, 2017). As of December 2017, the inflation rate had dropped to 15.37% (National Bureau of Statistics, 2017).

Recent developments in the world such as globalization, changes in policies (inflation targeting), among other factors have made forecasting of inflation to be difficult (Duncan & Martínez-García, 2018). Due to the importance of inflation forecasting in a modern economy, many researchers; for example, Aron & Muellbauer, 2012; Ogunc *et al*, 2013; Chen *et al*, 2014; Balcilar *et al*, 2015; Pincheira & Medel 2015; Medel *et al*, 2016; Altug & Cakmakli 2016 as well as Mandalinci 2017 have extended their studies to cover two or more countries. The difficulty of controlling inflation and the time lag of monetary policy suggest the need to maintain stable inflation. Most studies that tried to forecast inflation in Nigeria either used ARIMA (Adebiyi *et al.,* 2010; Olajide *et al*, 2012; Uko & Nkoro 2012; Etuk *et al*, 2012; Okafor & Shaibu 2013; Kelikume & Salami 2014; Mustapha & Kubalu 2016; Popoola *et al.,* 2017), SARIMA (Doguwa & Alade, 2013) or a combination of both (Otu *et al.,* 2014; John & Patrick, 2016).

This study is among the very few studies that used the ARMA, ARIMA and GARCH approaches to model annual inflation rate volatility in Nigeria. The rest of the paper is organized as follows. Section II is concerned with literature review. In Section III we show the methodology and models used in the study. We report and discuss the results of our findings in section IV. Finally, in Section V, we conclude and suggest relevant policy recommendations.

II. LITERATURE REVIEW

Theoretical Literature Review

One key role of monetary policy in any given economy is to ensure price stability and provide the environment for adequate credit expansion which will, in turn, promote growth and development. There are quite a number of theories of inflation. Some of these theories are the Monetarist theory, the Keynesian theory and the Neo – Keynesian theory among others. There is still no consensus among these theories on the root causes of inflation and how it should be controlled.

The Monetarist attributed the cause of long-run inflation to a growth in money supply which is not matched with output growth (Friedman 1956, 1960, 1971). The Keynesians did not agree with the postulation of the monetarist. To them, money creation has no direct impact on aggregate demand. The impact of money on aggregate demand can only be felt through interest rates. The interest rate on its own has a minimal impact on aggregate demand (Samuelson, 1971). According to the Keynesians, the velocity of money is not as stable as postulated by the monetarists. The Neo-Keynesians are basically divided inflation into three: Demand-pull, Cost-push and Structural inflation theorists. Demand-pull inflation occurs when there is an excess of demand over supply. When this excess occurs, there will be an inflationary gap. Cost-push theories attributed the increase in factor inputs and production costs in general as causes of inflation (Kavila & Roux, 2017). According to them, inflation is not a function of an increase in money supply as the monetarists claim. The Structuralist believed that structural rigidities, market imperfections and social tensions are the causes of inflation (Thirwell, 1974; Aghevei & Khan, 1977). They placed more emphasis on the supply side of the economy (Bernanke, 2005). Khan & Schimmelpfennig (2006) further considered food prices, administered prices, wages and import prices, as additional factors that drive inflation.

Empirical Literature Review

Lots of researches have been conducted on this theme over several decades. Given the specific focus of our paper on modelling and forecasting inflation in Nigeria, Table 1 below provides a fair sample of studies undertaken more recently:

Literature Summary on Modelling and Forecasting Inflation

Table 1

Author(s)/ Year	**Country**	**Period**	**Methodology**	**Major Finding(s)**
Yusif *et al*, (2015)	Ghana	1991:01 - 2010:12	Artificial Neural Network Model Approach, AR and VAR	Out-of-sample forecast error of Artificial Neural Network Model Approach is lower than other techniques.
Iftikhar & Iftikhar-ul-amin (2013)	Pakistan	1961 – 2012	ARIMA	ARIMA was found to be the most appropriate model
Mustapha & Kubalu (2016)	Nigeria	January 1995 to December 2013	ARIMA	ARIMA was the best-fitted model for explaining the relationship between past and current inflation rate.
Kabukcuoglu & Martnez-Garca	14 advanced countries.	1984:Q1-2015:Q1	Workhorse open-economy	Cross-country interactions yield

(2018)			New Keynesian framework	significantly more accurate forecasts of local inflation
Pincheira & Gatty (2016)	18 Latin American countries and 30 OECD countries	January 1994 to March 2013	FASARIMA, ARIMA, SARIMA and FASARIMAX	International factors help in forecasting Chilean inflation
Nyoni (2018k)	Zimbabwe	July 2009 to July 2018	GARCH	The AR (1) – IGARCH (1, 1) model is appropriate and the best for forecasting inflation in Zimbabwe.
Fwaga *et al.,* (2017)	Kenya	January 1990 – December 2015	EGARCH and GARCH	The inflation rate in Kenya can best be forecast with EGARCH.
Banerjee (2017)	41 countries comprising both developed and developing countries.	January 1958 – February 2016	GARCH	Developing countries have an inflation rate that is about 3.5% greater than that of developed countries.
Lidiema (2017)	Kenya	November 2011 to October 2016	SARIMA and Holt-Winters Triple Exponential Smoothing	SARIMA Model was a better model for forecasting inflation in Kenya than the Holt-winters triple exponential smoothing.
Otu *et al.,* (2014)	Nigeria	November 2003 to October 2013	ARIMA and SARIMA	SARIMA was a better model for forecasting inflation in Nigeria.
Ingabire & Mung'atu (2016)	Rwanda	2000Q1 to 2015Q1	ARIMA and VAR	ARIMA (3, 1, 4) model was better than the VAR model in predicting inflation in Rwanda.
Jere & Siyanga (2016)	Zambia	May 2010 to May 2014.	Holts exponential smoothing and ARIMA model	ARIMA ((12), 1, 0) model performed better than the Holts exponential smoothing.
Uwilingiyimana, *et al.* (2015)	Kenya	Monthly data from 2000 to 2014.	ARIMA and GARCH	The combination of both models, ARIMA (1, 1, 12) and GARCH (1, 2) provide the best result.
Udom & Phumchusri (2014)	Thailand	January 2004 and December 2012.	ARIMA method, Moving average method and Holt's and Winter exponential method.	ARIMA model was a better model when compared with other methods

Molebatsi & Raboloko (2016).	Botswana	January 2005 to December 2014	GARCH and ARIMA	Volatility for Botswana's CPI is low.
John & Patrick (2016)	Nigeria	Monthly data from 2000 to 2015	ARIMA and SARIMA	Inflation rates in Nigeria are seasonal and follow a seasonal ARIMA Model
Islam (2017)	Bangladesh	1971 – 2015	ARIMA	ARIMA model (1, 0, 0) was most appropriate for forecasting inflation in Bangladesh
Duncan & Martínez-García (2018).	14 emerging market economies	1980Q1 - 2016Q4	Bayesian VAR. Random-walk Model.	The random walk model tends to produce a lower root mean square prediction error than its competitors.
Ngailo *et al*, (2014).	Tanzania	January 1997 to December 2010	GARCH	GARCH(1,1) model is found to be the best model for forecasting inflation in Tanzania
Okafor & Shaibu (2013).	Nigeria	1981 – 2010	ARIMA	ARIMA (2,2,3) was the best model for forecasting.
Kelikume & Salami (2014).	Nigeria	Monthly data from 2003 to 2012	ARIMA and VAR	The VAR model was preferred to the ARIMA model because of smaller minimum square error.
Inam (2017)	Nigeria	1970 – 2012	VAR	Fiscal deficit, money supply, and output are not significant determinants of inflation in Nigeria.
Popoola *et al.*, (2017)	Nigeria	2006 – 2016	ARIMA	Discovered ARIMA (0,1,1) as the best model for forecasting inflation in Nigeria.

Source: Authors' computation from literature

III. MATERIALS & METHODS

The Moving Average (MA) model

Given:

$$\mathrm{NINF}_t = \alpha_0\mu_t + \alpha_1\mu_{t-1} + \cdots + \alpha_q\mu_{t-q} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [1]$$

where μ_t is a purely random process with mean zero and varience σ^2. We say that equation [1] is a Moving Average (MA) process of order q, commonly denoted as MA (q). NINF is the annual inflation rate in Nigeria at time t, $\alpha_0 \ldots \alpha_q$ are estimation parameters, μ_t is the current error term while $\mu_{t-1} \ldots \mu_{t-q}$ are previous error terms. Thus:

$$\mathrm{NINF}_t = \alpha_0\mu_t + \alpha_1\mu_{t-1} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [2]$$

is an MA process of order one, commonly denoted as MA (1). Owing to the fact that previous error terms are unobserved variables, we then scale them so that $\alpha_0=1$. Since:

$$\left.\begin{array}{c}E(\mu_t)=0\\ \forall t\end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [3]$$

Therefore, it implies that:

$$E(NINF_t)=0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [4]$$

and:

$$Var(NINF_t) \cong \left(\sum_{i=0}^{q}\alpha_t^2\right)\sigma^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

where μ_t is independent with a common varience σ^2. Thus, we can now re – specify equation [1] as follows:

$$NINF_t=\mu_t+\alpha_1\mu_{t-1}+\cdots+\alpha_q\mu_{t-q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [6]$$

Equation [6] can be re – written as:

$$NINF_t=\sum_{i=1}^{q}\alpha_i\mu_{t-i}+\mu_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [7]$$

We can also write equation [7] as follows:

$$NINF_t=\sum_{i=1}^{q}\alpha_i L^i\mu_t+\mu_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [8]$$

where L is the lag operator.

or as:

$$NINF_t=\alpha(L)\mu_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [9]$$

where:

$\alpha(L)=\theta(L)$[1] ……………………………………………………………………………………….. [10]

The Autoregressive (AR) model

Given:

$$NINF_t=\beta_1 NINF_{t-1}+\cdots+\beta_p NINF_{t-p}+\mu_t \ldots\ldots\ldots\ldots\ldots\ldots [11]$$

Where $\beta_1 \ldots \beta_p$ are estimation parameters, $CPI_{t-1} \ldots CPI_{t-p}$ are previous period values of the CPI series and μ_t is as previously defined. Equation [11] is an Autoregressive (AR) process of order p, and is commonly denoted as AR (p); and can also be written as:

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[1] *defined as in equation [22].*

$$\mathrm{NINF}_t = \sum_{i=1}^{p} \beta_i \mathrm{NINF}_{t-1} + \mu_t \dots \quad [12]$$

Equation [12] can be re – written as:

$$\mathrm{NINF}_t = \sum_{i=1}^{p} \beta_i L^i \mathrm{NINF}_t + \mu_t \dots \quad [13]$$

or as:

$$\beta(L)\mathrm{NINF}_t = \mu_t \dots \quad [14]$$

where:

$$\beta(L)=\phi(L)^2 \dots \quad [15]$$

or as:

$$\mathrm{NINF}_t = (\beta_1 L + \cdots + \beta_p L^p)\mathrm{NINF}_t + \mu_t \dots \quad [16]$$

Thus:

$$\mathrm{NINF}_t = (\beta_1 L)\mathrm{NINF}_t + \mu_t \dots \quad [17]$$

is an AR process of order one, commonly denoted as AR (1).

The Autoregressive Moving Average (ARMA) model

As initially postulated by Box & Jenkins (1970), an ARMA (p, q) process is simply a combination of AR (p) and MA (q) processes. Thus, combining equations [1] and [11]; an ARMA (p, q) process can be specified as follows:

$$\mathrm{NINF}_t = \beta_1 \mathrm{NINF}_{t-1} + \cdots + \beta_p \mathrm{NINF}_{t-p} + \mu_t + \alpha_1 \mu_{t-1} + \cdots + \alpha_q \mu_{t-q} \dots \quad [18]$$

or as:

$$\mathrm{NINF}_t = \sum_{i=1}^{p} \beta_i \mathrm{NINF}_{t-i} + \sum_{i=1}^{q} \alpha_i \mu_{t-i} + \mu_t \dots \quad [19]$$

by combining equations [7] and [12]. Equation [18] can also be written as:

$$\phi(L)\mathrm{NINF}_t = \theta(L)\mu_t \dots \quad [20]$$

where $\phi(L)$ and $\theta(L)$ are polynomials of orders p and q respectively, simply defined as:

$$\phi(L) = 1 - \beta_1 L \dots \beta_p L^p \dots \quad [21]$$

$$\theta(L) = 1 + \alpha_1 L + \cdots + \alpha_q L^q \dots \quad [22]$$

[2] *defined as in equation [23].*

It is essential to note that the ARMA (p, q) model, just like the AR (p) and the MA (q) models; can only be employed for stationary time series data; and yet in real life, many time series are non – stationary. For this simple reason, ARMA models are not suitable for describing non – stationary time series.

The Autoregressive Integrated Moving Average (ARIMA) model

ARIMA models are a set of models that describe the process (for example, CPI_t) as a function of its own lags and white noise process (Box & Jenkins, 1974). Making predicting in time series using univariate approach is best done by employing the ARIMA models (Alnaa & Ahiakpor, 2011). A stochastic process $NINF_t$ is referred to as an Autoregressive Integrated Moving Average (ARIMA) [p, d, q] process if it is integrated of order "d" [I (d)] and the "d" times differenced process has an ARMA (p, q) representation. If the sequence $\Delta^d NINF_t$ satisfies and ARMA (p, q) process; then the sequence of $NINF_t$ also satisfies the ARIMA (p, d, q) process such that:

$$\Delta^d NINF_t = \sum_{i=1}^{p} \beta_i \Delta^d NINF_{t-i} + \sum_{i=1}^{q} \alpha_i \mu_{t-i} + \mu_t \ldots \ldots [23]$$

which we can also re – write as:

$$\Delta^d NINF_t = \sum_{i=1}^{p} \beta_i \Delta^d L^i NINF_t + \sum_{i=1}^{q} \alpha_i L^i \mu_t + \mu_t \ldots \ldots [24]$$

where Δ is the difference operator, vector $\beta \in \mathbb{R}^p$ and $\alpha \in \mathbb{R}^q$.

The Autoregressive Conditionally Heteroskedastic (ARCH) model

In financial time series modelling and forecasting, it usually makes a lot of sense to take into account a model that describes how the varience of the errors evolves and such a model is non – other – than the ARCH model. The basic intuition behind ARCH family type models is that it is very rare that the varience of the errors will be constant over time and on such grounds, it is reasonable to consider models that do not assume that the varience is constant. To briefly explain the simple intuition behind the ARCH model, we start by defining the conditional varience of a random variable, μ_t:

$$\sigma_t^2 = var(\mu_t \mid \mu_{t-1}, \mu_{t-2}, \ldots) = E[\mu_t - E(\mu_t)^2 \mid \mu_{t-1}, \mu_{t-2}, \ldots] \ldots \ldots [25]$$

assuming that equation [3] also holds water in this case, such that:

$$\sigma_t^2 = var(\mu_t \mid \mu_{t-1}, \mu_{t-2}, \ldots) = E[\mu_t^2 \mid \mu_{t-2}, \ldots] \ldots \ldots [26]$$

Equation [26] indicates that the conditional varience of a zero mean normally distributed random variable μ_t is equal to the conditional expected value of the square of μ_t.

$$\sigma_t^2 = \varphi_0 + \varphi_1 \mu_{t-1}^2 \ldots \ldots [27]$$

Equation [27] is called an ARCH (1) model because the conditional varience depends only on one lagged squared error. Equation [27] cannot be seen as a complete model just because we haven't taken into account the conditional mean. The conditional mean, in this case; describes how the dependent variable, $NINF_t$; varies over time. As noted by Nyoni (2018k); there is no rule of thumb on how to specify the conditional mean equation; actually it takes any form deemed adequate by the researcher/s. Thus, the complete model consists of both the conditional mean equation and the ARCH specification as illustrated by Nyoni (2018k). Equation [27] can be generalized to a case where the error variance depends on p lags of squared errors as follows:

$\sigma_t^2=\varphi_0+\varphi_1\mu_{t-1}^2+\ldots+\varphi_p\mu_{t-p}^2$ ………………………………………………………………………….. [28]

Thus, equation [28] is an ARCH (p) model.

The Generalized ARCH (GARCH) model

The equation below:

$\sigma_t^2=\varphi_0+\varphi_1\mu_{t-1}^2+\lambda_1\mu_{t-1}^2$ ………………………………………………………………………….. [29]

is the "work – horse version" and yet most important case of a GARCH process, the GARCH (1, 1) model; where σ_t^2 is the conditional varience, φ_0 is the constant, $\varphi_1\sigma_{t-1}^2$ is the information about the previous period volatility, and $\lambda_1\sigma_{t-1}^2$ is the fitted varience from the model during the previous period. From equation [29], we deduce that:

$E_{t-1}[\mu_t^2]=\sigma_t^2$ ………………………………………………………………………………… [30]

such that:

$\sigma_t^2=\varphi_0+(\varphi_1+\lambda_1)\mu_{t-1}^2+\varepsilon_t-\lambda_1\varepsilon_{t-1}^2$ ……………………………………………………………….. [31]

which is apparently an ARMA (1, 1) model; this simply implies that indeed, a GARCH model can be expressed as an ARMA process of squared residuals. In this regard:

$\varepsilon_t=\mu_t^2-E_{t-1}[\mu_t^2]$ ………………………………………………………………………………… [32]

is the stochastic term. Given equation [31], we can use inference to conclude that the stationarity of the GARCH (1, 1) model requires:

$\varphi_1+\lambda_1<1$ ……………………………………………………………………………………………… [33]

Taking the unconditional expectation of equation [29], we get:

$\sigma^2=\varphi_0+\varphi_1\sigma^2+\lambda_1\sigma^2$ …………………………………………………………………………… [34]

so that:

$\sigma^2=\frac{\varphi_0}{1-\varphi_0-\lambda_1}$ …………………………… …………………………………………………………………… [35]

For this unconditional varience to exist, equation [33] must hold water and for it to be positive, then:

$\varphi_0>0$ …………………………………………………………………………………………………….. [36]

Equation [29] can be generalized into a GARCH (p, q) model where the current conditional varience is parameterized to depend upon p lags of the squared error and q lags of the conditional varience as shown below:

$\sigma_t^2=\varphi_0+\varphi_1\mu_{t-1}^2+\ldots+\varphi_p\mu_{t-p}^2+\lambda_1\sigma_{t-1}^2+\ldots+\lambda_q\sigma_{t-p}^2$ ………………………………………...……………… [37]

Equation [37] can also be written as follows:

$\sigma^2=\varphi_0+\varphi(L)\mu_t^2+\lambda(L)\sigma_t^2$ ……………………………………………………………………………… [38]

where φ(L) and λ(L) denote the AR and MA polynomials respectively, such that:

$\varphi(L)=\varphi_1 L+\ldots+\varphi_p L^p$ ………………………………...………………………………………………….. [39]

and:

$\lambda(L)=\lambda_1+\ldots+\lambda_q L^p$ ………………………………………………………………………………………. [40]

or as:

$\sigma_t^2=\varphi_0+\sum_{i=1}^{p}\varphi_i\mu_{t-i}^2+\sum_{j=1}^{q}\lambda_j\sigma_{t-j}^2$ …………………………...……………..……………………….……….. [41]

where condition [33] is now generalized as follows:

$\sum_{i=1}^{p}\varphi_i+\sum_{j=1}^{q}\lambda_j<1$ ……………………………………………………………………………..…………. [42]

Suppose all the roots of the polynomial:

$|1-\lambda(L)|^{-1}=0$ ………………………………………………………………..………………...….……… [43]

lie outside of the unit circle, then; we get:

$\sigma_t^2=\varphi_0|1-\lambda(L)|^{-1}+\varphi(L)|1-\lambda(L)|^{-1}\mu_t^2$ ………………………………..…………………………..………… [44]

which is indeed an ARCH (∞) process because the conditional varience linearly depends on all previous squared residuals. Therefore, the unconditional varience is expressed as follows:

$\sigma^2\equiv E(\mu_t^2)=\frac{\varphi_i}{1-\sum_{i=1}^{p}\varphi_i-\sum_{j=1}^{q}\lambda_j}$ ……………………………..……………………………………..………… [45]

Suppose:

$\varphi_1+\ldots+\varphi_p+\lambda_1+\ldots+\lambda_p=1$ ………………………………………………..………………………..……… [46]

then the unconditional varience will be ∞.

Conditions [33] and [42] basically mean the same thing. In a plethora of financial time series, these conditions are close to unity; indicating persistant volatility. Let's say:

$\varphi_1+\lambda_1=1$ ……………………………………...……………………………………..……………….. [47]

or more generally:

$\sum_{i=1}^{p}\varphi_i+\sum_{j=1}^{q}\lambda_j=1$ ……………………………………..……………………………..…………….. [48]

or simply:

$\varphi(L)+\lambda(L)=1$ ……………………………………...……..……...…………………………………… [49]

what it implies is that the resulting process is not covariance stationary. Such a process gives birth to what is called an Integrated GARCH or IGARCH model; a model in which current information remains vital when forecasting the volatility for all horizons.

Model Specification

Strictly based on our diagnostic tests and model evaluation criterion (see tables 2 – 19), we specify the following models:

ARMA (1, 0, 2) Model:

$$\left.\begin{array}{c} NINF_t = c + \beta_1 NINF_{t-1} + \alpha_1 \mu_{t-1} + \alpha_2 \mu_{t-2} + \mu_t \\ \text{where c is the model constant} \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [50]$$

ARIMA (1, 1, 1) Model:

$$\Delta NINF_{t-1} = c + \beta_1 \Delta NINF_{t-1} + \alpha_1 \mu_{t-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [51]$$

AR (3) – GARCH (1, 1) model:

The appropriate equations for the mean and varience were specified as follows:

$$\left.\begin{array}{c} NINF_t = c + \omega_1 NINF_{t-1} + \omega_2 NINF_{t-2} + \omega_3 NINF_{t-3} + \mu_t \\ \text{where: } \mu_t \cong N(0;\ \sigma_t^2) \text{ and} \\ \omega_1 \ldots\ \omega_3 \text{ are estimation parameters;} \\ \sigma_t^2 = \varphi_0 + \varphi_1 \mu_{t-1}^2 + \lambda_1 \sigma_{t-1}^2 \\ \text{where: } \varphi_0 \geq 0, \\ \varphi_1 \geq 0, \\ \lambda_1 \geq 0 \\ \text{Everything else remains as previously defined} \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [52]$$

The Box – Jenkins (1970) Methodology

The first step towards model selection is to difference the series in order to achieve stationarity. Once this process is over, the researcher will then examine the correlogram in order to decide on the appropriate orders of the AR and MA components. It is important to highlight the fact that this procedure (of choosing the AR and MA components) is biased towards the use of personal judgement because there are no clear – cut rules on how to decide on the appropriate AR and MA components. Therefore, experience plays a pivotal role in this regard. The next step is the estimation of the tentative model, after which diagnostic testing shall follow. Diagnostic checking is usually done by generating the set of residuals and testing whether they satisfy the characteristics of a white noise process. If not, there would be need for model re – specification and repetition of the same process; this time from the second stage. The process may go on and on until an appropriate model is identified (Nyoni, 2018i)

Data Collection

This study is based on Nigerian annual inflation rate data, from 1960 to 2016. All the data used in this study was gathered from the World Bank.

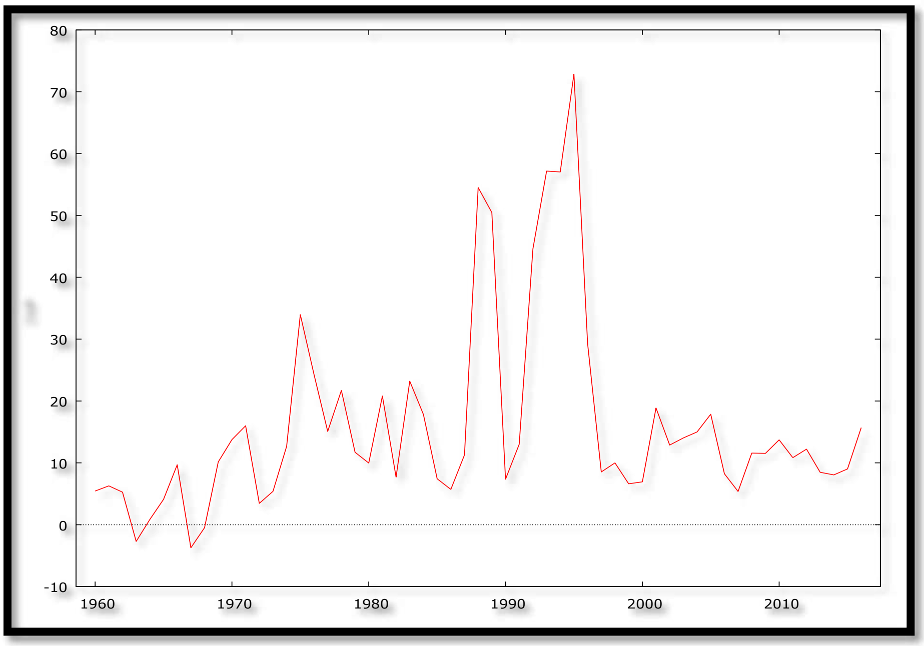
Diagnostic Tests and Model Evaluation

Stationarity Tests

Graphical Analysis

A time plot of the NINF series was graphically examined as shown below:

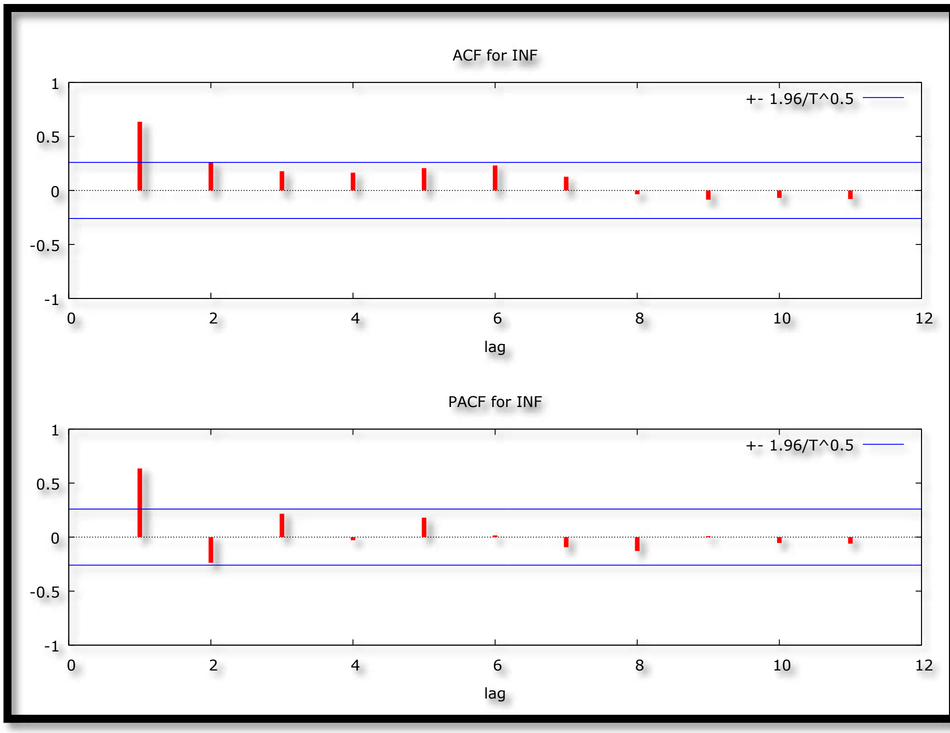
Figure 1



The above graph shows that the NINF series is likely to be stationary (when formally tested for stationarity) since it exhibits no particular trend. The implication is that the mean of NINF is generally not changing over time and hence we can safely conclude that the variance of NINF is basically constant over time.

The correlogram in levels

Figure 2



The figure above confirms the general stationarity of the NINF series as indicated by the autocorrelation coefficients, most of which are quite low at various lags.

The ADF test

The Augmented Dickey Fuller (ADF test) was used to check the stationarity of the NINF series. The general ADF test is done by running the following regression equation:

$NINF_t = c_t \gamma NINF_{t-1} + \sum_{i=1}^{p-1} \Delta NINF_{t-i} + \mu_t$ ………………………………………………………..……………….. [53]

Where c_t is a deterministic function of the time index t and $\Delta NINF_j = NINF_j - NINF_{j-1}$ is the differenced series of $NINF_t$. The null hypothesis H_0: $\gamma=1$ is tested against the alternative hypothesis H_a: $\gamma \leq 1$. If the null hypothesis is rejected, then the time series is stationary. The results of the ADF tests done in this study are shown below:

Levels: intercept

Table 2

Variable	**ADF Statistic**	**Probability**	**Critical Values**		**Conclusion**
NINF	-3.490778	0.0118	-3.552666	@1%	Not stationary
			-2.914517	@5%	Stationary
			-2.595	@10%	Stationary

Levels: trend & intercept

Table 3

Variable	**ADF Statistic**	**Probability**	**Critical Values**		**Conclusion**
NINF	-3.480478	0.0514	-4.130526	@1%	Not stationary

		-3.492149	@5%	Not stationary
		-3.174802	@10%	Stationary

Levels: without intercept and trend & intercept

Table 4

Variable	ADF Statistic	Probability	Critical Values		Conclusion
NINF	-2.265742	0.0239	-2.606911	@1%	Not stationary
			-1.946764	@5%	Stationary
			-1.613062	@10%	Stationary

Table 2 indicates that the NINF series is stationary at both 5% and 10% levels of significance. Table 3 indicates that the NINF series is only stationary at 10% level of significance. Table 4 shows that the NINF series is stationary at both 5% and 10% levels of significance. The most striking feature here is that all the tables 2 – 4 confirm and concur on the stationarity of the NINF series at 10% level of significance. However, we proceed to test for stationary in first differences because we want to achieve stationary at 1% and 5% levels of significance.

Correlogram at first differences

1st Difference: Intercept

Table 5

Variable	ADF Statistic	Probability	Critical Values		Conclusion
NINF	-7.666082	0.0000	-3.557472	@1%	Stationary
			-2.916566	@5%	Stationary
			-2.596116	@10%	Stationary

1st Difference: trend & intercept

Table 6

Variable	ADF Statistic	Probability	Critical Values		Conclusion
NINF	-7.607109	0.0000	-4.137272	@1%	Stationary
			-3.495295	@5%	Stationary
			-3.176618	@10%	Stationary

1st Difference: without trend and trend & intercept

Table 7

Variable	ADF Statistic	Probability	Critical Values		Conclusion
NINF	-7.739240	0.0000	-2.608490	@1%	Stationary
			-1.946996	@5%	Stationary
			-612934	@10%	Stationary

Tables 5 – 7 concur on the stationarity of the NINF series at all levels of significance when tested for stationarity after taking first differences.

Testing for ARCH / GARCH effects

In this study, ARCH / GARCH effects were tested using the Langrange Multiplier (LM) test as briefly described here: run the mean equation given by equation [] and save the residuals. Square the residuals and regress then on "p" own lags to test for ARCH effects of order "p". From this procedure, obtain R^2 and save it. The test statistic, TR^2 (number of observations multiplied by R^2) follows a $\chi^2(p)$ distribution and the null and alternative hypotheses are:

$$\left.\begin{array}{l} H_0: \gamma_1 = 0 \text{ and } \gamma_2 = 0 \text{ and } \gamma_3 = 0 \text{ and ... and } \gamma_p = 0 \\ H_1: \gamma_1 \neq 0 \text{ or } \gamma_2 \neq 0 \text{ or } \gamma_3 \neq 0 \text{ or } \gamma_p \neq 0 \end{array}\right\}$$

In this research paper, the ARCH / GARCH effects test was done for the AR (3) – GARCH (1, 1) model and the results are shown below:

Chi – square (2) = 5.94244 [0.0512409]

The p – value of [0.0512409] indicates a significance of this LM test result at 5% level of significance. This implies that there are (G) ARCH effects in the chosen model and therefore it is appropriate to estimate a GARCH model.

Evaluation of Various ARMA, ARIMA & GARCH Models

It is imperative to note that there are a number of model evaluation criterion in time series modelling and forecasting, for example; Mean Error (ME), Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE); however, this study will only be restricted to the most commonly used and highly celebrated criterion, that is; the Akaike's Information Criteria (AIC) and the Theil's U in order to select the best models (in terms of parsimony [AIC] and forecast accuracy [Theil's U]) to be finally presented in this study. A model with a lower AIC value is better than the one with a higher AIC value. Theil's U, as noted by Nyoni (2018l); must lie between 0 and 1, of which the closer it is to 0, the better the forecast method.

Evaluation of various ARMA models

Table 8

Model	**AIC**	**Theil's U**	**ME**	**RMSE**	**MAE**	**MAPE**
ARMA (1,0,1)	**448.1099**	*0.38397*	*0.027754*	*11.454*	*7.7045*	*74.895*
ARMA (0,0,1)	449.5590	0.58733	-0.01	11.811	8.3432	97.272
ARMA (1,0,0)	451.995	0.51166	0.11268	12.078	8.1804	93.057
ARMA (2,0,1)	449.9029	0.36554	0.037782	11.434	7.6867	73.559
ARMA (1,0,2)	*449.4435*	**0.34626**	*0.13182*	*11.383*	*7.8496*	*75.16*
ARMA (2,0,2)	451.2983	0.35274	0.15007	11.368	7.8311	75.615
ARMA (3,0,1)	451.6884	0.35945	0.060805	11.411	7.7084	72.999
ARMA (1,0,3)	451.2705	0.35441	0.15325	11.365	7.8258	75.678
ARMA (3,0,2)	453.8985	0.35664	0.16123	11.356	7.8448	76.076
ARMA (3,0,3)	453.8985	0.35648	0.070692	11.221	7.9094	78.014
ARMA (2,03)	453.1623	0.35116	0.13601	11.352	7.8288	75.989
ARMA (4,0,1)	453.5771	0.3559	0.078415	11.399	7.7635	73.835
ARMA (4,0,2)	455.0650	0.35901	0.17309	11.342	7.8966	77.255
ARMA (4,0,3)	455.8612	0.35585	0.081729	11.217	7.927	78.41
ARMA (1,0,4)	453.2130	0.35502	0.15559	11.358	7.8324	75.786
ARMA (2,0,4)	454.2317	0.35728	0.052125	11.255	7.8698	76.427
ARMA (3,0,4)	456.2085	0.3682	0.14226	11.253	7.8747	77.314

As shown in the table above, the ARMA (1,0,1) model has the lowest AIC value whilst the ARMA (1,0,2) model has the lowest Theil's U. In this study we finally present the ARMA (1, 0,

2) model due to its best forecast accuracy. From the analysis of tables 8 – 10, it is clear that the ARMA (1, 0, 2) model is the best in terms of forecast accuracy since has the lowest Theil's U value.

Evaluation of various ARIMA models

Table 9

Model	AIC	Theil's U	ME	RMSE	MAE	MAPE
ARIMA (1,1, 1)	*454.7004*	**0.70704**	*0.010732*	*13.047*	*8.6452*	*105.03*
ARIMA (0,1,0)	**453.5975**	0.98253	0	13.401	8.6162	91.575
ARIMA (1,1,0)	455.5886	0.97916	-0.0001	13.4	8.6435	92.501
ARIMA (0,1,1)	455.5579	0.96415	-0.0006	13.396	8.7259	95.627

As shown in the table above, the ARIMA (1,1,1) model has the lowest Theil's U value whilst the ARIMA (0,1,0) (or the random walk model) has the lowest AIC value. Since these models are essentially the same in terms of parsimony and yet quite different in terms of forecast accuracy, we only consider the ARIMA (1,1,1) model which has a better forecast accuracy as shown by a minimum Theil's U of 0.70704.

Evaluation of various GARCH models

Table 10

Model		AIC	Theil's U	ME	RMSE	MAE	MAPE
GARCH (1, 1)	AR (1)	440.1924	0.5068	0.79526	12.148	8.0997	90.264
GARCH (2, 2)	AR (1)	440.5544	0.49529	-0.11587	12.105	8.2075	94.977
GARCH (1, 2)	AR (1)	442.4653	0.50819	1.0397	12.179	8.0784	89.298
GARCH (2, 1)	AR (1)	438.7116	0.53814	0.33219	12.113	8.1209	87.956
GARCH (1, 0)	AR (1)	444.3211	0.43965	1.9019	12.649	8.1623	98.209
GARCH (0, 1)	AR (1)	440.1924	0.5068	0.79526	12.148	8.0997	90.264
GARCH (2, 0)	AR (1)	439.3446	0.48627	0.49965	12.136	8.1455	94.078
GARCH (3, 0)	AR (1)	434.1929	0.5043	0.27527	12.109	8.152	92.343
GARCH (1, 1)	AR (2)	434.2809	0.43659	0.93248	11.911	7.9768	82.308
GARCH (1, 1)	*AR (3)*	*428.3686*	**0.36229**	*0.66428*	*11.698*	*7.8866*	*75.333*
GARCH (1, 1)	AR (4)	422.5446	0.36775	0.7075	11.713	7.8856	72.379
GARCH (1, 0)	AR(2)	434.4141	0.41608	2.5618	12.633	8.0573	89.717
GARCH (1, 0)	AR (3)	426.8303	0.40789	1.8738	12.19	7.9493	80.2
GARCH (1, 0)	*AR (4)*	**420.8638**	*0.42136*	*1.9267*	*12.267*	*7.944*	*78.27*

As shown in the table above, the AR (3) – GARCH (1,1) model has the lowest Theil's U value whilst the AR (4) – GARCH (1,1) model has the lowest AIC value. While both models are quite good, in this study we will finally present the AR (3) – GARCH (1, 1) model due to its best forecast accuracy.

Residual & Stability Tests

ADF Test of the residuals of the ARMA (1,0,1) Model

Levels: intercept

Table 11

Variable	ADF Statistic	Probability	Critical Values	Conclusion

V_1	-7.410755	0.0000	-3.555023	@1%	Stationary
			-2.915522	@5%	Stationary
			-2.595565	@10%	Stationary

Levels: intercept and trend

Table 12

Variable	ADF Statistic	Probability	Critical Values		Conclusion
V_1	-7.380630	0.0000	-4.133838	@1%	Stationary
			-3.493692	@5%	Stationary
			-3.175693	@10%	Stationary

Levels: without intercept and trend & intercept

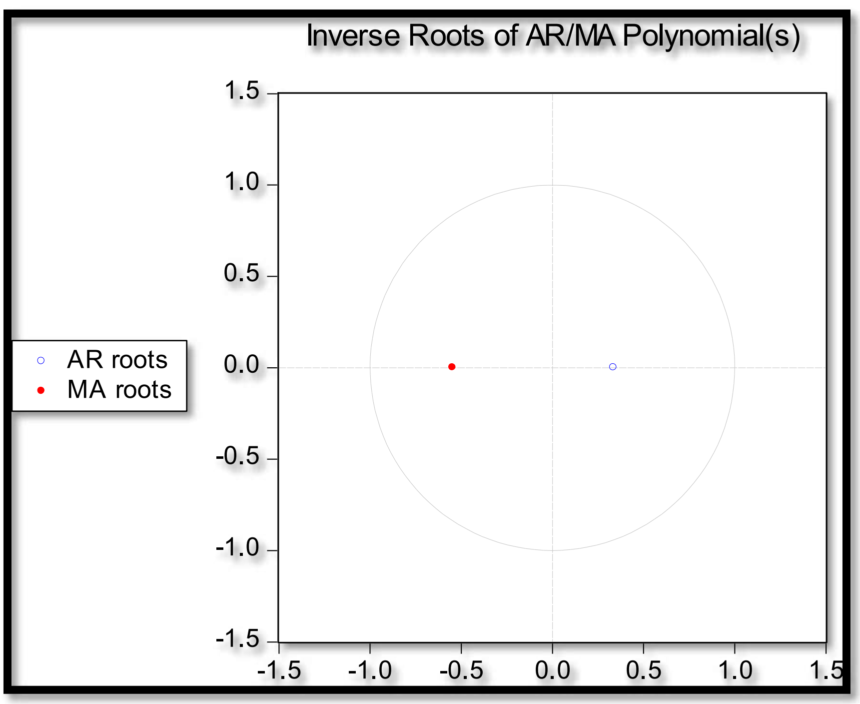
Table 13

Variable	ADF Statistic	Probability	Critical Values		Conclusion
V_1	-7.480299	0.0000	-2.607686	@1%	Stationary
			-1.946878	@5%	Stationary
			-1.612999	@10%	Stationary

Tables 11 , 12 and 13 indicate that the residuals of the ARMA (1, 0, 1) model are stationary and thus bear the features of a white – noise process.

Stability Test of the ARMA (1, 0, 1) Model

Figure 3



The figure above indicates that the ARMA (1, 0, 1) model is also stable since the corresponding inverse roots of the characteristic polynomial is in the unit circle.

ADF Test of the residuals of the ARMA (1, 0, 2) Model

Levels: Intercept

Table 14

Variable	ADF Statistic	Probability	Critical Values		Conclusion
V_2	-6.907861	0.0000	-3.555023	@1%	Stationary
			-2.915522	@5%	Stationary
			-2.595565	@10%	Stationary

Levels: intercept and trend

Table 15

Variable	ADF Statistic	Probability	Critical Values		Conclusion
V_2	-6.842560	0.0000	-4.133838	@1%	Stationary
			-3.493692	@5%	Stationary
			-3.175693	@10%	Stationary

Levels: without intercept and trend & intercept

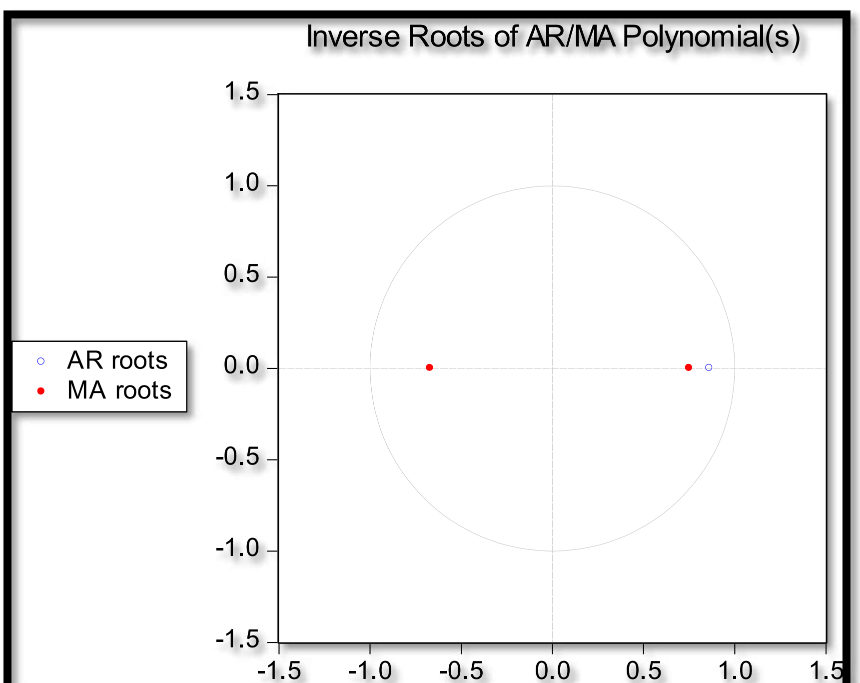
Table 16

Variable	ADF Statistic	Probability	Critical Values		Conclusion
V_2	-6.971262	0.0000	-2.607686	@1%	Stationary
			-1.946878	@5%	Stationary
			-1.612999	@10%	Stationary

Tables 14 , 15 and 16 indicate that the residuals of the ARMA (1, 0, 2) model are stationary and bear the characteristics of a white – noise process.

Stability Test of the ARMA (1, 0, 2) Model

Figure 4



The figure above shows that the ARMA (1, 0, 2) model is stable since the corresponding inverse roots of the characteristic polynomials are in the unit circle.

ADF Test of the residuals of the ARMA (1,1,1) Model

Levels: intercept

Table 17

Variable	ADF Statistic	Probability	Critical Values		Conclusion
V_3	-7.662356	0.0000	-3.560019	@1%	Stationary
			-2.917650	@5%	Stationary
			-2.596689	@10%	Stationary

Levels: intercept and trend

Table 18

Variable	ADF Statistic	Probability	Critical Values		Conclusion
V_3	-7.617701	0.0000	-4.140858	@1%	Stationary
			-3.496960	@5%	Stationary
			-3.177579	@10%	Stationary

Levels: without intercept and trend & intercept

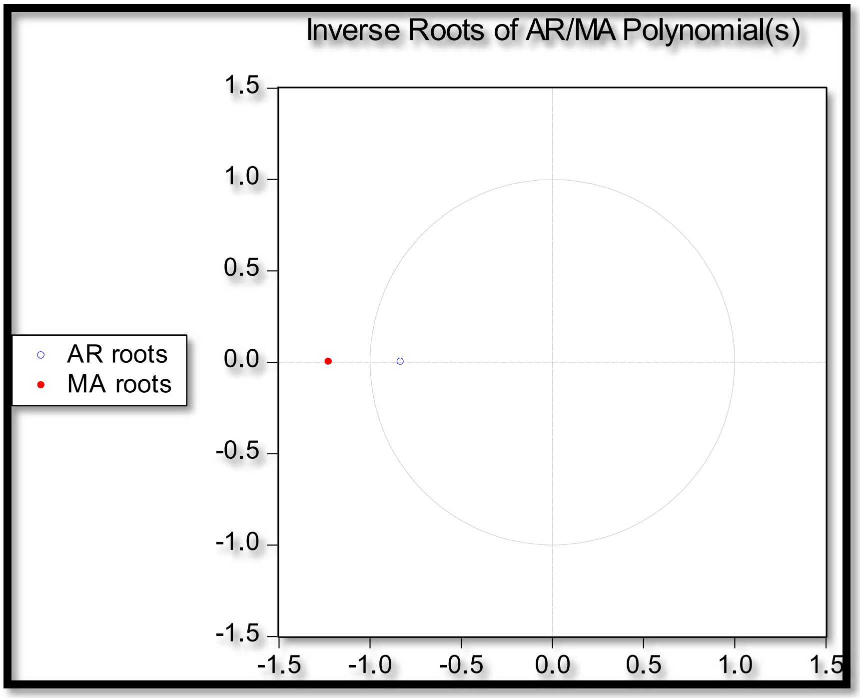
Table 19

Variable	ADF Statistic	Probability	Critical Values		Conclusion
V_3	-6.895562	0.0000	-2.609324	@1%	Stationary
			-1.947119	@5%	Stationary
			-1.612867	@10%	Stationary

Tables 17 , 18 and 19 indicate that the residuals of the ARIMA (1, 1, 1) model are stationary and thus bear the features of a white – noise process.

Stability Test of the ARIMA (1, 1, 1) Model

Figure 5



The figure above shows that the ARIMA (1, 1, 1) model is not stable since the corresponding inverse roots of the characteristic polynomials are not all found in the unit circle. The MA component falls outside the unit circle, hence confirming the instability of the ARIMA (1,1,1) model.

IV. RESULTS: PRESENTATION, INTERPRETATION & DISCUSSION

Descriptive Statistics

Table 20

Description	Statistic
Mean	15.941
Median	11.538
Minimum	-3.7263
Maximum	72.836
Standard Deviation	15.790
Skewness	1.9037
Excess Kurtosis	3.2084

As shown in table above, the mean is positive. The large difference between the maximum and the minimum confirms the sudden rise of inflation in Nigeria in 1995 which is likely to have been triggered by the political and economic instabilities that characterised Nigeria during the Sani Abacha era. The skewness is 1.9037 and the most important feature is that it is positive, implying that the NINF series has a long right tail and is non – symmetric. The rule of thumb for kurtosis is that it should be around 3 for normally distributed variables as reiterated by Nyoni & Bonga (2017h) and in this study, kurtosis has been found to be 3.2084. Therefore, the NINF series is normally distributed.

Results Presentation

Table 21

ARMA (1, 0, 2) Model:				
$NINF_t = 15.5 + 0.779NINF_{t-1} + 0.0445\mu_{t-1} - 0.382\mu_{t-2}$ … [54]				
P: 0.000 0.002 0.887 0.147				
S. E: 4.186 0.254 0.312 0.263				
Variable	**Coefficient**	**Standard Error**	**z**	**p – value**
Constant	15.4904	4.18598	3.701	0.0002***
AR (1)	0.779133	0.254036	3.067	0.0022***
MA (1)	0.0445126	0.312063	0.1426	0.8866
MA (2)	-0.381651	0.262942	-1.451	0.1467
ARIMA (1, 1, 1) Model:				
$\Delta NINF_{t-1} = 0.189 - 0.551\Delta NINF_{t-1} + 0.743\mu_{t-1}$ … [55]				
P: 0.923 0.112 0.008				
S. E: 1.955 0.346 0.28				
Variable	**Coefficient**	**Standard Error**	**z**	**p – value**
Constant	0.189231	1.95490	0.09680	0.9229
AR (1)	-0.550664	0.346489	-1.589	0.1120
MA (1)	0.742954	0.279996	2.653	0.0080***
AR (3) – GARCH (1, 1) Model				
$NINF_t = 6.06 + 0.769NINF_{t-1} - 0.3308NINF_{t-2} + 0.162NINF_{t-3}$ … … … … … … … … … … … … … … … … … … [56]				
P: 0.01 0.000 0.105 0.286				
S. E: 2.35 0.177 0.204 0.152				
$\sigma_t^2 = 22.8 + 0.679\varepsilon_{t-1}^2 + 0.147\sigma_{t-1}^2$ … [57]				
P: 0.2 0.000 0.115				
S. E: 17.856 0.161 0.094				
Variable	**Coefficient**	**Standard Error**	**z**	**p – value**
Constant	6.05789	2.35064	2.577	0.0100***
AR (1)	0.768795	0.176567	4.354	0.00000133***

AR (2)	-0.330778	0.204201	-1.620	0.1053
AR (3)	0.161850	0.151803	1.066	0.2863
φ_0	22.8427	17.8557	1.279	0.2008
ARCH (φ_1)	0.679485	0.160597	4.231	0.00000233***
GARCH (λ_1)	0.147326	0.0935581	1.575	0.1153
$\varphi_1 + \lambda_1$	0.826811			

, ** and * indicate statistical significance levels at 10%, 5% and 1% respectively.*

Interpretation & Discussion of Results

ARMA (1, 0, 2) model

The AR component is positive and statistically significant at 1% level of significance. This implies that previous period inflation rates are quite important in explaining current inflation rates in Nigeria.

ARIMA (1, 1, 1) model

The MA component is positive and statistically significant at 1% level of significance. This indicates that previous period shocks to inflation are quite imperative in explaining current inflation rates in Nigeria.

AR (3) – GARCH (1, 1) model

As theoretically expected, the constant of the mean equation, the ARCH term and the GARCH term are positive to ensure that the conditional varience is non – negative and thus the positivity constraint of the GARCH model is not violated. The ARCH term is statistically significant at 1% level of significance, indicating that strong G/ARCH effects are apparent. Thus a 1% increase in previous period volatility leads to an approximately 0.68% increase in current volatility of annual inflation rate in Nigeria. Since:

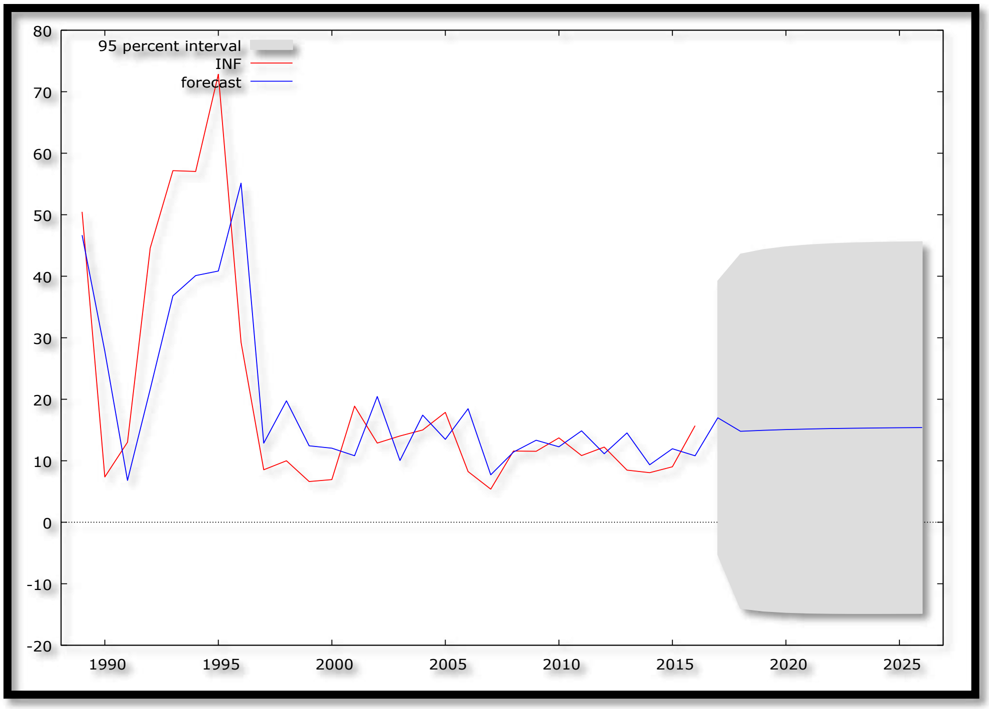
$$\varphi_1 + \lambda_1 < 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [58]$$

It implies the specified AR (3) – GARCH (1, 1) model is stationary. Thus the specified model is quite reliable in forecasting inflation volatility in Nigeria.

Forecast Graphs

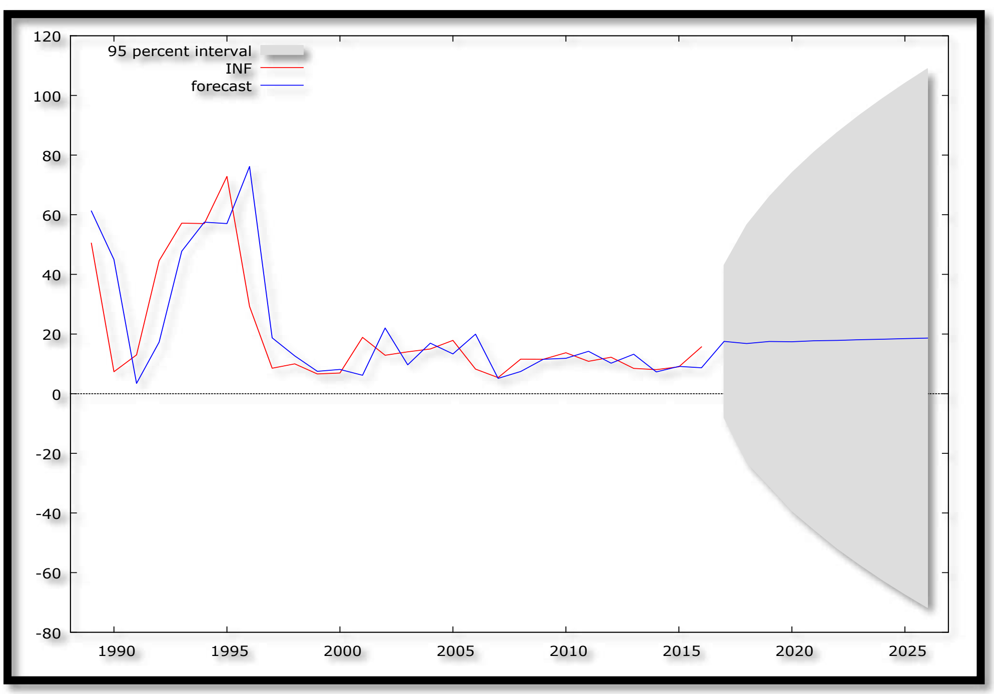
ARMA (1, 0, 2) model

Figure 6



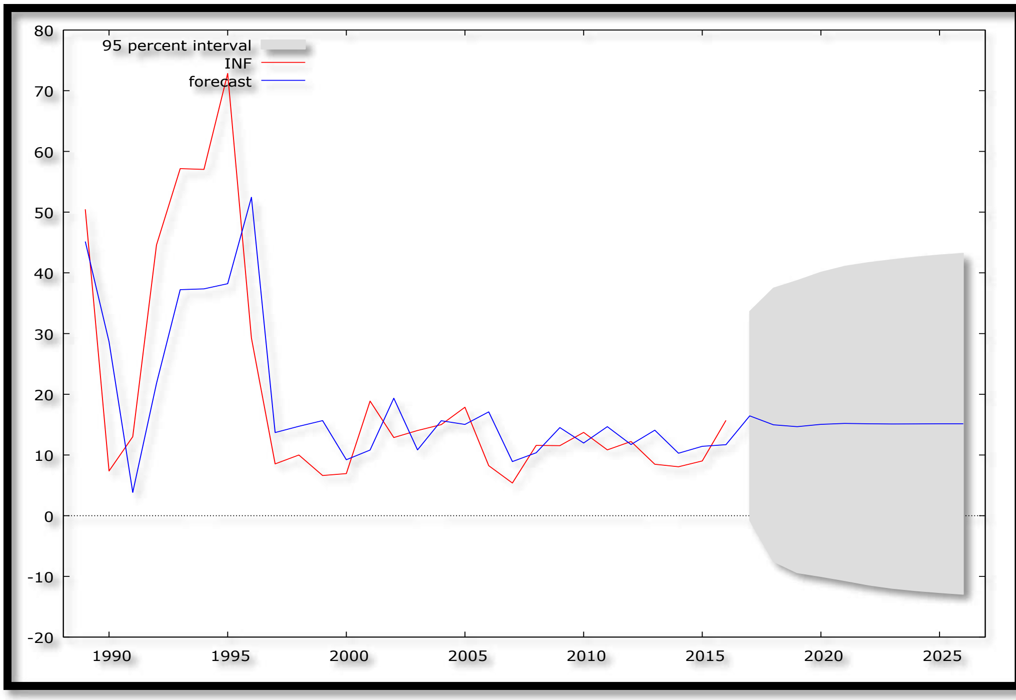
ARIMA (1, 1, 1) model

Figure 7



AR (3) – GARCH (1, 1) model

Figure 8



The figures 6 – 8 (with a forecast range of 10 years, that is; 2017 – 2027) indicate that inflation in Nigeria is likely to be stable (although relatively high), hovering around a general level of approximately 15% in the first half, that is between 2017 and 2021; after which it may likely rise to around 17%, of course; assuming that, in Nigeria; the current economic policy stance and other factors do not change significantly (or remain constant) over the forecast range. The most important feature of the figures 6, 7 and 8 is that they strongly concur in their forecasts; that inflation in Nigeria is well above 10% and may likely increase slightly [15% - 17% over the first half of the forecast range and probably beyond that in the second half] over the forecast range. Inflation that is less than 9% or generally low, is healthy for the economy and many authors, for example; Sergii (2009) and Marbuah (2010) have confirmed this. Therefore, in Nigeria; there is need to control inflation since it is quite high as shown by figures beyond 9%. Our forecasts justify the need for immediate policy intervention since inflation rates indicate that they may rise even to higher levels. Inflation has a well – known negative impact on growth, thus the need to control it.

V. CONCLUSION & RECOMMENDATIONS

Maintenance of price stability continues to be one of the main objectives of monetary policy for most countries in the world today and Nigeria is not an exception (Nyoni & Bonga, 2018a). The monetary policy of Nigeria can be more effective when it is forward – looking. This study envisages to enable the Central Bank of Nigeria (CBN) to have some "upper – hand" in the control of inflation in Nigeria by providing a reliable forecast of inflation in Nigeria. We use various ARMA, ARIMA and GARCH models to forecast inflation in Nigeria. The study prescribes the following recommendations:

i. The CBN, in line with the prescriptions of the monetarist school of economic thought; should engage on proper monetary management through the use of a fixed monetary growth rate rule, commensurate with GDP growth; in order to address inflation in Nigeria.

ii. The CBN can also make use of contractionary fiscal and monetary policy in order to reduce spending and inflationary pressures in the Nigerian economy.

iii. Policy makers in Nigeria should consider supply – side policies such as privatization and deregulation in order to improve long – term competitiveness, productivity and innovation in the country; that will in turn lower inflation.

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